

Overtakable capitalist growth paths*

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abstract

Comparing a process of labor- and capital-augmenting technical change directed by capitalists' maximization of profits with a counterfactual in which decentralized innovation decisions are governed by noncapitalist property relations, I claim that if the two economies start from the same technology and capital stock there's a date T such that after T per-capita consumption is forever strictly greater on the counterfactual.

Directed technical change, golden rule, growth-distribution duality

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1 Introduction

Where production is organized for profit, profitability directs the evolution of production technology. In this paper I argue that typical paths of profit-directed technical progress, though certainly progressive, are in one respect *less* progressive than some other socially accessible paths. Comparing the development of a stylized capitalist economy with a counterfactual in which decentralized innovation decisions are governed by a distinct set of property relations, I claim that if the two systems are seeded with the same initial technology and capital stocks, there's a time T such that the capitalist economy delivers at each date after T less consumption per person than is afforded at that date on the counterfactual.

This argument turns on the duality of wage-profit and consumption-growth relations in systems of production by means of labor and produced inputs.¹ By bringing this duality to bear on problems of ongoing technical change the paper shows that its interest extends beyond the better-studied issue of the one-shot choice of technique. Productive change is actually ongoing in capitalist economies, and the technical change mechanism that I discuss is a promising source of explanations for its historical profile, so I'm led to ask what that mechanism implies for the *evaluation* of our capitalist future. The paper isolates one way in which to pose this question and answers it by showing that people might do better to choose a different future.

¹The choice of technique is studied in light of that duality in Burmeister and Kuga (1970), von Weizsäcker (1971), and Roemer (1977).

2 Innovation directed by profitability

Consider a population of capitalists who hire labor and tie up nondepreciating stocks of a single good to produce more of that good. Time is continuous, and at any time t each capitalist runs a production activity described by a couple $(\rho(t), x(t))$, defined so that if $k(t)$ and $l(t)$ are the stock of the good committed and the flow of labor employed, the activity yields a flow of the good equal to

$$\min(\rho(t)k(t), x(t)l(t)).$$

Every capitalist can employ any amount of labor at a wage $w(t)$. Holding $k(t)$ she maximizes profits by hiring $[\rho(t)/x(t)]k(t)$ so that she takes

$$r(t) = \rho(t) [1 - w(t)/x(t)] \tag{1}$$

as the profit rate on her stock.

To collapse the population distribution of capitalists' technologies, I assume that all the capitalists follow the same rules for innovation and that they all start from a common technology. These assumptions allow me to reason about population aggregates by considering what happens to a single capitalist.

In particular I suppose that at every moment t each capitalist's operated activity obeys

$$\dot{\rho}(t) = \chi(t)\rho(t) \text{ and } \dot{x}(t) = \gamma(t)x(t) \tag{2}$$

where the capitalist chooses the profile of technical change $(\chi(t), \gamma(t))$ from an *innovation*

set

$$I \equiv \{(\chi, \gamma) \in \mathfrak{R}^2 \mid \gamma \leq g(\chi)\}$$

defined by a twice-differentiable, strictly decreasing, strictly concave g . Writing the *wage share* as $\omega(t) \equiv w(t)/x(t)$ you have from (1) that for any $\omega \in (0, 1)$ the capitalist can maximize the instantaneous change in her profit rate by picking (χ, γ) in I to maximize

$$\dot{r} = \rho[(1 - \omega)\chi + \omega\gamma], \quad (3)$$

a problem whose unique solution, characterized by the first-order condition

$$\omega g'(\chi) + 1 - \omega = 0, \quad (4)$$

can be represented by differentiable, respectively strictly decreasing and strictly increasing functions of the wage share $(\chi(\omega), \gamma(\omega))$.

Due in its outlines to Kennedy (1965) and lately revived by Duménil and Lévy (1995, 2003), Funk (2002), Acemoglu (2002), Foley (2003), and Julius (2005), this idealization of the innovation process gives pure expression to the idea that profitability acts as a social filter on technical change, selecting its tendential bias between labor and capital. I'm going to consider the resulting direction of productive change from a social point of view that I'll introduce in the next section.

3 Innovation to raise average consumption

If all the capitalists operate the activity $(\rho(t), x(t))$ while building up their stocks according to

$$\dot{k}(t) = g(t) k(t), \quad (5)$$

their investment per employed worker is $[x(t)/\rho(t)]g(t)$, which leaves

$$c(t) = x(t) [1 - g(t)/\rho(t)] \quad (6)$$

as the ratio of aggregate consumption to employed labor. By rearranging this relation between growth rates and per-worker consumption as

$$g(t) = \rho(t) [1 - c(t)/x(t)] \quad (7)$$

you find that it's identical to the relation between profit and wage rates (1) pointed out in the last section.

A simple thought experiment will bring out the importance of this duality. Suppose that the human population of this economy, $N(t)$, expands as in

$$N(t) = N_0 e^{nt}. \quad (8)$$

And suppose that production will be locked into the activity that emerges from a period of technical progress directed by a benevolent engineer. If the economy with its eventually stationary technology is to maintain a constant ratio of employed workers to its total population, the stocks available to production must also eventually grow at the proportional rate n . Let the engineer choose a direction of technical change to maximize,

at each moment, the rate of change of the level of consumption per capita that can be sustained on a path with a constant employment ratio. Defining the *investment weight*

$$\mu_n(t) \equiv \frac{n}{\rho(t)},$$

the engineer selects a $(\chi(t), \gamma(t))$ in I to maximize

$$\dot{c} = x[\mu_n \chi + (1 - \mu_n) \gamma].$$

The innovation that solves this problem can be written as $(\chi(1 - \mu_n), \gamma(1 - \mu_n))$ using the same functions that characterize the capitalists' decisions of section 1. It follows that the profit- and consumption-minded rules induce the same profile of technical change if and only if $r(t) = n$ so that the going shares of profits and wages $(1 - \omega(t), \omega(t))$ weigh the two components of innovation proportionally to the investment and consumption shares $(\mu_n(t), 1 - \mu_n(t))$. If the capitalists are to mimic the friendly engineer, distribution must respect “the golden rule” of Allais, Desrousseaux, Phelps, Robinson, Swan, and von Weizsäcker.

4 Long-run neutralization of technical change

To this point I have only reworked in differential form some old ideas about a one-time discrete choice of activity. But I had promised to evaluate trajectories of ongoing technical change. This requires that I first complete the model of section 2 with explicit dynamics for the wage and for capital accumulation.

Let wages follow a real Phillips motion in the ratio of employed workers to humans,

$$v(t) \equiv \frac{\rho(t) k(t)}{x(t) N(t)},$$

so that the wage share evolves according to

$$\dot{\omega} = \omega [\psi(v) - \gamma(\omega)] \tag{9}$$

for some $\psi(v)$ that has $\psi'(v) > 0$. If wages are entirely consumed, if each capitalist saves her profits in a constant proportion s , and if the population $N(t)$ grows like (8), the employment ratio obeys

$$\dot{v} = v [s(1 - \omega)\rho + \chi(\omega) - \gamma(\omega) - n] \tag{10}$$

while the augmentation of capital in technical change is governed by

$$\dot{\rho} = \chi(\omega)\rho. \tag{11}$$

The laws of motion (9, 10, 11), first put together by Shah and Desai (1981), make a complete dynamical system in ω, v, ρ . It turns out that by wedding endogenously directed technical change with the reserve-army wage-and-accumulation dynamics of Goodwin (1967) and Marx, this model arrives at a powerful explanation of the apparent long-run trendlessness of output-capital ratios and wage shares in growing capital economies.

To see how that goes, notice that since the boundary of the innovation set has $g'(0) < 0$, a

unique ω^* between 0 and 1 induces *Harrod-neutral* technical change,

$$\chi(\omega^*) = 0. \tag{12}$$

Let $\gamma^* \equiv \gamma(\omega^*)$ be the associated rate of labor augmentation; I assume that

$$0 \leq \psi^{-1}(\gamma^*) \leq 1. \tag{13}$$

Then the flow picked out by (9, 10, 11) has a locally asymptotically stable steady state² with: a constant employment ratio equal to $\psi^{-1}(\gamma^*)$; a constant wage share whose value, ω^* , is invariant with respect to saving and population growth rates; Harrod neutral progress at the rate γ^* ; capital accumulation at the rate $g^* \equiv \gamma^* + n$; and capital productivity constant at

$$\rho_R^* \equiv \frac{g^*}{s(1 - \omega^*)}. \tag{14}$$

Technical change, though it's intrinsically two-dimensional, degenerates to Harrod neutrality as the wage share approaches the value ω^* that annihilates its capital-augmenting component.

This model's fertility as a source of explanations for the Kaldorian regularities of capitalist growth paths invites attention to its implications for human wellbeing on those paths.

Here is one. Should you find yourself in the capitalist steady state trying to maximize the instantaneous rate of change of consumption per head compatible with accumulation at the rate g^* , the reasoning of section 3 would instruct you to apply the weights

$$\frac{g^*}{\rho_R^*}, 1 - \frac{g^*}{\rho_R^*}$$

²For a proof of its local stability see Shah and Desai (1981).

to capital and labor augmentation respectively. But positive capitalist consumption, in bounding the profit rate above g^* , bounds the profit share above the appropriate investment weight as from (14)

$$\frac{g^*}{\rho_R^*} = s(1 - \omega^*) < 1 - \omega^*. \quad (15)$$

You can tell already that, viewed from the standpoint of social consumption, the attracting income distribution puts too much weight on the augmentation of produced material inputs and too little on the augmentation of labor power.

5 Dueling filters

To bring this conclusion home I'll now compare the evolution of consumption per head in this system with an explicit consumption-driven alternative. No doubt this could be done by continuing section 3's fiction of the benevolent engineer, evaluating the development of (9, 10, 11) against the benchmark of the centrally promulgated solution to some Bellman or Pontryagin problem.³ However I think a more interesting comparison is to a decentralized economy in which innovation decisions are no better informed and no more powerfully computed than the capitalists' efforts.

For this reason I suppose that a group of coops, whose members exhaust a population that expands like (8), run production activities $(x_G(t), \rho_G(t))$.⁴ At every moment the coops hold on loan from the state a stock of means of production on which they pay interest at

³Nordhaus (1967) studies the intertemporal optimization of directed technical change.

⁴The thought that coops might be carriers of the golden rule was suggested by remarks in Nell (1976); von Weizsäcker and Samuelson (1971) expound the regulating role of a golden interest rate in socialist planning; a classic comparison of socialist and capitalist growth paths is Nuti (1970); and for golden-rule-based criticisms of technologically stationary capitalism see Goodwin (1972) or Thompson (2003).

the rate $i(t)$; the state also taxes coops' net income at some rate $\tau(t)$. The coop workers consume what they produce net of these interest and tax payments, and each coop chooses an innovation profile in I to maximize the rate of change in this consumption per worker due to innovation,

$$[1 - \tau(t)] x_G(t) [\mu(t) \chi + (1 - \mu(t)) \gamma],$$

where

$$\mu(t) \equiv \frac{i(t)}{\rho_G(t)}$$

is the interest share of coop revenue. The tax drops out of this problem, and the coops' constrained-best innovation is again described by $\chi(1 - \mu), \gamma(1 - \mu)$ with the consumption weight $1 - \mu$ playing the same role in their decisions as the wage share plays in the capitalists' decisions. Finally I assume that the coops continually apply for, though they might not receive, loans sufficient to employ all their members on their evolving production activities and that the bank observes their aggregate demand for credit.

Again my ploy requires that I populate this counterfactual economy with maximally naive actors, so I'll suppose that the bank officials pursue full employment and golden accumulation with simple rules of thumb. In particular let them control the interest rate by a rule

$$\dot{i} = \alpha [-\chi(1 - \mu) + \gamma(1 - \mu) + n - i] i, \quad (16)$$

that corrects its deviation from the growth rate of coops' demand for loans with an intensity of adjustment $\alpha > 0$, giving rise to a differential equation for the investment weight,

$$\dot{\mu} = [-(1 + \alpha) \chi(1 - \mu) + \alpha \gamma(1 - \mu) + \alpha n - \alpha i] \mu. \quad (17)$$

And I assume that the bank allows the stock on loan to the coops to accumulate at the

proportional rate $i(t) + z(v(t))$ for some $z(v(t))$ with $z' < 0$ and $z(1) = 0$, setting the enterprise tax equal to $[i(t) + z(v(t))]/\rho(t)$ so that its new loans are covered by its interest and tax revenue. Being in fact credit-rationed so long as $v < 1$, the coops accept these loans. The bank's policy and the coops' innovation rules together yield

$$\dot{v} = v[z(v) + i + \chi(1 - \mu) - \gamma(1 - \mu) - n], \quad (18)$$

as a law of motion for the ratio of the employed to the total population.

Appendix A shows that the dynamical system (16, 17, 18) has a unique globally asymptotically stable rest point characterized by full employment, Harrod-neutral progress at the rate γ^* , investment that accounts for a share $\mu^* = (1 - \omega^*)$ of national spending, and interest and accumulation rates constant at g^* . But the system approaches this configuration with a lower level of capital productivity,

$$\rho_G^* = \frac{g^*}{\mu^*} = \frac{g^*}{1 - \omega^*}, \quad (19)$$

than prevails in the capitalist long run, since it follows by comparison with (14) that if the capitalists do any consumption at all, $\rho_R^* > \rho_G^*$.

I won't pretend that the market syndicalism of (16, 17, 18) is an incentive-compatible, politically stable, or otherwise attractive socialism. *The Crime of Monsieur Lange* notwithstanding, I don't intend any affection for the coop form itself. I've only tried to sketch a sanely informed decentralization of the social pursuit of higher consumption per head, a foil to (9, 10, 11) that differs from it mainly in the decision rules that govern technical progress and in the property relations that give those rules currency.

In both economies ongoing technical change permits average social consumption to

increase forever. And the unemployment-stabilizing rate of accumulation is endogenous rather than selected by the given path of labor supply. So I can't ask about these economies the question I asked in section 3; another criterion is called for. For any two growth paths, say that one path *overtakes* the second if there's a date T such that after T per capita consumption is always strictly greater on the first path. And for any two economies A and B characterized by possibly distinct decision rules for innovation and accumulation and by identical paths of labor supply and an identical innovation set, say that A overtakes B if every growth path of B is overtaken by a growth path of A that shares that B -path's technology and capital stock at some date 0. I'll now argue that the economy (9, 10, 11) is indeed overtakable.

6 Overtakable capitalist growth paths

Though the nonlinearities in these systems preclude explicit comparisons of the time paths of consumption per head starting from arbitrary initial conditions, some indirect reasoning will establish overtaking in the most salient group of economies.

Suppose that at a time 0 a capitalist economy with $s < 1$ has already spent some time in its steady state. If labor productivity at 0 is x_0 , consumption per employed worker after 0 is

$$c_R(t) = x_0 \left(1 - \frac{\gamma^* + n}{\rho_R^*} \right) e^{\gamma^* t} = x_0 (1 - s(1 - \omega^*)) e^{\gamma^* t}. \quad (20)$$

And consider an alternative history in which property relations are reset at 0: Coops usurp capitalists' role in the organization of production, and a state bank succeeds them in control of the social surplus, setting in motion the system (16, 17, 18). This counterfactual path inherits a capital stock and technology that determine an initial value for the

employment ratio, and the new bank authorities, having observed their Harrod-neutral capitalist prehistory, start the interest rate out at the established rate of accumulation g^* .

A glance at (16) and (17) shows that

$$i(t) \leq g^*, \mu(t) < \mu^* \Rightarrow \dot{i}(t) > 0 \quad (21)$$

and

$$i(t) > g^*, \mu(t) \geq \mu^* \Rightarrow \dot{\mu}(t) < 0. \quad (22)$$

An orbit that starts from

$$i(0) = g^*, \text{ and } \mu(0) = \frac{g^*}{\rho_R^*} = s\mu^* < \mu^*, \quad (23)$$

will thus have $i(t) > g^*$ and $\mu(t) < \mu^*$ throughout its approach to the rest point, so that $\rho(t)$ converges to ρ_G monotonically from above with

$$\mu^* > \mu(t) \equiv \frac{i(t)}{\rho(t)} > \check{\mu}(t) \equiv \frac{g^*}{\rho(t)} \quad (24)$$

along that traverse.

The consumption per worker that's compatible with accumulation at the attracting rate g^* ,

$$c_G(t) \equiv x(t) (1 - \check{\mu}(t)),$$

itself increases at the proportional rate

$$\hat{c}_G \equiv \dot{c}_G/c_G = \frac{\check{\mu}}{1 - \check{\mu}} \chi (1 - \mu(t)) + \gamma (1 - \mu(t)). \quad (25)$$

From (20) and (25) it's clear that

$$\begin{aligned} \check{\mu}(t)\chi(1-\mu(t)) + [1-\check{\mu}(t)]\gamma(1-\mu(t)) &> [1-\check{\mu}(t)]\gamma(1-\mu^*) \\ \Rightarrow \hat{c}_G(t) &> \hat{c}_R(t). \end{aligned} \quad (26)$$

And the concavity of the innovation frontier $g(\cdot)$ implies that for any three investment weights μ_1, μ_2, μ_3 ,

$$\begin{aligned} \mu_1 &> \mu_2 > \mu_3 \\ \Rightarrow \mu_1\chi(1-\mu_2) + [1-\mu_1]\gamma(1-\mu_2) &> \mu_1\chi(1-\mu_3) + [1-\mu_1]\gamma(1-\mu_3). \end{aligned} \quad (27)$$

But (24) says that μ^* , $\mu(t)$, and $\check{\mu}(t)$ satisfy the antecedent of (27), which in turn ensures that they satisfy the antecedent of (26) and hence that $\hat{c}_G(t) > \hat{c}_R(t)$ throughout the transition from a capitalist to a socialist steady state.

Putting

$$\eta(t) \equiv \frac{c_G(t)}{c_R(t)} \quad (28)$$

and noting that $\hat{c}_G(t) \rightarrow \hat{c}_R(t) = \gamma^*$ as capital augmentation dies out you get that

$$\lim_{t \rightarrow \infty} \eta(t) \equiv \eta^* > 1. \quad (29)$$

Rates of accumulation go to g^* in both systems, so consumption per person is asymptotically the sustainable consumption per worker, $c_R(t)$ or $c_G(t)$, scaled down by the employment ratio. But that employment ratio, equal to $\psi^{-1}[\gamma^*]$ in the capitalist steady state, converges to 1 in the socialist counterfactual. So the ratio of consumption levels per person on the two paths approaches $\eta^*/\psi^{-1}[\gamma^*] \geq \eta^* > 1$ as time goes to infinity.

I've now shown that the steady-state trajectories of capitalist economies whose capitalists consume some of their profits are overtaken by counterfactual paths formed by switching to the laws of motion (16, 17, 18). Now suppose that the rest point of (9, 10, 11) is globally asymptotically stable. Then the capitalist *economy* is overtakable, since any one of its orbits is overtaken by the identically initially conditioned orbit of a hypothetical economy that makes the switch to (16, 17, 18) at a late-enough date.⁵

7 Late-capitalist growth and how to evaluate it

Beyond the direct conflict over the consumption of a given moment's product, and the garden-variety tradeoff between current consumption and the possible future consumption afforded by savers' accumulation of capital, this argument calls attention to a third, specifically *evolutionary* axis of antagonism between capitalists' and social interests in consumption. If you take the empirically plausible view that capital productivity is declining in the earlier stages of capitalist development, and if with a somewhat shakier basis in the facts you understand the recent growth of advanced capitalist countries as motion near the steady state of a profit-directed technical change system, you will conclude that, taxed as they are by capitalists' consumption, those economies have settled for Harrod neutrality and a retarded pace of labor productivity growth *too soon*.

This conclusion is a sort of bastard cousin of views often attributed to Marx. It's a fully spelled-out example of a scenario in which capitalist relations of production eventually

⁵Appendix B proves the last claim in the text. Since I have no proof of the global stability of (9, 10, 11), the possibility remains that the system has nonconvergent paths which are not overtakable. I doubt that this qualification subtracts much economic interest from the current claim. For one thing, (9, 10, 11) only bears consideration insofar as its orbits spend most of their time near its steady state so that the model can recover the rough long-run constancy of wage shares and output-capital ratios. Periodic or quasiperiodic orbits can't be dismissed on these grounds, but simulations and bifurcation analysis have failed to detect any in this version of the directed technical change model.

fetter the development of the forces of production. But it describes only a relative fettering, not an absolute stagnation, and this need not generate any endogenous pressure for a change in production relations.⁶ Moreover this scenario exactly inverts the pattern of technical change that's supposed to bring about Marx's falling rate of profit, since, far from being destroyed by innovation of the capital-using, labor-saving type, this system isn't getting *enough* of that kind of progress.

Though I won't be surprised if the empirical premises of this conclusion are rejected, I think the reasoning that leads up to it is independently interesting as an example of an underemployed strategy for evaluating long-run growth paths and the social arrangements that support them. The best-known procedure asks whether a path succeeds or fails in maximizing an intertemporal social welfare function. The search for a defensible function to be maximized leads mostly to chagrin. For example it's not clear why future utilities should be discounted as is required for the convergence of utility integrals on infinite paths with perpetually positive consumption. (I think it was mainly this problem that inspired the authors of the original overtaking criterion to propose it; see for example von Weizsäcker (1965).) But another difficulty arises where some function like that is embraced, namely that it doesn't tell us how to choose among the many decentralized economies whose actors have no hope of locating, let alone deliberately approaching, its maximum. This objection seems especially serious where the long-run development of a technically progressive economy is concerned, since people can find out what kinds of technical changes are available only by going out and running their technologies, and since they have no way of intelligibly reporting back their discoveries to a social planner.

A reader accustomed to the more familiar procedure might dismiss the arguments of this

⁶Miller (1981) introduces and Cohen (1987) develops this relative/absolute fettering distinction in the context of a technologically determinist interpretation of Marx's historical materialism.

paper, saying “Of course your capitalism is not optimal—you don’t allow the people who live in it to optimize intertemporally.” But since all the economies at which we might aim will fail the test that she mentions and for the same reason, we need another test. The alternative I’ve been trying out goes like this. Suppose that certain qualitative hypotheses about the laws of motion under some institutional set-up are true. Can we conclude that people unaided by any new capacity for computing and imposing social plans might rearrange their institutions in a way that makes them roughly better off? A conclusion like that is something to keep in mind.

Appendix A: global dynamics of (16, 17, 18)

I show that (16, 17, 18) has a unique nonzero rest point that attracts all trajectories with strictly positive initial conditions. This system decomposes as $v(t)$ does not appear in (16, 17). The $\chi(\cdot)$ terms in (16, 17) vanish at μ^* , so both righthand sides are sent to zero by $\mu = \mu^*$ and $i = g^* \equiv \gamma(1 - \mu^*) + n$. It’s readily checked from (16, 17) that the isoclines of that subsystem have

$$\frac{di}{d\mu} \Big|_{\dot{\mu}=0} < \frac{di}{d\mu} \Big|_{i=0} < 0 \quad (30)$$

everywhere, so that (μ^*, g^*) is the only interior rest point; that each isocline intersects both coordinate axes; and that both variables are increasing (decreasing) at all interior points below (above) their respective isoclines. Therefore the two open sets bounded by the isoclines and one of the axes are trapping regions in which all trajectories approach (μ^*, g^*) , and every trajectory that originates in the positive orthant but outside these trapping regions must enter one of them or approach (μ^*, g^*) directly. Therefore $\mu(t), i(t) \rightarrow \mu^*, g^*$ from any interior initial position and so by (18) $\dot{v}(t)/v(t) \rightarrow z(v(t))$. But $z(v) > 0$ for $v < 1$. So $v(t) \rightarrow 1$, and the steady state is asymptotically stable in the large.

Appendix B: overtaking from arbitrary initial conditions

If the rest point of (9, 10, 11) is globally asymptotically stable, then for any one of its trajectories and for $\omega(t_0), \rho_R(t_0)$ close enough to ω^*, ρ_R^* , a counterfactual with initial conditions

$$i(t_0) = s(1 - \omega(t_0))\rho_R(t_0) \quad (31)$$

$$\mu(t_0) = s(1 - \omega(t_0)) \quad (32)$$

has by (21) and (22)

$$g^* < i(t) \text{ and } \mu(t) < 1 - \omega(t)$$

for t in $(t_0 + q(t_0), t_0 + h(t_0))$ where $q(t_0)$ is the greatest q such that $g^* = i(q)$ if that equality is ever satisfied and t_0 otherwise, and where $h(t_0)$ is the smallest h such that $\mu(t_0 + h) = 1 - \omega(t_0 + h)$ if that equality is anywhere satisfied and an arbitrarily large H otherwise. It follows that in this interval,

$$\check{\mu}(t) = \frac{g^*}{\rho_G(t)} < \mu(t) = \frac{i(t)}{\rho_G(t)} < 1 - \omega(t) \quad (33)$$

and so by (27)

$$\check{\mu}(t)\chi(1 - \mu(t)) + [1 - \check{\mu}(t)]\gamma(1 - \mu(t)) > [1 - \check{\mu}(t)]\gamma(\omega(t)) \quad (34)$$

and therefore $\hat{c}_G(t) > \hat{c}_R(t)$. Let $\eta(t_0; t')$ be the ratio of sustainable socialist to capitalist consumption per worker at $t_0 + t'$ in the two systems, given that the change in property relations occurs at t_0 . Then for $t' \geq h(t_0)$ you can write

$$\ln \eta(t_0; t') = \int_{t_0}^{q(t_0)} (\hat{c}_G(t) - \hat{c}_R(t)) dt$$

$$+ \int_{q(t_0)}^{h(t_0)} (\hat{c}_G(t) - \hat{c}_R(t)) dt + \int_{h(t_0)}^{t'} (\hat{c}_G(t) - \hat{c}_R(t)) dt \quad (35)$$

As $t_0 \rightarrow \infty$, $q(t_0) \rightarrow 0$ so the first term vanishes. The second term is certainly bounded above zero. If there's an h such that $\mu(t_0 + h) = 1 - \omega(t_0 + h)$, the integrand of the third term is made arbitrarily small by taking t_0 toward infinity, ensuring that for t beyond $h(t_0)$, $1 - \omega(t)$ is arbitrarily close to μ^* and hence that $\mu(t)$, which can't exceed μ^* , remains arbitrarily close to $1 - \omega(t)$. If instead $\mu(t) \leq 1 - \omega(t)$ forever, the third term is nonnegative. Finally as t goes to infinity, the ratio of the two systems' employment levels approaches $(\psi^{-1}[\gamma^*])^{-1}$, and rates of accumulation in both systems approach g^* . So there are T_0 and T' such that the counterfactual formed from the capitalist economy at T_0 has greater per-capita consumption at all dates after $T \equiv T_0 + T'$.

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