The Rise in Returns to Education and The Decline in Household Savings*

Areendam Chanda
Louisiana State University

February 2005

Abstract

This paper explores the consequences of rising returns to human capital investment on the personal savings rate. Over the past two decades, the return to college education has increased relative to high school education leading economists to argue the presence of "skill biased technological progress". The literature explaining household savings has also burgeoned considerably, motivated by its declining rate in the US over the past couple of decades. Stylized facts suggest a negative relationship between returns to education and savings rates across most of the past century and also a negative relationship between education spending and savings rates across OECD countries. In this paper, we present a model where a declining savings rate emerges as an outcome of an exogenously driven increase in the return to education. The link between the two is attributed to optimizing behavior of altruistic households. The results of our model are robust to the inclusion of life cycle savings and unintentional bequests. Some of the interesting results of our model are (i) a rise in the return to education raises education spending ratio by less than what it reduces the aggregate savings rate (ii) for some parameter values it actually reduces both the education spending rate and the aggregate savings rate and finally, (iii) it also raises the return to capital due to physical capital-human capital complementarity.

Keywords: Skill Biased Technological Change, Savings, Education, Economic Growth.
JEL Classification: D91, E21, O33

*I have benefitted from the comments and advice of seminar participants at Brown, Calgary, Florida International, Georgia, North Carolina State, Southern Methodist University and Summer Meetings of the North American Econometric Society on earlier drafts of this paper. All errors are my own. E-mail: achanda@lsu.edu

1 Introduction

Our tools for measuring savings and investment are stone age definitions in the inforation age

-William Nordhaus (1995)

The personal savings rate of the US economy has been declining steadilly since the early eighties. Figure (1) shows the decline in both the household savings rate (as a share of personal disposable income) and the gross private savings rate (as a share of GDP). The former which hovered around 10% during the early eighties had fallen to less than 2.5% by the late nineties and was at 1.4% in 2003. As the graph shows, most of this decline took place in the nineties. Similarly the gross private savings rates declined from levels greater than 20% to levels below 15% by the late nineties. Over the past years, researchers have proposed a multitude of explanations for the decline with candidates such as the changing demographic structure, increasing ratio of wealth to income (reinforced by a belief of good times in the future), an increase in the discount rate, huge transfers of wealth to the aged (who have a higher marginal propensity to consume) through public institutions, increased access to credit allowing greater consumption smoothing, etc.[1] Gale and Sabelhaus (1999) take Nordhaus' quote above seriously and try to re-measure savings. After augmenting household savings with data on real capital gains, they suggest that savings is at its highest in the past forty years. They note that adding human capital, research and development, and other intangible capital would further raise the savings rate.[2] However as figure (1) suggests, the decline in real capital gains during the first years of the twenty first century clearly has not led to a rise in personal savings.[3]

From the early eighties onwards, the economy has also experienced what observers believe to be a rapid pace of technological change with the benefits of this change accruing disproportionately to those with higher levels of skills. This phenomenon of "skill-biased technological change" is considered to be the cause of a monotonic rise in income inequality experienced over the past two decades.[4] Though

[1]See Parker (1999) and Lusardi, Skinner and Venti (2001) for a thorough review of these arguments.

[2]Also see Lusardi et al (2001) for an exhaustive discussion of "wealth" and "mismeasurement" based explanations.

[3]The literature trying to explain the decline in savings is quite extensive. Gokhale, Kotlikoff and Sabelhaus (1996) decompose changes in savings due to cohort specific consumption propensities, those due to intergenerational redistribution of resources, changes in government spending patterns and changes in demographics. They argue that the redistribution of resources from the youth to the elderly and the higher consumption propensities of the latter group can explain the decline in savings. Parker (1999) undertakes a thorough examination of existing explanations of the savings decline. He argues that neither wealth changes nor the changing age distribution can completely explain the savings decline since they post date the consumption boom . He also rules out the importance of explanations based on income by age distribution, financial innovations and intergenerational fiscal transfers. Parker notes that financial innovation can explain a third of the decline and wealth changes another fifth. This implies that these two explanations are enough to account for more than half of the decrease. He speculates that since consumption is a forward looking variable, the consumption boom maybe linked to households learning about high levels of output in the future. Other possibilities include a shift in household preferences and finally he does not rule out that a combination of some of the arguments presented so far might explain the decline but questions the ability of a monocausal argument.

[4]For important theoretical contributions see Acemoglu (1998), Caselli (1999), Galor and Moav (2000), Galor and Tsiddon (1997). Increasing international trade is also often considered to be a cause of the increase in income inequality. However there now seems to be a greater consensus that skill biased technological change is the culprit.

traditionally average earnings for the more educated have been greater than that for the less educated, in the eighties and nineties the differences soared. Murphy and Welch (1992) noted that the college high school wage differential for all experience levels rose from 40% in 1963 to 48% in 1970 and then fell to 38% in the late 1970s only to soar to 58% in 1989. The dispersion in the returns to education is also reflected in the average returns which have undergone dramatic changes in the post-war era. Mincer (1998) notes that returns grew in the sixties, fell in the seventies to reach a low level of 4%, then rebounded in the eighties and reached heights of 12% or more in the first half of the nineties. Bound and Johnson (1992) and Katz and Murphy (1992) drew attention to the important role of the "demand" for more educated and skilled workers in driving these patterns. Kreuger (1993) suggested that the advent of the computer has had a role to play in these changes. Autor, Katz and Kreuger (1998) reconfirm the above findings by showing that the wage premium continued to rise in the nineties and that the most rapid skill upgrading has taken place in the more computer intensive sectors of the economy.[5]

Using these two widely observed movements in the US economy, we present a novel explanation for the decline in savings rates. We argue that rising returns to education may be an important, and as yet overlooked explanation for the decline in savings. Standard economic theory suggests that investments in education should rise in response to an increase in its rate of return- and this is the first element of our model. Indeed, Mincer (1998) shows that enrollment rates for the period 1967-1990 responded positively to the wage premia and predicted that an upward trend in enrollment was forthcoming. The second element of our model is the assumption that households are altruistic, and parents care about the amount they spend on their children's education and on the amount that they allocate as financial transfers. Finally, in order to maximize the effectiveness of their transfers, parents allocate funds to these two types of transfers until the returns are equalized- an assumption that is fairly standard since its introduction to the literature by Becker and Tomes (1986). In the presence of rising returns to education, parents find it more profitable to spend on their children's education rather than leave bequests. Following a permanent increase in the return to education, it is then possible to show that the ratio of bequests to output declines while the share of educational expenditures rise.

After establishing the main result, we then consider some important extensions to the model. A decline in the bequest ratio is not the same as a decline in the savings rate. Individuals save for a number of reasons with bequests being only one of them. Other important constituents include life cycle savings and precautionary savings. To check for the robustness of the model we then extend it to include life cycle savings. It turns out that the conditions which generate a decline in bequests will also generate a decline in life cycle savings and therefore a decline in the aggregate savings rate too. One of the interesting results of the model is that for some parameter values it might be possible for

[5] For recent challenges to the skill biased technological hypothesis see Card and DiNardo (2002) and Lemieux (2004). However this work is more concerned with "within group" inequality rather than "between group" inequality. The latter continued to rise in the nineties (see Figure 2, Autor, Katz and Kearney (2004)). Also see Autor et al (2004) for an assessment of the labor economics literature on this subject.

the reallocation towards human capital investment to occur without actually raising the educational investment ratio while at the same time causing the savings rate to decline. Further our numerical illustrations suggest that even a 1% increase in the human capital investment share can cause a 2% decline in the aggregate savings rate. Thus the model predicts that we do not need to look for a one for one substitution between the two ratios to test the theory and perhaps not even have to look for an increase in educational investment shares at all.

A concern of those working on the nature of bequests, has been the relative importance of intentional bequests to unintentional ones. To allow for both types of transfers, we further introduce uncertainty regarding the timing of death. The results continue to be robust. Allowing uncertain longevity means that the theoretical framework lends itself also to analyzing the effects of increasing life expectancy in the context of this framework. Increasing life expectancy implies that individuals will save more for retirement, will allocate less to their children and accidental bequests will also fall. Towards the end of our paper, we devote some space to analysing the relative effects of changes in life expectancy and changes in returns to education on the aggregate savings rate. We find that the latter has a greater effect on savings rates.

The theoretical framework in this paper relies on the assumption that intergenerational transfers are an important source of capital accumulation in the economy. The contribution of such transfers relative to life cycle savings as the primary source of wealth accumulation was first brought to attention by Kotlikoff and Summers (1981). They suggested that almost eighty percent of the stock of wealth accumulated by households in 1974 was attributable to intergenerational transfers. Within intergenerational transfers, one may distinguish between bequests and other types of transfers such as tuition support, loans and transfers of businesses to children. This question of inclusion is one of the main points of disagreement between the proponents of the intergenerational transfers and proponents of the life cycle savings story- best exemplified in the debate between Kotlikoff (1988) and Modigliani (1988). Modigliani is opposed to parental support in college education as being defined as an intergenerational transfer. He argues that this is a transfer of human wealth and should not be incorporated into any measure of non-human wealth. Kotlikoff responded by arguing that as long as the transfer is one of funds, the point is immaterial. Clearly this distinction is germane to the modeling strategy here. The model relies on a distinction between the two not only because the behavior of their respective returns differ but also because education expenditures, as noted by Modigliani, does not go towards asset accumulation but pays for current consumption.[6]Finally, and here there is less disagreement between the two, is the importance of intentional bequests versus unintentional bequests. They both agree that a large part of intergenerational transfers might be unintentional- precautionary savings and life cycle savings that were not consumed until death will become intergenerational transfers. We extend the model to include life cycle savings and uncertain life expectancy and the basic intuition still goes through. On the whole, elements of the debate are well captured in the framework adopted

[6]Though as mentioned above, it is more appropriately viewed as an investment

in this paper. Kotlikoff's argument that both types of intergenerational transfers matter implies that the entire optimization exercise carried out below is not over a variable of insignificant magnitude; Modigliani's recognition of the distinction between different types of transfers lies at the center of this paper and both of their observations on unintentional bequests is also taken care of.[7]

The basic insight of the model that parents might bequeath less when they perceive their children's income to be growing faster was noted also by Meade (1966). However he did not concern himself with issues of human capital and changes in the nature of transfers. In terms of the existing literature, the argument presented in this paper is clearly the closest in spirit to that of Bils and Klenow (2000). Their finding that it is not schooling which causes growth but the other way round is similar to the motivation here and provides us with some indirect support. Lambertini (2004) is also closely related to our work. She explores the consequences of skill biased technological change on public savings through its effects on funding of public education. The issue is important since a lot of the investment in human capital indeed takes place through the public sector - even in higher education. However, at least in the US, as we shall see, private spending has become quantitatively more important over the past two decades. Finally, a set of papers by Rangazas (2000, 2002) are the only ones that we are aware of which are actually interested in the implications of physical resources devoted to human capital investment and its role in economic growth.[8] Rangazas' exercises are similar to growth accounting exercises. The modelling structure is quite different (though at the core, he too assumes an optimization between human capital investments and physical capital investments) and the questions asked are different as well. However there are strong complementarities in terms of the overall research agenda.

In addition to suggesting a novel explanation for the savings decline, the model draws attention to the possible trade-off between physical capital accumulation and human capital accumulation. The macroeconomic consequences of whether intergenerational transfers are in the form of financial or human wealth was recognized by Blinder (1976). He observed that human capital transfers enter into the consumption expenditure side of the national income account whereas financial bequests enter the savings side. The recent work in economic growth (e.g. Mankiw, Romer and Weil (1992)) has argued that the human capital stock in an economy is an important factor in explaining differences in growth rates across nations. Our model suggests that the decline in the savings rate is nothing but the failure on the part of the national accounts to reflect a change in the nature of accumulation by the household: an endogenous substitution of human capital wealth for financial wealth in a period where technological improvements favor skills.

[7]Dynan, Skinner and Zeldes (2002) argue for a reconciliation of life cycle savings and bequests by suggesting that bequest motives exist but such savings decisions can be viewed as contingent contracts. Thus there is a bequest motive, but if in some states of the world there are unfavorable health shocks during old age, there are no bequests and the money is used for medical expenditures.

[8]Most papers such as that of Bils and Klenow (2000) are more concerned with time costs.

Our suggested explanation is both novel and yet can be perceived as being "obvious" once stated. To persuade the reader, in the next section we present some stylized relationships between educational and savings variables. Section 3 of the paper develops the theoretical model and in section 4, we expand it in logical steps to include life cycle savings and unintentional bequests. Section 5 of the paper concludes.

2 Stylized Facts on Education and Savings

Given that there are so many competing explanations for the savings decline, how much more of a story can human capital contribute? To consider the argument presented here, one needs to look at the magnitude of changes in investment in human capital as a share of GDP. In practice, the true measure of investment in human capital is anybody's guess.[9] The share of aggregate education expenditures, in GDP has hardly changed over the past few decades and hovers at around seven percent. However these are direct expenditures and since they do not include opportunity costs and activities which are complementary to human capital formation, they are often regarded with skepticism by researchers. This throws up an obvious problem- the easiest source of data cannot be used to test the basic theory. We are therefore left with a grab bag of stylized facts that we hope will convince the reader of a possible negative association between educational investment and savings rates. Many of these, we hope the reader will agree, are actually quite interesting and surprisingly seems to have gone unnoticed in the literature.

Two well known pieces in the national income measurement literature have tried to recalculate the investment and stock in human capital. Fraumeni and Jorgenson (1990) attempted to measure investment in education. If one looks at education's contribution only to market based activities, their estimate of aggregate investment in education as a share of GDP turns out to be 45%. Mincer (1990) in his comment on their paper noted that the rate of discount used by Jorgenson and Fraumeni was half of what it should have really been. That reduces the percentage roughly to 22.5%- still a very high number. Mincer himself prefers cost based approaches and notes that investment in education would end up being 14% of GNP (However, in his 1998 paper, Mincer quotes a lower share of 9.1% of GNP for 1980 of which 5% is in post secondary education). Adding on the job training takes this number up by another 6% to 20%. Though not related to the model presented here, one can conjecture that investment in on the job training is likely to rise during periods of rapid technological change. More indirect evidence of the importance of human capital stock comes from Kendrick's (1976) classic study which suggested that more than half of US capital stock is comprised of human capital. Given that physical capital investment in the US hovers at around 15% as a share of GDP, one would not be amiss in assuming that the official 7% share in national accounts data for education spending is a serious underestimate and the correct estimates maybe more in line with that of Mincer's.

[9] The well known paper on conditional convergence, Mankiw, Romer and Weil (1992) faced exactly the same problem and had to rely on extrapolations of enrollment rates as proxies for human capital investment rates.

Since investment in education numbers are hard to come by, one can instead look at the relationship between the returns to higher education and the savings rates. We already know that the relationship is negative during the late twentieth century. In figure (2), we plot the movements of returns to higher education, the personal savings rate and the private savings rate through the most of the previous century for the United States. The savings rates are five year moving averages based on annual data from the national accounts while the return to education are decade estimates for returns to college for young men from Goldin and Katz (1999). The graph suggests that a strong negative relationship might have persisted between the two up to 1950 as well and again in the seventies. Only during the fifties and the sixties does the negative relationship seem not to hold.[10]

To make an indirect assessment of the changes in investment in education, it is useful also to look at what happened to enrollments in higher education.[11] Figure (3) depicts enrollments in all higher education institutions for the 18-24 age group for the period 1960-2001. The overall trend is a monotonic increase in enrollment rates. During the seventies there is a stagnation which has already been the subject of the labor economics literature (see Card and Lemieux (2001)) and this coincided with a slight increase in savings rates and the fall in returns to higher education. From the early eighties the enrollment rates continued to rise and this coincided with falling savings rates and rising returns. As in the earlier graph we do still have a problem with the sixties. We use this figure to show another important aspect of human capital investments: the problems with national accounts data not withstanding, we have plotted the relative importance of private to public spending in higher education from 1960. Note that in the late sixties that share of private spending was considerably high and then through the late sixties and most of the seventies this ratio declined. Again this is a period when household savings rates went up and one could argue that perhaps the fact that the government's rising participation in higher education funding may have allowed household savings rates to rise. From the eighties onwards we again see the rising share of private spending and this coincides with a decline in the personal savings rate.

To derive an alternative estimate of the possible magnitude of the role of expenditures in education in explaining the savings decline, we looked at the weights given to education expenditures in the consumer price index over the years. These weights include only tuition, books and fees and no other expenditures and stand at a paltry 2.62% in 1998 (this translates to a 2.5% share in terms of personal disposable income). In 1980 this number was even lower at 0.87% (which translates to 0.76% of personal disposable income). Between 1980 and 1998 the savings rate as a share of personal disposable income fell by 6.5%. Based on these numbers increases in educational expenditures as a share of personal disposable income, can explain 28.2% of the savings decline. If we close in further to

[10]One can argue that for the first half of the twentieth century what should matter more are returns to schooling rather than college. In fact, the return to schooling also dropped drastically (from approximately 12% to 5%) between 1914 to 1950 (see figure 4, Goldin and Katz(1999)).

[11]We look at investment in higher education since that is where the larger increases are likely to take place given the relatively low higher education enrollment rates in the US.

the period 1984-98, which is at the heart of the savings decline, personal savings as a share of personal disposable income declined by 4.5 percentage points from 8.2% to 3.65%, while the share of education rose by 0.9% points. This also suggests a prima facie case for explaining almost 23% of the decline in the savings rate.[12]

Another set of evidence on the negative relationship between educational expenditures and savings rates comes from international comparisons. In figure (4) we plot the household savings rates for OECD countries and private education expenditure as a share of GDP for the year 2000. One can clearly see a strong negative relationship with the sole exception of one point on the far right (Korea). For the 14 OECD countries for which we have data, the correlation between these two variables is -0.30. Once one excludes Korea the correlation jumps to -0.63. This is indeed a strong negative association and we are not aware of any other study that has noticed this. Another outlier is of course the US which has the highest private spending on education as a share of GDP in the group and also one of the lowest savings rates. Once we drop the US in addition to Korea, the negative correlation drops to -0.48 - still very high.[13]

Clearly these exercises suggest that rising returns to education is at least worth investigating as a potential candidate in explaining the decline in the savings rate. We should also note, that while we have been searching for a negative relationship between educational expenditures and savings rates as motivating evidence for the theory, the theory itself will suggest that rising returns to human capital can actually reduce the aggregate savings rate without necesarilly raising the educational expenditure shares.

The next section of the paper develops a simple model with bequests being the sole constituent of savings. Section 4 of the paper incorporates life cycle savings. The first subsection deals with no uncertainty regarding life spans. The second subsection further develops the model by introducing uncertainty and therefore allowing unintentional bequests as well.

[12]One could easily argue that direct measures such as these do not capture the entire picture and can even be underestimates. Measures of investment in human capital do not generally include activities complementary to formal education. This is particularly true in the case of pre-college education. As the returns to college education rise, and securing admission becomes more competitive, parents to spend a lot more on their children's school education. This includes but is not restricted to costs incurred in tutoring and extra-curricular activities (which is often argued to increase the "cognitive skills" of the child) in order to enhance their children's prospects. In addition to underestimating investment in education in the accounting sense, there is anecdotal evidence suggesting increasing "time costs" have been ignored as well. See "The Parent Trap", Cover story in Newsweek (Jan 29, 2001). Adding on the job training would presumably increase the importance of education spending specially in a period during which rapid technological change forces firms to retrain their workers.

[13]The countries for which data are available include Australia, Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, Korea, Netherlands, Spain, UK and USA. The data for the private spending on education comes from OECD (2003) and the data on household savings rates comes from OECD (2004).

3 The Basic Model

Consider an overlapping generations economy with perfectly competitive markets. Economic activity extends over infinite discrete time. In every period the economy produces a single homogenous good that can be used for consumption, investment in physical capital and investment in human capital. The good is produced by physical capital and human capital augmented raw labor. The supply of the two factors of production, physical and human capital, is determined endogenously.

3.1 Production in the Economy

In every period the final good is produced under a constant returns to scale neoclassical production function that is subject to technological progress. The output produced at time t is,

$$Y_t = Y(K_t, A_t H_t) \equiv A_t H_t f(k_t);\ \ k_t \equiv K_t/(A_t H_t)$$

where A_t represents a human capital-augmenting productivity parameter. K_t and H_t represent physical and human capital stocks respectively in any given period, t. The production function $f(k_t)$ is strictly monotonic increasing, strictly concave satisfying the neoclassical boundary condition that ensure the existence of an interior solution to the producer's profit maximizing problem.

Producers operate in a perfectly competitive environment. Given the wage rate per efficiency unit of labor and the rate of return to capital at time t, w_t and r_t respectively, producers employ the optimal amount of capital K_t and human capital H_t, in order to maximize profits, i.e. $\{K_t, H_t\} = \arg\max[A_t H_t f(k_t) - w_t H_t - r_t K_t]$. Competitive factor markets ensure that factor prices equal their marginal products,

$$\begin{aligned} r_t &= f'(k_t) \\ w_t &= A_t[f(k_t) - f'(k_t)k_t] \equiv A_t w(k_t) \end{aligned} \tag{1}$$

H_t, the human capital stock is the economy's stock of raw Labor augmented by an efficiency parameter h,

$$H_t = h_t L$$

where L represents the population in the economy and is fixed and h represents the human capital augmenting factor. Without loss of generality, we assume that L is normalized to 1. The production function for human capital, is simple. Human capital for a single individual, in any period, h_t is a simple concave function of the educational expenditures in the past period, e_{t-1}[14],

[14] In practice, human capital production functions can be quite complicated and its correct specification is the subject of a wide literature. More comprehensive specifications would include time investments, human capital of teachers, and different specifications for different levels of education based on microeconomic evidence. For example see Bils and Klenow (2000) and Rangazas (2002). Also, here we assume that expenditures increase the quantity of human capital and not necesarilly the quality.

$$h_t = e_{t-1}^{\theta}, \quad \theta \in (0,1) \tag{2}$$

From equation(1) it follows that an individual receives a final wage of $w_t A_t h_t$.

Capital is assumed to depreciate completely in every period. This assumption ensures that there is a clear correspondence between inherited wealth in this model and household savings as defined in national accounts.[15] Technological progress is exogenous and takes place at a constant rate of growth, g,

$$A_{t+1} = (1+g)A_t$$

In this setup there are two ways to capture the effect of rising returns to education- through changes in parameter values of g and θ. Accelerating technological change (increases in g) could potentially raise the return to investing in human capital since higher A leads to higher wages. Alternatively rising θ reduces the extent of diminishing returns and thus encourage education spending. Galor and Moav (2000) use increasing g to characterize skill-biased technological change while Lambertini (2004) uses rising θ to characterize skill biased technological change.[16] In this paper we choose to focus on the latter. One problem with focussing purely on changes in g is we know for a fact that during the eighties there was a TFP slowdown and this would mean a slowdown in g - which would reduce the return to human capital and thus be counterfactual. Also in a closed economy changes in g will not affect the relative returns of physical and human capital anyways. Therefore, we choose to conduct our entire analysis with respect to changes in θ. However note that this is also simply an analytical device, a more full-blown model would incorporate the rising demand for college educated labor. This would surely add more realism but would needlessly complicate the basic story.

3.2 The household

In each period a generation is born. The economy is characterized by the coexistence of two generations simultaneously. Individuals are assumed to care about the total transfer they make to their children.[17]

[15] In fact in a closed economy version, the two will be exactly the same. The only difference arising from the small open economy assumption is that some of the inherited wealth may be invested abroad.

[16] It must be noted, however, that in Galor and Moav (2000), rising g has differential effects on the skilled versus the unskilled.

[17] Most overlapping generations models usually incorporate only one type of transfer -either human capital or bequests. Usually this depends on the focus of the model. Very few include both. Becker and Tomes (1986) takes cognizance of both types of transfers and it is their style which is adopted here. A similar preference structure in a dynamic setting is employed by Galor and Moav (2005) but abstracts from technological progress. Zilcha (2003) incorporated both bequests and human capital investments into the parents' utility function to study economic growth. However the functional form is of a constant returns Cobb-Douglas nature where human capital and bequests enter as separate arguments. In such a setup an individual's preferences structure might imply a relatively higher share of wealth being allocated to bequests relative to education spending or vice versa. One can then compare growth trajectories that emerge under the two different cases. Such a preference structure is obviously too restrictive to answer the question posed here. A lot of the work in overlapping generational models of human capital formation also usually assumes that individuals make their own educational investments. In practice however this is not completely correct. Rosenzweig and Wolpin (1993) show that intergenerational transfers begin with huge amounts earmarked by parents to support their children's human capital accumulation.

At any period t, members of one generation (generation, $t+1$), are economically inactive and spend time receiving an education. Members of another generation (generation, t), work and earn their livelihood. With their income and the bequest they receive, members of generation t in time period t consume the final good, support the younger generation's education and make bequest allocations. All transactions take place at the beginning of each period and shocks to the growth rate of technological progress also take place then. Therefore an individual is aware if her income changed because of a shock and adjusts expenditures accordingly.

Since an individual is not economically independent in the first period of her life, utility is defined over expenditures in the second period. In addition to her own consumption, she derives utility from the sum of the value of total transfers it makes to its offsprings. Transfers can take place in the form of investments in human capital and bequests. Altruism is directed only towards the young and not towards the old. In any period t, the only source of savings therefore are bequests of agents of period t.

The utility function is assumed to be of the logarithmic form and is separable in consumption and transfers. For a member of generation t, the optimization problem is as follows,

$$\max_{c_t, e_t, b_t} U_t^t = \alpha \ln c_t^t + \gamma \ln \omega_t \tag{3}$$

subject to,

$$c_t^t + \omega_t = w_t A_t h_t + r_t b_{t-1} \tag{4}$$

where

$$\omega_t = e_t + b_t \tag{5}$$

and

$$\alpha < 1, \gamma < 1 \text{ and } \alpha + \gamma = 1$$

ω_t is the total transfer made by a member of generation t. The budget constraint says that the sum of all expenditures is equal to the sum of an individuals labor earnings, $w_t A_t h_t$ and her inheritances, $r_t b_{t-1}$. A superscript t implies that the value of the variable in question was chosen by a member of generation t. A subscript tell us in which period that variable belongs to. The superscript is avoided for variables for which there is no such confusion.

Utility maximization implies that the following solutions for c_t^t and ω_t will hold,

$$c_t^t = \alpha \left(w_t A_t h_t + r_t b_{t-1} \right) \tag{6}$$

$$b_t + e_t = \gamma \left(w_t A_t h_t + r_t b_{t-1} \right) \tag{7}$$

Given the current specification, the model by itself does not provide a method for calculating how the total transfers should be divided between bequests and human capital investment. The decision regarding this is assumed to come from an arbitrage equilibrium. That is, the net rate of return on education and financial assets are equalized (Becker and Tomes (1986)). This implies that, investment in education will be undertaken until a point such that,

$$r_{e,t} = r_t$$

where $r_{e,t}$ denotes the marginal rate of return on human capital investments. Given the human capital production function, the condition can be rewritten as,

$$w_{t+1}A_{t+1}h'(e_t) \equiv r_{e,t+1} = r_{t+1} \tag{8}$$

$$\Rightarrow w_{t+1}A_{t+1}\theta(e_t)^{\theta-1} = r_{t+1}$$

$$\Rightarrow e_t^* = \left(\frac{w_{t+1}A_{t+1}\theta}{r_{t+1}}\right)^{\frac{1}{1-\theta}} \tag{9}$$

Since $\frac{w_{t+1}}{r_{t+1}}$ is increasing in k_{t+1},this of course means that the optimal amount of e_t^* is increasing in k_{t+1} as well. One can make some easy observations based on the above expression- from an individual's point of view (i.e the partial equilibrium) an increase in θ unambiguously raises the optimal level of investment. Further higher A_{t+1} also raises the optimal amount of investment.[18] To get a complete grasp of the general equilibrium effects, for the rest of the paper we will assume that the production function is Cobb-Douglas. This not only aids in obtaining closed form solutions but also makes it convenient to undertake numerical illustrations. Therefore,

$$Y_t = K_t^{\lambda}\left(A_t H_t\right)^{1-\lambda}$$

and

$$y_t = k_t^{\lambda}$$

This implies that

$$\frac{r_{t+1}}{w_{t+1}} = \frac{\lambda}{(1-\lambda)}k_{t+1}^{-1} = \frac{\lambda}{(1-\lambda)}\left(\frac{b_t}{A_{t+1}e_t^{\theta}}\right)^{-1}$$

and

$$\left(\frac{w_{t+1}}{r_{t+1}}\right) = \frac{(1-\lambda)}{\lambda}\left(\frac{b_t}{A_{t+1}e_t^{\theta}}\right) \tag{10}$$

Substituting equation (10) in equation (9) and after some rearranging we have both the optimal educational expenditures and the human capital per worker in t he economy as a function of bequests:

$$e_t^* = \frac{(1-\lambda)\,\theta}{\lambda}b_t \tag{11}$$

[18]Note that in a small open economy, since wages and interest rates are fixed, the above expression would provide the complete solution for equilibrium e_t^*.

$$h_{t+1} = \left(\frac{(1-\lambda)\,\theta}{\lambda} b_t\right)^{\theta} \tag{12}$$

Therefore,

$$k_{t+1} = \left(\frac{b_t^{1-\theta}}{A_{t+1}\left(\frac{(1-\alpha)\theta}{\alpha}\right)^{\theta}}\right) \tag{13}$$

i.e the capital per efficiency unit of labor is completely described by past bequests and the level of technology.

For the rest of the paper what we will care about are the ratios of education and savings to GDP per capita and to personal disposable income per capita (in the closed economy both of these are the same). The share of educational expenditures in GDP is defined as:

$$E_t = \frac{e_t^*}{y_t A_t h_t} = \frac{\frac{(1-\lambda)\theta}{\lambda} b_t}{y_t A_t h_t} \tag{14}$$

and the savings rate, which is the same as the bequest ratio, denoted by B_t, is

$$B_t \equiv \frac{b_t}{y_t A_t h_t} = \frac{\gamma\left(w_t A_t h_t + r_t b_{t-1}\right)}{y_t A_t h_t} - \frac{e_t}{y_t A_t h_t} \tag{15}$$

Substituting equations (11), (12) and (13) into the above expression and solving further gives us a clean expression for the bequest ratio:

$$B_t = \frac{\gamma\lambda}{(\lambda + (1-\lambda)\,\theta)} \tag{16}$$

Substituting this into equation (14), gives us the equilibrium education spending ratio in the economy:

$$E_t = \frac{\gamma\,(1-\lambda)\,\theta}{(\lambda + (1-\lambda)\,\theta)} \tag{17}$$

Therefore both bequests and education levels increase at the same rate as GDP and their shares remain constant unless preferences or production parameters change. Of particular interest to us is the effect of θ : an increase in theta produces the desired effect- it raises the education spending ratio and reduces the bequest ratio. As we had hinted earlier, the level of technology has no "ratio" effect since it's growth affects both GDP and these two variables identically. Thus declining TFP growth in this model is perfectly compatible with rising returns to education and a declining savings rate. Finally, we derive the expression of two additional variables: the growth rate of output per capita $(y_t A_t h_t)$ and the interest rate. Note, that growth rate of output per capita is the same as the growth rate of bequests, b_t, since the ratio of the two is always fixed. The growth rate of output per capita and bequests is

$$\frac{\Delta y_t A_t h_t}{y_{t-1} A_{t-1} h_{t-1}} = \frac{\Delta b_t}{b_{t-1}} = (1+g)^{\frac{1}{1-\theta}} - 1$$

and the interest rate is

$$r_t = \frac{(1+g)^{\frac{1}{1-\theta}} (\lambda + (1-\lambda)\theta)}{\gamma} \tag{18}$$

The derivation is done in the appendix. Note that an increase in θ unambiguously raises the interest rate. Therefore an increase in the return to education also raises the return to physical capital. This is not surprising, as it becomes more profitable to invest in education, individuals move away from investing in physical capital which raises the marginal product of the latter and moreover since the two factors are complementary inputs in production a higher level of human capital raises the return on physical capital. Finally, note that the growth rate of technology raises both the growth rate of output per capita and the interest rate. Of course, this result is simlar to standard Solow model except that we now have $\frac{1}{1-\theta}$-which reflects the endogenous response of human capital accumulation to changes in g. A tempting application of this result is the stock market returns during the eighties and particularly the nineties. We know for a fact that real returns to capital rose rapidly during this period. The above equation suggests that higher returns to education could have caused this increase. As discussed earlier, the literature on skill biased technological change uses the advent of information and communcation technologies as the manifestation of such technological change. There is also a literature that argues that stock market returns were tied to the information technology revolution (e.g. see Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001)). Based on all of this one can construct a couple of different scenarios- since the returns to capital were really high only in the nineties and not so much in the eighties, it is possible that in the eighties an increase in θ took place simultaneously with a decrease in g and thus interest rates were not affected. In the nineties however the higher θ coincided with a resurgence in g which led to a much higher return in physical capital. In either case, what happens to g does not affect the savings rate and the education expenditure ratio.

4 Introducing Old Age Consumption into the Closed Economy

As discussed in the earlier, the empirical literature on wealth accumulation is divided between proponents of the importance of life cycle savings and intergenerational transfers.[19] In this section we extend the model to include life cycle savings as well. The first subsection presents the model without any uncertainty involved. Therefore there are no unintentional bequests. The second subsection introduces uncertainty and allows such types of bequests to occur. This complicates the analysis somewhat but the story still goes through.

[19]It must be added that there is also considerable support for the precautionary savings hypothesis which we have not discussed in this paper.

4.1 No Uncertainty

The economy is now characterized by the coexistence of three generations simultaneously. At any period t, members of generation $t+1$, are economically inactive and spend time receiving an education as before. Members of generation t are working and earning their livelihood. With their income and also the bequest they receive they consume the final good, support the younger generation's education, make bequest allocations for the younger generation and save a fraction of their income for retirement. The third generation live off their own savings.[20]

An individual's utility is now defined over second and third period consumption and from expenditures on their children. As before, the utility function is assumed to be of the logarithmic form and is separable in consumption and transfers. For a member of generation t, the optimization problem is now,

$$\max_{c_t, c_{t+1}, \omega_t} U_t^t = \alpha \ln c_t^t + \beta \ln c_{t+1}^t + \gamma \ln \omega_t \tag{19}$$

subject to,

$$c_t^t + \omega_t + s_t = w_t A_t h_t + r_t b_{t-1} \tag{20}$$

$$r_{t+1} s_t = c_{t+1}^t \tag{21}$$

where

$$\alpha + \beta + \gamma = 1 \tag{22}$$

Utility maximization implies that the following solution for c_t^t, c_{t+1}^t and ω_t will hold,

$$c_t^t = \alpha \left(w_t A_t h_t + r_t b_{t-1} \right) \tag{23}$$

$$c_{t+1}^t = \beta r_{t+1} \left(w_t A_t h_t + r_t b_{t-1} \right) \tag{24}$$

$$b_t + e_t = \gamma \left(w_t A_t h_t + r_t b_{t-1} \right) \tag{25}$$

In the previous subsection where we did not have any life cycle savings, both personal disposable income and final output were identical. However now one needs to be more careful: there are two generations that receive income- the working age generation whose disposable income is $(w_t A_t h_t + r_t b_{t-1})$ and the elderly whose disposable income is $r_t s_{t-1}$. The sum of these two disposable incomes is equal

[20]Note that bequests here are not so in the strict sense of the term. Individuals receive them before the death of their parents and hence are literally *inter-vivos intergenerational transfers* which earn a return. It is easy to see that even if it were a bequest at death, the maximization problem and budget constraints would be unchanged. This is because the bequest allocation will earn a return for over two periods. That would be compensated by the fact that it needs to be discounted one period to make it comparable to the returns to investment in education -leaving the situation unaltered. The only way it would matter is that it would impose some additional liquidity constraints. Since the model will not incorporate issues of liquidity constraints on education expenditures, this assumption only plays a simplifying role. Further allowing perfect capital markets would remove this restriction as well. The evidence on borrowing constraints is ambiguous. Shea (1998), Cameron and Heckman (1998) and Cameron and Taber (2004) find no evidence that borrowing constraints play a role in college choices.

to final output. The working generation saves $s_t + b_t$ whereas the retired generation simply consumes all of its income. Therefore the appropriate aggregate savings rate in the economy, denoted by X_t, is the following

$$X_t = \frac{s_t + b_t}{w_t A_t h_t + r_t b_{t-1} + r_t s_{t-1}} \equiv \frac{s_t + b_t}{y_t A_t h_t} \tag{26}$$

This can be rewritten as

$$X_t = \frac{s_t + b_t}{w_t A_t h_t + r_t b_{t-1}} \frac{w_t A_t h_t + r_t b_{t-1}}{w_t A_t h_t + r_t b_{t-1} + r_t s_{t-1}} \tag{27}$$

To derive the aggregate savings rate we will derive the two fractions separately. This simplifies the problem considerably. The first fraction can be solved by using the results from the household optimization problem. Before we can proceed though, we need to recalculate the optimal amount of e_t^*. Note that now, $k_{t+1} = \left(\frac{b_t + s_t}{A_{t+1} e_t^\theta}\right)$. Using this in equation (9) and solving again gives us:

$$e_t^* = \left(\frac{(1-\lambda)}{\lambda}(b_t + s_t)\,\theta\right) \tag{28}$$

Now

$$\frac{b_t + s_t}{(w_t A_t h_t + r_t b_{t-1})} = (\gamma + \beta) - \frac{e_t}{(w_t A_t h_t + r_t b_{t-1})}$$

Substituting the value of the optimal amount of education (28) into the above expression and solving further gives us the savings rate of the young:

$$\frac{(b_t + s_t)}{(w_t A_t h_t + r_t b_{t-1})} = \frac{\lambda(\gamma + \beta)}{(\lambda + (1-\lambda)\,\theta)}$$

Like before, an increase in θ reduces the savings rate. Adding life cycle savings does not alter this basic result. This is not surprising given the logarithmic nature of the utility function which ensures that $\frac{s_t}{(w_t A_t h_t + r_t b_{t-1})} = \beta$ irrespective of what happens to the return to education.

Also note that the bequest ratio is:

$$\frac{b_t}{(w_t A_t h_t + r_t b_{t-1})} = \frac{\gamma\lambda - \beta(1-\lambda)\,\theta}{\lambda + (1-\lambda)\,\theta} \tag{29}$$

and the relative importance of life cycle savings to bequests (for the working age adults) is given by

$$\frac{s_t}{b_t + s_t} = \frac{\beta(\lambda + (1-\lambda)\,\theta)}{(\gamma + \beta)\,\lambda} \tag{30}$$

which is clearly an increasing function of θ. To derive the second component of X_t in equation (27), note that

$$\frac{r_t s_{t-1}}{(w_t A_t h_t + r_t b_{t-1} + r_t s_{t-1})} = \frac{\alpha k_t^{\alpha-1} s_{t-1}}{(1-\alpha) k_t^{\alpha} A_t h_t + \alpha k_t^{\alpha-1} (b_{t-1} + s_{t-1})}$$

Using the fact that $k_t = \frac{b_{t-1}+s_{t-1}}{A_t h_t}$ and equation (30) allows us to simplify the above expression to

$$\frac{r_t s_{t-1}}{(w_t A_t h_t + r_t b_{t-1} + r_t s_{t-1})} = \frac{\beta (\lambda + (1-\lambda)\theta)}{\gamma + \beta} \tag{31}$$

and since

$$\frac{w_t A_t h_t + r_t b_{t-1}}{w_t A_t h_t + r_t b_{t-1} + r_t s_{t-1}} = 1 - \frac{r_t s_{t-1}}{(w_t A_t h_t + r_t b_{t-1} + r_t s_{t-1})}$$

$$\Rightarrow \frac{w_t A_t h_t + r_t b_{t-1}}{w_t A_t h_t + r_t b_{t-1} + r_t s_{t-1}} = 1 - \frac{\beta (\lambda + (1-\lambda)\theta)}{\gamma + \beta} \tag{32}$$

This gives us the final expression for the aggregate savings rate:

$$X_t = \frac{\lambda (\gamma + \beta)}{(\lambda + (1-\lambda)\theta)} \left(1 - \frac{\beta (\lambda + (1-\lambda)\theta)}{\gamma + \beta}\right) \tag{33}$$

$$\Rightarrow X_t = \lambda \left(\frac{\gamma + (1-\lambda)\beta (1-\theta)}{(\lambda + (1-\lambda)\theta)}\right) \tag{34}$$

If $\beta = 0$, this reverts back to the bequest only case discussed above. Note that adding life cycle savings has two effects on the aggregate savings rate. First of all, by definition, it increases the aggregate savings rate of the working adults. On the other hand since the elderly do not save, this reduces the aggregate savings rate. The conflicting effects are best understood through equation (33). The first term tells us the savings rate of the working adults. This is multipled by a term that weighs their income in the total disposable income of the economy. Note that from the second term an increase in θ reduces the weight placed on the disposable income of the working adults. This is because for the retired a rise in the return to education also raises the return to capital which acts as a bonus for them. Thus rising returns to education now has a second effect that reduces the aggregate savings rate. We can also figure out the share of bequests in aggregate disposable income in this economy using equations (29) and (32):

$$B_t = \frac{\gamma\lambda - \beta (1-\lambda)\theta}{\lambda + (1-\lambda)\theta} \left(\frac{\gamma + (1-\lambda)\beta (1-\theta)}{\gamma + \beta}\right) \tag{35}$$

Again an increase in θ lowers the bequest ratio in the economy. Note that now it is possible for the bequest ratio to be negative - life cycle savings assures that physical capital accumulation will never be zero. Bequests will be negative if θ is high enough. The question of whether one should consider this possibility seriously or not is open to debate. Negative bequests means that parents borrow to finance their children's education and the loan is repaid by the child. This is not completely unrealistic- presumably with most college loans parents provide some kind of backing and guarantee even if it is the child that is the borrower, and presumably the entire family is involved in the decision.[21] Also note that the loans are repaid while the parents are still alive.

Finally, we derive the education ratio, interest rate and the growth rate of the economy - the details of which are worked out in the appendix. Education spending as a share of GDP is now:

$$E_t = ((1-\lambda)\theta)\left(\frac{\gamma+(1-\lambda)\beta(1-\theta)}{(\lambda+(1-\lambda)\theta)}\right) \tag{36}$$

An increase in θ no longer necesarilly implies increases in the education spending ratio. In fact it is possible for the education spending ratio to go down while life cycle savings goes down too in response to an increase in θ. From the expression above, clearly what happens depends on whether, $((1-\lambda)\theta)$ increases faster than the decline in $X_t = \left(\frac{\gamma+(1-\lambda)\beta(1-\theta)}{(\lambda+(1-\lambda)\theta)}\right)$. To investigate this further note that we can write the equation for E_t as:

$$\Rightarrow E_t = \left(\frac{(1-\lambda)\theta(\gamma+\beta)}{(\lambda+(1-\lambda)\theta)} - (1-\lambda)\theta\beta\right)$$

Comparing this to equation (17), the first term here in parentheses reflects the fact that young working adults will raise the education spending relative to their disposable income. However we have the additional $-(1-\lambda)\theta\beta$ term which creates the ambiguity. This reflects the fact that the retired generation benefit from a higher return on the life cycle savings, which raises their share in personal disposable income and since they do not spend on education, it works to reduce the total education spending as a share of total disposable income. Whether education spending as a share of GDP increases or not depends upon whether $\frac{\lambda(\gamma+\beta)}{(\lambda+(1-\lambda)\theta)^2} \gtreqless \beta$.[22] The left hand side is the marginal effect of higher theta on education spending rate by the economically active and the right hand side reflects the relative importance of life cycle savings which increases the share of the elderly in total income (see equation (28)). For low values of γ and high values of θ, the educational spending ratio may actually go down rather than go up. Low values of γ will tend to raise the interest rate and thus increase the share of the elderly's disposable income and high values of theta by reducing bequest transfers (and hence physical capital accumulation) will also raise the interest rate.

The interest rate and the growth rate both continue to increase following an increases in θ :

$$r = \frac{(1+g)^{\frac{1}{1-\theta}}(\lambda+(1-\lambda)\theta)}{\gamma+(1-\lambda)\beta(1-\theta)} \tag{37}$$

and

$$\frac{\Delta y_t A_t h_t}{y_{t-1}A_{t-1}h_{t-1}} = \frac{\Delta b_t}{b_{t-1}} = (1+g)^{\frac{1}{1-\theta}} - 1 \tag{38}$$

[21] Who do they borrow from? In the general equilibrium, parents "lend" some of their life cycle savings for education spending. Again this is all subject to the caveat that we have not considered the issue of collateralizing human capital investment loans.

[22] This can be easilly figured out by checking the condition for $\frac{\partial E_t}{\partial \theta} > 0$.

Though the model is much too sparse for a proper replication of the US economy, we pursue some numerical illustrations. We will examine the aggregate savings rate, the education spending rate, the bequest rate and the interest rate. The main question is of course choosing appropriate parameter values. For the illustration we assume that $\lambda = 1/3$ which is standard in the literature. For α we assume 0.60, $\beta = 0.25$ and for $\gamma = 0.15$. There are a couple of reasons why we assume such a high value for α and such a low value for β. First of all the model assumes equal lengths of time spent working and retiring. This is not true in reality. Even after college, an average adult can expect to work for more than 40 years and retire at the age of 65 for about 20 years. Thus an easy way out is to assign an alpha twice the value of beta. To this we can add two more things- the fact that the future is discounted and the fact that during the working period, an adult also needs to feed his offspring. With all this in mind $\alpha = 0.6$ seems to be a reasonable assumption. Also note that a value of β thats too high would imply a savings rate too that would be improbably high. Finally for γ we assume a relatively low weight to show that some of the numerical examples work even with limited altruism. The real problem is choosing a correct value for θ. Rangazas (2000) allows this to range from 0.10 to 0.15 based on observed shares. However his human capital production function is more specific in that it allows also for past human capital and existing stock of knowledge to matter and exhibits constant returns to scale in all three. Since our production function assumes decreasing returns to scale we will probably need slightly higher assumed values. One way to get around this is to use values of theta that are consistent with average real interest rates. For example, given that historically the long term real return on stocks in the US has been 6.5%, one can use the theta that corresponds between an average of this number and a long run risk free return. Therefore it might be reasonable to focus the sensitivity of the savings rate and other variables to changes in theta when the real interest rate lies between 4.5 to 6.5%. To calculate interest rates for various theta, we also need prior estimates of the rate of growth of A_t. Based on historical data, we assume an annual g value of 2.0% and also plot rates for a lower value of $g = 1\%$ (corresponding to lower TFP growth during the eighties). The latter number corresponds to the period of TFP slowdown. Finally, while the numbers discussed here are are annual rates, our model is an OLG model with longer time periods. When working with the calculations, we assume that each period lasts 25 years. Figure (5) plots the the Bequest ratio (B_t), the education spending ratio (E_t), the aggregate savings rate (X_t) and the interest rate (r_t).

<Insert Figure (5) here>

The first graph on the top left shows the bequest ratio as a share of GDP (which is the same as aggregate personal disposable income). As the graph suggests, the ratio falls off quite rapidly and reaches zero by the time $\theta = 0.3$. At that point, the aggregate savings rate is approximately 16% and the education spending ratio is 10% and the real interest rate corresponding to $g = 2\%$ (high g) is at 6%. These are all reasonable values though the aggregate savings rate does correspond more to the private savings rate rather than the personal savings rate. Notice that the education ratio begins to decline for high values of theta. At these values the bequest ratio is substantially negative (-0.04%) and the interest rate is substantially higher at almost 9% (corresponding to $\theta = 0.6$) and the savings rate is at 10%.

The obvious question to ask here is what is the reduction in the aggregate savings rate (and the change in the education spending ratio) when θ increases. Let's consider three examples. As a first scenario, consider the case when $\theta = 0.05$ and rises to 0.1. At such low levels of θ, the marginal return to human capital investment is very high and this is reflected in the fact the education spending to GDP ratio rises by about 2% points from less than 3% of GDP to 5% of GDP. The aggregate savings rate on the other hand drops from 27.5% to 25%- a slightly larger drop. Thus even at such low levels of θ the decrease in the savings rate is larger than the increase in education spending rate, albeit marginally. However the interest rate here is on the lower side- it increases from 4% to 4.5%.

The second scenario we consider is an increase of θ from 0.25 to 0.30. Here we begin with an aggregate savings rate of approximately 18.3% and education spending ratio of 9.2%. Following the increase in θ, we find that the savings rate has fallen to 16.7% and the education spending ratio has risen to only 10%- therefore a 1% increase in education rate leads to a 2% decline in the savings rate. The interest rate also increases by half a percentage point from 5.5% to 6%. If we allow g to fall to 1% simultaneously we can see that the interest rate would not rise but decline by half a percentage point to 5%. Of course in the nineties the growth rate of the economy went up. But in our model note that the growth rate is a positive function of both g and θ (see equation (38)). Therefore an increase in g is not necessary to raise the overall growth rate since we have assumed that theta has risen in any case.

Finally as a third exercise one can consider what happens when bequests are already in the negative range. Of course this exercise is subject to the caveat that we have not really bothered about collateralization of human capital and hence must be considered completely speculative. Suppose theta increased from 0.4 to 0.45. In this case initially the savings rate is 13.9% and the education spending ratio is 11.1%. Following an increase in theta, the savings rate drops to 12.7% and the education spending ratio rises only to 11.4%. This is an enormous effect- for every percentage point increase in education spending the savings rate declines by 4 points. Clearly what happens here is that the retired begin to benefit a lot from the sudden increase in theta which raises the return to capital. Also the fact that bequests are negative, puts further pressure on interest rates and hence the reallocation effect towards education spending is dwarfed by the factor price effects. At this level of theta, the interest rate is fairly high at 7% and increases to 7.5%.

Overall, one can conclude that the increase in education spending is usually lower than the decline in the aggregate savings rate following an increase in θ. As θ increases the tradeoff becomes higher- it is possible to see reductions in the savings rate with smaller and smaller increases in human capital investment rate.

4.2 Adding Uncertainty

To capture the role of educational returns in explaining the decline of savings rate, we have chosen a utility maximizing framework that assumes individuals optimize between financial bequests and human capital investments. The advantage of this is obvious in that it makes the arbitrage condition between the two investments the centerpiece of the model. A skeptic might be led to wonder if financial bequests are indeed important enough that increases in human capital investments are largely due to reductions in such bequests. Since Kotlikoff and Summers (1981) published their findings that bequests were the major source of capital accumulation in the US, the debate has largely been one of whether bequests are intentional or unintentional. On the whole the literature seems to suggest that unintentional bequests are more important that intentional ones. Clearly, if the theory presented here is to have any empirical significance, then it should be robust to the inclusion of unintentional bequests. Incorporating both unintentional bequests and intentional bequests in a unified model can be problematic, more so when one has human capital accumulation, endogenous interest rates and technological progress.[23] One has to now allow for heterogeneity of agents since wealth accumulation along a dynasty depends on its mortality history. Abel (1985) was probably the first to introduce unintentional bequests explicitly and derived the intra-cohort distribution of bequests, wealth and consumption. He then went on to analyze the effect of actuarially fair social security on national wealth. However intentional bequests were not included and neither were human capital investments. Zhang, Zhang and Lee (2003) developed the Abel model further to incorporate public education and re-examine fiscal policy issues. They also allow the interest rate to be endogenous. Both of these papers use overlapping generations frameworks and assume that individuals can live for two periods but a certain fraction of them die at the end of the first period. This creates unintentional bequests. It also means that even though agents maybe similar at the initial time period eventually there will be a distribution of wealth. The modeling strategy adopted here is similar in this respect but is quite different from then on. This is necessitated by the different preference structure assumed so far in our model: the optimal allocation between intentional bequests and human capital. Compared to those papers we are less interested in the actual distribution of wealth at every time period and more interested in the savings ratio in the aggregate. This does not mean that distributional issues can be ignored - after all the aggregate does depend on the way the underlying distribution evolves every time period. Fortunately, despite the distributional issues, modelling aggregate savings rate turns out to be relatively easy.

Households, as in section 2, live for three periods but are now faced with the probability that they will not survive beyond their second period in life, i.e. they live upto the end of their working lives but may die at the beginning of retirement. The probability that this may happen is p where $p \in (0, 1)$. Introducing this into the utility function does not mean simply multiplying the felicity during retirement with a $(1 - p)$ term. It would be reasonable that households now take into account the

[23] In fact, we are not aware of any other paper that has actually modelled all of these together in a tractable framework.

possibility that with the introduction of uncertain lifespans, that their descendants will receive an unintentional bequest in additional to the intentional bequest. If all that the household cares about is the total transfer made to their children, then they should choose a total intentional bequest lower than what they would have chosen with no uncertainty. Therefore they maximize an expected utility,

$$\max_{c_t^t, s_t} U_t^t = \alpha \ln c_t^t + (1-p)\beta \ln (r_{t+1} s_t) + (1-p)\gamma \ln \left(W_t - c_t^t - s_t\right) + p\gamma \ln \left(W_t - c_t^t\right) \tag{39}$$

where they expect the next generation to receive a bequest of $W_t - c_t$ if they themselves do not live for the second period and a bequest of $W_t - c_t^t - s_t$ if they do live through retirement. W_t here is the endowment at the beginning of period t -which would be wages and the return on bequests - bequests here could be either only intentional, or both intentional and unintentional. We continue to assume that liquidity constraints are not binding in the former case and hence wont bind in the latter case either. Also note that uncertain longevity does not affect investments in human capital.[24] Solving the above static problem subject to the regular budget constraints, now yields modified optimal values for consumption, life cycle savings and intentional transfers (ω_t (sum of intentional bequests and education spending)):

$$c_t^t = \frac{\alpha}{[\gamma + \beta(1-p) + \alpha]} W_t \tag{40}$$

$$s_t = \frac{\beta}{(\gamma + \beta)} \frac{(\gamma + \beta(1-p))}{[\gamma + \beta(1-p) + \alpha]} W_t \tag{41}$$

$$\omega_t = \left(\frac{\gamma}{(\gamma + \beta)} \frac{[\gamma + \beta(1-p)]}{[\gamma + \beta(1-p) + \alpha]} \right) W_t \tag{42}$$

The introduction of life expectancy does not affect the allocation between bequests and life cycle savings- presumably because of logarithmic utility. It does, of course, affect the allocation between (a) current consumption and (b) both future consumption and transfers by raising the weight on the former.

As stated before the problem is that now there are groups of individuals who have different W_t given their dynasty's mortality record. Consider one of these subgroups i. Within this subgroup, all individuals receive the same bequest, or b_{t-1}^i. They all face the same probability of death and have the same budget constraint. And importantly they all make the same decision regarding how much to invest in human capital, how much to set aside for a financial transfer and how much for life cycle savings. Lets assume that they all decide on intentional bequests of an amount b_t^i and life cycle savings, s_t^i. What happens next is that a fraction p of them will die at the beginning of the third period of their lives. Therefore a fraction p of their descendants receive bequests equal to $r_{t+1}\left(b_t^i + s_t^i\right)$ while another fraction receives bequests equal to $r_{t+1} b_t^i$. This means that in the next generation a fraction p make their decisions based on $r_{t+1}\left(b_t^i + s_t^i\right)$ and another fraction $(1-p)$ based on $r_{t+1} b_t^i$. Since this process repeats itself every time period, one might guess that the two variables (b_t^i and s_t^i)

[24]For developed economies, this seems to be a reasonable assumption.

might follow a well defined method of evolution in the aggregate. However to be able to do that, as a starting point, we need to make the following assumption:

Assumption 1 *At time $t = 0$ all individuals are alike and have the same endowment W_0.*

The above assumption allows for the economy in time period zero to be characterized by a representative agent. Further, both b_0 (bequests allocated at $t = 0$) and s_0 (life cycle savings allocated at $t = 0$) will be the same for all individuals. For the rest of the paper, IB_t is the aggregate *intentional* bequests and LS_t is aggregate life cycle savings. Since population has been normalized to one, "aggregate" here is equivalent to the weighted average of the bequest ratios of all groups in society at any given time period, t (it is easy to figure out that the number of subgroups in any period is 2^t under assumption (1)). In Appendix C it is shown that, indeed one can figure out well defined paths for both IB_t and LS_t and the results are derived more thoroughly. It turns out that bequests and life cycle savings is now characterized by a system of two difference equations,

$$IB_t = a_\omega(w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1}) - e_t \tag{43}$$

$$LS_t = a_s(w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1}) \tag{44}$$

where

$$a_\omega = \frac{\gamma}{(\gamma+\beta)} \frac{[\gamma+\beta(1-p)]}{[\gamma+\beta(1-p)+\alpha]}; a_s = \frac{\beta}{(\gamma+\beta)} \frac{[\gamma+\beta(1-p)]}{[\gamma+\beta(1-p)+\alpha]}$$

and the disposable income of the economically active generation is,

$$D_t^t = w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1} \tag{45}$$

Having derived these equations, they seem quite intuitive- every period a fraction, p, of life cycle savings in the economy ends up becoming unintentional bequests. It does not matter what the wealth distribution of the economy is- within every existing "type" the same fraction will die early and therefore the aggregate unintentional bequest is just a fraction of the sum of aggregate life cycle savings. Like before the optimal amount of educational expenditures remains,

$$e_t^* = \left(\frac{(1-\lambda)}{\lambda} (IB_t + LS_t)\,\theta \right) \tag{46}$$

and capital stock per efficiency unit of labor,

$$k_t = \frac{IB_{t-1} + LS_{t-1}}{A_t \left(\frac{(1-\lambda)}{\lambda} (IB_{t-1} + LS_{t-1})\,\theta \right)^\theta} = \frac{(IB_{t-1} + LS_{t-1})^{1-\theta}}{A_t \left(\frac{(1-\lambda)}{\lambda} \theta \right)^\theta}$$

As before, along the balanced growth path we continue to have $(IB_t + LS_t)$ growing at the rate $(1+g)^{\frac{1}{1-\theta}} - 1$. Personal disposable income for the entire economy does not change- it gets only reallocated. From equation (45),the personal disposable income of the adult generation is $w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1}$ and the personal disposable income of the retired is $(1-p)\, r_t LS_{t-1}$. Therefore aggregate disposable income is the same as GDP. Like before, the aggregate savings rate in the economy

$$X_t = \frac{(IB_t + LS_t)}{(w_t A_t h_t + r_t IB_{t-1} + r_t LS_{t-1})} \tag{47}$$

The savings rate for the economically active population is,

$$\frac{(IB_t + LS_t)}{(w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1})} = (a_w + a_s) - \frac{e_t}{(w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1})}$$

Substituting equation (46) and rearranging gives us

$$\frac{(IB_t + LS_t)}{(w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1})} = \frac{\lambda\,(a_w + a_s)}{(\lambda + (1-\lambda)\,\theta)}$$

$$\Rightarrow \frac{(IB_t + LS_t)}{(w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1})} = \frac{\lambda}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma + \beta\,(1-p))}{(\gamma + \beta\,(1-p) + \alpha)}$$

Combining this with equations (43) and (44) suggests that the allocation between life cycle savings and intentional bequests remains unchanged in the aggregate when compared to the complete certainty case (equation (30))

$$\frac{LS_t}{(IB_t + LS_t)} = \frac{\beta\,(\lambda + (1-\lambda)\,\theta)}{(\beta + \gamma)\,\lambda} \tag{48}$$

To figure out the aggregate savings rate in the economy (equation (47)) we use the earlier decomposition strategy,

$$X_t = \frac{(IB_t + LS_t)}{(w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1})} \frac{(w_t A_t h_t + r_t IB_{t-1} + pr_t LS_{t-1})}{(w_t A_t h_t + r_t IB_{t-1} + r_t LS_{t-1})}$$

$$\Rightarrow X_t = \frac{\lambda}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma + \beta\,(1-p))}{(\gamma + \beta\,(1-p) + \alpha)} \frac{(w_t A_t h_t + r_t IB_{t-1} + r_t p LS_{t-1})}{(w_t A_t h_t + r_t IB_{t-1} + r_t LS_{t-1})} \tag{49}$$

Now note that share of the retired generation's total disposable income

$$\frac{r_t (1-p) LS_{t-1}}{(w_t A_t h_t + r_t IB_{t-1} + r_t LS_{t-1})} = \frac{\lambda k_t^{\lambda-1} (1-p) LS_{t-1}}{((1-\alpha)\, k_t^{\lambda} A_t h_t + \lambda k_t^{\lambda-1}\,(IB_{t-1} + LS_{t-1}))}$$

which simplifies to

$$\frac{r_t (1-p) LS_{t-1}}{(w_t A_t h_t + r_t IB_{t-1} + r_t LS_{t-1})} = \frac{\lambda (1-p) LS_{t-1}}{(IB_{t-1} + LS_{t-1})}$$

Using this result and equation (48) in the equation for X_t (49) gives us,

$$X_t = \frac{\lambda}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma{+}\beta\,(1-p))}{(\gamma{+}\beta\,(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta\,(\lambda + (1-\lambda)\,\theta)}{(\gamma{+}\beta)}\right) \tag{50}$$

Note that again we have a negative effect of returns to education on aggregate savings. The equation above corresponds to equation (33) above. If $p = 0$, it would collapse to that result with no uncertainty. Life expectancy here clearly has two conflicting effects. The first effect is what might be called the utility based effect which is also the standard effect-higher life expectancy $(1-p)$ tends to raise the savings rate since it automatically places more weight on future consumption.[25] However now there is also a negative effect which is reflected in the term in the parentheses. Higher life expectancy also causes the savings rate to go down here. This is because the retired generation does not save anything but the fact that they live longer means that they account for a higher share of personal disposable income which causes the aggregate savings rate to decline. What about the growth rate and interest rates? The growth rate as one can anticipate continues to be the same as in the earlier examples:

$$\frac{\Delta y_t A_t h_t}{y_{t-1} A_{t-1} h_{t-1}} = \frac{\Delta IB_t}{IB_{t-1}} = (1+g)^{\frac{1}{1-\theta}} - 1$$

The interest rate is a little more complicated since now individuals do not put away as much in savings. The derivation is worked out in appendix D

$$r_t = \frac{(1+g)^{\frac{1}{1-\theta}}\,(\lambda + (1-\lambda)\,\theta)\,(\gamma{+}\beta\,(1-p)+\alpha)}{\left(1 - \frac{(1-p)\beta(\lambda+(1-\lambda)\theta)}{(\gamma+\beta)}\right)(\gamma{+}\beta\,(1-p))}$$

The two opposing factors that tend to raise and lower the savings rate, have opposite implications here.[26] Finally the bequest ratio can be derived by using equations (48) and (50):

$$B_t = \frac{1}{(\beta+\gamma)} \frac{\gamma\lambda - \beta\,(1-\lambda)\,\theta}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma{+}\beta\,(1-p))}{(\gamma{+}\beta\,(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta\,(\lambda + (1-\lambda)\,\theta)}{(\gamma{+}\beta)}\right)$$

and the education expenditure share (using equations (46) and (50)) is,

$$E_t = \frac{(1-\lambda)\,\theta}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma{+}\beta\,(1-p))}{(\gamma{+}\beta\,(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta\,(\lambda + (1-\lambda)\,\theta)}{(\gamma{+}\beta)}\right)$$

We are now in a position to repeat the numerical illustrations from the earlier section. For p we assume a value of 0.17. This is based on the 2002 US Life Tables (Arias (2004)) figures for the probability of surviving till the age of 65 which is 83%. Figure 6 presents the four graphs. Since life expectancy is so high in any case, there is very little difference compared to the earlier results. One feature that can be noticed easilly now is that the education spending ratio does not seem to decline atleast until $\theta = 0.7$. The interest rates are usually higher; for $\theta = 0.25$ earlier it was 5.5% and now

[25]This can be easilly seen by noting that an increase in $(1-p)$ causes $\frac{(\gamma+\beta(1-p))}{(\gamma+\beta(1-p)+\alpha)}$ to increase unambiguously.

[26]Note that the interest rate can be expressed in an abbreviated form: $r_t = \frac{(1+g)^{\frac{1}{1-\theta}}\lambda}{X_t}$.

it stands at 6%. This is obviously because of the fact that individuals choose to save less given the uncertainty. Other than that there is little difference. Repeating the earlier exercise for θ increasing from 0.25 to 0.3, now the aggregate savings rate falls from 18.4 to 16.8 % and is accompanied by an increase in aggregate education spending ratio from 9.2 to 10.1%. The trade-off is now slightly lower at 1.7. This is mainly because of the fact that the retired generation's share of disposable income is lower.

<Insert Figure 6 Here>

The incorporation of life expectancy into the model allows us to compare the effects of rising returns to education and changing life expectancy in a unified framework. The consequences of rising life expectancy on savings rates has been the subject of a long line of research and has obvious implications for domestic capital accumulation. As aging increases, a few offsetting effects may occur. For example, life cycles savings will rise while accidental bequests decline. Capital accumulation could go in either direction. The short run effect would be higher life cycle savings which would imply greater accumulation. However the higher life cycle savings are completely consumed in the next period. At the same time accidental bequests are lower. Accidental bequests however do not translate completely into retirement consumption. They are allocated to life cycle savings and intentional bequests. Therefore the reduction in accidental bequests implies that the medium run capital accumulation rate will be lower. Finally, as discussed above additional unintentional bequests make it easier to achieve the optimal amount of human capital investment in the next period without the model potentially going into "negative bequest" range. Given these potentially interesting possibilities, we also explored the consequences of rising life expectancy on aggregate savings rates. Figure 7 presents some illustrations. We have assumed three possible values for $\theta = 0.20, 0.22$ and 0.25. We have plotted life expectancy probabilities from a 50% chance of survival to a 100% chance of survival. As before we have the four different economic variables. The main inference that one can draw from these figures is the small variation in the variables despite huge variations in life expectancy irrespective of the value of theta.[27] The US is currently at around $1 - p = 0.83$. If $1 - p$ were to rise to say 0.90 then the savings rate would increase very insignificantly and for $\theta = 0.25$ would possibly begin a slight decline. However these differences are very small. Even if all uncertainty was wiped out the savings rate in the US would not change by more than 0.25%. On the other hand small changes in θ can clearly have much larger effects. Thus more than ageing it is the rising returns to education that can have more dramatic effects on savings rates.[28]

<Insert Figure 7 Here>

[27]The relative insensitivity of these variables are also true when one assumes more extreme values for θ. We found similar movements when $\theta = 0.1$ and $\theta = 0.4$.

[28]One disadvantage of the model is that it does not incorporate the possibility that longer life expectancy can itself encourage human capital investment. However, this would initially encourage more "spending" on education and reduce the savings rate but later on in life it would allow for greater savings but not necesarilly a greater savings rate.

5 Conclusion

The objective of this paper was to motivate the idea that, in a world where skills are becoming predominantly more important, one needs to think about how the composition of capital accumulation shifts. Given that it is generally agreed many developed countries in the world have been undergoing a period of rapid skill-biased technological progress, it is definitely worthwhile to think about the relative importance of human capital accumulation vis-a-vis physical capital accumulation. The fact that the savings ratio as measured by national accounts has been declining during the same period, makes the question all the more interesting. Based on our presentation of some stylized facts that have been ignored in the literature, a straightforward model highlighting the substitution between savings rate and education spending and some numerical illustrations we have tried to convince the reader of the importance of this issue. An interesting outcome of our model is the result that 1% reduction in the aggregate savings rate require less than (and sometimes a lot less than) a 1% increase in the education spending ratio. At the same time it is important to make a realistic assessment of the overall contribution that this could have had in explaining the savings decline. If we use the household savings rate, a 2% decline in savings rate because of rising returns means that we can approximately explain 20-25% of the overall decline. If we instead focus on the private savings rate we can potentially explain a 30-40% decline. While not overpowering, we hope the reader will agree that these are important numbers.

Regarding the model itself, there are other interesting lines along which it could be extended. For example, we have not included public education which forms a major chunk of educational expenditures in the U.S. An increase in public education expenditures implies that the incentive to reallocate towards private educational expenditures is less. However, as a matter of empirical observation the role of educational expenditures in public spending relative to private spending has fallen over the past two decades. Indeed it is possible that it in addition to rising returns, a lowering of public support forces individuals to increase private spending. If education can be viewed a product for which the demand is relatively inelastic, this could be an important explanation. However this would only imply a further reduction in the savings rate. We feel this is an important line for future research. Also, we have not considered the heterogeneity of agents. This can affect the results to the degree that borrowing constraints are an important consideration in education financing. However these could be incorporated and the results are unlikely to change substantially. The fact that enrollments have risen in college substantially indicates that there is a desire to acquire skills and increase spending on education. Secondly, as an empirical matter people have save far less for college than they typically have to borrow.[29] Therefore it is unlikely that liquidity constraints per se present a significant challenge

[29] We are grateful to Caroline Hoxby for drawing our attention to this fact and related data. A lot of interesting evidence on the drastic increase in borrowing for higher education can be found in the web site for the National Postsecondary Student Aid Study (http://nces.ed.gov/pubsearch/getpubcats.asp?sid=013). For example, The percentage of dependent undergraduates with family incomes of $50,000 or more who ever borrowed from federal loan programs increased between 1992-93 and 1995-96 at both public and private, not-for-profit 4-year institutions. Further, in 1992-93, 21 percent of

to the theory (which also explains why we have been liberal with ranges in our parameter values where education spending implies negative transfers). However this is again in interesting area for further research.

Finally, the paper draws attention to the measurement problem of human capital investment. As mentioned earlier the only available measures are the direct expenditures of education expenditures in GDP. However these are woefully inadequate. Opportunity costs of education are often considered to be equal in magnitude of direct expenditures.[30] Further in an economy like the US, a large part of housing investment is often an investment in human capital further underscoring the mismeasurement when relying on direct expenditures only.

References

[1] Abel, Andrew, 1985, Precautionary Savings and Accidental Bequests, *American Economic Review*, 75:777-791.

[2] Acemoglu, Daron, 1998, Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality, *Quarterly Journal of Economics*, 113 (4), November: 1055-89.

[3] Arias, Elizabeth, 2004, United States life tables, 2002, *National vital statistics reports*, 53 (6), Hyattsville, Maryland: National Center for Health Statistics.

[4] Autor, David H., Lawrence F. Katz and Melissa S. Kearney, 2004, Trends in U.S. Wage Inequality: Re-Assessing the Revisionists, mimeo, MIT.

[5] Autor, David H., Lawrence F. Katz and Alan B. Krueger, 1998, Computing Inequality: Have Computers Changed the Labor Market?, *Quarterly Journal of Economics*, 1169-1213.

[6] Becker, Gary S., and Nigel Tomes, 1986, Human Capital and the Rise and Fall of Families, *Journal of Labor Economics*, 4,no.3, pt2:S1-S39.

[7] Bils, Mark and Peter Klenow, 2000, Does Schooling Cause Growth?, *American Economic Review*, December, 90 (5), 1160-1183.

[8] Blinder Alan S., 1976, Intergenerational Transfers and Life Cycle Consumption, *American Economic Review (Papers and Proceedings of the American Economic Association)*, 66(2): 87-93.

[9] Bound, John and George Johnson, 1992, Changes in the Structure of Wages in the 1980s: An Evaluation of Alternative Explanations, *American Economic Review*, LXXXII: 371-392.

dependent undergraduates at public 4-year institutions from families making between $ 50,000 and $ 59,999 had ever borrowed. By 1995-96, 44 percent of undergraduates from families in that income range had borrowed.

[30] See Weil (2004)

[10] Cameron, Stephen and J. Heckman, 1998, Life Cycle Schooling and Educational Selectivity: Models and Choice", *Journal of Political Economy*, April.

[11] Cameron, Stephen and Christopher Taber, 2004, Estimation of Educational Borrowing Constraints Using Returns to Schooling, Journal of Political Economy, 112, 1, Part 1: 132-82.

[12] Card, David and John E. DiNardo, 2002. "Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles." *Journal of Labor Economics* 20 (October): 733-83.

[13] Card, David and Thomas Lemieux, 2001, "Dropout and Enrollment Trends in the Post War Period: What Went Wrong in the 1970s?" in J. Gruber (ed.) *An Economic Analysis of Risky Behavior Among Youth*, Chicago: University of Chicago Press for NBER, pp. 439-482

[14] Caselli, Francesco, 1999, Technological Revolutions, *American Economic Review*, 89(1): 78-102

[15] Dynan, Karen, Jonathan Skinner and Stephen Zeldes, 2002, The Importance of Bequests and Life Cycle Saving in Capital Accumulation: A New Answer, *American Economic Review*, May, 92 (2): 274-278.

[16] Gale, William G. and John Sabelhaus, 1999, Perspectives on the Household Savings Rate, *Brookings Paper on Economic Activity*,1:181-224.

[17] Galor, Oded, and Daniel Tsiddon, 1997, Technological Progress, Mobility, and Economic Growth, *American Economic Review*, 87(3): 363-82.

[18] Galor, Oded and Moav, Omer, 2000, Ability-Biased Technological Transition, Wage Inequality and Economic Growth, *Quarterly Journal of Economics*, 115(2): 469-497.

[19] Galor, Oded and Omer Moav, 2005, From Physical Capital to Human Capital Accumulation: Inequality in the Process of Development, *forthcoming in Review of Economic Studies*.

[20] Gokhale, Jagadeesh, Lawrence J. Kotlikoff and John Sabelhaus, 1996, Understanding the Post-War Decline in US Savings: A Cohort Analysis, *Brookings Paper on Economic Activity*, 1:315-407

[21] Goldin, Claudia and Lawrence F Katz, 1999, The Returns to Skill in the United States Across the Twentieth Century, NBER Working Paper No. 7126, Cambridge.

[22] Greenwood, Jeremy and Boyan Jovanovic, 1999, The Information-Technology Revolution and the Stock Market, American Economic Review, 89, 2 (May): 116-22.

[23] Hobijn and Jovanovic, 2001, The Information-Technology Revolution and the Stock Market: Evidence, American Economic Review, 91, 5 (December): 1203-20.

[24] Jorgenson, Dale W., and Barbara M. Fraumeni, 1990, Investment in Education and US Economic Growth, in Charls E. Walker et. al. ed., *The US Savings Challenge*, Boulder: Westview Press, 114-141.

[25] Katz, Larry and Kevin M. Murphy, Changes in Relative Wages, 1963-1987: Supply and Demand Factors, *Quarterly Journal of Economics*, CVII:35-78.

[26] Kendrick, J.W., 1976, The Formation and Stocks of Total Capital, New York: Columbia University Press.

[27] Kotlikoff, Laurence J.,1988, Intergenerational Transfers and Savings, *Journal of Economic Perspectives*, 2(2):41-58.

[28] Kotlikoff, Laurence J., and Lawrence H. Summers, 1981, The Role of Intergenerational Transfers in Aggregate Capital Accumulation, *Journal of Political Economy*, 89(4), 706-732.

[29] Krueger, Alan B., 1993, How Computers Have Changed the Wage Structure: Evidence from Micro Data, *Quarterly Journal of Economics*, CVIII: 33-60.

[30] Lusardi, Annamaria, Jonathan Skinner, and Steven Venti, 2001, Saving Puzzles and Savings Policies in the United States, *Oxford Review of Economic Policy*, Spring, 17(1), 95-115.

[31] Mankiw, Gregory N., David Romer and David N. Weil, 1992, A Contribution to the Empirics of Economic Growth, *Quarterly Journal of Economics*, May: 407-437.

[32] Lambertini, Luisa, 2004, Technological Change and Public Financing of Education, mimeo, Boston College.

[33] Lemieux, Thomas, 2004, Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?, Center for Labor Economics Working Paper No. 69, UC Berkeley, May.

[34] Meade, James, 1966, Life Cycle Savings, Inheritances and Economic Growth, *Review of Economic Studies*, 33: 61-78.

[35] Mincer, Jacob, 1990, Comment on Jorgenson and Fraumeni, in Charles E. Walker et. al. ed., *The US Savings Challenge*, Boulder: Westview Press, 143-147.

[36] Mincer, Jacob, 1998, Investment in US Education and Training as Supply Responses, *Research in Labor Economics*, Vol 17, 277-304

[37] Modigliani, Franco, 1988, The Role of Intergenerational Transfers and Life Cycle Saving in the Accumulation of Wealth, *Journal of Economic Perspectives*, 2(2):15-40.

[38] Murphy, Kevin and Finish Welch, 1992, The Structure of Wages, *Quarterly Journal of Economics*, CVII:285-326.

[39] Nordhaus, William D., 1995, How Should We Measure Sustainable Income?, *Cowles Foundation Discussion Paper 1101*, New Haven: Cowles Foundation, Yale University, May.

[40] OECD, 2004, OECD Economic Outlook, June, Issue 75, Paris.

[41] OECD, 2003, Education At a Glance, URL: www.oecd.org/edu/eag2003

[42] Parker, Jonathan A., 1999, Spendthrift in America? On Two Decades of Decline in the US Savings Rate, *NBER Macroeconomics Annual*.

[43] Rangazas, Peter, 2000, Schooling and economic growth:A King-Rebelo experiment with human capital, *Journal of Monetary Economics,* 46, pp 397-416.

[44] Rangazas, Peter, 2002, The Quantity and Quality of Schooling and U.S. Labor Productivity Growth (1870–2000), *Review of Economic Dynamics*, 5, 932–964.

[45] Rosenzweig, M.R. and K.I. Wolpin, 1993, Intergenerational Transfers and the Life-Cycle Incomes of Young Men and their Parents: Human Capital Investments, Coresidence and Intergenerational Financial Transfers, *Journal of Labor Economics*, 11:84-111.

[46] Shea, J., 2000, Does Parent's Money Matter?, *Journal of Public Economics*, Vol 77 (2), August, Pages 155-184

[47] Weil, David, 2004, *Economic Growth*, Addison Wesley.

[48] Zhang, Jie, Junsen Zhang and Ronald Lee, 2003, Rising Longevity, Public Education, Savings and Growth, Journal of Development Economics, Vol 70(1), pages 83-101.

[49] Zilcha, Itzhak, 2003, Intergenerational Transfers, Production and Income Distribution, *Journal of Public Economics*, 87:3-4: 489-513.

A Derivation of the Growth Rate and Interest Rate in the Absence of Life Cycle Savings

Beginning with equation (16),

$$B_t \equiv \frac{b_t}{y_t A_t h_t} = \frac{\gamma\lambda}{(\lambda + (1-\lambda)\,\theta)}$$

Substituting equations (13) and(12) into the above:

$$b_t\left(1 + \frac{(1-\lambda)\,\theta}{\lambda}\right) = \gamma\left(b_{t-1}^{\lambda+\theta(1-\lambda)} A_t^{1-\lambda}\left(\frac{(1-\lambda)\,\theta}{\lambda}\right)^{\theta(1-\lambda)}\right)$$

To calculate growth rates divide both sides by b_{t-1}

$$\begin{aligned}
\frac{b_t\left(1+\frac{(1-\lambda)\theta}{\lambda}\right)}{b_{t-1}} &= \frac{\gamma\left(b_{t-1}^{\lambda+\theta(1-\lambda)} A_t^{1-\lambda}\left(\frac{(1-\lambda)\theta}{\lambda}\right)^{\theta(1-\lambda)}\right)}{b_{t-1}} \\
&\Rightarrow \frac{b_t}{b_{t-1}} = \frac{\gamma(1+g)^{1-\lambda}\left(\frac{(1-\lambda)\theta}{\lambda}\right)^{\theta(1-\lambda)}}{\left(1+\frac{(1-\lambda)\theta}{\lambda}\right)}\left(\frac{A_{t-1}}{b_{t-1}^{1-\theta}}\right)^{1-\lambda} \qquad (51)
\end{aligned}$$

Therefore the only growth compatible with the balanced growth path is where A_{t-1} and $b_{t-1}^{1-\theta}$ grow at the same rate. Now $\frac{A_t}{A_{t-1}} = 1+g$.Therefore:

$$\left(\frac{b_t}{b_{t-1}}\right)^{1-\theta} = 1+g$$

So

$$\begin{aligned}
\frac{\Delta b_t}{b_{t-1}} &= (1+g)^{\frac{1}{1-\theta}} - 1 \\
&\Rightarrow \frac{\Delta y_t A_t h_t}{y_{t-1} A_{t-1} h_{t-1}} = (1+g)^{\frac{1}{1-\theta}} - 1
\end{aligned}$$

The next step is to calculate the interest rate which is plainly $r_t = \alpha k_t^{\alpha-1}$.
As per equation (13), we know that

$$k_{t+1} = \left(\frac{b_t^{1-\theta}}{A_{t+1}\left(\frac{(1-\alpha)\theta}{\alpha}\right)^{\theta}}\right)$$

We have already seen that $A_{t-1}/b_{t-1}^{1-\theta}$ is a fixed number (and hence $A_{t+1}/b_t^{1-\theta}$ after adjusting for the growth rate) , all that remains for us to do is to figure out this number. Using $\frac{b_t}{b_{t-1}} = (1+g)^{\frac{1}{1-\theta}}$ in equation (51),

$$\Rightarrow \quad (1+g)^{\frac{1}{1-\theta}} = \frac{\gamma(1+g)^{1-\lambda}\left(\frac{(1-\lambda)\theta}{\lambda}\right)^{\theta(1-\lambda)}}{\left(1+\frac{(1-\lambda)\theta}{\lambda}\right)}\left(\frac{A_{t-1}}{b_{t-1}^{1-\theta}}\right)^{1-\lambda}$$

$$\Rightarrow \quad \frac{(1+g)^{\frac{1}{1-\theta}}\left(1+\frac{(1-\lambda)\theta}{\lambda}\right)}{\gamma(1+g)^{1-\lambda}\left(\frac{(1-\lambda)\theta}{\lambda}\right)^{\theta(1-\lambda)}} = \left(\frac{A_{t-1}}{b_{t-1}^{1-\theta}}\right)^{1-\lambda}$$

Substituting this into equation (13) with appropriate adjustments and substituting that further into $r_t = \alpha k_t^{\alpha-1}$ gives us equation (18):

$$r_t = \frac{(1+g)^{\frac{1}{1-\theta}}\left(\lambda + (1-\lambda)\theta\right)}{\gamma}$$

B Derivation of the Growth Rate and Interest Rate in the Presence of Life Cycle Savings

Substituting the optimal value of education (equation (28)) in the expression for capital per effective unit of labor gives us:

$$k_t = \left(\frac{(b_{t-1}+s_{t-1})^{1-\theta}}{A_t\left(\frac{(1-\lambda)}{\lambda}\theta\right)^{\theta}}\right)$$

Along the balanced growth path, $(b_{t-1}+s_{t-1})^{1-\theta}$ will grow at the same rate as A_t.Therefore,

$$\frac{(b_t+s_t)}{(b_{t-1}+s_{t-1})} = (1+g)^{\frac{1}{1-\theta}}$$

Hence for output per capita:

$$\frac{y_t A_t h_t}{y_{t-1}A_{t-1}h_{t-1}} = \frac{\bar{k}^{\alpha} A_t\left(\frac{(1-\lambda)}{\lambda}\theta\right)^{\theta}(b_{t-1}+s_{t-1})^{\theta}}{\bar{k}^{\alpha} A_{t-1}\left(\left(\frac{(1-\lambda)}{\lambda}\theta\right)(b_{t-2}+s_{t-2})\right)^{\theta}}$$

$$\Rightarrow \frac{y_t A_t h_t}{y_{t-1}A_{t-1}h_{t-1}} = (1+g)^{\frac{1}{1-\theta}}$$

To figure out the education ratio we use equation ((28) in conjunction with (34),

$$\frac{e_t^*}{(w_tA_th_t + r_tb_{t-1} + r_ts_{t-1})} = \frac{\left(\frac{(1-\lambda)}{\lambda}\theta\right)(b_t + s_t)}{(w_tA_th_t + r_tb_{t-1} + r_ts_{t-1})}$$

$$\Rightarrow E_t = \left(\frac{(1-\lambda)}{\lambda}\theta\right) X_t$$

$$\Rightarrow E_t = ((1-\lambda)\theta)\left(\frac{\gamma + (1-\lambda)\beta(1-\theta)}{(\lambda + (1-\lambda)\theta)}\right)$$

To calculate the interest rate:

$$\lambda k_t^{\lambda-1} = \lambda \left(\frac{(b_{t-1} + s_{t-1})^{1-\theta}}{A_t\left(\frac{(1-\lambda)}{\lambda}\theta\right)^{\theta}}\right)^{\lambda-1}$$

We know that

$$X_t = \lambda\left(\frac{\gamma + (1-\lambda)\beta(1-\theta)}{(\lambda + (1-\lambda)\theta)}\right)$$

$$\Rightarrow X_t = \frac{b_t + s_t}{(y_tA_th_t)} = \lambda\left(\frac{\gamma + (1-\lambda)\beta(1-\theta)}{(\lambda + (1-\lambda)\theta)}\right)$$

Therefore,

$$b_t + s_t = X_t\left(k_t^{\lambda}A_t\left(\frac{(1-\lambda)}{\lambda}(b_{t-1} + s_{t-1})\theta\right)^{\theta}\right)$$

$$\Rightarrow \frac{b_t + s_t}{b_{t-1} + s_{t-1}} = X_t\frac{\left(k_t^{\lambda}A_t\left(\frac{(1-\lambda)}{\lambda}(b_{t-1} + s_{t-1})\theta\right)^{\theta}\right)}{(b_{t-1} + s_{t-1})}$$

$$\Rightarrow (1+g)^{\frac{1}{1-\theta}} = X_t\frac{\left(k_t^{\lambda}A_t\left(\frac{(1-\lambda)}{\lambda}\theta\right)^{\theta}\right)}{(b_{t-1} + s_{t-1})^{1-\theta}}$$

$$\Rightarrow (1+g)^{\frac{1}{1-\theta}} = X_t\left(k_t^{\lambda}\right)k_t^{-1}$$

$$\Rightarrow (1+g)^{\frac{1}{1-\theta}} = \lambda\left(\frac{\gamma + (1-\lambda)\beta(1-\theta)}{(\lambda + (1-\lambda)\theta)}\right)k_t^{\lambda-1}$$

$$\Rightarrow r = \lambda k^{\lambda-1} = \frac{(1+g)^{\frac{1}{1-\theta}}(\lambda + (1-\lambda)\theta)}{\gamma + (1-\lambda)\beta(1-\theta)}$$

C Deriving the Aggregate Savings Rate under Uncertainty

Under assumption (1) suppose everyone begins off with the same disposable income and for simplicity call this b_0 and s_0.Of these identical agents, a fraction p will die at the beginning of period1 and a fraction $1-p$ will continue to live. Consider the bequests that the two sets of descendents receive. Those whose parents die early will make the following choices,

$$s_1^p = a_s(w_1 A_1 h_1 + r_1(b_0 + s_0))$$

$$b_1^p = a_\omega(w_1 A_1 h_1 + r_1(b_0 + s_0)) - e_1$$

The superscript p represents the fraction whose parents did not live through retirement. From equations (41) and (42) a_s and a_ω represent,

$$\begin{aligned} a_\omega &= \frac{\gamma}{(\gamma+\beta)} \frac{[\gamma+\beta\,(1-p)]}{[\gamma+\beta\,(1-p)+\alpha]} \\ a_s &= \frac{\beta}{(\gamma+\beta)} \frac{[\gamma+\beta\,(1-p)]}{[\gamma+\beta\,(1-p)+\alpha]} \end{aligned}$$

For those whose parents did not die:

$$\begin{aligned} s_1^{1-p} &= a_s(w_1 A_1 h_1 + r_1 b_0) \\ b_1^{1-p} &= a_\omega(w_1 A_1 h_1 + r_1 b_0) - e_1 \end{aligned}$$

Therefore if we follow the aggregate *intentional* bequests (B_t) and aggregate life cycle savings in period $1(S_t)$:

$$B_1 = p b_1^p + (1-p) b_1^{1-p}$$

$$\Rightarrow B_1 = p\left[a_\omega(w_1 A_1 h_1 + r_1(b_0 + s_0)) - e_1\right] + (1-p)\left[a_\omega(w_1 A_1 h_1 + r_1 b_0) - e_1\right]$$

$$\Rightarrow B_1 = a_\omega(w_1 A_1 h_1 + r_1 b_0) + p a_\omega r_1 s_0 - e_1$$

$$\Rightarrow B_1 = a_\omega(w_1 A_1 h_1 + r_1 b_0 + p r_1 s_0) - e_1$$

and:

$$S_1 = p s_1^p + (1-p) s_1^{1-p}$$

$$\Rightarrow S_1 = p\left[a_s(w_1A_1h_1 + r_1(b_0 + s_0))\right] + (1-p)a_s(w_1A_1h_1 + r_1b_0)$$

$$\Rightarrow S_1 = a_s(w_1A_1h_1 + r_1b_0 + pr_1s_0)$$

Calculating the disposable income of the economically active generation:

$$D_1 = p(w_1A_1h_1 + r_1(b_0 + s_0)) + (1-p)(w_1A_1h_1 + r_1b_0)$$

$$\Rightarrow D_1 = w_1A_1h_1 + r_1b_0 + pr_1s_0$$

Therefore the ratios:

$$\begin{aligned} \frac{B_1}{D_1} &= a_\omega - \frac{e_1}{w_1A_1h_1 + r_1b_0 + pr_1s_0} \\ \frac{S_1}{D_1} &= a_s \end{aligned}$$

To consider what happens in period 2:

There will be now four types:

1. Agents with ancestral history of $(1-p, 1-p)$ (where the second (1-p) reflects the most recent history):

$$\begin{aligned} b_2^{1-p,1-p} &= a_\omega(w_2A_2h_2 + r_2b_1^{1-p}) - e_2 \\ s_2^{1-p,1-p} &= a_s(w_2A_2h_2 + r_2b_1^{1-p}) \end{aligned}$$

2. Agents with ancestral history of $(1-p, p)$

$$\begin{aligned} b_2^{1-p,p} &= a_\omega(w_2A_2h_2 + r_2b_1^{1-p} + r_2s_1^{1-p}) - e_2 \\ s_2^{1-p,p} &= a_s(w_2A_2h_2 + r_2b_1^{1-p} + r_2s_1^{1-p}) \end{aligned}$$

3. Agents with ancestral history of $(p, 1-p)$

$$\begin{aligned} b_2^{p,1-p} &= a_\omega(w_2A_2h_2 + r_2b_1^{p}) - e_2 \\ s_2^{p,1-p} &= a_s(w_2A_2h_2 + r_2b_1^{p}) \end{aligned}$$

4. Agents with ancestral history of (p, p)

$$\begin{aligned} b_2^{p,p} &= a_\omega(wA_2h_2 + r_2(b_1^p + s_1^p)) - e_2 \\ s_2^{p,p} &= a_s(wA_2h_2 + r_2(b_1^p + s_1^p)) \end{aligned}$$

Therefore Aggregate Bequests:

$$B_2 = (1-p)^2 b_2^{1-p,1-p} + (1-p)pb_2^{1-p,p} + p\,(1-p)\,b_2^{p,1-p} + p^2 b_1^{p,p}$$

$$\begin{aligned} \Rightarrow \quad & B_2 = (1-p)^2\left[a_\omega(w_2A_2h_2 + r_2b_1^{1-p}) - e_2\right] + (1-p)p\left[a_\omega(w_2A_2h_2 + r_2b_1^{1-p} + r_2s_1^{1-p}) - e_2\right] \\ & + p\,(1-p)\left[a_\omega(w_2A_2h_2 + r_2b_1^p) - e_2\right] + p^2\left[a_\omega(w_2A_2h_2 + r_2(b_1^p + s_1^p)) - e_2\right] \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad & B_2 = (1-p)\left[a_\omega(w_2A_2h_2 + r_2b_1^{1-p} + pr_2s_1^{1-p}) - e_2\right] \\ & + p\left[a_\omega(w_2A_2h_2 + r_2b_1^p + pr_2s_1^p) - e_2\right] \end{aligned}$$

$$\Rightarrow B_2 = a_\omega(w_2A_2h_2 + r_2B_1 + pr_2S_1) - e_2$$

Similarly Aggregate Savings:

$$S_2 = (1-p)^2 s_2^{1-p,1-p} + (1-p)ps_2^{1-p,p} + p\,(1-p)\,s_2^{p,1-p} + p^2 s_2^{p,p}$$

$$\Rightarrow S_2 = a_s(w_2A_2h_2 + r_2B_1 + pr_2S_1)$$

Disposable income in time period 2 will now be

$$D_2 = w_2A_2h_2 + r_2B_1 + pr_2S_1$$

One can now infer the emerging aggregate pattern. In every period, a fraction p of the earlier period's life cycle savings ends up as being an unintentional bequest. This obviously raises the disposable income as well.

Therefore we have the following basic structure:

$$\begin{aligned} B_t &= a_\omega(w_tA_th_t + r_tB_{t-1} + pr_tS_{t-1}) - e_t \\ S_t &= a_s(w_tA_th_t + r_tB_{t-1} + pr_tS_{t-1}) \\ D_t^t &= w_tA_th_t + r_tB_{t-1} + pr_tS_{t-1} \end{aligned}$$

D Deriving the Interest Rate under Uncertainty

Beginning with the aggregate savings rate given by equation (50)

$$X_t = \frac{\lambda}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma{+}\beta\,(1-p))}{(\gamma{+}\beta\,(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta\,(\lambda + (1-\lambda)\,\theta)}{(\gamma{+}\beta)}\right)$$

$$\begin{aligned}
\Rightarrow\ & \frac{(B_t + S_t)}{(w_t A_t h_t + r_t B_{t-1} + r_t S_{t-1})} = \frac{\lambda}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma{+}\beta\,(1-p))}{(\gamma{+}\beta\,(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta\,(\lambda + (1-\lambda)\,\theta)}{(\gamma{+}\beta)}\right) \\
\Rightarrow\ & (B_t + S_t) = \frac{\lambda}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma{+}\beta\,(1-p))}{(\gamma{+}\beta\,(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta\,(\lambda + (1-\lambda)\,\theta)}{(\gamma{+}\beta)}\right) y_t A_t h_t \\
\Rightarrow\ & (B_t + S_t) = \begin{array}{c} \frac{\lambda}{(\lambda+(1-\lambda)\theta)} \frac{(\gamma+\beta(1-p))}{(\gamma+\beta(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta(\lambda+(1-\lambda)\theta)}{(\gamma+\beta)}\right) \\ \times k_t^{\lambda} A_t \left(\frac{(1-\lambda)}{\lambda}\theta\,(B_{t-1} + S_{t-1})\right)^{\theta} \end{array} \qquad (52) \\
\Rightarrow\ & \frac{(B_t+S_t)}{(B_{t-1}+S_{t-1})} = \begin{array}{c} \frac{\lambda}{(\lambda+(1-\lambda)\theta)} \frac{(\gamma+\beta(1-p))}{(\gamma+\beta(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta(\lambda+(1-\lambda)\theta)}{(\gamma+\beta)}\right) \\ \times \frac{k_t^{\lambda} A_t \left(\frac{(1-\lambda)}{\lambda}\theta(B_{t-1}+S_{t-1})\right)^{\theta}}{(B_{t-1}+S_{t-1})} \end{array} \\
\Rightarrow\ & (1+g)^{\frac{1}{1-\theta}} = \begin{array}{c} \frac{\lambda}{(\lambda+(1-\lambda)\theta)} \frac{(\gamma+\beta(1-p))}{(\gamma+\beta(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta(\lambda+(1-\lambda)\theta)}{(\gamma+\beta)}\right) \\ \times \frac{k_t^{\lambda} A_t \left(\frac{(1-\lambda)}{\lambda}\theta\right)^{\theta}}{(B_{t-1}+S_{t-1})^{1-\theta}} \end{array} \\
\Rightarrow\ & (1+g)^{\frac{1}{1-\theta}} = \frac{\lambda}{(\lambda + (1-\lambda)\,\theta)} \frac{(\gamma{+}\beta\,(1-p))}{(\gamma{+}\beta\,(1-p)+\alpha)} \left(1 - \frac{(1-p)\beta\,(\lambda + (1-\lambda)\,\theta)}{(\gamma{+}\beta)}\right) k_t^{\lambda-1} \\
\Rightarrow\ & r_t = \lambda k_t^{\lambda-1} = \frac{(1+g)^{\frac{1}{1-\theta}}\,(\lambda + (1-\lambda)\,\theta)\,(\gamma{+}\beta\,(1-p)+\alpha)}{\left(1 - \frac{(1-p)\beta(\lambda+(1-\lambda)\theta)}{(\gamma+\beta)}\right)(\gamma{+}\beta\,(1-p))}
\end{aligned}$$

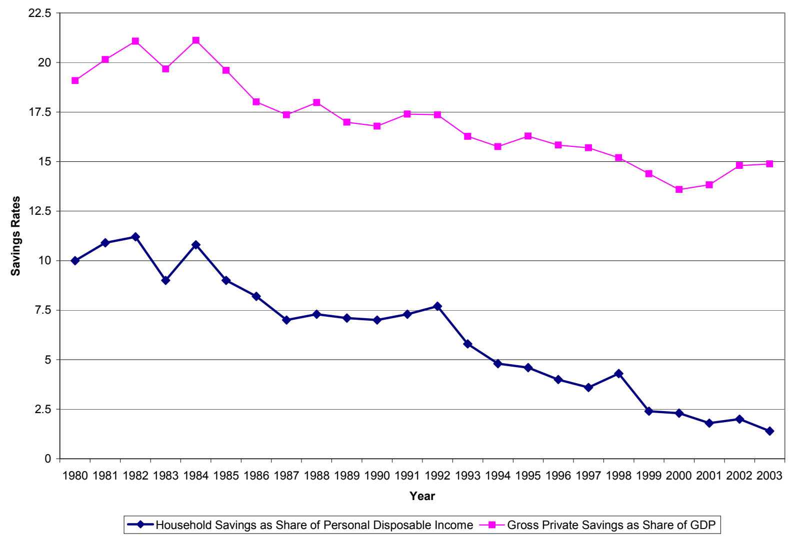

Figure 1: US Savings Rates (1980-2000)

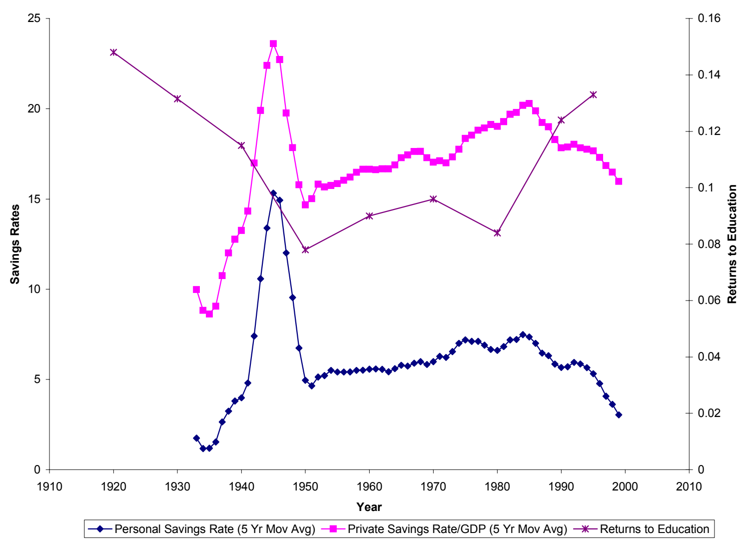

Figure 2: Returns to Education and Savings Rates in USA During the 20th Century

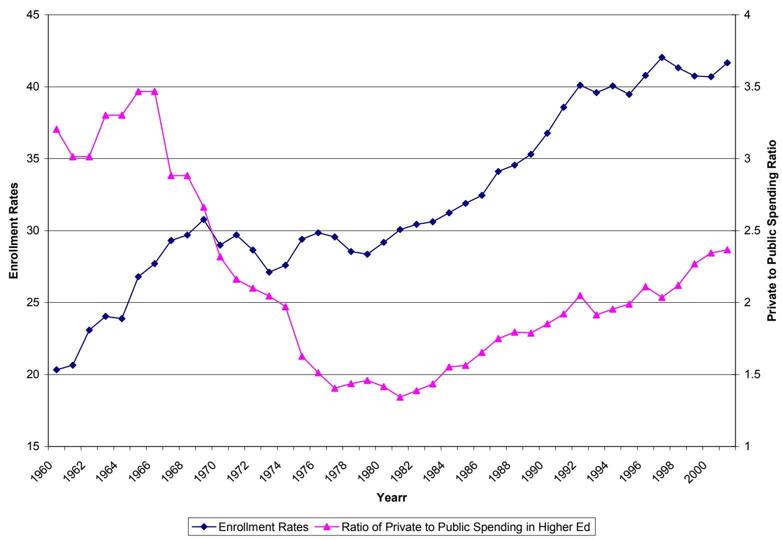

Figure 3: Enrollment Rates and the Ratio of Private to Public Expenditures in Higher Education.

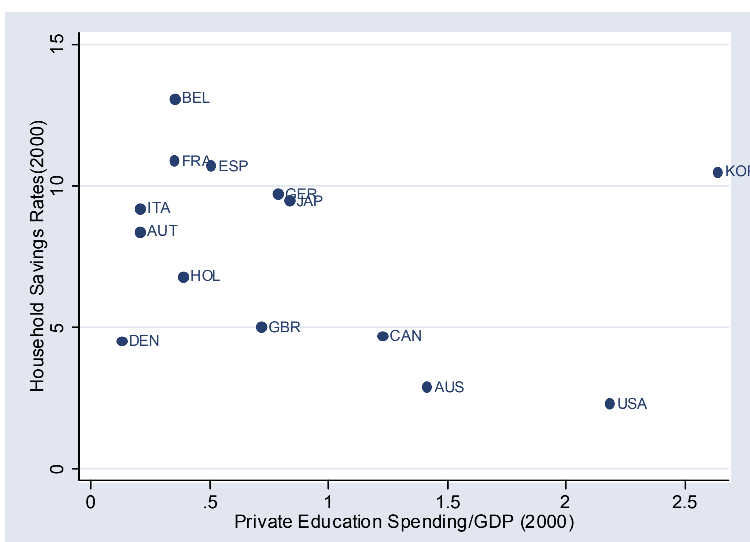

Figure 4: Private Education Spending and Household Savings Rates Across OECD Countries (2000)

Figure 5: Numerical Illustration with Life Cycle Savings

$$\alpha = 0.6, \beta = 0.25, \gamma = 0.15, g = 0.02$$

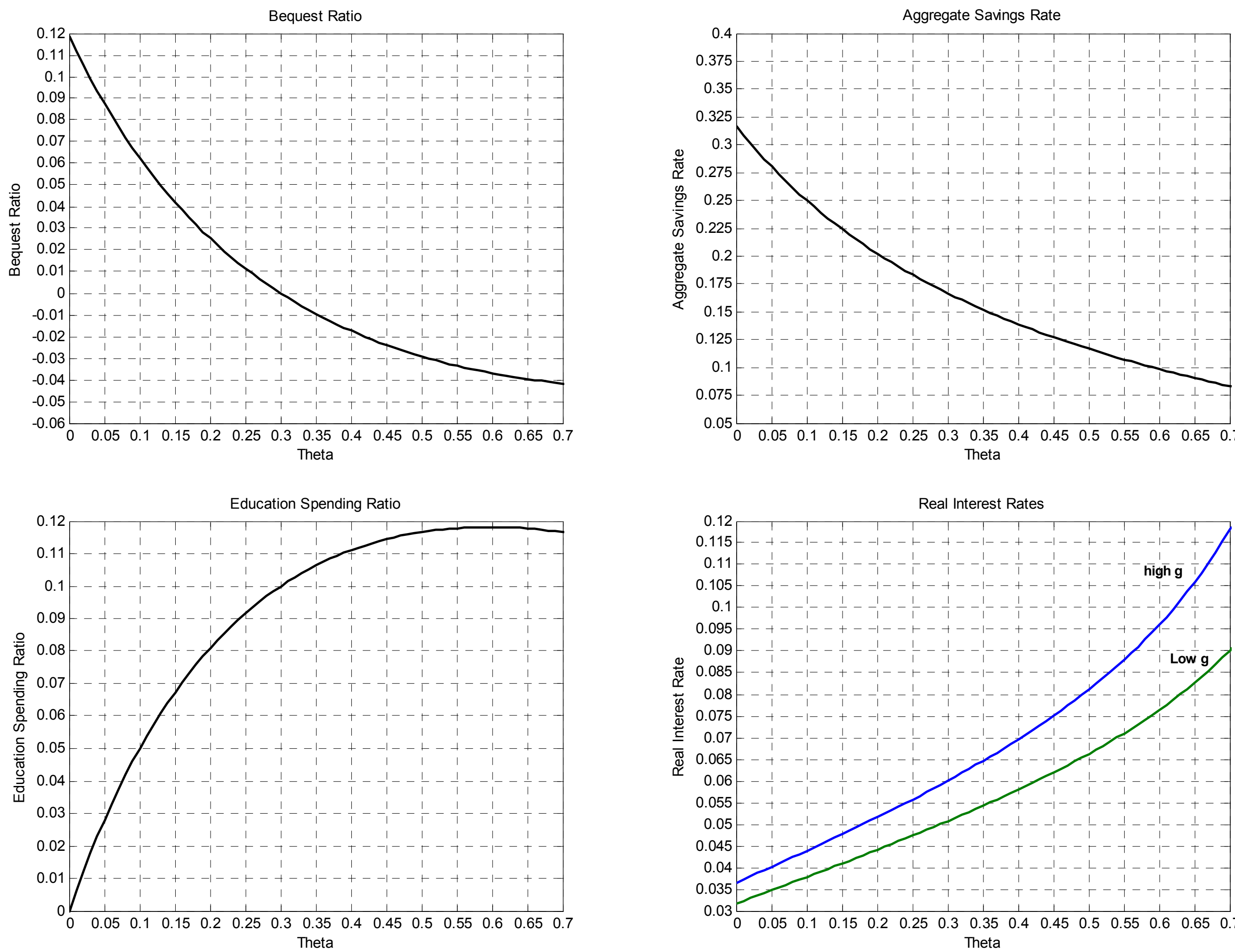

Figure 6: Numerical Illustrations with Life Cycle Savings and Unintentional Bequests
$\alpha = 0.6, \beta = 0.25, \gamma = 0.15, g = 0.02, p = 0.17$

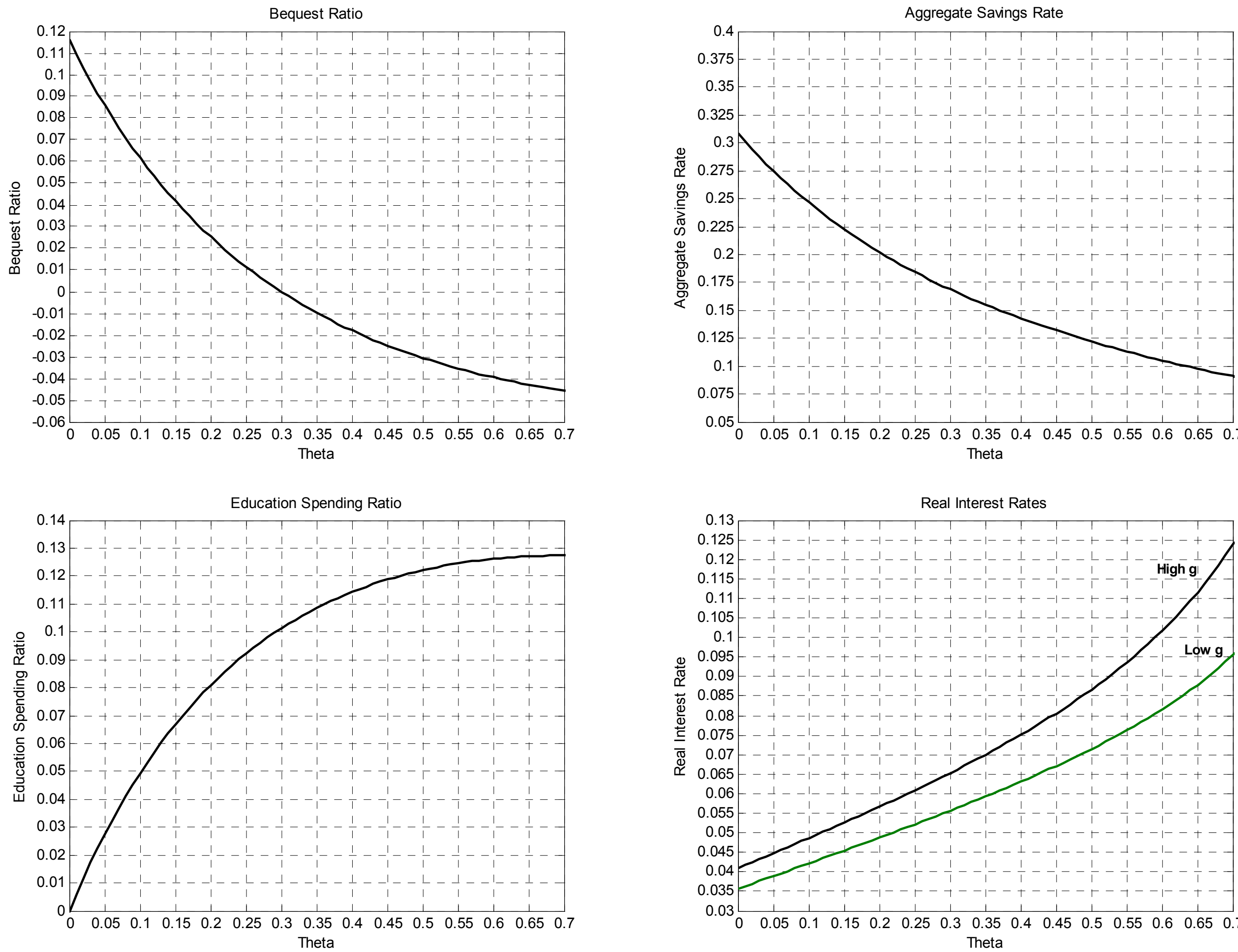

Figure 7: Changes in Life Expectancy