Optimal taxation with commitment in a two-sector neoclassical economy.

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Abstract:

This paper examines dynamic optimal income taxation problem in a two-sector neoclassical model where the government is able to commit to a sequence of tax plans for future. It finds that (1) while it is optimal to set a zero long run capital tax for the capital goods sector, steady state optimal capital tax can be nonzero in the consumption goods sector; (2) if the government faces an *ex ante* constraint of setting equal factor income taxes, the optimal levels of both capital tax rates are nonzero. The distortion created by the nonzero capital tax in consumption goods sector, given the other capital tax is set at zero, is in no way explosive in nature, since economic agents can avoid the compounding tax liabilities simply by shifting depreciated capital. The paper examines the optimal steady state capital tax in consumption goods sector with three popular classes of utility functions and finds that the set of conditions under which this tax is zero is in no way inferred by the model.

Keywords: Optimal taxation, *Ramsey* problem, Primal approach, Two-sector model.

***JEL* classification code:** C61, E13, E62, H21.

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1. Introduction.

The dynamic general equilibrium approach to the optimal taxation problem established in literature follows Ramsey's (1927) seminal paper that formally recognized that consumers and firms react to changes in fiscal policy. In addressing the issue of optimal choice of tax rates, the seminal work of Ramsey (1927) characterized the optimal levels for a system of excise taxes on consumption goods assuming that the government's objective was to choose these taxes to maximize social welfare subject to a set of constraints it faced. The set of constraints comprises of preset revenue target of the government and the economic agents' competitive equilibrium reaction to the tax policy. Each optimal tax plan from a standard *Ramsey* model therefore implies a feasible allocation of factor services and goods along with prices that fully reflects the optimal reaction of consumers and firms.

Literature on optimal taxation of factor income in dynamic settings with either infinitely-lived agents or overlapping generations, ever since its advancement and sophistication, has established a set of celebrated substantive results. In the context of standard neoclassical growth model with infinitely-lived individuals, Chamley (1986) and Judd (1985) establish that an optimal income-tax policy entails taxing capital at confiscatory rates in the short run and setting capital income taxes equal to zero in the long run. This result is judicious since a positive tax on the return from today's savings effectively makes consumption next period more expensive relative to consumption in the current period. In an infinitely-lived agent's model, therefore, a positive tax on capital income in the steady state implies that the implicit tax rate of consumption in future has an unbounded increasing trend. A current period nonzero tax on capital income therefore implies explosive distortions on future periods, which is unlike the uniform distortion created by period by period labor income tax, for instance. Chamley (1986) shows that with a steady state result of zero capital taxation in the scheme, it is possible for the government to announce high tax on capital income in period *0*. With no exogenous bounds on the magnitude of tax rates, this initial high tax rate on capital income may even be confiscatory.

While Chamley's (1986) approach to the problem was solving the programming problem period by period, an alternative approach, popularly known as the *primal* approach allows one to address the same problem with time *0* trading decisions (see for instance, Jones *et. al* (1997), Benhabib & Rustichini (1997), and Chari and Kehoe (1999)). This approach characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two simple conditions: a resource constraint and an *implementability* constraint. With both constraints depending only on allocation, this characterization implies that optimal allocations are solutions to a simple programming problem. The relative ascendancy of the *primal* approach, as it may appear, is that it allows direct characterization of the capital income tax rates instead of relying on interpretation of a set of shadow prices of constraints.

The current paper approaches the standard *Ramsey* problem using the *primal* approach in a dynamic general equilibrium set up of a two-sector model economy with demarcated features. It addresses the issue of optimal choice of capital income tax rates in a standard exogenous growth two-sector real business cycle model. The key finding is while it is optimal to tax capital income from capital producing sector at a zero rate, the steady state capital income tax for consumption goods sector may be nonzero and sustainable in a *Ramsey* equilibrium. Such a capital tax would not have potential compounding distortion effect, since economic agents have the option of shifting depreciated capital good to the sector where its income is untaxed and thus avoid the compounding tax liabilities. The set of policies at the government's disposal for which competitive equilibrium exists therefore implies conditional convergence of zero steady state capital income tax for the consumption goods sector. The paper presents some specific functional forms of utility functions and identifies the set of conditions for which the characterized steady state capital income tax for consumption goods sector converges to zero. Neither the model nor a specific economic intuition guarantees the fulfilment of these conditions. Hence the celebrated Chamley-Judd result of zero steady state tax on capital income cannot be unconditionally generalized for a class of neoclassical models.

The optimal taxation literature also includes significant contributions by Rebelo (1991), Jones & Manuelli (1992), Jones *et. al* (1993), and Ortigueira (1998), which examine endogenous growth models with taxes on physical and human capital. Empirical studies of welfare effects due to distortionary taxation under the dynamic general equilibrium setting for infinitely-lived agents are evolving encouragingly, a good example being Coleman II (2000)[1]. These studies, however, differ greatly in both the models that they analyze and the types of fiscal experiments undertaken. No matter how diverse the modelling approaches and spotlights of these studies are, three major findings emerge from the solutions to the *Ramsey* problem in representative infinitely-lived agent models. The famous paper by Chari & Kehoe (1999) presents a comprehensive survey of these findings in variants of environments. Of these, the current paper's focus is on the one that states capital income should not be taxed in the long run. The second interesting finding as may be found in Chari & Kehoe (1999) is that tax rates on labor income may be nonzero in the limit but should be roughly constant, and in no circumstances should be confiscatory. The third motivating finding from the literature on optimal taxation in infinitely-lived agent models stems directly from the time inconsistency of optimal policies. The fact that an optimal capital income tax plan can be time inconsistent was established seminally by Kydland & Prescott (1977). There has been a marked enthusiasm in relatively recent literature on political economy addressing such issues which are relevant to public policy choice of governments (see for instance, Persson *et. al* (1987), Chari & Kehoe (1990), Stokey (1991), Benhabib & Rustichini (1997) and Phelan & Stacchetti (2001)). For the current paper, the time inconsistency problem of optimal plans are suppressed by assuming that the society, at its disposal, has an effective commitment device, or a commitment technology, with which it can bind the government to continue with its initially announced tax plans.

The current paper proposes a general framework of a neoclassical model with infinitely-lived agents with identical preferences over consumption and labor supply, two factors of production which are labor and physical capital, two production sectors producing perishable consumption goods and new capital goods, and a benevolent government with preset revenue target to finance its consumption, linear income-tax instruments to furnish the expenditure and an effective *commitment* device to restrict itself from changing initially announced policies. It follows the primal approach to optimal taxation and examines the steady state properties of the optimal capital tax rules. In addition, it considers the case where the government faces an *ex ante* constraint of keeping the two labor income tax rates and the two capital income tax rates equal, i.e the constraint that factor income tax rates are not sector-specific. The reason why this experiment is important is as follows. Without such a constraint, the benchmark model prescribes that two different steady state capital income tax rates can be sustained in a *Ramsey* equilibrium, one of which should be kept zero in the long run. The key intuition behind this result is that a nonzero tax on capital income from consumption goods

[1] Ortigueira's (1998) study of transitional factor income taxation in an endogenous growth model involved numerical calibration of the benchmark economy to deduce, among others, welfare cost of distortionary taxation.

sector is not potentially explosive in creating distortions as long as the other capital tax instrument is set at zero. This is because since shifting capital is costless, households can shift the depreciated capital at end of each period to the sector where capital income is not taxed (capital goods sector) and avoid the compounding liabilities of capital tax. The nonzero capital income tax in the consumption goods sector becomes, in terms of consequences, a tax which has uniform distortion pattern, similar to a period by period consumption tax, for example. While this is theoretically proven to be the optimal sustainable choice of capital income tax rates for the government, it may be subject to criticism from a real world point of view. Governments in the real world often face the constraint of keeping income tax rates same irrespective of production and investment sectors. When such a constraint is imposed in the *Ramsey* problem, it induces a relatively more inefficient fiscal policy outcome. Restricting the government's choice of income taxes *ex ante* triggers an inefficient outcome with both nonzero capital income tax rates. Since capital tax compounds over time, this outcome cannot be sustained in a *Ramsey* equilibrium.

The set of policies which generates allocations that can be implemented as competitive equilibrium, as this paper advocates, prescribes that the optimal steady state capital income tax for capital goods sector is unambiguously zero, but the steady state optimal capital income tax for consumption goods sector is only conditionally zero. The set of conditions for which the celebrated Chamley-Judd result can be established, as characterized in three experiments using variants of utility functions, are neither inferred by the model nor justified by simple intuitions. In general, the steady state optimal capital tax for consumption goods sector can therefore be nonzero, and non-explosive in distortions. This result holds for a variant of commonly used utility functions with desirable properties, irrespective of separability and marginal rate of substitution of labor across sectors, intratemporal labor adjustment costs and varying types of labor. The three examples considered in this paper are in the spirits of Herrendorf & Valentinyi (2003), Huffman & Wynne (1999) and Jones *et. al* (1997), among others.

The paper, therefore, belongs more to the tradition of Jones *et. al* (1997) and Chari & Kehoe (1999), and is intended to complement the same dynasty. The remainder of the paper is organized as follows. Section *2* presents the underlying two-sector model, the representative household's and firms' problems and their solutions, and equilibrium definitions. Section *3* addresses the *Ramsey* problem using the *primal* approach and derives the optimal capital income tax rules. It also presents the constrained tax choice experiment and examines the optimal initial capital income taxes. Section *4* presents examples of utility functions and thereby characterizes the optimal steady state capital income tax rates. Section *5* concludes.

2.0 The economy.

To my knowledge, the prototype version of the two-sector neoclassical model was primarily proposed by Uzawa (1963) and Srinivasan (1964) to examine growth process and stability properties of the balanced growth equilibria. These studies considered standard neoclassical framework with two factors of production simultaneously in operation in two production sectors that produce perishable consumption goods and new capital goods, and focused on growth and equilibrium properties with varying factor intensities. Taxation under multi-sector neoclassical models of endogenous growth was examined primarily by Rebelo (1991), Jones *et al.* (1993) and Stokey & Rebelo (1995), followed by contributions such as Jones *et al.* (1997) which introduce a labor-leisure choice and a human capital accumulation process.

In this paper, the following dynamic general equilibrium environment is considered. Time is discrete and runs forever. The economy has two production sectors indexed by j, where $j = C, X$ denotes the consumption goods and capital goods sector, producing perishable consumption goods and new capital goods, respectively. There is a continua of measure one of identical, infinitely lived households, of identical firms in sector C that own a technology with which a perishable consumption good (c) and exogenously determined government consumption goods (g) can be produced, and of identical firms in sector X that own a technology with which new capital goods, x_c for the consumption producing sector and x_x for the capital producing sector, can be produced. The representative household is endowed with initial capital stock, with the property rights of the representative firms, and with one unit of time at each instant. Firms combine capital and labor, the two factors of production, for final production. At each point in time, five commodities are traded in sequential markets: the consumption goods, two new capital goods suitable for the production of consumption goods and new capital goods, working time in sector C, and working time in sector X.

All households have identical preferences over intertemporal consumption and labor services. The representative household derives utility from consumption (c_t) and disutility from effort given in terms of labor units in the two sectors of production (n_{ct} and n_{xt} in sectors C and X, respectively) at all time t, such that household's preferences for consumption and labor service streams $\{c_t, n_{ct}, n_{xt}\}_{t=0}^{\infty}$, can be defined by the utility function over infinite horizon:

$$\mathbf{U}(c_0, c_1, \ldots\ldots, n_{c0}, n_{c1}, \ldots\ldots, n_{x0}, n_{x1}, \ldots) = \sum_{t=0}^{\infty} \beta^t \, \mathbf{u}(c_t, n_{ct}, n_{xt}) \qquad (1)$$

where the subjective discount rate is β and $\beta \in (0,1)$.

Assumption 1: The current period utility function $\mathbf{u}:\mathbf{R}_{+}^{3} \rightarrow \mathbf{R}$ is bounded, continuously differentiable, strictly increasing in consumption (*c*), decreasing in labor (n_c and n_x), strictly concave, and satisfies Inada conditions, namely:

$$\lim_{c_t \rightarrow 0} \frac{\mathbf{u}_c(t)}{\mathbf{u}_{n_j}(t)} = \infty, \quad \text{and} \qquad \lim_{c_t \rightarrow \infty} \frac{\mathbf{u}_c(t)}{\mathbf{u}_{n_j}(t)} = 0,$$

for $n_j > 0$ where $j = C, X$. □

The household purchases new capital goods and rents capital to the firms for one period. Capital decays at the fixed rate $\delta \in (0,1)$. Firms return the rented capital stock next period net of depreciation δ, and pay unit cost of capital employed r_c and r_x, for capital stock employed in sector *C* and *X*, respectively[2]. Firms own nothing; they hire labor and capital on a rental basis, sell the output produced back to households, and return profits to shareholders. The technology for the representative firm in sector *C* producing (private) consumption good c_t and government consumption good g_t for all time *t* is:

$$c_t + g_t \leq \mathbf{F}^{\mathbf{c}}(k_{ct}, n_{ct}) \qquad (2.1)$$

and the technology for the representative firm in sector *X* producing new capital goods x_{ct} and x_{xt} for all time *t* is:

$$x_{ct} + x_{xt} \leq \mathbf{F}^{x}(k_{xt}, n_{xt}) \qquad (2.2)$$

Assumption 2: For sector *j*, with *j* = *C*, *X*, the technology $\mathbf{F}^{j}(k_{jt}, n_{jt})$ exhibits Constant Returns to Scale (CRTS), with $\mathbf{F}^{\mathbf{j}}:\mathbf{R}_{+}^{2} \rightarrow \mathbf{R}_{+}$ continuously differentiable, strictly increasing, strictly concave in both *k* and *n*, and satisfies Inada conditions, namely:

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[2] For simplicity, the depreciation rate is assumed to be sector-indifferent, although this assumption can easily be relaxed with two different constant depreciation rates δ_c and δ_x (see for instance Stokey & Rebelo (1995), and Herrendorf & Valentinyi (2003)), yielding zero marginal benefit to the analysis but incurring the cost of notational clutter.

$\lim_{k_{jt}\to 0}\mathrm{F}^{j}_{kj}(t)=\infty$ and $\lim_{k_{jt}\to\infty}\mathrm{F}^{j}_{kj}(t)=0$ for all $n_j > 0$. □

The level of government expenditures is considered to be given, and the expenditures program is assumed to converge to a constant level when time goes to infinity. The government finances the exogenous stream of consumption expenditures $\{g_t\}_{t=0}^{\infty}$ solely by linearly taxing income from capital and labor employed in both sectors. Throughout the paper, the assumption that the government has access to some commitment device, or a *commitment technology* that allows the government to commit itself once and for all to the sequence of tax rates announced at time *0*, is maintained. In other words, the commitment technology prevents the government from revising the path of fiscal instruments over time. This assumption allows one to avoid the general time inconsistency problem of optimal policies in dynamic settings. The benevolent government therefore is assumed to seek a tax system that provides revenues to finance g_t and to maximize household's welfare defined by *(1)*.

The government taxes labor income and capital income from sector j, with $j = C, X,$ at rates τ_t^j per unit and θ_t^j per unit, respectively. The government runs a balanced budget each period such that there are no government bonds in the economy. The set of analytical results this paper focuses on are insensitive to this assumption, which can be reconfirmed if one examines a one-sector bond economy analogue (see for instance, Chamley (1986) and Ljungqvist & Sargent (2000)). The government's budget constraint for all time t can be written as:

$$g_t = \tau_t^c w_{ct} n_{ct} + \tau_t^x w_{xt} n_{xt} + \theta_t^c r_{ct} k_{ct} + \theta_t^x r_{xt} k_{xt} \qquad (3)$$

where w_{jt} is the before tax return on per unit labor employed and r_{jt} is the before tax return on per unit capital employed in sector j, with $j = C, X$. In the first period, the government announces the program of tax rates and its expenditures. The representative household and firms are endowed with perfect foresight and behave competitively. Under the assumption of commitment technology, the announced program of tax rates cannot be changed at a later date.

2.1 Representative household's problem and its solution.

For the remainder of the paper, the consumption good will be treated as the numeraire. Let p_{jt} denote the relative price of a new capital good to be used in sector j and π_{jt} be the profits of representative firms in sector j, with $j = C, X$. The representative household's problem can be illustrated as program (4), as follows:

$$\max_{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}} \sum_{t=0}^{\infty} \beta^t \mathbf{u}(c_t, n_{ct}, n_{xt}) \tag{4.1}$$

$s.t.$

$$c_t + p_{ct}x_{ct} + p_{xt}x_{xt} \le \pi_{ct} + p_{xt}\pi_{xt} + (1-\theta_t^c)r_{ct}k_{ct} + (1-\theta_t^x)r_{xt}k_{xt} + (1-\tau_t^c)w_{ct}n_{ct} + (1-\tau_t^x)w_{xt}n_{xt} \tag{4.2}$$

$$\Delta k_{ct} = x_{ct} - \delta k_{ct} \tag{4.3}$$

$$\Delta k_{xt} = x_{xt} - \delta k_{xt} \tag{4.4}$$

$$k_{c0} > 0 \quad (given), \qquad k_{x0} > 0 \quad (given) \tag{4.5}$$

With CRTS technology in both sectors, competitive equilibrium profits are zero (and will be ignored in household's budget constraint hereafter). Using *(4.3)* and *(4.4)* to substitute for x_{ct} and x_{xt}, and defining $R_t^c \equiv [p_{ct}^{-1}(1-\theta_t^c)r_{ct} + (1-\delta)]$ and $R_t^x \equiv [p_{xt}^{-1}(1-\theta_t^x)r_{xt} + (1-\delta)]$, the household's budget constraint can be rewritten as:

$$c_t + p_{ct}k_{ct+1} + p_{xt}k_{xt+1} \le (1-\tau_t^c)w_{ct}n_{ct} + (1-\tau_t^x)w_{xt}n_{xt} + p_{ct}k_{ct}R_t^c + p_{xt}k_{xt}R_t^x \tag{4.2a}$$

The representative household's problem can now be illustrated as the problem of maximizing utility subject to *(4.2a)* and *(4.5)*. With $\beta^t \lambda_t$ as the Lagrange multiplier on time t budget constraint, the necessary conditions for the household's maximization problem are the period budget constraints *(4.2a)* along with the followings:

$$c_t \quad : \quad \mathbf{u}_c(t) = \lambda_t \tag{5.1}$$

$$n_{ct} \quad : \quad \mathbf{u}_{nc}(t) = -\lambda_t(1-\tau_t^c)w_{ct} \tag{5.2}$$

$$n_{xt} \quad : \quad \mathbf{u}_{nx}(t) = -\lambda_t(1-\tau_t^x)w_{xt} \tag{5.3}$$

$$k_{ct+1} \quad : \quad \frac{\lambda_t}{\lambda_{t+1}} = \beta \frac{p_{ct+1}}{p_{ct}} R_{t+1}^c \tag{5.4}$$

$$k_{xt+1} \quad : \quad \frac{\lambda_t}{\lambda_{t+1}} = \beta \frac{p_{xt+1}}{p_{xt}} R_{t+1}^x \tag{5.5}$$

and the Transversality conditions that put a restriction on the terminal value of the household's capital stocks in terms of utility:

$$\lim_{t\to\infty}[\lambda_t p_{jt} k_{jt+1}] = 0 \qquad \text{for } j = C, X. \qquad (5.6a)$$

The Transversality condition implies that the discounted lifetime utility is maximal when the terminal value of the capital stock in sector *j*, with *j = C, X,* is zero. For $u_c(0) > 0$ and using *(5.1)* and *(5.6a)*, the transversality conditions may be restated as

$$\lim_{t\to\infty} \frac{k_{ct+1}}{\Pi_{s=1}^{t} R_s^c} = 0 \qquad \text{and} \qquad \lim_{t\to\infty} \frac{k_{xt+1}}{\Pi_{s=1}^{t} R_s^x} = 0 \qquad (5.6b)$$

(5.6b) states that for an optimal consumption allocation the present discounted value of the household's capital stock must be zero as time goes to infinity. Consolidating the necessary conditions yields:

$$\frac{u_c(t)}{\beta u_c(t+1)}\left(\frac{p_{ct}}{p_{ct+1}}\right) = R_{t+1}^c \qquad (5.4a)$$
$$\frac{u_c(t)}{\beta u_c(t+1)}\left(\frac{p_{xt}}{p_{xt+1}}\right) = R_{t+1}^x \qquad (5.5a)$$

such that

$$\frac{R_{t+1}^c}{R_{t+1}^x} = \frac{p_{ct}}{p_{xt}}\left(\frac{p_{xt+1}}{p_{ct+1}}\right) \qquad (5.7)$$

$$\mathbf{u}_{nc}(t) = -\mathbf{u}_c(t)(1-\tau_t^c)w_{ct} \qquad (5.2a)$$
$$\mathbf{u}_{nx}(t) = -\mathbf{u}_c(t)(1-\tau_t^x)w_{xt} \qquad (5.3a)$$

such that

$$\frac{\mathbf{u}_{nc}(t)}{\mathbf{u}_{nx}(t)} = \frac{(1-\tau_t^c)w_{ct}}{(1-\tau_t^x)w_{xt}} \qquad (5.8)$$

Condition *(5.7)* combines the two Euler equations (*5.4a & b*) from the solutions to the household's maximization problem. The Euler equations state that household's one period ahead capital stock decision that maximizes utility is determined at the point where the household is indifferent between consuming today and saving for a later date. Condition *(5.8)* implies that the representative household will maximize its utility at the point where its marginal rate of substitution of labor across sectors is equal to the relative after tax wage rate of the two sectors.

2.2 Firms' problems and their solution.

Since the representative household's preferences are strictly monotone and all factors have strictly positive marginal products, $p_{jt} > 0$, $r_{jt} > 0$, and $w_{jt} > 0$, for all time *t*, for sector $j = C, X$. In sector *C*, problem of the representative firm producing private consumption goods and government consumption goods is:

$$\max_{c_t, k_{ct}, n_{ct}} \quad \pi_{ct} \equiv c_t + g_t - r_{ct}k_{ct} - w_{ct}n_{ct} \tag{6.1}$$

$$s.t. \quad c_t + g_t \le \mathrm{F}^c(k_{ct}, n_{ct}) \tag{6.2}$$

$$0 \le c_t, g_t, k_{ct}, n_{ct} \tag{6.3}$$

$$g_t = \overline{g}_t \tag{6.4}$$

Competitive pricing ensures that returns are equal to their marginal products. The necessary and sufficient conditions for the maximization problem are, therefore:

$$r_{ct} = \mathrm{F}^c_{kc}(t) \tag{6.5a}$$

$$w_{ct} = \mathrm{F}^c_{nc}(t) \tag{6.5b}$$

In sector *X*, problem of the representative firm producing new capital goods is:

$$\max_{x_{ct}, x_{xt}, n_{xt}, k_{xt}} \quad \pi_{xt} \equiv p_{ct}x_{ct} + p_{xt}x_{xt} - r_{xt}k_{xt} - w_{xt}n_{xt} \tag{7.1}$$

$$s.t. \quad x_{ct} + x_{xt} \le \mathrm{F}^x(k_{xt}, n_{xt}) \tag{7.2}$$

$$0 \le x_{ct}, x_{xt}, k_{xt}, n_{xt} \tag{7.3}$$

With competitive pricing, and for ℓ_t as the Lagrange multiplier associated with the problem, necessary and sufficient conditions for the maximization problem are:

$$p_{ct} = p_{xt} = \ell_t \qquad (7.4a)$$
$$r_{xt} = \ell_t \, \mathrm{F}^x_{kx}(t) \qquad (7.4b)$$
$$w_{xt} = \ell_t \, \mathrm{F}^x_{nx}(t) \qquad (7.4c)$$

Accordingly, I will simplify the model by denoting $p_{ct} = p_{xt} = p_t$ hereafter. Hence for both firms, inputs should be employed until the marginal revenue product of the last unit is equal to its rental price.

2.3 Competitive equilibrium.

For definitions in this subsection, symbols without time subscripts denote the one-sided infinite sequence for the corresponding variables, e.g. $n_c \equiv \{n_{ct}\}_{t=0}^{\infty}$.

Definition 2.3.1 (Competitive Equilibrium): A *competitive equilibrium* is an allocation *(c, g, n_c, n_x, x_c, x_x, k_c, k_x)*, a price system *(w_c, w_x, r_c, r_x, p)*, and a government policy $(\tau^c, \tau^x, \theta^c, \theta^x)$ such that

(a) Given the price system and the government policy, the allocation *(c, n_c, n_x, x_c, x_x, k_c, k_x)* solves the problem of the representative household.

(b) Given the price system, the allocation *(c, g, n_c, k_c)* solves the problem of the representative firm in sector *C*.

(c) Given the price system, the allocation *(x_c, x_x, n_x, k_x)* solves the problem of the representative firm in sector *X*.

(d) The markets clear, i.e. the two resource constraint defined by *(2.1)* and *(2.2)* hold simultaneously. □

Note that the government budget constraint did not appear in the definition *2.3.1*. Given the assumption about the utility function, the household's budget constraint is satisfied with equality in equilibrium. The government policy, the household's budget constraint and the two resource constraint defined by *(2.1)* and *(2.2)* imply that the government budget constraint *(3)* holds in equilibrium.

Given total time endowment at each instant for the household, define $\Im : \mathrm{R}^2_+ \rightarrow \mathrm{R}$ with $\Im$ (strictly) convex, such that the total time allocation constraint can be written as

$\Im(n_{ct}, n_{xt}) \le 1$. For (strict) convexity of the function $\Im : \mathrm{R}_{+}^{2} \to \mathrm{R}$, imposing separability, the household's utility function is (non) linear in labor. Combining the necessary conditions derived from the representative household's problem, the necessary conditions derived from the firms' problems, the resource and time allocation constraints, it can be shown that the (competitive) equilibrium dynamics is characterized by the Transversality conditions together with the following system of equations in the set of unknowns $\{c_t, k_{ct}, k_{xt}, n_{ct}, n_{xt}, r_{ct}, r_{xt}, w_{ct}, w_{xt}, p_t, \tau_t^c, \tau_t^x, \theta_t^c, \theta_t^x\}$:

$$\Im(n_{ct}, n_{xt}) \le 1 \tag{7.5a}$$

$$c_t + \bar{g}_t = \mathbf{F}^{\mathbf{c}}(k_{ct}, n_{ct}) \tag{7.5b}$$

$$x_{ct} + x_{xt} = \mathbf{F}^{x}(k_{xt}, n_{xt}) \tag{7.5c}$$

$$x_{ct} = k_{ct+1} - (1-\delta)k_{ct} \tag{7.5d}$$

$$x_{xt} = k_{xt+1} - (1-\delta)k_{xt} \tag{7.5e}$$

$$\mathbf{u}_{nc}(t) = -\mathbf{u}_{c}(t)(1-\tau_t^c)w_{ct} \tag{7.5f}$$

$$\mathbf{u}_{nx}(t) = -\mathbf{u}_{c}(t)(1-\tau_t^x)w_{xt} \tag{7.5g}$$

$$\frac{\mathrm{u}_c(t)}{\mathrm{u}_c(t+1)} = \frac{\beta p_{t+1}}{p_t} R_{t+1}^{c} \tag{7.5h}$$

$$\frac{\mathrm{u}_c(t)}{\mathrm{u}_c(t+1)} = \frac{\beta p_{t+1}}{p_t} R_{t+1}^{x} \tag{7.5i}$$

$$r_{ct} = \mathrm{F}_{kc}^{c}(t) \tag{7.5j}$$

$$w_{ct} = \mathbf{F}_{nc}^{c}(t) \tag{7.5k}$$

$$r_{xt} = p_t \mathrm{F}_{kx}^{x}(t) \tag{7.5l}$$

$$w_{xt} = p_t \mathbf{F}_{nx}^{x}(t) \tag{7.5m}$$

Equation *(7.5a)* represents the time allocation constraint. Equations *(7.5b)* and *(7.5c)* represent goods market clearing conditions. The next two are laws of motion for capital. The rest of the equations are the set of equilibrium conditions derived from household's and firms' optimization problems. A few observations deserve attention here. Note *(7.5h)* and *(7.5i)* together imply that after tax returns from capital are equal in a competitive equilibrium, which does not necessarily imply that pre-tax returns are equal. Note also that with *(7.5f)* and *(7.5g)*, a non-unitary marginal rate of substitution of labor across sectors would imply that after tax wage rates are not equal in equilibrium.

There are many competitive equilibria, indexed by different government policies. The multiplicity of competitive equilibrium motivates the *Ramsey* problem, defined as follows.

Definition 2.3.2 (*Ramsey* problem): Given the time *0* (initial) endowments of capital stocks, k_{c0} and k_{x0}, and the preset revenue target {g}, the *Ramsey* problem is to choose a competitive equilibrium that maximizes expression *(1)*. □

For a given welfare criterion, which the government uses to evaluate different allocations, the *Ramsey* problem for the government, therefore is to pick the fiscal policy (or one of them if there are many) that generates the competitive equilibrium allocation giving the highest value of the welfare criterion. This way of formulating the *Ramsey* problem was examined for a one-sector neoclassical model by Chamley (1986). An equivalent way of formulating the *Ramsey* problem is to let the government pick an allocation directly (rather than a set of taxes). However, the set of allocations from which the government is allowed to choose is restricted by the *implementability* constraints. Under any arbitrarily chosen fiscal policy, the optimal behaviour of the representative household and firms generates a competitive equilibrium allocation, which is one element in the set of allocations from which the government can choose. The literature usually refers to such an allocation as an *implementable* allocation. To implement this particular allocation as a competitive equilibrium, the government needs to choose the fiscal policy that generated it. The *Ramsey* problem with implementability constraint, therefore consists of choosing among all *implementable* allocations, the one that maximizes a welfare criterion. This way of formulating the *Ramsey* problem is commonly known as the *primal* approach (see for instance, Jones *et. al* (1997), Chari & Kehoe (1999), and Erosa & Gervais (2001)).

3.0 The *Ramsey* problem.

In the *primal* approach to the *Ramsey* problem, the government can be thought of as directly choosing a feasible allocation, subject to constraints that ensure the existence of prices and taxes such that the chosen allocation is consistent with the optimization behaviour of household and firms. This approach to the *Ramsey* problem was primarily used in Lucas & Stokey's (1983) analysis of an economy without capital. For a model economy with two (or more) factors comprising both physical and human capital, Jones *et. al* (1997) applies the *primal* approach replacing the period household budget constraint with a present value household budget constraint to characterize a set of *implementability* constraints to be incorporated in the government's optimization problem. Chari & Kehoe (1999) uses the *primal* approach in a more comprehensive set up and shows parallel results for static and dynamic environments under a neoclassical model economy[3].

For the *primal* approach to the *Ramsey* problem, therefore, it becomes customary to introduce a single present-value budget constraint for either the government or the representative household (one of them is redundant since both resource constraints are imposed). It turns out that the problem simplifies nicely if one chooses the present-value budget constraint of the household, in which future capital stocks can be (algebraically) eliminated. Note that since firms' problems are equivalent to a series of one-period maximization problems, in equilibrium $R_t \equiv R_t^c = R_t^x$. Consider, therefore, household's time *T* budget constraint:

$$c_T - (1-\tau_T^c) w_{cT} n_{cT} - (1-\tau_T^x) w_{xT} n_{xT} \leq p_T R_T [k_{cT} + k_{xT}] - p_T [k_{cT+1} + k_{xT+1}] \qquad (8a)$$

Let $\prod_{s=1}^{0} R_s \equiv 1$ be the numeraire. Divide *(8a)* by the period *T* term $p_T \prod_{s=1}^{T} R_s$ and evaluate the resulting expression at time *T-1*. Then add these two and evaluate the resulting expression at time *T-2*. Iterating this procedure (and finally adding the time *0* expression) and taking the limit of both sides of the sum as $T \rightarrow \infty$ results in the following expression:

$$\sum_{t=0}^{\infty} \frac{c_t - (1-\tau_t^c) w_{ct} n_{ct} - (1-\tau_t^x) w_{xt} n_{xt}}{p_t \prod_{s=1}^{t} R_s} \leq R_0 [k_{c0} + k_{x0}] \qquad (8b)$$

Where $\lim_{t\rightarrow\infty} k_{jt+1} \left(\prod_{s=1}^{t} R_s \right)^{-1} = 0$ is already imposed since the present discounted value of the capital stock in sector *j*, *j* = *C, X*, in period *t* evaluated using period *t* market prices is asymptotically zero as $t \rightarrow \infty$. Expression *(8b)* is the household's present-value budget constraint, which delivers the interpretation that the present value of consumption expenditures net of (net) labor earnings cannot exceed the value of the net initial assets. Assume that *(8b)* binds, i.e. there are no unused resources in the limit. Define the Arrow-Debreu price, $q_t^o \equiv p_t^{-1} \left(\prod_{s=1}^{t} R_s \right)^{-1}$ such that *(8b)* becomes:

[3] Among others, Benhabib & Rustichini (1997) also adopts the *primal* approach to dynamic optimal taxation in an economy with no commitment power at the society's disposal.

$$\sum_{t=0}^{\infty} q_t^o c_t = \sum_{t=0}^{\infty} q_t^o (1-\tau_t^c) w_{ct} n_{ct} + \sum_{t=0}^{\infty} q_t^o (1-\tau_t^x) w_{xt} n_{xt} + R_0^c k_{c0} + R_0^x k_{x0} \qquad (9)$$

with $q_0^o = p_0^{-1}$.

A summary of the *primal* approach to the *Ramsey* problem is as follows. In the first step, the necessary conditions from the household's maximization problem are derived by maximizing the representative household's utility subject to *(9)*. Then, the representative firms' problems and corresponding necessary conditions are reconsidered (which are necessarily same as derived before). The set of these (competitive) equilibrium conditions are solved for prices and taxes $\{q_t^o, r_{ct}, r_{xt}, w_{ct}, w_{xt}, \tau_t^c, \tau_t^x, \theta_t^c, \theta_t^x\}_{t=0}^{\infty}$ as functions of the allocations $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$. When these expressions are substituted into the household's present-value budget constraint *(9)*, that gives an intertemporal budget constraint involving only the *implementable* allocations. The government maximizes welfare subject to the two resource constraints and the adjusted intertemporal budget constraint, and solution to this problem characterizes the *Ramsey* allocation[4]. Once the *Ramsey* allocations are characterized, one can solve for the *Ramsey* equilibrium taxes and prices.

With λ^p as the Lagrange multiplier on the household's present-value budget constraint[5], solution to the household's problem yields the following necessary conditions with respect to changes in consumption and labor supply for all time *t*:

$$c_t : \beta^t \, \mathrm{u}_c(t) = \lambda^p q_t^o \qquad (10a)$$

$$n_{ct} : \beta^t \, \mathbf{u}_{nc}(t) + \lambda^p q_t^o (1-\tau_t^c) w_{ct} = 0 \qquad (10b)$$

$$n_{xt} : \beta^t \, \mathbf{u}_{nx}(t) + \lambda^p q_t^o (1-\tau_t^x) w_{xt} = 0 \qquad (10c)$$

With $q_0^o = p_0^{-1}$, the time *0* version of *(10a)* implies $\lambda^p = p_0 \, \mathrm{u}_c(0)$. Substituting for λ^p in *(10a)* gives the Arrow-Debreu price in terms of consumption allocations and initial relative price of capital goods, which is:

$$q_t^o = \frac{\beta^t \, \mathrm{u}_c(t)}{p_0 \, \mathrm{u}_c(0)} \qquad (10d)$$

[4] The definition of *Ramsey* allocation is given in subsection 3.1 (Definition 3.1.2).

[5] λ^p measures the value of additional units of resources available in the initial period evaluated in utility terms.

Using *(10d)* and $\lambda^p = p_0 \, \mathrm{u}_c(0)$ in *(10b)* and *(10c)*, the after tax wage rates for sector *C* and *X* for all time *t* in terms of consumption and labor supply allocations is given by:

$$(1-\tau_t^j)w_{jt} = \frac{-\mathbf{u}_{nj}(t)}{\mathbf{u}_c(t)} \qquad \text{for sector } j,\ j = C,\ X \qquad (10e)$$

The formulation of the representative firms' problems is unchanged, implying that the necessary conditions from firms' problem are also unchanged. With *(10d)* and *(10e)*, *(9)* may be rewritten as:

$$\sum_{t=0}^{\infty} \beta^t [\mathbf{u}_c(t)c_t + \mathbf{u}_{nc}(t)n_{ct} + \mathbf{u}_{nx}(t)n_{xt}] - p_0 \, \mathbf{u}_c(0)[R_0^c k_{c0} + R_0^x k_{x0}] = 0 \qquad (10f)$$

With $R_0^c = R_0^x$, the time *0* definition of R_t^j for sector $j = C, X$, gives:

$$p_0 = \frac{(1-\theta_0^c)\mathrm{F}_{kc}^c(0)}{(1-\theta_0^x)\mathrm{F}_{kx}^x(0)} \qquad (10g)$$

such that *(10f)* may be rewritten as:

$$\sum_{t=0}^{\infty} \beta^t [\mathbf{u}_c(t)c_t + \mathbf{u}_{nc}(t)n_{ct} + \mathbf{u}_{nx}(t)n_{xt}] - \Omega(c_0, n_{c0}, n_{x0}, \theta_0^c, \theta_0^x) = 0 \qquad (11)$$

where $\Omega(c_0, n_{c0}, n_{x0}, \theta_0^c, \theta_0^x) \equiv \left[\frac{(1-\theta_0^c)\mathbf{F}_{kc}^c(0)}{(1-\theta_0^x)\mathbf{F}_{kx}^x(0)}\right] \mathbf{u}_c(0)[R_0^c k_{c0} + R_0^x k_{x0}]$

Expression *(11)* is, therefore, the intertemporal constraint that involves only allocations and initial capital income tax rates that can be implemented in a competitive equilibrium, and is known in literature as the *implementability* constraint of the corresponding *Ramsey* problem. The *Ramsey* problem for the government, therefore, is to maximize *(1)* subject to the two (binding) resource constraints *(2.1)* and *(2.2)* and the *implementability* constraint defined by *(11)*.

3.1 Solution to the *Ramsey* problem.

Let $\Phi \geq 0$ be a Lagrange multiplier on *(11)*, and define[6]

$$\mathbf{V}(c_t, n_{ct}, n_{xt}, \mathbf{\Phi}) \equiv \mathbf{u}(c_t, n_{ct}, n_{xt}) + \mathbf{\Phi}[\mathbf{u}_c(t)c_t + \mathbf{u}_{nc}(t)n_{ct} + \mathbf{u}_{nx}(t)n_{xt}] \qquad (12.1)$$

The Lagrangian of the *Ramsey* problem can be written as:

$$\begin{aligned}\tilde{\mathbf{J}} = \sum_{t=0}^{\infty} \beta^t \{ & \mathbf{V}(c_t, n_{ct}, n_{xt}, \mathbf{\Phi}) \\ & + \chi_{1t}[\mathbf{F}^c(k_{ct}, n_{ct}) - c_t - g_t] \\ & + \chi_{2t}[\mathbf{F}^x(k_{xt}, n_{xt}) + (1-\delta)(k_{ct} + k_{xt}) - k_{ct+1} - k_{xt+1}] \} - \mathbf{\Phi}\Omega(c_0, n_{c0}, n_{x0}, \theta_0^c, \theta_0^x)\end{aligned}$$

$$(12.2)$$

where $\{\chi_{1t}, \chi_{2t}\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers on the two resource constraints. For given government revenue target $\bar{g}_t$ and initial capital endowments k_{c0} and k_{x0}, the problem is therefore to fix initial capital income tax rates θ_0^c and θ_0^x and maximize *(12.2)* with respect to $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$. The necessary conditions for an optimum for this problem due to changes in allocations are:

$$c_t : \mathrm{V}_c(t) = \chi_{1t}, \qquad \forall t \geq 1 \qquad (12.3a)$$

$$n_{ct} : \mathbf{V}_{nc}(t) = -\chi_{1t}\, \mathbf{F}_{nc}^c(t), \qquad \forall t \geq 1 \qquad (12.3b)$$

$$n_{xt} : \mathbf{V}_{nx}(t) = -\chi_{2t}\, \mathbf{F}_{nx}^x(t), \qquad \forall t \geq 1 \qquad (12.3c)$$

$$k_{ct+1} : \chi_{2t} = \beta[\chi_{1t+1}\, \mathrm{F}_{kc}^c(t+1) + \chi_{2t+1}(1-\delta)], \qquad \forall t \geq 0 \qquad (12.3d)$$

$$k_{xt+1} : \chi_{2t} = \beta\chi_{2t+1}[\mathrm{F}_{kx}^x(t+1) + (1-\delta)], \qquad \forall t \geq 0 \qquad (12.3e)$$

$$c_0 : \mathrm{V}_c(0) = \chi_{10} + \Phi\,\mathbf{\Omega}_{c0} \qquad (12.3f)$$

$$n_{c0} : \mathbf{V}_{nc}(0) = -\chi_{10}\, \mathbf{F}_{nc}^c(0) + \mathbf{\Phi}\,\Omega_{nc0} \qquad (12.3g)$$

$$n_{x0} : \mathbf{V}_{nx}(0) = -\chi_{20}\, \mathbf{F}_{nx}^x(0) + \mathbf{\Phi}\,\Omega_{nx0} \qquad (12.3h)$$

Consolidating *(12.3)* yields the following five equations:

---

[6]The following expression *(12.1)* is commonly referred to as the Pseudo utility function which combines the utility function and the infinite horizon part of the *implementability* constraint.

$$\mathbf{V}_c(t)\frac{\mathbf{F}_{kc}^c(t)}{\mathbf{F}_{kx}^x(t)} = \beta\,\mathbf{V}_c(t+1)\frac{\mathbf{F}_{kc}^c(t+1)}{\mathbf{F}_{kx}^x(t+1)}[\mathbf{F}_{kx}^x(t+1)+(1-\delta)], \qquad \forall t \geq 1 \qquad (13.1a)$$

$$\mathbf{V}_{nc}(t) = -\mathbf{V}_c(t)\,\mathbf{F}_{nc}^c(t), \qquad \forall t \geq 1 \qquad (13.1b)$$

$$\mathbf{V}_{nx}(t) = -\mathbf{V}_c(t)\frac{\mathbf{F}_{kc}^c(t)}{\mathbf{F}_{kx}^x(t)}\mathbf{F}_{nx}^x(t), \qquad \forall t \geq 1 \qquad (13.1c)$$

$$\mathbf{V}_{nc}(0) = [\mathbf{\Phi}\,\Omega_{c0} - \mathbf{V}_c(0)]\,\mathbf{F}_{nc}^c(0) + \mathbf{\Phi}\,\Omega_{nc0} \qquad (13.1d)$$

$$\mathbf{V}_{nx}(0) = -\mathbf{V}_c(0)\frac{\mathbf{F}_{kc}^c(0)}{\mathbf{F}_{kx}^x(0)}\mathbf{F}_{nx}^x(0) + \mathbf{\Phi}\,\Omega_{nx0} \qquad (13.1e)$$

The other three necessary conditions are the two (binding) resource constraints *(2.1)* and *(2.2)*, and the *implementability* constraint *(11)*, which are repeated here for convenience:

$$c_t + g_t = \mathbf{F}^{c}(k_{ct}, n_{ct}) \qquad (13.1f)$$

$$k_{ct+1} + k_{xt+1} - (1-\delta)(k_{ct} + k_{xt}) = \mathbf{F}^x(k_{xt}, n_{xt}) \qquad (13.1g)$$

$$\sum_{t=0}^{\infty}\beta^t[\mathbf{u}_c(t)c_t + \mathbf{u}_{nc}(t)n_{ct} + \mathbf{u}_{nx}(t)n_{xt}] - \Omega(c_0, n_{c0}, n_{x0}, \theta_0^c, \theta_0^x) = 0 \qquad (13.1h)$$

Note that the government is assumed to choose policy first and private agents are assumed to choose their actions then. Let N denote the set of policies for which a competitive equilibrium exists.

Definition 3.1.1 (*Ramsey* Equilibrium): A *Ramsey* equilibrium is a policy η in N, an allocation rule $\Gamma(.)$, and a price system $\mathrm{P}(.) = \{w_j(.), r_j(.), p(.)\}$ for *j* = *C, X,* such that

(a) The policy η maximizes the household's utility *(1)* subject to the resource constraints *(2.1)* and *(2.2)* and implementability constraint *(11)*.

(b) For every η', the allocation $\Gamma(\eta')$, the price system $\mathrm{P}(\eta')$, and the policy η' constitute a competitive equilibrium. □

Definition 3.1.2 (*Ramsey* Allocation): A *Ramsey* allocation corresponding to the *Ramsey* problem is a sequence $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$ that provides a solution to the system of difference equations *(13.1)* and characterizes the *Ramsey* equilibrium defined by *3.1.1.* □

First, note that a *Ramsey* equilibrium requires optimality by households and firms for all policies that the government might choose. Hence for a given value of the initial price level p_0 for which the Transversality condition *(5.6a)* is satisfied, an allocation $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$ and a multiplier Φ that satisfy the system of difference equations presented by *(13.1)* will characterize a *Ramsey* equilibrium. Using the resulting *Ramsey* allocation, one can then compute the *Ramsey* equilibrium values of all endogenous variables of the system. For instance, one can obtain q_t^o from *(10d)*, r_{ct} from *(6.5a)*, w_{ct} from *(6.5b)*, and τ_t^c from *(10e)*. Condition *(10e)* for sector *X* gives $(1-\tau_t^x)w_{xt}$, and so on.

3.2 Optimal capital income taxation.

Consider a case in which there is a $T \geq 0$ for which $g_t = \bar{g}$ for all $t \geq T$. Assume solution to the *Ramsey* problem converges to a time-invariant allocation, so that c, n_c, n_x, k_c and k_x are constant after some time. Then because $V_c(t)$ converges to a constant, the time invariant version of *(13.1a)* implies:

$$1 = \beta[F_{kx}^x + (1-\delta)] \qquad (14.1a)$$

Proposition 1: For a steady state solution to the *Ramsey* problem and a corresponding *Ramsey* allocation, the associated limiting tax rate on capital income from the capital goods producing sector *X* is zero.

Proof: With $\frac{q_{t+1}^o}{q_t^o} = \beta \frac{u_c(t+1)}{u_c(t)}$, as $t \to \infty$, $\frac{q_t^o}{q_{t+1}^o} \to \frac{1}{\beta}$

Also by definition, $\frac{q_t^o}{q_{t+1}^o} = \frac{p_{t+1}}{p_t}[(1-\theta_{t+1}^x)F_{kx}^x(t+1) + (1-\delta)]$

that implies as $t \to \infty$, $\frac{q_t^o}{q_{t+1}^o} \to [(1-\theta^x)F_{kx}^x + (1-\delta)]$.

Hence for $t \to \infty$

$$1 = \beta[(1-\theta^x)\mathbf{F}_{kx}^x + (1-\delta)] \qquad (14.1b)$$

(14.1a) and *(14.1b)* together imply $\theta^x = 0$. ■

Proposition 1 has analogy to the celebrated finding (often referred to as the Chamley-Judd result) of optimal taxation literature --- optimal steady state tax rate on capital income is zero. The finding for the case of capital goods producing sector is similar to what Judd (1985) and Chamley (1986) find using a one-sector model. This result is intuitive, since a nonzero tax rate on capital income in steady state would mean that distortions created by the tax evolves explosively, contrary to a uniform distortion that might be created by simple labor or consumption taxes (see Judd (1999) for details). One way the current modelling approach differs from a conventional one-sector competitive model is how savings and capital accumulation occurs across sectors. Note that households pay a strictly positive relative price for the new capital goods and rent it out to firms in anticipation of income from investment. Firms return the rented capital stock net of depreciation. Of these two installed capital stocks, only k_x is required to produce future capital goods. Hence if capital income from k_x is taxed in a steady state, this will induce compounding nature of distortions. The zero limiting tax rate of capital income from capital producing sector holds irrespective of specifications, as long as specified functions satisfy assumptions 1 and 2.

The result does not necessarily hold as robust for capital income tax in consumption goods producing sector. Consider the steady state capital income tax in the consumption producing sector. For $t \to \infty$, $\frac{q_t^o}{q_{t+1}^o} \to \left[(1-\theta^c)\frac{(1-\tau^x)\mathbf{F}_{nx}^x\mathbf{u}_{nc}}{(1-\tau^c)\mathbf{F}_{nc}^c\mathbf{u}_{nx}}\mathbf{F}_{kc}^c + (1-\delta)\right]$ which implies that $1 = \beta\left[(1-\theta^c)\frac{(1-\tau^x)\mathbf{F}_{nx}^x\mathbf{u}_{nc}}{(1-\tau^c)\mathbf{F}_{nc}^c\mathbf{u}_{nx}}\mathbf{F}_{kc}^c + (1-\delta)\right]$ holds for $t \to \infty$. Together with *(14.1a)*, this implies $\theta^c = 1 - \frac{\mathbf{F}_{kx}^x\mathbf{F}_{nc}^c}{\mathbf{F}_{kc}^c\mathbf{F}_{nx}^x}\left[\frac{(1-\tau^c)\mathbf{u}_{nx}}{(1-\tau^x)\mathbf{u}_{nc}}\right]$. The government's set of policies N for which a competitive equilibrium exists is therefore:

$$\mathrm{N} = \left\{(\tau^c, \tau^x, \theta^c, \theta^x)\middle|\theta^x = 0, \frac{\mathbf{F}_{kx}^x\mathbf{F}_{nc}^c}{\mathbf{F}_{kc}^c\mathbf{F}_{nx}^x}\left[\frac{(1-\tau^c)\mathbf{u}_{nx}}{(1-\tau^x)\mathbf{u}_{nc}}\right] = 1-\theta^c\right\} \qquad (14.1c)$$

Proposition 2: There exists a particular class of utility function and government's choice of labor income tax rates for which limiting tax rate on capital income from consumption goods producing sector C is zero. For all other cases, it is not zero.

Proof: Consider $\theta^c = 1 - \frac{\mathbf{F}_{kx}^x \mathbf{F}_{nc}^c}{\mathbf{F}_{kc}^c \mathbf{F}_{nx}^x} \left[\frac{(1-\tau^c)\mathbf{u}_{nx}}{(1-\tau^x)\mathbf{u}_{nc}} \right]$, and recall from *Ramsey* equilibrium system defined by *(13.1)*, $\frac{\mathbf{F}_{kx}^x \mathbf{F}_{nc}^c}{\mathbf{F}_{kc}^c \mathbf{F}_{nx}^x} = \frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}}$.

Since $\mathbf{V}_{nc} = \mathbf{u}_{nc} + \Phi[\mathbf{u}_{cnc}\, c + \mathbf{u}_{nc} + n_c\, \mathbf{u}_{ncnc} + n_x\, \mathbf{u}_{nxnc}]$, $\mathbf{V}_{nx} = \mathbf{u}_{nx} + \Phi[\mathbf{u}_{cnx}\, c + \mathbf{u}_{nx} + n_c\, \mathbf{u}_{ncnx} + n_x\, \mathbf{u}_{nxnx}]$, the term $\frac{\mathbf{F}_{kx}^x \mathbf{F}_{nc}^c}{\mathbf{F}_{kc}^c \mathbf{F}_{nx}^x} \left[\frac{(1-\tau^c)\mathbf{u}_{nx}}{(1-\tau^x)\mathbf{u}_{nc}} \right] = 1$ if and only if *(1)* there exists a particular (class of) utility function(s) for which $\frac{\mathrm{V}_{nc}}{\mathrm{V}_{nx}} = \frac{\mathrm{u}_{nc}}{\mathrm{u}_{nx}}$, and *(2)* the government sets labor income tax rates equal across sectors. Unless both conditions are satisfied simultaneously, $\theta^c \neq 0$. ■

A few clarifications deserve attention in the proof of proposition 2. First, for utility function defined by *(1)*, it is not explicitly assumed that utility is linear in labor, and that the marginal rate of substitution of labor across sectors is unitary. The first simplification is common in literature that deals with similar models, which (together with separability of utility function in consumption and labor) dramatically simplifies the expressions of $\mathbf{V}_{nj}$ by ruling out the second and cross derivatives of labor services. The second simplification (unitary marginal rate of substitution of labor) would imply that after tax wages are equal across sectors. The fact that workers may receive varying disutility from working in different sectors is empirically supported, evidence of which will be presented in the next section. Such simplifications are not obvious where there exists some intratemporal adjustment cost of labor across sectors (see for instance, Huffman & Wynne (1999)). For such a class of utility functions where $\Im : \mathrm{R}_+^2 \to \mathrm{R}$ is strictly convex, $\frac{\mathrm{V}_{nc}}{\mathrm{V}_{nx}} = \frac{\mathrm{u}_{nc}}{\mathrm{u}_{nx}}$ does not necessarily hold. Moreover, the additional condition for which the government taxes capital income from consumption goods sector at a zero rate is that government's *ex post* choice of labor income tax rates are equal across sectors, which is not inferred by the model. Unless both conditions, $\frac{V_{nc}}{V_{nx}} = \frac{u_{nc}}{u_{nx}}$ and $\tau^c = \tau^x$, hold simultaneously, $\theta^c \neq 0$.

This particular analytical result has a very sharp intuition. Since capital is produced in a different sector than the consumption goods sector, nonzero capital income tax in the consumption goods sector is similar, in terms of consequences, to a simple consumption tax which has uniform distortion pattern. The subscript *j* to capital stock and to labor denotes the level of capital and labor employed in a particular sector, and in no way restricts factors to be sector-specific. Since capital is freely movable between sectors, and following proposition 1, it is feasible for the household to purchase two new capital goods, invest the new capital k_x and both forms of the depreciated old capital goods in the capital goods sector. The next period capital to produce consumption goods is available through production of new capital goods. Hence, the depreciated capital good from consumption sector is transferred to capital goods sector for production. In this respect, a nonzero tax on capital income from capital goods sector would definitely have a chaotic distortionary effect, which cannot be optimal and duly recognized by the government. The household earns capital income from consumption goods sector in each period, gets taxed at a nonzero rate, and can avoid the compounding tax liabilities by shifting depreciated capital to the other sector. In other words, while the household invests in both sectors simultaneously, it makes savings of capital stock for only the capital goods sector, income from which is untaxed.

3.3 Constrained tax choice.

The previous analysis concluded that the government's optimal choice of steady state capital tax rates *may* vary across sectors. Consider, for instance, a class of utility functions for which $\frac{V_{nc}}{V_{nx}} = \frac{u_{nc}}{u_{nx}}$ holds[7]. The government's set of policies for which a competitive equilibrium exists would then be:

$$\widetilde{N} = \left\{ (\tau^c, \tau^x, \theta^c, \theta^x) \middle| \theta^x = 0, 1 - \theta^c = \frac{(1-\tau^c)}{(1-\tau^x)} \right\}$$

[7] One may consider the utility function as $u(.) = \ln c_t + [1 - n_{ct} - n_{xt}]$ which is supported by the lottery argument of Hansen (1985). This functional form is popular in real business cycle literature, as may be found in Herrendorf & Valentinyi (2003), among others. I will introduce it more formally in the next section.

implying that the government sets a limiting zero tax on capital income from consumption goods sector if and only if it sets labor income tax rates equal across sectors. Hence given that particular class of utility functions, for any subset of *Ramsey* policy that prescribes varying labor income tax rates across sectors, the optimal steady state tax on capital income from consumption goods sector is nonzero and non-explosive in nature.

While this may be a robust theoretical proposal for fiscal policy choice, it may be subject to criticisms from a realistic point of view. Real world governments often face a constraint of keeping same tax rates for the same factor across different sectors. Considering the proposed model, it becomes interesting to test what happens to government's optimal capital tax choice in the event when the government, *ex ante*, faces an additional constraint of keeping all factor-specific tax rates same, i.e. same labor income tax rates and same capital income tax rates across sectors. Same labor income tax rates across sectors can be empirically justified, since it is observed in most tax plans laid out by governments. In the present context, equal labor income tax rates across sectors would imply (in general) that in a competitive equilibrium the marginal rate of substitution of labor across sectors equals the before-tax wage ratio.

In principle, it is predictable that such additional constraints in the *Ramsey* problem *(12.2)* would necessarily yield an inefficient *Ramsey* equilibrium outcome relative to the one proposed earlier. While the celebrated Chamley-Judd result of zero steady state capital tax is typically claimed to be the most efficient outcome in a tax distorted one sector economy, the two sector analogue of this result would suggest that any nonzero tax on capital income from capital goods sector would be inefficient. The prescription of a nonzero tax on capital income from consumption sector is backed up by a clear intuition that such a capital tax will not have compounding distortion effects as long as the government keeps the other capital tax zero. A nonzero tax on capital income from consumption goods sector is therefore sustainable in a *Ramsey* equilibrium. If the government's choice of capital tax rates is constrained to be same *ex ante*, the only efficient optimal rule for the government would be that both capital tax rates are zero. Hence in a *Ramsey* problem with constrained capital tax choice, any nonzero optimal tax on capital income would be an inefficient outcome for the government.

To test it formally, note that since the after tax returns to capital are equal across sectors in a competitive equilibrium, constraining capital income taxes to be same is tantamount to constraining pre-tax returns to capital across sectors to be same. In other words, one can test the restriction of equal capital income taxes across sectors by incorporating the additional constraint $\mathbf{F}^c_{kc}(t) = p_t\,\mathbf{F}^x_{kx}(t), \forall t$ in the *Ramsey* problem *(12.2)*. Substituting for the equilibrium relative price of new capital goods, and imposing the constraint that government keeps the labor income tax rates same across sectors, the additional constraint becomes $\dfrac{\mathbf{F}^c_{kc}}{\mathbf{F}^x_{kx}} \cdot \dfrac{\mathbf{F}^x_{nx}}{\mathbf{F}^c_{nc}} = \dfrac{\mathbf{u}_{nx}}{\mathbf{u}_{nc}}$.

Consider, therefore, the Lagrangian form of *Ramsey* problem with constrained tax choice for the government,

$$
\begin{aligned}
\mathbf{J} = \sum_{t=0}^{\infty} \beta^t \{ & \mathbf{V}(c_t, n_{ct}, n_{xt}, \mathbf{\Phi}) \\
& + \chi_{1t}[\mathbf{F}^c(k_{ct}, n_{ct}) - c_t - g_t] \\
& + \chi_{2t}[\mathbf{F}^x(k_{xt}, n_{xt}) + (1-\delta)(k_{ct} + k_{xt}) - k_{ct+1} - k_{xt+1}] \\
& + \chi_{3t}\left[\frac{\mathbf{F}^c_{kc}}{\mathbf{F}^x_{kx}} \cdot \frac{\mathbf{F}^x_{nx}}{\mathbf{F}^c_{nc}} - \frac{\mathbf{u}_{nx}}{\mathbf{u}_{nc}}\right] \} - \mathbf{\Phi}\,\Omega(c_0, n_{c0}, n_{x0}, \theta^c_0, \theta^x_0)
\end{aligned}
$$

(14.2)

where $\{\chi_{1t}, \chi_{2t}, \chi_{3t}\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers on the two resource constraints and the additional *ex ante* tax choice constraint. The necessary conditions for an optimum for this problem for changes in consumption, labor supply and one period ahead capital stocks are:

$$c_t : \mathrm{V}_c(t) = \chi_{1t}, \qquad \forall t \geq 1 \qquad \textit{(14.3a)}$$

$$n_{ct} : \mathbf{V}_{nc}(t) = -\chi_{1t}\,\mathbf{F}^c_{nc}(t) - \chi_{3t}\left[\frac{\mathbf{F}^x_{nx}(t)}{\mathbf{F}^x_{kx}(t)}\left\{\frac{\mathbf{F}^c_{kcnc}(t)}{\mathbf{F}^c_{nc}(t)} - \frac{\mathbf{F}^c_{kc}(t)\mathbf{F}^c_{ncnc}(t)}{[\mathbf{F}^c_{nc}(t)]^2}\right\} - \left\{\frac{\mathbf{u}_{nxnc}(t)}{\mathbf{u}_{nc}(t)} - \frac{\mathbf{u}_{nx}(t)\mathbf{u}_{ncnc}(t)}{[\mathbf{u}_{nc}(t)]^2}\right\}\right], \qquad \forall t \geq 1 \qquad \textit{(14.3b)}$$

$$n_{xt} : \mathbf{V}_{nx}(t) = -\chi_{2t}\,\mathbf{F}^x_{nx}(t) - \chi_{3t}\left[\frac{\mathbf{F}^c_{kc}(t)}{\mathbf{F}^c_{nc}(t)}\left\{\frac{\mathbf{F}^x_{nxnx}(t)}{\mathbf{F}^x_{kx}(t)} - \frac{\mathbf{F}^x_{nx}(t)\mathbf{F}^x_{kxnx}(t)}{[\mathbf{F}^x_{kx}(t)]^2}\right\} - \left\{\frac{\mathbf{u}_{nxnx}(t)}{\mathbf{u}_{nc}(t)} - \frac{\mathbf{u}_{nx}(t)\mathbf{u}_{ncnx}(t)}{[\mathbf{u}_{nc}(t)]^2}\right\}\right], \qquad \forall t \geq 1 \qquad \textit{(14.3c)}$$

$$k_{ct+1} : \chi_{2t} = \beta\left\{\chi_{1t+1}\,\mathbf{F}^c_{kc}(t+1) + \chi_{2t+1}(1-\delta) + \chi_{3t+1}\left[\frac{\mathbf{F}^x_{nx}(t+1)}{\mathbf{F}^x_{kx}(t+1)}\left\{\frac{\mathbf{F}^c_{kckc}(t+1)}{\mathbf{F}^c_{nc}(t+1)} - \frac{\mathbf{F}^c_{kc}(t+1)\mathbf{F}^c_{nckc}(t+1)}{[\mathbf{F}^c_{nc}(t+1)]^2}\right\}\right]\right\} \qquad \textit{(14.3d)}$$

$$k_{xt+1}: \chi_{2t} = \beta\left\{\chi_{2t+1}[\mathbf{F}_{kx}^{x}(t+1)+(1-\delta)] + \chi_{3t+1}\left[\frac{\mathbf{F}_{kc}^{c}(t+1)}{\mathbf{F}_{nc}^{c}(t+1)}\left\{\frac{\mathbf{F}_{nxkx}^{x}(t+1)}{\mathbf{F}_{kx}^{x}(t+1)} - \frac{\mathbf{F}_{nx}^{x}(t+1)\mathbf{F}_{kxkx}^{x}(t+1)}{[\mathbf{F}_{kx}^{x}(t+1)]^{2}}\right\}\right]\right\}$$

(14.3e)

Consolidating *(14.3)* yields three necessary conditions for a *Ramsey* equilibrium, and the one of interest is:

$$\mathrm{V}_c(t)\frac{\mathrm{F}_{kc}^{c}(t)}{\mathrm{F}_{kx}^{x}(t)} = \beta\left[\mathrm{V}_c(t+1)\frac{\mathrm{F}_{kc}^{c}(t+1)}{\mathrm{F}_{kx}^{x}(t+1)}[\mathrm{F}_{kx}^{x}(t+1)+(1-\delta)] + \chi_{3t+1}[\Theta_{t+1}]\right] - \chi_{3t}[\Lambda_t]$$

(14.3f)

Where Θ_{t+1} and Λ_t are terms comprising derivatives of $\mathrm{F}^c(.)$ and $\mathrm{F}^x(.)$, evaluated at time *t+1* and *t*, respectively, defined as[8]:

$$\Theta_{t+1} \equiv \left\{\frac{\mathbf{F}_{nx}^{x}}{\mathbf{F}_{kx}^{x}}\left[\frac{\mathbf{F}_{kckc}^{c}}{\mathbf{F}_{nc}^{c}} - \frac{\mathbf{F}_{kc}^{c}\mathbf{F}_{nckc}^{c}}{[\mathbf{F}_{nc}^{c}]^{2}}\right]\left[\frac{\mathbf{F}_{kx}^{x}+(1-\delta)}{\mathbf{F}_{kx}^{x}}\right] + \frac{\mathbf{F}_{kc}^{c}}{\mathbf{F}_{nc}^{c}}\left[\frac{\mathbf{F}_{nxkx}^{x}}{\mathbf{F}_{kx}^{x}} - \frac{\mathbf{F}_{nx}^{x}\mathbf{F}_{kxkx}^{x}}{[\mathbf{F}_{kx}^{x}]^{2}}\right]\left[\frac{\delta-1}{\mathbf{F}_{kx}^{x}}\right]\right\}$$

$$\Lambda_t \equiv \left\{\frac{\mathbf{F}_{nx}^{x}}{\mathbf{F}_{kx}^{x}}\left[\frac{\mathbf{F}_{kckc}^{c}}{\mathbf{F}_{nc}^{c}} - \frac{\mathbf{F}_{kc}^{c}\mathbf{F}_{nckc}^{c}}{[\mathbf{F}_{nc}^{c}]^{2}}\right] - \frac{\mathbf{F}_{kc}^{c}}{\mathbf{F}_{nc}^{c}}\left[\frac{\mathbf{F}_{nxkx}^{x}}{\mathbf{F}_{kx}^{x}} - \frac{\mathbf{F}_{nx}^{x}\mathbf{F}_{kxkx}^{x}}{[\mathbf{F}_{kx}^{x}]^{2}}\right]\right\}\frac{1}{\mathbf{F}_{kx}^{x}}$$

Recall the otherwise equivalent condition derived from *Ramsey* problem *(12.2)* where factor income tax choices were not constrained for the government. For a $T \geq 0$ for which $g_t = \bar{g}$ for all $t \geq T$, and assuming convergence of the solution to the *Ramsey* problem to a time-invariant allocation, the time invariant version of *(13.1a)* implied $1 = \beta[\mathrm{F}_{kx}^{x} + (1-\delta)]$, which acted instrumentally for the proof of proposition 1. With the current *Ramsey* problem, for $t \to \infty$, $1 = \beta[(1-\theta^{x})\mathbf{F}_{kx}^{x} + (1-\delta)]$ still holds in a *Ramsey* equilibrium. Unless $1 = \beta[\mathrm{F}_{kx}^{x} + (1-\delta)]$ holds from the time invariant version of *(14.3f)* corresponding to *Ramsey* problem *(14.2)*, it is trivial that $\theta^{x} \neq 0$ *vis a vis* $\theta^{c} \neq 0$. In proposition 3, it is formally proved that $1 = \beta[\mathrm{F}_{kx}^{x} + (1-\delta)]$ does not hold in *Ramsey* equilibrium with constrained factor income tax, resulting in a relatively inefficient *Ramsey* equilibrium outcome with two nonzero steady state capital income tax rates.

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[8] The time notations attached to the derivatives are omitted in defining Θ_{t+1} and Λ_t, without loss of generality, just to avoid notational clutter.

Consider a $T \geq 0$ for which $g_t = \overline{g}$ for all $t \geq T$, and assume that the solution to the *Ramsey* problem *(14.2)* converges to a time-invariant allocation. The time invariant version of *(14.3f)* is:

$$\mathrm{V}_c \frac{\mathrm{F}^c_{kc}}{\mathrm{F}^x_{kx}} = \beta \mathrm{V}_c \frac{\mathrm{F}^c_{kc}}{\mathrm{F}^x_{kx}} [\mathrm{F}^x_{kx} + (1-\delta)] + \chi_3 \Sigma \qquad (14.4a)$$

Where

$$\Sigma \equiv \left\{ \frac{\mathbf{F}^x_{nx}}{\mathbf{F}^x_{kx}} \left[\frac{\mathbf{F}^c_{kckc}}{\mathbf{F}^c_{nc}} - \frac{\mathbf{F}^c_{kc}\mathbf{F}^c_{nckc}}{[\mathbf{F}^c_{nc}]^2} \right] \left[\frac{\beta[\mathbf{F}^x_{kx} + (1-\delta)] - 1}{\mathbf{F}^x_{kx}} \right] + \frac{\mathbf{F}^c_{kc}}{\mathbf{F}^c_{nc}} \left[\frac{\mathbf{F}^x_{nxkx}}{\mathbf{F}^x_{kx}} - \frac{\mathbf{F}^x_{nx}\mathbf{F}^x_{kxkx}}{[\mathbf{F}^x_{kx}]^2} \right] \left[\frac{1-\beta(1-\delta)}{\mathbf{F}^x_{kx}} \right] \right\}$$

In order to prove that both capital tax rates are nonzero, it is sufficient to prove that $\Sigma \neq 0$, which in turn implies $1 \neq \beta[\mathbf{F}^x_{kx} + (1-\delta)]$ in a *Ramsey* equilibrium with constrained tax choice.

Proposition 3: For a steady state solution to the *Ramsey* problem *(14.2)* and a corresponding *Ramsey* allocation, the two associated limiting tax rates on capital income are nonzero.

Proof: Suppose not, and hence $\Sigma = 0$ such that *(14.4a)* implies $1 = \beta[\mathrm{F}^x_{kx} + (1-\delta)]$.

Given the underlying parameter restrictions and assumption 2, $\frac{\mathbf{F}^x_{nx}}{\mathbf{F}^x_{kx}} \left[\frac{\mathbf{F}^c_{kckc}}{\mathbf{F}^c_{nc}} - \frac{\mathbf{F}^c_{kc}\mathbf{F}^c_{nckc}}{[\mathbf{F}^c_{nc}]^2} \right] < 0$, $\frac{\mathbf{F}^c_{kc}}{\mathbf{F}^c_{nc}} \left[\frac{\mathbf{F}^x_{nxkx}}{\mathbf{F}^x_{kx}} - \frac{\mathbf{F}^x_{nx}\mathbf{F}^x_{kxkx}}{[\mathbf{F}^x_{kx}]^2} \right] > 0$ and $[1-\beta(1-\delta)] > 0$. Hence for $\Sigma = 0$, it must be that $\beta[\mathbf{F}^x_{kx} + (1-\delta)] - 1 > 0$, which is a contradiction. ■

Thus if the government faces an *ex ante* constraint of keeping factor income tax rates same across sectors, the *Ramsey* equilibrium outcome comprises taxing capital income from both sectors at a strictly nonzero rate. This is a relatively inefficient outcome since with nonzero capital income tax in both sectors, the distortions created by the taxes would be compounding in nature. With this tax plan in the scheme, the household will not be able to avoid the compounding tax liabilities by simply shifting depreciated capital.

3.4 Taxation of initial capital.

Given the Ramsey problem *(12.2)*, if the government is free to choose θ_0^j for $j = C, X,$ how does the government tax income from initial capital?[9] Since steady state tax rate on capital income is *0* for capital goods sector, there exists a strong impetus for the government to set a scheme involving high taxation of initial capital income, likely due to high values of government consumption expenditure. A few observations deserve attention in this context. Since household's preferences are strictly monotone, a high tax on income from initial capital employed in consumption goods sector must accompany high tax on income from initial capital employed in capital goods sector. Without exogenous bounds on the tax rates, the high initial tax rates may even be confiscatory, i.e. either $\theta_0^j > 1$ for both sectors holds in equilibrium, or both initial capital income tax rates are fractions. In either case, of course, these two tax rates are not *necessarily* equal, and *(10g)* implies $\theta_0^j \neq 1$ for $j = C, X,$ since p_0 is finite and strictly positive.

Denote the *maximum value* Lagrangian associated with the *Ramsey* problem as $\tilde{J}^*$. The derivative of $\tilde{J}^*$ with respect to θ_0^x is:

$$\tilde{J}^*_{\theta_0^x} = \Phi(1-\theta_0^c)\,F_{kc}^c(0)\,u_c(0)(k_{c0}+k_{x0})\left[\frac{(\delta-1)}{(1-\theta_0^x)^2\,F_{kx}^x(0)}\right] \qquad (14.5)$$

which is strictly positive for $\theta_0^c > 1$ (and consequently $\theta_0^x > 1$) and $\Phi > 0$. Ljungqvist & Sargent (2000) defines the non-negative Lagrange multiplier Φ as a measure of the utility costs of raising government revenues through distortionary taxes. Without distortionary taxes, a competitive equilibrium would attain first-best outcome for the representative household, and Φ would be zero, so that the household's present-value budget constraint would not exert any additional constraining effect on welfare maximization beyond what is present in the economy's set of technologies. Contrary to the first-best outcome, when the government has to use some tax device, the multiplier Φ is strictly positive and can be interpreted as the welfare cost of the distorted margin, implicit in the implementability constraint *(11)*.

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[9] Hereafter, I will consider the *Ramsey* problem *(12.2)* as the benchmark, and *Ramsey* problem with constrained tax choice defined by *(14.2)* as a special experimental case. I will extend all further analysis on the basis of the benchmark *Ramsey* problem defined by *(12.2)*.

With the presence of distortionary taxation and $\Phi > 0$, $\widetilde{\mathrm{J}}^{*}_{\theta_0^x} > 0$ if $\theta_0^c > 1$ (and consequently $\theta_0^x > 1$). This result is fairly intuitive. By raising θ_0^j for *j = C, X*, and thereby increasing the revenues from taxation of the initial capital stocks employed in the two sectors, the government reduces its future need to rely on distortionary factor income taxation. With this policy, the value of the utility cost of raising government revenue through distortionary taxation, Φ, falls. Hence the implication of *(14.5)* is that the government should set θ_0^j for *j = C, X*, high enough to drive down Φ quickly to zero. With zero optimal limiting capital income tax rate for capital goods sector, the government should raise *bulk* of the exogenous revenue through a time *0* capital levy and for time onwards should only use non-distortionary tax instruments.

4.0 Specific utility functions.

In this section I will characterize the optimal steady state capital tax for consumption goods sector associated with the *Ramsey* equilibrium *(13.1)* with a variant of commonly used utility functions. A few observations from established literature deserve special attention in this context. The utility function defined in (*1*) is standard in literature that adopts models with consumption-labor choice. There exist a handful of studies that consider a class of utility functions which are separable, logarithmic in consumption and linear in labor (see for instance, Herrendorf & Valentinyi (2003)). Such utility functions aids theoretical tractability of models, although their empirical justification can be contentious. Huffman & Wynne (1999) propose a class of utility functions that captures the idea of intratemporal labor adjustment cost assuming that shifting labor across sectors is costly. Their proposed functional form characterizes strict convexity of the function $\Im : \mathrm{R}_{+}^{2} \rightarrow \mathrm{R}$ relevant to the current paper. Jones *et. al* (1997) present a useful specification of a utility function where the planner is unable to distinguish between income from two types of labor. I will consider these utility function specifications for experimenting the key analytical results, acknowledging that there may be many other interesting cases to consider.

4.1 Equal marginal disutility of labor:

Consider the broader class of utility functions:

$$U(c_t, n_{ct}, n_{xt}) = \frac{[c_t \exp(1 - n_{ct} - n_{xt})]^{1-\sigma} - 1}{1-\sigma} \qquad (15.1a)$$

with $\sigma \geq 0$, the inverse of elasticity of intertemporal substitution. Consider u(.) as a special case of $U(.)$ where $\sigma \to 1$. As $\sigma \to 1$, using l'Hôpital's rule, it is possible to show that

$$\mathbf{u}(c_t, n_{ct}, n_{xt}) = \ln c_t + (1 - n_{ct} - n_{xt}) \qquad (15.1b)$$

Specification *(15.1b)* that characterizes utility linear in labor services can be justified by the lottery argument of Hansen (1985). In the context of the current paper's analytical tractability, such utility functions simplify the expressions of $\mathbf{V}_{nj}$ by ruling out the second and cross derivatives of labor services. The specific form *(15.1b)* also exhibit unitary marginal rate of substitution of labor across sectors. While this simple assumption that workers receive equal marginal disutility from different sectors is typically held in a subset of multisector general equilibrium models established in literature, empirically, there is strong evidence against it for the case of the US industrial sector. The BLS survey 2002 reports suggests that injury related incidence per 100 worker varies greatly across different industrial sectors, and incidence rates are relatively higher in goods-producing sector as compared to the service producing sector. Hence, one can argue that such utility functions are increasingly stylized and ignores the empirically supported evidence of varying disliking for jobs across sectors.

The set of policies for the government which can be implemented in a competitive equilibrium, given *(15.1b)*, is presented by:

$$\tilde{\mathrm{N}} = \left\{ (\tau^c, \tau^x, \theta^c, \theta^x) \middle| \theta^x = 0, 1 - \theta^c = \frac{(1-\tau^c)}{(1-\tau^x)} \right\}$$

which states that the optimal steady state capital income tax for consumption goods sector is zero if and only if the government keeps the two labor income tax rates equal across sectors. Now consider competitive equilibrium condition which states that the marginal rate of substitution of labor must equal the relative after tax wage rates. Given specification *(15.1b)*, the marginal rate of substitution of labor across sectors is one. This implies the after tax wage rates across sectors are equal (and not the tax rates). Hence for $\tilde{\mathrm{N}}$, the government's optimal choice of labor income tax rates may or may not be equal across sectors, although both choices will generate allocations which can be implemented as a competitive equilibrium. In the particular policy choice where labor income tax rates vary, the government taxes capital income from consumption goods sector at a nonzero rate. Moreover, if the government's choice of labor income tax rates are bounded such that $\tau^j \in (0,1)$, the government subsidizes capital income from consumption sector for a policy choice of $\tau^x > \tau^c$.

A possible extension to this specification may be to consider varying marginal disutility of labor across sectors maintaining the assumption that utility is linear in labor services. The simplest form that specifies this idea is perhaps $\mathbf{u}(c_t, n_{ct}, n_{xt}) = \mathbf{ln}(c_t) + [1 - \boldsymbol{\upsilon}(n_{ct}, n_{xt})]$ where $\boldsymbol{\upsilon} : \mathbf{R}_+^2 \rightarrow \mathbf{R}$ is a convex function and linear in its two arguments, such that $\boldsymbol{\upsilon}_{n_j n_j} = 0$ for $j = C, X$ and $\boldsymbol{\upsilon}_{n_c n_x} = \boldsymbol{\upsilon}_{n_x n_c} = 0$. In order to incorporate the non-unitary marginal rate of substitution of labor in this functional form, one can define a parameter $\mu > 0$ such that $\upsilon_{n_c} = \mu \upsilon_{n_x}$. Due to the empirical evidence from US industrial sector, it is sensible to assume that $\mu \neq 1$. Invoking this specification yields the same policy set for the government as given by $\widetilde{\mathrm{N}}$, and same conclusion holds.

4.2 Intratemporal labor adjustment cost:

This functional form, as mentioned earlier, is in the spirit of Huffman & Wynne (1999). Assume there exist some intratemporal adjustment cost of labor across sectors, and consider the following utility function:

$$\mathbf{u}(c_t, n_{ct}, n_{xt}) = \mathbf{ln}(c_t) + \{1 - \zeta[\psi n_{ct}^{-\omega} + (1-\psi) n_{xt}^{-\omega}]^{-\frac{1}{\omega}}\} \qquad (15.1c)$$

Where $\omega \leq -1, \zeta > 0$ and $1 \geq \psi \geq 0$. This specification of the utility function allows for the idea that it is costly to reallocate labor from one sector to the other. Note that with $\omega = -1, \zeta = 2$ and $\psi = \frac{1}{2}$, *(15.1c)* reduces to $\mathbf{ln}(c_t) + \{1 - n_{ct} - n_{xt}\}$, which exhibits unitary marginal rate of substitution of labor across sectors, and is tantamount to saying that the household receives equal disutility from labor services from the two sectors. In the context of the current setting, the restrictions $\omega = -1$ and $\psi = \frac{1}{2}$ together imply that marginal rate of substitution of labor across sectors is equal to one. There is an issue, of course, that how these costs should be interpreted here, which I will not focus in detail. The form is useful for experiments of the key analytical result, hence it is reasonable to abstract from these rather irrelevant but otherwise important details.

The marginal rate of substitution of labor across sectors for this specification, for all permissible values of ω, is:

$$\frac{\mathbf{u}_{nc}}{\mathbf{u}_{nx}} = \frac{\psi\zeta n_c^{-\omega-1}[\psi n_c^{-\omega} + (1-\psi)n_x^{-\omega}]^{-\frac{1}{\omega}-1}}{(1-\psi)\zeta n_x^{-\omega-1}[\psi n_c^{-\omega} + (1-\psi)n_x^{-\omega}]^{-\frac{1}{\omega}-1}}$$

For any $\omega < -1$, which can be interpreted as the adjustment cost parameter, the optimal steady state tax rate for capital income from consumption goods sector is:

$$\theta^c = 1 - \left[\frac{\psi\zeta n_c^{-\omega-1}[\psi n_c^{-\omega} + (1-\psi)n_x^{-\omega}]^{-\frac{1}{\omega}-1}(1+\Phi) - \Phi[n_c\,\mathbf{u}_{ncnc} + n_x\,\mathbf{u}_{nxnc}]}{(1-\psi)\zeta n_x^{-\omega-1}[\psi n_c^{-\omega} + (1-\psi)n_x^{-\omega}]^{-\frac{1}{\omega}-1}(1+\Phi) - \Phi[n_c\,\mathbf{u}_{ncnx} + n_x\,\mathbf{u}_{nxnx}]}\right]\left[\frac{(1-\tau^c)\,\mathbf{u}_{nx}}{(1-\tau^x)\,\mathbf{u}_{nc}}\right]$$

With $\mathbf{u}_{ncnc} \neq 0$, $\mathbf{u}_{nxnx} \neq 0$, $\mathbf{u}_{ncnx} \neq 0$ and $\mathbf{u}_{nxnc} \neq 0$. This implies the set of policies at the government's choice which can be implemented in competitive equilibrium comprises of θ^c which is nonzero, even in the case when the government sets labor income tax rates equal across sectors.

4.3 Two types of labor:

This particular functional form where labor services are of two specific types is due to Jones *et. al* (1997), and is intended to represent the case where the planner is unable to distinguish between income from two types of labor. A probable *rationale* for this utility function may be the often realized and empirically supported fact that producing capital goods is typically more skill-intensive than producing manufacturing consumption goods. The example considered therefore features one household that sells two types of labor in the market. Jones *et. al* (1997) invoke this specification with an *ex ante* restriction on the choice of labor income tax rates. I will consider the unconstrained version of the optimal choice of capital tax rate with the specification, i.e. the associated limiting tax rate on capital income from consumption goods sector corresponding to *Ramsey* equilibrium *(13.1)*. Consider the following utility function:

$$\mathbf{u}(c_t, 1-n_{ct}, 1-n_{xt}) = \frac{c_t^{1-\sigma}(1-n_{ct})^{\gamma_c}(1-n_{xt})^{\gamma_x}}{1-\sigma} \qquad (15.1d)$$

with $\sigma \geq 0$, and $\gamma_j < 0$ for $j = C, X$.

This utility function can be interpreted as that of a household with two members each of which is able to supply one unit of leisure to the market each period. The marginal rate of substitution of labor across sectors with specification *(15.1d)* is:

$$\frac{\mathbf{u}_{nc}}{\mathbf{u}_{nx}} = \frac{\gamma_c(1-n_x)}{\gamma_x(1-n_c)}$$

Since now the utility function has cross derivatives of consumption and labor supply, it is useful to state the following expression:

$$\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}} = \frac{\gamma_c}{\gamma_x}\left[\frac{\Phi\{\mathbf{Y}\}-(1+\Phi)(1-n_x)^{\gamma_x}(1-n_c)^{\gamma_c-1}}{\Phi\{\mathbf{Z}\}-(1+\Phi)(1-n_c)^{\gamma_c}(1-n_x)^{\gamma_x-1}}\right]$$

where

$$\mathbf{Y} \equiv n_c(1-n_x)^{\gamma_x}(\gamma_c-1)(1-n_c)^{\gamma_c-2} + n_x(1-n_x)^{\gamma_x-1}\gamma_x(1-n_c)^{\gamma_c-1} - (1-\sigma)(1-n_x)^{\gamma_x}(1-n_c)^{\gamma_c-1}$$

$$\mathbf{Z} \equiv n_c(1-n_x)^{\gamma_x-1}\gamma_c(1-n_c)^{\gamma_c-1} + n_x(1-n_x)^{\gamma_x-2}(\gamma_x-1)(1-n_c)^{\gamma_c} - (1-\sigma)(1-n_x)^{\gamma_x-1}(1-n_c)^{\gamma_c}$$

It is straightforward to notice that for all permissible values of the parameter γ, the condition $\frac{\mathrm{V}_{nc}}{\mathrm{V}_{nx}} = \frac{\mathrm{u}_{nc}}{\mathrm{u}_{nx}}$ does not hold. This implies the set of policies at the government's disposal for which a competitive equilibrium exists (i.e. which can be implemented in a competitive equilibrium), $\mathrm{N} = \left\{(\tau^c, \tau^x, \theta^c, \theta^x) \middle| \theta^x = 0, \frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}}\left[\frac{(1-\tau^c)\mathbf{u}_{nx}}{(1-\tau^x)\mathbf{u}_{nc}}\right] = 1-\theta^c\right\}$, prescribes that an *ex post* choice of equal labor income tax rates is not sufficient to guarantee zero steady state tax on capital income from consumption goods sector. The *Ramsey* equilibrium consequences due to an *ex ante* restriction of equal factor-specific tax rates have already been discussed in subsection 3.3. Since the proof of proposition 3 holds for any utility function specification as long as it satisfies assumption 1, it holds for *(15.1d)*.

5.0 Concluding remarks.

The paper formulated a two-sector neoclassical production model with infinitely-lived agents in order to analyze the optimal income taxation problem (the *Ramsey* problem) and examine celebrated optimal capital taxation principles established in one-sector and endogenous growth analogues. The extension of one-sector model to a two-sector version with endogenous capital good's price makes it convenient to scrutinize sector specific optimal capital income taxes in the steady state. The analysis reached a startling robust conclusion. While it is optimal to set a long run zero tax on capital income from capital goods sector, the optimal steady state capital income tax for consumption goods sector can be nonzero. For a standard class of utility functions that has desirable properties, this result holds, and the set of conditions for which this tax rate is zero is in no way inferred by the model. The set of feasible policies which generate competitive equilibrium allocation (from which the government can choose) prescribes that both optimal steady state capital taxes are zero if and only if the utility function is separable in consumption and labor services and linear in labor services, and the government sets the *ex post* optimal labor income tax rates equal across sectors. Unless both conditions hold simultaneously, it is not zero, i.e. for all other cases, the optimal steady state capital tax from consumption goods sector is nonzero and non-explosive in nature of distortion.

An experiment of adding a constraint that restricts the government to keep factor-specific income tax rates same across sectors *ex ante* was conducted, which resulted in an inefficient outcome with two nonzero long run capital income taxes. Hence restricting the government's choice of income tax rates *ex ante* eventually forces the government to choose two nonzero capital income taxes optimally, which cannot be sustained in a *Ramsey* equilibrium. Initial tax rates on capital income and their inherent properties are also analyzed. The celebrated result of confiscatory taxation of initial capital in the absence of exogenous tax bounds is re-established. The optimal steady state capital tax in consumption goods sector is characterized using three popular classes of utility functions, all of which cohere the key analytical finding of nonzero capital taxation in consumption goods sector. Since there is no explicit inference from the model that the government chooses equal labor income taxes across sectors optimally, the optimal steady state capital tax from consumption goods sector is nonzero in general.

The model, as may be argued, is sensitive to the simplifying assumption of commitment technology, which enables avoidance of potential problems of time inconsistency of optimal policies typical in a dynamic general equilibrium setting with distortions. In the case where commitment power is not perfect, both the limit tax rate and the steady state allocation of capital are different from their levels found in second best outcome. The assumption of a commitment technology is hardly acceptable in extreme form. Within the context of the current paper, the commitment device is not explicitly modelled, although one might simply consider that the government can commit to its future actions by a restriction on its constitution. It is therefore acknowledged that relaxing the commitment assumption or modelling it formally in the current setting may be important for another stream of literature.

This paper advocates that the government's long run tax policy may comprise of three income tax instruments --- the two labor income tax rates and nonzero tax on capital income in the consumption goods sector --- all of which have uniform distortion pattern. Capital income from consumption goods sector can be taxed at a nonzero rate optimally without creating compounding distortions in the long run as long as the other capital tax is set at zero. This allows economic agents to shift depreciated capital to the untaxed sector and avoid the compounding capital tax liabilities. Although a one-sector analogue of this model with similar assumptions would possibly provide consensus to the celebrated result of zero limiting capital income taxation, the current paper suggests that simple observant arguments claiming to clarify the result are less likely to be very useful. The result of limiting zero capital income tax cannot be unconditionally generalized for a wider class of neoclassical production models. A more functional path to understanding the basic forces that drive the properties of optimally chosen tax rates is to demarcate features of the economy which are conjectured to account for the result. The deterministic convex model presented in this paper incorporated two factors and corresponding two tax instruments at the disposal of the government to finance its exogenously determined consumption expenditure. This set up is probably the simplest form of two-sector dynamic general equilibrium model of optimal income taxation which enables one to realize the distinct features of optimal tax rules in a more realistic setting. The experiment with an additional constraint that restricts the government's choice of factor income tax rates to be same across sectors produces reassuring results and reinforces the key analytical findings from the benchmark model.

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