The (Much Understated) Quantitative Role of Capital Accumulation and Saving *

Comments welcome

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Abstract

Can factor accumulation still help us understand differences in capital inflows and income across countries? This paper offers a quantitative evaluation of neoclassical models of growth with collateral constraints. Previous work has found evidence that supports the qualitative predictions of this class of models for the direction of capital flows — they are driven by domestic scarcity — and the role of domestic savings — they act as complements rather than substitutes to capital inflows. In this paper, I estimate the factor shares implied by the long-term dynamics of external debt observed in the data. I find that a model with constant-elasticity-of substitution technology and a collateral constraint can generate plausible capital shares and cross-country distributions of debt-to-GDP ratios. This suggests that capital accumulation may play a more important role than suggested by the recent literature on growth, even in a world with limited financial integration.

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1 Introduction

There are surprisingly large and well-documented differences in standards of living across countries (see for example Hall and Jones [1999]). Neoclassical growth models predict that these differences should disappear over time, particularly in a world with capital mobility in which we should observe flows from capital-abundant to capital-deprived countries. We do not see these flows even though differences in rates of return are predicted to be quite large by standard neoclassical models (Lucas [1990]). The most recent literature on growth attributes income differences to cross-country variations in total factor productivity (TFP) not in factor accumulation (Klenow and Rodriguez-Clare [1997], Hall and Jones [1999], Parente and Prescott [2000]). In this paper, I argue that factor accumulation and saving play an important role in determining a country's ability to attract capital flows. The role of saving is not only qualitatively but also quantitatively important. If capital flows tend to accelerate convergence and/or provide an opportunity for technology transfer, the role of saving may be more important than previously thought.

In a well-known paper, Lucas [1990] has argued that neoclassical models of growth do poorly in explaining capital flows without some form of capital market imperfection. One class of models has focused on imperfections in the form of collateral constraints. These types of constraints have received some attention both theoretically within a neoclassical framework [1] and empirically [2]. The most intriguing prediction of this class of models is that domestic saving acts as a complement to capital inflows, which ceteris paribus, should still be driven by decreasing returns and domestic scarcity: countries that save more attract more capital inflows. Newly constructed data on net foreign liabilities (Lane and Milesi-Ferretti [2001]) have recently allowed a more in-depth investigation of these questions. In Verdier [2005], I use these data and find qualitative support for both these mechanisms — the complementary role of savings and the neoclassical force of decreasing returns. The objective of this paper is to evaluate the quantitative relevance for explaining long-term capital flows of a class of models that features these two forces. More specifically, this paper estimates the capital shares implied by the data on net external debt. The results imply that the role of saving and capital accumulation in explaining differences in income and capital flows may be much more important than generally accepted in the growth literature.

One example of such a framework is the model developed by Barro, Mankiw and Sala-i-Martin [1995] (BMS hereafter). In this model, there are two types of capital, foreign and domestic. Constrained countries can borrow freely on world markets for their foreign capital needs, but must save in order to accumulate domestic capital. Since foreign and domestic capital are complementary in production however, the collateral constraint slows down the rate of income convergence compared to the standard open-economy neoclassical model. Perhaps more importantly, this complementarity in production combined with the collateral constraint leads to a complementarity between saving and foreign financing: one capital input is the result of domestic saving while the other must come from abroad, and production must combine these two factors.

[1] See Barro, Mankiw and Sala-i-Martin [1995], Lane [2001].

[2] For example, Adda and Eaton [1998], Lane and Milesi-Ferretti [2001].

Economies that save more are rewarded by additional capital inflows. This model offers a natural way to examine cross-country long-run debt accumulation.

In Verdier [2005], I find qualitative support for this model's predictions for debt dynamics. In the framework developed by BMS, debt is proportional to output. Therefore, the model predicts that debt should exhibit the same type of convergence-like dynamics as income. Capital should flow to locations where it is scarce in efficiency units, i.e. for constant technology levels. When convergence equations for debt derived from the model are estimated, I find evidence of decreasing returns and of a complementarity between domestic savings and debt accumulation. These results are robust to the econometric specification, the sample of countries considered as well as assumptions about technology and the measurement of domestic savings. The model also predicts that the debt-to-GDP ratio should be constant, i.e. that debt and output should have the same dynamics. This prediction is not supported by the data.

These results prompt some additional questions. They suggest that a simple model of growth that features decreasing returns and domestic saving complementarity is useful in thinking about long-term movements in capital. This conclusion however, is *qualitative*. How relevant is this framework *quantitatively*? The objective of this paper is to answer this question. More specifically, I estimate the factor shares implied by the convergence equations estimated in Verdier [2005] by matching the regression coefficients from the data to their simulated counterparts in the model. To obtain these simulated data, the model must be solved. Non-linear model solution methods are often appropriate when countries to which the model applies are far from their steady state. Since this is likely to be the case for many of the countries in our sample, the model is solved non-linearly and simulated for all countries in the sample. These simulated data are then used to determine whether reasonable technology parameters can be found to match the convergence equations.

I consider three versions of the model. In the first version, saving behavior is endogenous and does not vary across countries. Under the assumption of identical preferences, the potential complementary role of domestic savings is obscured. Consequently, the focus is on matching the convergence rate. When the model with endogenous savings behavior is used, I find that the model implies unrealistic shares of capital, that are either too low or too high. In a second Solow-type version of the model, I allow saving behavior to differ across countries; for estimation, it is captured by the average savings rate in the data. In this framework, I can estimate the factor shares by matching both the effect of decreasing returns and of domestic savings. This estimation yields more plausible factor shares. Nevertheless, two potential problems remain after this modification. First, the model cannot both match the convergence rate and the effect of the savings rate on debt accumulation. In particular, the simulated coefficient on the savings rate is low compared to the data. In addition, the estimated share of foreign capital implies unrealistically high debt-to-GDP ratios, unless one allows for very high institutional disincentives to capital accumulation. These results highlight the weakness of the model: the tight relationship between debt and output. The form of the credit constraint combined with the Cobb-Douglas technology implies that debt and output are proportional, and that their ratio is constant and uniform across countries. A successful model of debt dynamics must allow for a more complex relationship between debt and output while retaining the prediction of savings complementarity and convergence.

One way to achieve this objective is to relax the assumption of unit elasticity between foreign and domestic capital imposed by the choice of a Cobb-Douglas production function, which I consider in a third version of the model. By assuming instead that the technology is best described by a constant-elasticity-of-substitution (CES) production function, the model can generate plausible capital shares as well as levels and cross-country distributions of debt-to-GDP ratios.

The intriguing results from this exercise is that a model with only two important features — decreasing returns and saving complementarity — can teach us so much about variations in capital inflows across small open economies. In addition, this model can match the behavior of debt-to-GDP ratios across countries, something few standard neoclassical open-economy models of growth seem able to achieve (see Verdier [2003]). The recent literature on growth has mostly focused on the small contribution of capital accumulation in accounting for cross-country income differences (see for example Klenow and Rodriguez-Clare [1997], Hall and Jones [1999] and Parente and Prescott [2000]). As a consequence the gains from capital flow liberalization can also be small (see Gourinchas and Jeanne [2003]). Given the inability of accumulation models that assume exogenous growth to explain income differences, the result that savings rates play an important role in explaining capital inflows may be surprising. But the failure of the accumulation view may be due to the assumption of exogenous growth. A recent paper by Córdoba and Ripoll [2005] argues that in a world with endogenous TFP the role of factor accumulation can be quite large. In fact, they find that most cross-country income differences can be accounted by differences in savings rates and human capital. The present paper leaves cross-country variations in saving rates and TFP unexplained. However, it underscores the importance of understanding these differences even in a world with limited financial integration and exogenous productivity growth.

The paper is organized as follows. Section 2 describes the BMS growth model. Section 3 presents the method and results of estimation. Section 4 concludes.

2 The Model

The objective of this paper is to evaluate the quantitative performance of neoclassical models with quantity constraints. What form should this constraint take? In order to achieve this objective — the estimation of capital shares — a simple modelling device is needed. The credit constraint should not however, be completely arbitrary, static or unrelated to the economic fundamentals of the model. For these reasons, the framework developed by BMS is appropriate. In the BMS model, the debt ceiling is a direct function of the capital stock, and thus, changes over time as the economy develops. I start by describing a version of the BMS model with endogenous savings which focuses on the role of convergence. The initial objective of the model was to emphasize how a simple collateral constraint could help a neoclassical model match the observed income convergence. A Solow-type version of the model which allows for a possible role for savings is introduced in a later section.

In the BMS model, credit-constrained small open economies can use foreign financing to accumulate part of their capital and must save in order to finance the remaining fraction. There are three inputs in production: labor L, and two types of capital, K and Z. Technology takes the form of a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} Z_t^{\eta} (\theta_t L_t)^{1-\alpha-\eta} \tag{1}$$

where $\alpha, \eta > 0$, $\alpha + \eta < 1$. θ_t is the exogenous source of technology and grows at rate g while raw labor grows at rate n. The production function can be expressed in units of effective labor (where $x_t = \frac{X_t}{\theta_t L_t}$):

$$y_t = k_t^{\alpha} z_t^{\eta} \tag{2}$$

Profit Maximization then requires that factor prices equal the marginal productivities of inputs so that

$$\begin{aligned} R_{kt} &= \alpha k_t^{\alpha-1} z_t^{\eta} = \alpha \frac{y_t}{k_t} \\ R_{zt} &= \eta k_t^{\alpha} z_t^{\eta-1} = \eta \frac{y_t}{z_t} \\ w_t &= k_t^{\alpha} z_t^{\eta} - R_{kt} k_t - R_{zt} z_t = (1-\alpha-\eta) y_t \end{aligned} \tag{3}$$

where R_{kt} is the rental rate of k, R_{zt} is the rental rate of z and w_t is the wage rate.

Households collect income from labor and capital services. They consume and accumulate capitals k and z, as well as debt, d, on which households pay the constant world interest rate, r. The budget constraint faced by the infinitely-lived representative consumer is

$$(1+g)(1+n)(k_{t+1} + z_{t+1} - d_{t+1}) = (1 + R_{kt} - \delta)k_t + (1 + R_{zt} - \delta)z_t - (1+r)d_t + w_t - c_t \tag{4}$$

where I have assumed that foreign and domestic capital depreciate at the same rate, δ. If the economy is open, small relative to the rest of the world and takes world prices as given, domestic returns will be determined by the world interest rate r, so that $r = R_k - \delta$.

The difference between the two types of capital lies in the economy's capacity to finance their accumulation. A credit-constrained economy can use capital k as collateral, but cannot use z. This means that it can finance the accumulation of k through foreign financing, but must save in order to accumulate the domestic capital z. k thus represents capital that can easily be borrowed abroad. z on the other hand can represent capital foreign investors may find too risky to invest in. One obvious example is human capital. But z can represent other forms of capital for which information asymmetries render foreign financing difficult [3]. For example, foreign investors may finance machinery and equipment, which are easier to repossess, whereas domestic savers invest in building structures. Differences between the two types of capital may also be sectoral. In that case, foreign capital is used in the formal sector of the economy, while domestic capital

[3] Domestic firms might find it difficult to convince foreign investors to channel capital to their projects because of moral hazard problems, or risk of debt repudiation (see Cohen and Sachs [1986] or Gertler and Rogoff [1990]).

operates in the informal sector: firms in the informal sector may not easily collateralize their assets. In all these examples — human vs physical capital, structures vs equipment or informal vs formal capital sectors — k and z can be complements in production.

Whether a country is constrained or not is determined by initial asset holdings relative to steady state capital z^*. More specifically, if $k_0 + z_0 - d_0 \geq z^*$, the constraint does not bind and the model behaves as the open-economy Ramsey model with infinite speed of convergence. However, if $k_0 + z_0 - d_0 < z^*$, the constraint binds ($k_t = d_t$). In that case the combination of the credit constraint, the small-open-economy assumption and profit maximization (equation (3)) imply that

$$d_t = k_t = \frac{\alpha}{r+\delta} y_t \tag{5}$$

so that $\frac{k}{y}$ has a constant path to the steady state. The production function can therefore be written as

$$y_t = B z_t^{\varepsilon} \tag{6}$$

where $B \equiv \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}$ and $\varepsilon \equiv \frac{\eta}{1-\alpha}$.

Under these assumptions, the household problem takes the form:

$$\begin{array}{rcl} \underset{\{c_t, z_{t+1}\}_{t=0}^{\infty}}{Max} & & \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ s.t. & & \\ (1+n)(1+g)z_{t+1} & = & (1-\alpha)Bz_t^{\varepsilon} + (1-\delta)z_t - c_t \end{array} \tag{7}$$

The dynamics of the system are governed by the Euler equation

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta^* \left((1-\alpha)\varepsilon B z_{t+1}^{\varepsilon-1} + 1 - \delta\right) \tag{8}$$

where $\beta^* = \frac{\beta}{(1+n)(1+g)}$, and market clearing

$$(1+n)(1+g)z_{t+1} = (1-\alpha)Bz_t^{\varepsilon} + (1-\delta)z_t - c_t \tag{9}$$

Note that with the exception of the gross national product term (here $(1-\alpha)Bz^{\varepsilon}$ is GNP and $-\alpha B z^{\varepsilon}$ is net factor payments), these two equations in consumption c and domestic capital z are nearly identical to those of a closed-economy neoclassical growth model. The model will therefore behave like one. The convergence rate however, is finite — contrary to what it would be in a world with perfect capital markets — though higher than in a closed economy. z has the standard transition to its steady state because of the collateral constraint. Consequently, k does not jump immediately to its steady state since domestic and foreign capital are complementary in production. However, k will initially be high relative to z since it can be borrowed from abroad, causing $\frac{k}{z}$ to fall during the transition.

In this model $\varepsilon = \frac{\eta}{1-\alpha}$ governs how fast decreasing returns set in, and hence the magnitude of the convergence rate. Individual capital shares α and η govern the relative importance of domestic *vs* foreign capital, or the degree of capital mobility. α is the income share of capital that can be used as collateral. A higher α is thus synonymous with a higher degree of capital mobility. As α rises, the relative importance of the foreign sector increases and the economy behaves more like an open economy. A higher η corresponds to a more closed economy, in which the relative importance of the domestic sector is greater. In the extreme case when $\alpha = 0$, the economy is closed; when $\eta = 0$, capital markets are perfect and the economy has an infinite rate of convergence. Finally, raising $\frac{\alpha}{\eta}$ for a given $\alpha + \eta$ increases the degree of capital mobility as it increases $\frac{\alpha}{\alpha+\eta}$, the share of capital that can be used as collateral. This will raise the rate of convergence.

This system is easily solved. Log-linearizing the system and approximating around the steady state gives

$$\log z_t = \lambda^t \log z_0 + (1 - \lambda^t) \log z^* \tag{10}$$

where $1 - \lambda$ is the convergence rate. This implies that the change in net foreign debt takes the form

$$\log d_t - \log d_0 = -(1 - \lambda^t) \log d_0 + (1 - \lambda^t) \log d^* \tag{11}$$

where

$$d^* = \frac{\alpha}{r + \delta} B \left[\frac{\frac{1}{\beta^*} - (1 - \delta)}{(1 - \alpha)\varepsilon B} \right]^{\frac{\varepsilon}{\varepsilon - 1}} \tag{12}$$

The model thus predicts that countries that have low debt initially — which in this model is equivalent to low initial income — will accumulate debt faster. This prediction is a direct result of the assumption of decreasing returns. Countries that have low income and debt are constrained and have high return to capital. Now consider the role of the effective discount factor β^*. As β^* increases, consumers put a higher weight on future consumption and more importance on current saving. It can be characterized as the their 'propensity' to save. In this model, steady state debt is a positive function of the discount factor, i.e. $\frac{\partial d^*}{\partial \beta^*} > 0$. Countries that have a higher propensity to save tend to have higher debt levels. In the context of a convergence equation, this also means that countries that save more during the transition will attract more capital flows. In the BMS model, domestic savings and foreign investment are *complements*. This prediction is the result of the combination of two assumptions: the credit constraint and the complementarity of foreign and domestic capital in production. Countries can borrow foreign capital freely. It remains true however, that capital flows where it is most scarce, i.e. the model predicts convergence. Because of the complementarity, higher savings in the domestic capital increases the marginal product of foreign capital, thereby attracting more capital flows from abroad.

Manipulating this equation yields a convergence equation for debt of the form

$$\begin{aligned} \log d_t - \log d_0 &= (1 - \lambda^t) \log \frac{\alpha B}{r + \delta} + (1 - \lambda^t) \frac{\varepsilon}{1 - \varepsilon} \log B(1 - \alpha)\varepsilon\beta - (1 - \lambda^t) \log d_0 \\ &- (1 - \lambda^t) \frac{\varepsilon}{1 - \varepsilon} \log \left((1 + n)(1 + g) - \beta(1 - \delta) \right) \end{aligned} \tag{13}$$

This is the reduced form I will use to estimate the capital shares implied by the model. To understand the results, it will be useful to write the constant as a function of the original capital shares as follows

$$\log d_t - \log d_0 = (1 - \lambda^t) \log \beta + (1 - \lambda^t) \log \eta + (1 - \lambda^t) \frac{1 - \eta}{1 - \alpha - \eta} \log \frac{\alpha}{r + \delta} \tag{14}$$

$$- \quad (1-\lambda^t)\log d_0 - (1-\lambda^t)\frac{\varepsilon}{1-\varepsilon}\log\left((1+n)(1+g)-\beta(1-\delta)\right)$$

3 A Quantitative Assessment

3.1 Methodology

How far off is this model quantitatively? Can we find reasonable values of the technology parameters that can reproduce the data? One way to determine whether the model is quantitatively relevant is to use moments of the data and see how closely they can be reproduced using the model. A first step then is to find appropriate moments and a second, to decide on an estimation method. I address each of these questions below.

3.1.1 The moments

In Verdier [2005], I estimate convergence equations of a form similar to equation (15) and find qualitative support for the predictions of the model. In particular, I find that countries with low levels of initial debt did accumulate more debt subsequently — which suggests convergence. More specifically, an equation of the form

$$\frac{1}{T}\left(\log\frac{D_{iT}}{\theta_{iT}L_{iT}} - \log\frac{D_{i0}}{\theta_{i0}L_{i0}}\right) = -\frac{(1-\lambda^T)}{T}\log\frac{D_{i0}}{\theta_{i0}L_{i0}} + \frac{(1-\lambda^T)}{T}\log d^* \tag{15}$$

— where $\frac{D_{iT}}{\theta_{iT}L_{iT}} \equiv d_{iT}$ is debt per efficiency units of labor — is estimated using least squares. T is the period of time over which averages are taken. I choose these least squares coefficients as the moments to reproduce.

In order to estimate this equation, some simplifying assumptions must be made. First, since I observe debt-per capita $\frac{D}{L}$ and not debt per efficiency units $\frac{D}{\theta L}$, I must make some assumption about technology. Second, I must find variables to control for the steady state level d^*. Finally, since the model applies to credit-constrained economies, we must find a way to identify them in the sample countries we use. I address these issues here briefly, but a more elaborate discussion can be found in Verdier [2005].

Estimation is first undertaken under the assumption that the initial level of technology is identical across countries as in Mankiw, Romer and Weil [1992] (MRW hereafter), i.e.

$$\log\theta_{i0} = c$$

with

$$\frac{\theta_{it}}{\theta_{i0}} = (1+g)^t$$

This assumption is relaxed later.

In order to control for the steady state, I will restrict the variables to those suggested by the model. In the version of the model developed in the previous section, this corresponds to the labor force growth rate. In a subsequent section, I will consider a version of the model with a fixed savings rate, and add it as a control variable [4].

[4] In Verdier [2005] I have found that additional controls do not improve the fit of the regression.

Finally, I must find a way to discriminate between constrained and unconstrained countries. First, I focus on countries that have positive net liabilities, and that are small open economies. These however, may include high-income countries. In the model, countries are constrained if they have low levels of initial wealth. As a second step, samples will be divided according to the level of initial income. A more thorough discussion of sample composition is provided in the data section below.

3.1.2 The Matching Exercise

The quantitative assessment offered here will attempt to match the coefficients from the convergence equation. Specifically, I can simulate the model for all countries in the sample and estimate the convergence equation data for debt using the simulated data. I can then choose the values of the capital shares that minimize the distance (by some metric) between actual and simulated moments.

Under what assumptions should the model be simulated? In the model, countries can be different along many dimensions. The convergence equation for debt suggests two dimensions in which we may want to allow differences in the simulated data: first, initial conditions; second, steady states. I turn to each in turn.

In order to simulate the data, I must provide a set of initial conditions for z. I do not have direct measures of the initial distribution of capital that was saved for domestically. Since GDP data is the most reliable, we use the distribution of initial output in the data to generate a distribution of initial domestic capital stocks across countries [5]. Countries may also have different initial levels of technology, θ_0. I will first assume initial technology levels are identical across countries, and relax this assumption below.

Countries also have different steady states. First, I only allow the steady state to be defined by the labor force growth rate taken from the data, an assumption which is relaxed later.

Since there are only a limited set of moments, I cannot estimate all the model parameters. I choose to estimate the technology parameter η and α, the shares of domestic and foreign capital in income. This quantitative exercise can therefore allow us to determine the magnitude of decreasing returns necessary to reproduce the variation in debt accumulation observed in the data.

The complete algorithm for estimating $\Gamma = \begin{pmatrix} \alpha \\ \eta \end{pmatrix}$ is thus as follows:

1. Estimate a vector of moments $\hat{m}$ from the data. Here these moments are the least squares coefficients in the convergence equations on debt, i.e. the least squares coefficients obtained by regressing $\frac{1}{T}(\log d_T - \log d_0)$ on

$$X = \begin{bmatrix} 1 & \log d_0 & \log(1+n)(1+g) - \beta(1-\delta) \end{bmatrix}$$

$\hat{m}$ is the vector of least squares coefficients from this regression.

[5] See section C in the Appendix for details on the choice of initial conditions.

2. Given values for n, z_0 taken from the data, and an initial value for Γ, solve and simulate the non-linear model for each country, i.e. obtain the transition path for consumption, domestic capital, output and debt:

   - Find decision rules for consumption $c = f_i(z)$ in each country i. Obtain the simulated series by computing

$$
\begin{aligned}
c_{it}^s &= f_i(z_{it}^s) \\
y_{it}^s &= B(z_{it}^s)^{\varepsilon} \\
d_{it}^s &= \frac{\alpha}{r+\delta} y_{it}^s \\
z_{it+1}^s &= \frac{1}{(1+n)(1+g)} \left((1-\alpha)y_{it}^s + (1-\delta)z_{it}^s - c_{it}^s\right) \\
\frac{D_{it}^s}{L_{it}^s} &= \theta_{it}^s d_{it}^s
\end{aligned}
\qquad (16)
$$

   for $t = 1, 2, ..., T$ for each country i given z_{i0} , and where the superscript s denotes the simulated series.

3. Use the simulated data to estimate $\tilde{m}(\Gamma)$, the same moments as in the data, that is regress $\frac{1}{t}\left(\log d_t^s - \log d_0^s\right)$ on

$$X^s = \begin{bmatrix} 1 & \log d_0^s & \log(1+n)(1+g) - \beta(1-\delta) \end{bmatrix}$$

   $\tilde{m}(\Gamma)$ is the vector of least squares coefficients from this regression.

4. Choose Γ to minimize

$$J(\Gamma) = \left(\hat{m} - \tilde{m}(\Gamma)\right)^{\mathtt{T}} \mathtt{W} \left(\hat{m} - \tilde{m}(\Gamma)\right)$$

   $\mathtt{W}$ is a positive semi-definite weighing matrix that can take any value. In this exercise, more weight will be placed on the more precisely estimated coefficient, i.e., $\mathtt{W}$ is a diagonal matrix with the inverse of the variance of the least squares coefficients on the diagonal.

Since the BMS model is non-linear, Step 2 requires a numerical solution method to approximate the decision rule for consumption. A non-linear approach is used for two reasons. First, linear approximations are notoriously inaccurate when used to approximate decision rules for economies far away from their steady state. Since I have no reason to believe these countries are near their steady state, a non-linear method seems appropriate. Second, since I want to determine how much variation in debt can be explained by concavity in the production function, it seems important to capture the curvature in the decision rule correctly. Details on the solution method are available in the Appendix.

Since I cannot estimate all the parameters of the model, I must calibrate some preference and technology parameters. As in MRW, I choose a value of 3 per cent for the depreciation rate and of 2 per cent for the exogenous rate of technological progress. Several authors estimate that the world interest rate has been between 0 and 5 per cent over the past thirty years (see IMF [1995], Allsopp and Glyn [1999] and Chadha and Dimsdale [1999]). I choose $r = 0.05$, a value at the high range of these estimates. Since the samples considered here will be dominated by low-income and potentially credit-constrained economies, it seems appropriate to choose a higher value for r. Finally, the data are simulated under the assumptions that the intertemporal elasticity of substitution σ is 1.5 and the discount factor β is 0.95. These are standard values in the business cycle literature and are also in line with other studies where models are calibrated for developing countries (see Chari, Kehoe and McGrattan [1997]).

Finally, note that during the estimation of the capital shares, both α and η are constrained to be within the interval $[0.05, 0.95]$. [6]

3.2 The Data

The data on net external debt are taken from Lane and Milesi-Ferretti [2001]. This database contains net foreign assets for 66 countries between 1970 and 1998, constructed by the authors using data on current account balances supplemented by available stock data on foreign direct investment, portfolio equity and debt assets and liabilities. These measures are adjusted for valuation effects such as exchange rate changes, variations in the price of capital goods and changes in the values of stock market indices [7]. In all regressions, debt per worker is computed as

$$\frac{D}{L} = \frac{-NFA}{pL}$$

where NFA is a measure of net foreign assets in US dollars, p is the US GDP deflator, and L is working-age population from the Penn World Tables.

Two measures of NFA are used [8]. The first, `ACUMCA`, is based on cumulative current accounts. It is available for both industrial and developing countries. The second, `ACUMFL`, is based on direct stock measures of the various assets included in debt and is available for developing countries. The main difference between these two measures is the treatment of unrecorded capital flows. `ACUMCA` implicitly assumes that unrecorded capital flows reflect accumulation of foreign debt assets by domestic residents. The second measure, `ACUMFL`, only includes unrecorded capital flows to the extent that they are recorded in net errors and omissions. If capital flight is important and often goes unreported, `ACUMFL` will tend to overstate external debt levels.

As noted above, the BMS model is only relevant for indebted credit-constrained small-open economies. Three samples will be considered. First, estimation will be restricted to countries with positive net external debt in both beginning and end of sample. Note that this implies a sample that excludes long-term creditors and countries that have switched from being net lenders to net borrowers, and vice versa such as Japan

[6] This is done for computational reasons. As α and η approach 0 or 1, the computer can no longer distinguish them from these values. Since the model has no solution when the parameters take on these values, the computation becomes difficult.

[7] More details on the debt measures, as well as all other data used in the paper are available in the Appendix.

[8] Arguably, the measures of net foreign assets used here may include a lot of public debt. Since the BMS model applies mostly to private debt, this could be problematic. As shown in Verdier [2005] however, the results are robust to using a measure of private debt.

and the U.S.. The remaining group of countries also excludes important members of the G7 countries, such France and the U.K.

These restrictions reduce the sample size to 42 observations for the `ACUMCA` measure (Sample I) and 29 observations for the `ACUMFL` measure (Sample II). A third sample corresponds to the poorest half of the `ACUMCA` sample in terms of income in 1970 (Sample III). Sample compositions are described in the Appendix. Note that among developing nations, the samples are dominated by middle-income countries from Latin America and Asia.

The data on output and the labor force are taken from the Penn World Tables 6.0.

3.3 The Model with Identical Cross-Country Saving Behavior

3.3.1 Common Technology

What should we expect from the model for 'reasonable' capital shares? For illustrative purposes, Figures 1 and 2 show the approximated consumption decision rules and the transition paths to the steady state for each country in Sample I when $\alpha = \eta = 0.35$. The decision rule function has the expected features: it is concave and increasing in domestic capital z. In Figure 2, each variable is expressed as a ratio to its steady state value. The transition paths are similar to those obtained from a closed-economy Ramsey model, as expected. Table 1 shows the magnitudes of the least squares coefficients for the basic model with identical cross-country saving behavior. For $\alpha = \eta = 0.35$, the fraction of capital that can be used as collateral $\left(\frac{\alpha}{\alpha+\eta}\right)$ is one half. The convergence rate is approximately 3 per cent, a bit higher than estimated in the income growth literature. As noted in BMS, the model comes close to reproducing the convergence rates observed in the data on output with reasonable parameter values.

Table 2 shows the result of the matching exercise for α and η. The capital shares are chosen so as to match the reference regressions estimated by ordinary least squares. How does matching occur? Recall that what matters for matching the convergence rate is $\varepsilon = \frac{\eta}{1-\alpha}$, the coefficient on domestic capital in the collapsed production function, or alternatively the total share of capital $\alpha + \eta$. The higher the share of broad capital, the lower the speed of convergence since a high share of capital means that diminishing returns set in more slowly. Individual shares however, can serve as a measure of how open the economy is. If $\alpha = 0$, none of the accumulated capital can serve as collateral, and it is the equivalent of a closed economy. If $\eta = 0$, all capital serves as collateral, and the model exhibits infinite convergence. Raising $\frac{\alpha}{\eta}$ for a given $\alpha + \eta$ raises the degree of capital mobility because it raises $\frac{\alpha}{\alpha+\eta}$, the fraction of capital that serves as collateral.

Consider first the results shown in Table 2. The standard errors are shown in the Data column in parenthesis below the least square estimates. These, squared, serve as weights in estimation: more weight is put on matching the estimates that are more precisely estimated. In all estimations, the convergence rate has the lowest variance, and hence receives more weight in estimating the capital shares. Compared to Table 1, the estimated convergence rate is low in all samples. To match it, we must choose a higher value of ε

and increase the share of broad capital. For Sample I, this tendency to increase ε is reinforced by the size of the coefficient on labor force growth, which is large in absolute value relative to Table 1. Increasing the share of broad capital in the model will increase the relative importance of the domestic sector, and raise the negative effect of labor force growth: as ε increases, each new generation must be equipped with more capital. An increase in ε is better achieved by an increase in η. The estimate of the constant in Sample I however, is low compared to the standard case. Recall from equation (15), that the constant is partly a function of the debt-to-GDP ratio $\frac{d}{y} = \frac{\alpha}{r+\delta}$. The model partly interprets the constant as this ratio, and lowers α to match it. In Samples I and II, α hits the lower constraint imposed on estimation of the capital shares. In Sample III, the constant is fairly high and this results in a high value of α and a labor share that is around 0.3. Sample III shows more plausible shares.

3.3.2 Varying Technology

How seriously should we take these estimates? Although the convergence rate is precisely estimated, the other least square coefficients have high variance. So far, I have considered matching a regression under very strict assumptions about preferences and technology. I have assumed there are no differences in saving behavior across countries, that there is unit elasticity of substitution between foreign and domestic capital — two assumptions I relax later — and that initial technology levels are identical — an issue I address now. The regression in Table 2 is not a very good fit, in part perhaps because we have imposed the assumption of common technology levels across countries. This however, is not a prediction of the model. Indeed, the model predicts that capital should flow where it is most scarce *in efficiency units*. If there are uncontrolled differences in θ_0, i.e. if efficiency units differ across countries, the least square coefficients will be biased. Suppose technology takes the following form:

$$\log \theta_{i0} = c + \log A_{i0}$$

with

$$A_{it} = (1+g)^t A_{i0}$$

I can control for these differences by using traditional growth accounting methods. An estimate of total factor productivity A_{i0} is computed under the assumption that $\alpha = 0.3$[9]. Debt accumulation has dynamics of the form:

$$\begin{aligned} \frac{1}{T}(\log d_T - \log d_0) &= \frac{(1-\lambda^T)}{T}\log\beta + \frac{(1-\lambda^T)}{T}\log\eta + \frac{(1-\lambda^T)}{T}\frac{1-\eta}{1-\alpha-\eta}\log\frac{\alpha}{r+\delta} \\ &- \frac{(1-\lambda^T)}{T}\log d_0 + \frac{(1-\lambda^T)}{T}\log A_0 \\ &- \frac{(1-\lambda^T)}{T}\frac{\varepsilon}{1-\varepsilon}\log\left((1+n)(1+g) - \beta(1-\delta)\right) \end{aligned} \tag{17}$$

Table 3 shows the least squares coefficients predicted by the model for $\alpha = \eta = 0.35$. Note from equation (17) that the coefficients on d_0 and A_0 should be equal in absolute value. Since the convergence equation

[9]The results are robust to using other values for α. See Appendix for details.

is a property of the linearized model however, there are some discrepancies between the two coefficients on these variables. Other than the coefficient on technology, the coefficients here are similar to those in the case of common technology levels.

The estimates of the foreign capital share shown in Table 4 are similar to those in obtained under the assumption of common technology, with the exception of Sample III. For all three samples, estimate of α hits the lower constraint imposed on estimation.

How reasonable are these shares? Standard estimates from the National Income and Product Accounts usually put the labor share of income anywhere between 0.05 to 0.8 — a share that applies to a composite of raw labor and human capital. Low-income countries often have low estimates of the labor share. These estimates typically measure labor shares by the ratio of employee compensation to national income. Recent work by Gollin [2002] and Bernanke and Gürkaynak [2001] however, suggests that employee compensation understates total labor compensation, more so in low-income countries since these economies devote a large portion of their labor force to self-employment or employment outside of corporate businesses. Taking this into account, these authors find that the labor share has much less variation across countries [10] and ranges between 0.65 and 0.8. This implies a share for physical capital between 0.2 and 0.35. If we interpret α and η narrowly as shares of physical and human capital as in the original BMS paper, our estimate for α is outside the range suggested by these authors. By this measure, these estimates are not plausible.

Even if α corresponds only to physical capital however, it is likely that η applies to a composite of physical and human capital. Alternatively, I can judge the plausibility of these estimates by looking at the share of total capital $\alpha + \eta$. In the growth literature, estimates of the broad capital share vary between 0.6 and 0.8 (see MRW and Barro and Sala-i-Martin [1995]). The results here are consistent with these estimates except in Sample I.

Should we conclude that the model is consistent with the data on debt? One prediction of the data provides another way of judging whether these estimated factor shares are plausible. The model predicts that the debt-to-GDP ratio is a constant function of the foreign share α, i.e. $\frac{d}{y} = \frac{\alpha}{r+\delta}$. Can we reproduce the debt-to-GDP ratios observed in our sample? As an illustration of the external positions of the countries considered, Figure 3 shows histograms of debt-to-GDP ratios in all three samples in 1998. The highest ratio observed is less than 2 and the average ratio is approximately 0.4. For all three Samples, the estimated $\alpha = 0.05$ implies a debt-to-GDP ratio of 0.63 which is close to the average observed ratio. α however, is constrained to be no lower than 0.05 in estimation, which implies that had it not been constrained, the estimation would have chosen a negative value for α. The resulting debt-to-GDP ratio is thus not believable.

These results seem to imply that while the model seems to find plausible values for the broad share of capital, they are inconsistent with observed debt-to-output ratios [11]. The moments I have considered however, are poorly estimated, often with large variances. With the exception of the convergence rate, the

[10] Hence, justifying the use of a constant and unique-factor-shares production function like Cobb-Douglas technology.

[11] A similar point is made by Duczynski [2000]. He does not however, provide estimates of the capital shares. His criticism of the BMS model does not take into account the prediction for savings, which I address below.

model does not appear to have great relevance for the the debt data considered here. I have assumed however, that saving behavior was identical across countries and could be captured in a variable encompassing both the labor force and discounting effects $(1+n)(1+g) - \beta(1-\delta)$. In previous work [12], I presented evidence that a model of the kind considered here with exogenous savings was a useful framework in which to examine debt data. I turn to this next.

3.4 The Model with Variation in Cross-Country Saving Behavior

I can constrain the behavior of savings by assuming that domestic consumers save a fixed fraction of income. This allows for cross-country variations in saving behavior in the matching exercise, and therefore can provide an extra moment to match. Let s_y denote the domestic savings rate, or the rate at which consumers save out of gross domestic product y to accumulate domestic capital so that $s_y y_t = y_t - c_t$. Since domestic savings must equal investment in human capital ($i_t^h = (s_y - \alpha)Bz_t^{\varepsilon}$), we have

$$(1+n)(1+g)z_{t+1} = sBz_t^{\varepsilon} + (1-\delta)z_t \tag{18}$$

where $s = s_y - \alpha$. In the steady state, $z_{t+1} = z_t = z^*$ so that

$$z^* = \left[\frac{sB}{(1+n(1+g) - (1-\delta)}\right]^{\frac{1}{1-\varepsilon}} \tag{19}$$

Since $k_t = d_t = \frac{\alpha}{r+\delta} y_t$,

$$d^* = \frac{\alpha B}{r+\delta}\left(\frac{sB}{(1+n)(1+g) - (1-\delta)}\right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{20}$$

and the convergence equation for debt is

$$\begin{aligned} \log d_t - \log d_0 &= (1-\lambda^t)\frac{1-\eta}{1-\alpha-\eta}\log\frac{\alpha}{r+\delta} - (1-\lambda^t)\log d_0 \\ &+ (1-\lambda^t)\frac{\varepsilon}{1-\varepsilon}\log s - (1-\lambda^t)\frac{\varepsilon}{1-\varepsilon}\log\left((1+n)(1+g) - (1-\delta)\right) \end{aligned} \tag{21}$$

As before, the model predicts that countries that initially have low debt levels will accumulate it faster, which in this model, is equivalent to convergence. A second mechanism however, comes into play in this version of the model: the role of domestic savings. A standard view of capital inflows is that they act as substitutes to domestic savings. Open economies can increase investment with no cost in current consumption. In this model, domestic savings act as *complements* to capital inflows. This is the result of the combination of two features of the model: first, the complementarity in production of the two types of capital; and second, the collateral constraint. Production cannot take place without both factors, and these factors have different sources, one foreign and the other domestic. An increase in domestic saving raises the marginal product of foreign capital, which increases an economy's ability to attract financing from abroad.

The algorithm for estimation is essentially the same as before:

[12]See Verdier [2005]

1. Estimate a vector of moments $\hat{m}$ from the data. Here these moments are the least squares coefficients in the convergence equations on debt, i.e. the least squares coefficients obtained by regressing $\frac{1}{T}\left(\log d_t - \log d_0\right)$ on

$$X = \left[\begin{array}{cccc} 1 & \log d_0 & \log\left(1+n\right)\left(1+g\right)-\left(1-\delta\right) & \log s \end{array}\right]$$

with the possible addition of $\log A_0$. $\hat{m}$ is the vector of least squares coefficients from this regression.

2. Given values for n, s, z_0 and A_0 taken from the data, and an initial value for Γ simulate the non-linear model for each country, i.e. obtain the transition path for consumption, domestic capital, output and debt, i.e.

   - Obtain the simulated series by computing

$$\begin{aligned} y^s_{it} &= B(z^s_{it})^{\varepsilon} \\ d^s_{it} &= \frac{\alpha}{r+\delta} y^s_{it} \\ z^s_{it+1} &= \frac{1}{(1+n)(1+g)}\left(s y^s_{it} + (1-\delta) z^s_{it}\right) \\ \frac{D^s_{it}}{L^s_{it}} &= A^s_{it} d^s_{it} \end{aligned} \qquad (22)$$

   for $t = 1, 2, ..., t$ for each country i given z_{i0} and A_{i0}.

3. Use the simulated data to estimate $\tilde{m}(\Gamma)$, the same moments as in the data, that is regress $\log d^s_t - \log d^s_o$ on

$$X^s = \left[\begin{array}{cccc} 1 & \log d^s_0 & \log\left(1+n\right)\left(1+g\right)-\left(1-\delta\right) & \log s \end{array}\right]$$

with the possible addition of $\log A_0$. $\tilde{m}\left(\Gamma\right)$ is the vector of least squares coefficients from this regression.

4. Choose Γ to minimize

$$J(\Gamma) = \left(\hat{m} - \tilde{m}(\Gamma)\right)^{\mathtt{T}} \mathtt{W} \left(\hat{m} - \tilde{m}(\Gamma)\right)$$

Results with Varying Saving Behavior As noted above, the data are simulated using the average values of n and s in the data. s is measured as $1 - \frac{c}{y}$ which is taken from the Penn World Tables 6.0 [13]. If however, the least squares coefficients from the data, are estimated using instruments, I can no longer use the raw data for savings and labor force growth in simulation. I therefore use the first-stage

[13] The model suggests that a better measure of savings would be the rate at which consumers save out of gross national product, not gross domestic product. Availability of GNP data from the Penn World Tables however, is more limited than for GDP. In addition, the regression results are fairly robust to the use of GNP. See Verdier [2005].

fitted values of these variables, i.e. the values of n and s used in simulation are the fitted values obtained by regressing $\log(1+n)(1+g)-(1-\delta)$ and $\log s$ on the matrix of instruments $\mathcal{Z}$ where

$$\mathcal{Z} = \begin{bmatrix} 1 & \log d_0 & \log(1+\bar{n}_{1960-1969})(1+g)-(1-\delta) & \log s_{1960-1969} \end{bmatrix}$$

Table 5 shows the results for the model with exogenous savings for $\alpha = \eta = 0.35$. The convergence rate is now lower than in the model with endogenous savings, and the coefficient on savings is around 0.03. How does this version of the model perform?

The column entitled Data in Table 6 is a reproduction of the results in Verdier [2005]. The first result to note is that the fit of the regression is much better than before. Both A_0 and the savings rate consistently enter the equation significantly. As noted in Verdier [2005], these results are robust to the samples considered, the assumptions about technology and to additional controls. In all three Samples, the constant is fairly low, causing the model to choose a low value of α. In fact, the level of the constant causes the estimation to hit the lower constraint for the capital shares. Are these shares reasonable? One potential problem is that they vary according to the measure of debt used. In Samples II and III, two groups of relatively low income, the fraction of foreign capital ($\frac{\alpha}{\alpha+\eta}$), or the degree of openness varies between 9 and over 45 per cent.

As noted before, equation (21) implies two restrictions on the least square coefficients: the coefficient on d_0 and A_0 as well as those on s and $(1+n)(1+g)-(1-\delta)$ are equal in absolute value respectively. I can therefore use a more parsimonious specification where I impose these restrictions [14]. There are two positive results from estimation under these restrictions as shown in Table 7. First, the factor shares are similar across samples. Second, the estimated broad capital share is between 0.6 and 0.8. Many authors have argued that a broad share of capital (that includes both physical and human capital) of this magnitude is reasonable (e.g. MRW, Barro and Sala-i-Martin [1995]. Are these shares individually plausible? If α is strictly interpreted as the share of physical capital, these estimates are at the low end of the parameters used in the literature [15].

It is difficult to judge whether these estimates are individually plausible since they cannot strictly be interpreted as human and physical capital shares. Even if we could interpret them as such, and found them plausible on that basis, some problems remain, namely the simulated coefficient on the savings rate, and the level of the implied debt-to GDP ratio. I turn to each of these issues in turn.

The Effect of Saving The estimated coefficient on the savings rate appears high relative to what was predicted in Table 5 for standard values of the capital shares. The simulated value for this coefficient is consistently below what is estimated in the data. Why is the simulated effect of the savings rate so low? As shown in equation (21), to increase the coefficient on savings in the simulated data, a higher value of $\frac{\varepsilon}{1-\varepsilon}$ is needed, a goal better achieved by lowering $\frac{\alpha}{\eta}$. This however, would tend to reduce the convergence rate: recall that for a given $\alpha+\eta$, reducing α reduces the convergence rate because the fraction of capital from

[14] An F-test shows that these restrictions are not rejected by the data.

[15] For example, Klenow and Rodriguez-Clare [1997] argue that a share of $\frac{1}{3}$ is reasonable.

abroad has decreased. Thus, the model cannot fully match both the effect of savings and the convergence rate. This is illustrated in Table 8 where each panel shows a pair of two coefficients being matched together. Compare the first and second panels. To match the effect of the savings rate, ε must be close to or above 0.8. To match the convergence rate, ε is much lower. As shown in Table 9, matching both is a compromise that requires lower values of η.

The level of the debt-to-GDP ratio Putting aside the model's difficulty in matching both the effects of saving and convergence, it can find plausible values of the capital shares. There is another way of judging the validity of the model: the level and dynamics of the debt-to-GDP ratio. Although the capital shares of the model were not chosen to match the debt-to-GDP ratio, are the resulting estimates consistent with the bahavior of that variable? So far, it seems like this is not possible since the model predicts that this ratio should be constant.

The model can still be judged however, on the basis of the level of the debt-to-GDP ratio. The implied level debt-to-GDP ratio is still outside the observed range in the data with a value well over 2. In the original BMS model, the authors interpret α as the share of physical capital. For a plausible value of 0.3, they noted that the model implied high levels of the current account, levels not often observed in developing economies. The paper offers two potential explanation for this. First, the authors note that certain low-income countries may be insufficiently productive to be credit-constrained. The samples considered here are dominated by middle income developing countries, which suggests that this explanation is unlikely to apply. Second, the authors observe that the fraction of capital that can be used as collateral is likely to be less than the share of physical capital. My estimate of α however, corresponds to the share of capital that comes from abroad, not necessarily the share of physical capital.

A third explanation proposed in the BMS paper is that firms pay a proportional tax τ on output, so that profits are

$$(1-\tau)k_t^\alpha z_t^\eta - w_t - R_{kt}k_t - R_{tz}z_t$$

This tax is meant to include any disincentive to invest in capital such as low levels of property rights protection. In this case the market-clearing condition becomes

$$(1+n)(1+g)z_{t+1} = s(1-\tau)Bz_t^\varepsilon + (1-\delta)z_t \tag{23}$$

and the constraint takes the form $k_t = d_t = (1-\tau)\frac{\alpha}{r+\delta}y_t$, so that in the steady state

$$z^* = \left[\frac{(1-\tau)sB}{(1+n)(1+g)-(1-\delta)}\right]^{\frac{1}{1-\varepsilon}} \tag{24}$$

$$d^* = \frac{\alpha B}{r+\delta}\left(\frac{(1-\tau)sB}{(1+n)(1+g)-(1-\delta)}\right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{25}$$

and the convergence equation for debt is

$$\log d_t - \log d_0 \quad = \quad (1-\lambda^t)\frac{1-\eta}{1-\alpha-\eta}\log\frac{(1-\tau)\alpha}{r+\delta} - (1-\lambda^t)\log d_0 \tag{26}$$

$$+ \quad (1-\lambda^t)\frac{\varepsilon}{1-\varepsilon}\log s - (1-\lambda^t)\frac{\varepsilon}{1-\varepsilon}\log\left((1+n)(1+g)-(1-\delta)\right)$$

Can such a modification lower the value of α predicted by the model? Table 10 shows the results of estimation. The top panel shows the results under the assumption that $\tau = 0.3$, the value suggested in BMS. To lower the resulting debt-to-GDP ratio, the value of its denominator in the model has also been increased. As suggested in BMS, the world interest rate is assumed to be 0.06. The value of the the depreciation rate δ has also been raised to 0.06 from 0.03, to be more in line with other calibrations in the literature (see Chari, Kehoe and McGrattan [1997]). The three factor shares have plausible values for all three samples, between 0.2 and 0.5. The fraction of foreign capital has a believable value, below 0.5 although it is highest in the low-income Sample III. The value of the the debt-to-GDP ratio however, is still high at close to 1.5.

It is possible that the value of τ that we have chosen is too low. Many of the countries in the sample are known for severe controls on capital flows. This parameter however, is difficult to calibrate. Chari, Kehoe and McGrattan [1997] attempt to explain cross-country income differences by variations in disincentives to invest in capital. They measure these disincentives with the price of investment relative to consumption. The average value of this relative price is 0.78 in the samples they use. The bottom panel of Table 10 uses this value for τ. Note that this is a very high value as it implies producers appropriate less than thirty per cent of their output. The resulting estimate of the share of broad capital is now close to 0.8, while $\frac{d}{y}$ is close to one, at the high end of observed values of the debt-to-GDP ratio.

Therefore, a model with Cobb-Douglas technology cannot match the level or dynamics of the debt-to-GDP ratio. The objective of the next section is to determine whether a model with a more general production function can better match cross-country variations in debt-to-GDP ratios.

3.5 The Model with CES Technology

This section considers a simple modification of the model which involves adopting a more general technology representation in the form of a CES production function. Suppose that firms have access to the following production function

$$Y_t = V_t^{\eta}\left(\theta_t L_t\right)^{1-\eta} \tag{27}$$

where L_t is labor and V_t is composite capital:

$$V_t = \left[aK_t^{\rho} + (1-a)Z_t^{\rho}\right]^{\frac{1}{\rho}} \tag{28}$$

As before K_t is foreign capital and Z_t is domestic capital. Output per efficiency units is

$$\begin{aligned} y_t &= \left[ak_t^{\rho} + (1-a)z_t^{\rho}\right]^{\frac{\eta}{\rho}} \\ &= v_t^{\eta} \end{aligned} \tag{29}$$

The parameter ρ captures the degree of complementarity in production. The elasticity of substitution between foreign and domestic capital is $\frac{1}{1-\rho}$. As $\rho \to -\infty$, the elasticity of substitution approaches 0, i.e. foreign and domestic capital are perfect complements. $\rho = 0$ corresponds to the Cobb-Douglas technology for which the elasticity of substitution is 1. Finally, when $\rho = 1$, foreign and domestic capital are perfect substitutes (the production function is linear).

Profit maximization implies

$$\begin{aligned} R_{kt} &= a\eta\left(\frac{k_t}{v_t}\right)^{\rho}\frac{y_t}{k_t} \\ R_{zt} &= (1-a)\eta\left(\frac{z_t}{v_t}\right)^{\rho}\frac{y_t}{z_t} \\ w_t &= y_t - R_{kt}k_t - R_{zt} \end{aligned}$$

The combination of the credit constraint ($d_t = k_t$), the small-open-economy assumption ($R_{kt} = r + \delta$) and profit-maximization implies that the market-clearing condition takes the following form:

$$\begin{aligned} (1+n)(1+g)z_{t+1} &= y_t - f_k(k_t, z_t)k_t + (1-\delta)z_t - c_t \\ &= y_t - (r+\delta)k_t + (1-\delta)z_t - c_t \end{aligned} \tag{30}$$

Unlike the model with Cobb-Douglas technology, this version of the model does not lend itself easily to assuming exogenous saving behavior. This is because a model with CES technology can easily generate endogenous growth if model parameters (such as the labor force growth rate and the exogenous saving rate) do not satisfy fairly restrictive conditions (see Barro and Sala-i-Martin [1995]). Households are therefore assumed to optimally choose the path of consumption. The problem faced by the representative household is

$$\begin{aligned} \underset{\{c_t, z_{t+1}, k_{t+1}\}_{t=0}^{\infty}}{Max} \quad & \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ s.t. \quad & \\ (1+n)(1+g)z_{t+1} &= [ak_t^{\rho} + (1-a)z_t^{\rho}]^{\frac{\eta}{\rho}} - (r+\delta)k_t + (1-\delta)z_t - c_t \end{aligned} \tag{31}$$

which implies the following Euler equation

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta^* \left[f_z(k_{t+1}, z_{t+1}) + (1-\delta)\right] \tag{32}$$

where $\beta^* \equiv \frac{\beta}{(1+n)(1+g)}$

Equilibrium satisfies

$$\begin{aligned} \left(\frac{c_{t+1}}{c_t}\right)^{\sigma} &= \beta^* \left[f_z(k_{t+1}, z_{t+1}) + (1-\delta)\right] \\ (1+n)(1+g)z_{t+1} &= [ak_t^{\rho} + (1-a)z_t^{\rho}]^{\frac{\eta}{\rho}} - (r+\delta)k_t + (1-\delta)z_t - c_t \\ a\eta\left(\frac{k_t}{v_t}\right)^{\rho}\frac{y_t}{k_t} &= R_{kt} = r + \delta \end{aligned}$$

If we assume $\frac{1}{\beta^*} = 1 + r$, then the world real interest rate is the same as the rate that would occur in the closed economy that is, the economy is neither more patient nor less patient than the world economy. In that case, the representative agent will use the two types of capital in a way that ensures that their returns are equated in the steady state, i.e.

$$\begin{aligned} R_k &= R_z \\ a\eta \left(\frac{k_t}{v_t}\right)^{\rho} \frac{y_t}{k_t} &= (1-a)\eta \left(\frac{z_t}{v_t}\right)^{\rho} \frac{y_t}{z_t} \\ \frac{k^*}{z^*} &= \left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}} \end{aligned} \tag{33}$$

Then the steady state satisfies

$$\begin{aligned} \frac{1}{\beta^*} &= 1 + r \\ (1+n)(1+g)z^* &= [ak^{*\rho} + (1-a)z^{*\rho}]^{\frac{\eta}{\rho}} - (r+\delta)k^* + (1-\delta)z^* - c^* \\ a\eta \left(\frac{k^*}{v^*}\right)^{\rho} \frac{y^*}{k^*} &= r + \delta \\ \frac{k^*}{z^*} &= \left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}} = \mu \end{aligned} \tag{34}$$

which can be solved to obtain k^*, z^*, c^* as follows:

$$\begin{aligned} z^* &= \left(\frac{\Omega}{r+\delta}\right)^{\frac{1}{1-\eta}} \\ k^* &= \mu z^* \\ y^* &= [ak^{*\rho} + (1-a)z^{*\rho}]^{\frac{\eta}{\rho}} \\ c^* &= y^* - (r+\delta)k^* - [(1+n)(1+g) - (1-\delta)]\, z^* \end{aligned} \tag{35}$$

where $\Omega \equiv a\eta[a\mu^{\rho} + 1 - a]^{\frac{\eta-\rho}{\rho}} \mu^{\rho-1}$.

Since saving is optimally chosen, I must find a way to link observed saving behavior to a parameter of the model. I choose to focus on the discount factor. Since I dot not observe the discount factor directly, I assume it is reflected in the observed long-run saving rate. I will choose the discount factor β so that the observed average savings rate $\bar{s}$ corresponds to the steady state savings rate. I can therefore choose β by implementing the following algorithm:

1. Given the parameters of the model, the observed labor force growth rate n, and an initial guess for the discount factor, β_0, compute the steady state for $\frac{k^*}{y^*}$, $\frac{i^*}{k^*}$ as defined by equations (35).

2. Let $\frac{\bar{c}}{\bar{y}} = 1 - \bar{s}$ where $\bar{s}$ is the observed average savings rate taken from the data. Then choose β as follows

$$\begin{aligned} &\underset{\beta}{Min} \quad \frac{\bar{c}}{\bar{y}} - \frac{c^*}{y^*} \\ &s.t. \end{aligned} \tag{36}$$

$$0 < \beta \leq 0.999$$
$$\beta^* \equiv \frac{\beta}{(1+n)(1+g)} \geq 0.9$$

where $\frac{c^*}{y^*} = 1 - (r+\delta)\frac{k^*}{y^*} - [(1+n)(1+g) - (1-\delta)]\frac{z^*}{y^*}$

That is, choose β by minimizing the difference between the consumption-output ratio $\frac{\bar{c}}{\bar{y}} = 1 - \bar{s}$ implied by the data and its steady state value as implied by the model $\frac{c^*}{y^*}$. We know that a model with decreasing returns and saving complementarity does a good job of explaining patterns of capital flows using variations in saving rates for plausible capital shares. Here the relevant parameters are η, the share of composite capital, and ρ the parameter that captures the elasticity of substitution. These will be chosen to minimize the distance between observed and simulated moments.

The algorithm for matching is modified as follows:

1. Estimate a vector of moments $\hat{m}$ from the data. Here these moments are the least squares coefficients in the convergence equations on debt, i.e. the least squares coefficients obtained by regressing $\frac{1}{T}(\log d_t - \log d_0)$ on

$$X = \begin{bmatrix} 1 & \log \frac{d_0}{A_0} & \log \frac{s}{(1+n)(1+g)-(1-\delta)} \end{bmatrix}$$

   $\hat{m}$ is the vector of least squares coefficients from this regression.

2. Given values for n,s, z_0 taken from the data, and an initial value for Γ, solve and simulate the non-linear model for each country, i.e. obtain the transition path for consumption, domestic capital, output and debt:

   - Find the value of β_i by using the algorithm described above.
   - Find decision rules for consumption $c = f_i(z)$ and foreign capital $k = g_i(z)$ in each country i. Obtain the simulated series by computing

$$\begin{aligned} c^s_{it} &= f_i(z^s_{it}) \\ k^s_{it} &= g_i(z^s_{it}) \\ y^s_{it} &= [ak^{s\rho}_{it} + (1-a)z^{s\rho}_{it}]^{\frac{\eta}{\rho}} \\ d^s_{it} &= k^s_{it} \\ z^s_{it+1} &= \frac{1}{(1+n)(1+g)}(y^s_{it} - (r+\delta)k^s_{it} + (1-\delta)z^s_{it} - c^s_{it}) \\ \frac{D^s_{it}}{L^s_{it}} &= A^s_{it}d^s_{it} \end{aligned} \tag{37}$$

   for $t = 1, 2, ..., T$ for each country i given z_{i0} , and where the superscript s denotes the simulated series.

3. Use the simulated data to estimate $\tilde{m}(\Gamma)$, the same moments as in the data, that is regress $\frac{1}{t}\left(\log d_t^s - \log d_0^s\right)$ on

$$X^s = \left[\begin{array}{ccc} 1 & \log \frac{d_0^s}{A_0} & \log \frac{s}{(1+n)(1+g)-(1-\delta)} \end{array} \right]$$

$\tilde{m}(\Gamma)$ is the vector of least squares coefficients from this regression.

4. Choose Γ to minimize

$$J(\Gamma) = (\hat{m} - \tilde{m}(\Gamma))^{\mathtt{T}} \mathtt{W} (\hat{m} - \tilde{m}(\Gamma))$$

As before a non-linear method is used to solve the model. This is of particular importance for this version of the model since contrary to the Cobb-Douglas case, the decision rule for foreign capital does not have a closed-form solution. Non-linearities therefore play an important role. See the Appendix for more details on the solution method.

Results with CES Technology Table 11 shows the results of estimation. This version of the model seems much more able to match both the convergence coefficient and saving complementarity. In all three samples, the simulated convergence coefficient is around 3 per cent. In addition, the coefficient on saving is much higher than in previous model versions. In order to achieve this matching, the elasticity of substitution has to be very low, particularly in Sample III, where it is $\frac{1}{1-\rho} = 0.02$. That is, the model predicts that the collateral constraint is particularly binding for low-income countries.

How does matching occur? Recall that in the model with Cobb-Douglas technology, it was difficult to match both the convergence and saving coefficients. This is because the elasticity of substitution between foreign and domestic capital was constrained to be 1. When this assumption is relaxed, it is much easier for the model to match the data. When the elasticity of substitution is high — when k and z are close substitutes — decreasing returns set in more slowly. An increase in z leads to a relative reduction in k; the offsetting movements in their marginal products reduces the speed of convergence. Since the elasticity of substitution is fixed for a Cobb-Douglas technology, the only way for the model to match the 3 per cent observed convergence rate is to increase the share of foreign capital α. But this also reduces the degree to which the collateral constraint is binding and decreases saving complementarity. Thus the model with Cobb-Douglas technology has difficulty matching both convergence and saving complementarity. When the elasticity of substitution is low — when k and z are close complements — decreasing returns set in more quickly. An increase in z also raises k; the reinforcing movements in their marginal products raise the speed of convergence. In the model with CES technology, when the elasticity of substitution between domestic and foreign capital is different from 1, the model can match both high convergence and high saving complementarity by choosing a low elasticity of substitution. The estimate of the composite capital share is now much lower – around 0.4 — but still plausible.

What about the debt-to-GDP ratio? Figure 4 shows initial and steady-state debt-to-GDP distributions in the model. Although this ratio now varies across countries and has more plausible dynamics, these distributions have a much higher average than those observed in the data in Figure 3. This however, seems to be the result of a particular choice for the parameter κ which measures the share of foreign capital in total capital ($\kappa = \frac{k}{k+z}$). I chose $\kappa = 0.5$. It is difficult to determine whether this is a reasonable choice for κ since I have no information on the relative sizes of domestic vs foreign capital for the countries in the sample. This parameter however, plays a crucial role in determining the size of the debt-to-GDP ratio in the steady state. In this model, the size of debt corresponds to the size of foreign capital when the collateral constraint binds. As a result, if we observe low debt-to-GDP ratios in the data, the explanation provided by the model is that the share of foreign capital is low.

Table 12 shows the results when $\kappa = 0.2$. The matching results are fairly similar to when $\kappa = 0.5$. The elasticity of substitution is slightly higher in Samples I and III and slightly lower in Sample II. As shown in Figure 5 however, the distribution of the debt-to-GDP ratio is much closer to that observed.

What can we conclude from these results? This model can match the convergence rate implied by debt data with plausible values of factor shares for Cobb-Douglas technology. These values however, are inconsistent with at least two data regularities. First, to match the effect of savings on debt accumulation, the model needs very high values for the capital shares. Second, it predicts implausibly high values of the debt-to-GDP ratio. A model in which the assumption of a unit elasticity between domestic and foreign capital is relaxed however, is much more successful at matching these data regularities while still predicting decreasing returns and saving complementarity.

4 Conclusion

This paper has attempted to offer a quantitative assessment of neoclassical models with collateral constraints. Previous work has shown that the predictions of these models for debt accumulation are supported qualitatively. First, the accumulation of debt does seem to be partly driven by decreasing returns and the domestic scarcity of capital. Second, domestic savings do seem to positively affect debt and act as complements to capital inflows. These results however, are not sufficient to determine how relevant such a framework might be.

In order to offer a quantitative evaluation, this paper takes the BMS model seriously and estimates the factor shares implied by the convergence equation on debt. As a first step, I emphasize the convergence rate in a model with endogenous saving decisions. In general, this version of the model tends to produce factor estimates that are either too high or too low. When I consider a model in which saving behavior varies across countries, the estimates of the capital and labor shares fall within a range that seems reasonable at first pass. The share of foreign capital is around 0.2 and the share of domestic capital is approximately 0.5.

The estimated share of foreign capital however, implies a debt-to-GDP of over 200 per cent, a level rarely observed in the data. In fact, in the sample considered here, the debt-to-GDP ratio rarely exceeds 100 per cent. To reduce the debt-to-GDP implied by the model, I must assume that an implausibly large portion of output cannot be appropriated by producers due to corruption, taxes and disincentives to capital accumulation. What can we conclude from these results? Perhaps surprisingly, when I only attempt to match the convergence rate and the effect of savings, the model does well in producing reasonable capital shares. This suggests that these two mechanisms — decreasing returns and the complementarity of savings — may play an important role in the long-term dynamics of external debt. Both the strength and the weakness of the model however, are encompassed in the same feature. It is the collateral constraint combined with the Cobb-Douglas technology that leads to the complementarity of savings. At the same time, this rigid constraint implies that debt and output are proportional, that their ratio is constant and identical across countries and is a function of the share of foreign output — three predictions we know to be false. The modelling challenge is to retain the predictions of savings complementarity and convergence while allowing for a more flexible relationship between debt and output.

This can be accomplished by relaxing the assumption of unit elasticity between domestic and foreign capital. Once the model is modified to accommodate a more general production function in the form of CES technology, it is more successful in matching both the convergence and saving complementarity, while producing more plausible distributions for the debt-to-GDP ratio. Although much recent research has suggested that factor accumulation and decreasing returns are of limited importance in explaining cross-country income differences, this paper suggest that a country's ability to attract capital flows may directly be linked to its savings rate. This result seem to be both qualitatively and quantitatively relevant. In a world in which growth is endogenous, it is possible that the gains from higher saving may be even larger even with imperfect capital markets. This suggests that much effort should be spent in understanding differences in savings rates across countries. It is probable that many factors which produce low productivity in poor countries — such as weak institutions — also reduce the incentive to save.

Figure 1: Decision Rules - Sample I

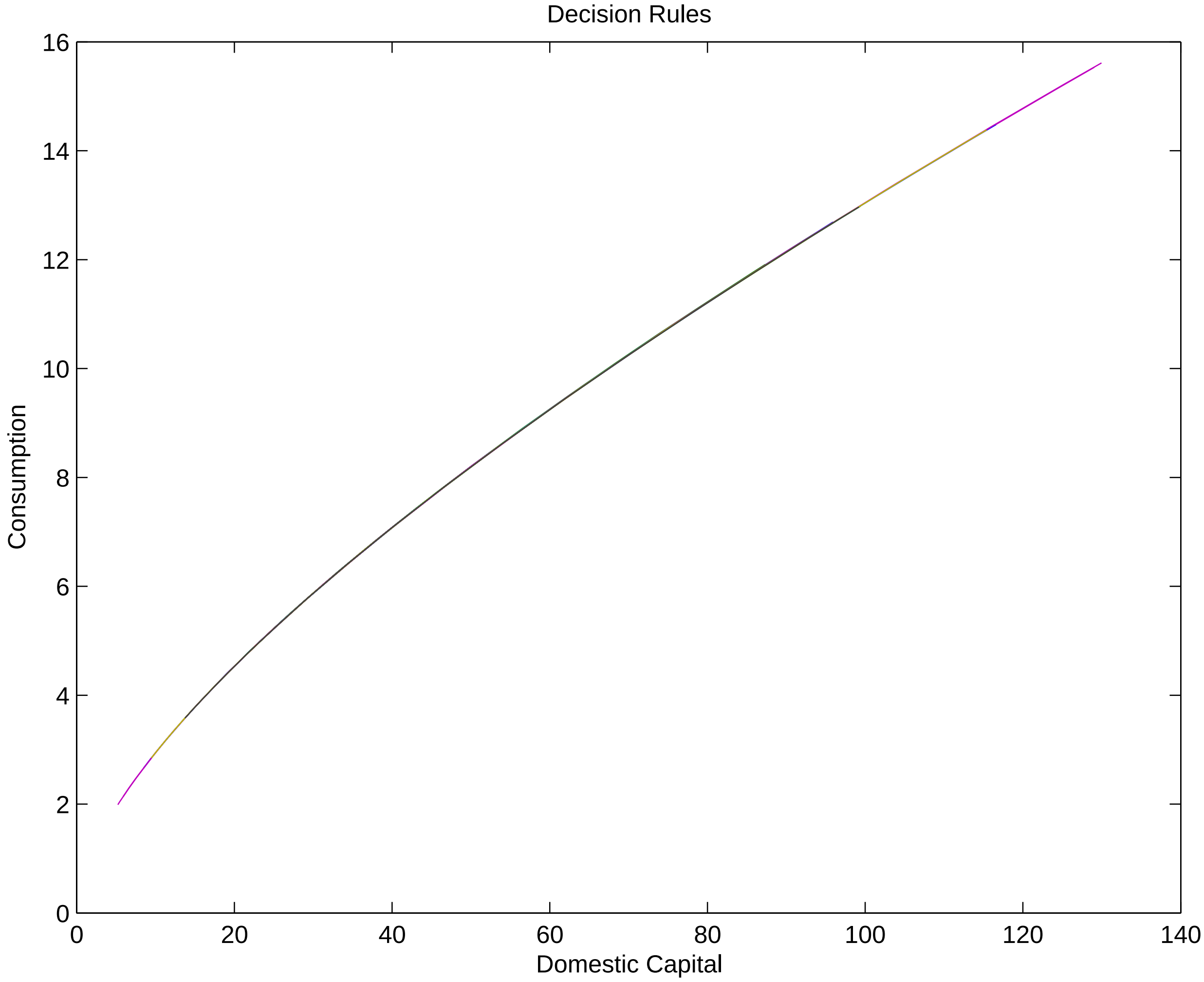


Figure 2: Transition Paths - Sample I

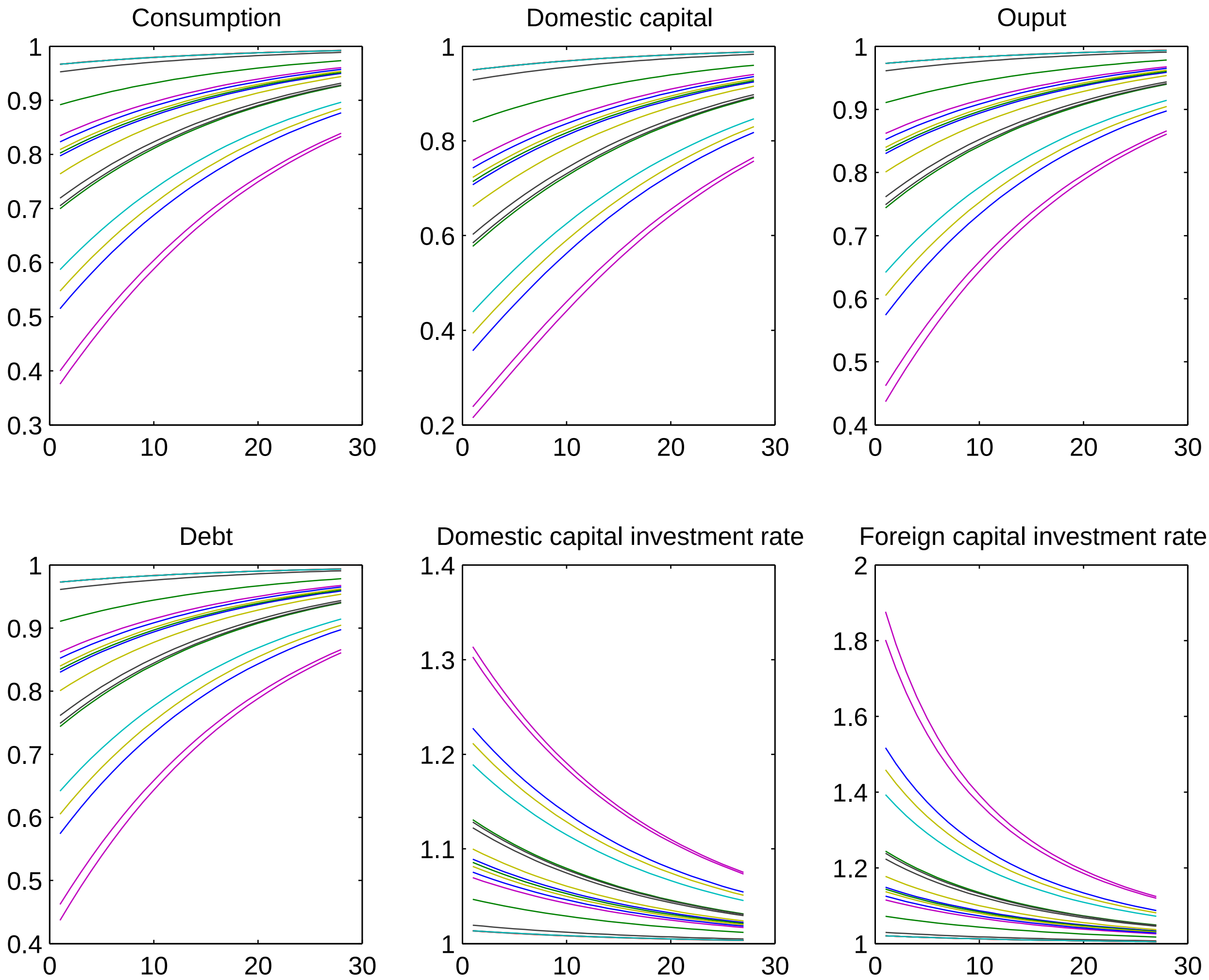


Table 1: Endogenous Savings: Common Technology - Parametrised capital shares

OLS estimation	Sample I	Sample II	Sample III
constant	0.073	0.056	0.056
d_0	-0.032	-0.031	-0.029
$(1+n)(1+g)-(1-\delta)$	-0.032	-0.038	-0.035
α	0.350	0.350	0.350
η	0.350	0.350	0.350
$\frac{\alpha}{\alpha+\eta}$	0.500	0.500	0.500
ε	0.538	0.538	0.538

Table 2: Endogenous Savings: Common Technology - Estimated Parameters

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
OLS estimation	Data	Simulation	Data	Simulation	Data	Simulation
constant	-0.046	-0.023	-0.063	-0.027	0.120	0.120
	(0.145)	–	(0.180)	–	(0.220)	–
d_0	-0.024	-0.023	-0.022	-0.021	-0.033	-0.033
	(0.004)	–	(0.005)	–	(0.006)	–
$(1+n)(1+g)-\beta(1-\delta)$	-0.142	-0.047	-0.138	-0.052	-0.097	-0.027
	(0.068)	–	(0.081)	–	(0.102)	–
α	–	0.050	–	0.050	–	0.485
	–	(0.002)	–	(0.001)	–	(0.007)
η	–	0.665	–	0.689	–	0.232
	–	(0.002)	–	(0.002)	–	(0.004)
$1-\alpha-\eta$	–	0.285	–	0.261	–	0.282
$\frac{\alpha}{\alpha+\eta}$	–	0.070	–	0.068	–	0.676
ε	–	0.700	–	0.725	–	0.452
$\frac{d}{y}$	–	0.625	–	0.625	–	6.069
J	–	2.003	–	1.184	–	0.482

Note: Standard errors are in parenthesis. The standard errors of the least square coefficients act as weight in the estimation of the parameters.

Table 3: Endogenous Savings: Varying Technology - Parametrised capital shares

OLS estimation	Sample I	Sample II	Sample III
constant	0.106	0.079	0.076
d_0	-0.028	-0.031	-0.030
$(1+n)(1+g)-(1-\delta)$	-0.028	-0.037	-0.035
A_0	0.025	0.030	0.030
α	0.350	0.350	0.350
η	0.350	0.350	0.350
$\frac{\alpha}{\alpha+\eta}$	0.500	0.500	0.500
ε	0.538	0.538	0.538

Table 4: Endogenous Savings: Varying Technology - Estimated Parameters

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
OLS estimation	Data	Simulation	Data	Simulation	Data	Simulation
constant	-0.175	0.001	-0.206	-0.018	-0.023	-0.015
	(0.136)	–	(0.201)	–	(0.275)	–
d_0	-0.035	-0.034	-0.028	-0.028	-0.038	-0.034
	(0.005)	–	(0.006)	–	(0.008)	–
$(1+n)(1+g)-\beta(1-\delta)$	-0.091	-0.002	-0.149	-0.043	-0.107	-0.020
	(0.063)	–	(0.079)	–	(0.103)	–
A_0	0.055	0.034	0.030	0.027	0.026	0.033
	(0.017)	–	(0.020)	–	(0.030)	–
α	–	0.050	–	0.050	–	0.050
	–	(0.017)	–	(0.021)	–	(0.007)
η	–	0.050	–	0.588	–	0.360
	–	(0.381)	–	(0.001)	–	(0.008)
$1-\alpha-\eta$	–	0.900	–	0.362	–	0.590
$\frac{\alpha}{\alpha+\eta}$	–	0.500	–	0.078	–	0.122
ε	–	0.053	–	0.619	–	0.379
$\frac{d}{y}$	–	0.625	–	0.625	–	0.625
J	–	5.324	–	2.672	–	0.999

Note: Standard errors are in parenthesis. The standard errors of the least square coefficients act as weight in the estimation of the parameters.

Figure 3: Sample distribution of debt-to-GDP ratios, 1998

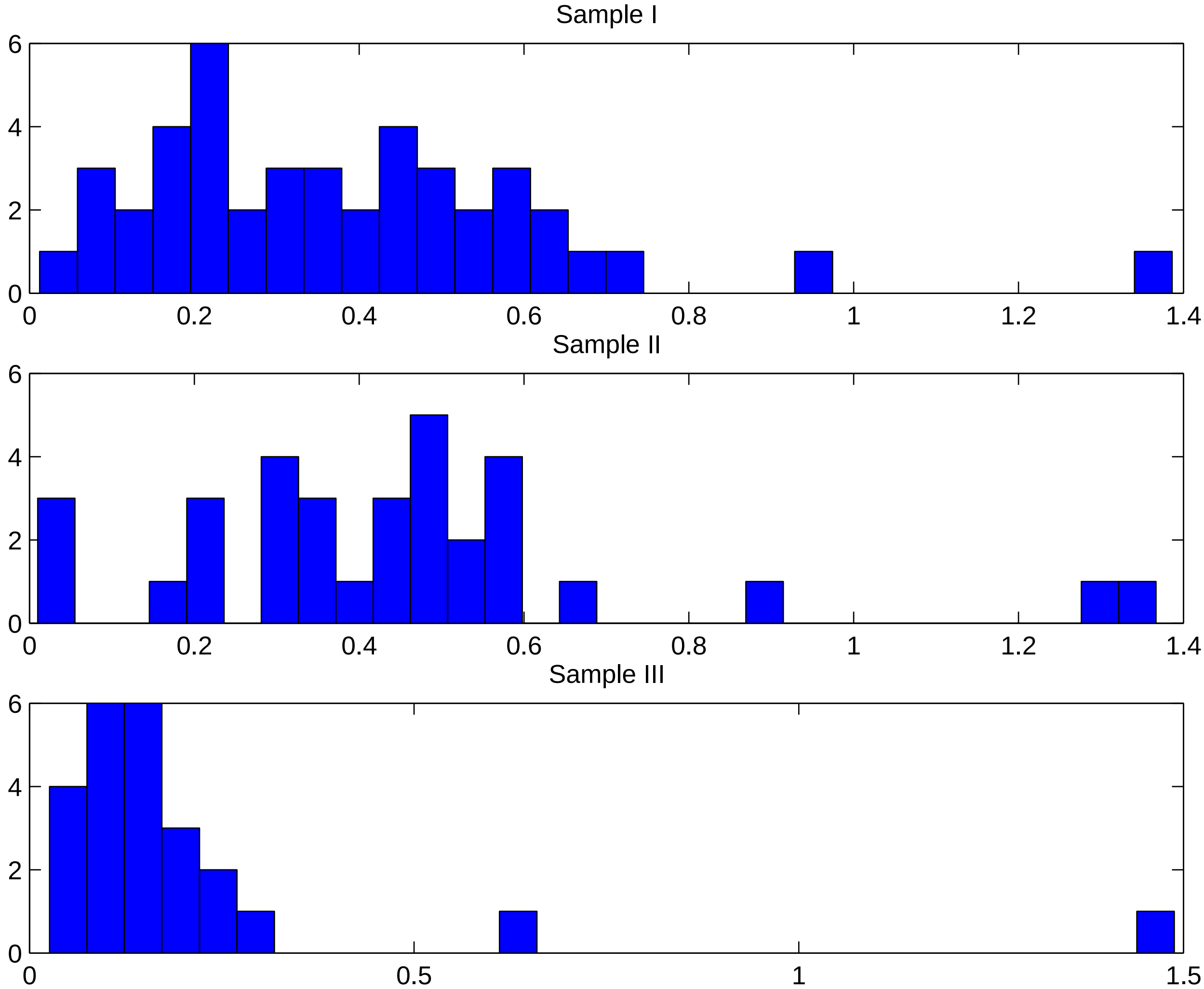


Table 5: Exogenous Savings: Varying Technology - Parametrised capital shares

OLS estimation	Sample I	Sample II	Sample III
constant	0.118	0.108	0.101
d_0	-0.024	-0.026	-0.025
$(1+n)(1+g)-(1-\delta)$	-0.023	-0.027	-0.028
s	0.028	0.030	0.029
A_0	0.022	0.026	0.026
α	0.350	0.350	0.350
η	0.350	0.350	0.350
$\frac{\alpha}{\alpha+\eta}$	0.500	0.500	0.500
ε	0.538	0.538	0.538

Table 6: Exogenous Savings: Varying Technology

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
OLS estimation	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.011	0.001	-0.114	-0.000	-0.143	-0.016
	(0.105)	–	(0.139)	–	(0.204)	–
d_0	-0.036	-0.027	-0.029	-0.026	-0.037	-0.037
	(0.005)	–	(0.006)	–	(0.007)	–
$(1+n)(1+g)-(1-\delta)$	-0.025	-0.001	-0.081	-0.030	-0.074	-0.002
	(0.034)	–	(0.040)	–	(0.052)	–
s	0.055	0.001	0.050	0.032	0.066	0.003
	(0.016)	–	(0.015)	–	(0.025)	–
A_0	0.056	0.026	0.042	0.026	0.065	0.038
	(0.015)	–	(0.018)	–	(0.029)	–
α	–	0.050	–	0.050	–	0.050
	–	(0.153)	–	(0.260)	–	(0.167)
η	–	0.050	–	0.534	–	0.062
	–	(0.027)	–	(0.185)	–	(0.019)
$1-\alpha-\eta$	–	0.900	–	0.416	–	0.888
$\frac{\alpha}{\alpha+\eta}$	–	0.500	–	0.086	–	0.446
ε	–	0.053	–	0.562	–	0.065
$\frac{d}{y}$	–	0.625	–	0.625	–	0.625
J	–	4.927	–	2.678	–	2.303

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
IV estimation	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.053	0.001	-0.114	0.000	-0.156	-0.016
	(0.107)	–	(0.139)	–	(0.205)	–
d_0	-0.037	-0.026	-0.029	-0.026	-0.037	-0.037
	(0.005)	–	(0.006)	–	(0.007)	–
$(1+n)(1+g)-(1-\delta)$	-0.016	-0.001	-0.080	-0.028	-0.074	-0.002
	(0.034)	–	(0.040)	–	(0.052)	–
s	0.072	0.001	0.058	0.031	0.071	0.003
	(0.018)	–	(0.019)	–	(0.026)	–
A_0	0.057	0.025	0.044	0.026	0.068	0.038
	(0.015)	–	(0.018)	–	(0.030)	–
α	–	0.050	–	0.050	–	0.050
	–	(0.154)	–	(0.255)	–	(0.169)
η	–	0.050	–	0.523	–	0.061
	–	(0.020)	–	(0.192)	–	(0.017)
$1-\alpha-\eta$	–	0.900	–	0.427	–	0.889
$\frac{\alpha}{\alpha+\eta}$	–	0.500	–	0.087	–	0.450
ε	–	0.053	–	0.551	–	0.064
$\frac{d}{y}$	–	0.625	–	0.625	–	0.625
J	–	5.208	–	2.570	–	2.411

Note: Standard errors are in parenthesis. The standard errors of the least square coefficients act as weight in the estimation of the parameters. s and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969.

Table 7: Exogenous Savings: Varying Technology

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
OLS estimation	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.055	0.055	0.039	0.039	0.051	0.051
	(0.023)	–	(0.026)	–	(0.034)	–
$\frac{d_0}{A_0}$	-0.033	-0.028	-0.027	-0.023	-0.035	-0.027
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.051	0.027	0.051	0.037	0.054	0.026
	(0.013)	–	(0.013)	–	(0.019)	–
α	–	0.250	–	0.200	–	0.244
	–	(0.103)	–	(0.096)	–	(0.136)
η	–	0.380	–	0.502	–	0.379
	–	(0.090)	–	(0.093)	–	(0.127)
$1-\alpha-\eta$	–	0.369	–	0.298	–	0.376
$\frac{\alpha}{\alpha+\eta}$	–	0.397	–	0.285	–	0.391
ε	–	0.507	–	0.628	–	0.502
$\frac{d}{y}$	–	3.130	–	2.499	–	3.051
J	–	4.872	–	1.600	–	4.181

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
IV estimation	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.034	0.034	0.031	0.031	0.046	0.046
	(0.026)	–	(0.033)	–	(0.037)	–
$\frac{d_0}{A_0}$	-0.034	-0.027	-0.027	-0.024	-0.035	-0.027
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.065	0.026	0.057	0.033	0.058	0.024
	(0.015)	–	(0.019)	–	(0.021)	–
α	–	0.172	–	0.165	–	0.222
	–	(0.116)	–	(0.125)	–	(0.149)
η	–	0.414	–	0.493	–	0.375
	–	(0.111)	–	(0.115)	–	(0.136)
$1-\alpha-\eta$	–	0.414	–	0.343	–	0.403
$\frac{\alpha}{\alpha+\eta}$	–	0.293	–	0.250	–	0.372
ε	–	0.500	–	0.590	–	0.482
$\frac{d}{y}$	–	2.144	–	2.057	–	2.777
J	–	8.717	–	1.909	–	4.301

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis. s and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969.

Table 8: Exogenous Savings: Limited matching I

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
Matching constant and $\frac{d_0}{A_0}$	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.034	0.034	0.031	0.031	0.046	0.046
	(0.026)	–	(0.033)	–	(0.037)	–
$\frac{d_0}{A_0}$	-0.034	-0.034	-0.027	-0.027	-0.035	-0.035
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.065	0.010	0.057	0.027	0.058	0.003
	(0.015)	–	(0.019)	–	(0.021)	–
α	–	0.165	–	0.162	–	0.217
	–	(0.097)	–	(0.119)	–	(0.124)
η	–	0.213	–	0.422	–	0.071
	–	(0.144)	–	(0.120)	–	(0.084)
$1-\alpha-\eta$	–	0.622	–	0.417	–	0.712
$\frac{\alpha}{\alpha+\eta}$	–	0.436	–	0.277	–	0.754
ε	–	0.255	–	0.503	–	0.090
$\frac{d}{y}$	–	2.058	–	2.022	–	2.713
J	–	0.000	–	0.000	–	0.000
Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
Matching constant and $\frac{s}{(1+n)(1+g)-(1-\delta)}$	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.034	0.034	0.031	0.031	0.046	0.046
	(0.026)	–	(0.033)	–	(0.037)	–
$\frac{d_0}{A_0}$	-0.034	-0.010	-0.027	-0.012	-0.035	-0.015
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.065	0.065	0.057	0.057	0.058	0.058
	(0.015)	–	(0.019)	–	(0.021)	–
α	–	0.160	–	0.156	–	0.223
	–	(0.188)	–	(0.146)	–	(0.182)
η	–	0.733	–	0.700	–	0.623
	–	(0.206)	–	(0.136)	–	(0.170)
$1-\alpha-\eta$	–	0.106	–	0.143	–	0.154
$\frac{\alpha}{\alpha+\eta}$	–	0.179	–	0.183	–	0.263
ε	–	0.873	–	0.830	–	0.801
$\frac{d}{y}$	–	2.004	–	1.956	–	2.785
J	–	0.000	–	0.000	–	0.000

Table 9: Exogenous Savings: Limited matching II

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
Matching $\frac{d_0}{A_0}$ and $\frac{s}{(1+n)(1+g)-(1-\delta)}$	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.034	-0.006	0.031	0.265	0.046	-0.006
	(0.026)	–	(0.033)	–	(0.037)	–
$\frac{d_0}{A_0}$	-0.034	-0.027	-0.027	-0.024	-0.035	-0.027
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.065	0.026	0.057	0.033	0.058	0.024
	(0.015)	–	(0.019)	–	(0.021)	–
α	–	0.050	–	0.673	–	0.050
	–	(0.057)	–	(0.026)	–	(0.072)
η	–	0.475	–	0.193	–	0.457
	–	(0.124)	–	(0.043)	–	(0.162)
$1-\alpha-\eta$	–	0.475	–	0.134	–	0.492
$\frac{\alpha}{\alpha+\eta}$	–	0.095	–	0.777	–	0.099
ε	–	0.500	–	0.590	–	0.482
$\frac{d}{y}$	–	0.625	–	8.408	–	0.628
J	–	8.717	–	1.909	–	4.301

Note: Standard errors are in parenthesis. The standard errors of the least square coefficients act as weight in the estimation of the parameters. s and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969.

Table 10: Exogenous Savings: Varying Technology - Institutional Tax

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
$\tau = 0.3$	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.034	0.034	0.031	0.031	0.046	0.046
	(0.026)	–	(0.033)	–	(0.037)	–
$\frac{d_0}{A_0}$	-0.034	-0.027	-0.027	-0.024	-0.035	-0.027
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.065	0.026	0.057	0.033	0.058	0.024
	(0.015)	–	(0.019)	–	(0.021)	–
α	–	0.250	–	0.251	–	0.297
	–	(0.114)	–	(0.118)	–	(0.137)
η	–	0.375	–	0.442	–	0.339
	–	(0.100)	–	(0.103)	–	(0.123)
$1-\alpha-\eta$	–	0.375	–	0.307	–	0.365
$\frac{\alpha}{\alpha+\eta}$	–	0.399	–	0.362	–	0.467
ε	–	0.500	–	0.590	–	0.482
$\frac{d}{y}$	–	3.119	–	3.133	–	3.711
J	–	8.717	–	1.909	–	4.301
Dependent variable: $\log d_t - \log d_0$	**Sample I**		**Sample II**		**Sample III**	
$\tau = 0.78$	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.034	0.034	0.031	0.031	0.046	0.046
	(0.026)	–	(0.033)	–	(0.037)	–
$\frac{d_0}{A_0}$	-0.034	-0.027	-0.027	-0.024	-0.035	-0.027
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.065	0.026	0.057	0.033	0.058	0.024
	(0.015)	–	(0.019)	–	(0.021)	–
α	–	0.453	–	0.461	–	0.481
	–	(0.071)	–	(0.067)	–	(0.083)
η	–	0.273	–	0.318	–	0.250
	–	(0.068)	–	(0.070)	–	(0.089)
$1-\alpha-\eta$	–	0.273	–	0.221	–	0.269
$\frac{\alpha}{\alpha+\eta}$	–	0.624	–	0.592	–	0.658
ε	–	0.500	–	0.590	–	0.482
$\frac{d}{y}$	–	5.667	–	5.767	–	6.017
J	–	8.717	–	1.909	–	4.301

Note: Standard errors — which act as weight in the estimation of the parameters — are in parenthesis. s and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969. The data are simulated under the assumption that $r+\delta = 0.12$

Table 11: CES Technology — $\kappa = 0.5$

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
IV estimation	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.034	0.007	0.031	0.014	0.046	-0.001
	(0.026)	–	(0.033)	–	(0.037)	–
$\frac{d_0}{A_0}$	-0.034	-0.033	-0.027	-0.028	-0.035	-0.034
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.065	0.041	0.057	0.031	0.058	0.044
	(0.015)	–	(0.019)	–	(0.021)	–
η	–	0.367	–	0.426	–	0.392
	–	(0.001)	–	(0.000)	–	(0.000)
ρ	–	-27.084	–	-3.527	–	-20.052
	–	(0.000)	–	(0.000)	–	(0.001)
$\frac{1}{1-\rho}$	–	0.036	–	0.221	–	0.048
κ	–	0.500	–	0.500	–	0.500
J	–	3.556	–	2.082	–	1.980

Note: Standard errors are in parenthesis. The standard errors of the least square coefficients act as weight in the estimation of the parameters. s and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969.

Table 12: CES Technology — $\kappa = 0.2$

Dependent variable: $\log d_t - \log d_0$	Sample I		Sample II		Sample III	
IV estimation	Data	Simulation	Data	Simulation	Data	Simulation
constant	0.034	-0.007	0.031	0.007	0.046	-0.012
	(0.026)	–	(0.033)	–	(0.037)	–
$\frac{d_0}{A_0}$	-0.034	-0.031	-0.027	-0.031	-0.035	-0.033
	(0.004)	–	(0.005)	–	(0.006)	–
$\frac{s}{(1+n)(1+g)-(1-\delta)}$	0.065	0.042	0.057	0.035	0.058	0.042
	(0.015)	–	(0.019)	–	(0.021)	–
η	–	0.422	–	0.500	–	0.418
	–	(0.000)	–	(0.000)	–	(0.000)
ρ	–	-17.033	–	-20.529	–	-20.567
	–	(0.000)	–	(0.000)	–	(0.001)
$\frac{1}{1-\rho}$	–	0.055	–	0.046	–	0.046
κ	–	0.200	–	0.200	–	0.200
J	–	5.106	–	2.526	–	3.024

Note: Standard errors are in parenthesis. The standard errors of the least square coefficients act as weight in the estimation of the parameters. s and $(1+n)(1+g)-(1-\delta)$ are instrumented using the average savings rate and average labour force growth between 1960 and 1969.

Figure 4: Simulated distribution of debt-to-GDP ratios — $\kappa = 0.5$

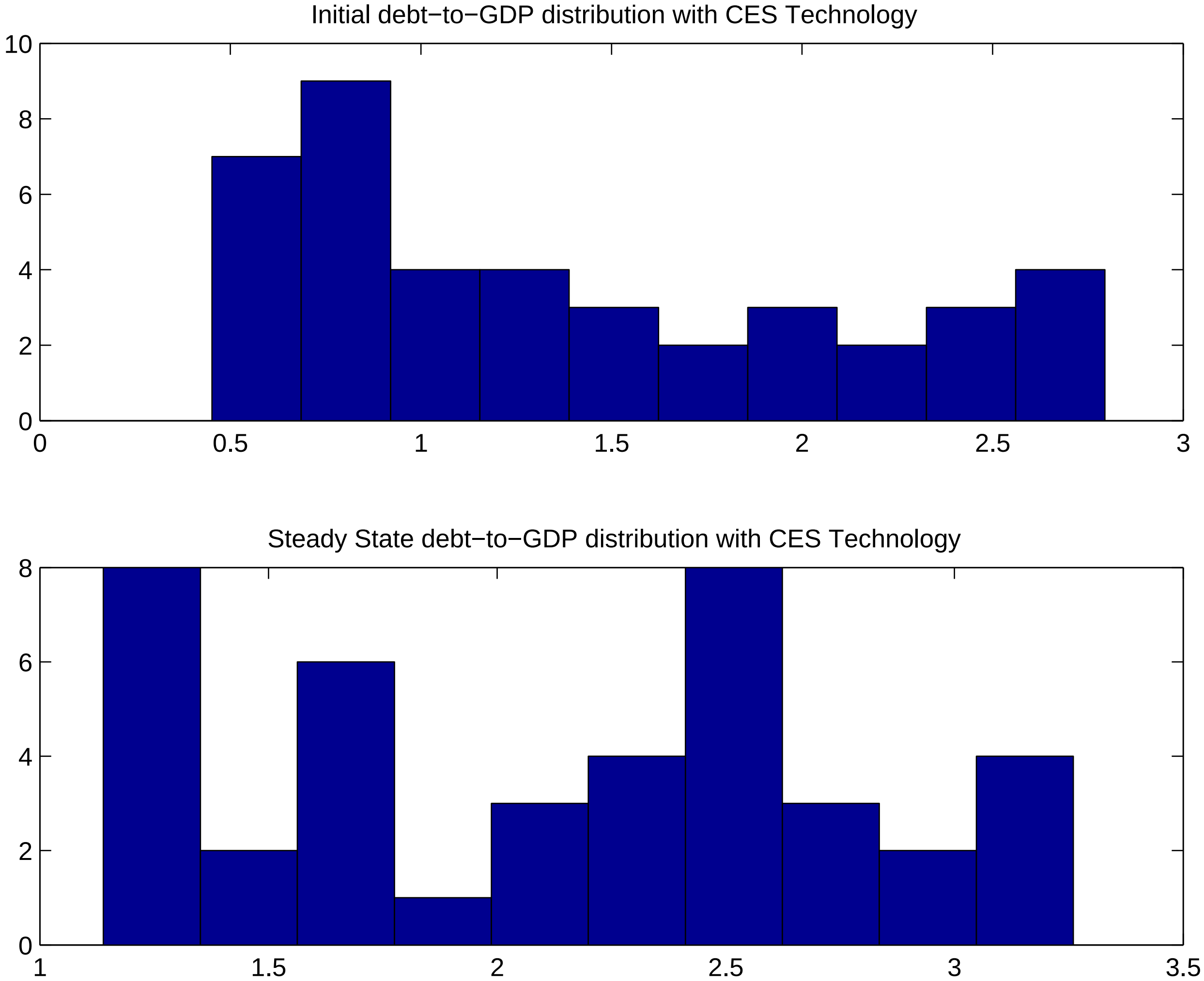
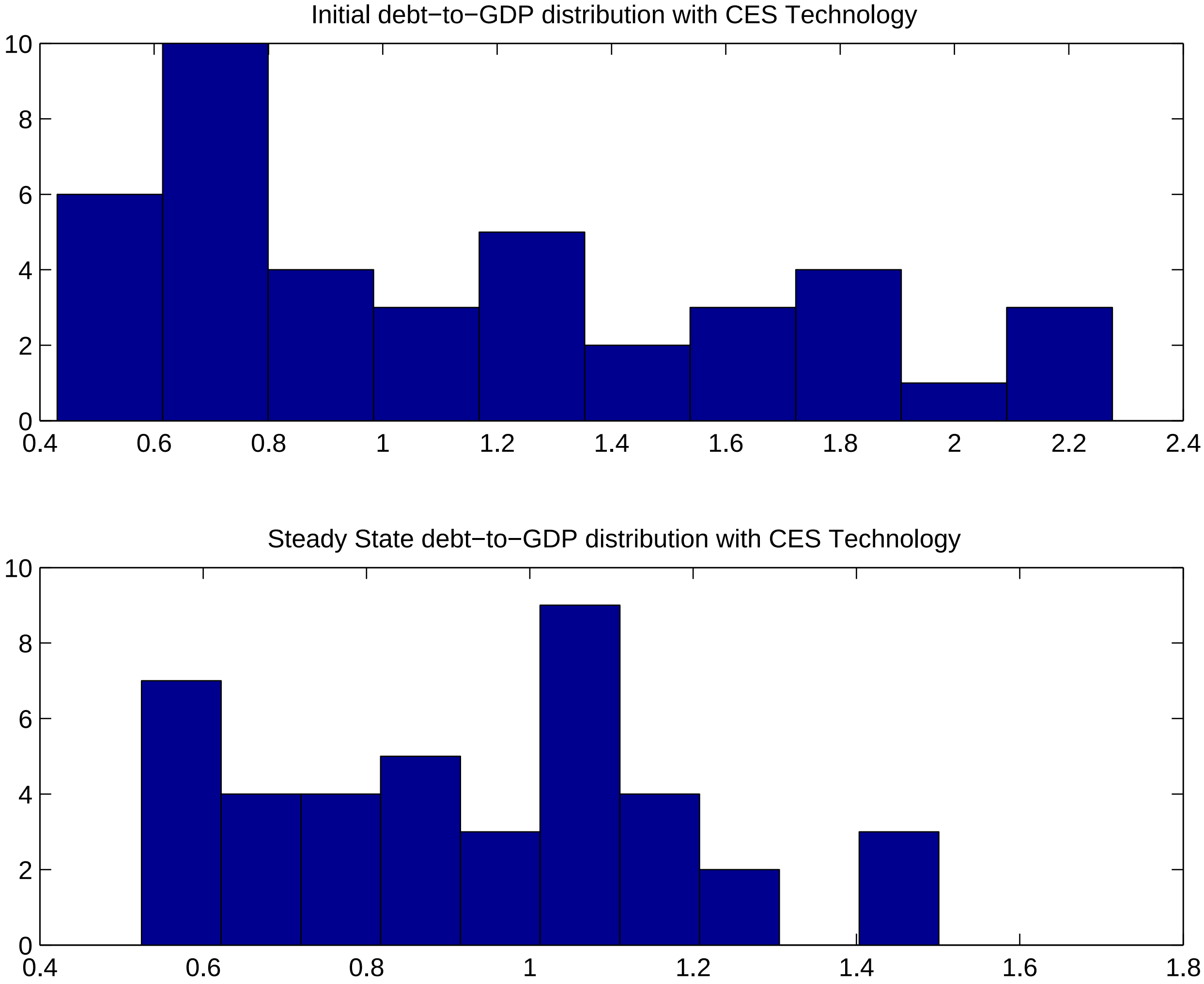


Figure 5: Simulated distribution of debt-to-GDP ratios — $\kappa = 0.2$



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Appendix

A Solution Method

B A Non-Linear Solution Method for the BMS Model with Endogenous Savings

Non-linear methods for approximating decision rules are useful when one wants to ask about the behavior of an economy away from the steady state. Indeed, Taylor approximation methods remain local in nature. A popular approach to approximate non-linear decision rules is to use least squares methods. In its simplest form, this method involves choosing a non-linear function of the state variables (e.g. a polynomial of physical capital for consumption). This however, is often problematic since powers of the state variables are co-linear. An alternative is least squares orthogonal polynomial approximation. This involves choosing functions of the state variables that are orthogonal to each other so as to avoid co-linearity. Chebychev polynomial are an example of such functions.

The reduced form of the model takes the following form:

$$z_{t+1} = \left(\frac{1}{(1+n)(1+g)}\right)\left((1-\alpha)Bz_t^{\varepsilon} + (1-\delta)n_t - c_t\right) \tag{B.1}$$

$$c_{t+1}^{\sigma} = \beta^* c_t^{\sigma}\left((1-\alpha)\varepsilon B z_{t+1}^{\varepsilon-1} + 1 - \delta\right) \tag{B.2}$$

We want to find a decision rule for consumption as a function of current domestic capital which satisfies the Euler equation and the market-clearing condition. Here we assume that over the domain $[\overline{z}, \underline{z}]$ the consumption decision rule takes the form

$$c_t = \phi(z_t, \theta) = \sum_{i=0}^{n_z} \theta_i T_i\left(\varphi(z_t)\right) \tag{B.3}$$

where T_i is the Chebychev polynomial of order $i = 0, ..., n_h$ and $\varphi(z)$ is a linear function mapping the domain of z into $[-1, 1]$. Chebychev polynomials are computed as

$$T_n(x) = \cos(n \cos^{-1}(x)) \tag{B.4}$$

Let the Euler equation residual be

$$R\left(z_{jt}; \phi, \theta\right) \equiv \phi\left(z_{jt+1}(z_{jt}, \theta), \theta\right)^{\sigma} - \phi\left(z_{jt}, \theta\right)^{\sigma} \beta^* \left((1-\alpha)\varepsilon B z_{jt+1}(z_{jt}, \theta)^{\varepsilon-1} + 1 - \delta\right) \tag{B.5}$$

where $z_j \in [\overline{z}, \underline{z}]$. Then the problem is to choose θ such that

$$\sum_{j=1}^{node_z} R\left(z_{jt}; \phi, \theta\right) T_i\left(\varphi(z_j)\right) = 0 \tag{B.6}$$

for $i = 0, ..., n_h$. This can be written as

$$\mathcal{T}\left(\varphi(z)\right) R\left(z; \phi, \theta\right) = 0 \tag{B.7}$$

where

$$\mathcal{T}(z) = \begin{pmatrix} T_0\left(\varphi(z_1)\right) & \dots & T_0\left(\varphi(z_{node_z})\right) \\ \vdots & \ddots & \vdots \\ T_{n_z}\left(\varphi(z_1)\right) & \dots & T_{n_z}\left(\varphi(z_{node_z})\right) \end{pmatrix} \tag{B.8}$$

The algorithm is as follows

1. Choose the order of approximation n_z. We choose $n_z = 5$
2. Choose an interval $[\underline{z}, \overline{z}]$ over which to estimate the decision rule [16].

[16] See Section C for a discussion of the choice of the interval

3. Set up a grid of $node_z$ ($node_z = 20$)data points $\{z_i\}_{i=1}^{node_z}$ over $[\underline{z}, \overline{z}]$.
4. Compute the $node_z > n_z$ roots of the Chebychev polynomial of order $node_z > n_z$ as

$$x_i = \cos \frac{(2i-1)\pi}{2node_z}$$

for $i = 1, \ldots, node_z$.
5. Compute the matrix

$$a\mathcal{T}(z) = \begin{pmatrix} T_0\left(\varphi(x_1)\right) & \ldots & T_0\left(\varphi(x_{node_z})\right) \\ \vdots & \ddots & \vdots \\ T_{n_h}\left(\varphi(x_1)\right) & \ldots & T_{n_z}\left(\varphi(x_{node_z})\right) \end{pmatrix} \tag{B.9}$$

6. Choose an initial value for θ [17].
7. Compute z_i as

$$z_i = \underline{z} + (x_i + 1)\frac{(\overline{z} - \underline{z})}{2}$$

for $i = 1, \ldots, node_z$ to map $[-1, 1]$ into $[\underline{z}, \overline{z}]$.
8. Compute $R\left(z_{jt}; \phi, \theta\right)$ for $j = 1, \ldots, node_z$ and evaluate

$$\mathcal{T}\left(\varphi(z)\right) R\left(z; \phi, \theta\right)$$

9. If it is close enough to zero, i.e. if

$$\mathcal{T}\left(\varphi(z)\right) R\left(z; \phi, \theta\right) < tol$$

where tol is the tolerance level, then stop and form

$$c_t = \phi(z_t, \theta) = \sum_{i=0}^{n_z} \theta_i T_i\left(\varphi(z_t)\right)$$

else update θ and go back to Step 7.

C Choice of the grid

The estimation of α and η requires a solution for consumption for each country. In the simulated data, countries are assumed to be different in two dimensions. First, they have different labor force growth rates, the values of which are taken from the data. Second, they start at different initial values for human capital. We cannot directly use data values for human capital since the model has no scale. Data values must therefore be scaled down in some fashion. Here we will use the distribution of income as a scale. The country with the highest level of per worker GDP at the beginning of the sample is chosen as the standard country. Specifically, for each country j we construct the distribution of income relative to the standard country s at the beginning of the sample

$$\nu_{j0} = \frac{y_{j0}}{y_{s0}}$$

where y_{j0} is income at the beginning of the sample for country j and y_{s0} is income at the beginning of the sample for the standard country. We also construct the distribution of income implied by labor force growth rates in the steady state

$$\nu^* = \frac{y_j^*}{y_s^*}$$

where the '$*$' denotes the steady state value. If we assume that the standard country is $1 - \varphi$ away from its steady state at the beginning of the sample period [18], we can compute the deviation of country j from its steady state as

$$\Delta_j = \varphi \frac{\nu_{j0}}{\nu^*}$$

[17] See Section D for a discussion of the choice of initial values
[18] We choose $\varphi = 0$

For each country j the approximation grid is then taken to be in the interval $[\underline{z}, \overline{z}]$ where

$$\underline{z} = (1 - \Delta_j)z^*$$

and

$$\overline{z} = (1 + \Delta_j)z^*$$

We choose $node_z$ equally spaced points within this interval.

D Choice of initial value

One fairly straightforward way of obtaining an initial value for θ is to log-linearize the model around the steady state:

1. Find a solution to the model by using standard first-order Taylor approximation methods. That is, solve the following log-linear system

$$\begin{aligned}
\hat{y}_t &= \varepsilon \hat{z}_t \\
\hat{y}_t - \frac{i^*}{i^* + c^*}\hat{i}_t &= \frac{c^*}{i^* + c^*}\hat{c}_t \\
\hat{i}_t &= (1+n)(1+g)\frac{z^*}{i^*}\hat{z}_{t+1} - (1-\delta)\frac{z^*}{i^*}\hat{z}_t \\
(1-\beta^*(1-\delta))\hat{z}_{t+1} + \sigma\hat{c}_{t+1} - \sigma\hat{c}_t &= (1-\beta^*(1-\delta))\hat{y}_{t+1}
\end{aligned} \tag{D.10}$$

   where $\hat{q}$ denotes the percentage deviation of q from its steady state. The steady state is defined by

$$\begin{aligned}
\frac{y^*}{z^*} &= \frac{1}{(1-\alpha)\varepsilon}\left(\frac{1}{\beta^*} - (1-\delta)\right) \\
z^* &= \left(\frac{1}{B}\frac{y^*}{z^*}\right)^{\frac{1}{\varepsilon - 1}} \\
y^* &= Bz^{*\varepsilon} \\
i^* &= \left[(1+n)(1+g) - (1-\delta)\right] z^* \\
c^* &= (1-\alpha)y^* - i^*
\end{aligned} \tag{D.11}$$

2. For each value of $z_i \in [\underline{z}, \overline{z}]$, find the corresponding value of c_i using the linear solution.
3. Regress $\log c$ on $T_i(\varphi(z_t))$ and use the OLS coefficients as initial values for θ.

E Solving the BMS Model with Exogenous Savings

Let s denote the domestic savings rate, or the rate at which consumers save out of output to accumulate domestic capital. Since domestic savings must equal investment in domestic capital ($i_t^h = sBh_t^\varepsilon$):

$$(1+n)(1+g)z_{t+1} = sBz_t^\varepsilon + (1-\delta)z_t \tag{E.12}$$

Solving this equation yields

$$z^* = \left(\frac{sB}{(1+n)(1+g) - (1-\delta)}\right)^{\frac{1}{1-\varepsilon}} \tag{E.13}$$

and the steady state is defined by

$$\begin{aligned}
y^* &= Bz^{*\varepsilon} \\
k^* &= \frac{\alpha}{r+\delta}y^* \qquad (E.14) \\
d^* &= k^* \qquad (E.15)
\end{aligned}$$

F Convergence rate

F.1 Endogenous Savings

Reducing the linearized system defined by (D.11), we have

$$\begin{aligned} \hat{c}_{t+1} + \frac{1-\varepsilon}{\sigma}\left(1-\beta^*(1-\delta)\right)\hat{z}_{t+1} &= \hat{c}_t \\ \hat{z}_{t+1} &= \frac{1}{\beta}\hat{z}_t - \frac{1}{(1+n)(1+g)}\frac{c^*}{z^*}\hat{c}_t \end{aligned} \tag{F.16}$$

or

$$\begin{bmatrix} \hat{z}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{1}{(1+n)(1+g)}\frac{c^*}{z^*} \\ \frac{\varepsilon-1}{\beta\sigma}\left(1-\beta^*(1-\delta)\right) & \frac{1-\varepsilon}{\sigma}\frac{\left(1-\beta^*(1-\delta)\right)}{(1+n)(1+g)}\frac{c^*}{z^*}+1 \end{bmatrix} \begin{bmatrix} \hat{z}_t \\ \hat{c}_t \end{bmatrix} \tag{F.17}$$

The convergence rate is governed by the eigenvalue of the transition matrix that solves

$$\begin{aligned} 2\lambda &= \frac{1}{\beta}+1+\left(\frac{1-\varepsilon}{\sigma}\right)\frac{\left(1-\beta^*(1-\delta)\right)}{(1+n)(1+g)}\frac{c^*}{z^*} \\ &- \left[\left(\frac{1}{\beta}+\left(\frac{1-\varepsilon}{\sigma}\right)\frac{\left(1-\beta^*(1-\delta)\right)}{(1+n)(1+g)}\frac{c^*}{z^*}\right)^2 - 4\left(\frac{\varepsilon-1}{\sigma\beta}\right)\frac{\left(1-\beta^*(1-\delta)\right)}{(1+n)(1+g)}\frac{c^*}{z^*}\right]^{\frac{1}{2}} \end{aligned} \tag{F.18}$$

As $\varepsilon \to 1$, $\lambda \to 1$ and the model does not exhibit convergence.

F.2 Exogenous Savings

Linearizing equation (E.12) yields

$$\hat{z}_{t+1} = \frac{\varepsilon s B {z^*}^{\varepsilon-1}+1-\delta}{(1+g)(1+n)}\hat{z}_t \tag{F.19}$$

so that

$$\hat{z}_{t+1} = \left[\varepsilon + \frac{(1-\varepsilon)(1-\delta)}{(1+n)(1+g)}\right]\hat{z}_t \tag{F.20}$$

with solution

$$\hat{z}_t = \hat{z}_0\left[\varepsilon + \frac{(1-\varepsilon)(1-\delta)}{(1+n)(1+g)}\right]^t \tag{F.21}$$

Let $\lambda = \varepsilon + \frac{(1-\varepsilon)(1-\delta)}{(1+n)(1+g)}$. Again, as $\varepsilon \to 1$, $\lambda \to 1$ and the model does not exhibit convergence.

G Solving the BMS model with a CES Production Function

G.1 Firms

$$\begin{aligned} y_t &= [ak_t^\rho + (1-a)z_t^\rho]^{\frac{\eta}{\rho}} \\ &= v_t^\eta \end{aligned} \tag{G.22}$$

Profit maximization implies

$$\begin{aligned} R_{kt} &= \eta a k_t^{\rho-1}[ak_t^\rho + (1-a)z_t^\rho]^{\frac{\eta}{\rho}-1} \\ &= a\eta\left(\frac{k_t}{v_t}\right)^\rho \frac{y_t}{k_t} \end{aligned}$$

$$\begin{aligned} R_{zt} &= \eta(1-a)z_t^{\rho-1}[ak_t^\rho + (1-a)z_t^\rho]^{\frac{\eta}{\rho}-1} \\ &= (1-a)\eta\left(\frac{z_t}{v_t}\right)^\rho \frac{y_t}{z_t} \\ w_t &= y_t - R_{kt}k_t - R_{zt} \end{aligned}$$

The combination of the credit constraint ($d_t = k_t$), the small-open-economy assumption ($R_{kt} = r + \delta$) and profit-maximization implies that the market-clearing condition takes the following form:

$$\begin{aligned} (1+n)(1+g)z_{t+1} &= y_t - f_k(k_t, z_t)k_t + (1-\delta)z_t - c_t \\ &= y_t - (r+\delta)k_t + (1-\delta)z_t - c_t \end{aligned} \tag{G.23}$$

G.2 Households

$$\begin{aligned} \underset{\{c_t, z_{t+1}, k_{t+1}\}_{t=0}^\infty}{Max} & \quad \sum_{t=0}^\infty \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ s.t. & \\ (1+n)(1+g)z_{t+1} &= [ak_t^\rho + (1-a)z_t^\rho]^{\frac{\eta}{\rho}} - (r+\delta)k_t + (1-\delta)z_t - c_t \end{aligned} \tag{G.24}$$

The first-order conditions

$$\begin{aligned} c_t^{-\sigma} - \lambda_t &= 0 \\ R_{kt} &= r + \delta \\ -\lambda_t(1+n)(1+g) + \beta\lambda_{t+1}\left[f_z(k_{t+1}, z_{t+1}) + (1-\delta)\right] &= 0 \end{aligned}$$

The Euler equation

$$\left(\frac{c_{t+1}}{c_t}\right)^\sigma = \beta^*\left[f_z(k_{t+1}, z_{t+1}) + (1-\delta)\right] \tag{G.25}$$

where $\beta^* = \frac{\beta}{(1+n)(1+g)}$

G.3 Equilibrium

Equilibrium satisfies

$$\begin{aligned} \left(\frac{c_{t+1}}{c_t}\right)^\sigma &= \beta^*\left[f_z(k_{t+1}, z_{t+1}) + (1-\delta)\right] \\ (1+n)(1+g)z_{t+1} &= [ak_t^\rho + (1-a)z_t^\rho]^{\frac{\eta}{\rho}} - (r+\delta)k_t + (1-\delta)z_t - c_t \\ a\eta\left(\frac{k_t}{v_t}\right)^\rho \frac{y_t}{k_t} &= R_{kt} = r + \delta \end{aligned}$$

G.4 Steady State

Closed-Economy Steady State for the Open-Economy If we assume $\frac{1}{\beta^*} = 1 + r$, then the world real interest rate is the same as the rate that would occur in the closed economy that is the economy is neither more patient or less patient than the world economy. In that case, the representative agent will use the two types of capital in a way that ensures that their return are equated in the steady state, i.e.

$$\begin{aligned} R_k &= R_z \\ a\eta\left(\frac{k_t}{v_t}\right)^\rho \frac{y_t}{k_t} &= (1-a)\eta\left(\frac{z_t}{v_t}\right)^\rho \frac{y_t}{z_t} \\ \frac{k^*}{z^*} &= \left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}} \end{aligned} \tag{G.26}$$

Then the steady state satisfies

$$\begin{aligned}
\frac{1}{\beta^*} &= 1+r \\
(1+n)(1+g)z^* &= [ak^{*\rho} + (1-a)z^{*\rho}]^{\frac{\eta}{\rho}} - (r+\delta)k^* + (1-\delta)z^* - c^* \\
a\eta\left(\frac{k^*}{v^*}\right)^{\rho}\frac{y^*}{k^*} &= r+\delta \\
\frac{k^*}{z^*} &= \left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}} = \mu
\end{aligned} \tag{G.27}$$

which can be solved to obtain k^*, z^*, c^* as follows:

$$\begin{aligned}
a\eta k^{*\rho} v^{*\eta-\rho} k^{*-1} &= r+\delta \\
z^* &= \left(\frac{\Omega}{r+\delta}\right)^{\frac{1}{1-\eta}}
\end{aligned} \tag{G.28}$$

where $\Omega = a\eta[a\mu^\rho + 1 - a]^{\frac{\eta-\rho}{\rho}}\mu^{\rho-1}$

$$\begin{aligned}
k^* &= \mu z^* \\
i^* &= (1+n)(1+g)z^* - (1-\delta)z^* \\
c^* &= y^* - (r+\delta)k^* - i^*
\end{aligned}$$

G.5 Log-linear Solution

The system to solve is

$$\begin{aligned}
y_t &= Av_t^\eta \\
v_t &= [ak_t^\rho + (1-a)z_t^\rho] \\
i_t &= (1+n)(1+g)z_{t+1} - (1-\delta)z_t \\
y_t &= i_t + c_t + (r+\delta)k_t \\
\left(\frac{c_{t+1}}{c_t}\right)^\sigma &= \beta^*\left((1-a)\eta\left(\frac{z_{t+1}}{v_{t+1}}\right)^\rho \frac{y_{t+1}}{z_{t+1}} + 1 - \delta\right) \\
a\eta\left(\frac{k_t}{v_t}\right)^\rho \frac{y_t}{k_t} &= r+\delta
\end{aligned}$$

The log-linear system is

$$\begin{aligned}
\hat{y}_t &= \eta\hat{v}_t \\
\hat{v}_t &= a\left(\frac{k^*}{v^*}\right)^\rho \hat{k}_t + (1-a)\left(\frac{z^*}{v^*}\right)^\rho \hat{z}_t \\
\hat{i}_t &= (1+n)(1+g)\frac{z^*}{i^*}\hat{z}_{t+1} - (1-\delta)\frac{z^*}{i^*}\hat{z}_t \\
\hat{y}_t &= \frac{i^*}{y^*}\hat{i}_t + \frac{c^*}{y^*}\hat{c}_t + (r+\delta)\frac{k^*}{y^*}\hat{k}_t \\
\sigma\hat{c}_{t+1} - \sigma\hat{c}_t &= \beta^*(1-a)\eta(\rho-1)\left(\frac{z^*}{v^*}\right)^\rho \frac{y^*}{z^*}\hat{z}_{t+1} - \beta^*(1-a)\eta\rho\left(\frac{z^*}{v^*}\right)^\rho \frac{y^*}{z^*}\hat{v}_{t+1} \\
&\quad + \beta^*(1-a)\eta\left(\frac{z^*}{v^*}\right)^\rho \frac{y^*}{z^*}\hat{y}_{t+1} \\
(\rho-1)\hat{k}_t - \rho\hat{v}_t + \hat{y}_t &= 0
\end{aligned}$$

The solution to this linearized system is used as a initial value for the non-linear algorithm described in the first part of the Appendix.

H Data

H.1 Debt

The debt data are from Lane and Milesi-Ferretti (2001). They construct net foreign asset positions for 66 countries between 1970 and 1998. Their approach essentially consists in using available stock data and supplementing it with flows from balance-of-payments data. More specifically, they note that the balance-of-payments identity implies that the sum of the current account (CA), financial flows – which include foreign direct investment, portfolio equity, debt flows and capital transfers (e.g. debt forgiveness) – and the change in reserves equals zero plus net errors and omissions. The change in the value of net foreign assets thus corresponds to the sum of the current account, capital transfers and capital gains or losses on the stock of assets. The first measure used in this paper, `ACUMCA`, corresponds to the cumulative sum of current account balances. It is available for industrial and developing countries between 1970 and 1998. The second measure `ACUMFL` corresponds to the sum of stock measures of the various assets and liabilities. These measures are either cumulative flows or direct stock measures. `ACUMFL` is available for developing countries between 1970 and 1998. Both measures are adjusted for debt reductions and forgiveness. In addition, these measures take into account valuations changes, such as exchange rate changes, and variations in the price of capital goods, as well as changes in stock market values.

The main difference between the two measures is the treatment of unrecorded capital flows. By cumulating current accounts, the `ACUMCA` measure implies that unrecorded capital flows – including but over and above net errors and omissions – correspond to assets held by domestic investors abroad. On the other hand, `ACUMFL` only reflects unrecorded capital outflows to the extent that they are recorded in net errors and omissions. In countries with periods of unrecorded capital flight, debt measured by `ACUMFL` will tend to be larger than debt measured by `ACUMCA` since the latter records a larger portion of unrecorded capital holdings. The debt per worker measure used in this paper corresponds to $\frac{D}{L} = \frac{D^m}{p_{US}L}$ where $D^m = -\texttt{ACUMCA}$ or $D^m = -\texttt{ACUMFL}$. The debt data are measured in US dollars. To obtain a real value, they are divided by p_{US}, the US GDP deflator obtained from the IMF's *International Financial Statistics*.

The dependent variable is the average annual growth rate of debt

$$\Delta d = \frac{\log d_T - \log d_t}{T - t}$$

where $[t; T]$ is the sample period.

H.2 Labour Force and Savings

L corresponds to the labour force. It is measured by the population between 15 and 64 computed from output and population data from the Penn World Tables, version 6.0 as

$$L = \frac{\texttt{RGDPL}}{\texttt{RGDPW}} \times \texttt{POP}$$

where `RGDPL` is real per capita chain GDP, `RGDPW` is real per worker chain GDP, and `POP` is total population.

The labour force growth rate corresponds to the average annual growth rate of L computed as

$$1 + n = \left(\frac{\mathtt{L}_T}{\mathtt{L}_t}\right)^{\frac{1}{T-t}}$$

The labour force growth rate variable used in the regression is the log of $(1 + n)(1 + g) - (1 - \delta)$. I follow Mankiw, Romer and Weil (1992) and assume a growth rate of technological progress of $g = 0.02$ and a depreciation rate $\delta = 0.03$.

The savings rate is measured as $1 - \frac{c}{y}$ where $\frac{c}{y}$ corresponds to the average value of `kc` between 1970 and 1997 in the Penn World Tables 6.0.

H.3 TFP

PWT 6.0 does not provide estimates of the stock of physical capital. To compute total factor productivity at the beginning of sample, capital per worker in 1970 is estimated using the permanent inventory scheme

$$(1+\bar{n})\frac{K_{t+1}}{L_{t+1}} = \frac{I_t}{L_t} + (1-\delta)\frac{K_t}{L_t}$$

Under the assumption that capital and output per worker grow at the same rate g — as they do in the model —, the initial physical capital stock is

$$\frac{K_0}{L_0} = \frac{\frac{I_0}{L_0}}{(1+\bar{n})(1+g)-(1-\delta)}$$

where we estimate initial investment per worker as

$$\frac{I_0}{L_0} = \left(\frac{1}{10}\sum_{t=1970}^{1980} \mathtt{KI}_t\right)\mathtt{RGDPW}_{1970}$$

and labour force growth as

$$\bar{n} = \frac{\log(\mathtt{L}_{1980}) - \log(\mathtt{L}_{1970})}{10}$$

The TFP measure is computed as

$$A_0 = \log(\mathtt{RGDPW}_0) - \alpha \log\frac{K_0}{L_0}$$

with $\alpha = 0.3$.

H.4 Missing Values

Many variables are not available for all the years and countries in the full Lane and Milesi-Ferretti database. In order to retain the largest number of countries for estimation, the sample period differs across countries from a maximum of 28 years to a minimum of 15 years. This justifies the use of annual averages for both levels and growth rates.

H.5 Sample Composition

Sample I (CUMCA)	Sample II (NFA)	Sample III (CUMCA) [a]
Argentina (ARG)	Argentina (ARG)	Bolivia (BOL)
Australia (AUS)	Bolivia (BOL)	Brazil (BRA)
Austria (AUT)	Brazil (BRA)	Colombia (COL)
Bolivia (BOL)	Chile (CHL)	Costa Rica (CRI)
Brazil (BRA)	Colombia (COL)	Dominican Republic (DOM)
Canada (CAN)	Costa Rica (CRI)	Ecuador (ECU)
Chile (CHL)	Dominican Republic (DOM)	Egypt (EGY)
Colombia (COL)	Ecuador (ECU)	El Salvador (LSV)
Costa rica (CRI)	Egypt (EGY)	Guatemala (GTM)
Denmark (DNK)	El Salvador (SLV)	India (IND)
Dominican Republic (DOM)	Guatemala (GTM)	Indonesia (IDN)
Ecuador (ECU)	India (IND)	Ivory Coast (CIV)
Egypt (EGY)	Indonesia (IDN)	Jamaica (JAM)
El Salvador (SLV)	Israel (ISR)	Korea (KOR)
Finland (FIN)	Ivory Coast (CIV)	Malaysia (MYS)
Greece (GRC)	Jamaica (JAM)	Mauritius (MUS)
Guatemala (GTM)	Jordan (JOR)	Morocco (MAR)
Iceland (ISL)	Korea (KOR)	Pakistan (PAK)
India (IND)	Malaysia (MYS)	Sri Lanka (LKA)
Indonesia (IDN)	Mauritius (MUS)	Thailand (THA)
Ireland (IRL)	Mexico (MEX)	Turkey (TUR)
Israel (ISR)	Morocco (MAR)	
Ivory Coast (CIV)	Pakistan (PAK)	
Jamaica (JAM)	Peru (PER)	
Jordan (JOR)	Philippines (PHL)	
Korea (KOR)	Sri Lanka (LKA)	
Malaysia (MYS)	Syria (SYR)	
Mauritius (MUS)	Thailand (THA)	
Mexico (MEX)	Tunisia (TUN)	
Morocco (MAR)	Turkey (TUR)	
New Zealand (NZL)		
Pakistan (PAK)		
Peru (PER)		
Portugal (PRT)		
Spain (ESP)		
Sri Lanka (LKA)		
Sweden (SWE)		
Syria (SYR)		
Thailand (THA)		
Tunisia (TUN)		
Turkey (TUR)		
Uruguay (URY)		
42 countries	30 countries	24 countries

[a]The low-income Sample III corresponds to countries whose 1970 GDP per worker is lower than the median for that variable in that year in the whole CUMCA sample