The Pro-cyclical R&D Puzzle:

Technology Shocks and Pro-cyclical R&D Expenditure

Taiji Harashima

University of Tsukuba
Cabinet Office of Japan

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Abstract

Empirically R&D expenditure moves pro-cyclically, but the pro-cyclicality is a puzzle from the Schumpeterian point of view. The paper examines the cyclical property of R&D expenditure in the context of endogenous growth, and concludes that (i) substitutability between investing in physical capital and investing in technology/knowledge is a key of the cyclical property of R&D, (ii) basically technology shocks accompany counter-cyclical R&D and demand shocks accompany pro-cyclical R&D, and (iii) the easiest way to solve the pro-cyclical R&D puzzle is to abandon the conjecture that business cycles are generated mainly by technology shocks.

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Correspondence: Taiji Harashima:
Graduate School of Systems and Information Engineering, University of Tsukuba,
1-1-1 Tenoudai, Tsukuba, Ibaraki 305- 8573, Japan
Email: tharashm@sk.tsukuba.ac.jp
t-harashima@mve.biglobe.ne.jp

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I. INTRODUCTION

It has been reported in many empirical studies that R&D expenditure moves pro-cyclically. Geroski and Walters (1995), Fátas (2000), and Rafferty and Funk (2004) conclude that R&D expenditure has a pro-cyclical property. Wälde and Woitek (2004) examine R&D expenditure in G7 countries and conclude that it is fair to argue that there is stronger evidence for pro-cyclical rather than counter-cyclical behavior of R&D expenditure. Empirical evidence as a whole suggests that R&D expenditure is in fact pro-cyclical.

However, the observed pro-cyclical R&D expenditure is a puzzle from the Schumpeterian point of view. It is argued in the literature on the Schumpeterian notion that productivity improving activities compete with production activities for resources and recessions are associated with a higher pace of productivity improving activities. In the Schumpeterian growth notion, opportunity costs in recessions are so important that counter-cyclicality of R&D activities is a natural consequence. Theoretical researches on endogenous growth and short-run fluctuations like Bental and Peled (1996), Matsuyama (1999), and Wälde (2002) deal with this opportunity cost effect and predict counter-cyclical R&D expenditure, which sharply contradicts the observed pro-cyclical property. To solve this puzzle, Barlevy (2004) explores a modified Schumpeterian growth model but it needs to assume irrational activities of entrepreneurs. The pro-cyclical R&D expenditure is still a puzzle from the Schumpeterian point of view.

On the other hand, some economists argue that the pro-cyclical property of R&D activities is a natural consequence of imperfections in financial markets. They argue that, because it has been observed that the R&D expenditure in a small firm is positively correlated with the cash flow of the firm, the pro-cyclical property emerges due to imperfections in financial markets that generate pro-cyclical cash flows in small firms. The literature on the cash flow effect includes Hall (1992), Himmelberg and Petersen (1994), Hall et al. (1998), Mulkay, Hall and Mairesse (2001), and Rafferty and Funk (2004), and they commonly predict pro-cyclical R&D expenditure in case of demand shocks. From their point of view, the pro-cyclical R&D expenditure is not a puzzle.

Which view is correct? Is the pro-cyclical R&D expenditure really a puzzle? The reason of the different predictions may be because the two views have been studied from completely different standpoints without considering each other. The most important difference between them is that the former assumes a frictionless economy and the latter assumes financial frictions. However, there is another noticeable difference between them. The models based on the Schumpeterian view implicitly assume technology shocks and the studies on cash-flow effects assume basically demand shocks. This difference suggests that the cyclical property of R&D expenditure may depend on types of shocks. The observed pro-cyclical R&D expenditure may reflect the type of shocks that dominates actual business cycles. Hence, it may be necessary to examine effects of various shocks on the cyclical property of R&D expenditure on the basis of a common framework. The paper explores this possibility and examines how different the cyclical property of R&D expenditure is according to types of shocks, i.e. technology shocks and demand shocks, based on a common endogenous growth model.

Results are previewed as follows: (i) as has been stressed in the Schumpeterian literature, substitutability between investments in k_t and in A_t is a key that determines cyclical property of R&D expenditure, (ii) technology shocks basically accompany counter-cyclical R&D expenditure and demand shocks basically accompany pro-cyclical R&D expenditure, and (iii) the easiest way to solve the pro-cyclical R&D puzzle is to abandon the conjecture that business cycles are generated mainly by technology shocks.

The paper is organized as follows. In section II, firstly it is shown that empirical evidence suggests that R&D expenditure is in fact pro-cyclical. Secondly, an endogenous growth model in which substitutability between investing in physical capital and investing in R&D is incorporated is constructed, and effects of technology shocks and demand shocks on the cyclical property of R&D expenditure are examined. It is shown that basically technology shocks accompany counter-cyclical R&D expenditure and demand shocks accompany pro-cyclical R&D expenditure. In section III, some possible reasons for the observed pro-cyclical R&D expenditure in case of technology shocks are considered. Finally some concluding remarks are offered in section IV.

II. THE PRO-CYCLICAL R&D PUZZLE

1. Empirical evidence

Many empirical researches conclude that R&D expenditure has a pro-cyclical property. Geroski and Walters (1995) conclude that there is some pro-cyclical behavior of R&D expenditure in the UK, and Fátas (2000) argues that in the U.S. R&D expenditure is pro-cyclical. Wälde and Woitek (2004) examine R&D expenditure in G7 countries comprehensively and conclude that it is fair to argue that there is stronger evidence for pro-cyclical rather than counter-cyclical behavior of R&D expenditure. Rafferty and Funk (2004) show that firm level R&D data provide evidence of a strong positive correlation between firm's sales and its R&D expenditure, which implies a pro-cyclical property of R&D expenditure. Comin and Gertler (2004) argue that R&D expenditure in the U.S. is especially pro-cyclical over the medium term cycle. Exceptionally Saint-Paul (1993) that is one of the earliest works on this subject concludes that there remains very little evidence of any pro- or counter-cyclical behavior of R&D.[1] There is little evidence that R&D expenditure is counter-cyclical.[2] As a whole, empirical evidence suggests that R&D expenditure is in fact pro-cyclical.

2. The model

[1] The result in Saint-Paul (1993) is criticized for it resting on inappropriate identification restrictions in VAR estimation.

[2] Rafferty and Funk (2004) find some evidence of a small counter-cyclical component in large firms. However they conclude that it appears to work only during expansions.

R&D activities are the most important driving force of economic growth, and thus to analyze movements of R&D expenditure correctly, they should be examined in the context of endogenous growth that is achieved by successive R&D activities. In addition, the feature of substitutability between investing in physical capital and investing in R&D should be incorporated in endogenous growth models that are used for this analysis. Physical capital and knowledge/technology/idea are equally capital inputs in the sense that they are used to produce outputs, and thus investments in physical capital and in knowledge/technology/idea can be substituted each other. Investors decide in each period whether to invest in physical capital or in knowledge/technology/idea capital and after comparing profitability of each investment, investors choose the most profitable investment. Hence, investments in physical capital and investments in knowledge/technology/idea capital are not decided independently but they are allocated through arbitrage between them. As a result, without considering the feature of substitutability between them, it seems impossible to examine correctly how much investments in knowledge/technology/idea capital, i.e. R&D investments, are allocated and what cyclical property R&D expenditure has. The feature of substitutability between investing in physical capital and investing in R&D therefore is explicitly incorporated in the model in the paper.[3]

The production function is assumed to be $Y_t = F(A_t, K_t, L_t)$, where Y_t (≥ 0) is outputs, K_t (≥ 0) is capital inputs, L_t (≥ 0) is labor inputs, and A_t (≥ 0) is knowledge/technology/idea inputs in period t. The model is based on the following assumptions.

Assumptions:

(A1) The accumulation of capital and knowledge/technology/idea is $\dot{K}_t = Y_t - C_t - v\dot{A}_t - \delta K_t$, where $v(> 0)$ is a constant and a unit of K_t and $\frac{1}{v}$ of a unit of A_t are produced using the same

[3] The original model is developed in Harashima (2004). This model has a very important advantage that it is free from both scale effects and the influence of population growth. See also Harashima (2005a, 2005b).

amounts of inputs, and δ is the rate of depreciation.[4]

(A2) Every firm is identical and has the same size, and for any period, $m = \frac{M_t^{\rho}}{L_t} = \text{constant}$ where M_t is the number of firms and $\rho(>1)$ is a constant.

(A3) $\frac{\partial Y_t}{\partial K_t} = \frac{1}{M_t^{\rho}} \frac{\partial Y_t}{\partial (vA_t)}$ and thus $\frac{\partial y_t}{\partial k_t} = \frac{1}{mv} \frac{\partial y_t}{\partial A_t}$.

Assumption (A1) is standard one in the literature of endogenous growth. Assumption (A2) simply assumes that the number of population and the number of firms in an economy are positively related, which seems intuitively natural. Substitutability between investing in physical capital and investing in knowledge/technology/idea capital is incorporated in the model by assumption (A3). In assumption (A3), the paper assumes that returns to investing in K_t and investing in A_t for a firm are kept equal. In addition, it is also assumed in (A3) that a firm that invents a new technology can not obtain all the returns to investing in A_t. This means that investing in A_t increases Y_t but returns of an individual firm that invests in A_t is only a fraction of the increase of Y_t such that $\frac{1}{M_t^{\rho}} \frac{\partial Y_t}{\partial (vA_t)} = \frac{1}{mL_t} \frac{\partial Y_t}{\partial (vA_t)}$. The reason why only a fraction of the increase in Y_t the returns of an individual firm is, is uncompensated knowledge spillovers to other firms.

More specifically, the production function is assumed to have the following functional form: $Y_t = F(A_t, K_t, L_t) = A_t^{\alpha} f(K_t, L_t)$, where $\alpha(0 < \alpha < 1)$ is a constant. Let $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $c_t = \frac{C_t}{L_t}$ and $n_t = \frac{\dot{L}_t}{L_t}$ and assume that $f(K_t, L_t)$ is homogenous of degree one. Thereby

[4] Hence, like Jones' (1995) non-scale model, A_t, as well as K_t, is produced less as A_t and L_t increase if the usual production function of homogeneous of degree one is assumed.

$y_t = A_t^{\alpha} f(k_t)$, and $\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t - \delta k_t$. By assumptions (A2) and (A3), $A_t = \frac{\alpha f(k_t)}{m v f'(k_t)}$ because $\frac{\partial y_t}{m v \partial A_t} = \frac{\partial y_t}{\partial k_t} \Leftrightarrow \frac{\alpha}{mv} A_t^{\alpha-1} f(k_t) = A_t^{\alpha} f'(k_t)$. Since $A_t = \frac{\alpha f}{m v f'}$, then $y_t = A_t^{\alpha} f = \left(\frac{\alpha}{mv}\right)^{\alpha} \frac{f^{1+\alpha}}{f'^{\alpha}}$ and $\dot{A}_t = \frac{\alpha}{mv} \dot{k}_t \left(1 - \frac{f f''}{f'^2}\right)$.

For simplicity, the growth rate of population is assumed to be positive and constant, i.e. $n_t = n > 0$ hereafter, and in the paper, only the case of Harrod neutral technological progress such that $y_t = A_t^{\alpha} k_t^{1-\alpha}$ and thus $Y_t = K_t^{1-\alpha}(A_t L_t)^{\alpha}$ is examined.[5] Because the production function is Harrod neutral and because $A_t = \frac{\alpha f(k_t)}{m v f'(k_t)}$ and $f = k_t^{1-\alpha}$, then $A_t = \frac{\alpha}{m v (1-\alpha)} k_t$ and $\frac{f f''}{f'^2} = -\frac{\alpha}{1-\alpha}$. The accumulation of capital thereby proceeds by $\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n k_t - \delta k_t = \left(\frac{\alpha}{mv}\right)^{\alpha} \frac{f^{1+\alpha}}{f'^{\alpha}} - c_t - \frac{\alpha}{m L_t} \dot{k}_t \left(1 - \frac{f f''}{f'^2}\right) - n k_t - \delta k_t$. Hence,

$$\dot{k}_t = \frac{\left(\frac{\alpha}{mv}\right)^{\alpha} \frac{f^{1+\alpha}}{f'^{\alpha}} - c_t - n k_t - \delta k_t}{1 + \frac{\alpha}{m L_t}\left(1 - \frac{f f''}{f'^2}\right)} = \frac{m L_t (1-\alpha)}{m L_t (1-\alpha) + \alpha} \left\{ \left[\left(\frac{\alpha}{mv}\right)^{\alpha} (1-\alpha)^{-\alpha} - n - \delta \right] k_t - c_t \right\}.$$

Since the problem of scale effects in endogenous growth models is not a focal point in the paper, it is assumed for simplicity that the population L_t is sufficiently large and thus $\frac{m L_t (1-\alpha)}{m L_t (1-\alpha) + \alpha} = 1$ hereafter.

The optimization problem of a representative household therefore is:

Max $E_0 \int_0^{\infty} u(c_t) \exp(-\theta t) dt$,

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[5] As is well known, only Harrod neutral technological progress matches the stylized facts presented by Kaldor (1961).

subject to

$$\dot{k}_t = \left[\left(\frac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - n - \delta \right] k_t - c_t \,.$$

Let Hamiltonian H be

$$H = u(c_t)\exp(-\theta t) + \lambda_t \left\{ \left[\left(\frac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - n - \delta \right] k_t - c_t \right\}$$

where λ_t is a costate variable, and thus the optimality conditions are

(1) $\dfrac{\partial u(c_t)}{\partial c_t} \exp(-\theta t) = \lambda_t$,

(2) $\dot{\lambda}_t = -\dfrac{\partial H}{\partial k_t}$,

(3) $\dot{k}_t = \left[\left(\dfrac{\alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - n - \delta \right] k_t - c_t$,

(4) $\lim_{t \to \infty} \lambda_{1t} k_{1t} = 0$.

Before examining the cyclical property of R&D expenditure, the basic nature of the model is examined. First, the condition for a steady state growth path is examined.

Lemma 1: If and only if $\dfrac{\dot{c}_t}{c_t} = \dfrac{\dot{k}_t}{k_t}$ = constant, all the optimality conditions are satisfied.

Proof: See Appendix 1.

Unquestionably rational households will select the initial consumption that leads to a growth path that satisfies all the conditions, i.e. a growth path such that $\dfrac{\dot{c}_t}{c_t} = \dfrac{\dot{k}_t}{k_t}$ = constant. Hence, it is assumed that given the initial A_0 and k_0, a representative household sets the initial consumption so as to achieve a growth path that satisfies all the conditions, i.e. a growth path of $\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} =$ constant, while firms adjust k_t so as to achieve $\frac{\partial Y_t}{\partial K_t} = \frac{1}{M_t^{\rho}} \frac{\partial Y_t}{\partial (vA_t)}$. As a result of rational behavior of households and firms, the following steady state growth path is achieved.

Lemma 2: $\frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \text{constant}$

Proof: See Appendix 2.

3. Substitutability between investing in k_t and in A_t

For any endogenous growth model that can achieve a steady state growth path, the condition such that $\frac{A_t}{k_t} = \text{constant}$ must be satisfied. Without this condition, an economy can not grow at a constant rate. The endogenous growth model in the paper of course satisfies this condition.[6] However, the model in the paper satisfies not only this condition but a stricter condition such that $\frac{A_t}{k_t}$ = a unique constant. The model has this feature because investments in k_t and in A_t are substitutable, which is assumed by assumption (A3). It is shown in the following proposition.

Proposition 1: Even if there is a shock that changes k_t and/or A_t, eventually the ratio $\frac{A_t}{k_t}$

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[6] On the steady state growth path, $\frac{A_t}{k_t} = \frac{\alpha}{mv(1-\alpha)} = \text{constant}$.

returns to a unique constant that is same as before the shock, i.e. $\frac{A_t}{k_t} = \frac{\alpha}{mv(1-\alpha)}$.

Proof: By assumption (A3), lemma 1 and lemma 2, the relation $\frac{\partial y_t}{\partial k_t} = \frac{\alpha}{mv}\frac{\partial y_t}{\partial A_t}$ is held on the steady state growth path such that $\frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t}$ = constant. Hence, $\frac{A_t}{k_t} = \frac{\alpha}{mv(1-\alpha)}$ on the steady state growth path. Here, parameters α, m and v have unique constant values and thus $\frac{A_t}{k_t} = \frac{\alpha}{mv(1-\alpha)}$ = a unique constant.

Even if there is a shock that changes k_t and/or A_t, eventually the economy returns to the steady state growth path such that $\frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t}$ = constant by lemma 2. As a result, even if there is a shock that changes k_t and/or A_t, eventually the ratio $\frac{A_t}{k_t}$ returns to a unique constant that is same as before the shock, i.e. $\frac{A_t}{k_t} = \frac{\alpha}{mv(1-\alpha)}$.

Q.E.D.

The nature $\frac{A_t}{k_t}$ = a unique constant shown in proposition 1 will strictly restrain movements of k_t and A_t after shocks and thus will have an significant influence on the cyclical property of R&D expenditure. After any shock that changes k_t and/or A_t, k_t and A_t must be adjusted in order to return to the unique ratio of $\frac{A_t}{k_t}$. If the equation $\frac{A_t}{k_t}$ = a unique constant can be restored in the period when the shock occurred, the nature shown in proposition 1 may not bind movements of k_t and A_t severely after the period. However, if the equation $\frac{A_t}{k_t}$ = a unique constant is far from restored in the period when the shock occurred and thus the adjustment process continues after the period, the equation $\frac{A_t}{k_t}$ = a unique constant will bind and alter the movements of both k_t and A_t significantly in the following period after the shock. In this sense, whether the equation $\frac{A_t}{k_t}$ = a unique constant can be restored in the period when the shock occurred or not seems essential for the cyclical property of R&D expenditure. The equation $\frac{A_t}{k_t}$ = a unique constant is therefore a key that determines the cyclical movements of R&D expenditure.

For example, if A_t increases 1 % additionally by a shock, k_t must also be increased 1 % eventually to restore the equation $\frac{A_t}{k_t}$ = a unique constant. In many modern economies, the capital/output ratio is 2-3. This means that after an additional 1 % increase of A_t by the shock, the stock of capital k_t must be increased 1 % additionally, which is equivalent to 2-3 % of output. However, the additional 1 % increase of A_t increases output y_t only by α % of output that can be used to increase k_t additionally to restore the equation $\frac{A_t}{k_t}$ = a unique constant. In many modern economies, the share of labor input α is 0.6-0.7. It is easily recognized that it is impossible to fill the necessary increase of k_t that is equivalent to 2-3 % of output with only 0.6-0.7 % of output. Hence, the necessary increase of k_t will not be achieved in the period when the shock occurred and the process to restore the equation $\frac{A_t}{k_t}$ = a unique constant will take several periods after the shock. During the adjustment period, investments in k_t should grow faster than before but those in A_t should grow slower than before in order to restore the equation $\frac{A_t}{k_t}$ = a unique constant, and thus they will show very different cyclical patterns. This example suggests that the nature that the equation $\frac{A_t}{k_t}$ = a unique constant is restored eventually is really playing an essential role for the cyclical property and that whether the equation $\frac{A_t}{k_t}$ = a unique constant can be restored in the period when the shock occurred is an important criterion to judge how cyclically R&D expenditure moves.

This essential equation $\frac{A_t}{k_t}$ = a unique constant is held because investments in k_t and in A_t are substitutable and thus the returns to them must be equal in any time. The substitutability between them therefore is a deeper source of the cyclical property of R&D expenditure. In this sense, to include the substitutability into models properly seems indispensable when the cyclical property of R&D expenditure is examined.

Remark 1: If investments in k_t and in A_t are not substitutable, i.e. assumption (A3) is not held and thus $\frac{\partial y_t}{\partial k_t} \neq \frac{1}{mv}\frac{\partial y_t}{\partial A_t}$, after a shock that changes k_t and/or A_t, the ratio $\frac{A_t}{k_t}$ does not necessarily return to that before the shock.

Keeping this important nature in mind, effects of various shocks on the cyclical property are examined in the following sub-sections. The focal point is whether the criterion that, after a shock, the equation $\frac{A_t}{k_t}$ = a unique constant can be restored in the period when the shock occurred, is satisfied. First the cyclical property in case of shocks on A_t and secondly that in case of shocks other than shocks on A_t are examined. In those analyses, it is assumed for simplicity that (i) before each shock, an economy is on the steady state growth path and thus $\frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t}$ = constant by lemma 2, and (ii) investments that were planned before a shock are not changed in the period when the shock occurred.

4. Technology shocks

Whether the criterion that after a shock on A_t the equation $\frac{A_t}{k_t}$ = a unique constant can be restored in the period when the shock occurred is satisfied is examined. Here, when A_t increases by zA_t ($0 < z$) by a positive shock on A_t, output y_t increases by $zA_t \frac{\partial y_t}{\partial A_t}$ due to the increase of A_t, and this increase of output $zA_t \frac{\partial y_t}{\partial A_t}$ is allocated to the increase of consumption and the increase of investments in k_t and in A_t. It is assumed that $wzA_t \frac{\partial y_t}{\partial A_t}$ is allocated to the increase of consumption and $(1-w)zA_t \frac{\partial y_t}{\partial A_t}$ is allocated to the increase of investments in k_t and in A_t where $0 \leq w \leq 1$. Since consumption is pro-cyclical, w may be roughly same as the share of consumption in output on the steady state growth path. An important point that should be examined is whether the increase of investments $(1-w)zA_t \frac{\partial y_t}{\partial A_t}$ is large enough to restore the unique ratio $\frac{A_t}{k_t} = \frac{\alpha}{mv(1-\alpha)}$ that is required when proceeding on the steady state growth path as was shown in proposition 1.

Proposition 2: After a positive shock on A_t, if and only if $\alpha(1-w)\frac{y_t}{k_t} \geq 1$, k_t can be more than the necessary quantity to hold the equation $\frac{A_t}{k_t}$ = a unique constant in the period when the shock occurred.

Proof: Let the shock makes A_t change by zA_t. In order to hold the equation $\frac{A_t}{k_t}$ = a unique constant, the increase of k_t initiated by the shock on A_t in the period when the shock occurred needs to be more than zk_t that can make the equation $\frac{A_t}{k_t}$ = a unique constant be held, and thus the condition $(1-w)zA_t\frac{\partial y_t}{\partial A_t} \geq zk_t$ must be satisfied in the period when the shock occurred in order to hold the equation $\frac{A_t}{k_t}$ = a unique constant, because, by assumption, investments that were planned before the shock are not changed in the period when the shock occurred. Here, $(1-w)zA_t\frac{\partial y_t}{\partial A_t} \geq zk_t \Leftrightarrow \alpha(1-w)\frac{y_t}{k_t} \geq 1$. Hence, if and only if $\alpha(1-w)\frac{y_t}{k_t} \geq 1$, k_t can be more than the necessary quantity to hold the equation $\frac{A_t}{k_t}$ = a unique constant in the period when the shock occurred.

Q.E.D.

Firms' investment activities will change significantly if k_t is less than the necessary quantity to hold the equation $\frac{A_t}{k_t}$ = a unique constant in the period when the shock occurred.

Corollary 1: After a positive shock on A_t, if $\alpha(1-w)\frac{y_t}{k_t} < 1$, (i) investments in k_t are more profitable than those in A_t in the periods after the shock until recovering the steady state growth path, i.e. $\frac{\partial y_t}{\partial k_t} > \frac{\alpha}{mv}\frac{\partial y_t}{\partial A_t}$, and (ii) the growth rate of investments in A_t is lower than that in k_t in the periods after the shock until recovering the steady state growth path.

Proof: (i) On the steady state path, $\frac{\partial y_t}{\partial k_t} = \frac{\alpha}{mv}\frac{\partial y_t}{\partial A_t}$. By proposition 2, in the period when the shock occurred, if $\alpha(1-w)\frac{y_t}{k_t} < 1$, k_t is below the necessary quantity to be on a steady state growth path while A_t is over the necessary quantity. Hence $\frac{\partial y_t}{\partial k_t} > \frac{\alpha}{mv}\frac{\partial y_t}{\partial A_t}$ until recovering the steady state growth path.

(ii) It is self-evident by (i) and proposition 1 and 2.

Q.E.D.

If investments in k_t are more profitable than those in A_t, firms will invest more in k_t and less in A_t compared with investments before the shock. As a result, the growth rates of investments in k_t and in A_t change oppositely and, R&D expenditure responds negatively after a positive shock on A_t.

5. Demand shocks

Secondly, the cyclical property of R&D expenditure in case of a shock that changes investments in k_t but is independent from shocks on A_t is examined. The focal point is whether the criterion that, after a shock of this type, the equation $\frac{A_t}{k_t}$ = a unique constant can be restored in the period when the shock occurred, is satisfied. This type of shocks can be interpreted as demand shocks, because most shocks that are independent from shocks on A_t seem to originate in the demand side such as changes of parameter values in utility function, monetary policy, fiscal policy etc. Here, when k_t increases by zk_t ($0 < z$) by a positive shock of this type, output y_t increases by $zk_t\frac{\partial y_t}{\partial k_t}$ due to the increase of k_t, and this increase of output $zk_t\frac{\partial y_t}{\partial k_t}$ is allocated to the increase of consumption and the increase of investments in k_t and in A_t. It is assumed, like the case of shocks on A_t, that $wzk_t\frac{\partial y_t}{\partial k_t}$ is allocated to the increase of consumption and $(1-w)zk_t\frac{\partial y_t}{\partial k_t}$ is allocated to the increase of investments in k_t and in A_t where $0 \le w \le 1$. As was examined in case of shocks on A_t, it is examined whether the increase of investments $(1-w)zk_t \frac{\partial y_t}{\partial k_t}$ is large enough to restore the unique ratio $\frac{A_t}{k_t} = \frac{\alpha}{mv(1-\alpha)}$ that is required when proceeding on the steady state growth path as was shown in proposition 1.

Proposition 3: After a positive shock of this type, i.e. a shock that increases investments in k_t but is independent from shocks on A_t, if and only if $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} \geq 1$, A_t can be more than the necessary quantity to hold the equation $\frac{A_t}{k_t}$ = a unique constant in the period when the shock occurred.

Proof: Let the shock makes k_t change by zk_t. In order to hold the equation $\frac{A_t}{k_t}$ = a unique constant, the increase of A_t initiated by this shock in the period when the shock occurred needs to be more than zA_t that can make the equation $\frac{A_t}{k_t}$ = a unique constant be held, and thus the condition $(1-w)zk_t \frac{\partial y_t}{\partial k_t} \geq zA_t$ must be satisfied in the period when the shock occurred in order to hold the equation $\frac{A_t}{k_t}$ = a unique constant, because, by assumption, investments that were planned before the shock are not changed in the period when the shock occurred. Here, $(1-w)zk_t \frac{\partial y_t}{\partial k_t} \geq zA_t \Leftrightarrow (1-\alpha)(1-w) \geq \left(\frac{A_t}{k_t}\right)^{1-\alpha} \Leftrightarrow (1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} \geq 1$. Hence, if and only if $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} \geq 1$, A_t can be more than the necessary quantity to hold the equation

$\frac{A_t}{k_t}$ = a unique constant in the period when the shock occurred.

Q.E.D.

Like the case of shocks on A_t, firms' investment activities will change significantly if A_t is less than the necessary quantity to hold the equation $\frac{A_t}{k_t}$ = a unique constant in the period when the shock occurred.

Corollary 2: After a positive shock of this type, i.e. a shock that increases investments in k_t but is independent from shocks on A_t, if $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} < 1$, (i) investments in k_t are less profitable than those in A_t in the periods after the shock until recovering the steady state growth path, i.e. $\frac{\partial y_t}{\partial k_t} < \frac{\alpha}{mv}\frac{\partial y_t}{\partial A_t}$, and (ii) the growth rate of investments in A_t is higher than that in k_t in the periods after the shock until recovering the steady state growth path.

Proof: (i) On the steady state path, $\frac{\partial y_t}{\partial k_t} = \frac{\alpha}{mv}\frac{\partial y_t}{\partial A_t}$. By proposition 3, in the period when the shock occurred, if $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} < 1$, A_t is below the necessary quantity to be on a steady state growth path while k_t is over the necessary quantity. Hence $\frac{\partial y_t}{\partial k_t} < \frac{\alpha}{mv}\frac{\partial y_t}{\partial A_t}$ until recovering the steady state growth path.

(ii) It is self-evident by (i) and proposition 1 and 2.

Q.E.D.

What should be stressed is that, contrary to the case of shocks on A_t, R&D expenditure responds positively by a positive shock of this type if the necessary quantity of A_t to hold the equation $\frac{A_t}{k_t}$ = a unique constant is not obtained in the period when the shock occurred. Corollary 1 and corollary 2 indicate opposite directions with regard to movements of R&D expenditure after shocks. That is, after a positive shock on A_t, the growth rate of investments in A_t decreases, but after a positive shock of this type, the growth rate of investments in A_t increases. Technology shocks and demand shocks therefore lead to completely different consequences with regard to cyclicality of R&D expenditure.

6. Calibration

What proposition 2 and 3 imply is that the cyclical property of R&D expenditure depends on values of α, w and $\frac{y_t}{k_t}$ because the conditions such that $\alpha(1-w)\frac{y_t}{k_t} \geq 1$ and $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} \geq 1$ are essential in order that the criterion that the equation $\frac{A_t}{k_t}$ = a unique constant can be restored in the period when the shock occurred is satisfied. Among α, w and $\frac{y_t}{k_t}$, the value of w seems difficult to estimate, but it is assumed for the time being that w is the ratio of consumption to output. Other possibilities of the value of w are considered later. The values of the share of labor input α, the ratio of consumption to output w and the ratio of output to capital $\frac{y_t}{k_t}$ appear to take roughly common values across times and economies, and here the following particular values are used, which are roughly same as those in the U.S.

The share of labor input α: 0.7

The ratio of consumption to output w: 0.6

The ratio of output to capital $\frac{y_t}{k_t}$: 0.4

First, the cyclical property in case of technology shocks is examined. By corollary 1, if the condition $\alpha(1-w)\frac{y_t}{k_t} \geq 1$ is not satisfied, investments in A_t shows a counter-cyclical property. However, this condition $\alpha(1-w)\frac{y_t}{k_t} \geq 1$ is hard to satisfy because, when α, w and $\frac{y_t}{k_t}$ take the above particular values, $\alpha(1-w)\frac{y_t}{k_t}$ = 0.112, which is far below unity that is required by the condition. The difference of a figure, i.e. 1 versus 0.112, will not be reconciled by minor adjustments of the values of α, w and $\frac{y_t}{k_t}$ or the functional form of production function. This result therefore will hold for a wide range of parameter values and functional forms and it is highly likely that the condition $\alpha(1-w)\frac{y_t}{k_t} \geq 1$ is not satisfied in most economies. Furthermore this large difference of a figure suggests that the adjustment period to restore the equation $\frac{A_t}{k_t}$ = a unique constant will persist for a long period of time.

The result that the condition $\alpha(1-w)\frac{y_t}{k_t} \geq 1$ is far from satisfied indicates that after a positive shock on A_t, investments in A_t basically respond negatively by corollary 1, and thus that if outputs fluctuate solely due to shocks on A_t, R&D expenditure (= investments in A_t) has basically a counter-cyclical nature.

Remark 2: Business cycles that are generated by technology shocks basically accompany counter-cyclical R&D expenditure.

Remark 2 is not a new finding but confirms the prediction of the Schumpeterian notion. Any Schumpeterian growth model has a counter-cyclical property because Schumpeterian growth models are based on substitutability between investments in k_t and in A_t and assume that business fluctuations are solely attributed to shocks on A_t. By a technology shock, a new opportunity is generated and it can be exploited by expanding production capacity. It appears rational for a firm to exploit this opportunity generated by the technology shock by increasing investments in k_t that exploit the new opportunity and suspending new R&D expenditure for a while.

Next, the cyclical property in case of demand shocks is examined. By corollary 2, if the condition $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} \geq 1$ is not satisfied, investments in A_t shows a pro-cyclical property. Like the condition $\alpha(1-w)\frac{y_t}{k_t} \geq 1$ in case of shocks on A_t, the condition $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} \geq 1$ is difficult to satisfy. The values of parameter set above result in $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} = 0.178$ which is far from unity the condition $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} \geq 1$ requires. Like shocks on A_t, minor adjustments of parameter values or functional forms therefore will not change this result and thus the result will hold for a wide range of parameter values and functional forms.

The result that the condition $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}}$ is far from satisfied indicates that after a positive shock of this type, investments in A_t basically respond positively by corollary 2, and thus that if outputs fluctuate solely due to this type of shocks, R&D expenditure (= investments in A_t) has basically a pro-cyclical nature.

Remark 3: Business cycles that are generated by demand shocks basically accompany pro-cyclical R&D expenditure.

Remark 3 is in sharp contrast to remark 2. The cyclical property of R&D expenditure is completely different according to types of shocks. Technology shocks basically generate counter-cyclical R&D expenditure but demand shocks basically generate pro-cyclical R&D expenditure.

III. DISCUSSION

1. Some explanations

Empirically R&D expenditure moves pro-cyclically. It is predicted theoretically that demand shocks are basically consistent with the observed pro-cyclical R&D expenditure but technology shocks are not. Hence, in case of technology shocks, the observed pro-cyclical R&D expenditure is a puzzle. This pro-cyclical R&D puzzle appears a big headache to complete a scenario of technology shock driven business fluctuations. Several possibilities to solve this pro-cyclical R&D puzzle are considered in this section. First, a possibility that investments in k_t and in A_t are not substitutable is considered. Since the results in the paper crucially depend on the substitutability as was shown in proposition 1 and remark 1, the picture will be completely different without the substitutability. If it is not possible to substitute investments in k_t for those in A_t, shocks on A_t may accompany pro-cyclical R&D expenditure because A_t need not be adjusted to hold a unique ratio of $\frac{A_t}{k_t}$. If A_t can shift independently from k_t, a positive/negative shock on A_t merely means that the ratio of $\frac{A_t}{k_t}$ shifts upwards/downwards and A_t is not affected through the channel of keeping this ratio. Barlevy (2004) explores this type of solution to the pro-cyclical R&D puzzle. However, the presumption of non-substitutability requires that the returns to investments in k_t and in A_t are usually different, which implies that agents act irrationally in some respects and do not exploit opportunities fully. Barlevy (2004) thus argues that entrepreneurs act short-sightedly and fail to respond optimally to aggregate shocks. Introducing irrationality, however, does not seem a compelling idea and may destroy the foundation of models. Hence, for the analysis of the cyclical property of R&D expenditure, models that deny substitutability between them seem erroneous, although these models may be used for other purposes.

Another possibility is that investments in k_t that are initiated to exploit opportunities generated by a technology shock necessitates additional R&D expenditure, i.e. investments in k_t and in A_t are complementary. A positive technology shock may induce additional R&D expenditure in firms that intend to enjoy uncompensated knowledge spillovers. However, the distribution of R&D intensity (the ratio of R&D expenditure to Sales) over firms is highly skewed and many firms invest in k_t with little R&D activity.[7] This fact implies that investments in k_t basically do not require additional R&D activities. Hence, it appears unlikely that there is a strong causal relationship from investing in k_t to investing in A_t.

Next, there is a possibility that in the statistics of R&D expenditure, a significant amount of expenditure that is irrelevant to the increase of A_t is accounted as R&D expenditure. If R&D expenditure excluding these ingredients has a counter-cyclical property, it may be argued that "true" R&D expenditure is counter-cyclical. Bental and Peled (1996) and Francois and Lloyd-Ellis (2003) argue that some R&D activities seem to move counter-cyclically. However, if this story is true, a significant amount of R&D expenditure must be irrelevant to the accumulation of A_t in order that R&D expenditure can be pro-cyclical. If a large part of R&D expenditure is irrelevant to the accumulation of A_t, for what purpose firms take such kind of R&D activities that contribute neither to the increase of knowledge/technology/idea nor the

7 See, e.g. Cohen, Levin and Mowery (1987).

accumulation of physical capital? This expenditure may be seen as simply wasting money. Do rational firms intentionally waste money? As a whole, it seems difficult to accept the argument *a priori* that a large part of the observed R&D expenditure is irrelevant to the accumulation of A_t.

Another possibility is that the parameter w is not properly calibrated, e.g. w may be near zero. In the calibration, w is set to be 0.6 as the ratio of consumption to output. However, it is not clear how much households consume out of the increase of output caused by a positive shock on A_t. Because a positive technology shock is basically a permanent shock, rational households increase their consumption after the shock but it is difficult to show analytically how much they increase their consumption. Hence, a possibility that w is near zero can not be denied *a priori*. However, if w is near zero, $\alpha(1-w)\frac{y_t}{k_t}$ = 0.28, which is still far below unity. Hence, even if w is near zero, the whole picture does not change.

Finally, there is a possibility that there are some frictions in markets that make R&D expenditure pro-cyclical. In the model in this paper, no friction is assumed, but if some kinds of frictions are introduced into the model, technology shocks may coexist with pro-cyclical R&D expenditure.[8] The most intensively studied friction with regard to R&D expenditure is the imperfection in financial markets, the effect of which is called "cash flow effects." It is argued that firms that attempt to invest in R&D face external cash flow constraints due to some kinds of imperfections in financial markets. Hall (1992), Himmelberg and Petersen (1994), Hall et al. (1998), Mulkay, Hall and Mairesse (2001), and Rafferty and Funk (2004) study this possibility and conclude that the cash flow and R&D expenditure in small firms are closely and positively related.[9] A weak point of the argument is that although cash flow constraints may be important

[8] Of course, demand shocks implicitly assume some kinds of frictions. However, the model in the paper assumes that those frictions exist outside the model and thus are exogenous to the model.

[9] Hall (2002) surveys the recent literature on cash flow effects.

for small firms, large firms may not face the constraint and thus in macro level, not in firm level, it is not clear how significant cash flow constraints are. Opportunity cost effects in large firms that do not seem to face cash flow constraints may overwhelm cash flow effects in small firms in macro level.

Even if cash flow effects are sufficiently large and important in macro level, it raises another problem for technology shocks. Many empirical researches conclude that cash flow constraints are commonly important for both physical investments and R&D investments.[10] In these researches, there is basically no significant difference between them. Rather it is reported in some researches that physical investments are more responsive to cash flow disturbances than R&D investments.[11] Hence, if the cash flow constraint is an essential factor, investments in physical capital may also be affected significantly by this constraint, which implies that business cycles as a whole are affected significantly by financial imperfections and that monetary disturbances are more important than technology disturbances in business cycles. The role financial frictions play for business fluctuations is particularly stressed in the credit view of the monetary transmission mechanism and has been the subject of a large literature, e.g. Bernanke and Gertler (1989).

To sum up, if a frictionless economy is assumed, no counter argument that the observed pro-cyclical R&D expenditure is consistent with technology shocks seems sufficiently persuasive.[12] The imperfection in financial markets seems to be a probable source of pro-cyclical R&D expenditure and may solve the pro-cyclical R&D puzzle, but it may in reverse cast doubt on importance of technology shocks in business cycles.

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[10] See e.g. Hall (1992), Himmelberg and Petersen (1994), Hall et al. (1998), and Mulkay, Hall and Mairesse. (2001).

[11] See e.g. Hall (1992) and Himmelberg and Petersen (1994).

[12] Another but very unlikely possibility is that firms' expectation of technology shocks is made by an adaptive manner. Hence, a positive technology shock will make firms' expectation of success probability of R&D higher. However, adaptive expectations do not appear compelling at all.

2. Technology shocks and pro-cyclical R&D

The easiest way to solve the pro-cyclical R&D puzzle is to abandon the conjecture that business cycles are mainly generated by technology shocks. As remark 3 shows, demand shocks are consistent with pro-cyclical R&D expenditure, and thus if business cycles are driven mainly by demand shocks, the pro-cyclical R&D puzzle does not exist. Among many criticisms to Real Business Cycle models, the criticism that Solow residuals consist of many other elements than technology shocks and true technology shocks are much smaller is regarded as the most formidable one Real Business Cycle models has been facing and the Achilles heal of the RBC literature.[13] Recently another problem that positive technology shocks appear to lead to decline in labor input is disputed.[14] In addition to these criticisms, the pro-cyclical R&D puzzle seems to be one of the problems that should be solved if business cycles are modeled to be driven mainly by technology shocks.

IV. CONCLUDING REMARKS

Empirical evidence suggests that R&D expenditure moves pro-cyclically. However, the observed pro-cyclical R&D expenditure is a puzzle from the Schumpeterian point of view. In the Schumpeterian growth notion, opportunity costs in recessions are so important that counter-cyclicality of R&D activities is a natural consequence. On the other hand, some economists argue that because it has been observed that R&D expenditure in a small firm is positively correlated with the cash flow of the firm, the pro-cyclical property emerges due to imperfections in financial markets that generate pro-cyclical cash flows in small firms. From their point of view, the pro-cyclical R&D expenditure is not a puzzle. Which view is correct? Is

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[13] See e.g. Burnside, Eichenbaum and Rebelo (1996) and King and Rebelo (1999).

[14] See e.g. Francis and Ramey (2002).

the pro-cyclical R&D expenditure really a puzzle? The cyclical property of R&D expenditure may depend on types of shocks. The paper examined how different the cyclical property of R&D expenditure is according to types of shocks, i.e. technology shocks and demand shocks, based on a common endogenous growth model.

The results of the paper are as follows:

(i) As has been stressed in the Schumpeterian literature, substitutability between investments in k_t and in A_t is a key that determines cyclical property of R&D expenditure. After any shock that changes k_t and/or A_t, k_t and A_t must be adjusted to return to a unique ratio of $\frac{A_t}{k_t}$. If the equation $\frac{A_t}{k_t}$ = a unique constant is far from restored in the period when the shock occurred and thus the adjustment process continues after the period, the nature $\frac{A_t}{k_t}$ = a unique constant will bind and alter the movements of both k_t and A_t significantly in the following period after the shock. In this sense, whether the equation $\frac{A_t}{k_t}$ = a unique constant can be restored in the period when the shock occurred or not is essential for the cyclical property of R&D expenditure.

(ii) Technology shocks basically accompany counter-cyclical R&D expenditure and demand shocks basically accompany pro-cyclical R&D expenditure. Because for a wide range of parameter values and functional forms the conditions $\alpha(1-w)\frac{y_t}{k_t} \geq 1$ in case of technology shock and $(1-\alpha)(1-w)\left(\frac{y_t}{k_t}\right)^{\frac{\alpha-1}{\alpha}} \geq 1$ in case of demand shock are not satisfied, after a positive shock investments in A_t basically respond negatively in case of technology shock and positively in case of demand shock. Hence, the cyclical property of R&D expenditure is completely different according to types of shocks.

(iii) If a frictionless economy is assumed, no counter argument that the observed pro-cyclical

R&D expenditure is consistent with technology shocks seems sufficiently persuasive. The imperfection in financial markets seems to be a probable source of pro-cyclical R&D expenditure and may solve the pro-cyclical R&D puzzle, but it may in reverse cast doubt on importance of technology shocks in business cycles. The easiest way to solve the pro-cyclical R&D puzzle is to abandon the conjecture that business cycles are generated mainly by technology shocks.

Appendix

1. Proof of lemma 1

(Step 1) By equation (3), $\frac{\dot{k}_t}{k_t} = \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n - \delta\right] - \frac{c_t}{k_t}$. On the other hand, by equation (2), $\frac{\dot{\lambda}_t}{\lambda_t} = -\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n - \delta\right]$.

Here, $\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{k}_t}{k_t} = -\left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n - \delta\right] + \left[\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n - \delta - \frac{c_t}{k_t}\right] = -\frac{c_t}{k_t}$. Thereby if $\frac{c_t}{k_t} > 0$, then $\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{k}_t}{k_t} < 0$. Hence, the transversality condition (4) $\lim_{t\to\infty} \lambda_t k_t = 0$ is not satisfied if and only if $\frac{c_t}{k_t} = 0$ (Because $c_t \geq 0$ and $k_t \geq 0$).

(Step 2) By equation (1), (2) and (3) $\frac{\dot{c}_t}{c_t} = \frac{\left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n - \delta - \theta}{\varepsilon}$ = constant, and by equation (3) $\frac{\dot{k}_t}{k_t} = \left(\frac{\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - n - \delta - \frac{c_t}{k_t}$. If $\frac{\dot{k}_t}{k_t} > \frac{\dot{c}_t}{c_t}$, then $\frac{c_t}{k_t}$ diminishes as time passes, then $\frac{\dot{k}_t}{k_t}$ increases. Hence, eventually $\frac{c_t}{k_t}$ diminishes to zero. Therefore, by (step 1), the transversality condition (4) is not satisfied. If $\frac{\dot{k}_t}{k_t} < \frac{\dot{c}_t}{c_t}$, then $\frac{c_t}{k_t}$ increases as time passes, then $\frac{\dot{k}_t}{k_t}$ diminishes and eventually becomes negative. Hence, k_t decreases and eventually becomes negative which violate the condition $k_t \geq 0$. However, if $\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t}$, then $\frac{c_t}{k_t}$ is constant and thus $\frac{\dot{k}_t}{k_t}$ and $\frac{\dot{c}_t}{c_t}$ continue to be constant and identical.

Q.E.D.

2. Proof of lemma 2

(Step 1) Because $\dot{y}_t = \left(\frac{A_t}{k_t}\right)^{\alpha}\left[(1-\alpha)\dot{k}_t + \alpha\frac{k_t}{A_t}\dot{A}_t\right]$ and $\dot{A}_t = \frac{\alpha}{mv}\dot{k}_t\left(1-\frac{f f''}{f'^2}\right) = \frac{\alpha}{mv(1-\alpha)}\dot{k}_t$,

$\dot{y}_t = \dot{k}_t\left(\frac{A_t}{k_t}\right)^{\alpha}\left[(1-\alpha)+\frac{\alpha^2}{mv(1-\alpha)}\frac{k_t}{A_t}\right]$, and thus $\frac{\dot{y}_t}{y_t} = \frac{\dot{k}_t}{k_t}\left[(1-\alpha)+\frac{\alpha^2}{mv(1-\alpha)}\frac{k_t}{A_t}\right]$. Because

$A_t = \frac{\alpha}{mv(1-\alpha)}k_t$, $\frac{\dot{y}_t}{y_t} = \frac{\dot{k}_t}{k_t}[(1-\alpha)+\alpha] = \frac{\dot{k}_t}{k_t}$. Hence $\frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t}$ = constant.

(Step 2) Because $\dot{y}_t = \left(\frac{A_t}{k_t}\right)^{\alpha}\left[(1-\alpha)\dot{k}_t + \alpha\frac{k_t}{A_t}\dot{A}_t\right]$ and $\dot{A}_t = \frac{\alpha}{mv(1-\alpha)}\dot{k}_t$, $\dot{y}_t = \dot{A}_t\left(\frac{A_t}{k_t}\right)^{\alpha}\left[\frac{mv(1-\alpha)^2}{\alpha}+\alpha\frac{k_t}{A_t}\right]$,

and thus $\frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{k_t}\frac{mv(1-\alpha)^2}{\alpha}+\alpha\frac{\dot{A}_t}{A_t}$. Because $A_t = \frac{\alpha}{mv(1-\alpha)}k_t$, $\frac{\dot{y}_t}{y_t} = (1-\alpha)\frac{\dot{k}_t}{k_t}+\alpha\frac{\dot{A}_t}{A_t}$. Hence,

$\frac{\dot{y}_t}{y_t} = \frac{\dot{k}_t}{k_t} = (1-\alpha)\frac{\dot{k}_t}{k_t}+\alpha\frac{\dot{A}_t}{A_t}$ and thus $\frac{\dot{k}_t}{k_t} = \frac{\dot{A}_t}{A_t}$. Therefore $\frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t}$ =constant.

Q.E.D.

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