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Development Power and Its Power Model: The Analytic Approach for Continuous Motivity of Economic Growth

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ABSTRACT: Based on the Partial Distribution [F. Dai, 2001] and the theory of Development Power [F. Dai, 2004], this paper discusses the power model of relation between development power (DP) and productivity. The power model also supports the hypothesis [F. Dai, 2005] that there are three kinds of energy states in economy, i.e. normal state, strong state and super state, and DP is the continuous motivity to economic growth. By the power model of DP, we could interpret in analytic way that the diffusion of DP and the diversifications of economic development also might be occurred after the super state. Finally, the conclusions in this paper are researched in the empirical way, the results indicate the power model is better than the exponential model of DP in many cases, and we could get the inimitable outcomes in describing the macroeconomic process by the power model of DP.

KEYWORDS: Development Power (DP), Partial Distribution, power model, macroeconomic analysis

1 Introduction

In order to study macroeconomic process in a manner which is different from the business cycle theory [R. E. Lucas, 1981], the real business cycle theory [E. Prescott, 1982], new growth theory [P. M. Romer, 1986] and so on, the author has put forward the theory of development power [F. Dai, etc., 2004]. The theory of development power (DP) indicates:

● The productivity is the visible behavior ability for mankind to develop economy, and the DP is the invisible behavior motivity for mankind to develop economy. DP is also the developing force for economic growth continuously.

● If we regard the productivity as the hardware of economic development, then DP is the software of economic development. Both of them are a pair of most basic factors in economic development. The movement of DP will influence the level of productivity, and vice verse.

● The accumulation and release of DP will bring on the fluctuations of economic developing force. Just the movement of DP results in the growth or recession of economic productivity, so it is the motive force of economic development.

● The reason that there are huge differences between the developed nations and backward regions in economic growth rate and the level of economic development is the difference in the abilities to accumulate and release DP.

● The DP is scaled by economic development energy, and measured by fluctuation rate of level of economic production (i.e. productivity). So we can say that the DP is the basic energy of economic growth.

● If we describe the basic level of productivity with Gross Nation Product (GDP), then the fluctuation rate of GDP could describes DP.

The general model [F. Dai, 2005] has been put forward which can describe the relation between the DP and productivity. This model includes three kind of basic models, i.e., exponential model, power model and logarithm model. So we might say that there are three kind of basic modes in the macroeconomic process, and they occur generally in combinative way. Then we have a new approach to study the macro-economy. We know, from reference [20], that there are three kinds of states of economic energy in economic development, i.e., normal state, strong state and super state in economic energy. The reference [20] has also given the time-variant models which can describe the changing process of DP and productivity based on the exponential model. Further more, this paper will discuss the power model of relation between DP and productivity, and give the time-variant models for describing the changing process of DP and productivity based on the power model. The discussion about power model also supports the hypothesis that there are three kinds of states of economic energy in economic development. The method of calculating the critical time of strong energy and super energy based on power model will be given, and by which we can prove in theory that the economic energy will diffuse and the industrial development, merchandises production and market services will present the trend of diversification after super state in energy occurs.

By use of the DP models in this paper, we will explain that the developed nations could renew their economy rapidly after suffering the important balefulness like nature disaster or a large scale war, the reason is that huge DP accumulated in economy has been released effectively and sufficiently. Germany's and Japan's economic development process after War II are the better examples.

We shall make an empirical analysis on the results in this paper by means of the US GDP data from 1940 to 2004. The empirical results will explain that the effect of power model fitting really the economic process is better than exponential model under many circumstances, and not better in some other circumstances. So it will make know that the exponential model and the power model can be completed one with another, but can not be substituted one with another.

2 The Basic Model of DP

2.1 Notations and descriptions of models

If note:

$\mu(t)$: the basic economic level at the time t, namely a measuring index for basic productivity, the basic level for short, $\mu(t)\geq 0$.

$\sigma(t)$: the fluctuation range of the basic economic level at the time t, i.e., the standard variance of the basic economic level, $\sigma(t)>0$. $\sigma(t)$ can describe the absolute energy of economic development at the time t.

$v(t)=\sigma(t)/\mu(t)$: the fluctuation rate of the basic economic level at the time t, is a measuring index of DP (Development Power), $\mu(t)\geq 0$. $v(t)$ can describe the developing energy of economy, generally $0<v(t)<1$.

$X(t)$: the real economic level at the time t, real level for short. $X(t)$ is a non-negative random variable for any $t\geq 0$.

According to assumptions in references [17]-[20], the real level $X(t)$, for any $t\in[0, \infty)$, follows the Partial Distribution[21]-[23] as

$$f(x)=\begin{cases} e^{-\frac{[x-\mu(t)]^2}{2[v(t)]^2}} \Big/ \int_0^{\infty} e^{-\frac{[x-\mu(t)]^2}{2[v(t)]^2}}\,dx & x\geq 0 \\ 0 & x<0 \end{cases}$$

At this time, we denote: $X(t)\in P(\mu(t), v^2(t))$. $X(t)$ is actually a Partial Process[24].

Because the economic growth is essentially the increase for level of productivity, and the economic recession is essentially the decline for level of productivity, we do not distinguish the economic level from productivity in concept in the following discussion, i.e. regard them as the same. Also we do not distinguish DP from the developing energy of economy. And the $\mu(t)$, $v(t)$ and $X(t)$ can be separately noted μ, and X for short.

$\mu(t)$ and $v(t)$ may be not continuous about the time t, namely when $0 \le t_0 < t_1 < \cdots < t_{n+1} < \cdots < \infty$, we have

$$\mu(t) = \begin{cases} \mu_0 & t_0 \le t < t_1 \\ \vdots & \\ \mu_n & t_n \le t < t_{n+1} \\ \vdots & \end{cases}$$, where , $\mu_i \ge 0$, i=1, 2, ⋯

Also, we have

$$\sigma(t) = \begin{cases} \sigma_0 & t_0 \le t < t_1 \\ \vdots & \\ \sigma_n & t_n \le t < t_{n+1} \\ \vdots & \end{cases}$$, where, $\sigma_i > 0$, i=1, 2, ⋯

thus, $X(t_i) \in P(\mu(t_i), [v(t_i)]^2)$ is a dispersed process, where $\mu(t_{i+1}) = X(t_i)$, $t = t_0, t_1, \cdots$.

2.2 The basic model of DP

In general, the time t is considered, and the tiny margin of v(value of DP) should have something to do with the tiny margin of μ (value of productivity) and the current states of DP and productivity, namely, Δv has something to do with $v\Delta\mu$, $\frac{1}{\mu}\Delta\mu$ and $\frac{v}{\mu}\Delta\mu$, the $\frac{1}{\mu}$ in the two parts later indicates that growth rate of v is of reverse ratio to the current μ. Thus, we can suppose that there is a function $V = G(x_1, x_2, x_3, x_4)$ which is continuous about x_1, x_2, x_3, x_4, and

$$\Delta v = G(v\Delta\mu, \frac{1}{\mu}\Delta\mu, \frac{v}{\mu}\Delta\mu, \Delta t)$$
$$= G(0,0,0,0) + \frac{\partial G}{\partial x_1} v\Delta\mu + \frac{\partial G}{\partial x_2}\frac{1}{\mu}\Delta\mu + \frac{\partial G}{\partial x_3}\frac{v}{\mu}\Delta\mu + \frac{\partial G}{\partial t}\Delta t + \circ(\Delta\mu) + \circ(\Delta t) + \circ(\Delta t \cdot \Delta\mu)$$

where, $\circ(\Delta\mu)$ is a small quantity to $\Delta\mu$ in a higher rank, $\circ(\Delta t)$ is a small quantity to Δt in a higher rank, and $\circ(\Delta t \cdot \Delta\mu)$ is a quantity to $\Delta t \cdot \Delta\mu$ in a same rank.

Because no differences of basic level and time would bring on no difference of DP, i.e. $\Delta v = 0$ when $\Delta\mu = 0$ and $\Delta t = 0$, $G(0,0,0,0) = 0$. Letting $\Delta\mu \to 0$ and $\Delta t \to 0$, and overlooking the $\circ(\Delta\mu)$, $\circ(\Delta t)$ and $\circ(\Delta t \cdot \Delta\mu)$, we have

$$dv = \left(\frac{\partial G}{\partial x_1} v + \frac{\partial G}{\partial x_2}\frac{1}{\mu} + \frac{\partial G}{\partial x_3}\frac{v}{\mu}\right) d\mu + \frac{\partial G}{\partial x_4} dt \quad \left(\frac{\partial G}{\partial x_j} = \frac{\partial G(0,0,0,0)}{\partial x_j}, j=1,2,3,4\right)$$

If we suppose that the DP has something to do with the time in implicit way, and let $\frac{\partial G(0,0,0,0)}{\partial x_4} = 0$, then we have

$$\frac{dv}{d\mu} = av + b\frac{1}{\mu} + c\frac{v}{\mu} \qquad (1)$$

where, $a = \frac{\partial G(0,0,0,0)}{\partial x_1}$, $b = \frac{\partial G(0,0,0,0)}{\partial x_2}$, $c = \frac{\partial G(0,0,0,0)}{\partial x_3}$.

From expression (1), we obtain its solution as follows:

$$v = v_0 \left(\frac{\mu}{\mu_0}\right)^c e^{a(\mu - \mu_0)} + b\mu^c e^{a\mu} \int_{\mu_0}^{\mu} \frac{e^{-au}}{u^{c+1}} du \qquad (2)$$

where, μ_0 and v_0 are separately the original values of $\mu(t)$ and $v(t)$.

Uniting $X(t) \in P(\mu(t), [v(t)]^2)$ with expression (2), we have the general model of relation between DP and economic level, namely

$$\begin{cases} v(t) = v_0\left(\dfrac{\mu(t)}{\mu_0}\right)^c e^{a(\mu(t)-\mu_0)} + b(\mu(t))^c e^{a\mu(t)} \displaystyle\int_{\mu_0}^{\mu(t)} \dfrac{e^{-au}}{u^{c+1}} du \\ X(t) \in P(\mu(t), v^2(t)), \quad t \in [0, \infty) \\ \mu_0 = \mu(0), v_0 = v(0), \end{cases} \tag{3}$$

Specially, if μ_0=$\mu(0)$ and $\mu(t)$>0, from (3)

1) If a=γ, b=c=0, we have the exponential model of relation between DP and economic level as

$$v(t) = v_0 e^{\gamma[\mu(t)-\mu_0]} \tag{4}$$

2) If b =γ, a =c=0, we have the logarithm r model of relation between DP and economic level as

$$v(t) = v_0 + \gamma \ln\left(\frac{\mu(t)}{\mu_0}\right) \tag{5}$$

3) If c=γ, a=b=0, we have the power model of relation between DP and economic level as

$$v = v_0\left(\frac{\mu(t)}{\mu_0}\right)^\gamma \tag{6}$$

where, the constant $\gamma(\neq 0)$ is characteristic exponent of DP, DP exponent for short. DP exponent can reflect the character of DP movement. The results above indicate that there are three kind of basic modes of relation between DP and productivity in the macroeconomic process, i.e., exponential mode, power mode and logarithm mode, and the general economic process could be described generally in way of combination of the three modes.

The exponential model (4) and its applications have been discussed in references [18-21] and model (5) will be discussed in other papers. We will discuss the power model (6) as follow.

According to model (6), if v_0= $v(0)$>0, we obtain $\mu(t) = \mu_0\left(\dfrac{v(t)}{v_0}\right)^{\frac{1}{\gamma}}$. This equation expresses how DP, as the economic developing energy $v(t)$, influences productivity $\mu(t)$. So we could think that DP is the engine for economic growth.

The DP exponent γ in expression (4), (5) and (6) have the following meanings:

1) In the process of economic growth, i.e. the productivity $\mu(t)$ is increasing by degree,

$$\gamma = \begin{cases} >0, \text{ DP is accumulating} \\ =0, \text{ DP is stable} \\ <0, \text{ DP is releasing} \end{cases}$$

2) In the process of economic recession, i.e. the productivity $\mu(t)$ is of descending

$$\gamma = \begin{cases} >0, \text{ DP is releasing} \\ =0, \text{ DP is stable} \\ <0, \text{ DP is accumulating} \end{cases}$$

If DP has the different characters in the different periods of time $[t_i, t_{i+1})$ (i=0, 1, ···), the DP exponent γ can be a sectioned function as follows

$$\gamma(t)=\begin{cases}\gamma_0 & t_0\le t<t_1\\ & \vdots\\ \gamma_n & t_n\le t<t_{n+1}\end{cases}$$

where, γ_i is DP exponent, i=0,1,2, ···.

3 The Power Model of DP

3.1 The time-variant model based on DP's power model

The equation (6) can be expressed as

$$v(t)=\frac{v_0}{\mu_0^{\gamma}\varphi^2}\left(\pm\varphi\mu(t)^{\frac{\gamma}{2}}\right)^2 \tag{7}$$

where, "+" is corresponding to economic growth, "-" is corresponding to economic recession, scaling parameter $\varphi(>0)$ is a constant and waiting for determining.

Expression (4) can be regard as a kinetic energy model according to physics, we could let

$$\frac{d\mu(t)}{dt}=\pm\varphi\mu(t)^{\frac{\gamma}{2}} \tag{8}$$

Then, by solving the equation (5), we obtain the time-variant model for productivity:

$$\mu(t)=\begin{cases}\mu_0 e^{\pm\varphi\cdot t} & \gamma=2\\ \mu_0\left(1\pm\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t\right)^{\frac{2}{2-\gamma}} & \gamma\ne 2\end{cases}$$

and combining the equation (6), we have the time-variant model for DP:

$$v(t)=\begin{cases}v_0 e^{\pm 2\cdot\varphi\cdot t} & \gamma=2\\ v_0\left(1\pm\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi\cdot t\right)^{\frac{2\gamma}{2-\gamma}} & \gamma\ne 2\end{cases}$$

3.2 The time-variant models of productivity and DP in economic growth

From (6) and (8), we know that the productivity $\mu(t)$ can be expressed as

$$\mu(t)=\begin{cases}\mu_0 e^{\varphi\cdot t} & \gamma=2\\ \mu_0\left(1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t\right)^{\frac{2}{2-\gamma}} & \gamma\ne 2\end{cases} \tag{9}$$

and DP $v(t)$ can be expressed as

$$v(t)=\begin{cases}v_0 e^{2\cdot\varphi\cdot t} & \gamma=2\\ v_0\left(1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi\cdot t\right)^{\frac{2\gamma}{2-\gamma}} & \gamma\ne 2\end{cases} \tag{10}$$

In the expression (9) and (10), γ is larger than zero ($\gamma>0$) for economic growth process with DP accumulation. At this time, if $0<\gamma\le 2$, thus $0\le t<\infty$; and if $\gamma>2$, thus, $0\le t<\frac{2}{\varphi(\gamma-2)}\mu_0^{\frac{2-\gamma}{2}}$. γ is smaller than zero ($\gamma<0$) for economic growth process with DP release, at this time, if $\gamma<0$, thus $t>0$.

3.3 The time-variant models of productivity and DP in economic recession

In similar, from (6) and (8), we know that the productivity $\mu(t)$ can be expressed as

$$\mu(t)=\begin{cases}\mu_0 e^{-\varphi\cdot t} & \gamma=2\\ \mu_0\left(1-\dfrac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t\right)^{\frac{2}{2-\gamma}} & \gamma\neq 2\end{cases} \tag{11}$$

and DP $v(t)$ can be expressed as

$$v(t)=\begin{cases}v_0 e^{-2\cdot\varphi\cdot t} & \gamma=2\\ v_0\left(1-\dfrac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi\cdot t\right)^{\frac{2\gamma}{2-\gamma}} & \gamma\neq 2\end{cases} \tag{12}$$

In the expression (11) and (12), γ is smaller than zero ($\gamma<0$) for economic recession process with DP accumulation, $0\le t<\dfrac{2}{\varphi(2-\gamma)}\mu_0^{\frac{2-\gamma}{2}}$; and γ is larger than zero ($\gamma>0$) for economic recession process with DP release. At this time, if $0<\gamma<2$, thus $0\le t<\dfrac{2}{\varphi(\gamma-2)}\mu_0^{\frac{2-\gamma}{2}}$; and if $\gamma\ge2$, thus, $0\le t<\infty$. γ is smaller than zero ($\gamma<0$) for economic growth process with DP release, at this time, if $\gamma<0$, thus $t>0$.

3.4 The computing method of scaling parameter φ

For the economic growth process, according to (9), we have

$$\varphi=\begin{cases}\ln\mu_1-\ln\mu_0 & \gamma=2\\ \dfrac{2}{2-\gamma}\mu_0^{\frac{2-\gamma}{2}}\left[\left(\dfrac{\mu_1}{\mu_0}\right)^{\frac{2-\gamma}{2}}-1\right] & \gamma\neq 2\end{cases} \tag{13}$$

similarly, for the economic recession process, according to (11), have

$$\varphi=\begin{cases}\ln\mu_0-\ln\mu_1 & \gamma=2\\ \dfrac{2}{2-\gamma}\mu_0^{\frac{2-\gamma}{2}}\left[1-\left(\dfrac{\mu_1}{\mu_0}\right)^{\frac{2-\gamma}{2}}\right] & \gamma\neq 2\end{cases} \tag{14}$$

Of course, there are many other methods to compute φ.

4 The Analysis for DP's Motion Characters

4.1 The analysis for DP's motion characters in economic growth process

Definition 1 (the energy state in economic development). If the t_τ and t_ε are real numbers, $t_\varepsilon>0$ and $t_\tau>t_\varepsilon$, then

1) When the time $t\in(0, t_\varepsilon)$, and at this time, if DP $v(t)<1$, we call the economic state the normal state in economic energy, and the normal state for short.

2) When the time $t\in[t_\varepsilon, t_\tau)$, and at this time, if DP $v(t)\ge1$, we call the state of economy the strong state in economic energy, and the strong state for short. t_ε is the critical time of strong state.

3) If the time $t=t_\tau$, and at this time, $v(t)=\infty$, we call the state of economy the super state in economic energy, and the super state for short. t_τ is the critical time of super state.

In the process of economic growth with DP accumulating, $v>1$ means the DP may start to diffuse, so t_ε, the critical time of strong state, is also the critical time of DP diffusing; and $v>\mu$ indicates that the current productivity is difficult to satisfy the need to economic growth, at the same time, economic production is like to present an important transform, namely the super state in energy is like to occur. In this case, the time $t=T$, determined by $v(T)=\mu(T)$, is the critical time for economic production to present an important transform. The important transform here means the great transform in economic policy or economic technology.

Definition 2 (the critical time of important transform and important disaster in economy). In the process of economic DP accumulating, if economy is growth and the equation $v(T)=\mu(T)>0$ is correct, than the T is called the critical time for important economic transform; if economy is recession and the equation $v(T)=\mu(T)>0$ is correct, than the T is called the critical time for important economic disaster.

In general, the macro-economy presents a moral state. If DP becomes larger and larger, the strong state might occur. The super state might occur if both strong state and important transform occurred.

According to (9) and (10), we have

Proposition 1. In the process of economic growth with DP accumulating, i.e. $\mu(t)$ increases by degree, $r>0$ and $r\neq 2$:

1) The strong state may occur, and its critical time is

$$t_\varepsilon = \frac{2}{(\gamma-2)\varphi}\mu_0^{\frac{2-\gamma}{2}}\left(1-v_0^{\frac{\gamma-2}{2\gamma}}\right) \tag{15}$$

2) If $r>1$, the important economic transform may occur, and its critical time is

$$T = \frac{2}{(\gamma-2)\varphi}\mu_0^{\frac{2-\gamma}{2}}\left[1-\left(\frac{\mu_0}{v_0}\right)^{\frac{2-\gamma}{2(\gamma-1)}}\right] \tag{16}$$

3) If $r>2$, the super state may occur, and its critical time is

$$t_\tau = \frac{2}{(\gamma-2)\varphi}\mu_0^{\frac{2-\gamma}{2}} \tag{17}$$

4) If $0<r<2$, the super state can not occur at a finite time.

According to proposition 1, we see that the important economic transform will generally occur after the strong state occurs if both of them must occur. Also in the process of economic growth with DP accumulating, and if $r>2$ and $t>t_\tau$ in the model (9) and (10), the expressions of DP $v(t)$ and productivity $\mu(t)$ are separately

$$v(t) = v_0\left(\left|1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t\right|\right)^{\frac{2\gamma}{2-\gamma}}\cdot(-1)^{\frac{2\gamma}{2-\gamma}} \tag{18}$$

and

$$\mu(t) = \mu_0\left(\left|1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t\right|\right)^{\frac{2}{2-\gamma}}\cdot(-1)^{\frac{2}{2-\gamma}} \tag{19}$$

From the knowledge of complex function, we know that $(-1)^\alpha=e^{(2k+1)\alpha\pi i}=\cos((2k+1)\alpha\pi)+i\sin((2k+1)\alpha\pi)$ (α is a real number, $i=\sqrt{-1}$ is the imaginary unit, $k=0,\pm 1,\pm 2,\cdots$), thus both $(-1)^{\frac{2}{2-\gamma}}=e^{\frac{2(2k+1)\pi}{2-\gamma}i}$ and $(-1)^{\frac{2\gamma}{2-\gamma}}=e^{\frac{2(2k+1)\pi\gamma}{2-\gamma}i}$ are multiple valued, and both (18) and (19) are multi-valued functions. If like this, expression (18) means that the energy of economic development diffuses completely, well and obviously. And expression (19) means that the diversifications of economic development and producing mode are occur based on diffusing of economic energy and important economic transform.

What we need to say is, at the beginning of diversifications, the different industries of economy and production have the same DP one and another. Because of the differences in developing environment, developing policy or developing approach and so on, they have the different results in development, some develops well, some develops difficultly, and some bowls out.

Proposition 2. In the process of economic growth with DP accumulating, if r=2,

1) The strong state may occur, and its critical time is

$$t_\varepsilon = -\frac{\ln v_0}{2\varphi}$$

2) The important economic transform may occur, and its critical time is

$$T = \frac{1}{\varphi}\ln\left(\frac{\mu_0}{v_0}\right)$$

3) The super state can not occur at a finite time.

Proposition 1 and proposition 2 also show that although strong state and the important economic transform occur, the super state not always occurs, i.e., economic diversifications not always occur.

For the process of economic growth with DP releasing, we have the following proposition 3:

Proposition 3 In the process of economic growth with DP releasing, i.e. $\mu(t)$ increases by degree, r<0. The strong state and the super state will never occur.

Proof: Because $\gamma<0$, i.e. $1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t>1$ and $\frac{2\gamma}{2-\gamma}<0$, there is not the strange point t_τ which make the $v(t)$ in expression (10) be ∞, namely super state will never occur; and if let $v_0\left(1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi\cdot t\right)^{\frac{2\gamma}{2-\gamma}}\geq 1$, we have $\left(1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi\cdot t\right)^{\frac{2\gamma}{\gamma-2}}\leq v_0$, owing to $1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t>1$, this contradicts with $v_0<1$. So, the strong state will never occur.

4.2 The analysis for DP's moving characters in economic recession process

Proposition 4. In the process of economic recession with DP releasing, i.e. $\mu(t)$ descends by degree and $\gamma>0$. Both the strong state and the super state will never occur if $\gamma\geq 2$; and if $0<\gamma<2$, the economy may have a start of slow growth in a way of selectivity after DP passes through the low valley time $t_0=\frac{2}{(2-\gamma)\varphi}\mu_0^{\frac{2-\gamma}{2}}$.

Proof: from (11) and (12), we know, $\mu(t)$ and $v(t)$ descend about the time t if $\gamma\geq 2$, so the strong state and the super state will never occur. If $0<\gamma<2$, $\mu(t)$ and $v(t)$ descend when the time t is in the field of [0, $\frac{2}{(2-\gamma)\varphi}\mu_0^{\frac{2-\gamma}{2}}$]; $\mu(t)=v(t)=0$ at the $t_0=\frac{2}{(2-\gamma)\varphi}\mu_0^{\frac{2-\gamma}{2}}$; when $t>t_0=\frac{2}{(2-\gamma)\varphi}\mu_0^{\frac{2-\gamma}{2}}$, DP $v(t)$ and the productivity $\mu(t)$ have the following expressions separately

$$v(t) = v_0\left(\left|1-\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t\right|\right)^{\frac{2\gamma}{2-\gamma}}\cdot e^{\frac{2(2k+1)\pi\gamma}{2-\gamma}i} \tag{20}$$

and

$$\mu(t) = \mu_0\left(\left|1-\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t\right|\right)^{\frac{2}{2-\gamma}}\cdot e^{\frac{2(2k+1)\pi}{2-\gamma}i} \tag{21}$$

where, $k=0,\pm 1,\pm 2,\cdots$. The equations (20) and (21) are multi-valued functions, these means the new process of economic development, after economic recession with DP releasing and the losing of ability to produce for a short time, will be very hard and face manifold difficult choice.

Proposition 5. In the process of economic recession with DP accumulation, i.e. $\mu(t)$ descends by degree and $\gamma<0$, there might be the strong state and super state which are strange and not normal. At the same time, the critical time of strong state is $t_\varepsilon = \frac{2}{(2-\gamma)\varphi}\mu_0^{\frac{2-\gamma}{2}}\left(1-v_0^{\frac{\gamma-2}{2\gamma}}\right)$, the critical time of super state is

$t_\tau = \frac{2}{(2-\gamma)\varphi}\mu_0^{\frac{2-\gamma}{2}}$, and the critical time of important disaster is $T=\frac{2}{(2-\gamma)\varphi}\mu_0^{\frac{2-\gamma}{2}}\left[1-\left(\frac{\mu_0}{v_0}\right)^{\frac{2-\gamma}{2(\gamma-1)}}\right]$.

Some explanations about proposition 5:

1) In the process of economic recession with DP accumulation, if suffering the important balefulness like ruinous nature disaster or destructive war at the critical time T, then economic DP might be accumulated rapidly, and result in the strong state or even the super state.

2) At the time $t_\tau = \frac{2}{(2-\gamma)\varphi}\mu_0^{\frac{2-\gamma}{2}}$, the productivity $\mu(t)=0$, but DP $v(t)=\infty$, these mean that although producing ability loses for a little, the latent energy of economic development, i.e. DP, is very huge, and the foreground of economic renewing might be very good.

3) When $t>t_\tau$, the expressions of productivity $\mu(t)$ and DP $v(t)$, in (20) and (21) separately, are all the multi-valued functions. These means there are many ways for DP diffusion and production choice in economic renewing.

Germany's and Japan's economic renewing process after War II are the better examples for proposition 5.

Putting the discussions above together, we have got some conclusions as follow:

1) The process of economic growth with DP accumulation is a fine mode, it enable economy to develop continuously. This kind of process could result in the strong state in economic energy; and the important economic transform may occur if DP exponent $\gamma>1$; further more, the super state in economic energy may occur if $\gamma>2$. The super state might cause the economic energy to diffuse and economic productions to diversify.

2) The process of economic growth with DP release is a process which makes economy growing by use of economic energy. The economic growth can not be kept on for a long term in this kind of economic process. There are two possible results at the end of this process, one is economic recession, and one is another process of economic growth with DP accumulation. So, if at the time like this, we need to take some measures to accumulate DP, the economic energy, and enter into a process of economic growth with DP accumulation.

3) The process of economic recession with DP release is not an optimistic mode. A long economic recession with DP release occurs easily in the no enterprising and underdeveloped country or in the declined industries. Most of efforts must be made in order to take apart from the economic recession with DP release. Otherwise, the economy in the country or the industries will perish.

4) The process of economic recession with DP accumulation occurs because the economy disables to grow continuously or the economic environment becomes bad. For developed nations, this kind of process comes generally after the economy is growing for a long term or a large scale war or nature disaster starts. When war or nature disaster ends, these developed nations accumulate a huge of economic energy to renew their economy, and could use their original technology, knowledge, ability and idea to choose the appropriate ways for economic development. In their renewing economy, DP should be diffused and production should

be diversified in many ways and directions.

5 The Empirical Researches for DP's Power Model

Here we take US GDP (chained) price index [25] (Fiscal Year 2000 = 1.000, the GDP index for short) in the period of 1940-2004 as the scale of productivity and the empirical samples. Though US economy, productivity $\mu(t)$, has been growing from end of World War II to 2004, DP fluctuates always. There are many different characteristics of DP in this period. We have the notations and expressions as follow:

$\mu(t)$: The basic level of economic productivity of the year t, t=1940, 1941, ···, 2004, measured by the GDP index.

$v(t)$: The scale value of DP of the year t, measured by the fluctuating ratio of the productivity level. $v(t)=|\mu(t)-\mu(t-1)|/\mu(t)$, t=1941, 1942, ···, 2004.

$X(t)$: The actual level of productivity of the year t, a non-negative stochastic variable. $X(t)\in P(\mu(t),[v(t)]^2)$.

The interval of time unit for sampling data GDP is a year, the stability of data is higher, and the difference between GDP price index of one year and that of last year can nicely describe the economic fluctuation, so we adopt the formulas of $v(t)$ mentioned above. The curves of DP and productivity in US economy are shown in figure 1. In figure 1, the proportion of the real indexes of $\mu(t)$ to indexes drawn is 1:10.

We could see, from figure 1, the economic energy (i.e. DP) would go down if it is higher in a certain degree, namely DP will release if it is accumulated to a certain degree. Also, the DP will accumulate if it is released to a certain degree.

In the following, we will give the analysis about DP motion separately on the data from 1989 ~1998 (an economic growth process with DP releasing), 1998~2001 (an economic growth process with DP accumulating), and the data of local higher points after 1980 (1981, 1989, 1991, 2001, 2004). the analysis include estimating the parameters, comparing the actual fitting effects between exponential model [20] and power model, and forecasting the trend of US economy in future years.

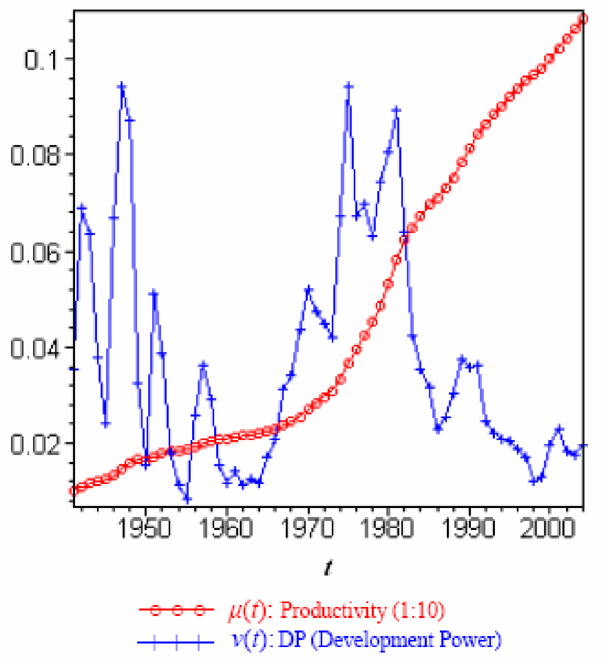

Fig. 1 The curves of US economic productivity $\mu(t)$ and DP $v(t)$ in the time field from 1941 to 2004. $\mu(t)$ is valued by GDP (chained) price index (Fiscal Year 2000=1.000). The proportion of the real indexes of $\mu(t)$ to drawn indexes is 1:10. Though productivity $\mu(t)$ has been growing approximately, DP fluctuates always.

5.1 Estimating the Parameters in DP models

Here, we take the years (t=1989~1998) and the years (t=1998~2001) as our analytic samples for DP $v(t)$, and estimate parameters by least square method.

5.1.1 Estimating the Parameters in DP models from 1989 to 1998. From 1989 to 1998, DP $v(t)$ gets smaller and smaller, namely is a economic growth with DP releasing. From expression (6), we have the following estimated results for power model of DP:

$$\bar{v}_p(t) = 0.01272893321\,[\mu(t)]^{-4.919879375}$$

The estimated error is

$$s_p = \frac{1}{10}\sqrt{\sum_t (v(t)-\bar{v}_p(t))]^2}$$ =0.0009399010355, t=1989~1998.

The coefficient of determination: R^2=0.8904225584.

According to the method in reference [20], we have the following estimated results for exponential model of DP:

$$\bar{v}_e(t) = 3.469717623e^{-5.644970610\mu(t)}$$

The estimated error: $s_e = \frac{1}{10}\sqrt{\sum_t (v(t)-\bar{v}_e(t))]^2}$ =0.0008927384778, t=1989~1998.

The coefficient of determination: R^2=0.8983647312.

We see, at this time, the fitting effect of exponential model is better than that of power model by comparing the above estimated results.

5.1.2 Estimating the Parameters in DP models from 1998 to 2001. From 1998 to 2001, DP $v(t)$ gets larger and larger, namely is an economic growth with DP accumulating. From expression (6), we have the following estimated results for power model of DP:

$$\bar{v}_p(t) = 0.01674590577[\mu(t)]^{9.130608441}$$

The estimated error: $s_p = \frac{1}{4}\sqrt{\sum_t (v(t)-\bar{v}_p(t))]^2}$ =0.0008869103168, t=1998~2001.

The coefficient of determination: R^2=0.8743832421.

In other hand, we have the following estimated results for exponential model of DP:

$$\bar{v}_e(t) = 0.000001963020774e^{9.047523134\mu(t)}$$

The estimated error: $s_e = \frac{1}{4}\sqrt{\sum_t (v(t)-\bar{v}_e(t))]^2}$ =0.0009034276422, t=1998~2001.

R^2=0.8691761064.

We see, at this time, the fitting effect of power model is better than that of exponential model by comparing the above estimated results.

5.2 The empirical analysis for economic state in energy and its evolution

5.2.1 The analysis for DP motion states in US economy and its trend forecast on 1989~1998. We know it is a process of economic growth with DP releasing in US from 1989 to 1998. Whether how long the process keeps on, according to proposition 3, the strong state and super state can not occur. This can be shown in figure 2. In the figure 2, DP values (after 1998) on power model is estimated by expression (10), and DP values (after 1998) on exponential model is estimated by method in reference [20]; the productivity after 1998 is estimated by the power expression in section 5.1.1.

5.2.2 The analysis for DP motion states in US economy and its trend forecast on 1998~2001. Because the fitting effect of power model is better than that of exponential model at this time, we make the corresponding analysis and forecast on power model. According to section 6.1.2 and formula (9) and (10), we have the expressions of productivity $\mu(t)$ and DP $v(t)$based on power model as follow

$$\mu(t) = \mu_0\left(1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi t\right)^{\frac{2}{2-\gamma}} \quad \text{and} \quad v(t) = v_0\left(1+\frac{2-\gamma}{2}\mu_0^{\frac{\gamma-2}{2}}\varphi\cdot t\right)^{\frac{2\gamma}{2-\gamma}}$$

where, DP exponent γ=9.092183212, scaling parameter φ=0.01433570717, and $\mu_0=\mu(1998)$=0.9675, $\mu_1=\mu(1999)$=0.9802, $v_0=v(1998)$= 0.01198966408.

By formula (12), (13) and (14), we compute separately:

The critical time of strong state is t_c=18.09861977 (probably, Jan. to Feb., 2016), The critical time of important transform is : T=18.80131354(probably, Sep. to Oct., 2016)

The critical time of super state is t_τ=22.01115994 (probably, 2020)

According to the calculating results above, US economy might come into the strong state if going on developing for 18 years (probably at 2016) in the way of 1998~2001, and might come into the super state if go on developing for 22 years (probably at 2020) in the way of 1998~2001. And some important transforms might be occurred in US economy if going on developing for 17 years (probably at 2017) in the way of 1998~2001. But, US economy kept on developing for 4 years in the way of 1998~2001 only, namely DP of US economy starts releasing after 2001, the difference is very larger. The curves of the productivity $\mu(t)$ and DP $v(t)$ determined separately by expression (9) and (10) are shown in figure 3.

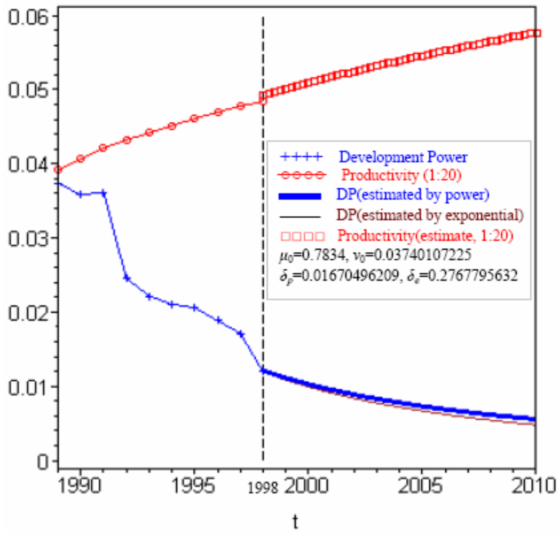

Fig.2 Energy analysis for US economic development from 1989 to 1998. US economy is in a process of economic growth with DP releasing from 1989 to 1998. If an economic process, which has the same characters like that of the process from 1989 to 1998, keeps on, the strong state and super state never occur. The productivity after 1998 is estimated by the power expression in section 6.1.1.

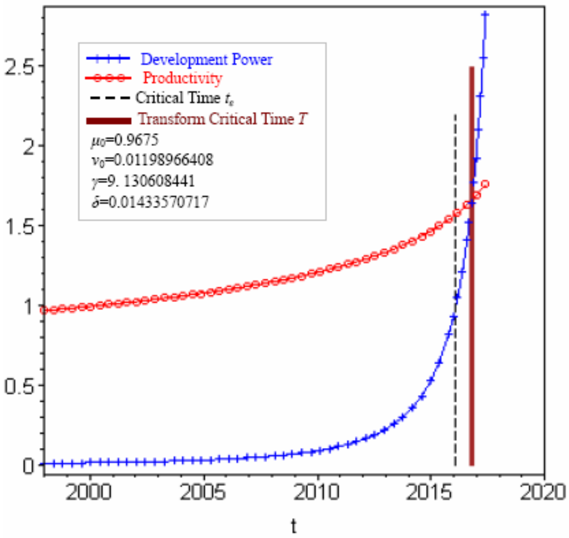

Fig.3 Energy analysis for US economic development from 1998 to 2001. US economy is in a process of economic growth with DP accumulating from 1998 to 2001. If an economic process, which has the same characters like that of the process from 1998 to 2001, keeps on, the strong state and super state may occur. The critical time of strong state is t_c≈18 (probably, 2016), and the critical time of super state is t_τ≈22 (probably, 2020). But this process keeps on four years only.

5.3 The motion analysis for local tops of DP in US economy recent twenty years or more

According to the figure 1, we see the local top values of DP $v(t)$ in US economy descend gradually after 1980. This indicates that US economy is in the growth with DP releasing as a whole. Here we take the local top values of DP after 1980(i.e. 1981, 1989, 1991, 2001, 2004) as the samples for analysis. The data are listed in table 1, where,* means the local top values of DP.

Table 1. The data of productivity and DP (1981-2004)

Year t	Productivity $\mu(t)$	DP $v(t)$	Year t	Productivity $\mu(t)$	DP $v(t)$
1981	0.5830	0.08919382504*	1993	0.8838	0.02217696311
1982	0.6229	0.06405522556	1994	0.9028	0.02104563580
1983	0.6504	0.04228167282	1995	0.9218	0.02061184639
1984	0.6744	0.03558718861	1996	0.9395	0.01883980841
1985	0.6963	0.03145196036	1997	0.9559	0.01715660634
1986	0.7125	0.02273684211	1998	0.9675	0.01198966408
1987	0.7311	0.02544111613	1999	0.9802	0.01295653948
1988	0.7541	0.03049993370	2000	1.0000	0.01980000000
1989	0.7834	0.03740107225*	2001	1.0236	0.02305588120*
1990	0.8125	0.03581538462	2002	1.0426	0.01822367159
1991	0.8430	0.03618030842*	2003	1.0614	0.01771245525
1992	0.8642	0.02453135848	2004	1.0825	0.01949191686*

Note:

The data source of GDP (Chained) Price Index (Fiscal Year 2000 = 1.000): http://www.whitehouse.gov.

* The higher value of DP in the local years.

According to the power model (6) and exponential model [21], we have the following estimated results for power model of DP:

$\bar{v}_p(t)$ =0.02344567993$[\mu(t)]^{-2.389469545}$

The estimated error: $s_p = \frac{1}{5}\sqrt{\sum_t (v(t)-\bar{v}_p(t))^2}$ =0.001257987548

The coefficient of determination: R^2=0.9871749827.

And estimated results for exponential model is

$\bar{v}_e(t)$ =0.4350885010$e^{-2.912478361\mu(t)}$

The estimated error: $s_e = \frac{1}{5}\sqrt{\sum_t (v(t)-\bar{v}_e(t))^2}$ =0.002397790794

R^2=0.9659184631.

where, t=1981,1989,1991,2001,2004.

Comparing the estimated results, we see, the fitting effect of power model is better than that of exponential model.

The samples foldgram and fitting curves of power model and exponential model, based on the local top values of DP from 1981 to 2004, are drawn in the figure 4.

We also see, from the figure 4, the local top values of DP in US economy descend gradually in recent twenty years or more. If US economy develops in the current trend and up to 2010, the top value of DP is only v_p(2010)=0.01478588751 or v_e(2010)=0.01272136724. So low the values means DP of US would have been released sufficiently. Because DP can not be illimitably released, US economy should have a large scale of accumulation for DP in a farsighted standpoint. This process should be an economic growth with DP accumulating, or an economic recession with DP accumulating. In order to avoid the economic recession, US government needs to do a lot of works, such as economic innovation, technology innovation, policy innovation, system innovation, market innovation, financial innovation, education innovation, management innovation, etc., and the workloads would be very enormous. If like that, US economy may keep on growing stably in next decade or more.

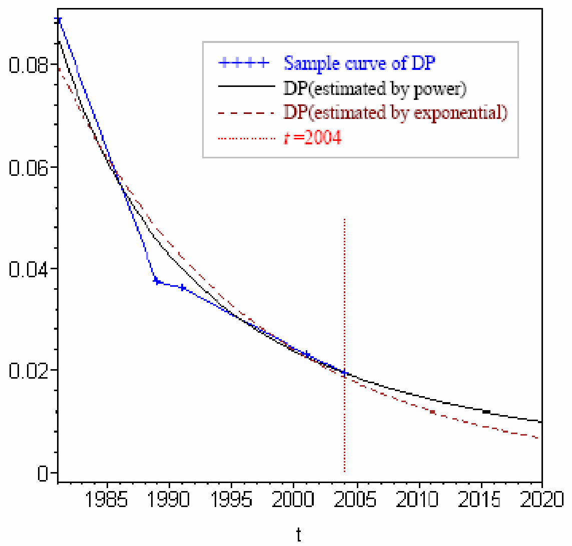

Fig.4 The motion process of local top values of DP in US economy from 1981 to 2004 and the analysis on its trend in the future. The local top values of DP $v(t)$ in US economy descend gradually after 1980.According to its current trend, the local top value of DP is only v_p(2010)= 0.01478588751 or v_e(2010)=0.01272136724 up to 2010. So, US economy needs to have a larger scale of accumulation in DP. This process may be an economic growth with DP accumulating or an economic growth with DP accumulating.

6 Conclusions and Remarks

This paper points out, based on DP theory and models[17-20], economic process includes three kinds of essential law, namely exponential low, power law and logarithm law, and do the works as follow:

1) The basic model of relation between the productivity and DP, (2) or (3), is put forward based on Partial

Distribution [21-24]. The power model of DP, a special case in model (3), has been discussed. So we can affirm further that DP (economic energy) is the basic motivity to push continuously the economy to develop and progress.

2) The time-variant models of productivity and DP, (9)-(12), are given based on the power model. They could be applied to describe the motion processes and analyze the important characters of productivity and DP. And we also can measure DP and forecast the trend of economy developing by these models.

3) The basic energy states in economic development, i.e. the state of normal state, strong state and super state in economic energy, are discussed further based on reference [20], and the approaches to calculate the critical times of strong state, super state and the important transform are also given.

4) The multi-values of DP model and productivity model, in an economic growth, also are occurred after the critical time of super state as they are in expression (18) and (19). The expression (18) means the diffusion of DP (economic energy), and the expression means the diversifications of economic development and producing mode. So we could say that the DP theory and DP models can interpret the economic diversifications and the diffusion of economic energy in analytic way.

5) It is expressed, by means of the power model of DP as in expressions (20) and (21), that the diffusion of DP and the diversifications of economic development also might be occurred at the beginning of a new economic growth after the economic recession.

6) The empirical analysis for the analytic results given in this paper is done by use of the US GDP data from 1940 to 2003. We see, by the empirical researches, the DP's power models and exponential models can not be replaced mutually. In fact, the estimating effect of power model is better than that of exponential model in many cases.

What we need point out is the error of models (9)-(12) might become larger along with the time becomes longer. There are two ways as follow to solve that problem.

1) Renewing the DP exponent γ in models. We could renew constantly the DP exponent γ in the models of (9)-(12) by use of the newer data if need.

2) Adjusting the scaling parameter in models. The scaling parameter φ in expression (13) and (14) is computed on $\mu_0=\mu(0)$ and $\mu_1=\mu(1)$. In practice, we could compute φ by the approaches as follow if need.

To economic growth:

$$\varphi = \begin{cases} \dfrac{\ln \mu(\Delta t) - \ln \mu_0}{\Delta t} & \gamma = 2 \\ \dfrac{2}{(2-\gamma)\Delta t} \mu_0^{\frac{2-\gamma}{2}} \left[\left(\dfrac{\mu(\Delta t)}{\mu_0} \right)^{\frac{2-\gamma}{2}} - 1 \right] & \gamma \neq 2 \end{cases}$$

To economic recession:

$$\varphi = \begin{cases} \dfrac{\ln \mu_0 - \ln \mu(\Delta t)}{\Delta t} & \gamma = 2 \\ \dfrac{2}{(2-\gamma)\Delta t} \mu_0^{\frac{2-\gamma}{2}} \left[1 - \left(\dfrac{\mu(\Delta t)}{\mu_0} \right)^{\frac{2-\gamma}{2}} \right] & \gamma \neq 2 \end{cases}$$

where, $\Delta t>0$.

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