

Sigma Convergence versus Beta Convergence: Evidence from U.S. County-Level Data

Andrew T. Young*
Department of Economics
University of Mississippi
University, MS 38677
atyoung@olemiss.edu
Tel: 662-915-5829, Fax: 404-894-6030

Matthew J. Higgins
College of Management
Georgia Institute of Technology
Atlanta, GA 30308-0520
mhiggins@mgt.gatech.edu
Tel: 404-894-4368. Fax: 404-894-6030

Daniel Levy
Department of Economics
Bar-Ilan University
Ramat-Gan 52900, Israel
levyda@mail.biu.ac.il
Tel: +972-3-531-8331, Fax: +972-3-535-3180

JEL Codes: O40, O11, O18, R11

Key Words: σ -convergence, β -convergence, Solow growth model, speed of convergence, U.S. county-level data, income distribution, Gini coefficient, equality

Last Revision: June 19th, 2006

* Corresponding Author.. We thank Jordan Rappaport for kindly sharing with us his data and computer codes and for providing helpful suggestions throughout the study.

Sigma Convergence versus Beta Convergence: Evidence from U.S. County-Level Data

Abstract

In this paper we outline (i) why σ -convergence may not accompany β -convergence, (ii) discuss evidence of β -convergence in the U.S., and (iii) use U.S. county-level data containing over 3,000 cross-sectional observations to demonstrate that σ -convergence has not occurred at the county-level across the U.S., or within the vast majority of the individual U.S. states considered separately.

I. Introduction

Barro and Sala-i-Martin (1995) and Sala-i-Martin (1996) draw a useful distinction between two types of convergence in growth empirics: σ -convergence and β -convergence. When the dispersion of real per capita income (henceforth, simply “income”) across a group of economies falls over time, there is σ -convergence. When the partial correlation between growth in income over time and its initial level is negative, there is β -convergence.¹

When economists refer to the “convergence literature,” they refer to the large literature, typified by the seminal papers by Barro and Sala-i-Martin (1992) and Mankiw et al. (1992), exploring β -convergence. Sala-i-Martin (1996, p. 1326), surveying this literature, concludes that “the estimated speeds of [β -]convergence are so surprisingly similar across [cross-sectional] data sets, that we can use a mnemonic rule: *economies converge at a speed of two percent per year.*” In other words, economies close the gap between their present level of income and their balanced growth level by, on average, 2 percent each year. Panel data studies find even higher rates of β -convergence – see Islam (1995) and Evans (1997a) – as do the county-level U.S. studies of Higgins et al. (2006) and Young et al. (2006).

However, β -convergence is not a sufficient condition for σ -convergence. Quah (1993) and Friedman (1992) both suggest that σ -convergence should be of interest since it speaks directly as to whether the distribution of income across economies is becoming more equitable. Still, β -convergence has remained a primary focus of growth empirics, perhaps because, intuitively, it would seem to be necessary for σ -convergence.

¹ Sala-i-Martin (1996) makes a distinction between *conditional* β -convergence (as described above) and *absolute* β -convergence, where poor economies simply grow faster than wealthy ones. For simplicity, and since absolute β -convergence can be a specific case of conditional β -convergence where balanced growth paths are identical across economies, we focus on the conditional concept and call it β -convergence.

In this paper we demonstrate that β -convergence is indeed a necessary but *not* sufficient condition for σ -convergence. Then we discuss evidence of β -convergence in the U.S. using county-level data covering 1970 to 1998 and containing over 3,000 cross-sectional observations. We demonstrate, using the same data, that σ -convergence did not occur during that time period in the U.S. or within the vast majority of the individual U.S. states considered separately. If we accept the estimated β -convergence effects, one interpretation is that balanced growth paths for rich counties are higher than those of poor counties: *rich counties have maintained growth rates comparable to poor economies because they are comparably below their balanced growth paths.*

The paper is organized as follows. Section II explains why σ -convergence need not accompany β -convergence. Section III discusses the existing empirical evidence from the U.S. indicating that β -convergence exists in the U.S., including at the county-level. Section IV describes the U.S. county-level data. Section V demonstrates that σ -convergence did not occur across the U.S., or within a large majority of the individual U.S. states, from 1970 to 1998. Section VI reports Gini coefficients for the same county-level data that are consistent with a lack of σ -convergence. Section VII concludes.

II. β -Convergence versus σ -Convergence

Following Sala-i-Martin's (1996) exposition, assume that β -convergence holds for economies $i = 1, \dots, N$. Log-income of the i -th economy can be approximated by

$$\log(y_{it}) = a + (1 - \beta)\log(y_{i,t-1}) + u_{it}, \quad (1)$$

where $0 < \beta < 1$ and u_{it} has mean zero, finite variance, σ_u^2 , and is independent over t and

i . Manipulating (1) yields,

$$\log\left(\frac{y_{it}}{y_{i,t-1}}\right) = a - \beta \log(y_{i,t-1}) + u_{it}. \quad (1)'$$

Thus, $\beta > 0$ implies a negative correlation between growth and initial log income.

The sample variance of log income in t is given by

$$\sigma_t^2 = \left(\frac{1}{N}\right) \sum_{i=1}^N [\log(y_{it}) - \mu_t]^2, \quad (2)$$

where μ_t is the sample mean of (log) income. The sample variance is close to the population variance when N is large, and (1) can be used to derive the evolution of σ_t^2 :

$$\sigma_t^2 \cong (1 - \beta)^2 \sigma_{t-1}^2 + \sigma_u^2. \quad (3)$$

Only if $0 < \beta < 1$ is the difference equation stable, so β -convergence is necessary for σ -convergence.² Given $0 < \beta < 1$, the steady-state variance is,

$$(\sigma^2)^* = \frac{\sigma_u^2}{[1 - (1 - \beta)^2]}. \quad (4)$$

Thus, the cross-sectional dispersion falls with β but rises with σ_u^2 . Combining (3) and (4) yields,

$$\sigma_t^2 = (1 - \beta)^2 \sigma_{t-1}^2 + [1 - (1 - \beta)^2] (\sigma^2)^*, \quad (5)$$

which is a first-order linear difference equation with constant coefficients. Its solution is given by,

$$\sigma_t^2 = (\sigma^2)^* + (1 - \beta)^{2t} [\sigma_0^2 - (\sigma^2)^*] + c(1 - \beta)^{2t}, \quad (6)$$

² If $\beta \leq 0$ the variance increases over time. If the $\beta = 1$ the variance is constant and if $\beta > 1$ the partial correlation between (log) income and its previous-period value would be negative and the series would oscillate, potentially from positive to negative values and back (making little economic sense).

where c is an arbitrary constant. Thus, as long as $0 < \beta < 1$, we have $|1 - \beta| < 1$, which implies that

$$\lim_{t \rightarrow \infty} (1 - \beta)^{2t} = 0. \quad (7)$$

This ensures the stability of σ_t^2 because it implies that,

$$\lim_{t \rightarrow \infty} \sigma_t^2 = (\sigma^2)^*. \quad (8)$$

Moreover, since $(1 - \beta) > 0$, the approach to $(\sigma^2)^*$ is monotonic.

It follows, therefore, that the variance will increase or decrease towards its steady-state value depending on the initial σ_0^2 . Intuitively, consider two economies, A and B, where both economies begin at the same level of income. However, assume that B begins on its balanced growth path while A begins far below its balanced growth, and assume that β -convergence holds. The initial variance (σ_0^2) will be zero, but σ_t^2 will grow over time as A grows faster than B and approaches a higher balanced growth path. Indeed, β -convergence is the reason for the increasing variance.

The above example is stylized. In real economies, σ -convergence would also depend on whether or not disturbances are correlated, and have constant variances, across time and economies. Still, even in the stylized example, β -convergence is necessary but not sufficient for σ -convergence.

III. β -Convergence

Many studies have documented β -convergence in the U.S. Barro and Sala-i-Martin (1992), Evans and Karras (1996a and 1996b), Sala-i-Martin (1996), and Evans

(1997a and 1997b) find statistically significant β -convergence effects using U.S. state-level data. The present authors use U.S. county-level data to document statistically significant β -convergence effects across the U.S. (Higgins et al., 2006), and within many individual U.S. states in and of themselves (Young et al., 2006). See Table 1.

Using a consistent three stage least squares (3SLS) estimation method, we estimate the β -convergence rate to be between 6 and 8 percent for the U.S. as a whole and, for individual U.S. states, β -convergence rate point estimates range from just under 4 percent to just over 14 percent. (See Table 1, column 3.) Even considering ordinary least squares (OLS) estimates, β -convergence rate estimates are always positive when significant. (See Table 1, column 2.)

Clearly, considerable evidence supports the existence of β -convergence, which is a necessary condition for σ -convergence. Below we explore whether or not σ -convergence is occurring using the same county-level data that were used by Higgins et al. (2006) and Young et al. (2006).

IV. U.S. County-Level Data

Higgins et al. (2006) and Young et al. (2006) focus on the U.S. income growth from 1970 to 1998. The data set includes 3,058 county-level observations, and 50 individual state samples of various sizes, also at the county-level. See Figure 1.

The personal income measure is defined by the U.S. Bureau of Economic Analyses (BEA). The personal income measure is adjusted to be net of government transfers and is expressed in per capita 1992 dollars using the U.S. GDP deflator. Population measures from the U.S. Census are used to construct per capita amounts. Real

per capita income levels are expressed as natural logs and values are considered for both 1970 and 1998.³

The measure used for personal income is that of the U.S. BEA.⁴ The definitions that are used for the components of personal income at the county-level are essentially the same as those used for national measures. For example, the BEA defines “personal income” as the sum of wage and salary disbursements, other labor income, proprietors’ income (with inventory valuation and capital consumption adjustments), rental income (with capital consumption adjustment), personal dividend income and personal interest income.

V. σ -convergence

To our knowledge, the only study of U.S. regional σ -convergence is Tsionas (2000). He examines real Gross State Products (RGSPs) and finds that “...the cross sectional variance has fluctuated very little in the 20-year period from 1977 to 1996” (pp. 235-236). In contrast, the time period we cover is nearly a decade longer. Moreover, we have over 3,000 county-level cross-sectional observations while Tsionas uses 50 state-level observations.⁵ However, our findings are ultimately consistent with Tsionas’.

Table 2 reports 1970 and 1998 cross-sectional standard deviations of (log) income for the entire sample of U.S. counties, and for each of the 50 U.S. states. The 1998 standard deviation for the full U.S. sample (0.2887) is about 5.8 percent greater than that

³ For a more detailed discussion of the data, see Higgins et al. (2006) or an appendix available from the authors. Also, see U.S. BEA (2001) for the personal income data concept and data gathering methods. The original data set contained 3,066 observations. Eight counties, however, were excluded from the data set for various reasons. Primarily, counties were excluded for lack of data.

⁴ The data and their measurement methods are described in detail in “Local Area Personal Income, 1969–1992” published by the BEA under the Regional Accounts Data, February 2, 2001.

⁵ As well, Tsionas apparently (and inexplicably) did not convert RGSPs into per capita measures.

of 1970 (0.2728). In only 3 out of 50 states (Kansas, Kentucky, and Oklahoma) is the 1998 standard deviation less than that of 1970. Thus, for the vast majority of the individual states, as well as for the full U.S., σ -divergence occurred from 1970 to 1998.

Some have suggested that interpreting measures of dispersion may not be straightforward if the distributions are not unimodal, e.g., Quah (1997) and Desdoigts (1999). However, as Figure 2 demonstrates, for the U.S. county-level data the distribution of income is unimodal for both 1970 and 1998.⁶ Figure 2 also allows one to confirm, visually, that σ -convergence is not present.

VI. Has σ -divergence Implied Greater Income Inequality?

Another measure we report that is associated with σ -convergence (in the sense that it deals with the distribution of income) is the Gini coefficient associated with U.S. counties' 1970 and 1998 (log) incomes: 0.0167 and 0.0165 respectively – a decrease of about 1.2 percent. See Table 3. Recall that Gini coefficient is a number between 0 (perfect equality) and 1 (perfect inequality).

Interestingly, at the county-level, although the distribution of U.S. per capita income became a bit more dispersed from 1970 to 1998, it became a bit more equal.⁷ However, the change in both the standard deviation and the Gini coefficient are small enough to suggest that both dispersion and equality remained essentially the same.

To try to understand further the evolution of the U.S. county-level income distribution, Table 3 summarizes two additional statistics computed from the 1970 and 1998 income distributions. From 1970 to 1998, the skewness of the distribution

⁶ Figure 1 is generated using income data, rather than log-income data. The latter was used in constructing the figures reported in Table 1.

⁷ This statement is not to be confused with one concerning the distributions of U.S. *individuals'* incomes.

increased from -0.2244 (to the left) to 1.7240 (to the right). At the same time, kurtosis increased from 3.4334 to 10.3237, implying that the distribution has become more peaked. This suggests that these two effects have been offsetting to a great extent.

VII. Conclusion

What are we to make of the presence of β -convergence and the lack of σ -convergence? One interpretation is that the U.S. is approaching its steady-state real per capita income variance from below.⁸ This implies that the initial distribution of income was narrow relative to the distribution of balanced growth paths.

Another interpretation is that the variance of the balanced growth paths is itself increasing. However, one may consider this second interpretation unlikely considering the relative institutional homogeneity of counties across the U.S. This is certainly the case within given states where the same β -convergence versus σ -convergence results hold in the majority of cases.

A third – and perhaps the most unlikely – interpretation is that rich counties have balanced growth *rates* that are higher than those of poor counties. There is little reason to think, however, that the long-run growth rates of technological know-how remain divergent across U.S. counties.

In either case, the evolution of skewness and kurtosis suggests that there may be an underlying σ -convergence for a “majority club” of U.S. counties *but* that there is another “minority club” that is evolving into a long right-hand tail of the distribution, preventing σ -convergence in the aggregate.

⁸ A related issue, which we do not address in this paper directly, is whether or not the cross-sectional distribution of log per capita income is ergodic (Evans, 1996). That would mean that the cross-sectional variance is stationary around a mean or is converging asymptotically toward a constant mean.

References

- Barro, R. J. and Sala-i-Martin, X. X. 1992. Convergence. *Journal of Political Economy* 100, 223–251.
- Barro, R. J. and Sala-i-Martin, X. X. 1995. *Economic Growth*. MIT Press, Cambridge.
- Desdoigts, A. 1999. Patterns of Economic Development. *Journal of Economic Growth* 4, 305–330.
- Evans, P. 1996. Using Cross-Country Variances to Evaluate Growth Theories. *Journal of Economic Dynamics and Control* 20, 1027–1049.
- Evans, P. 1997a. How Fast Do Economies Converge? *Review of Economics and Statistics* 36, 219–225.
- Evans, P. 1997b. Consistent Estimation of Growth Regressions. Working Paper, Ohio State University.
- Evans, P. and Karras, G. 1996a. Do Economies Converge? Evidence from a Panel of U.S. States. *Review of Economics and Statistics* 78, 384–388.
- Evans, P. and Karras, G. 1996b. Convergence Revisited. *Journal of Monetary Economics* 37, 249–265.
- Friedman, M. 1992. Do Old Fallacies Ever Die? *Journal of Economics Literature* 30, 2129–2132.
- Higgins, M. J., Levy, D., and Young, A. T. 2006. Growth and Convergence Across the U.S.: Evidence from County-Level Data. *Review of Economics and Statistics* 88, forthcoming.
- Islam, N. 1995. Growth Empirics: A Panel Data Approach. *Quarterly Journal of Economics* 110, 1127–1170.
- Mankiw, N. G., Romer, D. H. and Weil, D.N. 1992. A Contribution to the Empirics

- of Economic Growth. *Quarterly Journal of Economics* 107, 407–437.
- Quah, D. T. 1993. Galton’s Fallacy and the Convergence Hypothesis. *Scandinavian Journal of Economics* 95, 427–443.
- Quah, D. T. 1997. Empirics for Growth and Distribution: Stratification, Polarization, and Convergence Clubs. *Journal of Economic Growth* 2, 27–59.
- Sala-i-Martin, X. X. 1996. Regional Cohesion: Evidence and Theories of Regional Growth and Convergence. *European Economic Review* 40, 1325–1352.
- Tsionas, E.G. 2000. Regional Growth and Convergence: Evidence from the United States. *Regional Studies* 34, 231–238.
- U.S. Bureau of Economic Analysis. 2001. *Local-Area Personal Income, 1969–1992*. Regional Accounts Data.
- Young, A.T., Levy, D., and Higgins, M. J. 2006. Heterogeneity in Convergence Rates and Income Determination across U.S. States: Evidence from County-Level Data, working paper.

Table 1: Asymptotic Convergence Rates – Point Estimates & 95% Confidence Intervals

<u>State</u>	<u>Number of Counties</u>	<u>OLS Estimates & 95% C.I.</u>	<u>3SLS Estimates & C.I.</u>
United States	3,058	0.0239 (0.0224, 0.0255)	0.0658 (0.0632, 0.0981)
Alabama	67	0.0424 (0.0036, 0.1080)	0.0931 (0.0492, 0.1466)
Arkansas	74	0.0479 (0.0166, 0.1098)	0.0738 (0.0570, 0.1363)
California	58	0.0457 (0.0046, 0.1249)	0.0375 (0.0178, 0.0868)
Colorado	63	0.0166 (0.0031, 0.0384)	0.0759 (0.0426, 0.1009)
Florida	67	0.0268 (0.0010, 0.1109)	0.0767 (0.0480, 0.1174)
Georgia	159	0.0230 (0.0109, 0.0413)	0.1043 (0.0699, 0.1142)
Idaho	44	0.0892 (0.0021, 0.1566)	0.0913 (0.0471, 0.1145)
Illinois	102	0.0434 (0.0213, 0.1168)	0.0537 (0.0337, 0.1062)
Indiana	92	0.0067 (-0.0054, 0.0245)	0.0622 (0.0354, 0.1221)
Iowa	99	0.0570 (0.0224, 0.1176)	0.0574 (0.0175, 0.0954)
Kansas	106	0.0560 (0.0360, 0.1086)	0.0639 (0.0434, 0.1228)
Kentucky	120	0.0431 (0.0233, 0.0922)	0.1054 (0.0561, 0.1160)
Louisiana	64	0.0341 (0.0128, 0.0955)	0.1555 (0.0989, 0.1940)
Michigan	83	0.0121 (-0.0043, 0.0427)	0.1152 (0.0536, 0.1659)
Minnesota	87	0.0202 (0.0053, 0.0459)	0.0454 (0.0305, 0.0719)
Mississippi	82	0.0249 (0.0009, 0.1509)	0.1405 (0.0455, 0.1923)
Missouri	115	0.0230 (0.0094, 0.0452)	0.0817 (0.0387, 0.1132)
Montana	56	0.0359 (0.0099, 0.0996)	0.0865 (0.0367, 0.1566)
New York	62	0.0111 (-0.0238, 0.0284)	0.0465 (0.0285, 0.0853)
North Carolina	100	0.0228 (0.0078, 0.0491)	0.1302 (0.0966, 0.1574)
North Dakota	53	0.0528 (0.0103, 0.1247)	0.0761 (0.0353, 0.1102)
Ohio	88	0.0170 (-0.0005, 0.0520)	0.0503 (0.0299, 0.1059)
Oklahoma	77	0.0415 (0.0139, 0.1136)	0.1152 (0.0574, 0.1437)
Pennsylvania	67	0.0240 (0.0043, 0.0707)	0.0705 (0.0291, 0.1099)
South Carolina	46	0.0142 (-0.0147, 0.1259)	0.0960 (0.0243, 0.1315)

Note: based on results originally reported in Higgins et al. (2006) and Young et al. (2006)..

Table 2: Standard Deviations U.S. Counties' Log Per Capita Incomes, 1970 vs 1998

<u>Region</u>	<u>Number of Counties</u>	<u>1970 Per Capita Income Standard Deviation</u>	<u>1998 Per Capita Income Standard Deviation</u>
United States	3,058	0.2728	0.2887
Alabama	67	0.1949	0.2073
Alaska	9	0.4785	0.4798
Arizona	9	0.2136	0.2987
Arkansas	74	0.1904	0.1911
California	58	0.1646	0.3328
Colorado	63	0.2862	0.3282
Connecticut	8	0.1491	0.2411
Delaware	3	0.2062	0.2886
Florida	67	0.2575	0.3360
Georgia	159	0.2065	0.2304
Hawaii	4	0.1513	0.2441
Idaho	44	0.2003	0.2098
Illinois	102	0.2044	0.2263
Indiana	92	0.1263	0.1819
Iowa	99	0.1089	0.1415
Kansas	106	0.2279	0.1804
Kentucky	120	0.3171	0.3151
Louisiana	64	0.2195	0.2389
Maine	16	0.1233	0.2002
Maryland	24	0.2213	0.2927
Massachusetts	14	0.1355	0.2155
Michigan	83	0.1966	0.2663
Minnesota	87	0.1887	0.1963
Mississippi	82	0.1929	0.2464
Missouri	115	0.2408	0.2464
Montana	56	0.1870	0.1911
Nebraska	93	0.1645	0.3475
Nevada	17	0.1853	0.2150
New Hampshire	10	0.0941	0.1444
New Jersey	20	0.1379	0.2768
New Mexico	32	0.2770	0.3055
New York	62	0.2028	0.2995
North Carolina	100	0.1971	0.2184
North Dakota	53	0.1562	0.2361
Ohio	88	0.1681	0.2241
Oklahoma	77	0.2724	0.2180
Oregon	36	0.1534	0.2163
Pennsylvania	67	0.1692	0.2214
Rhode Island	5	0.0830	0.1239
South Carolina	46	0.1924	0.2251
South Dakota	66	0.2091	0.3476
Tennessee	97	0.2136	0.2641
Texas	254	0.2744	0.3035
Utah	29	0.1732	0.2522
Vermont	14	0.0949	0.1934
Virginia	84	0.2408	0.3006
Washington	39	0.1672	0.2213
West Virginia	55	0.2318	0.2436
Wisconsin	70	0.1940	0.2177
Wyoming	23	0.1623	0.2308

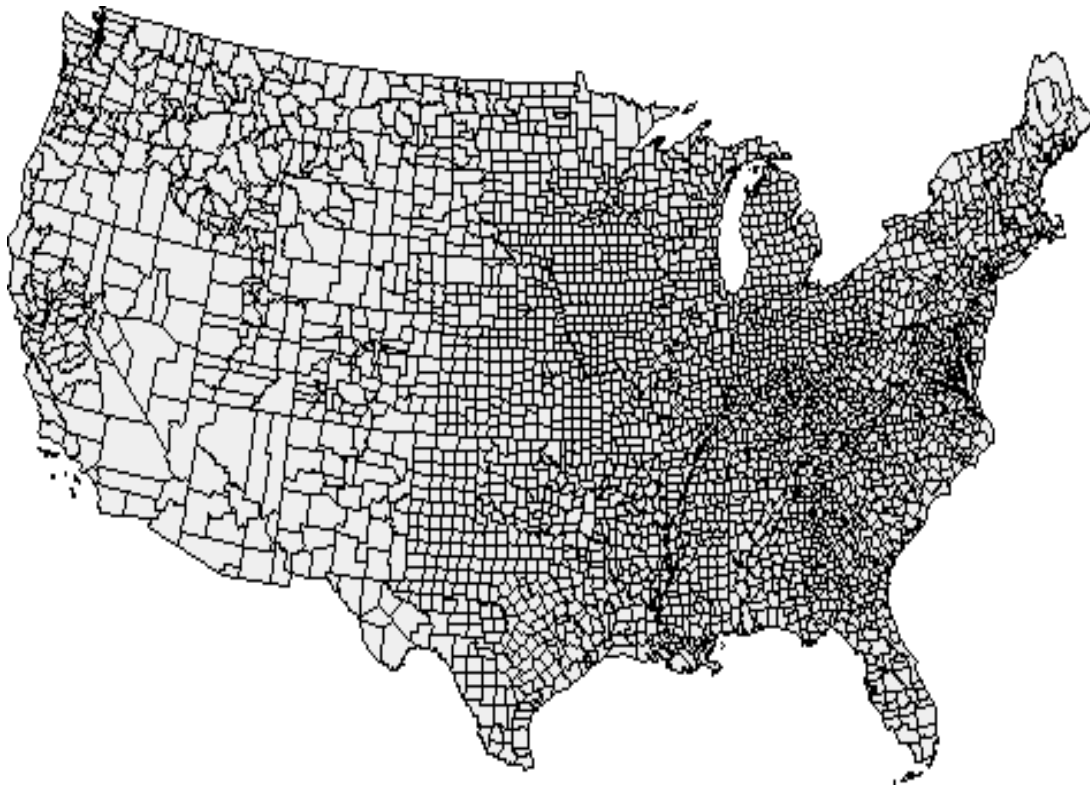
Note: per capita income figures are in natural log form.

Table 3: Summary Statistics for Distribution of U.S. Counties' Log Per Capita Incomes, 1970 vs 1998

<u>Statistic</u>	<u>1970 Per Capita Income</u>	<u>1998 Per Capita Income</u>
Standard Deviation	0.2728	0.2887
Gini Coefficient	0.1666	0.1654
Skewness	-0.2244	1.7240
Kurtosis	3.4334	10.3237

Note: per capita income figures are in natural log form.

Figure 1: Continental U.S. Counties



Note: excluded from the figure, but included in the analysis, are the counties of Alaska and Hawaii.

Figure 2: Distribution of U.S. Counties' Log Per Capita Incomes, 1970 vs 1998

