INTEREST RATE RULES AND THE RESPONSE TO THE OUTPUT GAP

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ABSTRACT

Modern monetary policy analysis is built around the concept of an interest rate rule that responds to both inflation and output. This paper evaluates the quantitative implications of having a policy rule target different definitions of the output gap in a New Keynesian model with endogenous capital. One crucial result is that different model specifications result in alternative values for potential output, raising the issue of which output gap to target. The results of this paper suggest that targeting the true output gap can be well approximated by a rule that only reacts to inflation.

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1. Introduction

Ever since Taylor (1993) analysed monetary policy within the context of a Taylor rule, the incorporation of some measure of the output gap is a common feature in both empirical and theoretical research. Nevertheless, the concept used to denote the output gap has rarely been consistent among researchers and policymakers, as several definitions have been provided. However, the use of different output gap concepts will lead to different paths for the monetary policy instrument in its aim at stabilising output and inflation, with consequences not only for these two variables, but also for all other endogenous variables in the models considered. This issue was highlighted by McCallum (2001) within the context of a simple New Keynesian model without capital and an inelastic labour supply. The purpose of this paper is to extend McCallum's framework to consider the quantitative importance of responding to the output gap in a model with a richer supply side.

Most central banks attempt to stabilise not only the inflation rate, but also the output level around its potential, even if this element is unobserved. Furthermore, to the extent that the output gap, however it is defined, is related to real marginal costs, this will also have an effect on inflation via the New Keynesian Phillips Curve (NKPC). As argued by the ECB (2000, p. 37) "(...) particular caution is required when drawing conclusions with regard to policy which are based on estimates of the level of the output gap". As the ECB again argues, potential output should not measure the level of output at which all inputs would be fully utilised. This argument makes sense not only because marginal costs rise steeply at high degrees of factor utilisation, but also because labour input will vary over the cycle, even in the absence of nominal rigidities but in response to changes in real variables.

At its core, discussions regarding the output gap can be divided into two, those that measure them as the temporary fluctuations in output, and those that make the distinction that treat the output gap as the difference between cyclical output and its flexible price counterpart. The former concept is commonly used in applied studies (e.g., Mehra, 1993 and Clarida Gali and Gertler, 2000), whereas the latter seems to feature more prominently in theoretical research (e.g. Clarida, Gali and Gertler, 1999 and McCallum and Nelson, 1999) . From a theoretical perspective it is clear that the difference between current (cyclical) output and flexible price output is the correct measure, even if many current discussions reveal the fact that this issue is still not well understood[2]. Nevertheless, it is also possible that the error that arises from using cyclical output as a proxy for the output gap is quantitatively small, although given the considerable amount of dissatisfaction in using de-trended output to estimate the NKPC[3] . Furthermore, an additional problem may arise even when there is consensus regarding the appropriate definition of the output gap, since models with different specifications will result in different measures of flexible price output. Therefore, the above discussion regarding the output gap is not one of uncertain measurement, but of different concepts.

The above considerations suggest that analysing the quantitative consequences of using alternative definitions of the output gap in a monetary policy rule[4] can highlight its

[2] Indeed, at a recent economic seminar at HM Treasury this was an issue of debate and many commentators argued that the government should try to minimise all output fluctuations in output. From a theoretical standpoint, this objective is clearly undesirable.

[3] See for example, Galí and Gertler (1999).

[4] For the purposes of this paper it is not necessary that the monetary authorities be maximising any particular welfare function.

practical importance. With this purpose in mind, this paper develops a New Keynesian, sticky-price model with endogenous capital and habit formation in consumption. An advantage of using this framework is that the results regarding the output gap are more general than those in McCallum and Nelson (1999) and McCallum (2001).

The paper is organised as follows. Section 2 presents the model; section 3 discusses the alternative definitions of the output gap to be used; section 4 will compare the model's performance as the monetary policy rule reacts to the different concepts of the output gap, with emphasis not only on the relative volatility of the endogenous variables, but also the persistence of shocks. Section 5 concludes.

2 The model

2.1 Households

The economy is populated by a large number of households making decisions in the present given expectations about the future. The representative household, which produces a differentiated good, maximises at *t* its intertemporal utility function

$$E_t \Sigma_{j=0}^{\infty} \beta^j \left[\frac{\sigma - 1}{\sigma} \left(\frac{c_{t+j}}{c_{t+j-1}^{h}} \right)^{\frac{\sigma-1}{\sigma}} + \gamma(m_t) - \chi \frac{n_t^{(s)1+\xi}}{1+\xi} \right] \tag{1}$$

where c_t is a composite consumption good, m_t are real money balances and n_t denotes labour input. β is the household's discount factor and *h* [0,1) measures the degree of habit formation in consumption (see Fuhrer, 2000). Real money balances enter the utility function as these reduce the amount of resources required to undertake market transactions.

The representative household produces its differentiated good given a production function $y_t = f(k_t, n_t z_t)$ where z_t represents a technology shock, and k_t and n_t denote the capital stock and labour input, respectively. Demand for the household's good is given by $Y_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta}$ where $P_t(i)$ is the nominal price of the good produced by the household, P_t is the aggregate price level and Y_t is aggregate demand.

The household maximises lifetime utility (1) at *t* by choosing a sequence for consumption, labour, money balances, one period bonds (b_{t+1}) with a real price of $(1+r_t)^{-1}$ and capital (k_{t+1}) subject to its real budget constraint for period *t*:

$$\left(\frac{P_t(i)}{P_t}\right)^{1-\theta} Y_t - T_t = c_t + k_{t+1} - [1-\delta]k_t + C(x_t, k_t) + w_t(n_t^d - n_t^s) + m_t - \frac{m_{t-1}}{1+\pi_t} + \frac{b_{t+1}}{1+r_t} - b_t \tag{2}$$

where δ is the depreciation rate, w_t is the real wage, T_t are lump-sum taxes, n_t^d and n_t^s are the amount of labour demanded and supplied by the household, respectively and π_t represents the inflation rate ($P_t / P_{t-1} - 1$). It is now well known that sticky price with endogenous capital can lead to unrealistically high volatility in the endogenous variables at very high frequency (Ellison and Scott, 2001). Consequently, investment incurs adjustment costs, $C(x_t, k_t)$, as in Casares and McCallum (2000), where x_t, investment, is related to the capital stock by the equation

$$k_{t+1} = [1-\delta]k_t + x_t \tag{3}$$

and the specific formulation of the adjustment costs is given by

$$C(x_t, k_t) = \psi \frac{x_t^{\eta+1}}{k_t^{\eta}} \tag{4}$$

Furthermore, it is also assumed that production is determined by the quantity demanded:

$$f(k_t, n_t z_t) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} \tag{5}$$

with the production function being

$$f(k_t, n_t z_t) = k_t^{\alpha} n_t^{\alpha} e^{z_t} \tag{6}$$

Letting λ_t and μ_t denote the Lagrange multipliers on (2) and (5), respectively, the optimality conditions are (2), (5) and[5]:

$$c_t: \quad \left(\frac{c_t}{c_{t-1}^{h}} \right)^{\frac{\sigma-1}{\sigma}} \frac{1}{c_t} - \lambda_t - h\beta E_t \left(\frac{c_{t+1}}{c_t^{h}} \right)^{\frac{\sigma-1}{\sigma}} \frac{1}{c_t} = 0 \tag{7}$$

$$m_t: \quad \gamma'(m_t) - \lambda_t + \beta E_t \left(\frac{\lambda_{t+1}}{1+\pi_{t+1}} \right) = 0 \tag{8}$$

$$n_t^s: \quad -\chi n_t^{\xi} + \lambda_t w_t = 0 \tag{9}$$

[5] It is also assumed that the transversality conditions hold.

$$n_t^d: \quad -\lambda_t w_t + \mu_t f_{n,t} = 0 \tag{10}$$

$$b_{t+1}: \quad \lambda_t - \beta E_t \lambda_{t+1}(1+r_t) = 0 \tag{13}$$

$$k_{t+1}: \quad -\lambda_t\left[\frac{\partial C(x_t, k_t)}{\partial k_{t+1}} - 1\right] + \beta E_t \lambda_{t+1}\left[1 - \delta - \frac{\partial C(x_{t+1}, k_{t+1})}{\partial k_{t+1}}\right] + \beta E_t \mu_{t+1} f_{k,t+1} = 0$$

(14)

$$P_t(i): \quad (1-\theta)\lambda_t Y_t \frac{P_t^{-\theta}(i)}{P_t^{1-\theta}} + \theta\mu_t Y_t \frac{P_t^{-(1+\theta)}}{P_t^{-\theta}} = 0 \tag{15}$$

2.2 Market clearing.

For general equilibrium, the labour market clears and the economy is subject to the aggregate resource constraint:

$$n_t^s = n_t^d \tag{15}$$

$$y_t = c_t + x_t + C(x_t, k_t) + g_t \tag{16}$$

Fiscal policy is given by an exogenous process for government expenditure and the central bank's monetary policy instrument is a short-term interest rate, given by $(1+R_t)=(1+r_t)(1+E_t\pi_{t+1})$.

2.3 Linear Model

In order to analyse the model's dynamic properties, the optimality conditions obtained above are linearised around the steady state[6]. The result is a linear rational expectations model with the following equations[7]:

$$h\beta\left(\frac{\sigma-1}{\sigma}\right)E_t\hat{c}_{t+1}+\left[1-h\beta-\left(\frac{\sigma-1}{\sigma}\right)\left(1+h^2\beta\right)\right]\hat{c}_t+\left(\frac{\sigma-1}{\sigma}\right)h\hat{c}_{t-1}+\left(1-h\beta\right)\hat{\lambda}_t=0 \qquad (17)$$

$$\xi\hat{n}_t=\hat{\lambda}_t+m\widehat{c}_t+f_{n,t} \qquad (18)$$

$$(1+C')U^{\varpi}\varpi\hat{u}_t+U^{\varpi}C'\eta(x_t-k_t)=\left(\frac{\theta-1}{\theta}\right)f_k\left(\hat{f}_{k,t}+m\hat{c}_t\right) \qquad (19)$$

$$\hat{x}_t=\frac{1}{1+\delta}E_t\hat{x}_{t+1}+\frac{\delta}{1+\delta}k_t+E_t\frac{\Theta f_k\left(m\hat{c}_{t+1}+\hat{f}_{k,t+1}\right)-r_t}{(1+\delta)\eta C'} \qquad (20)$$

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[6] "Hats" denote the percentage deviation from steady state and capital letters represent steady-state values.

[7] C' represents $\frac{\partial C(x_t,k_t)}{\partial x_t}$ and $\Theta=\frac{\theta-1}{\theta}$

$$R_t = r_t + E_t \pi_{t+1} \tag{21}$$

$$\hat{y}_t = \alpha\left(\hat{k}_t + \hat{u}_t\right) + (1-\alpha)\hat{n}_t + z_t \tag{22}$$

$$\hat{y}_t = \frac{C}{Y}\hat{c}_t + \frac{G}{Y}\hat{g}_t + \frac{X + C(x,k)}{Y}\hat{x}_t \tag{23}$$

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta\hat{x}_t \tag{24}$$

$$m\hat{c}_t = \hat{w}_t - \hat{f}_{n,t} \tag{25}$$

There are nine equations and eleven variables and the two remaining equations to complete the model pertain to pricing decisions and the monetary policy rule. Regarding the former, sticky prices á la Calvo (1983) will be posited, so as to generate a New Keynesian Phillips curve (NKPC), which would be purely forward looking. In the case of linearisation around a non-zero steady-state and allowing firms that are unable to re-optimise to simply adjust their prices by the previous period's inflation rate yields a Phillips curve of the type proposed by Fuhrer and Moore. Given the considerable amount of debate concerning the suitability of each specification both formulations will be considered. Hence, the initial Phillips curve formulation will be:

$$\pi_t = \phi_0 E_t \pi_{t+1} + (1-\phi_0)\pi_{t-1} + \phi_1 m\hat{c}_t \tag{26}$$

With ϕ_1 ($\succ 0$) reflecting the degree of price rigidity.

2.4 Monetary Policy.

The general approach to modelling monetary policy consists of the setting of a short-term nominal interest rate that responds to deviations from some target for both inflation and output, and also allows for some interest rate smoothing[8]:

$$R_t = (1-\mu_3)\left[\mu_1 \pi_t + \mu_2 \tilde{y}_t\right] + \mu_3 R_{t-1} + \varepsilon_{R,t} \qquad (27)$$

$\tilde{y}_t$ represents the output gap that the monetary authorities respond to, and $\varepsilon_{R,t}$ represents an i.i.d. monetary policy shock. One could of course argue, as in McCallum (2001) and McCallum and Nelson (1999) that the above rule is not operational, but modifying it will not alter the main results of this paper. What is necessary, however, is that the coefficient on inflation, μ_1, be greater than unity to ensure stability.

2.5 Sources of shocks.

The model includes four shocks: technology, cost-push, policy rule shocks and exogenous changes in government expenditure. As shown by Clarida, Galí and Gertler (1999), under perfect information, with purely forward-looking inflation and in the absence of cost-push shocks the monetary authorities could use the policy rule to fully offset all other shocks and perfectly stabilise both output and inflation. This ignores the shocks to the policy rule itself and for the purposes of the analysis presented here a rule such as (27) is a good approximation to actual monetary policy, even if it is not derived from optimising behaviour. Technology shocks lead to an increase in output (fall in the

[8] Representative among these are McCallum and Nelson (1999), Clarida, Galí and Gertler (2000).

output gap) and a fall in the inflation rate. To the extent that the monetary authorities want to stabilise employment, they will attempt to offset temporary technology shocks, but this will exacerbate the negative effect on the output gap, and in this sense there is a trade-off. Another trade-off arises with the cost-push shock, which leads to both an increase in inflation and a fall in output, whereas this does not arise with fiscal and monetary policy shocks.

3. Calibration.

This section assigns parameter values to the above equations to make them directly comparable to the literature on NK business cycle analysis, such as Casares and McCallum (2001) and Rotemberg and Woodford (1999).

σ, which determines the sensitivity of consumption to the real interest rate and the degree of consumption smoothing, will be set to 0.2, as in McCallum and Nelson (1999)[9].

For *h*, the standard value of 0.8, as in Fuhrer (2000) will be adopted. The two parameters ψ and η that determine the adjustment costs are chosen, as in Casares and McCallum (2001), to lead to an elasticity of investment with respect to the real interest rate of 2.5 per cent, and to amount to 1 per cent of output in the steady state.

The labour share will be set at 0.3, and the time discount factor β to 0.995, implying a steady-state real interest rate of 2 per cent and ξ equals 1, as in Galí (2003).

The δ will be set at 0.025.

[9] McCallum (2001) argues in favour of 0.4, but in the context of a model without capital or investment.

Table 1: Calibration	
Parameter	Value
α	0.3
β	0.995
δ	0.025
η	2.55
θ	6
ρ_z	0.95
σ_z	0.0925
σ_{eR}	0.0135
μ_1	1.5
μ_2	0.1
ϕ_0	0.5
ϕ_1	0.1
$\frac{G}{Y}$	0.2
μ_3	0.8

4. Output gap concepts.

How is the output gap to be defined? McCallum (2001) makes the definition clear[10]: it is the difference between cyclical output and its flexible price level. Under fully flexible prices the mark-up is constant, so that $w_t = \Theta f_{n,t}$. In a model without capital[11] and an inelastic labour supply, as in McCallum and Nelson (1999), the output gap is directly related to movements in labour. However, in a more fully developed model with endogenous investment and allowing for some degree of labour supply elasticity, such as the one presented above, this is no longer the case. Labour supply will fluctuate even in the absence of nominal rigidities, in response to real shocks. Furthermore, capacity output is variable, but as Neiss and Nelson (2001) and Woodford (2004) argue, it is not clear what kind of capacity output should be used to define the natural level of output. One could obtain the level of output under fully flexible prices and use that as the natural and capacity rate. However, this poses the problem that capacity output is dependent of the labour supply under flexible prices and the *existing* capital stock, instead of its hypothetical flex-price level. Neiss and Nelson (2001) argue in favour of using the fully flexible level in order to avoid the potentially destabilising effects of monetary policy shocks, but from an applied perspective this approach is unappealing. A further similar problem arises when models embody habit formation in consumption. In this case, lagged consumption levels have an effect on labour supply, so when considering capacity output should one take the lagged level of consumption that would have prevailed under flexible prices or the *actual* lagged value? Again, to the extent that labour supply is affected by

[10] Other definitions, all implying much the same concept can be found in Neiss and Nelson (2002, fn. 7).
[11] Or, alternatively, exogenous capital.

previous values, households will not consider what they would have chosen in a hypothetical flex-price world, but their actual values in the previous period.

It is clear then that although there is an emerging consensus on what the concept of the should be, allowing for endogenous state variables (in this case, the current period capital stock and lagged consumption) creates a potential source of conflict when trying to define capacity output. Nevertheless, an issue that Neiss and Nelson (2001) do not consider is whether the difference between the two concepts is quantitatively important, and this will be discussed further below.

Therefore, this paper will consider four different concepts of the output gap and their effects: the deviation of output from a linear trend, that is, defining cyclical output[12] as the output gap, deviations of employment from the steady state level, as in McCallum and Nelson (1999) and the "true" output gap, where both definitions of capacity output described above will be considered.

5. Monetary Policy and the Output Gap: Quantitative Implications.

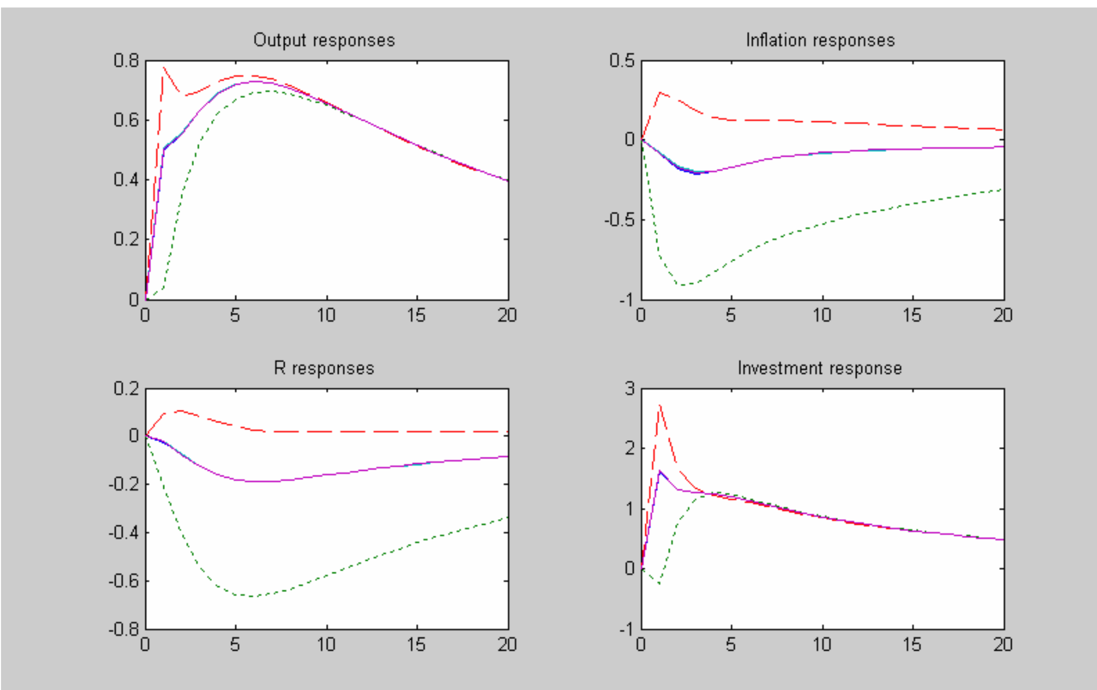
5.1 Impulse responses.

This section evaluates the consequences of using alternative concepts of the output gap in the monetary policy rule by analysing the impulse responses to each of the four shocks considered above and also for the volatility of the endogenous variables.

[12] HP filtering would lead to the same conclusion.

All models share the same calibrated values as presented in Table 1, except for the monetary policy rule. In model 1 the parameter μ_2, reflecting the reaction of the interest rate to the output gap, is set to zero; in model 2, the central bank reacts to cyclical output, that is, $\tilde{y}_t = \hat{y}_t$. In the third model, the central bank react to changes in the employment level, as in McCallum and Nelson (1999) assumed for the case of a sticky-price economy with no capital and an inelastic labour supply. In models 4 and 5 the output gap represents the deviation of output from its flexible price level, except that in the former the fully flexible price model is considered, whereas for the latter actual lagged consumption and current capital affect capacity output.

Figure 1: Technology Shock.



The dashed lines represent model 3, the dotted lines model 2 and the remaining models (indistinguishable) are represented by the solid line. The reason that the variables respond differently in this context is a direct consequence of the systematic component of monetary policy. A positive technology shock generally leads to an increase in output and a fall in inflation. However, in model 2, because the monetary authorities are trying to dampen all fluctuations in output, regardless of their source, output falls by less than in the other models, but with the secondary effect of a greater effect on inflation. In the case of model 3, as is common in sticky price models, a temporary technological shock leads to an increase in output, but accompanied by a fall in employment. To offset the effect of z on employment output is stimulated further, so that output increases even further and the effect on inflation is positive[13]. Interestingly, when the monetary authorities do not react to any output gap concept, the response is virtually indistinguishable from that of reacting to the theoretical gaps.

5.2 Consequences of alternative rules on the variables' volatilities.

Table 2 presents the volatilities of five variables under each of the model specifications. Model 2 is the most successful in achieving its task of reducing fluctuations in cyclical output (and consequently, investment), but this comes at the cost of over a tripling in the volatility of the inflation rate and the nominal interest rate[14], so that the costs associated with this rule are very high. Model 3, which responded to movements in employment, results in the highest volatility of output and investment, for the reasons discussed above, but its performance is not that different from the other models, except in the lower volatility of inflation. Most importantly however, is the fact that if the central bank

[13] The effect on inflation is different in McCallum and Nelson (1999) because in their model reacting to employment is equivalent to targeting the true output gap.

[14] It should be recalled that one of the arguments put forward to support the existence of interest rate smoothing was the monetary authorities' desire to maintain a stable interest rate.

responds to one of the two theoretical output gap concepts or does not target output at all lead a an indistinguishable performance in terms of the volatilities of the variables. One reaches very similar conclusions when the model without habit formation in consumption is analysed, as shown on Table 3. It is worth emphasising however, that in the present context a lower degree of inflation volatility will in general improve welfare[15], but that this is not necessarily the case for output, since the latter will also be subject to real shocks.

Two key results emanate from this analysis. Firstly, if the central bank responds to the output gap, as measured by the departure of output from its flexible price level, then it does not matter quantitatively whether it is the theoretical level of output than one uses or the level of output conditional on past values.

TABLE 2:STANDARD DEVIATIONS

	σ_Y	σ_x	σ_π	σ_R
Model1	1.84	2.99	**0.49**	0.47
Model2	**1.74**	**2.71**	1.81	1.59
Model3	1.93	3.39	0.55	**0.22**
Model4	1.84	2.98	**0.51**	0.47
Model5	1.84	2.98	**0.50**	0.46

Secondly, ignoring output altogether leads to a level of economic performance that is virtually indistinguishable from one that targets the correct output gap. Moreover, given the additional uncertainties that would arise if the policy rule analysed above were made operational and ignorance about the flexible price level of output, the costs, in terms of volatilities, of reacting to output gaps are likely to be even higher. Clearly, by raising the

[15] As argued by McCallum (2001), an explicit welfare is not always necessary when analysing alternative monetary policy rules.

coefficient on the output gap in the policy rule the above results would become even more pronounced.

TABLE 3:STANDARD DEVIATIONS (NO HABIT FORMATION)

	σ_Y	σ_x	σ_π	σ_R
Model1	1.37	3.27	**0.52**	**0.53**
Model2	1.28	**3.03**	1.45	1.32
Model3	1.53	3.83	1.00	0.57
Model4	1.37	3.26	**0.53**	**0.53**
Model5	**1.35**	3.22	**0.52**	**0.52**

6. Conclusion.

Monetary policy rules that include the deviation of the inflation rate and output from some target measure have become a common feature of monetary policy analysis. By including some measure of the output gap, including detrended output, in the policy rule, it is believed that the effects will be beneficial, in the form of reduced fluctuations. This ignores the fact that some of these fluctuations in output arise as a consequence of real shocks in the economy, and trying to offset these should not be an objective for policymakers.

In 2001 McCallum concluded that the monetary authorities should not respond strongly to the output gap. Aside from issues of operationality or the correct measurement of the output gap, using the wrong concept of the output gap leads to a serious worsening of the model's behaviour. But the true level of potential output is model specific, so that by varying the modelling of the labour market or the inclusion of capital, one obtains a different measure of output under flexible prices, just like real business cycles would. Consequently, reacting to McCallum and Nelson's (1999) measure of the output gap yields results that are worse than not targeting the gap at all. Given the above, one is driven to conclude not that it is undesirable to respond strongly to the output gap, but that one *should not* respond to the output gap.

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