The Inflation Dynamics of Pegging Interest Rates

ABSTRACT

A relatively simple analysis of central banks pegging interest rates applies whenever prices are determined in a price-flexible model where the central bank pursues a singular price-level or nominal-income target. Applying the model empirically in the U.S. and find that prior to 1980, the Federal Reserve would have met its price-level or nominal-income targets best by using the M1 definition of money. However, after 1982, the Federal Reserve would have more effectively met is targets by pegging the interest rate. We also further the analysis in a general-equilibrium, cash-in-advance model with explicit state-contingent securities that complete markets.

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I. Introduction:

The literature on “inflation dynamics,“ goes back to Sargent and Wallace (1975), Lucas (1976), and Sargent (1989), and continues through more recent work such as Woodford (2003). Much of that literature deals with the issue of a central bank pegging the interest rate. Sargent and Wallace (1975) argued that a central bank pegging the interest rate to meet a price-level target[1] leaves prices indeterminate. McCallum (1981) and Woodford (2003) argued that prices are determinate when the central bank pegs interest rates based on certain feedback rules based on either the price level or inflation rate such as Taylor’s (1993) rule. The analyses of Sargent and Wallace (1975) and Woodford (2003) are based on Sargent’s (1979) precedent of solving expectational difference equations in economies with infinite horizons. Sargent’s precedent is to solve any diverging expectational difference equation forward and assume its solution is bounded. However, Eagle and Murff (2005) review Sargent’s analysis and (i) find that Sargent never did prove that his precedent works in all cases, (ii) find situations where Sargent’s precedent leads to incorrect solutions, and (iii) argue that Woodford’s (2003) use of Sargent’s precedent is fallacious; that in essence the veil of infinity hides the incompleteness of Woodford’s model of a cashless economy. Also, Eagle (2005b) revisits the issue of price indeterminacy issue in light of Eagle and Murff’s (2005) findings.

Our analysis does not address the issue of whether or not prices are determined when the central bank targets interest rates in an infinite economy; however, we do address for a finite

[1] Actually, Sargent and Wallace (1975) start out with the central bank’s objective function involving prices and output. However, as a result of their famous irrelevance principle that money supply cannot systematically affect output; the only variable remaining under the central bank’s control is the price level.

economy. Also, even for infinite economies when prices are flexible[2] and the central bank has a singular goal such as the price level or nominal income, then this paper's analysis applies. In addition to providing a new theoretical analysis for the inflation dynamics of pegging interest rates, this paper empirically applies that theoretical analysis to U.S. data between 1959 and 2004. Based on this analysis, we compile statistics on the one-step-ahead forecast errors between 1970 and 2004 had the Federal Reserve pursued either a price-level or nominal-income target. For this hypothetical analysis, the Federal Reserve uses either the money supply or the interest rate as its instrument. We find that between 1970 and 1980, the forecast errors would have been less had the Federal Reserve targeted the M1 definition of money. However, after 1982, the forecast errors would have been less had the Federal Reserve pegged the interest rate rather than the money supply.

The next section clarifies the terminology distinctions between "pegging the interest rate" and "targeting the interest rate." Section III presents our analysis which is based on a Fisher-like equation, the same basic equation that forms the basis of other economists' analyses (e.g., Wordford, 2003). In addition to discussing the analysis in terms of equations, section III presents the analysis in graphical terms. Section IV explains our empirical analysis and results. To validate the analysis theoretically, we add a cash-in-advance constraint to a finite-horizon pure-exchange Arrow-Debreu economy. Section V briefly reviews the Arrow-Debreu pure-exchange economy, whereas Section VI adds the cash-in-advance constraint and furthers the analysis. In section VII, we summarize our findings and reflect upon our analysis and findings.

[2] Our restricting the focus of this paper to price-flexible models should not be misconstrued as implying that we are claiming that prices are in fact flexible.

II. Terminology:

We make the distinction between "pegging the interest rate" and "targeting the interest rate." In this paper, targeting refers to the central bank's goal. A central bank's short-term goal may be in terms of the price level, the inflation rate, the output level, or the level of nominal aggregate demand. Even so, the central bank may *peg* the interest rate in its attempt to achieve that short-term goal. In this case, the nominal interest rate is not the short-term goal; instead, it is the instrument the central bank uses to try to meet its short-term goal. This strategy is what we call "pegging the nominal interest rate."

On the other hand, a central bank may have long-term goals and choose to set the nominal interest-rate in the short-term to meet those long-term goals. In such a situation, the central bank's short-term goal may in fact be the nominal interest rate. This strategy is what we call "targeting the nominal interest rate."

In the real world, "pegging the interest rate" and "targeting the interest rate" become blurred. However, in theoretical modeling, the distinction is very important. For example, consider a central bank that pursues price stability. Two of the most important ways for a central bank to pursue price stability is either through price-level targeting or inflation targeting. Assume the central bank in our example targets the price level. Define $\hat{P}_t$ to be the central bank's targeted price level at time t and i_t^* is the nominal interest rate set by the central bank. Let $E_t[P_t \mid i_t^*]$ be the central bank's expectations at time t of the price level at time t, conditional on the nominal interest rate being pegged at i_t^*. If the central bank uses an interest-rate instrument to pursue a short-term price-level target, it would set the nominal interest rate i_t^* such that $E_t[P_t \mid i_t^*] = \hat{P}_t$. We call such a strategy by the central bank "short-term price-level targeting" even though the central bank uses the nominal interest rate as its instrument in its attempt to achieve that short-term price-level target.[3]

On the other hand, assume the central bank sets i_t^* so that $E_t[P_{t+n}] = \hat{P}_{t+n}$ for some n>0. Thus, the central bank targets a long-term price level, but its only short-term target is the nominal interest rate itself. We call such a strategy "short-term interest-rate targeting." Basically, we use the term "pegging the interest rate" whenever the central bank has some short-term goal other than the interest rate. When the only short-term goal is to set the interest rate, we say the central bank is "targeting the interest rate."

III. Basic Analysis:

Our analysis is based on an Euler equation that Woodford (2003) refers to as a Fisher equation. We present this Fisher-Euler equation as an equality of "expected bangs per buck" where a "buck" will be one unit of the currency in the economy, whether that currency be euros, yens, pounds, dollars, or whatever. The following Fisher-Euler equation states that the marginal utility of consumption per "buck" this period must equal one plus the nominal interest rate times the expected marginal utility of consumption per "buck" next year:

$$\frac{\xi_{jt} U'_{jt}(c_{jt})}{P_t} = (1+i_t)\beta_j E_t\left[\frac{\xi_{j,t+1} U'_{j0}(c_{j,t+1})}{P_{t+1}}\right] \tag{1}$$

where i_t is the nominal interest rate, P_t is the price level, c_{jt} is individual j's consumption at time t, β_j is j's time preference factor, and $\xi_{jt} U_{j_t}(c_{jt})$ is individual j's utility function at time t.

[3] See the appendix in Eagle (2005b) for a technical correction of Sargent and Wallace's objective function needed in order that $E_t[P_t \mid i_t^*] = \hat{P}_t$ apply.

The term ξ_{jt} is a positive random variable with mean 1 representing shocks at time t to individual j's utility function. Therefore, $\xi_{jt}U'_{jt}(c_{jt})$ is individual j's marginal utility of consumption at time t.

Woodford calls his version of (1) a Fisher equation because he rewrites it in a way where the nominal interest rate is related to expected inflation and a real expected return related to the expected marginal utilities of consumption. (See Woodford, 2003, p. 71 and p. 79).

We rewrite (1) differently depending on whether the central bank targets the price level or nominal aggregate demand in the short run.[4]

Price-Level Targeting

When the central bank targets the price level, the relevant relationship is between the nominal interest rate and the price level. Rewriting the Fisher-Euler equation (1), we get:

$$(1+i_t)P_t = \tilde{\mu}_t \equiv \frac{\xi_{jt}U'_{jt}(c_{jt})}{\beta_j \cdot E_t\left[\dfrac{\xi_{j,t+1}U'_{j,t+1}(c_{j,t+1})}{P_{t+1}}\right]} \tag{2}$$

where we call $\tilde{\mu}_t$ the "P-Fisher coefficient". The "P" indicates that this relationship is more relevant when the central bank targets the price level. Solving (2) for P_t, we get:

$$P_t = \frac{\tilde{\mu}_t}{(1+i_t)} \tag{3}$$

When the central bank targets the price level, (3) is the essence of how pegging the nominal interest rate affects the price level. We refer to (3) as the "iP-Fisher Relationship."

[4] A central bank could target things other than the price level or nominal income. However, we have chosen to focus this paper only on price-level target and nominal-income (or nominal-aggregate-demand) targeting.

The equation of exchange states that MV=N=PY where M is the money supply, V is the income velocity[5] of money, N is nominal aggregate demand, P is the price level, and Y is real aggregate supply. Assume $V_t = V(i_t, \Psi_t, \eta_t)$ where Ψ_t is a vector of other variables in the system that affect velocity, and η_t is a stochastic term with mean zero.. Solving the equation of exchange for the price level gives $P_t = \frac{MV(i_t, \Psi_t, \eta_t)}{Y_t}$. Substitution this into (3) and solving for M gives:

$$M_t^d = \frac{\tilde{\mu}_t Y_t}{(1+i_t)V(i_t, \Psi_t, \eta_t)} \tag{4}$$

We use M_t^d because we use (4) to determine the amount of money demanded for any particular nominal interest rate the central bank pegs. We call (4) the P-Fisher M^d function.[6]

If we take the equation of exchange, MV=PY and solve for P, we get $P = \frac{MV}{Y}$. If we assume that real aggregate supply (Y) is given, then $\text{var}[P \mid M] = \left(\frac{M}{Y}\right)^2 \text{var}[V \mid M]$ is the variance of the price level when the central bank sets the money supply. On the other hand, (3) implies that $\text{var}[P \mid i] = \frac{\text{var}[\tilde{\mu} \mid i]}{(1+i)^2}$ is the variance of the price level when the central bank targets the nominal interest rate. Therefore, the variance of the price level will be the same for both the money supply and interest rate instruments when the following condition holds:

$$\text{var}[\tilde{\mu} \mid i] = \left(\frac{M(1+i)}{Y}\right)^2 \text{var}[V \mid M] \tag{5}$$

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[5] The term "income velocity of money" is really mislabeled as income is associated with aggregate supply not aggregate demand. However, money times velocity equals nominal aggregate demand, which equals nominal income only in equilibrium.

[6] We are not saying that this is "the money demand function" that would be consistent with a demand function based on microeconomic principles.

When the left side of (5) is less than the right side, then pegging the nominal interest rate will lead to more price stability than setting the money supply. On the other hand, setting the money supply will lead to more price stability when the right side of (5) is less than the left side. That the issue of which instrument leads to greater stability depends on the direction of the inequality of the two sides (5) is similar to Poole's (1970) conclusion that the issue depends on the relative stability of the IS curve versus the relative stability of the LM curve.

To more generally determine which instrument is more effective, (5) should take into account a stochastic level of real aggregate supply. This more general issue can more easily be analyzed in logarithmic form. Taking the natural logarithm of both sides of (3) gives $p_t = \ln(\tilde{\mu}_t) + \ln(1+i_t)$ where p_t is $\ln(P_t)$. The variance of p_t conditional on a pegged i_t is then $\mathrm{var}[p_t \mid i_t] = \mathrm{var}[\ln(\tilde{\mu}_t) \mid i_t]$. Taking the logarithm of the equation of exchange gives $m_t + v_t = p_t + y_t$ where $m_t = \ln(M_t)$, $v_t = \ln(V_t)$, and $y_t = \ln(Y_t)$. Therefore, the variance of p_t conditional on a set money supply is $\mathrm{var}[p_t \mid m_t] = \mathrm{var}[y_t \mid m_t] + \mathrm{var}[v_t \mid m_t] - 2\,\mathrm{cov}[v_t y_t \mid m_t]$. Therefore, in order for pegging the interest rate and setting the money supply to have equal effectiveness, the following condition must hold:

$$\mathrm{var}[\ln(\tilde{\mu}_t) \mid i_t] = \mathrm{var}[y_t \mid m_t] + \mathrm{var}[v_t \mid m_t] - 2\,\mathrm{cov}[v_t y_t \mid m_t] \qquad (6)$$

If the left side of (6) is greater than the right side, then setting the money supply is more effective, whereas if the left side of (6) is less than the right side then pegging the interest rate is more effective.

Figure 1 shows how pegging the interest rate leads to price stability when the P-Fisher coefficient $\tilde{\mu}_t$ is stable even though velocity is unstable. The graph in the upper left quadrant plots the P-Fisher M^d curve, which is stochastic because of the unstable velocity. A decrease in velocity shifts the P-Fisher M^d curve to the left (an increase), whereas a velocity increase shifts the curve to the right. When the central bank pegs the nominal interest rate at i_t^*, the quantity of money demanded will decrease (increase) when velocity increases (decreases). The graph in the lower left quadrant shows the N=MV relationship where N is nominal aggregate demand. Since the slope of this relationship is velocity, an increase in velocity will pivot the N=MV curve counterclockwise (an increase), whereas a velocity decrease will pivot the curve clockwise. Note that the shifts in the P-Fisher M^d curve and N-MV offset each other in this example, leading to the level of nominal aggregate demand being the same regardless of velocity. The lower right quadrant of Figure 1 plots the equilibrium condition N=PY curve, whose slope is Y. Since the level of nominal aggregate demand is unaffected by changes in velocity in this example, the price level is also unaffected.

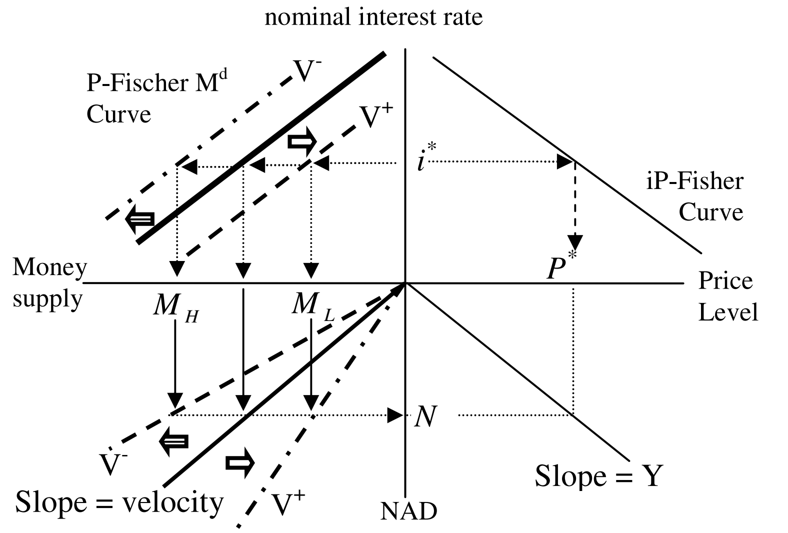

Figure 1. Why Pegging the Nominal Interest Rate can Precisely Determine the Price Level Despite Unstable Velocity

The upper right quadrant of Figure 1 summarizes the other three quadrants. When the P-Fisher coefficient is stable, the price level is invariant to changes in velocity when the central bank targets the nominal interest rate.

Figure 2 shows how the central bank setting the money supply leads to price instability when velocity is unstable even though the P-Fisher coefficient $\tilde{\mu}_t$ is stable. For a given money supply, the stochastic changes in velocity will cause nominal aggregate demand to fluctuate as

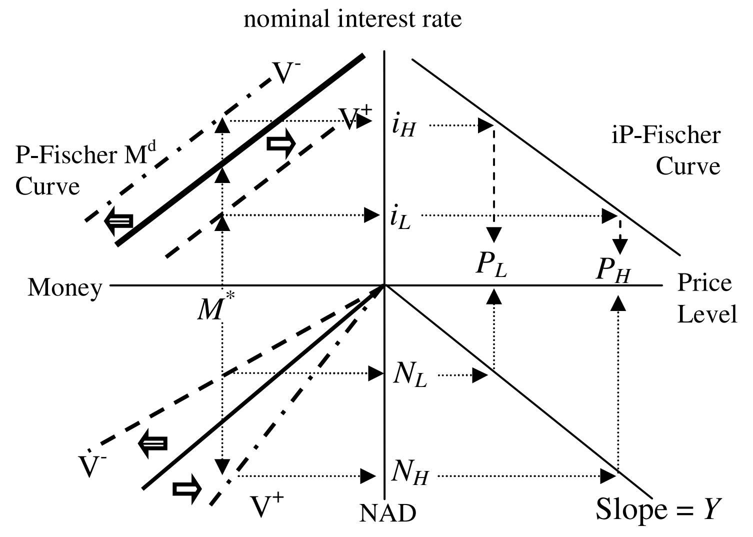

Figure 2. Price Instability Resulting from Volatility in Velocity

shown in the lower left quadrant. In the right quadrant, the fluctuating nominal aggregate demand leads to fluctuating price levels when real aggregate supply remains the same.

The upper quadrants of Figure 2 also shows why fluctuating velocity leads to price instability when the central bank sets the money supply. For a given money supply M^*, the fluctuating P-Fisher M^d curves lead to instability in the interest rate which through the iP-Fisher curve means price instability.

Nominal-Aggregate-Demand Targeting (Similar to Nominal-Income Targeting)[7]

The previous section analyzed pegging the interest rate under price-level targeting. However, Eagle and Domian (2003a and 2003b) argue that the central bank should target nominal income or nominal aggregate demand instead of price-level targeting. This section discusses pegging the interest rate under nominal-aggregate-demand targeting. What is relevant under nominal-aggregate-demand targeting is the relationship between the nominal interest rate and nominal aggregate demand. Rewriting the Fisher-Euler equation (1) and replacing P_t with N_t/Y_t, we get:

[7] Nominal income is associated with nominal aggregate supply. Since the theory we present is in terms of nominal aggregate demand, not nominal aggregate supply; we use the term "nominal-aggregate-demand targeting" rather than the term "nominal income targeting."

$$(1+i_t)N_t = \mu_t \equiv \frac{\xi_{jt}U'_{jt}(c_{jt})Y_t}{\beta \cdot E\left[\frac{\xi_{j,t+1}U'_{j,t+1}(c_{j,t+1})Y_{t+1}}{N_{t+1}}\right]} \tag{7}$$

where we call μ_t "the N-Fisher coefficient". We use the "N" prefix because (7) is most applicable to when the central bank targets nominal aggregate demand. Solving (7) for N_t gives:

$$N_t = \frac{\mu_t}{1+i_t} \tag{8}$$

When the central bank targets nominal aggregate demand, (8) is the essence of how pegging the nominal interest rate affects nominal aggregate demand. We refer to (8) as the "iN-Fisher Relationship."

Substituting $M_t V(i_t, \Psi_t, \eta_t) = N_t$ into (8) and solving for M_t gives:

$$M_t^d = \frac{\mu_t}{(1+i_t)V(i_t, \Psi_t, \eta_t)} \tag{9}$$

We call (9) the N-Fisher M^d function as it will determine the quantity of money demanded when the central bank targets nominal aggregate demand.[8]

Since N=MV, $\text{var}[N \mid M] = (M)^2 \text{ var}[V \mid M]$. Taking the variance of both sides of (8) conditional on the nominal interest rate gives $\text{var}[N_t \mid i_t] = \frac{\text{var}[\mu_t \mid i_t]}{(1+i_t)^2}$. Therefore, the variance of nominal aggregate demand will be the same for both the money supply and interest rate instruments when the following condition holds:

$$\text{var}[\mu_t \mid i_t] = \left(\frac{M}{1+i_t}\right)^2 \text{var}[V \mid M] \tag{10}$$

[8] In fact (4) and (9) are the same because $\mu_t = \tilde{\mu}_t Y_t$. However, μ_t is likely to be more stable under nominal aggregate demand targeting than is $\tilde{\mu}_t$, whereas $\tilde{\mu}_t$ is likely to be more stable under price-level targeting than is μ_t. This is discussed later in the paper.

When the left side of (10) is less than the right side, then pegging the interest rate will lead to more stability in the level of nominal aggregate demand than will setting the money supply. On the other hand, setting the money supply will lead to more such stability when the right side of (10) is less than the left side. As opposed to (5), equation (10) is very general and applies even when real aggregate supply is stochastic.

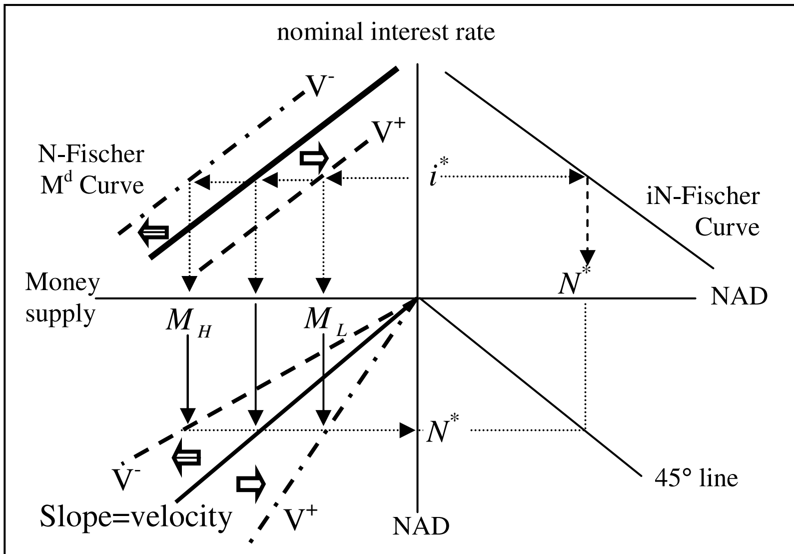

Figure 3. Why Pegging the Nominal Interest Rate Can Precisely Determine Nominal Aggregate Demand (NAD) When Velocity is Unstable But the iN-Fischer Curve is Stable

Figure 3 shows how pegging the interest rate leads to stability in nominal aggregate demand when the N-Fisher coefficient μ_t is stable even though velocity is unstable. Similarly, Figure 4 shows how setting the money supply leads to instability in the level of nominal aggregate demand under the same conditions. Because these graphs are similar to those in Figures 1 and 2, we consider them to be self explanatory.

IV. Emirical Results:

In the previous section, we identified the conditions that determine whether the central bank's instrument should be the interest rate or the money supply. In this section, we apply the analysis of the previous section to address this issue empirically.

First, consider price-level targeting. According to the analysis of the previous section, if the central bank uses the interest rate as its instrument, it should use equation (3) to do so. If its price target is P_t^*, then it should set its pegged interest rate i_t^* so that:

$$1 + i_t^* = \frac{E_t[\tilde{\mu}_t]}{P_t^*}. \tag{11}$$

While the P-Fisher coefficient $\tilde{\mu}_t$ can vary over time, $\tilde{\mu}_t$ depends on the expectations of the future as is shown in (1). From (3) and (11), we conclude that if the central bank were to target the interest rate, the difference between the actual price level and the targeted price level would equal:

$$\tilde{P}_t - P_t^* = \frac{\tilde{\mu}_t - E_{t-1}[\tilde{\mu}_t]}{1 + i_t^*} \tag{12}$$

where $\tilde{P}_t$ is price level that would have resulted had the central bank targeted the interest rate. However, we only will observe the pegged interest rate if the central bank is in fact pegging the interest rate. As a result, instead of (12), we measure these forecast errors by $\dfrac{\tilde{\mu}_t - E_{t-1}[\tilde{\mu}_t]}{1 + i_t}$ where i_t is actual 3-month Treasury-Bill rate.[9] The difference between (12)

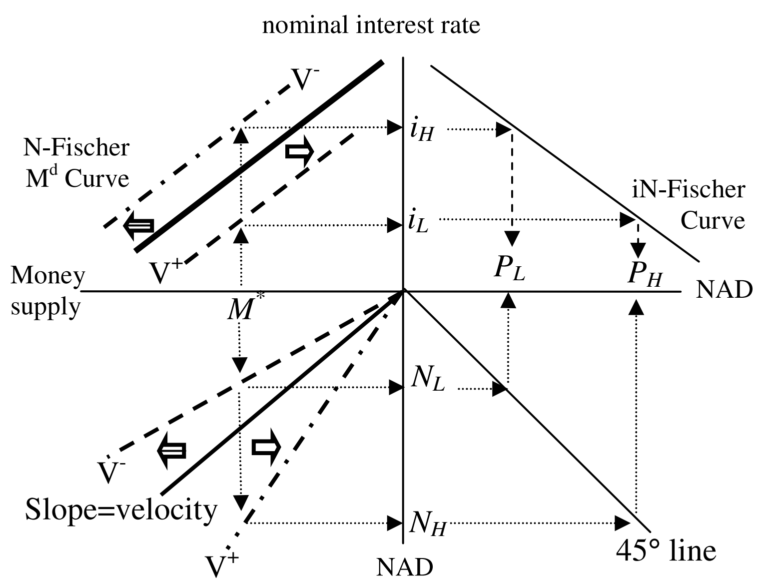

Figure 4. Why Setting the Money Supply Leads to Instability in Nominal Aggregate Demand (NAD) when Velocity is Unstable, but the iN-Fisher Curve is Stable.

[9] Since our analysis is quarterly, we use the average of the 3-month T-bill rates for the last month of the previous quarter and the 3-month T-Bills for first two months of the present quarter as reported by the Federal Reserve.

and this measure should be very insignificant. We next divided these forecast errors the current price level so that these would represent percentage one-step-ahead forecast errors. By (3), we can write this percentage measure as:

$$\left(P_t - \frac{E_{t-1}[\tilde{\mu}_t]}{1+i_t}\right)/P_t = 1 - \frac{E_{t-1}[\tilde{\mu}_t]}{(1+i_t)P_t} \tag{13}$$

We estimate $E_{t-1}[\tilde{\mu}_t]$ by assuming that the central bank's and public's expectation of $\tilde{\mu}_t$ is based on past values of $\tilde{\mu}_t$ as shown below:

$$E_{t-1}[\tilde{\mu}_t] = \sum_{k=1}^{8} \tilde{b}_k \tilde{\mu}_{t-k} + \sum_{s=1}^{4} \tilde{a}_s D_{st} \tag{14}$$

where the D_{qt} variables are seasonal dummies. Since one does observe past interest rates and past price levels, one can determine the past P-Fisher coefficients using equation (3).

For comparison purposes, we also estimated the one-step-ahead forecast errors had the central bank set the money supply instead. The equation of exchange is $M_t V_t = P_t Y_t$. Solving for P_t, we get $P_t = M_t \frac{V_t}{Y_t}$. To pursue its price-level target of P_t^*, the central bank would set the money supply so that $P_t^* = M_t^* E_{t-1}\left[\frac{V_t}{Y_t}\right]$. Therefore, the one-step-ahead forecast errors of prices if the central bank uses the money supply instrument rather than the interest rate instrument would be $\tilde{\tilde{P}}_t - P_t^* = M_t^*\left(\frac{V_t}{Y_t} - E_{t-1}\left[\frac{V_t}{Y_t}\right]\right)$ where $\tilde{\tilde{P}}_t$ is the price level that would have resulted had the central bank used the money supply as its instrument rather than the interest rate. However, if the central bank did not set the money supply, we cannot observe its theoretical value. Therefore, we use $M_t\left(\frac{V_t}{Y_t} - E_{t-1}\left[\frac{V_t}{Y_t}\right]\right)$ as a proxy measure of these one-step-ahead forecast errors where M_t is the actual money supply. Again we divide by P_t to make this a percentage one-step-ahead-forecast error.

$$M_t\left(\frac{V_t}{Y_t}-E_{t-1}\left[\frac{V_t}{Y_t}\right]\right)/P_t=1-E_{t-1}\left[\frac{V_t}{Y_t}\right]/P_t \tag{15}$$

We assume that the central bank's and public's expectations of the ratio of velocity to real aggregate supply is given by:

$$E_{t-1}\left[\frac{V_t}{Y_t}\right]=\sum_{k=1}^{8}\beta_k\frac{V_t}{Y_t}+\sum_{s=1}^{4}\alpha_s D_{st} \tag{16}$$

Where ε_t is the one-step-ahead percentage forecast error at time t, the term "standard one-step-ahead forecast error" (abbreviated std. OSA% forecast error) is defined as:

$$\sqrt{\sum_{t=1}^{n}\frac{\varepsilon_t^2}{n}} \tag{17}$$

where this statistic pertains to the period from t=1 to t=n. We used (13) and (14) to estimate the OSA % forecast errors, and then use (17) to compute the std. OSA% forecast error to represent the hypothetical situation of the central bank pegging the interest rate as it targeted the price level. We use (15) and (16) and (17) to compute the std. OSA% forecast error that represents the hypothetical situation of the central bank setting the money supply as it targeted the price level. We use Ordinary Least Squares to estimate the initial conditions of (14) and (16) for the period from 1959 through the fourth quarter of 1959. From 1970 on, we estimate the OSA % forecast errors and then re-estimate the coefficients of (14) and (16) to reflect the new information.

Table 1 reports this analysis for the monetary instruments of the interest rate (the 3-month T-bill rate), and the M1, M2, and M3 definitions of money. In bold is the instrument with the lowest std. OSA% forecast error for the period analyzed. During the 1970s, Table 1 indicates that the central bank setting the M1 definition of money would have minimized the standard one-

instrument	1970:1 to 2004:4	1970:1 to 1979:4	1980:1 to 2004:4	1983:1 to 2004:4
interest rate	1.47%	0.95%	1.63%	**0.55%****
M1	1.05%	**0.74%**	1.14%	1.05%
M2	0.88%	1.07%	0.80%	0.78%
M3	**0.85%**	1.07%	**0.75%**	0.78%

Table 1: Standard One-Step-Ahead GDP-Deflator % Forecast Error by Period and Instrument

step-ahead forecast error in the price level. However, the M1's std. OSA% forecast error is not statistically significantly less than it would have been had the central bank pegged the interest rate. (We later explain how we determine statistical significance.)

Because the rather turbulent time from 1980-1982 can be considered a fairly turbulent time, it may be more appropriate to consider the time after 1982 rather than the time after 1980. Also, the effects of DICMCA of 1980 may have needed some time for the economy to adjust to it. After 1982, the central bank would have minimized the standard one-step-ahead forecast error by targeting the interest rate rather than any money supply measure. In fact, the 0.55% standard OSA% forecast error for interest rate targeting is statistically significantly less than the std. OSA% forecast error for the M3 definition of money, which had the second lowest std. OSA% forecast error.

We also empirically studied the relative effectiveness of monetary instruments when the central bank targets nominal income or nominal aggregate demand. The OSA forecast error when the central bank pegs the interest rate under nominal-aggregate-demand targeting will be $\tilde{N}_t - N_t^* = \dfrac{\mu_t - E_{t-1}[\mu_t]}{1+i_t^*}$ where N_t^* is the targeted nominal aggregate demand and $\tilde{N}_t$ is the level of nominal aggregate demand that would have resulted had the central bank pegged the interest rate. Using the actual interest rate instead of what the central bank would have targeted

instrument	1970:1 to 2004:4	1970:1 to 1979:4	1980:1 to 2004:4	1983:1 to 2004:4
interest rate	2.01%	1.48%	2.18%	**0.76%**
M1	1.35%	**1.22%*****	1.40%	1.24%
M2	1.19%	1.40%	1.08%	0.90%
M3	**1.13%**+++	1.39%	**1.01%**+++	0.91%

+++ means significant at the 1% level for a special bootstrapping testing (See Table 3 for more details)

Table 2: Standard One-Step-Ahead Nominal GDP % Forecast Error by Period and Instrument

it to be and then dividing by the price level and using (8) gives the following measure of this one-step-ahead percentage forecast error:

$$1-\frac{E_{t-1}[\mu_t]}{N_t\left(1+i_t^*\right)} \tag{18}$$

Similar to what we did under price-level targeting, we assumed:

$$E_{t-1}[\mu_t]=\sum_{k=1}^{8} b_k\mu_{t-k}+\sum_{s=1}^{4} a_s D_{st} \tag{19}$$

For the central bank setting the money supply while pursuing a nominal-aggregate-demand target, we measured the one-step-ahead percentage forecast errors by

$$M_t\,(V_t-E_{t-1}[V_t])/N_t=1-E_{t-1}[V_t]/N_t \tag{20}$$

and we assumed that the central bank's and public's expectations of velocity is given by:

$$E_{t-1}[V_t]=\sum_{k=1}^{8}\tilde{\beta}_k V_t+\sum_{s=1}^{4}\tilde{\alpha}_s D_{st} \tag{21}$$

In summary, we used (18), (19), and (17) to determine the std. OSA% forecast error that would have applied had the central bank pegged the interest rate as it pursued a nominal-aggregate-demand target. We used (20), (21), and (17) to determine the std. OSA% forecast error that would have applied had the central bank set the money supply instead as it pursued a nominal aggregate-demand target.

Table 2 presents the std. OSA % forecast errors for Nominal GDP for different periods and for different instruments. In the 1970s, the M1 definition of money would have provided the

Time Period	Targeting Objective	Comparison	Combined Distribution	interest-rate distribution only	money-supply Distribution only
1970:1 to 1979:4	Price Level	M1** vs. interest rate	1.5%**	4.29%**	0.07%***
1982:1 to 2004:4	Price Level	interest rate*** vs. M1	0.00%***	0.00%***	0.02%***
1982:1 to 2004:4	Price Level	interest rate** vs. M2	3.20%**	0.02%***	8.79%*
1982:1 to 2004:4	Price Level	interest rate*** vs. M3	0.15%***	0.02%***	0.42%***
1970:1 to 1979:4	NAD	M1 vs. interest rate	28.21%	37.94%	10.43%
1982:1 to 2004:4	NAD	interest rate*** vs. M1	0.84%***	0.00%***	2.65%**
1982:1 to 2004:4	NAD	interest rate vs. M2	26.85%	23.64%	29.70%
1982:1 to 2004:4	NAD	interest rate vs. M3	17.47%	18.50%	15.92%
1970:1 to 2004:4	Price Level	M3 vs. interest rate	48.63%	61.33%	0.01%***
1970:1 to 2004:4	NAD	M3 vs. interest rate	59.77%	70.35%	0.50%***
1980:1 to 2004:4	Price Level	M3 vs. interest rate	31.22%	46.77%	0.00%***
1980:1 to 2004:4	NAD	M3 vs. interest rate	46.73%	60.38%	0.00%***

Table 3: Bootstrapping P-values.

lowest std. OSA% forecast error, which was 1.22%. This was statistically significantly less than the 1.48% std. OSA% forecast error, that pertains to if the central bank had pegged the interest as it pursued nominal-aggregate-demand targeting. After 1982, pegging the interest rate resulting with a lower std. OSA% forecast error, although it was not statistically significantly less.

To assess the significance of the differences, we employed a bootstrapping method. The null hypothesis was that the central bank will experience the same std. forecasting error regardless whether it pegged the interest rate or targeted the money supply. We first formed a distribution of the possible OSA% forecasting errors by pooling the OAS% forecasting errors from both the money-supply analysis and the interest-rate analysis. We then randomly sampled

30,000 computations of a pair of std. OSA% forecasting errors and then computed the percent of those computations where the difference of the std. OSA% forecasting errors were equal to or greater than those that were observed in each case. The percentages resulting from these simulations, we treat as the P-values for the tests of the null hypothesis that the true std. OSA% forecasting errors are the same.

Table 3 presents the results of this bootstrapping. In addition to the bootstrapping where we combined both sets of OSA% forecasting errors in the pool, we also looked at the bootstrapping results when we kept the forecasting errors separate and used one of the separate pools (but not both).,

V. Review Of An Arrow-Debreu Pure-Exchange Economy:

Section III discussed our theoretical analysis and Section VI used our theoretical analysis to empirically investigate which instrument, the interest rate or the money supply, would have better served the Federal Reserve in its price-level targeting or nominal-aggregate-demand targeting. However, until readers see this analysis applied to a general equilibrium model, they may be skeptical of the analysis' validity. This section briefly reviews a standard Arrow-Debreu pure exchange economy without storage consisting of one nonstorable consumption good. Section VI then adds the cash-in-advance constraint to this model and then furthers the analysis of section III.

Assume each consumer j's time-separable utility function is:

$$\xi_{j0}U_{j0}(c_{j0})+\sum_{t=1}^{T}\beta^{t}\sum_{s=1}^{S_t}\pi_{st}\xi_{jst}U_{jst}(c_{jst}) \qquad (22)$$

where c_{j0} is j's consumption at time 0, c_{jst} is j's consumption in state s at time t, β is the time discount factor, and π_{st} is the probability of state s occurring at time t. The T represents the last period of the economy and S_t is the number of possible states at time t. The functions $U_{j0}(c_{j0})$ and $U_{jst}(c_{jst})$ are continuous, twice differentiable, strictly concave, and strictly increasing. To rule out corner solutions, assume $\lim_{c\to 0} U'_{j0}(|c|) = \lim_{c\to 0} U'_{jst}(|c|) = +\infty$. The time frame for the s subscript is determined by the t subscript next to the s subscript. For example, the s in c_{js1} refers to one of the possible states that can occur at time 1. For notional convenience, we assume a common time discount factor β.

At time 0, consumers can buy or sell state-contingent securities. These state-contingent securities are prepaid securities where the buyer pays the seller the price of the security at time 0. Let x_{jst} represent individual j's demand at time 0 for the state-contingent security that delivers one consumption good at time t iff state s occurs at time t. Define Ω_{st} so that the price of this security equals $P_0 \pi_{st} \Omega_{st}$. With it so defined, Ω_{st} represents the real pricing kernel.

Each consumer j chooses x_{jst} for all s and t to maximize (22) subject to:

$$P_0 c_{j0} + P_0 \sum_{t=1}^{T} \sum_{s=1}^{S_t} \pi_{st} \Omega_{st} x_{jst} = P_0 y_{j0} \tag{23}$$

$$P_{st} c_{jst} = P_{st} \left(y_{jst} + x_{jst} \right) \tag{24}$$

where (24) applies for all states s=1,2,…,S_t for all t=1,2,…,T where S_t is the finite number of states of nature at time t.

The market clearing conditions are that $\sum_{j=1}^{n} c_{j0} = Y_0$, $\sum_{j=1}^{n} c_{jst} = Y_{st}$, and $\sum_{j=1}^{n} x_{jst} = 0$ for all states s at time t and for t=1,2,…T, where the aggregate supply of the consumption good is represented by Y_0 at time 0 and Y_{st} in state s at time t respectively. Consumer j's optimization problem is satisfied when $\frac{\xi_{j0}U'_{j0}(c_{j0})}{P_0} = \frac{\beta^t \pi_{st}\xi_{jst}U'_{jst}(c_{jst})}{P_0\pi_{st}\Omega_{st}}$ for all s=1,2,…,S_t and for all t=1,2,…,T, which implies that

$$\Omega_{st} = \frac{\beta^t \xi_{jst}U'_{jst}(c_{jst})}{\xi_{j0}U'_{j0}(c_{j0})} \tag{25}$$

The left side of (25) is the real pricing kernel and the right side is the intertemporal marginal rate of substitution. Some literature mistakenly defines the pricing kernel as the intertemporal marginal rate of substitution (See, for example, Campbell, Lo, and MacKinlay, 1997, p. 294). The equality between the real pricing kernel and the intertemporal marginal rate of substitution shown in (25) is an equilibrium condition, not a definition.

Since this is a standard one-good Arrow-Debreu pure-exchange economy with well behaved utility functions, a unique competitive equilibrium exists and that competitive equilibrium is Pareto efficient. Also, the following property holds:

Consumption-Aggregate-Supply Invariance Property: Let 1 and 2 represent any two different states of nature. If real aggregate supply and each consumer's utility function U_{jst}(.) is the same in both states of nature (i.e., there are no utility shocks), then every individual's consumption will be the same in both states of nature.

> Proof by contradiction. Assume there is some consumption allocation in a competitive equilibrium where for some states 1 and 2, each consumer's utility function is the same for both states 1 and 2, $Y_{1t}=Y_{2t}$, and there are two individuals j and k such that $c_{j1t} < c_{j2t}$ and $c_{k1t} > c_{k2t}$. Since this is an Arrow-Debreu competitive equilibrium, the consumption allocation must be Pareto efficient. Define $\tilde{c}_{j1t} \equiv \frac{1}{2}\left(c_{j1t} + c_{j2t}\right)$ and $\tilde{c}_{k1t} \equiv \frac{1}{2}\left(c_{k1t} + c_{k2t}\right)$. Define a new consumption allocation where for all consumers, for all states of nature, and for all time periods, the new consumption equals the old consumption except that j's consumption in states 1 and 2 are both $\tilde{c}_{j1t}$ and k's consumption in states and 2 are both $\tilde{c}_{k1t}$. The new consumption allocation is obviously feasible since the original allocation

was feasible. Because both j and k are strictly risk averse, they are both better off with this new consumption allocation. However, that contradicts the statement that the original consumption allocation is Pareto efficient. We, therefore, conclude that the consumption allocation must be the same as long as neither aggregate output nor the form of the utility functions changes. Q.E.D.

In the next section, we add a cash-in-advance constraint to an Arrow-Debreu economy which causes consumers to hold money during the period but not between periods. We also add consumers' holding nominal bonds. However, in order for consumers to choose to hold nominal bonds, they must either expect to receive or pay nominal subsidies or taxes. In other words, the only reason consumers hold nominal bonds in the following economy is to hedge against the nominal risk they face from nominal subsidies or taxes. Such a conclusion follows directly from the consumption-real-aggregate-supply invariance property. In the model that follows the central bank will be able to make changes in nominal aggregate demand even when real aggregate supply does not change. However, the consumption-real-aggregate-supply invariance property states that if real aggregate supply does not change, then the consumption allocation must stay the same. Nominal contracts expose both parties to the risk that nominal aggregate demand will change when nominal real aggregate supply does not change. Therefore, the only reason a consumer would choose to hold a nominal contract is to hedge against another nominal contract. For example, a consumer expected to pay nominal taxes in the future will buy a nominal bond to hedge against the nominal risk. Similarly, a consumer expecting to receive nominal subsidies in the future would issue a nominal bond to hedge against this nominal risk.

VI. Adding A Cash-In-Advance Constraint to An Arrow-Debreu Economy:

We add a cash-in-advance (CIA) constraint similar to Lucas (1982). However, we assume that there are both cash transactions and credit transactions. Sometimes it is more convenient to make a cash transaction and other times it is more convenient to make a credit transaction where one in essence borrows the money for the transaction concurrently with the spending of that money. To make the notion of “convenience” well defined in the model, assume that for some transactions there is no transactions cost for using cash, but there is a positive and significant transactions cost for using credit. For other transactions, there is a positive and significant transactions cost for using cash, but there is no transactions cost for using credit. As a result, consumers will always use the approach for which there is no transactions cost.

The fraction of the transactions for which cash is more convenient equals $\left(f_t(i_{st}, Y_{st}) + \varepsilon_{st}\right)$. The error term ε_{st} is stochastic with mean zero, and is serially uncorrelated. Therefore, the cash-in-advance constraint for each consumer j will equal:

$$M_{jst} \geq \left(f_t(i_{st}, Y_{st}) + \varepsilon_{st}\right) P_{st} c_{jst} \tag{26}$$

Under the assumptions we make, (26) will in fact hold with equality.

The timing of the information at the very beginning of this economy is very important. At the very beginning, all variables except the money supply and velocity are known. That is the time when the central bank must decide what to do. It must announce the interest rate if it pegs the interest rate, and it must announce the money supply if it pegs the money supply. The central bank is unaware of the realized value of velocity when it pegs the money supply or the interest rate. A second later, after the central bank has announced its money-supply or interest-rate peg, consumers become aware not only of the central bank’s announcement, but also the fraction of the value of their consumption transactions that will be more convenient in cash than in credit. The consumers then acquire the money they will need, buy and sell securities, which include state-contingent securities, and nominal bonds. They therefore acquire the exact amount of cash they will need.

When the consumers undertake credit transactions, instead of spending money they issue nominal bonds in exchange for the goods. Since they know at the beginning of the period how much their credit-transactions that period will be, they take this into account at the beginning of the period in the regular credit market. For example, suppose a consumer wanted to hold neither a negative nor positive position in nominal bonds from the current period to the next period, but she knows she will buy 100 “bucks” worth of goods on credit. She, therefore, buys 100 “bucks” worth of nominal bonds at the beginning of the period so that her net position in nominal bonds would be zero at the end of the period.

Since consumers know the amount of cash they will need, the amount of money they choose to hold at the beginning of the period will equal the amount they will need as long as the nominal interest rate is positive (which we will assume that it is).[10] With no money being held between periods, and no transactions costs being incurred; there will be no distortions to the Arrow-Debreu economy.[11]

Given one’s information, consumer j’s optimization problem at time 0 is to maximize (22) subject to:

$$M_{j0} + \left(1 - \{f_o(i_0, Y_0) + \varepsilon_0\}\right) P_0 c_{j0} + B_{j0} + P_0 \sum_{t=1}^{T} \sum_{s=1}^{S_t} \Omega_{st} x_{jst} = P_0 y_{j0} + Z_{j0} \tag{27}$$

---

[10] The assumption that the nominal interest rate is an assumption of a result of the model. We do need to be careful with such an assumption as it does restrict the more basic assumptions underlying the model. The reason we need this assumption is the well recognized problem of money in models where people will choose to hold money instead of nominal bonds when the nominal interest rate is negative. A negative nominal interest rate therefore impinges on the economy’s ability to move to Pareto efficiency. So that we do not get distracted on this long-standing difficult issue, we just assume that it does not apply to any economy we are considering here.

[11] We purposely made the assumptions in the manner that we did so that we would have a stochastic velocity in an economy where there is no distortion on the Arrow-Debreu economy. That way the Pareto-efficiency results of the Arrow-Debreu economy extend to the economy with money added.

$$M_{jst} + \left(1 - \{f_t(i_{st}, Y_{st}) + \varepsilon_{st}\}\right)P_{st}c_{jst} + B_{jst} = P_{st}(y_{jst} + x_{jst}) + Z_{jst} + B_{js,t-1}(1 + i_{js,t-1}) \qquad (28)$$

and (26) where (26) and (28) apply for all states s at time t and for t=1,2,...,T.

Equation (28) is the budget constraint for each period except time 0. The consumers receive funds from four sources: (i) the value of his/her endowment in period t, (ii) the value of the state-contingent securities he/she purchased at time 0 that matures at time t, (iii) the nominal subsidies they receive (negative subsidies are taxes)., and (iv) the principal and interest from nominal bonds they held from the previous period. For periods t=1,2,...,T; the consumer spends those funds on either goods or nominal bonds. For the cash transactions, the consumer uses the funds to obtain money at the beginning of the period. The value of these cash transactions equals $\left(f_t(i_{st}, Y_{st}) + \varepsilon_{st}\right)P_{st}c_{jst}$. The consumer invests the rest of the funds into nominal bonds. The amount of these nominal bonds purchased at the beginning of the period equals $\left(1 - \{f_t(i_{st}, Y_{st}) + \varepsilon_{st}\}\right)P_{st}c_{jst} + B_{jst}$. When the consumer engages in the credit transactions, he/she issues bonds equal to the value of the credit transactions, which equals $\left(1 - \{f_o(i_0, Y_0) + \varepsilon_0\}\right)P_{st}c_{j0}$. Note that the variable B_{jst} is the net amount of nominal bonds j holds from period t to period t+1.

Equation (27) is consumer j's budget constraint for time 0. This is basically the same as (28) with two differences. First, the consumer issues or purchases state-contingent securities at time 0. Second, there are no bonds carried over from before time 0 because the economy did not exist prior to time 0.

Prices are determined in this finite economy. By working with a finite economy we avoid the controversial debate between Eagle and Murff (2005) and Sargent and Wallace (1975), Sargent (1978), Woodford (2003). With a finite economy, there can be no interest rate in the final period as there is no future period with which to borrow or lend. As a result, the central bank has no choice but use the money supply in that final period. Setting the money supply in that final period will determine price by the aggregate of (26) holding with equality. If the central bank pegged interest rates prior to time T, then we can use (3) or (8) working backwards in time with expectations operators to conclude that prices will be determined.

To avoid confusion, it is important that the taxes and subsidies to the Arrow-Debreu economy of the previous section do not redistribute wealth and do not interject any risk other than nominal risk. An easy way to achieve these results is by assuming that for all consumers j, their taxes and subsidies satisfy the following condition:

$$Z_{jst} = \rho_j \sum_{k=1}^{m} Z_{kst} \tag{29}$$

where ρ_j does not change by time period, m is the number of consumers, and $\sum_{j=1}^{m} \rho_j = 1$. What (29) means is that regardless whether or not the government taxes or pays subsidies, consumer will always receives the same fraction of those taxes or subsidies. That this condition leads to no wealth redistribution is clear in the proof of the following proposition:

The Nominal-Hedging Proposition: Under the above assumptions, the CIA consumption allocation is Pareto efficient and the following conditions hold:

$$B_{j0} = Z_{j0} \tag{30}$$

$$B_{jst} = Z_{jst} + B_{js,t-1}(1 + i_{js,t-1}) \tag{31}$$

Proof: Subtracting (30) from (27) and recognizing that (26) holds with equality gives (23). Subtracting (31) from (28) and recognizing that (26) holds with equality gives (24). Therefore, the consumers' optimization problem of this section's CIA economy is the same as that in the previous section's Arrow-Debreu economy. Hence, the consumption allocations in this section's CIA economy are the same as in the previous section's Arrow-Debreu economy. Since the previous section's Arrow-Debreu consumption allocation was Pareto efficient so must be the consumption allocation resulting from this section's CIA economy under these assumptions.

Remaining to show is that (30) and (31) are feasible for the government as well as for consumers. Recognize that the government's budget constraints for period 0 and for any state s at any time t are:

$$\sum_{j=1}^{m} B_{j0} = \sum_{j=1}^{m} Z_{j0} \tag{32}$$

$$\sum_{j=1}^{m} B_{jst} = \sum_{j=1}^{m} Z_{jst} + (1+i_{s,t-1})\sum_{j=1}^{m} B_{js,t-1} \tag{33}$$

Substituting (29) into (30) and (31) and then summing over all consumers shows that (32) and (33) hold, meaning the government's budget constraints hold. Q.E.D.

The Nominal-Hedging Proposition shows that the only reason that consumers hold nominal bonds between periods in this model is to hedge against the nominal-aggregate-demand risk in the nominal taxes and subsidies. By the Consumption-Real-Aggregate-Supply Property of the previous section, the Pareto-efficient consumption allocation remains unchanged when real aggregate supply remains unchanged as long as no utility shocks occur and no wealth redistribution takes place. Therefore, Pareto-efficient consumption should be unaffected by changes in nominal aggregate demand. Therefore, in the absence of government taxes and subsidies, then the consumers would neither hold nor issue nominal bonds in equilibrium. However, if the government issues nominal taxes or nominal subsidies, consumers will choose to issue or hold nominal bonds to offset the nominal risk they face in the nominal taxes or nominal subsidies.

The preceding analysis is very revealing about the common completes-market assumption made by monetary economists such as Woodford (2003). If markets are truly complete, then monetary policy cannot have any real positive impact on the economy unless it redistributes wealth. The only way that monetary policy can result in a Pareto improvement in the economy is if markets are incomplete. The logic behind these statements is simple. **If markets are complete, then the resulting allocation is Pareto efficient, which means a**

Pareto improvement is impossible. Thus, if we believe that monetary policy can affect Pareto efficiency, then monetary policy can only do so when markets are incomplete. This means that any role for monetary policy to make Pareto improvements must be in its ability to help complete markets. For monetary economists to argue that they will let other financial markets deal with the incompleteness of markets so that monetary policy can focus on its "true" role is a non sequitur. The only possible Pareto-efficiency role of monetary policy is to help complete markets, to help make contracts behave in a manner consistent with complete markets. See Eagle (2005a) and Eagle and Domian (2005a and 2005b) for a discussion of the role of monetary policy in completing markets.

Since markets are complete in the model of this section, the resulting consumption allocation is Pareto-efficient regardless of monetary policy. Since nominal contracts expose one to the risk that the real payments on the nominal contracts will change when nominal aggregate demand changes even though neither changes in real aggregate supply nor utility shocks occur. That nominal exposure then would violate the Consumption-Aggregate-Supply-Invariance Property of the previous section. As a result, the only reason consumers would choose to enter nominal contracts when markets are complete would be to hedge against other nominal contracts. In a model with complete markets, other contracts will dominate the nominal contracts for non-hedging purposes. In this model, the state-contingent securities will dominate the nominal contracts as the state-contingent securities will be able to identify changes in nominal aggregate demand in their different states. On the other hand, under the assumptions in Eagle and Domian (2005a and 2005b), quasi-real contracts complete the markets and therefore dominate nominal contracts. Under more general assumptions, Eagle (2005a) shows that quasi-real contracts in conjunction with three other types of contracts can approximately complete markets. In an economy with nominal contracts instead of quasi-real contracts, Eagle (2005a) shows that monetary policy can help complete markets by the central bank targeting nominal aggregate demand.

While monetary policy does not have Pareto impacts in this model, it nevertheless does have inflation impacts, which is the subject of this paper. These inflation impacts are as discussed in the analysis of section III. We now further that analysis as it applies to the model of this section. We will begin this discussion by looking to the utility function (22) being logarithmic without utility shocks as is shown below:

$$\ln(c_{j0}) + \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S_t} \pi_{st} \ln(c_{jst}) \qquad (34)$$

For this utility function, we can use (25) to determine the real pricing kernel, which is:

$$\Omega_{st} = \frac{\beta^t c_{j0}}{c_{jst}} \qquad (35)$$

Multiplyint by c_{jst}, summing across all consumers, noting that the sum of all consumption equals real aggregate supply in equilibrium, and then dividing both sides by Y_{jst} we conclude that:

$$\Omega_{st} = \frac{\beta^t Y_{j0}}{Y_{jst}} \qquad (36)$$

This in tern implies that for all s and t:

$$\frac{c_{jst}}{Y_{st}} = \frac{c_{j0}}{Y_0} \qquad (37)$$

This is the result that forms the basis of the analysis by Eagle and Domian (2005b).

We will first study nominal-aggregate-demand targeting. Then (7) and (37) implies that

$\mu_t = \dfrac{Y_t}{c_{jt}\beta E_t\left[\dfrac{Y_{t+1}}{c_{j,t+1}}\dfrac{1}{N_{t+1}}\right]}$. Substituting $\dfrac{c_{j0}}{Y_0}$ for $\dfrac{c_{jt}}{Y_t}$ and $\dfrac{c_{j,t+1}}{Y_{t+1}}$,which we can do by (37), we get:

$$\mu_t = \frac{1}{\beta E_t\left[\dfrac{1}{N_{t+1}}\right]} \tag{38}$$

This implies that $\mu_t = E_t[\mu_t]$, which means the central bank and the public know μ_t with certainty at time t. The CB will therefore set the interest rate so that $1+i_t^* = \dfrac{\mu_t}{N_t^*}$. By (8), we conclude that $N_t = \dfrac{\mu_t}{1+i_t^*} = \dfrac{E_t[\mu_t]}{1+i_t^*} = N_t^*$. In other words, the central bank is able to perfectly meet its nominal aggregate-demand target.

This is even clearer when we assume that $N_{t+1} = N_{t+1}^*$, because then (38) implies that $\mu_t = \dfrac{N_{t+1}^*}{\beta}$, which is clearly a constant when the public knows the central bank's nominal-aggregate-demand targets.

This result of the central bank being able to perfectly target nominal aggregate demand does apply more generally than just the logarithmic utility function. Let $\overline{\rho}$ be the weighted average of consumers coefficients of relative risk aversion where each consumer's coefficient is weighted by the derivative of the consumer's consumption with respect to real aggregate supply:

$$\overline{\rho}_t \equiv \sum_{j=1}^{m}\left(\tilde{\rho}_{jt}\cdot\frac{dc_{jt}}{dY_t}\right) \tag{39}$$

where $\tilde{\rho}_{jt}$ is consumer j's coefficient of relative risk aversion at time j.

<u>N-Fisher-Coefficient-Variance Proposition</u>: If $\overline{\rho}$<1, then $\frac{\partial \mu_t}{\partial Y_t}$>0. If $\overline{\rho}$=1, then $\frac{\partial \mu_t}{\partial Y_t}$=0. If $\overline{\rho}$>1, then $\frac{\partial \mu_t}{\partial Y_t}$<0.

<u>Proof</u>: By (7), the definition of the N-Fisher coefficient is $\mu_t \equiv \frac{U'_{jt}(\tilde{c}_{jt})Y_t}{\beta \cdot E_t\left[\frac{U'_{j,t+1}(\tilde{c}_{j,t+1})Y_{t+1}}{N_{t+1}}\right]}$ where we added "~" marks to consumption to show that this is Pareto-efficient consumption; this symbolization is consistent to that in Eagle (2005a). Taking the partial derivative of μ_t with respect to Y_t gives:

$$\frac{\partial \mu_t}{\partial Y_t} = \frac{U''_{jt}(\tilde{c}_{jt})\frac{\partial c_{jt}}{\partial Y_t}Y_t + U'_{jt}(\tilde{c}_{jt})}{\beta \cdot E_t\left[\frac{U'_{j,t+1}(\tilde{c}_{j,t+1})Y_{t+1}}{N_{t+1}}\right]} = \frac{\left(\frac{U''_{jt}(\tilde{c}_{jt})}{U'_{jt}(\tilde{c}_{jt})}\frac{\partial \tilde{c}_{jt}}{\partial Y_t}Y_t + 1\right)U'_{jt}(\tilde{c}_{jt})}{\beta \cdot E_t\left[\frac{U'_{j,t+1}(\tilde{c}_{j,t+1})Y_{t+1}}{N_{t+1}}\right]} = \frac{\left(-\tilde{\rho}_{jt}\frac{\partial \tilde{c}_{jt}}{\partial Y_t}\frac{Y_t}{\tilde{c}_{jt}} + 1\right)U'_{jt}(\tilde{c}_{jt})}{\beta \cdot E_t\left[\frac{U'_{j,t+1}(\tilde{c}_{j,t+1})Y_{t+1}}{N_{t+1}}\right]}$$

In the last step, we used the definition of the j's correlation of relative risk aversion, which is $\tilde{\rho}_{jt} \equiv -\frac{U''_{jt}(\tilde{c}_{jt})}{U'_{jt}(\tilde{c}_{jt})}\tilde{c}_{jt}$. Next, we use equation (11) from Eagle (2005a), which says that $\tilde{c}'_{jt}(Y_t) = \frac{\partial \tilde{c}'_{jt}}{\partial Y_t} = \frac{1}{\tilde{\alpha}_{jt}}\frac{\tilde{c}_{jt}}{Y_{it}}$ where $\tilde{\alpha}_{jt} \equiv \frac{\tilde{\rho}_{jt}}{\overline{\rho}}$. Substituting these into the above expression for $\frac{\partial \mu_t}{\partial Y_t}$ gives $\frac{\partial \mu_t}{\partial Y_t} = \frac{\left(-\tilde{\rho}_{jt}\frac{\overline{\rho}}{\tilde{\rho}_{jt}}\frac{\tilde{c}_{jt}}{Y_t}\frac{Y_t}{\tilde{c}_{jt}} + 1\right)U'_{jt}(\tilde{c}_{jt})}{\beta \cdot E_t\left[\frac{U'_{j,t+1}(\tilde{c}_{j,t+1})Y_{t+1}}{N_{t+1}}\right]} = \frac{(-\overline{\rho}+1)U'_{jt}(\tilde{c}_{jt})}{\beta \cdot E_t\left[\frac{U'_{j,t+1}(\tilde{c}_{j,t+1})Y_{t+1}}{N_{t+1}}\right]}$. Since $U'_{jt}(\tilde{c}_{jt})$>0 and the denominator is greater than zero, then how μ_t changes with Y_t depends on how $\overline{\rho}$ compares to one. If $\overline{\rho}$<1, then $\frac{\partial \mu_t}{\partial Y_t}$>0. If $\overline{\rho}$=1, then $\frac{\partial \mu_t}{\partial Y_t}$=0. If $\overline{\rho}$>1, then $\frac{\partial \mu_t}{\partial Y_t}$<0. Q.E.D

This proposition shows that as long as the average relative risk aversion for the whole economy is one, changes in real aggregate supply cannot affect the N-Fisher coefficient. By the Consumption-Aggregate-Supply Invariance Property, the consumption allocation will not change unless the level of real aggregate supply changes. Therefore, when the average relative risk aversion equals one,[12] the N-Fisher coefficient will be stable and the central bank will be able to perfectly meet its nominal-aggregate-demand targets as long as no utility shocks occur.

When the average relative risk aversion of the whole economy exceeds one or is less than one, then changes in real aggregate supply can affect the N-Fisher coefficient. Under these assumptions of non-unity average relative risk aversion, uncertainty concerning real aggregate supply will be reflected in instability of the N-Fisher coefficient. Even when average relative risk aversion is non-unity, the only reason for instability in the N-Fisher coefficient is because of uncertainty in real aggregate supply. Again, if real aggregate supply does not change and no utility shocks occur, then the consumption allocation cannot change as long as markets are complete (See the Consumption-Aggregate-Supply Invariance Property).

We therefore conclude that if markets are complete, then the only sources of instability to the N-Fisher coefficient μ_t will be due either to potential utility shocks or changes in the real aggregate supply. Also, potential changes in real aggregate supply can contribute to the instability to the N-Fisher coefficient only if the average relative risk aversion differs from one. This analysis then indicates a strong theoretical argument in favor of the possibility that the N-Fisher coefficient could be relatively stable, which would mean that a central bank targeting nominal aggregate demand will be very successful.

P-Fisher Coefficient Variance Proposition: For the P-Fisher coefficient $\tilde{\mu}_{jt}$, $\frac{\partial \tilde{\mu}_{jt}}{\partial Y_t} < 0$.

[12] The relative risk aversion associated with the logarithmic utility function is one.

Proof: By (2) $\tilde{\mu}_t \equiv \dfrac{U'_{jt}(c_{jt})}{\beta \cdot E_t\left[\dfrac{U'_{j,t+1}(c_{j,t+1})}{P_{t+1}}\right]}$. Taking the partial derivative of the P-Fisher constant with respect to real aggregate supply gives $\dfrac{\partial \tilde{\mu}_t}{\partial Y_t} \equiv \dfrac{U''_{jt}(c_{jt})}{\beta \cdot E_t\left[\dfrac{U'_{j,t+1}(c_{j,t+1})}{P_{t+1}}\right]}$. Since $U''_{jt}(c_{jt}) < 0$, and the denominator is positive, the result follows. Q.E.D.

This proposition shows that uncertainty concerning real aggregate supply will be reflected as instability in the P-Fisher coefficient. However, the Consumption-Aggregate-Supply Invariance proposition still implies that only changes in real aggregate supply or utility shocks can cause the instability in this coefficient. Hence, the only reasons for the central bank not meeting its price target is because of potential changes in real aggregate supply or utility shocks.

VII. Conclusions and Reflections

This paper discussed both theoretical and empirical issues related to central banks using either an interest-rate or money-supply instrument to pursue either price-level targeting or nominal-aggregate-demand (or nominal-income) targeting. When prices are determined, this analysis applies. Whether or not a central bank will do better setting the money supply or pegging the interest rate depends on the stability of velocity compared to the stability of the P-Fisher coefficient in case of price-level targeting or the N-Fisher coefficient in the case of nominal-income targeting. We found that when markets are complete, the only possible sources of instability in the P-Fisher and N-Fisher coefficients are possible utility shocks or possible changes in real aggregate supply. If average relative risk aversion equals one, then even possible changes in real aggregate supply will not affect the stability of the N-Fisher coefficient and hence will not interfere with the central bank pursuing nominal-aggregate-demand targeting.

However, regardless of the value of average relative risk aversion, changes in real aggregate supply will affect the stability of the P-Fisher coefficient and hence will interfere with the central bank pursing a price-level target.

Empirically, we find that in the 1970s, setting the money supply would have resulted with better success at both price-level targeting and nominal-aggregate-demand targeting. However, after 1982, pegging the interest rate would have resulted with better success, especially so for price-level targeting.

Some side notes of this paper are also important to note. In particular, we found that when markets are complete, consumers only will enter into nominal contracts to hedge against their other nominal contracts. Because nominal contracts expose their parties to nominal risk, other securities will dominate nominal contracts except for nominal hedging purposes.

A second very important side note has to do with complete markets and the role of monetary policy. If markets are complete, then monetary policy can not make Pareto improvements to the economy since complete markets mean we already have Pareto efficiency. Also, if we agree that the role for monetary policy should be to make Pareto improvements in the economy, then that role for monetary policy must be to help complete markets. Thus, the monetary economics literature should follow the lead of Eagle (2005a) and Eagle and Domian (2005a and 2005b) to see how monetary policy can contribute to complete markets. Some monetary economists argue that we should let other financial markets complete the markets so that monetary policy should be left to handle what its true role should be. Such a belief makes no sense since any Pareto-improving role for monetary policy must involve helping completing markets.

References

David Eagle (2005a). "Completing Markets in a One-Good, Pure Exchange Economy Without State-Contingent Securities," Finance 0501009, Economics Working Paper Archive at WUSTL.

David Eagle (2005b). "Price Indeterminacy Reinvented: Pegging Interest Rates While Targeting Prices, Inflation, or Nominal Income," Macroeconomics 0501028, Economics Working Paper Archive at WUSTL.

David Eagle & Elizabeth Murff (2005). "Logical Pitfalls of Assuming Bounded Solutions to Expectational Difference Equations," GE, Growth, Math methods 0501002, Economics Working Paper Archive at WUSTL.

David Eagle & Dale Domian (2003a). "Sounding the Alarm on Inflation Indexing and Strict Inflation Targeting," Macroeconomics 0312010, Economics Working Paper Archive at WUSTL.

David Eagle & Dale Domian, (2003b). "Quasi-Real Indexing -- The Pareto-Efficient Solution to Inflation Indexing," Macroeconomics 0312012, Economics Working Paper Archive at WUSTL, revised 31 Dec 2003.

Lucas, Robert E. (1976). "Econometric Policy Evaluation: A Critique," *Carnegie-Rochester Conference Series on Public Policy* 1, 19-46.

Lucas, Robert E. (1982). "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics*, 10(3), Nov. 1982, pp. 335-359.

McCallam, Bennett T. (1981), "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations," Journal of Monetary Economics, 8:319-329.

Poole, William, (1970). "Optimal Choice of Monetary Policy Instrument in a Simple Stochastic Macro Model," *Quarter Journal of Economics*, 84(#2):197-216.

Sargent, Thomas J. (1979). *Macroeconomic Theory – Economic Theory, Econometrics, and Mathematical Economics* (Academic Press – New York).

Sargent, Thomas J. (1986). *Dynamic Macroeconomic Theory* (Harvard University Press – Cambridge, Massachusetts).

Sargent, Thomas J. and Neil Wallace (1975). "'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy* 83:241-254.

Woodford, Michael (2003). *Interest and Prices – A Foundation of a Theory of Monetary Policy* (Princeton University Press – Princeton, New Jersey).