Learning the inflation target

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Abstract

The New Keynesian model with rational expectations unrealistically predicts that unanticipated credible changes in the inflation target lead to an immediate jump in the inflation level while the output gap is unaffected. We set up a theoretical model where agents learn the behaviour of the economy. In this context, a permanent change in the inflation target leads inflation to respond sluggishly while the output gap is temporarily affected. We extend the model to allow for both *learners* and forward looking agents to coexist. The calibrated model explains quite well transition dynamics during the Volker disinflation.

1 Introduction

The New Keynesian (NK) model with Rational Expectations (RE) became the workhorse of modern monetary economics. This model allows for a short-run role of money since firms do not freely update prices every period. However, the NK model predicts that an unanticipated fully credible disinflation is accomplished with no output cost and that inflation jumps immediately to its new target. Symmetrically, if a credible central bank suddenly announces a higher inflation target (which can be interpreted as an expansionary policy), inflation is still predicted to jump immediately to target and output is predicted to be unchanged. These results are clearly at odds with conventional wisdom; Ball (1995b) shows that disinflations very frequently cause recessions, moreover inflation adjusts slowly to target.

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We model the disinflation as a change in the central bank's inflation target. We maintain the widely used NK model as our benchmark and we examine how the model's predictions change when a proportion of the private sector follows a learning algorithm. Learning techniques are quite suitable for a disinflation environment, as it is reasonable to expect that the private sector takes time to understand the economy's new mechanics. There are two other arguments that motivate a departure from RE. Firstly, Estrella and Fuher (2003) show that RE induce counterfactual observations in a general class of macroeconomic models, which include the NK framework. Secondly, the econometric evidence from survey expectations suggests that expectations are not fully rational and usually a backward looking component is found to be present.

We depart from RE and model the private sector to be econometricians, as it is standard to assume in the learning literature. This learning option seems more in line with reality where agents only possess limited knowledge about the economy's behaviour. The expectation mechanism that we employ has stronger empirical support than RE, and we show that it reconciles the NK model transition dynamics with the data. When a disinflation is pursued, a recession will take place while inflation is reduced gradually to target. If the inflation target is raised, the economy will experience a temporary boom while inflation rises to its new level. Departing from RE does not introduce additional degrees of freedom in the model since we impose an endogenous consistency criteria in our learning mechanism. Besides explaining disinflation stylized facts we set the initial and final inflation target to match the Volker disinflation, making the model dynamics comparable with the data.

The paper is structured as follows: section 2 presents a brief literature review, section 3 introduces the theoretical model and the learning mechanism, section 4 evaluates the model under learning, section 5 presents our main model where *learners* and forward looking agents coexist, and section 6 concludes.

2 Literature Review

In the last two decades, RE became the most widely used expectations mechanism in economics. Nevertheless, RE assume an amazing knowledge of the economy, knowledge that in practice economic agents do not possess. Learning, as in Marcet and Sargent (1989a,b,c), assumes that economic agents behave as econometricians because they have limited information about the underlying economic model. The learning literature was first devoted to analyze convergence properties of learning algorithms. Under certain assumptions it was shown that the learning equilibrium would converge to RE. Learning was not commonly employed as an expectations mechanism *per se*, instead it was seen as a refinement criteria in models with multiple RE equilibria. The literature has only recently started to pose learning as an expectation process. Marcet and Nicolini (2003) show that stylized facts on hyperinflations can be explained when expectations are formed with a learning process. Sargent (1999) assumes the central bank is a *learner* providing an explanation for variations in the inflation rate. Orphanides and Williams (2003) assume that agents are *learners* and study optimal monetary policy in such context. Bischi and Marimon (2001) study stability conditions in a model where agents are *learners* and monetary authorities pursue an inflation target. Our main model assumes that one part of the private sector are *learners* and another part are forward looking. In a similar setup to Bishi and Marimon (2001), Evans *et al* (2001) also address the issue of heterogeneous expectations. Other papers introduce the issue of heterogenous expectations, for instance Giannitsarou (2003) studies a variety of heterogenous forms in a learning framework. Coupling rational or forward looking agents with *learners* or backward looking agents in theoretical models is uncommon, an exception is Evans *et al* (1993).

The NK model with RE can not properly explain disinflation dynamics. However, there have been attempts to reconcile the NK model with the data. Ball (1995a) explains disinflation dynamics by modelling the central bank to be non-credible. The credibility approach has only had limited success. Under RE the private sector can not make systematic mistakes, so on average the private sector will have correct beliefs about the central bank objectives. Consequently, during disinflations, recessions are as likely as booms, which is a counterfactual observation. Combining imperfect credibility with staggered price adjustments yields the prediction that if credibility is sufficiently low a recession will always occur. However, even quite credible central banks (e.g. Germany) did not manage to avoid recessions when pursuing a disinflation. The previous observation casts doubt that credibility alone can explain disinflation dynamics.[1] Galí and Gertler (1999) consider the NK model when the firms that update prices may compute optimal prices or simply use a rule of thumb. This formulation introduces a backward looking component in the model's reduced form. The authors estimate the parameters in the model concluding that backward looking behaviour is statistically significant. In Christiano *et al* (2001), in periods that firms do not compute optimal prices, a rule of thumb indexation is used. The framework of Christiano *et al* (2001) and Galí and Gertler (1999) are very similar and can be seen as special cases of the model presented in this paper.[2] Even though Galí and Gertler (1999) and Christiano *et al* (2001) do not analyze disinflations, both models imply that they are costly. Fuhrer (1997) also proposes on empirical grounds a Phillips curve with forward and backward looking behaviour. The novelty of our model is that the backward looking mechanism is a learning algorithm which is suitable for regime changes in general and for a disinflation episode in particular.

[1] See Clarida and Gertler (1997) for further details.

[2] If one assumes fully rational firms, Christiano *et al* (2001) model becomes equivalent to Yun (1996) model where disinflations are costless; that is to say, the crucial assumption is the departure from RE while indexation is irrelevant.

The learning algorithm uses the same structural form of RE, being theoretically more defensible and providing a more robust justification of backward looking behaviour.

There are other proposals in the literature that can explain disinflations but depart from the NK framework. Fuher and Moore (1995) propose a wage contracting model that can account for costly disinflations. As Roberts (1998) discusses Fuher and Moore (1995) framework is also observationally equivalent to a model where expectations are not fully rational. Mankiw and Reis (2002) consider a flexible price model where agents form expectations rationally but only revise them periodically. In such a setting, disinflations are costly but the assumption of flexible prices is crucial. The authors consider that economic agents face a fixed probability of being able to update their information set. In the NK model, firms face a fixed probability of being able to update prices and when doing so firms always have the most recent information set. If in Mankiw and Reis (2002) setting one would assume sticky prices then there would be no difference from the NK framework where disinflations are costless. It is important to note that the econometric evidence in Galí and Gertler (1999) suggests that there is some degree of price stickiness and one should not ignore this empirical observation. Ball (1994) considered a model where prices are not fixed and firms could choose a predetermined time-varying path for prices until the next adjustment, as in Mankiw and Reis (2002). Ball (1994) showed that this feature would be an improvement upon previous specifications in his paper but remarked that "...time-varying prices are not a convincing explanation because they are uncommon in the real world" and that "Economists should aim for a theory of disinflation that is consistent with the prevalence of fixed prices.". The learning algorithm presented in this paper implies costly disinflations with sticky and flexible prices.

Erceg and Levine (2003) build a model to explain disinflation dynamics. The authors assume that wages and prices are determined by staggered four-quarter nominal contracts, capital is subject to quadratic adjustment costs, and the inflation target is both subject to transitory and persistence shocks. In addition, the private sector makes use of the Kalman filter to infer the value of the unobservable inflation target. The authors analyze the Volker disinflation concluding that the model can account for the empirical observations in the data. We do stress that our paper maintains the widely used NK benchmark avoiding building a new model to explain disinflations. Moreover, Erceg and Levine (2003) assumptions introduce additional degrees of freedom making it easier to match the model with the data.

Modern macroeconomic models have been reluctant to assume departures from RE. The common arguments are that too many degrees of freedom are introduced, agent's expectations are inconsistent with the model and that expectations formation does not change with policy. We address the previous criticisms by considering a learning algorithm where no degrees of freedom are introduced.

The learning algorithm lies in the class with the same functional form of RE and the only free parameter is made endogenous to the model and to policy. Also the literature has recognized that during regime changes RE are unrealistic and that a learning mechanism is more suitable, Clarida *et al* (1999) sketch the previous argument and specifically point out that a disinflation is an example of a regime change that should be analyzed under learning. The most crucial observation that motivated us to consider a small departure from RE is that the econometric evidence suggests that expectations are not completely rational. Roberts (1998) uses survey expectations on inflation data to show that expectations are not perfectly rational. The empirical literature does not support the view that expectations are completely rational but also does not support large deviations from rationality.[3] The expectation mechanism employed in the present paper is consistent with the empirical literature.

3 Description of the model

We use the NK model as derived in Woodford (2003) and first suggested by Calvo (1983). Since the derivation of the model is available in the previous references we will just describe the reduced form equations. The Aggregate Supply (AS) curve is given by

$$\pi_t = \kappa z_t + \beta E_t \pi_{t+1} + u_t \tag{1}$$

where π_t denotes inflation, z_t denotes the output gap and u_t is an iid shock. It is common to denote the described shock in the AS curve by cost push shock. The Investment-Saving (IS) curve is described as

$$z_t = E_t z_{t+1} - \sigma^{-1}(r_t - r_t^n - E_t \pi_{t+1}) \tag{2}$$

where r_t is the interest rate set by the central bank and r_t^n is the natural interest rate. Usually it is assumed that the natural interest rate follows an AR(1) process

$$r_t^n = \rho r_{t-1}^n + \varepsilon_t \tag{3}$$

The model is closed with an equation for the interest rate. We will assume a Taylor rule[4] of the type

$$r_t = \pi^* + \varphi_\pi(\pi_t - \pi^*) + \varphi_z z_t \tag{4}$$

where the constant term in the interest rate rule is set to make the inflation target π^* consistent in equilibrium. Plugging the interest rate rule in the IS

[3]For further details on this point see Roberts (1998) and the literature reviewed therein.
[4]For details see Taylor (1993).

equation and rearranging the system one obtains

$$y_t = a + bE_t y_{t+1} + \varkappa r_t^n \tag{5}$$

where $y_t = [z_t, \pi_t]'$, $a = \frac{1}{\sigma+\varphi_z+\kappa\varphi_\pi}[\pi^*(\varphi_\pi - 1), \kappa\pi^*(\varphi_\pi - 1)]'$, $\chi = \frac{1}{\sigma+\varphi_z+\kappa\varphi_\pi}[1, \kappa]'$ and

$$b = \frac{1}{\sigma + \varphi_z + \kappa\varphi_\pi} \begin{bmatrix} \sigma & 1 - \beta\varphi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \varphi_z) \end{bmatrix} \tag{6}$$

Taking the results of Blanchard and Kahn (1980), Bullard and Mitra (2002) find that if $\kappa(\varphi_\pi - 1) + (1 - \beta)\varphi_z > 0$ then there is a unique solution to the model described in equation 5. Note that Bullard and Mitra (2002) consider a model where $a = 0$. If a is a constant matrix, condition (1c) in Blanchard and Kahn (1980) must be satisfied. Since this condition is trivially met, uniqueness conditions are equivalent.

When the natural interest rate is observed and follows an AR(1) process the Minimal State Variable (MSV) solution takes the form of $y_t = A + Cr_t^n$.[5] We assume that economic agents do not know the RE solution but do know the functional form of the MSV solution. At each moment in time, the private sector will use the available data to estimate A and C with a learning algorithm; we denote the estimates obtained at time t of A by A_t and of C by C_t. Therefore, at time t the private sector will think that the economy behaves as $y_t = A_t + C_t r_t^n$, this equation is known as the Perceived Law of Motion (PLM). Given the estimates of the private sector, expectations are formed as $E_t y_{t+1} = A_t + C_t \rho r_t^n$. Expectations influence endogenous variables through equation 5. Inserting expectations in equation 5 and rearranging the terms to fit the functional form of the MSV solution yields $y_t = a + bA_t + (bC_t\rho + \varkappa)r_t^n$. The later equation describes the Actual Law of Motion (ALM) of the economy. The mapping from the PLM to the ALM is called the T-map, and in this model it is given by

$$T(A, C) = (a + bA, bC\rho + \chi) \tag{7}$$

where we dropped the time subscripts for convenience. The fixed point in the T-map is the RE solution, where the PLM and ALM are equal. So, we assume that agents can not directly solve for RE but know the functional form of the MSV and using a learning algorithm update A_t and C_t.

[5]Usually there is more than one functional form for a solution to a RE model. That is to say, when solving for RE one first imposes a functional form for the solution, and then solves for the parameters in that functional form. The MSV solution is the functional form capable of solving the RE model and that has the minimum number of variables. Note that Blanchard and Kahn (1980) determinacy approach is restricted to the class of MSV solutions. For further details on MSV solutions see McCallum (1983).

It is common in the learning literature to analyze if a given solution is E-stable. The E-stability concept means that at a given fixed point of the T-map the following equation is locally stable

$$\frac{d}{d\tau}(A, C) = T(A, C) - (A, C) \tag{8}$$

Frequently, if a solution is E-stable then the learning equilibrium converges to the RE solution. Bullard and Mitra (2002) compute the E-stability conditions for the model where r_t^n is an AR(1) process. Interestingly, for our model the condition for E-stability is equal to the condition for uniqueness.

We still did not describe the way that the parameters are updated, we assume that private agents use a Recursive Least Squares (RLS) formula given by

$$\phi_t = \phi_{t-1} + \alpha_t R_t^{-1} x_{t-1}(y_{t-1} - \phi_{t-1}' x_{t-1})' \tag{9}$$
$$R_t = R_{t-1} + \alpha_t (x_{t-1} x_{t-1}' - R_{t-1}) \tag{10}$$

where $\phi_t = \begin{bmatrix} A_{z,t} \, A_{\pi,t} \\ C_{z,t} \, C_{\pi,t} \end{bmatrix} = \begin{bmatrix} A_t' \\ C_t' \end{bmatrix}$, $x_{t-1} = \begin{bmatrix} 1 \\ r_{t-1}^n \end{bmatrix}$.

If $\alpha_t = t^{-1}$ RLS is equivalent to Ordinary Least Squares. It is also common in the literature to assume $\alpha_t = \alpha$, where $\alpha \in (0, 1)$. Assuming a constant tracking parameter means that recent observations are given more weight, such rules may be optimal under regime changes. Later we will consider different tracking parameters for the equations relative to inflation and output.

In section 5.2, we will consider the NK model when the natural interest rate is i.i.d. normally distributed. The appendix shows that, for our purposes, both cases are similar but when the natural interest rate is i.i.d. convergence is slightly faster. The MSV solution when r_t^n is i.i.d. takes the form $y_t = [A_z, A_\pi]'$. At time t agents have a Perceived Law of Motion (PLM) of the type $y_t = [A_{z,t}, A_{\pi,t}]'$ and so $E_t y_{t+1} = [A_{z,t}, A_{\pi,t}]'$ inserting the later expectations in the system one obtains the T-map

$$T(A) = (a + bA) \tag{11}$$

It is easy to check that when r_t^n is i.i.d. the determinacy and E-stability conditions are exactly the same as before. In this model the learning algorithm is simpler. At time t the private sector runs the regressions

$$A_{\pi,t} = A_{\pi,t-1} + \alpha_t(\pi_{t-1} - A_{\pi,t-1}) \tag{12}$$
$$A_{z,t} = A_{z,t-1} + \alpha_t(z_{t-1} - A_{z,t-1}) \tag{13}$$

If $\alpha_t = t^{-1}$ then the specified rules are equivalent to

$$A_{\pi,t} = \frac{\sum_{i=0}^{t-1} \pi_i}{t} \tag{14}$$

$$A_{z,t} = \frac{\sum_{i=0}^{t-1} z_i}{t} \tag{15}$$

Since agents recognize that the MSV solution for inflation and the output gap only depend on a constant the private sector just computes an average of realized values.

4 Model evaluation

We will analyze how the model behaves during a disinflation. We will use the parameters suggested for the USA by Clarida *et al.* (2000) where $\kappa = 0.3$, $\sigma = 1$, and $\rho = 0.35$. There is no consensus in the literature on the value of κ, the value of 0.3 is consistent with Roberts (1995), which used survey expectations in the estimation procedure. Usually β is set to be a number very close to one, in later simulations we use $\beta = 1$.[6] For the interest rate rule we assume $\varphi_\pi = 1.5$ and $\varphi_z = 0.5$ as in "the" Taylor rule.[7] For these parameters the solution is unique and E-stable. We assume that at the beginning of the disinflation the economy is at steady state and inflation is 15.3%, then the inflation target is lowered to 3.7%. These values correspond to filtered inflation 1980 first quarter and 1984 fourth quarter as can be observed in figure 1, using other values does not change the results.[8] The NK model is linearized around a zero steady state value of inflation. If one would assume a disinflation from 11.5% to 0%, i.e. keeping the magnitude of the disinflation, then the simulated series would only differ by a constant. In accordance with the previous observation, Ball (1995b) concludes that the initial level of inflation has no clear effect on the sacrifice ratio. We keep the option of a disinflation to a non-zero inflation target to make the model dynamics comparable with the Volker disinflation.

During the Volker disinflation the economy went into recession, as can be seen by observing the output gap on figure 2. Figure 3 computes the Federal Funds interest rate, which rose significantly in this period to reduce the inflation rate. Ball (1995b) reports a sacrifice ratio for the Volker disinflation using quarterly data of 0.018. The sacrifice ratio is computed as the sum of the annualized output gap divided by the change in inflation. For the data series presented in this paper,

[6]This option was also taken in Mankiw and Reis (2002) and Ball (1994). In practical terms, setting the discount factor to unity makes a zero output gap consistent with a steady state value of inflation above zero. Results presented later in the paper are very similar if one considers $\beta = 0.99$, as in Clarida *et al.*(2000).

[7]Clarida *et al.* (2000) present evidence that the Taylor principle was satisfied during a time period that contains the Volker disinflation.

[8]The data appendix contains the description of the data series used throughout the paper.

and treating as zero the observations where the output gap is positive, the sacrifice ratio is 0.006. Ball (1995b) computations of the sacrifice ratio pretend to identify the relative costs of different episodes in a systematic way. The author draws attention that the sacrifice ratio is extremely sensitive to the output gap measure and the assumptions used.[9]

For exposition purposes we will evaluate the model when the entire private sector behaves as *learners*, our main model will be presented in a later section. We set the standard deviation of the cost push shock and the innovation in the natural interest rate to be 0.02. In figure 4, we plot the average inflation and output gap generated by 5000 simulations of the model for ($\alpha_{inf} = 0.5, \alpha_{out} = 0.5$), figure 5 computes the interest rate. Even though, we allow our *learners* to use different tracking parameters for the equations relative to inflation and relative to output it is optimal for them to use the same value of 0.5. Qualitatively the model accounts well for what one would expect, inflation is reduced sluggishly converging to the new target and in the short run the economy experiences a recession. Doing the symmetric experience, i.e. raising the inflation target, inflation rises sluggishly to the target while the economy experiences a boom. As Mankiw and Reis (2002) emphasize, in the NK model under RE an unanticipated credible disinflation results in an immediate reduction of inflation and there are no output costs. It is usually inferred that price stickiness is translated to inflation stickiness but such inference is not correct. However, as this section shows once learning is introduced in the NK model, transition dynamics become similar to the data. The drawbacks of this first model is that convergence is too slow and that expectations are completely backward looking.

In order to obtain a more precise notion of convergence we will report some statistics. We will run the model 5000 times and compute the mean of all realizations. Firstly we will report the first time period when mean inflation and mean output gap differ by less than 0.005 from their steady state levels. Secondly, we will also report the following measure that takes into account volatility,

$$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n} - x^*)^2}{N}} \tag{16}$$

where N is the total number of simulations, $x_{t,n}$ is the n-th realization at a

[9]Indeed Ball (1995b) computes the output gap in an "extreme" way, the author assumes that trend output grows log-linearly from the start of the disinflation to the end plus four quarters. The output gap is the difference from log-output to trend output. The justification of the author is that at the peak inflation, the change in inflation is zero and the natural level of output is often defined as the level consistent with stable inflation. The author also assumes that output is back at its trend only 4 quarters after the trough in inflation. Even though, these definitions do serve the authors purposes of identifying the relative costs of disinflations they must be taken with care when considering these values as an absolute measure. As a matter of example, by assuming that trend output grows log-linearly from peak to trough the sacrifice ratio drops to 0.012.

chosen t and x^* is steady state value of x. Note that the following formula can be decomposed in

$$\sqrt[2]{\frac{\sum_{n=1}^{N}[(x_{t,n}-\bar{x}_t)^2+(\bar{x}_t-x^*)^2+2(x_{t,n}-\bar{x}_t)(\bar{x}_t-x^*)]}{N}} \tag{17}$$

where $\bar{x}_t$ is the mean value across realizations at time t. The third term is zero and for large enough t one expects the second term to be very small. We will report the computations for $t = 16$ and $t = 100$. For the previous model and for $N = 5000$, the following table summarizes convergence statistics.

	out		inf	
	$t = 16$	$t = 100$	$t = 16$	$t = 100$
$\bar{x}_t$	-0.0283	-0.0003	0.0737	0.0377
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-\bar{x}_t)^2}{N}}$	0.0246	0.0246	0.0231	0.0222
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-x^*)^2}{N}}$	0.0375	0.0246	0.0431	0.0222
$t : \bar{x}_t - x_t < 0.005$	36		37	

Table 1

At $t = 16$ average inflation and average output are still far from steady state, while at $t = 100$ the series are quite close to steady state. The fourth row in the table shows that variability around the mean is quite similar for $t = 16$ and $t = 100$. The fourth and fifth rows are very similar for $t = 100$, this is because mean values are very close to the steady state. The sacrifice ratio assuming that the disinflation lasts 16 and 20 quarters is 0.012 and 0.014 respectively. These results broadly confirm that convergence is relatively slow when compared with the Volker disinflation. In a later section, we will consider a model where some agents are *learners* while others are forward looking. Such model will converge faster and in line with the data. The next subsection determines the consistent tracking parameters.

4.1 Learning speed

At this stage, our main uncertainty relates to the vector of the tracking parameters $\alpha = (\alpha_{inf}, \alpha_{out})$. To make α endogenous we will employ the concept of Internal Consistency (IC) first introduced by Marcet and Nicolini (2003). Before introducing the formal concept we need some notation. Let $y_t(\alpha)$ denote the values generated in the economy when economic agents use α as tracking parameters, $E_t^{\alpha'} y_{t+1}(\alpha)$ denote the expected value at t of y_{t+1} when all agents in the economy use α as tracking parameters and the predictions are made using α'. So, for a given time horizon T and a small number close to zero ε the vector α is consistent if

$$E\left(\frac{1}{T}\sum_{t=1}^{T}(y_{t+1}(\alpha)-E_t^{\alpha}y_{t+1}(\alpha))^2\right)\leq\min_{\alpha'}E\left(\frac{1}{T}\sum_{t=1}^{T}(y_{t+1}(\alpha)-E_t^{\alpha'}y_{t+1}(\alpha))^2\right)+\varepsilon \tag{18}$$

That is to say, α is internally consistent if all economic agents use it and predictions made by these tracking parameters are "good enough" when compared with predictions made by other tracking parameters α'. We chose the time horizon to be 20 quarters, which is the duration of the Volker disinflation episode. We computed results for $\varepsilon = 0.00001$ which approximately corresponds to 2% of the total MSE for output and inflation when the tracking parameters are $(\alpha_{inf} = 0.5, \alpha_{out} = 0.5)$. Expectations on equation 18 are computed by Monte Carlo integration using 500 simulations. Even though it is common to assume a unique tracking parameter, during the disinflation the private sector may realize that the output gap is more stable than the level of inflation, hence the private sector may choose different tracking parameters for output and inflation.[10] We present results for inflation in figure 6 and for output in figure 7. The horizontal axis represents the tracking parameters $(\alpha_{inf}, \alpha_{out})$, and the vertical axis represents $(\alpha'_{inf}, \alpha'_{out})$. In figure 6 a value of 1 means that condition 18 is met, a value of 0 means that $(\alpha'_{inf}, \alpha'_{out})$ is inefficient given that all agents use $(\alpha_{inf}, \alpha_{out})$. If a 1 occurs in the diagonal it means that $(\alpha_{inf}, \alpha_{out})$ is internally consistent. By examining figure 6 one can see that when agents predict inflation the internally consistent tracking parameters are 0.5, 0.7 and 0.9 no matter what the tracking parameter for output is. In the last column, one can see that 0.5 predicts well more often. By examining figure 7, when agents only predict output the optimal values are always 0.5. The parameters found in our analysis are higher than the values that the literature usually assumes, the main reason is that the large bulk of the literature does not focus on regime changes. The appendix on the learning speed analyzes in further detail the consistency criteria and provides sensitivity analysis.

5 *Rationals* and *learners*

5.1 Motivation of the model

Up to now, we have analyzed an economy inhabited solely by *learners*, which use past data to form expectations. The econometric evidence from surveys shows

[10]For the estimation of the variance-covariance matrix the private sector is assumed to use a low tracking parameter of 0.05. It is optimal for *learners* to do so because even with the regime change the steady state values of this matrix are not altered. Assuming other values for the variance-covariance matrix tracking parameter does not change the results.

that expectations incorporate more information than the one contained in past data. Note that α was determined using the IC requirement making our *learners* not to be purely backward looking agents. Nevertheless, we will build a model where a proportion of the private sector clearly forms expectations using more information than the one contained in past data. In a simplified version, we consider that a proportion of the private sector is completely rational. In a more elaborate model, we introduce a simplification that enables an explicit solution but where a proportion of the private sector is forward looking although not completely rational. Introducing agents with forward looking behaviour makes aggregate expectations to have both an adaptive and a forward looking component as it is observed in survey data.

The literature has proposed models, on empirical grounds, where the private sector has both backward and forward looking behaviour. Even though the capability of explaining disinflations has not been explicitly studied, such setups imply costly disinflations. In the model that we present in this section, part of the private sector uses a learning algorithm. In fact, this introduces a foundation for backward looking behaviour, i.e. if agents do not have sufficient knowledge to compute RE, they can still use available data optimally to form expectations. Doing so, backward looking behaviour is introduced. During the disinflation, using a consistency criterion we determine endogenously the optimal tracking parameter, leaving no free parameters.

5.1.1 Comparison with previous models

We will first analyze both the evidence suggested in Roberts (1997,1998), Galí and Gertler (1999) and Christiano *et al* (2001) as special cases where heterogeneous learning rules are followed by the private sector. After having established the link between these papers and our approach we will develop a theoretical model where both *learners* and forward looking agents coexist.

Remember that in our model with no autocorrelated shocks, inflation expectations are given by

$$E_t\pi_{t+1} = E_{t-1}\pi_t + \alpha_{inf}(\pi_{t-1} - E_{t-1}\pi_t) \tag{19}$$

Considering a constant tracking parameter and solving backwards one can write expectations of inflation as

$$E_t\pi_{t+1} = \alpha_{inf}\sum_{j=0}^{\infty}(1-\alpha_{inf})^j(\pi_{t-1-j}) \tag{20}$$

Roberts (1997,1998) analyzes the following equation to describe the evolution of inflation expectations

$$S_t\pi_{t+1} = P(L)\pi_{t-1} + [1 - P(1)]M_t\pi_{t+1} \tag{21}$$

Survey expectations $S_t\pi_{t+1}$ are assumed to be a weighted average between rational (or Mathematical) expectations $M_t\pi_{t+1}$ and lagged values of inflation, where P(.) is a polynomial and L is the lag operator. Roberts (1998) says that this equation is a fairly good description of the Livingston and Michigan survey on inflation expectations. Note that if one would say that agents form expectations by using a weighted average between tracking and rational expectations one would write

$$S_t\pi_{t+1} = \psi\alpha_{inf}\sum_{j=0}^{\infty}(1-\alpha_{inf})^j(\pi_{t-1-j}) + (1-\psi)M_t\pi_{t+1} \tag{22}$$

or allowing for varying tracking parameters

$$S_t\pi_{t+1} = \psi P(L)(\pi_{t-1}) + (1-\psi)M_t\pi_{t+1} \tag{23}$$

A difference of minor importance between equation 23 and Roberts (1997,1998) approach is that a priori we expect the polynomial P(L) to place less weight on older inflation data. A bigger difference is that the weight on rational expectations is $(1-\psi)$ and not $1-P(1)$. Interestingly, Roberts (1997) concluded that the weight $1-P(1)$ was not a convincing estimate.

Roberts (1998) also proposes another equation for the formation of expectations

$$S_t\pi_{t+1} = \psi S_{t-1}\pi_t + (1-\psi)M_t\pi_t \tag{24}$$

If we would consider the coexistence of *trackers* and *rationals* we could write

$$S_t\pi_{t+1} = \psi\left[S_{t-1}\pi_t(1-\alpha_{inf}) + \alpha_{inf}\pi_{t-1}\right] + (1-\psi)M_t\pi_{t+1} \tag{25}$$

So indeed the two equations are very similar. The only difference is that Roberts (1998) does not include in his equations last period inflation which theoretically should be given a small weight.[11] From this discussion we can see that the analysis of Roberts (1997,1998) can be rethought to be close to our framework where *trackers* and *rationals* coexist.

Clarida *et al* (1999) also point out that the NK Phillips curve is somewhat at odds with the data but that the following generalization of the IS and AS equations provide better outcomes

$$\pi_t = kz_t + \tau\pi_{t-1} + \beta(1-\tau)E_t\pi_{t+1} \tag{26}$$

$$z_t = \theta z_{t-1} + (1-\theta)E_t z_{t+1} - \sigma^{-1}(r_t - r_t^n - E_t\pi_{t+1}) \tag{27}$$

[11]We considered before $\alpha_{inf} = 0.5$ but if a regime change is not in place the optimal α will be very small. In fact, $\alpha = t^{-1}$ becomes optimal when there are no regime changes.

where $\tau, \theta \in (0,1)$. Once again these equations are similar to our framework, the above equations do assume that a proportion of agents follow adaptive expectations $E_t\pi_{t+1} = \pi_{t-1}$ and others follow rational expectations. Considering our suggested framework we would write

$$\pi_t = kz_t + \beta\tau\alpha(L)\pi_{t-1} + \beta(1-\tau)E_t\pi_{t+1} \tag{28}$$

$$z_t = \theta\alpha(L)z_{t-1} + (1-\theta)E_t z_{t+1} - \\ - \sigma^{-1}(r_t - r_t^n - \tau\alpha(L)\pi_{t-1} - (1-\tau)E_t\pi_{t+1}) \tag{29}$$

In practical terms it is the AS equation that is subject to empirical tests and since in empirical framework it is often assumed that $\beta = 1$, our AS equation is a generalization of the standard backward-forward looking AS curve.

Galí and Gertler (1999) argue correctly that the NK Phillips curve already assumes a relationship between marginal cost and the output gap. When deriving the NK Phillips curve an intermediate step yields an equation with marginal costs

$$\pi_t = \zeta mc_t + \beta E_t\pi_{t+1} \tag{30}$$

Using this equation it is shown that known problems in the literature with the NK Phillips curve disappear. The authors address the issue that some firms may have adaptive expectations and estimate the following equation

$$\pi_t = \zeta mc_t + \gamma_b\pi_{t-1} + \gamma_f E_t\pi_{t+1} \tag{31}$$

where $\gamma_b + \gamma_f = 1$ if $\beta = 1$. The authors conclude that the fraction of firms with backward looking behavior is smaller than the ones with forward looking behavior, but statistically significant. So once again the results of Galí and Gertler (1999) suggest that the New Keynesian model has both adaptive and rational expectations. Christiano *et al* (2001) assume that the firms that do not optimize prices adjust them using a rule of thumb, that is prices are indexed to previous period inflation. The model in reduced equation is very similar to equation 31, with $\gamma_b = \gamma_f = 0.5$. Yun (1996) proposed a model of indexation assuming fully rational expectations. While Christiano *et al* (2001) model has several nice properties with respect to inflation persistence, Yun (1996) model does not; that is to say the crucial hypothesis is not indexation but the departure from RE.

According to Roberts (1998) the proportion of backward looking agents is estimated to be between 0.2 and 0.4 depending on the set of instruments used and the sample period. According to Galí and Gertler (1999) the weight on backward looking behaviour is also estimated to be between 0.2 and 0.4.

5.2 A theoretical model of learning and rational expectations

In the previous section we showed that the coexistence of backward and forward looking agents has strong empirical support. This section analyzes the NK model with i.i.d shocks when both *learners* and *rationals* coexist. Assuming that *learners* use a constant tracking parameter as in our disinflation simulation, this model allows for an explicit solution for rational agents.

The model in reduced form yields

$$y_t = a + bE_t y_{t+1} + \varkappa r_t^n \tag{32}$$

but now assume that aggregate expectations are a weighted average between expectations of *Learners* and *Rationals*

$$E_t y_{t+1} = \psi E_t^L y_{t+1} + (1-\psi) E_t^R y_{t+1} \tag{33}$$

Agents, that follow tracking, form their expectations as

$$E_t^L y_{t+1} = E_{t-1}^L y_t + \alpha(y_{t-1} - E_{t-1}^L y_t) \tag{34}$$

Now we assume that a fixed proportion of agents are completely rational and therefore they do know how *learners* form expectations and the proportion of *learners* and *rationals* in the economy.

In order to solve the rational agents problem we conjecture that the MSV solution takes the form

$$y_t = A + BE_t^L y_{t+1} \tag{35}$$

Hence

$$y_{t+1} = A + BE_{t+1}^L y_{t+2} \tag{36}$$

$$y_{t+1} = A + B(E_t^L y_{t+1}(1-\alpha) + \alpha y_t) \tag{37}$$

Since $E_t^R y_{t+1} = A + B(E_t^L y_{t+1}(1-\alpha) + \alpha y_t)$ the ALM will be

$$y_t = a + b\psi E_t^L y_{t+1} + b(1-\psi)A + b(1-\psi)B(E_t^L y_{t+1}(1-\alpha) + \alpha y_t) + \varkappa r_t^n \tag{38}$$

Rearranging terms

$$\begin{aligned} y_t = {} & (I - b(1-\psi)B\alpha)^{-1}[(a + b(1-\psi)A)] + \\ & + (I - b(1-\psi)B\alpha)^{-1}[(b\psi + b(1-\psi)(1-\alpha)B)E_t^L y_{t+1}] \end{aligned} \tag{39}$$

So to solve for RE we need to consider the following two equations

$$A = (I - b(1-\psi)B\alpha)^{-1}(a + b(1-\psi)A) \tag{40}$$

$$B = (I - b(1-\psi)B\alpha)^{-1}(b\psi + b(1-\psi)(1-\alpha)B) \tag{41}$$

The second equation is a quadratic matrix equation on B that can be solved using the generalized eigenvalues method.[12] After computing B solving for A is a trivial problem.

The MSV solution changed when we introduced rational agents. Adaptive *learners* could then realize that their expectations are taken into account by *rationals* ... so *learners* should think that *rationals* know what they think. This would make *learners* to estimate a different MSV and in its turn *rationals* would estimate again another MSV and ... this problem would be taken to infinity. To avoid this complication we will show that if *learners* behave as if the *rationals* did not exist then their expectations do converge to an equilibrium. For this economy, stability conditions under tracking can be computed by examining directly the learning algorithm.[13] The learning algorithm is

$$E_t^L y_{t+1} = E_{t-1}^L y_t + \alpha(y_{t-1} - E_{t-1}^L y_t) \tag{42}$$

Using equations 39-41 the equation just described can be written as

$$E_t^L y_{t+1} = (I(1-\alpha) + \alpha B)E_{t-1}^L y_t + \alpha A + \alpha(I - b(1-\psi)B\alpha)^{-1}\varkappa r_{t-1}^n \tag{43}$$

The previous system will be stable as long as the matrix $(I(1-\alpha)+\alpha B)$ has eigenvalues with absolute value smaller than one. The asymptotic mean of expectations is $Ey = (I-B)^{-1}A$ which corresponds to the RE equilibrium $[0,\pi^*]$. The asymptotic variance is $vec\sum = (I - (I(1-\alpha)+\alpha B)\otimes(I(1-\alpha)+\alpha B))^{-1}vec(\alpha(I-b(1-\psi)B\alpha)^{-1}\varkappa\sigma(\alpha(I-b(1-\psi)B\alpha)^{-1}\varkappa)')$. Using the normality assumption for r_t^n one concludes $E_t^L y_{t+1} \sim N([0,\pi^*],\sum)$.

So, in this model aggregate expectations are somewhat backward looking which makes a disinflation to be costly, but rational private agents become aware that a regime change occurred. Rational agents are aware that not everybody is rational and predict inflation and the output gap accordingly. If $\psi = 0$ then rational agents predict that inflation goes immediately to its new target and that no recession occurs. On the other hand, if the proportion of *learners* is nearly one then *rationals* should predict as *learners* do.

We assume that the weight on backward looking behavior is 0.3, which is the midpoint in the results of Galí and Gertler (1999) and Roberts (1998). In our first model, the consistent tracking parameters were estimated to be $\alpha_{inf} = \alpha_{out} = 0.5$. Nevertheless, the presence of rational agents could change the consistent tracking parameter, for the time being let's assume the same tracking parameters.[14] For the parametrization presented before and considering that $\psi = 0.3$ and $\alpha = 0.5$ the learning algorithm is indeed stable. Figure 8 plots a 5000 average for the paths

[12] For a discussion see Uhlig (1999)

[13] This approach closely follows Evans and Honkapohja (2001) section 3.3.

[14] We will skip the internal consistency analysis for this model. The reason is that the model in the next subsection is a more elaborate version of this one and we will proceed to internal consistency analysis therein.

of inflation and the output gap, figure 9 plots the interest rate.[15] Convergence is faster as the number of rational agents and the tracking parameter increase. The following table presents convergence statistics, which show that the introduction of rational agents accelerates convergence.[16]

	out		inf	
	$t = 16$	$t = 100$	$t = 16$	$t = 100$
$\bar{x}_t$	0.0003	0	0.0376	0.0378
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-\bar{x}_t)^2}{N}}$	0.0208	0.0208	0.0247	0.0249
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-x^*)^2}{N}}$	0.0208	0.0208	0.0248	0.0249
$t : \bar{x}_t - x_t < 0.005$	8		10	

Table 3

5.3 ***Learners*** **and** ***near rationals***

In the last section, the fact that agents were only predicting a constant for output and inflation made the analysis of coupling *learners* and *rationals* easier. It is possible to extend the framework where *learners* and *rationals* coexist to more complicated models, but in principle one has to recur to numerical techniques to do so. The technical difficulty with fully rational agents is that when predicting future variables, rational agents must take into account the learners' expectations formation process. Once *learners* estimate more than an average an explicit MSV where a fixed point exists is not so easily obtained. In addition, when a numerical solution is used one can not analytically prove long run convergence properties. In this paper we opt to introduce a simplifying assumption that will enable an explicit solution. We make the assumption that *rationals* do not update the way learners think when predicting future variables, for future reference we will denote these agents as *near rationals*. *Near rationals* will still be aware of regime shifts and if these agents would be the sole inhabitants of our NK economy disinflations would still be costless.

In this section we will assume again that the natural interest rate is autocorrelated, as it is standard to assume in the NK model. If *learners* proportion in the economy is given by ψ then the economy would now be described by

$$y_t = a + b\psi E_t^L y_{t+1} + b(1-\psi)E_t^R y_{t+1} + \varkappa r_t^n \tag{44}$$

[15] We assume that two shocks affect the economy, one shock affects the AS equation and the other the IS equation. The standard deviation of both shocks was set to be 0.02.

[16] The appendix examines convergence for an economy with i.i.d. shocks but only populated with *learners*.

Near rationals will solve the following equation

$$y_t = A^R + B^R A_t^L + C^R r_t^n + D^R C_t^L r_t^n \tag{45}$$

where variables with R upper script are variables for *near rationals* and variables with L upper script are variables for learners MSV, $y_t = A_t^L + C_t^L r_t^n$. Expectations for *near rationals* are formed as $E_t^R y_{t+1} = A^R + B^R A_t^L + C^R \rho r_t^n + D^R C_t^L \rho r_t^n$, i.e. the way that learners think is not taken to evolve. So plugging these expectations back in the ALM yields

$$\begin{aligned} y_t = a + b(1-\psi)A^R + b(I\psi + (1-\psi)B^R)A_t^L + \\ + (b(1-\psi)C^R\rho + \varkappa)r_t^n + b(I\psi + (1-\psi)D^R)\rho C_t^L r_t^n \end{aligned} \tag{46}$$

The solution must satisfy $A^R = (I - b(1-\psi))^{-1}a$, $B^R = (I - b(1-\psi))^{-1}b\psi$, $C^R = (I - b(1-\psi)\rho)^{-1}\varkappa$, $D^R = (I - b(1-\psi)\rho)^{-1}b\psi\rho$. The way that this problem was solved ensures that once learners converge to RE equilibrium so do *near rationals*. Hence, the relevant question to be posed is whether *learner's* expectations will converge to equilibrium.

The relevant T-map is

$$\begin{aligned} T(A^L, C^L) = (a + b(1-\psi)A^R + b(I\psi + (1-\psi)B^R)A^L, \\ b(1-\psi)C^R\rho + b(I\psi + (1-\psi)D^R)C^L\rho + \chi) \end{aligned} \tag{47}$$

The fixed point in the T-map is

$$A^L = (I - b(I\psi + (1-\psi)B^R))^{-1}(a + b(1-\psi)A^R) \tag{48}$$

$$C^L = (I - b(I\psi + (1-\psi)D^R)\rho)^{-1}(\varkappa + b(1-\psi)C^R\rho)) \tag{49}$$

The fixed point for A^L and C^L corresponds to the RE equilibrium, namely $A^L = [0, \pi^*]$. This result is not surprising, it means that the presence of *near rationals* does not alter the long run behavior of the economy.

E-stability is obtained if the matrices $b(I\psi + (1-\psi)B^R) - I$ and $b(I\psi + (1-\psi)D^R)\rho - I$ have all eigenvalues with negative real parts. The previous conditions can be written as $B^R - I$, $D^R - I$.

Considering the same parametrization as before and $\psi = 0.3, \rho = 0.35$ yields

$$B^R = \begin{bmatrix} 0.1875 & -0.3125 \\ 0.1875 & 0.6875 \end{bmatrix}, D^R = \begin{bmatrix} 0.0599 & -0.0397 \\ 0.0238 & 0.1233 \end{bmatrix} \tag{50}$$

The previous matrices have all eigenvalues with real parts smaller than one, hence the fixed point is E-stable.[17]

[17]Evans and Honkapohja (1998) showed that E-stability implies local convergence of the

We conducted the internal consistency analysis for this economy, the details of the analysis are presented in the appendix. We present results in figures 10 and 11, which have a very clear interpretation. For inflation, $\alpha_{inf} = 0.5$ is always optimal. For output, $\alpha_{out} = 0.3$, is always optimal. The optimal value for output is lower in this model compared to the first model in this paper. The reason is that *near rationals* make the economy go to its new steady state relatively fast and hence recession is not as sharp, making the output gap more stable.

With the consistent tracking parameters at hand we can assess how far from rationality *near rationals* are. The former agents do commit prediction errors during the disinflation episode, systematically over-predicting inflation. We simulated the model 5000 times and computed average realizations and average predictions by *near rationals*. During the first 25 periods after the beginning of the disinflation, average prediction error is 0.15% for output and 0.34% for inflation. Prediction errors are bigger for initial periods, being the maximum error 1.48% for inflation at period 3.[18] Note that as *learners* expectations converge to the new equilibrium *near rationals* mistakes are reduced. The Survey of Professional Forecasters shows that during the Volker disinflation even Professional Forecasters made systematic prediction mistakes as *near rationals* do in this model.

Figure 12 plots the average paths of inflation and output gap for this economy under the disinflation episode considered previously, where $\alpha_{out} = 0.3$ and $\alpha_{inf} = 0.5$. Figure 13 plots the interest rate and figure 14 plots a typical realization. The sacrifice ratio assuming the disinflation lasts for 16 and 20 periods are 0.0054 and 0.0056 respectively, a value quite similar to the computations in this paper but lower than the estimates of Ball(1995b). The economy where *learners* and *near rationals* coexist converges in line with the data. There is still a temporary recession while inflation gradually moves to target, more importantly convergence is not too slow. The following table supports the previous claim.

learning algorithm in a class of models that contain the NK framework. The difference between this economy and the NK framework are the matrices that constitute the T-map. Evans and Honkapohja (1998) results can be applied to the model presented in this section. Also note that when the economy converges, as it is assumed in the E-Stability concept, *near rationals* do not commit mistakes, being completely rational. Convergence conditions under recursive least squares and tracking are not always the same; simulation based results suggest that if agents use a tracking algorithm the economy also converges to equilibrium.

[18]We considered the first 25 periods where rational agents would not commit mistakes, i.e. predictions for period 3 to 27. The error prediction for period 2 ($E_1^R y_2 - y_2$) is equal for *rationals* and *near rationals* since the disinflation is unanticipated. From period 2 to period 3 the coefficients for learners change the most causing the biggest prediction error for *near rationals*. At period 3 average inflation is 10.19% and the average prediction ($E_2^R y_3$) is 11.67%.

	out		inf	
	$t = 16$	$t = 100$	$t = 16$	$t = 100$
$\bar{x}_t$	-0.004	-0.0004	0.0403	0.0375
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-\bar{x}_t)^2}{N}}$	0.0211	0.0208	0.0191	0.0187
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-x^*)^2}{N}}$	0.0214	0.0208	0.0194	0.0187
$t : \bar{x}_t - x_t < 0.005$	15		15	

Table 4

Figure 15 plots aggregate expectations of *learners* and *near rationals* for next quarter versus current and one quarter ahead series. The prediction error is the vertical distance between the expectation for next quarter and one quarter ahead realizations. So the model implies that there is a systematic overprediction of inflation. The Survey of Professional Forecasters is the only survey that reports one quarter ahead expectations, unfortunately only the predictions for the GNP deflator cover the entire period of the Volker disinflation.[19] Figure 16 computes one quarter ahead expectations from the Survey of Professional Forecasters versus current and one quarter ahead inflation. For most of the disinflation period, expectations systematically overpredict inflation as is implied by the present model. Moreover, figure 16 also shows that, in periods of rising inflation, expectations underpredict inflation; hence the overall picture is consistent with some degree of backward looking expectations.

6 Conclusions

Building on the results of Marcet and Sargent (1989c), this paper analyzes the NK model under a disinflation when part of the private sector forms expectations using a learning algorithm. The NK model under RE is not able to account for the observed inflation persistence when an unanticipated credible disinflation is under way. This paper shows that when learning is introduced in the NK model, transition dynamics during an unanticipated disinflation become consistent with the data. Assuming that the private sector learns is specially suited for regime changes since RE unrealistically assume that the private sector expectations catch up immediately. Moreover, the learning mechanism uses the same functional form of RE providing a foundation for backward looking behaviour. Using the internal consistency requirement makes tracking parameters to be endogenous, leaving no free parameters in the expectations formation process.

Evidence from surveys on inflation expectations conclude that expectations are neither purely backward looking nor purely forward looking. To incorporate

[19]In the data the difference between the GNP deflator and the GDP deflator is almost imperceptible.

the previous observation we assumed that, an empirically plausible, small proportion of the private sector is not forward looking during the regime change. Aside from the empirical appeal of our formulation we show that the advantages also spill over to a theoretical formulation. Even though learning is suitable to analyze regime changes, convergence is usually too slow under this assumption. Our empirically consistent heterogenous framework solves this problem since a part of the private sector still learns but convergence is not too slow. In addition, our model generates persistence in inflation and a recession, which are patterns observed during the Volker disinflation.

We make an important contribution to the literature by showing that learning techniques can be useful to describe transition dynamics. We managed to explain disinflation dynamics with the widely used NK model as our benchmark, we thus avoid introducing arbitrary features into the model to achieve our goal. Moreover, our results do not hinge on specific assumptions; we used the NK framework and our results also carry over to a less appealing flexible price model. Hence, this paper suggests a robust explanation for disinflation dynamics in general and for the Volker disinflation in particular.

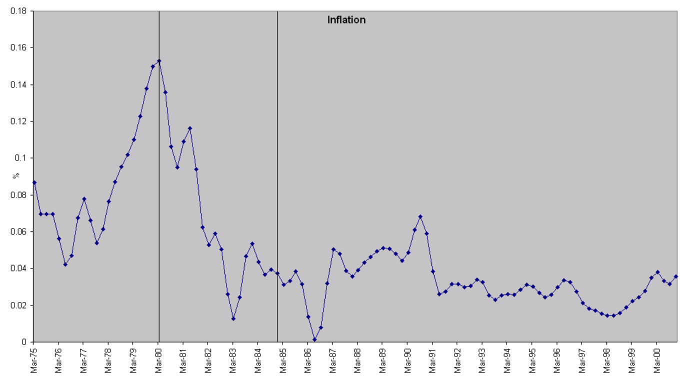

Figure 1: Inflation

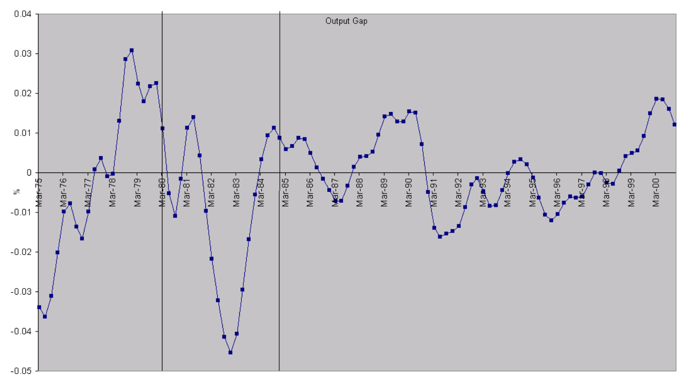

Figure 2: Output Gap

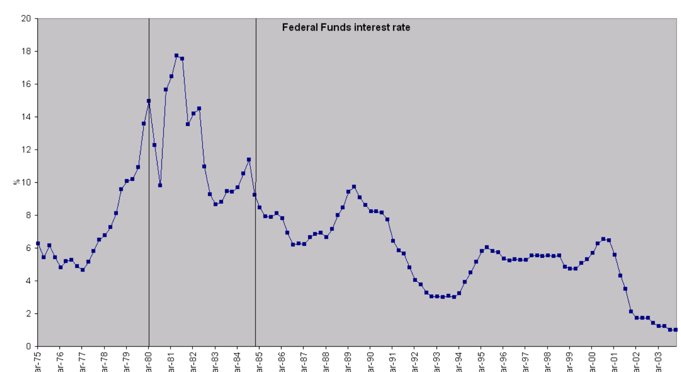

Figure 3: Federal Funds interest rate

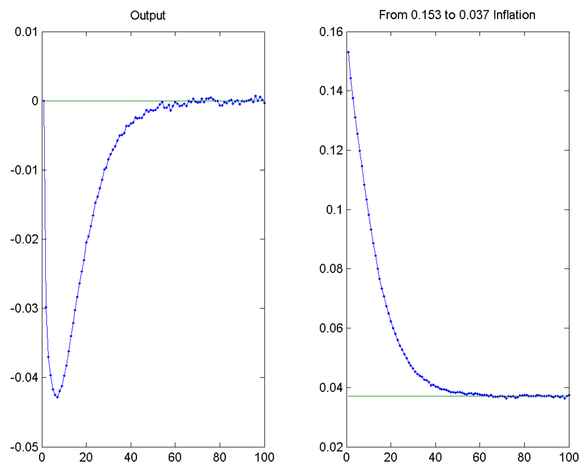

Figure 4: *Learners* economy - output and inflation.

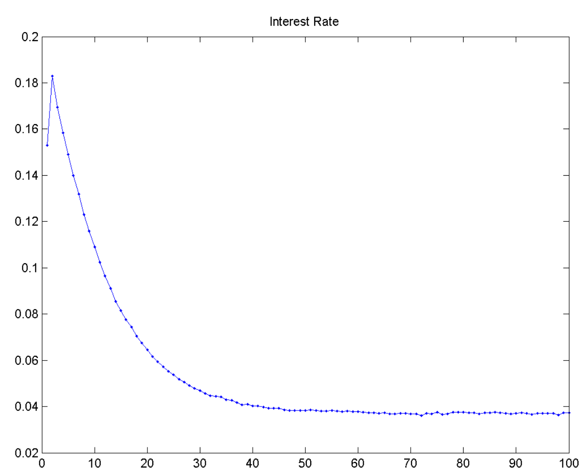

Figure 5: *Learners* economy - interest rate.

out	inf		predicting inflation					e=0.00001				shocks=0.02,0.02					J=500						T=20				sum
0.9	0.9	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	4
0.9	0.7	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	5
0.9	0.5	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	1	11
0.9	0.3	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	5
0.9	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.9	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	4
0.7	0.7	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	5
0.7	0.5	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	1	11
0.7	0.3	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	5
0.7	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.9	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	4
0.5	0.7	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	5
0.5	0.5	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	1	11
0.5	0.3	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	5
0.5	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.3	0.9	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	4
0.3	0.7	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	5
0.3	0.5	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	1	11
0.3	0.3	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	5
0.3	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.9	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	4
0.1	0.7	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	5
0.1	0.5	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	1	11
0.1	0.3	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	5
0.1	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0.1	0.1	0.1	0.1	0.1	0.3	0.3	0.3	0.3	0.3	0.5	0.5	0.5	0.5	0.5	0.7	0.7	0.7	0.7	0.7	0.9	0.9	0.9	0.9	0.9	out
		0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	inf

Figure 6: Internal Consistency Table for inflation - *Learners*.

out	inf		predicting output					e=0.00001				shocks=0.02,0.02					J=500						T=20				sum
0.9	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
0.7	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
0.7	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
0.7	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
0.7	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
0.5	0.9	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	15
0.5	0.7	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	15
0.5	0.5	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	15
0.5	0.3	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	15
0.5	0.1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	15
0.3	0.9	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	10
0.3	0.7	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	10
0.3	0.5	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	10
0.3	0.3	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	10
0.3	0.1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	10
0.1	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0.1	0.1	0.1	0.1	0.1	0.3	0.3	0.3	0.3	0.3	0.5	0.5	0.5	0.5	0.5	0.7	0.7	0.7	0.7	0.7	0.9	0.9	0.9	0.9	0.9	out
		0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	inf

Figure 7: Internal Consistency Table for output - *Learners*.

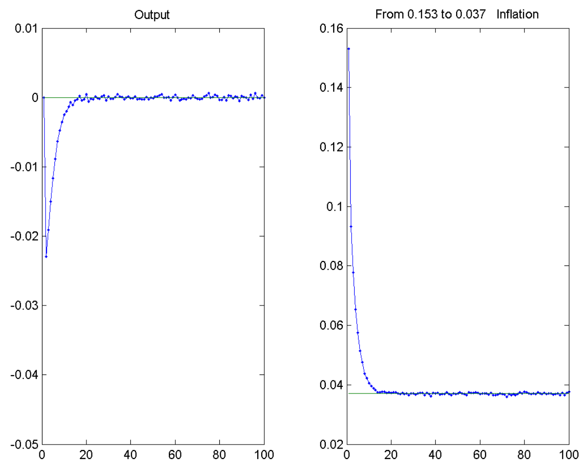

Figure 8: *Learners* and *Rationals* economy - output and inflation.

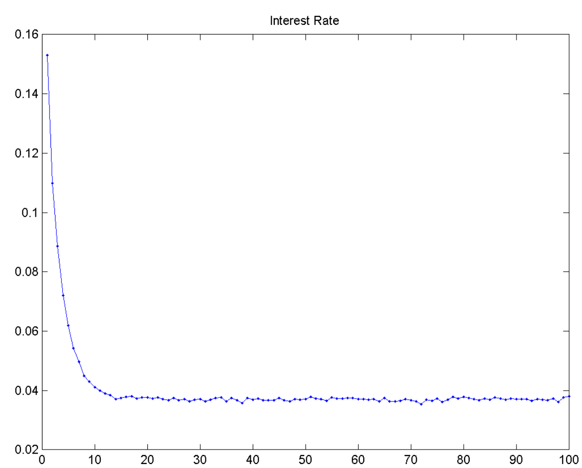

Figure 9: *Learners* and *Rationals* economy - interest rate.

out	inf		predicting inflation				e=0.00001				shocks=0.02,0.02					J=500						T=20				sum
0.9	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.7	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2
0.9	0.5	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	24
0.9	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.7	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2
0.7	0.5	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	24
0.7	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.7	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2
0.5	0.5	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	24
0.5	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.3	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.3	0.7	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2
0.3	0.5	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	24
0.3	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.3	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.7	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2
0.1	0.5	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	24
0.1	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0.1	0.1	0.1	0.1	0.1	0.3	0.3	0.3	0.3	0.3	0.5	0.5	0.5	0.5	0.5	0.7	0.7	0.7	0.7	0.7	0.9	0.9	0.9	0.9	0.9	out
		0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	inf

Figure 10: Internal Consistency Table for inflation - *Near Rationals*.

out	inf		predicting output				e=0.00001				shocks=0.02,0.02					J=500						T=20				sum
0.9	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.7	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.5	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.3	0.9	1	1	1	0	0	1	1	1	1	0	1	1	1	1	0	1	1	1	0	0	1	1	1	1	0	18
0.3	0.7	1	1	1	0	0	1	1	1	1	0	1	1	1	1	0	1	1	1	0	0	1	1	1	1	0	18
0.3	0.5	1	1	1	0	0	1	1	1	1	0	1	1	1	1	0	1	1	1	0	0	1	1	1	1	0	18
0.3	0.3	1	1	1	0	0	1	1	1	1	0	1	1	1	1	0	1	1	1	0	0	1	1	1	1	0	18
0.3	0.1	1	1	1	0	0	1	1	1	1	0	1	1	1	1	0	1	1	1	0	0	1	1	1	1	0	18
0.1	0.9	0	0	1	1	1	0	0	0	1	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1	1	14
0.1	0.7	0	0	1	1	1	0	0	0	1	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1	1	14
0.1	0.5	0	0	1	1	1	0	0	0	1	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1	1	14
0.1	0.3	0	0	1	1	1	0	0	0	1	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1	1	14
0.1	0.1	0	0	1	1	1	0	0	0	1	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1	1	14
		0.1	0.1	0.1	0.1	0.1	0.3	0.3	0.3	0.3	0.3	0.5	0.5	0.5	0.5	0.5	0.7	0.7	0.7	0.7	0.7	0.9	0.9	0.9	0.9	0.9	out
		0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	0.1	0.3	0.5	0.7	0.9	inf

Figure 11: Internal Consistency Table for output - *Near Rationals*.

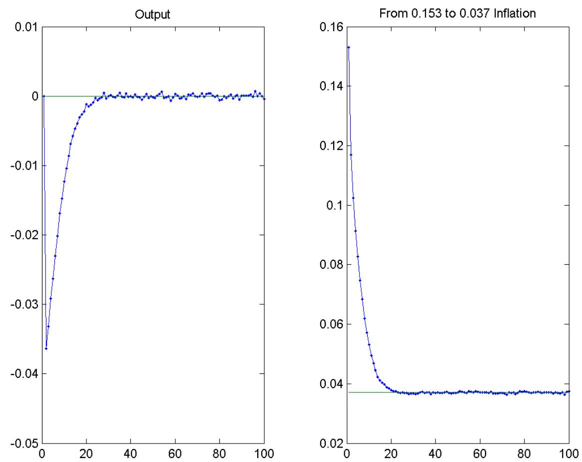

Figure 12: Learners and near rationals economy

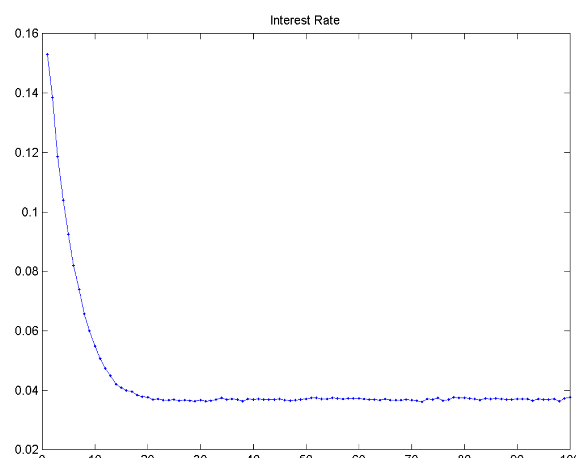

Figure 13: Learners and near rationals economy

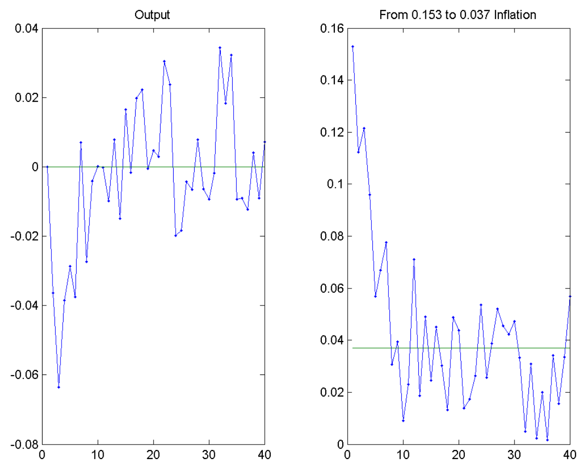

Figure 14: Learners and near rationals economy - Typical Realization

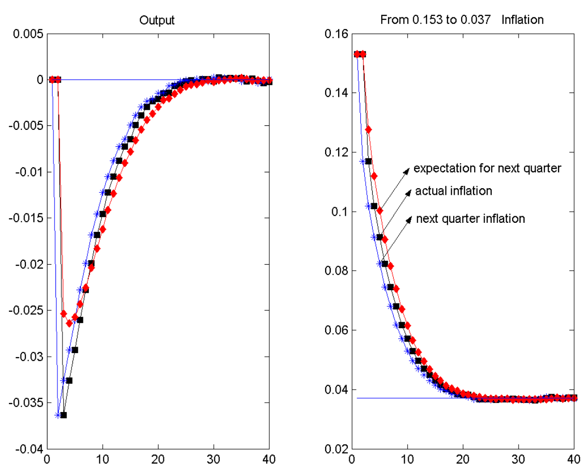

Figure 15: Model expectations - *Near Rationals*.

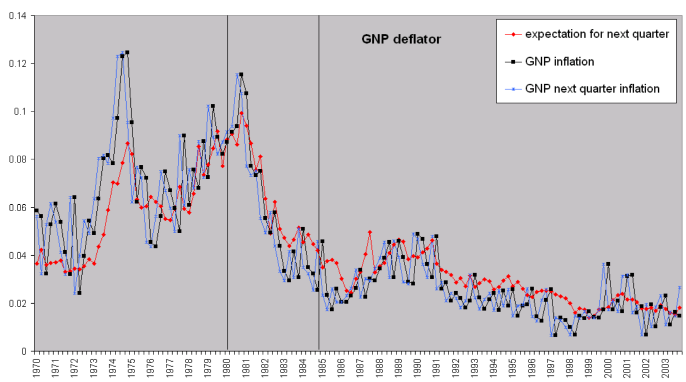

Figure 16: Survey of Professional Forecasters GNP deflator expectations

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A-1 Appendix on the role of shocks

This appendix will analyze whether a bigger variance in the shocks lead to a faster convergence to the new equilibrium; in addition we will also analyze convergence difference between an economy with i.i.d. shocks and autocorrelated shocks.

Carroll (2003) reports that when more news about inflation are available, expectations of the general public catch up faster with expectations of professional forecasters. More news of inflation are likely to be correlated to an higher inflation variance. It is known that when there is more variability in the regressed variables the estimators become more precise, on the other hand variability on the error term leads to smaller precision. In an economy of learners bigger precision may imply faster convergence. So in an economy with i.i.d. shocks the bigger variance of the error term is expected to be translated into a lower convergence speed. If the shocks are not i.i.d., since shocks are also a regressor, the effect should be mixed.

Time of	convergence	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
No shocks	output	157	53	32	23	18
	inflation	161	54	33	24	18
st.d.=0.01	output	0	0	0	0	0
	inflation	-1	0	0	+1	+1
st.d.=0.02	output	-2	-1	0	+1	0
	inflation	-1	0	-1	-1	+1

Table I: Convergence time for an economy with i.i.d. shocks

Time of	convergence	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
st.d.=0.01	output	+5	+4	+4	+4	+4
	inflation	+4	+4	+4	+3	+4
st.d.=0.02	output	+5	+5	+4	+4	+4
	inflation	+4	+4	+4	+3	+4

Table II: Convergence time for an economy with correlated shocks

The previous tables show that an higher variance of shocks does not lead to an increase in the time of convergence.[20] The first two rows in table I display the time of convergence for different tracking parameters. All the other values in table I and II display the time of convergence difference with respect to the

[20]We considered that the economy had converged when inflation minus the inflation target was less than 0.005 and the output gap was less than 0.005. For the stochastic economy, 5000 realizations of the series of shocks were simulated; the convergence criteria is the same but for average inflation and average output gap across all realizations. As in section 5.2, for table I two uncorrelated shocks are assumed to influence the ALM directly. As in section 5.3, for table II a correlated natural interest rate and an uncorrelated cost push shock are assumed.

non-stochastic economy. One can see that higher variances have no effect on convergence. Table II shows that when shocks are autocorrelated convergence is slightly slower.

A-2 Internal Consistency Appendix

We conducted sensitivity analysis considering alternative values of T; for T=16 and T=24, being these values still plausible for our purposes, the results remain unaltered. Smaller values of ε lead to a smaller number of "good" predictors but the results are mainly unchanged.

Note that the magnitude of shocks influences the prediction performance of tracking parameters. Insofar as there is a regime change it is optimal to give more importance to recent observations, this is precisely what high tracking parameters do. On the other hand, shocks create noise in the economy, and if one gives more importance to recent observations then predictions will be harmfully influenced by recent shocks. We first considered that besides the autocorrelated natural interest rate there is a non-autocorrelated cost-push shock. We also did sensitivity analysis by considering the presence of other shocks, being the results robust. In the alternative specification we considered that both the IS and AS equation could be influenced by non-autocorrelated shocks. We do not assume that the shocks are correlated because the correlations between the output gap and inflation are influenced in this time period by the disinflationary episode.

We estimated the data standard deviation of the output gap and inflation from 1980 to 1984 to be 0.02 and 0.04. We set the magnitude of shocks so that for the internal consistent tracking parameter the model yields plausible variances when compared with the data during the Volker disinflation.

A-2.1 *Learners'* model

We first guessed that ($\alpha_{inf} = 0.5, \alpha_{out} = 0.5$) will be internally consistent. Assuming ($\alpha_{inf} = 0.5, \alpha_{out} = 0.5$), we computed the variance of the shocks that would yield plausible variances in the model. We considered the standard deviations for the natural interest rate innovation and the cost-push shock to be 0.02; for 500 realizations and over 20 quarters the average standard deviation of output is 0.023 and of inflation is 0.033. When we considered that besides the natural interest rate two non-autocorrelated shocks were present we could adjust the standard deviations in the model to match perfectly the standard deviations in the data, doing so the main results did not change.

A-2.2 Main model

To determine the internally consistent tracking parameters we first guessed that $\alpha_{inf} = 0.5$ and $\alpha_{out} = 0.3$ are the best internal consistent parameters. We assume that the natural interest rate is autocorrelated and there is a cost-push shock. Dropping the cost push shock and considering other uncorrelated shocks (as we did in our first model) does not change the results. We set the standard deviation for the cost-push shock and the innovation in the natural interest rate to be 0.02. If the tracking parameter for output is 0.3 and for output is 0.5, for an average of 500 simulations and for a range of 20 periods, the average standard deviation for output is 0.023 and for inflation is 0.036. As before we choose the time range T to be 20 quarters and $\varepsilon = 0.00001$, being the results robust to other values of these parameters.

A-3 Data Appendix

All variables refer to the USA. Inflation was computed using the seasonally adjusted monthly Consumer Price Index (CPI) for all urban consumers and all items. The source is the United States Department of Labor: Bureau of Labor Statistics and the series code is CPIAUCSL. Quarter inflation is computed as the sum of the months in the quarter divided by the sum of the CPI of the months of the previous quarter. The reported series were filtered using the band pass filter eliminating components with periodicity smaller than 4 quarters.[21]

The series for Gross Domestic Product at constant prices and seasonally adjusted is the official series of the Federal Reserve System. A first series was computed by eliminating the components with periodicity smaller than 32 quarters. A second series was computed by eliminating components smaller than 4 quarters. The output gap is computed by the log of the second series divided by the first series.

The federal funds interest rate source is the Federal Reserve System. The monthly annualized rates were transformed to quarterly annualized rates using a geometric average.

The Survey of Professional Forecasters reports the mean values for the GNP deflator prediction for the current quarter and the following quarter. Expected inflation is computed as the annualized change from the current quarter prediction to the next quarter prediction. The GNP deflator source is the Federal Reserve System.

[21] For a description of the band pass filter see Baxter and King (1999).