The Sixth of Working papers for Researches about Development Power (DP)
The Completed Time: 2005-05-25

The Logarithm Model of Development Power: A Tool to Analyze the Motivity of Economic Growth

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ABSTRACT: After the discussions to exponential and power model [F. Dai, 2005], this paper points out there are three kinds of basic modes in the macroeconomic process, i.e., the exponential, power and logarithm mode, and discusses the logarithm model of Development Power (DP). By the analysis on logarithm model of DP, we will see the reasons, of which there are anomaly cycles in economic process, are just the alternate motion of DP accumulating and releasing. And that is also the reasons that there are the economic fluctuations in production markets. The logarithm model of DP can also describe the different characters of DP motion at the different stage, and indicates in analytic way that the diffusion of DP and the diversifications of economic productivity also might occur after an economic recession. The empirical researches are done about the conclusions in this paper, and the results express that the logarithm model is better than the power model and exponential model of DP in many cases. These three models of DP can not be replaced one another.

KEYWORDS: Development Power (DP), Partial Distribution, logarithm model, macroeconomic analysis

1 Introduction

There are many outstanding macroeconomic theories, such as the business cycle theory [R. E. Lucas, 1981], the real business cycle theory [E. Prescott, 1982], new growth theory [P. M. Romer, 1986] and so on. differing from the basic thoughts of those theories in economy, the theory of Development Power (DP) is put forward by Feng Dai (2004), and the exponential and power model of DP have been discussed [Feng Dai, 2005]. DP theory thinks

- The productivity is the visible behavior ability for mankind to develop economy, and the DP is the invisible behavior motivity for mankind to develop economy. DP is also the developing force for economic growth continuously.
- If we regard the productivity as the hardware of economic development, then DP is the software of economic development. Both of them are a pair of most basic factors in economic development. The motion of DP will influence the development of productivity, and vice verse.
- The accumulation and release of DP will bring on the fluctuations of economic production and market. So DP is the motive force of economic development.
- The reason that there are huge differences between the developed nations and backward regions in economic growth rate and the level of economic development is the difference in the abilities to accumulate and release DP.
- DP can be scaled by economic development energy, and measured by fluctuation rate of level of economic production (i.e. productivity). So we can say that the DP is the basic energy of economic growth.

The following conclusions are gained by use of the exponential model and power model of DP:

There are three kinds of states of energy in economic process, i.e., normal state, strong state and super state in economic energy. The economic development is generally in the normal state of economic energy, so the normal state is an ordinary state of economy. When DP is accumulated to a higher level, economy might come into the strong state in economic energy. At the same time, all of DP in policy, technology, management, education, idea and information, etc, step up, and the economic production throbs. Later, the important transform related to economy might occur; the original mode, environment and system of economic development will not adapt to the future economy, an epochal economic process will start and a new economic environment will come into being, then economy might come into the super state in developing energy. Under the super state of economic energy, we can not evaluate DP in a finite way, the new structure economy has a boundless vitality, DP (economic energy) would diffuse sufficiently and the industrial development, merchandises production and market services would present the diversification. Of course, the time when economy is in the super state is generally brief. The critical times of strong state, important transform and super state can be computed in two ways.

Speaking in comparison, exponential model can describe the economic process which DP changes in a quicker rate, and power model can describe the economic process in which DP changes in a mezzo rate. But, we need a model which can describe the economic process in which DP changes in a slower rate. The logarithm model, which will be discussed in this paper, is just the model.

Based on the DP theory and the works in references [18-21], this paper will discuss the logarithm model of relation between DP and productivity. By means of discussion to logarithm model, we will know further that the continuous alternations between DP accumulating and releasing are the essencial reasons that the irregular cycle in economic process and the fluctuations in economic production occur. in the defferent stages of economic development, like the economic growth with DP accumulating or releasing or the economic recession with DP accumulating or releasing, the DP motions present the different characters, and influence the economic development in different ways. The logarithm model will also indicate that a large scale diffussion in DP and the diversification of economic growth might occur after a economic recession.

We shall make the empirical analysis on the results in this paper by means of the US GDP data from 1940 to 2004. The empirical results will explain that the effect of logarithm model fitting really the economic process is better than exponential model and logarithm model under many circumstances, and not better in some other circumstances. So these three models can be reinforced and not replaced one another.

2 The Basic Model of DP

2.1 Notations and descriptions of models

If note:

$\mu(t)$: the basic economic level at the time t, namely a measuring index for basic productivity, the basic level for short, $\mu(t)\geq 0$.

$\sigma(t)$: the fluctuation range of the basic economic level at the time t, i.e., the standard variance of the basic economic level, $\sigma(t)>0$. $\sigma(t)$ can describe the absolute energy of economic development at the time t.

$v(t)=\sigma(t)/\mu(t)$: the fluctuation rate of the basic economic level at the time t, is a measuring index of DP (Development Power), $\mu(t)\geq 0$. $v(t)$ can describe the developing energy of economy, generally $0<v(t)<1$.

$X(t)$: the real economic level at the time t, real level for short. $X(t)$ is a non-negative random variable for any $t\geq 0$.

According to assumptions in references [17]-[20], the real level $X(t)$, for any $t\in[0, \infty)$, follows the Partial Distribution[21]-[23] as

$$f(x)=\begin{cases} e^{-\frac{[x-\mu(t)]^2}{2[v(t)]^2}} \Big/ \int_0^\infty e^{-\frac{[x-\mu(t)]^2}{2[v(t)]^2}}dx & x\ge 0 \\ 0 & x<0 \end{cases}$$

At this time, we denote: $X(t)\in P(\mu(t), v^2(t))$. $X(t)$ is actually a Partial Process[24].

Because the economic growth is essentially the increase for level of productivity, and the economic recession is essentially the decline for level of productivity, we do not distinguish the economic level from productivity in concept in the following discussion, i.e. regard them as the same. Also we do not distinguish DP from the developing energy of economy. And the $\mu(t)$, $v(t)$ and $X(t)$ can be separately noted μ, and X for short.

$\mu(t)$ and $v(t)$ may be not continuous about the time t, namely when $0\le t_0<t_1<\ldots< t_{n+1}<\ldots<\infty$, we have

$$\mu(t)=\begin{cases} \mu_0 & t_0\le t<t_1 \\ & \vdots \\ \mu_n & t_n\le t<t_{n+1} \\ & \vdots \end{cases}$$, where , $\mu_i\ge 0$, $i=1, 2, \cdots$

Also, we have

$$\sigma(t)=\begin{cases} \sigma_0 & t_0\le t<t_1 \\ & \vdots \\ \sigma_n & t_n\le t<t_{n+1} \\ & \vdots \end{cases}$$, where, $\sigma_i>0$, $i=1, 2, \cdots$

thus, $X(t_i)\in P(\mu(t_i), [v(t_i)]^2)$ is a dispersed process, where $\mu(t_{i+1})=X(t_i)$,$t=t_0, t_1, \cdots$.

2.2 The basic model of DP

If we suppose that there is a function $V=G(x_1,x_2,x_3,x_4)$ which is continuous and differentiable about x_1, x_2, x_3, x_4, and from references [20-21], thus we have

$$dv=\left(\frac{\partial G}{\partial x_1}v+\frac{\partial G}{\partial x_2}\frac{1}{\mu}+\frac{\partial G}{\partial x_3}\frac{v}{\mu}\right)d\mu+\frac{\partial G}{\partial x_4}dt \quad \left(\frac{\partial G}{\partial x_j}=\frac{\partial G(0,0,0,0)}{\partial x_j}, j=1,2,3,4\right)$$

If we suppose that the DP has something to do with the time in implicit way, and let $\frac{\partial G(0,0,0,0)}{\partial x_4}=0$, then we have

$$\frac{dv}{d\mu}=av+b\frac{1}{\mu}+c\frac{v}{\mu} \tag{1}$$

where, $a=\frac{\partial G(0,0,0,0)}{\partial x_1}$, $b=\frac{\partial G(0,0,0,0)}{\partial x_2}$, $c=\frac{\partial G(0,0,0,0)}{\partial x_3}$.

Solving the expression (1) and uniting it with $X(t)\in P(\mu(t), [v(t)]^2)$, we have the general model of relation between DP and economic level as follows:

$$\begin{cases} v(t)=v_0\left(\frac{\mu(t)}{\mu_0}\right)^c e^{a(\mu(t)-\mu_0)}+b(\mu(t))^c e^{a\mu(t)}\int_{\mu_0}^{\mu(t)}\frac{e^{-au}}{u^{c+1}}du \\ X(t)\in P(\mu(t), v^2(t)), \quad t\in[0,\infty) \\ \mu_0=\mu(0), v_0=v(0), \end{cases} \tag{2}$$

where, μ_0 and v_0 are separately the original values of $\mu(t)$ and $v(t)$.

Specially, if $\mu(t)>0$, from (2)

1) If $a=\gamma$, $b=c=0$, we have the exponential model of relation between DP and economic level as

$$v(t)=v_0 e^{\gamma[\mu(t)-\mu_0]} \tag{3}$$

2) If $b=\gamma$, $a=c=0$, we have the logarithm model of relation between DP and economic level as

$$v(t)=v_0+\gamma\ln\left(\frac{\mu(t)}{\mu_0}\right) \tag{4}$$

3) If $c=\gamma$, $a=b=0$, we have the power model of relation between DP and economic level as

$$v=v_0\left(\frac{\mu(t)}{\mu_0}\right)^{\gamma} \tag{5}$$

where, the constant $\gamma(\neq 0)$ is characteristic exponent of DP, DP exponent for short. DP exponent can reflect the character of DP movement. The results above indicate that there are three kinds of basic modes of relation between DP and productivity in the macroeconomic process, i.e., exponential mode, logarithm mode and logarithm mode, and the general economic process could be described generally in way of combination of the these three modes.

The exponential model (3) and logarithm model (5) and their applications have been discussed in references [20-21]. We will discuss the logarithm model (4) as follow.

According to model (4), we have $\mu(t)=\mu_0 e^{\frac{v(t)-v_0}{\gamma}}$. This equation expresses how DP, as the economic developing energy $v(t)$, influences productivity $\mu(t)$. So we could think that DP is the engine for economic growth.

The DP exponent γ in expression (4), (5) and (6) have the following meanings:

1) In the process of economic growth, i.e. the productivity $\mu(t)$ is increasing by degree,

$$\gamma=\begin{cases} >0, \text{ DP is accumulating} \\ =0, \text{ DP is stable} \\ <0, \text{ DP is releasing} \end{cases}$$

2) In the process of economic recession, i.e. the productivity $\mu(t)$ is of descending

$$\gamma=\begin{cases} >0, \text{ DP is releasing} \\ =0, \text{ DP is stable} \\ <0, \text{ DP is accumulating} \end{cases}$$

If DP has the different characters in the different periods of time $[t_i, t_{i+1})$ (i=0, 1, ···), the DP exponent γ can be a sectioned function as follows

$$\gamma(t)=\begin{cases} \gamma_0 & t_0\le t<t_1 \\ & \vdots \\ \gamma_n & t_n\le t<t_{n+1} \end{cases}$$

where, γ_i is DP exponent, i=0,1,2, ···.

3 The Logarithm Model of DP

3.1 The basic logarithm model of DP

The equation (4) can be expressed as

$$v(t)=\frac{1}{\varphi^2}\left(\pm\varphi\sqrt{v_0+\gamma\ln\left(\frac{\mu(t)}{\mu_0}\right)}\right)^2 \tag{6}$$

where, "+" is corresponding to economic growth, "-" is corresponding to economic recession, scaling parameter φ(>0) is a constant and waiting for determining.

Expression (4) can be regard as a kinetic energy model according to physics, we could let

$$\frac{d\mu(t)}{dt}=\pm\varphi\sqrt{v_0+\gamma\ln\left(\frac{\mu(t)}{\mu_0}\right)} \tag{7}$$

If $\gamma\neq 0$, and according to (4), we have $\mu(t)=\mu_0 e^{\frac{v(t)-v_0}{\gamma}}$. Then $\frac{d\mu(t)}{dt}=\frac{\mu_0}{\gamma}e^{\frac{v(t)-v_0}{\gamma}}\frac{dv(t)}{dt}$, bring it into (7) and by use of (4), we obtain

$$\frac{\mu_0}{\gamma}e^{\frac{v(t)-v_0}{\gamma}}\frac{dv(t)}{dt}=\pm\varphi\sqrt{v(t)}$$

Note: $\sqrt{v(t)}=x$ and $c=\frac{\gamma\varphi}{2\mu_0}e^{\frac{v_0}{\gamma}}$, have $e^{\frac{x^2}{\gamma}}dx=\pm cdt$, namely $\pm ct=\int_{\sqrt{v_0}}^{\sqrt{v(t)}}e^{\frac{x^2}{\gamma}}dx$. When $v(t)$ increases by degree, we get $v(t)>v_0$, then $ct=\int_{\sqrt{v_0}}^{\sqrt{v(t)}}e^{\frac{x^2}{\gamma}}dx$; and when $v(t)$ is descending, $v(t)<v_0$ is got, then $-ct=\int_{\sqrt{v_0}}^{\sqrt{v(t)}}e^{\frac{x^2}{\gamma}}dx$.

According to the proof method in reference [22], we have

$$\int_{\sqrt{v_0}}^{\sqrt{v(t)}}e^{\frac{x^2}{\gamma}}dx=\int_{0}^{\sqrt{v(t)}}e^{\frac{x^2}{\gamma}}dx-\int_{0}^{\sqrt{v_0}}e^{\frac{x^2}{\gamma}}dx=\begin{cases}\sqrt{\frac{\gamma\pi}{2}\left(e^{\frac{4}{\pi}\frac{v(t)}{\gamma}}-1\right)}-\sqrt{\frac{\gamma\pi}{2}\left(e^{\frac{4}{\pi}\frac{v_0}{\gamma}}-1\right)} & \gamma>0\\ \sqrt{\frac{|\gamma|\pi}{2}\left(1-e^{\frac{4}{\pi}\frac{v(t)}{\gamma}}\right)}-\sqrt{\frac{|\gamma|\pi}{2}\left(1-e^{\frac{4}{\pi}\frac{v_0}{\gamma}}\right)} & \gamma<0\end{cases}$$

Go a step further

$$v(t)=\begin{cases}\frac{\pi\gamma}{4}\ln\left[1+\left(\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}+\mathrm{sgn}(v(t)-v_0)\sqrt{\frac{\gamma}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right] & \gamma>0\\ \frac{\pi\gamma}{4}\ln\left[1-\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}-\mathrm{sgn}(v(t)-v_0)\sqrt{\frac{|\gamma|}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right] & \gamma<0\end{cases} \tag{8}$$

where, $\mathrm{sgn}(v(t)-v_0)=\begin{cases}1 & v(t)>v_0\\ 0 & v(t)=v_0\\ -1 & v(t)<v_0\end{cases}$.

3.2 The time-variant models of productivity and DP in economic growth

From (4) and (8), we have, based on logarithm law, the following time-variant models for economic growth process:

3.2.1 The models of productivity and DP in economic growth with DP accumulation. At this time, $v(t)$ increases by degree and $\gamma>0$, DP model is as

$$v(t)=\frac{\pi\gamma}{4}\ln\left[1+\left(\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}+\sqrt{\frac{\gamma}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right] \tag{9}$$

Uniting (9) to (4), we have the productivity as follows

$$\mu(t)=\mu_0 e^{-\frac{v_0}{\gamma}}\left[1+\left(\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}+\sqrt{\frac{\gamma}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right]^{\frac{\pi}{4}} \tag{10}$$

where, $0\le t<\infty$.

3.2.2 The models of productivity and DP in economic growth with DP release. At this time, $v(t)$ increases by degree and $\gamma<0$, DP model is as

$$v(t)=\frac{\pi\gamma}{4}\ln\left[1-\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}-\sqrt{\frac{|\gamma|}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right] \tag{11}$$

Uniting (11) to (4), we have the productivity as follows

$$\mu(t)=\mu_0 e^{-\frac{v_0}{\gamma}}\left[1-\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}-\sqrt{\frac{|\gamma|}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right]^{\frac{\pi}{4}} \tag{12}$$

where, $0<t\le\sqrt{\frac{2\pi}{|\gamma|}}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}\left(1+\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}\right)$.

3.3 The time-variant models of productivity and DP in economic recession

In similar, from (4) and (8), we have, based on logarithm law, the following time-variant models for economic recession process:

3.3.1 The models of productivity and DP in economic recession with DP accumulation. At this time, $v(t)$ is descending and $\gamma<0$, DP model is as

$$v(t)=\frac{\pi\gamma}{4}\ln\left[1-\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}+\sqrt{\frac{|\gamma|}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right] \tag{13}$$

Uniting (13) to (4), the productivity $\mu(t)$can be expressed as

$$\mu(t)=\mu_0 e^{-\frac{v_0}{\gamma}}\left[1-\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}+\sqrt{\frac{|\gamma|}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right]^{\frac{\pi}{4}} \tag{14}$$

where, $0<t\le\sqrt{\frac{2\pi}{|\gamma|}}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}\left(1-\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}\right)$.

3.3.2 The models of productivity and DP in economic recession with DP release. At this time, $v(t)$ is descending and $\gamma>0$, DP model is as

$$v(t)=\frac{\pi\gamma}{4}\ln\left[1+\left(\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}-\sqrt{\frac{\gamma}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right] \tag{15}$$

Uniting (15) to (4), the productivity $\mu(t)$can be expressed as

$$\mu(t)=\mu_0 e^{-\frac{v_0}{\gamma}}\left[1+\left(\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}-\sqrt{\frac{\gamma}{2\pi}}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}t\right)^2\right]^{\frac{\pi}{4}} \tag{16}$$

where, $0\le t<\infty$.

3.4 The computing method of scaling parameter *φ*

For the economic growth process with DP accumulation, according to (10), we have

$$\varphi(t)=\sqrt{\frac{2\pi}{\gamma}}\frac{\mu_0}{t}e^{-\frac{v_0}{\gamma}}\left(\sqrt{\left(\frac{\mu(t)}{\mu_0}\right)^{\frac{4}{\pi}}e^{\frac{4v_0}{\pi\gamma}}-1}-\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}\right)\quad (t>0) \tag{17}$$

In expression (17), t could be any fixed value. Specially, let $t=1$, have

$$\varphi=\sqrt{\frac{2\pi}{\gamma}}\mu_0 e^{-\frac{v_0}{\gamma}}\left(\sqrt{\left(\frac{\mu_1}{\mu_0}\right)^{\frac{4}{\pi}}e^{\frac{4v_0}{\pi\gamma}}-1}-\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}\right)\quad (\mu_1=\mu(1)) \tag{18}$$

Similarly, for the economic growth process with DP release, according to (12), we have

$$\varphi=\sqrt{\frac{2\pi}{|\gamma|}}\mu_0 e^{-\frac{v_0}{\gamma}}\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}-\sqrt{1-\left(\frac{\mu_1}{\mu_0}\right)^{\frac{4}{\pi}}e^{\frac{4v_0}{\pi\gamma}}}\right) \tag{19}$$

For the economic growth recession process with DP accumulation, according to (14), we have

$$\varphi=\sqrt{\frac{2\pi}{|\gamma|}}\mu_0 e^{-\frac{v_0}{\gamma}}\left(\sqrt{1-\left(\frac{\mu_1}{\mu_0}\right)^{\frac{4}{\pi}}e^{\frac{4v_0}{\pi\gamma}}}-\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}\right) \tag{20}$$

And for the economic growth recession process with DP accumulation, according to (16), we have

$$\varphi=\sqrt{\frac{2\pi}{\gamma}}\mu_0 e^{-\frac{v_0}{\gamma}}\left(\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}-\sqrt{\left(\frac{\mu_1}{\mu_0}\right)^{\frac{4}{\pi}}e^{\frac{4v_0}{\pi\gamma}}-1}\right) \tag{21}$$

4 The Analysis for DP's Motion Characters

4.1 The analysis for DP's motion characters in economic growth process

Definition 1 (the energy state in economic development). If the t_τ and t_ε are real numbers, $t_\varepsilon>0$ and $t_\tau>t_\varepsilon$, then

1) When the time $t\in(0, t_\varepsilon)$, and if DP $v(t)<1$, we call the economic state the normal state in economic energy, and the normal state for short.

2) When the time $t\in[t_\varepsilon, t_\tau)$, and if DP $v(t)\ge 1$, we call the state of economy the strong state in economic energy, and the strong state for short. t_ε is the critical time of strong state.

3) If the time $t=t_\tau$, and $v(t)=\infty$, we call the state of economy the super state in economic energy, and the super state for short. t_τ is the critical time of super state.

In the process of economic growth with DP accumulating, v>1 means the DP may start to diffuse, so t_ε, the critical time of strong state, is also the critical time of DP diffusing; and v>μ indicates that the current productivity is difficult to satisfy the need to economic growth, at the same time, economic production is like to present an important transform, namely the super state in energy is like to occur. In this case, the time $t = T$, determined by $v(T)=\mu(T)$, is the critical time at which an important transform might occur in economic production. The important transform here means the great transform in economic policy or system or great innovation for economic technology.

Definition 2 (the critical time of important transform and important disaster in economy). In the process of economic DP accumulating, if economy is growth and the equation $v(T)=\mu(T)>0$ is correct, than the T is called the critical time for important economic transform; if economy is recession and the equation $v(T)=\mu(T)>0$ is correct, than the T is called the critical time for important economic disaster.

In general, the macro-economy presents a moral state. If DP becomes larger and larger, the strong state might occur. The super state might occur if both strong state and important transform occurred.

From (9), we have

Proposition 1 In the process of economic growth with DP accumulating, i.e. $\mu(t)$ increases by degree and r>0, then

1) The strong state may occur, and its critical time is

$$t_\varepsilon = \sqrt{\frac{2\pi}{\gamma}\frac{\mu_0}{\varphi}}\, e^{-\frac{v_0}{\gamma}}\left(\sqrt{e^{\frac{4}{\pi\gamma}}-1}-\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}\right) \tag{22}$$

2) The important economic transform may occur, and its critical time T is determined by the group of equations as follow

$$\begin{cases} v(T) = \mu_0 e^{\frac{v(T)-v_0}{\gamma}} \\ v(T) = \dfrac{\pi\gamma}{4}\ln\left[1+\left(\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}+\sqrt{\dfrac{\gamma}{2\pi}\dfrac{\varphi}{\mu_0}}\, e^{\frac{v_0}{\gamma}}T\right)^2\right] \end{cases} \tag{23}$$

3) The super state never occurs in a finite time, i.e. its critical time is $t_\tau=\infty$.

Proposition 1 indicates that economic development in logarithm law is a kind of mild economic process; the important transform is commonly in a small scale if it occur in economy corresponding to logarithm law. So the transform could not bring in super state in a finite time.

For process of economic growth with DP accumulating and from (9) and (10), let $v(T)= \mu(T)$. If note:

$$S=1+\left(\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}+\sqrt{\frac{\gamma}{2\pi}\frac{\varphi}{\mu_0}}\, e^{\frac{v_0}{\gamma}}T\right)^2, \text{ have } S=\left(\frac{\pi\gamma}{4\mu_0}e^{\frac{v_0}{\gamma}}\ln(S)\right)^{\frac{4}{\pi}}.$$

Thus, the critical time of important economic transform, T, can be gained by the following algorithm 1.

Algorithm 1 μ_0, v_0, φ, γ, S_0 (>1) are given. i=0, and for any δ>0,

1) Compute $S_{i+1}=\left(\dfrac{\pi\gamma}{4\mu_0}e^{\frac{v_0}{\gamma}}\ln(S_i)\right)^{\frac{4}{\pi}}$.

2) Evaluate $R=S_{i+1}- S_i$.

3) If $|R|<\delta$, go to 6); if $|R|>\delta$, go to 4).

4) If $R>\delta$, let $S_{i+1}= S_{i+1}-\delta|R|$; if $R<-\delta$, let $S_{i+1}= S_{i+1}+\delta|R|$.

5) $i+1\rightarrow i$, go to 1).

6) $T=\sqrt{\frac{2\pi}{\gamma}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}}\left(\sqrt{S-1}-\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}\right)$, finish.

For process of economic growth with DP releasing and from (9), we have the proposition 2 as follows.

Proposition 2 In the process of economic growth with DP releasing, i.e., $\mu(t)$ increases by degree and γ<0,

1) Strong state and super state will never occur.

2) If the continuous time of economic growth is larger than $t=\sqrt{\frac{2\pi}{|\gamma|}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}}\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}$, the economic recession with DP accumulating might occur.

***Proof*:**

1) If γ<0 and $\mu(t)$ increases, as we know From (4), $v(t)$ is a descending function about $\mu(t)$. So, the strong state and super state will never occur.

2) From expression (11), $v(t)$ reaches its minimum ($v(t)$=0) when $t=\sqrt{\frac{2\pi}{|\gamma|}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}}\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}$. Afterward, $v(t)$ will increases by degree. From expression (12), $\mu(t)$ reaches its maximum ($\mu(t)=\mu_0 e^{-\frac{v_0}{\gamma}}$) when $t=\sqrt{\frac{2\pi}{|\gamma|}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}}\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}$, and $\mu(t)$ will be of descending afterward.

Proposition 2 tells us, in the economic growth, DP need to be accumulated after it released. at the same time, the process of economic growth with DP releasing is finished, and the process of economic recession with DP accumulating may start.

4.2 The analysis for DP's motion characters in economic recession process

Here, suppose the economic process is in a recession, i.e., productivity $\mu(t)$ is descending. Then we have two corresponding propositions.

Proposition 3 In the process of economic recession with DP accumulating, i.e., $\mu(t)$ is descending and r<0. when the continuous time of recession exceeds over the

$$t=\sqrt{\frac{2\pi}{|\gamma|}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}}\left(1-\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}\right) \tag{24}$$

then a large scale diffussion in DP and the diversification of economic growth might occur after the economic recession.

***Proof*:** according to (13) and (14), and when $t>\sqrt{\frac{2\pi}{|\gamma|}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}}\left(1-\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}\right)$, we have separately

$$v(t)=\frac{\pi\gamma}{4}\ln\left|\left|1-\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}+\sqrt{\frac{|\gamma|}{2\pi}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}}\,t\right)^2\right|\right|+\frac{\pi\gamma}{2}\ln i\quad \left(t<\sqrt{\frac{2\pi}{|\gamma|}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}}\left(\sqrt{2}-\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}\right)\right)$$

and

$$\mu(t)=\mu_0 e^{-\frac{v_0}{\gamma}}\left[\left|1-\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}+\sqrt{\frac{|\gamma|}{2\pi}\frac{\varphi}{\mu_0}e^{\frac{v_0}{\gamma}}}\,t\right)^2\right|\right]^{\frac{\pi}{4}}\cdot(-1)^{\frac{\pi}{4}}.$$

From theory of complex function, we know $\ln i=(\frac{1}{2}+k)\pi i$ (k=0,1,…) and $(-1)^{\frac{\pi}{4}}=e^{\frac{(2k+1)\pi^2}{4}i}$ = $\cos(\frac{(2k+1)\pi^2}{4})+i\sin(\frac{(2k+1)\pi^2}{4})$ (k=0, ± 1, ± 2,⋯). Where, $i=\sqrt{-1}$ is the imaginary unit. And at the same time, the DP $v(t)$ and productivity $\mu(t)$ are multi-valued, and these mean diffussion in DP and the diversification of economic production. In addition, $v(t)$ is descending and $\mu(t)$ increases by degree on any single-valued branch, these mean DP is releasing and economy is growing.

Proposition 3 indicates that

1) After a period of economic recession with DP accumulating, DP needs to have a release. And this release may invert economic recession to economic growth. And the inverted time could be decided by formula (24).

2) DP $v(t)$ and productivity $\mu(t)$ are multi-valued means that DP releasing and economic growth may exceed the original ways, and come into their new ways.

Proposition 4 In the process of economic recession with DP releasing, i.e. $\mu(t)$ is descending and $\gamma>0$. When the continuous time of recession exceeds over the

$$t=\sqrt{\frac{2\pi}{\gamma}}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}\left(\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}-1\right) \tag{25}$$

then the process of economic growth with DP accumulating might come.

Because the t in expression (25) makes the formula (15) minimum, i.e., $v(t)$=0, the results in proposition 4 can be easily obtained. Proposition 4 indicates the economy may grow when economic recession go to a certain degree. So it is important for us to know, by expression (25), the optimal starting time of DP to accumulate in the economic recession.

Putting the discussions above together, we have got some conclusions on logarithm law as follow:

1) The process of economic growth with DP accumulation is a fine mode, it enable economy to develop continuously. In this kind of process, strong state in economic energy may occur, after that, the important economic transform may occur. Afterward, the process of economic growth with DP release will present when the DP accumulates to a higher degree because DP accumulated must be released.

2) The process of economic growth with DP release is a process which makes economy growing by use of economic energy. The economic growth can not be kept on for a long term in this kind of economic process. There are two possible results at the end of this process, one is economic recession with DP accumulating, and one is another process of economic growth with DP accumulating. So, if at the time like this, we need perhaps to take some measures to accumulate DP, the economic energy, and enter into a process of economic growth with DP accumulation.

3) There may be a large scale diffussion in DP and the diversification of economic growth after the economic recession with DP accumulating. Then, economic growth with DP release or economic recession with DP release may occur.

4) The process of economic growth with DP accumulating may start at the end of the economic recession with DP releasing.

Putting the four aspects above together, we have a completed cycle of economic development. Both of the rates and intensity of DP accumulating or releasing are distinguished in the different process of economy, so that the continuous times of each economic process are different. All of these result in the anomaly cycle in economic process.

5 The Empirical Researches for DP's Logarithm Model

Here we take US GDP (chained) price index [26] (Fiscal Year 2000 = 1.000, the GDP index for short) in the period of 1940-2004 as the scale to evaluate the productivity and the empirical samples. Though US economy, productivity $\mu(t)$, has been growing from end of World War II to 2004, DP fluctuates always. There are many different characteristics of DP in this period. We have the notations and expressions as follow:

$\mu(t)$: The basic level of economic productivity of the year t, t=1940, 1941, ···, 2004, measured by the GDP index.

$v(t)$: The scale value of DP of the year t, measured by the fluctuating ratio of the productivity level. $v(t)=|\mu(t)-\mu(t-1)|/\mu(t)$, t=1941, 1942, ···, 2004.

$X(t)$: The actual level of productivity of the year t, a non-negative stochastic variable. $X(t)\in P(\mu(t),[v(t)]^2)$.

The interval of time unit for sampling data GDP is a year, the stability of data is higher, and the difference between GDP price index of one year and that of last year can nicely describe the economic fluctuation, so we adopt the formulas of $v(t)$ mentioned above. The curves of DP and productivity in US economy are shown in figure 1. In figure 1, the proportion of the real indexes of $\mu(t)$ to indexes drawn is 1:10.

We could see, from figure 1, the economic energy (i.e. DP) would go down if it is higher in a certain degree, namely DP will release if it is accumulated to a certain degree. Also, the DP will accumulate if it is released to a certain degree.

In the following, we will give the analysis about DP motion separately on the data from 1989 ~1998 (an economic growth process with DP releasing), 1998~2001 (an economic growth process with DP accumulating), and the data of local top points after 1980 (1981, 1989, 1991, 2001, 2004). The empirical analysis include estimating the parameters, comparing the actual fitting effects of exponential model [20], power model [21] and logarithm model, and forecasting the DP and economic level of US in future years.

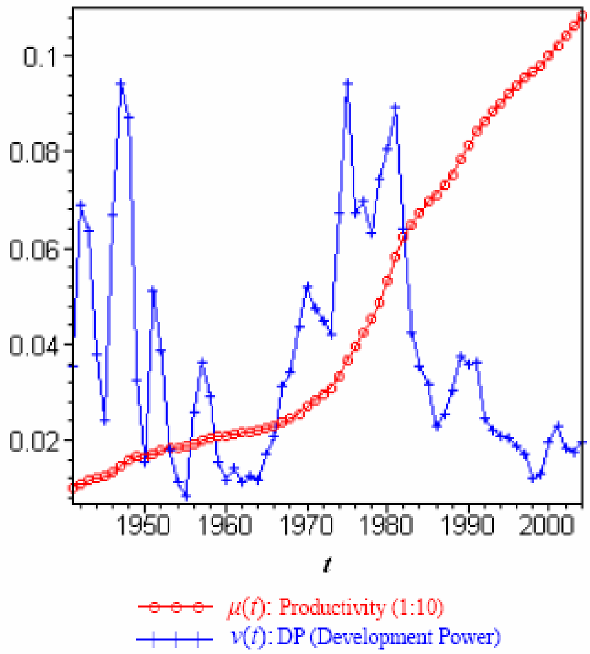

Fig. 1 The curves of US economic productivity $\mu(t)$ and DP $v(t)$ in the time field from 1941 to 2004. $\mu(t)$ is valued by GDP (chained) price index (Fiscal Year 2000=1.000). The proportion of the real indexes of $\mu(t)$ to drawn indexes is 1:10. Though productivity $\mu(t)$ has been growing approximately, DP fluctuates always.

5.1 Estimating the Parameters in DP models

5.1.1 Estimating the Parameters in DP models from 1989 to 1998. From 1989 to 1998, DP $v(t)$ gets smaller and smaller, namely is a economic growth with DP releasing. From expression (4), we have the following estimated result for logarithm model of DP:

$$\bar{v}_l(t)=0.009982990030-0.1199747377\ln\mu(t).$$

The estimated error:

$$s_l=\frac{1}{10}\sqrt{\sum_t(v(t)-\bar{v}_l(t))^2}=0.0007974065745,\ t=1989\sim1998.$$

The coefficient of determination: R^2=0.9099057130.

According to the results in reference [20], we have the following estimated result for exponential model of DP:

$\bar{v}_e(t)$=3.469717623$e^{-5.644970610\mu(t)}$

The estimated error:

$$s_e = \frac{1}{10}\sqrt{\sum_t (v(t)-\bar{v}_e(t))^2}$$ =0.0008927384778, t=1989~1998.

The coefficient of determination: R^2=0.8983647312.

According to the results in reference [21], we have the following estimated result for power model of DP:

$\bar{v}_p(t)$=0.01272893321$[\mu(t)]^{-4.919879375}$.

The estimated error:

$$s_p = \frac{1}{10}\sqrt{\sum_t (v(t)-\bar{v}_p(t))^2}$$ =0.0009399010355, t=1989~1998.

The coefficient of determination: R^2=0.8904225584.

Comparing the above estimated results, we see that the priority orders of fitting effect are the logarithm model, exponential model and power model.

5.1.2 Estimating the Parameters in DP models from 1998 to 2001. From 1998 to 2001, DP $v(t)$ gets larger and larger, namely is an economic growth with DP accumulating. From expression (4), we have the following estimated result for logarithm model of DP:

$\bar{v}_l(t)$=0.01739141530+0.1557804863ln$[\mu(t)]$.

The estimated error:

$$s_l = \frac{1}{4}\sqrt{\sum_t (v(t)-\bar{v}_l(t))^2}$$ =0.0007203865098, t=1998~2001.

The coefficient of determination: R^2=0.9034015490

Correspondingly, we have the following estimated result for exponential model of DP:

$\bar{v}_e(t)$=0.000001963020774$e^{9.047523134\mu(t)}$.

The estimated error:

$$s_e = \frac{1}{4}\sqrt{\sum_t (v(t)-\bar{v}_e(t))^2}$$ =0.0009034276422, t=1998~2001.

R^2=0.8743832421.

And, we also have the following estimated result for power model of DP:

$\bar{v}_p(t)$=0.01674590577$[\mu(t)]^{9.130608441}$.

The estimated error:

$$s_p = \frac{1}{4}\sqrt{\sum_t (v(t)-\bar{v}_p(t))^2}$$ =0.0008869103168, t=1998~2001.

R^2=0.8691761064.

Comparing the above estimated results, the priority orders of fitting effect are the logarithm model, power model and exponential model.

5.1.3 Estimating the Parameters in DP models on the local tops of DP in recent twenty years. According to the figure 1, we see the local top values of DP $v(t)$ in US economy descend gradually after 1980. This indicates that US economy is in the growth with DP releasing as a whole. Here we take the local top values of DP after 1980(i.e. 1981, 1989, 1991, 2001, 2004) as the samples for analysis. The data are listed in table 1, where,* means the local top values of DP.

According to the logarithm model (4) and references [20-21], we have the following estimated result for logarithm model of DP:

$\bar{v}_l(t)$=0.02246709772-0.1091570256ln$[\mu(t)]$.

The estimated error:

$$s_l = \frac{1}{5}\sqrt{\sum_t (v(t)-\bar{v}_l(t))^2} = 0.003254822736.$$

R^2=0.9157449808.

The estimated result for exponential model is

$$\bar{v}_e(t) = 0.4350885010e^{-2.912478361\mu(t)}.$$

The estimated error:

$$s_e = \frac{1}{5}\sqrt{\sum_t (v(t)-\bar{v}_e(t))^2} = 0.002397790794.$$

R^2=0.9659184631.

The estimated result for power model is

$$\bar{v}_p(t) = 0.02344567993[\mu(t)]^{-2.389469545}.$$

The estimated error:

$$s_p = \frac{1}{5}\sqrt{\sum_t (v(t)-\bar{v}_p(t))^2} = 0.001257987548.$$

R^2=0.9871749827.

where, t=1981,1989,1991,2001,2004.

Table 1. The data of productivity and DP (1981-2004)

Year t	Productivity $\mu(t)$	DP $v(t)$	Year t	Productivity $\mu(t)$	DP $v(t)$
1981	0.5830	0.08919382504*	1993	0.8838	0.02217696311
1982	0.6229	0.06405522556	1994	0.9028	0.02104563580
1983	0.6504	0.04228167282	1995	0.9218	0.02061184639
1984	0.6744	0.03558718861	1996	0.9395	0.01883980841
1985	0.6963	0.03145196036	1997	0.9559	0.01715660634
1986	0.7125	0.02273684211	1998	0.9675	0.01198966408
1987	0.7311	0.02544111613	1999	0.9802	0.01295653948
1988	0.7541	0.03049993370	2000	1.0000	0.01980000000
1989	0.7834	0.03740107225*	2001	1.0236	0.02305588120*
1990	0.8125	0.03581538462	2002	1.0426	0.01822367159
1991	0.8430	0.03618030842*	2003	1.0614	0.01771245525
1992	0.8642	0.02453135848	2004	1.0825	0.01949191686*

Note:

The data source of GDP (Chained) Price Index (Fiscal Year 2000 = 1.000): http://www.whitehouse.gov.

* The higher value of DP in the local years.

Comparing the above estimated results, we see that the priority orders of fitting effect are the power model, exponential model and logarithm model. From all of the fitting effects above, we know that power model, exponential model and logarithm model can be perfected and not be replaced one another.

5.2 The empirical analysis for economic state in energy and its evolution

5.2.1 The analysis for DP motion states in US economy and its trend forecast on 1989~1998. We know it is a process of economic growth with DP releasing in US from 1989 to 1998. Whether how long the process keeps on, according to proposition 3, the strong state and super state can not occur. This can be shown in figure 2. In the figure 2, DP values (after 1998) on logarithm model are estimated by expression (11), and DP values (after 1998) on exponential model and power model are estimated by method in reference [20-21]. The productivity after 1998 is estimated by the logarithm expression (12). Where, $\mu_0=\mu(1989)=0.7834$, $\mu_1=\mu(1990)=0.8125$, $v_0=v(1989)=0.03740107225$, $\varphi_l=0.2217304514$, $\varphi_e=0.2767795632$, $\varphi_p=0.01670496209$.

From proposition 2, we have the following formula to compute the time of low point of DP on logarithm model:

$$t=\sqrt{\frac{2\pi}{|\gamma|}}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}$$

The computed result is t=19.98809184≈20. That means the DP release may be reversed to DP accumulation at 2009 (1989+20), and economic growth may inverse to economic recession.

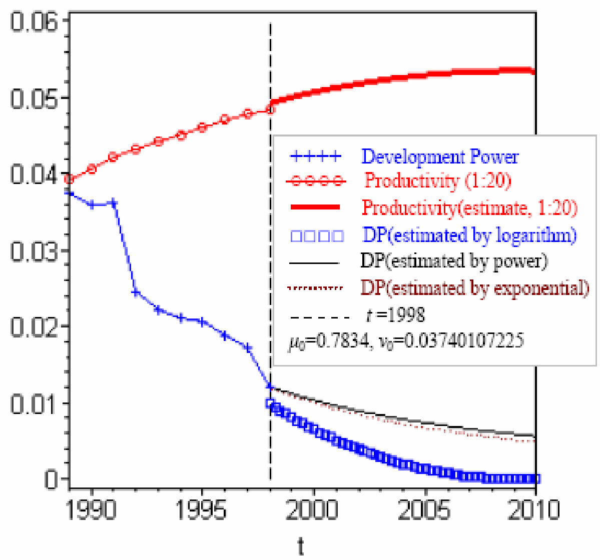

Fig.2 Energy analysis for US economic development from 1989 to 1998. US economy is in a process of economic growth with DP releasing. If an economic process, which has the same characters like that of the process from 1989 to 1998, keeps on, the strong state and super state never occur. The productivity after 1998 is estimated by the logarithm model in expression (12).

5.2.2 The analysis for DP motion states in US economy and its trend forecast on 1998~2001. Because the fitting effect of logarithm model is better than exponential and power model at this time, the strong state may occur according to proposition 1. The critical time of strong state can be computed by following formula

$$t_\varepsilon=\sqrt{\frac{2\pi}{\gamma}}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}\left(\sqrt{e^{\frac{4}{\pi\gamma}}-1}-\sqrt{e^{\frac{4v_0}{\pi\gamma}}-1}\right)$$

=2146.062621

where, μ_0=0.9675, v_0=0.01198966408,

γ=0.1557804863, φ=0.1569697947.

That means strong state may occur at 2146 if US economy develop continuously in the trend of 1998~2001. It is too long time for us to think the strong state comes into being.

5.2.3 The motion analysis for local tops of DP in US economy recent twenty years or more. Based on the estimated results in section 5.1.3, we have drawn the figure 3. There are samples foldgram, fitting curves of logarithm , exponential and power model from 1981 to 2004. We see, in figure 3, the local top values of DP $v(t)$ in US economy descend gradually after 1980. This indicates that US economy is in the growth with DP releasing as a whole. if giving a forecast in the current trend, we have the following results by use of logarithm, exponential and power model:

up to 2010, DP higher points are separately

v_l(2010)= 0.00140685195

v_e(2010)= 0.01272136726

v_p(2010)= 0.01478588751

All of forecast result show DP is too low, and these means DP of US economy has been released sufficiently. According to proposition 2, the economic recession with DP accumulating might occur. The critical time is

$$t=\sqrt{\frac{2\pi}{|\gamma|}}\frac{\mu_0}{\varphi}e^{-\frac{v_0}{\gamma}}\sqrt{1-e^{\frac{4v_0}{\pi\gamma}}}$$

=27.72802351≈28.

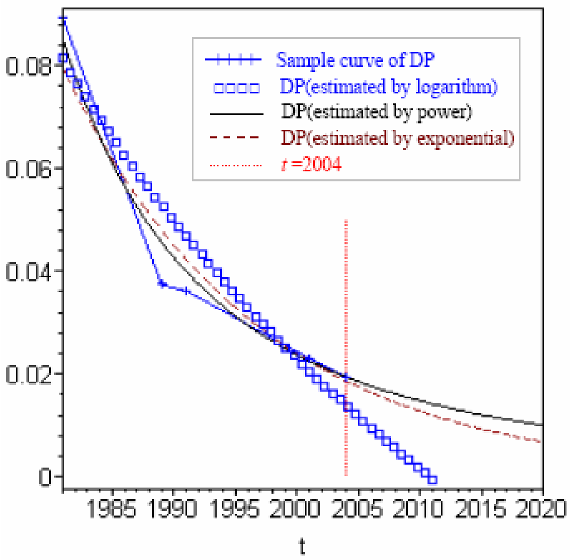

Fig. 3 The motion process of local top values of DP in US economy from 1981 to 2004. The local top values of DP $v(t)$ in US economy descend gradually after 1980. According to its current trend and based on logarithm, exponential and power model , the local top value of DP is only v_l(2010)=0.00140685195, v_e(2010)=0.01272136726, or v_p(2010)=0.01478588751 up to 2010. So, US economy needs to have a larger scale of accumulation in DP in the future. This process may be an economic growth with DP accumulating or an economic recession with DP accumulating.

That means the DP release may be reversed to DP accumulation at 2009 (1981+28), and economic growth may inverse to economic recession at the same time. This result is corresponding to the forecast result in 5.2.1. Because DP can not be illimitably released, US economy should have a large scale of accumulation for DP in a farsighted standpoint. This process should be an economic growth with DP accumulating, or an economic recession with DP accumulating. In order to avoid the economic recession, US government needs to do a lot of works, such as economic innovation, technology innovation, policy innovation, system innovation, market innovation, financial innovation, education innovation, management innovation, cultural development, etc., and the workloads would be very enormous. If like that, US economy may keep on growing stably in next decade or more.

As a whole of empirical researches, the logarithm model is better in fitting DP of US economy than the exponential model and logarithm model, so we could say that US economy develop in a mild process. But, DP is lower now whether the analysis is based on logarithm, exponential or logarithm model. Therefore, it is important for US government to accumulate the economic DP.

6 Conclusions and Remarks

Based on based on DP theory and models [17-21], This paper discusses emphatically the logarithm model of DP, points out again economic process includes three kinds of essential mode, namely exponential mode, logarithm mode and logarithm mode. In general, economic process is a mixed process of exponential law, logarithm law and logarithm law.

As for logarithm model itself, we get the conclusions as follow:

1) The basic logarithm model of relation between the productivity and DP, formula (4), is put forward based on Partial Distribution [22-24], and has been discussed. So we can affirm further that DP (economic energy) is the basic motivity to push continuously the economy to develop and progress.

2) The time-variant models of productivity and DP, (9)-(16), are given based on the logarithm model. They could be applied to describe the motion processes and analyze the important characters of productivity and DP. And we also can measure DP and forecast the trend of economy developing by these models.

3) If we say, the exponential model is suited to describe the economic process of which the speed is quick in rising and falling; the power model is suited to describe the economic process in which the speeds of growth or recession is mezzo; then the logarithm model is suited to describe the economic process of which the speed is slow in rising and falling. So these three models can be reinforced and not replaced one another.

4) DP accumulating and DP releasing are always alternated one with another. This alternated process pushes the economic growth to be replaced by economic recession or the opposition. These are the reasons that there are the cycles in economic development process and the economic fluctuation in market.

5) Based on the references [21-22], the three kinds of energy states in economy (i.e., normal state, strong state and super state) are analyzed further. The strong state may occur in an economic growth with DP accumulation if it follows the logarithm law, but super state never occurs. The strong state and super state may occur in an economic recession with DP accumulation if it follows the logarithm law. the method of computing the critical time of the corresponding strong state, super state or the important transform are given.

To a whole cycle of DP motion process, we have

1) The process of economic growth with DP accumulation is a fine mode if it follows the logarithm law, it enable economy to develop continuously. But DP accumulated must be released, i.e., the process of economic growth with DP release will present when the DP accumulates to a higher degree.

2) The process of economic growth with DP release is a process which makes economy growing by use of economic energy. The economic growth can not be kept on for a long term in this kind of economic process. There are two possible results at the end of this process, one is economic recession with DP accumulating, and one is another process of economic growth with DP accumulating.

3) When the economic recession with DP accumulating ends, there may be a large scale diffussion in DP and the diversification of economic growth.

4) The process of economic growth with DP accumulating may start at the end of the economic recession with DP releasing.

Exponential mode, logarithm mode and logarithm mode are the basic models to describe the economic process and DP. Also, we could regard the economic process following generally the integration of the three models. Of course, this hypothesis should be validated further.

What we need point out is the error of models (9)-(16) might become larger along with the time becomes longer. There are two ways to solve that problem [21].

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