The theory and measurement of cash payments: a case study of the Netherlands

EDGAR L. FEIGE

1. Introduction

One of the more intractable problems in the area of monetary economics is the measurement of cash payments. In recent years, interest in cash payments has been revived as a direct result of their alleged role in lubricating the 'underground' economy. Because cash payments rarely leave a 'paper' trail, they are an effective medium of exchange for those seeking to avoid the payment of income or consumption taxes. But the importance of cash payments goes well beyond the issue of the underground economy. Indeed, the appropriate measurement of cash payments, and more particularly, the turnover or velocity of cash, is essential for the coherent development of monetary theory, and for measuring the effects of monetary changes on all macroeconomic activity.

Since the stock of currency (C), is well defined and easily measured, the estimation of total cash payments ($C \cdot V_c$) reduces to the problem of estimating the number of times the average unit of currency turns over (V_c) in any given period. The issue can be traced back to Jevons who in 1875 wrote:

> 'I have never met with any attempt to determine in any country the average rapidity of circulation, nor have I been able to think of any means whatever of approaching the investigation of the problem, except in the inverse way. If we knew the amount of exchanges effected, and the quantity of currency used, we might get by division the average number of times the currency is turned over; but . . . the data are quite wanting.'[1]

The implications of the intellectual barrier established by the absence of a viable method for estimating V_c can not be underestimated. For example, the modern version of the quantity theory of money,[2] although intellectually

[1] Jevons, W.E. (1875) *Money and the Mechanism of Exchange* quoted in R. Selden (1956).
[2] Friedman, M. (1956) *The Quantity Theory of Money.* A restatement in M. Friedman (1956).

R.D.H. Heijmans and H. Neudecker (eds.), The Practice of Econometrics. ISBN 90-247-3502-5.

rooted in Fisher's pioneering elaboration of the Equation of Exchange,[3] departs from Fisher's emphasis on monetary flows, and reformulates the theory in terms of the *stock* of money. The commonplace emphasis on monetary stocks in both monetary and macroeconomic theory rather than on *payment flows,* is both arbitrary and pernicious. Arbitrary, because it assumes that the work money does is proportional to the money stock; pernicious, insofar as it destroys the independent behavioral role of payment velocity in monetary theory.

The essential work done by money as the medium of exchange is directly proportional to the volume of payments effected by money, not to the stock itself, except, of course, in the limiting case where the payment velocity of money is constant. It is the absence of independent estimates of payment velocity, (V_c), that necessitates the assumption, implicit in fifty years of monetary theory, that the services of money are proportional to its stock. Moreover, without independent estimates of V_c, theorists have been forced to specify the theory of money demand from the limiting perspective of income velocity rather than payment velocity.

The central role of V_c was clearly recognized by Fisher, who also proposed an ingenious method for measuring this elusive magnitude.[4] Fisher argued that:

> 'The importance of such accurate determinations (of the velocity of currency) can scarcely be overestimated. When we know statistically the velocity of circulation of money we shall be in a position to study inductively the 'quantity theory' of money, to discover the significance of that velocity in reference to crises, accumulations of wealth, density of population, rapid transit and communication, as well as many other conditions. In fact a new realm in monetary statistics will have been opened.' p. 618.

Yet, despite Fisher's claims for the theoretical and empirical importance of the payment velocity of currency, and his early efforts to devise a method for calculating V_c, little further attention was given to the problem for almost sixty years. The intervening years did, however, produce a plethora of studies of[5] 'income velocity', often in the guise of studies of the demand for money. But, what is commonly known as 'income velocity', namely, the ratio of observed income to the money stock, is neither a correct measure of the number of times money is spent to purchase currently produced goods and services, no is it a useful theoretical construct for predicting the consequences of changes in the

[3] Fisher, I. (1911).
[4] Fisher, I. (1909).
[5] See Selden (1956) for an excellent review.

money supply on economic activity. What is remarkable, is the tenacity with which economists continue to cling to the 'income'-velocity concept despite Keynes' early warning, that income velocity is 'a hybrid conception having no particular significance'[6] whose use, 'has led to nothing but confusion.'[7] Perhaps the same can be said for our commonplace definitions of 'the money supply', (M1; M2; . . . M_i), concepts whose usefulness are limited by their total neglect of the weights required for meaningful monetary aggregation, namely, the payment velocities of the media of exchange.

It was not until 1970, that Laurent (1970),[8] apparently unaware of Fisher's earlier work, examined the issue of currency payments, and established the rudiments of a method for estimating currency tranfers. This work might have been sadly ignored, had it not been for the fortuitous circumstance that it was found handy as a means for implementing the 'transactions' method[9] for estimating the size and growth of the 'underground' economy.

The purpose of this paper is to elaborate Laurent's conceptual framework in order to gain a greater understanding of both the theory and measurement of currency payments and more particularly, the payment velocity of currency. Section 1 reviews Fisher's early efforts to develop a procedure for estimating cash payments and currency velocity. This method, reincarnated by Cramer[10] has been employed by the Netherlands Central Bank in order to obtain estimates of currency payments in the Netherlands as part of a study that attempted to apply Feige's (1979; 1980) 'transaction' method for estimating the underground economy in the Netherlands. The procedures and results of the Netherlands Central Bank are evaluated and are later compared with the alternative set of estimates developed in Section 4. Section 2 reviews the essential features of Laurent's (1970) approach to the estimation of currency payments, and Section 3 elaborates this framework to produce a theoretical model of denomination-specific currency payments. The model incorporates institutional variables such as the currency-quality standard that is a decision variable of the monetary authority. It also incorporates engineering factors that pertain to the physical characteristics of the currency itself. Both of these theoretical innovations are relied upon when the model is empirically implemented in Section 4. The empirical section includes estimates of denomination specific cash payments in the Netherlands from 1950 to 1984. Empirical estimates are also obtained for the total number of lifetime transactions that a unit of currency can on average effect, as well as estimates of the average lifetime of each denomination and its payment velocity.

[6] Keynes, J.M. (1930), Vol. II, p. 24.
[7] Keynes, J.M. (1936), p. 299.
[8] Laurent, R. (1970).
[9] Feige, E. (1979).
[10] Cramer, J.S. (1981a).

The final section of the paper summarizes the results of the study and suggests further applications of the methodology to some fundamental issues of monetary and macroeconomics.

2. The Fisher cash loop method

At the beginning of the century, Fisher (1909) devised an ingenious approximation for estimating the velocity of cash payments. Fisher constructed a model of cash flows that described the various types of currency exchanges that take place between the time individuals initially withdraw cash from the banking system and subsequently redeposit it. Sectoring his hypothetical economy into firms, individual depositors and non-depositors, Fisher examined all possible cash exchanges between the three sectors, and then derived an expression for the value of cash payments effected in the acquisition of goods and services.[11]

Fisher assumed that individuals exclusively acquire cash by withdrawing it from banks.[12] Given information on the value of total withdrawals (ω), and an estimate of the number of payments that occurred between cash withdrawals and subsequent redeposits [the 'cash loop' (λ)], the value of cash payments (P) is simply:

$$P = C \cdot V_c = \omega \cdot \lambda \tag{2.1}$$

where:

P represents the value of cash payments.
C represents the stock of currency in circulation.
V_c represents the average velocity or turnover of currency.
ω represents cash withdrawals from the banking system.
λ represents the 'cash loop'.

Fisher's model specification was conditioned by the institutional structure of the payments mechanism as it functioned during the early years of the 20th century. The approximate formula he derived for the 'total circulation of cash in exchange for goods' equaled the total amount of cash deposited in banks plus total wages paid. Given this estimate of cash payments, and information on the stock of currency outstanding, Fisher estimated that the average velocity of currency was approximately 18 turnovers per year, implying an average holding period between cash exchanges of about three weeks, and a 'cash loop' (λ) approximately equal to 1.6.

[11] In the more general model, presented in Section 3, total cash payments include cash payments made for the acquisition of goods and services as well as the acquisition of real and financial assets.

[12] A model of currency payments that includes real and financial assets other than 'money' would also permit the acquisition of cash from sales of real and financial assets. Such 'sales' would of course include cash withdrawals from time and savings deposits.

Seventy years elapsed before Fisher's loop length estimate was utilized by Cramer (1981a)[13] in his efforts to measure the volume of cash payments in the Netherlands. Cramer obtained rough estimates of the volume of cash withdrawals in the Netherlands and applied Fisher's estimate of the 'cash loop' ($\lambda \approx 2$), to obtain estimates of cash payments in the Netherlands. This same procedure was subsequently adopted by the Netherlands Central Bank (1984) in its efforts to estimate the value of cash payments.[14] Boeschoten and Fase, (1984) obtained estimates of cash withdrawals from banks and giro institutions for several years and multiplied these estimates by the assumed 'cash loop' ($\lambda = 2$) to obtain currency payments. In 1980, this procedure yielded an estimate of average currency velocity of 15.25 implying that the average currency note is held for 24 days before being spent.

3. The simple average lifetime method

Laurent (1970) suggested an alternative framework for estimating the volume of cash payments (P) and hence, the velocity of currency (V_c). If it is assumed that notes can effect a total of G payments during their lifetime, namely, between the time a note is initially issued and finally withdrawn from circulation, and one can obtain an estimate of the average lifetime of notes at time t L(t), it follows that:

$$V_c(t) = G/L(t). \tag{3.1}$$

and

$$P(t) = V_c(t) \cdot C(t) = G/L(t) \cdot C(t). \tag{3.2}$$

Laurent was able to obtain empirical estimates of the average lifetime of currency notes from data on issues and redemptions of currency and could therefore identify changes in cash payments over time in terms of the unknown parameter (G).[15] In order to obtain estimates of (G), Laurent employed Fisher's equation of exchange:

$$C(t) \cdot \frac{G}{L(t)} + D(t) \cdot V_d(t) = [PT](t) \tag{3.3}$$

and assumed that:

$$\frac{[PT](t)}{[py](t)} = k_0 \tag{3.4}$$

[13] Cramer, J.S. (1981a).

[14] Boeschoten, W.C. and M.M.G. Fase (1984), pp. 20–21.

[15] In principle, G could be empirically estimated from well-designed engineering study that tracked note exchange in a controlled environment that simulated real-world conditions.

where:

D = Checkable deposits.

V_d = Velocity or turnover of checkable deposits.

PT = Value of total transactions.

py = Value of current income or product (GNP).

k_0 = Constant.

Assumption (3.4) enabled Laurent to substitute nominal income as a proxy for total transactions in the equation of exchange (3.3). He then selected that value of G which maximized the correlation between the two sides of equation (3.3). Laurent found that an estimate of $G \approx 129$ produced the best fit for his modified equation of exchange.

Assumption (3.4) has been implicitly and explicitly utilized in monetary economics for more than half a century, and was rarely questioned prior to its use as a key assumption in the 'transactions' method for estimating the size and growth of unrecorded and unreported income.[16] Indeed, the assumption plays an important role in most studies of the demand for money that are specified in terms of income rather than transactions. However, the past decade has witnessed an unprecedented degree of financial innovation that may have significantly raised the ratio of transactions to income. Due to the paucity of data on gross financial flows for most nations, it is currently difficult to determine the extent to which the ratio has risen as a result of the understatement of income[17] (the unrecorded 'income hypothesis') or as the result of a vigorous increase in gross financial transactions. Regardless of how that issue is finally resolved, one is left with the uncomfortable conclusion that (3.4) is a tenuous assumption for inference in monetary economics. Unfortunately, the abandonment of this assumption calls into question much of the empirical literature on the demand for money that relies on (3.4) to permit the substitution of income for transactions in the money demand function. It will be recalled that the remarkable 'monetarist' counter-revolution rested on the cornerstone of a stable demand function for money. Yet in the United States, predictions from conventional money-demand functions became highly unreliable during the mid 1970's, precisely at the time when the rising ratio of (PT) to (py) was producing very high estimates of unrecorded and unreported income. It is possible, as we shall later suggest, that Laurent's use of assumption (3.4) was reasonable during a large part of the period he studied, but is no longer tenable.

[16] Efforts to estimate the value of total transactions [PT] have been undertaken by Cramer (1981a, 1981b) for the Netherlands and the United Kingdom, and by Feige (1980; 1981; 1985a) for the U.S.; the U.K. and Sweden.

[17] The Bureau of Economic Analysis has recently released a major revision of the U.S. National Income and Product Accounts that incorporates an upward revision of National Income of $ 147.5 billion for 1984 to take account of misreported income on tax returns.

A recent major survey of currency usage in the U.S. sheds some light on this issue. Undertaken by the Board of Governors of the Federal Reserve System,[18] the survey of household currency usage estimated that the average turnover of currency for product expenditures in 1984 was between 50 and 55 turnovers per year. Taking the lower figure, in conjunction with estimates of the average lifetimes of U.S. currency notes, Feige, (1986) estimated that $G \approx 155$ in 1984.[19]

Since this recent survey was specifically designed to obtain an estimate of currency velocity, its findings can be used to obtain estimates of the total number of lifetime transactions (G), that notes of various denominations can sustain before being judged unfit for further circulation. Given estimates of (G) for the U.S., and an estimated relationship between (G) and certain physical characteristics of paper currency as measured by folding test machines that are used in both the U.S. and the Netherlands, it is possible to infer a value of (G) appropriate for the currency of the Netherlands. These calculations are fully described in Section 5.

4. A denomination-specific model of currency velocity and cash payments

In order to gain the maximum benefit from the point estimate of the average velocity of currency obtained by the recent Federal Reserve survey, it is necessary to specify a denomination-specific model of currency payments over time. The average velocity of currency ($V_c(t)$) for any time period t, is defined as:

$$V_c(t) = \frac{P(t)}{C(t)} = \frac{1}{C(t)} \cdot \sum_{i=1}^{d} V_i(t) \cdot C_i(t) = \sum_{i=1}^{d} V_i(t) \cdot w_i(t), \qquad i = 1 \ldots d. \tag{4.1}$$

Similarly,

$$V_i(t) = \frac{P_i(t)}{C_i(t)} \tag{4.1^1}$$

where:

P(t) is the total volume of payments effected by all denominations of currency during period t.

[18] Spindt et al (1985); Avery et al (1985).

[19] The earlier estimate of currency velocity employed by Feige, (1980) (based on the Laurent method modified by information pertaining to changes in the physical characteristics of notes) ranged between 55 and 61 turnovers per year. The Federal Reserve Board survey results thus lend credence to the Laurent procedure.

$V_c(t)$ is the average velocity of currency during period t.

$C(t)$ is the average stock of currency of all denominations held during period t.

$w_i(t)$ is the share of the ith currency denomination. $(C_i(t)/C(t))$.

The institutional setting for the theory of cash payments is characterized by a monetary authority that issues notes (I) to the banking system. Firms and individuals withdraw currency from the banking system and use the currency notes to effect payments for assets, goods and services. Currency notes are eventually redeposited with the banking system, which returns the notes to the monetary authority. The monetary authority, being responsible for maintaining the fitness standard of the outstanding currency supply, sorts the notes received from circulation, and determines which notes are fit for recirculation and which notes are unfit and thus require redemption (R). The redeemed notes are those, that upon inspection, are deemed to be sufficiently worn to warrant destruction. For given fitness standards and physical characteristics of currency notes at time t, there exists a total number of payments $(G_i(t))$[20] that notes of denomination i can make during their lifetime before being redeemed. The number of cumulative payments $(N^*_i(t))$ performed by notes of denomination i by period t is:

$$N^*_i(t) = G_i(t) \cdot \left\{ \int_0^t R_i^n(s) \cdot ds + \int_0^t \{I_i^n(s) - R_i^n(s)\} \cdot \gamma_i(s) \cdot ds \right\} \tag{4.2}$$

where:

$N^*_i(t)$ is the number of cumulative payments performed by notes of denomination i by t.

$G_i(t)$ is the total number of payments that notes of denomination i can make in their lifetime before being destroyed, given the fitness standards and physical characteristics of paper currency in existence at time t.

$I_i^n(s)$ is the number of new notes of denomination i issued during period s.

$R_i^n(s)$ is the number of notes of denomination i redeemed during period s.

$\gamma_i(s)$ is the average fraction of lifetime payments performed by notes of denomination i as of date s.

The cumulative value of payments effected by notes of denomination i up to period t is:

$$P^*_i(t) = G_i(t) \cdot \left\{ \int_0^t R_i(s) \cdot ds + \int_0^t \{I_i(s) - R_i(s)\} \cdot \gamma_i(s) \cdot ds \right\} \tag{4.3}$$

where:

$R_i(s) = R_i^n(t) \cdot D_i$; where D_i is the nominal value of denomination i (e.g. Fl. 5; 10; 25; 50; 100; 1000); and similarly,

$I_i(s) = I_i^n(t) \cdot D_i$.

[20] $G_i(t)$ can be viewed as the mean of the distribution of lifetimes.

The total cumulative sum of payments effected by all denominations by period t is then:

$$\sum_{i=1}^{d} P^*_i(t) = \sum_{i=1}^{d} G_i(t) \cdot \left\{ \int_0^t R_i(s) \cdot ds + \int_0^t \{I_i(s) - R_i(s)\} \cdot \gamma_i(s) \cdot ds \right\} \quad (4.4)$$

Combining equations (4.1) and (4.3) it is possible to express the velocity of any denomination i as:

$$V_i(t) = \frac{\frac{d}{dt} \left\{ G_i(t) \cdot \left\{ \int_0^t R_i(s) \cdot ds + \int_0^t \{I_i(s) - R_i(s)\} \cdot \gamma_i(s) \cdot ds \right\} \right\}}{C_i(t)} \quad (4.5)$$

If we assume that the average note outstanding has performed one half of its total lifetime payments, differentiation with respect to t yields:

$$V_i(t) = \frac{.5 \cdot G_i(t) \cdot [R_i(t) + I_i(t)] + \frac{\dot{G}_i(t)}{G_i(t)} \cdot P^*_i(t)}{C_i(t)} \quad (4.6)$$

where:

$$\dot{G}_i(t) = \frac{d}{dt} G_i(t).$$

The actual number of lifetime payments ($G_i(t)$) that notes of denomination i can effect, depends upon *both* the physical characteristics (Q^*_p) of the notes and the quality fitness standard (Q^*_s) established by the monetary authority. If (G^*) denotes the number of lifetime payments that notes with physical characteristics (Q^*_p) can sustain under the quality fitness standard (Q^*_s), then the effective number of lifetime payments ($G_i(t)$) that notes of denomination i can perform is:

$$G_i(t) = \delta(t) \cdot G^*(t) \quad (4.7a)$$

where,

$$E\{\delta_i(t)\} = 1 \quad (4.7b)$$

and

$$\delta_i(t) \begin{cases} = 1 \text{ if } Q_s = Q^*_s \\ <1 \text{ if } Q_s > Q^*_s \\ >1 \text{ if } Q_s < Q^*_s \end{cases} \quad (4.7c)$$

The monetary authority attempts to maintain the quality of currency in circulation by maintaining a quality fitness standard Q^*_s which is expected to prevail over time. Temporary departures from the fitness standard will change the effective number of lifetime transactions that, on average, units of currency can effect. For example, when the monetary authority wishes to introduce a new series of currency into circulation, it might temporarily raise fitness standards and retire old series notes before they have reached the normal level of deterioration. In this case, the actual fitness standard $Q_s > Q^*_s$ and $\delta < 1$, implying that old-series notes that have been prematurely redeemed will effect a smaller actual number of total lifetime transactions (G). Alternatively, if for reasons of economy, the monetary authority temporarily permits the quality of the outstanding stock of currency to deteriorate, $Q_s < Q^*_s$ and $\delta > 1$, and the effective number of lifetime transactions will increase, as notes, normally regarded as unfit for circulation, are recirculated into the payments system.

Incorporating the above specifications into (4.1), the average velocity of currency can now be written as:

$$V_c(t) = \sum_{i=1}^{d} w_i(t) \cdot \frac{\delta_i(t) \cdot .5G^*_i(t)[R_i(t) + I_i(t)] + \left\{\frac{\dot{G}_i(t)}{G(t)} + \frac{\dot{\delta}_i(t)}{\delta(t)}\right\} P^*_i(t)}{C_i(t)} \tag{4.8}$$

If we assume that:

a) $G^*_i(t) = G^*(t)$ for all i; namely, that all denominations have identical physical characteristics,

and,

b) $\dot{G}^*_i(t) = \dot{\delta}_i(t) = 0$; namely, that during the period t there are no changes in physical characteristics nor in fitness standards, then:

$$V_c(t) = \frac{1}{C(t)} \cdot .5G^*(t) \cdot \sum_{i=1}^{d} \delta_i(t)[R_i(t) + I_i(t)]. \tag{4.9}$$

If it is also the case that the monetary authority maintains uniform fitness standards for all currency denominations, then;

c) $\delta_i(t) = \delta(t)$ for all i,

and:

$$G^*(t) = \frac{V_c(t)}{\frac{1}{C(t)} \cdot \delta(t) \cdot .5 \sum_{i=1}^{d} [R_i(t) + I_i(t)]} \tag{4.10}$$

Equations (4.9) and (4.10) are instructive insofar as they specify the precise set of variables required to obtain estimates of the average velocity of currency (V_c) and the total number of lifetime payments of currency notes (G^*). Data on the stock of currency ($C_i(t)$) are readily available as is information on the value of new issues ($I_i(t)$) and redemptions ($R_i(t)$) of currency. Quantitative information on fitness standards (δ) could in principle be supplied by the monetary authority since it uses highly sophisticated automated sorting machines whose fitness-standard calibration is both set and recorded by the monetary authority. Thus, estimates of currency velocity require some specific knowledge of G*, the total number of payments that the average note can sustain during its lifetime. Conversely, an estimate of G* requires an independent estimate of V_c.

Fortunately, the aforementioned Federal Reserve survey provides an independent estimate of V_c for the United States in 1984. If we assume that the quality of U.S. currency during that year was approximately at its normal level (i.e., $\delta \approx 1$), equation (4.10) can be used to obtain a direct estimate of G* for the United States, since the necessary information on C(t); $R_i(t)$ and $I_i(t)$ is readily available.[21]

In order to obtain a provisional estimate of V_c for the Netherlands, it is necessary to take account of any differences in the currency quality standards that are maintained in the two countries and any differences in the physical characteristics of the paper which makes up the currency issue of the two countries.

An exact measure of the relative currency quality standard maintained by the monetary authorities in the Netherlands and the United States could in principle be obtained by an engineering study that examined the differential percentage of notes sorted as unfit from a prearranged test deck of currency that was sorted by the soil-detection equipment used in each country.[22] In the absence of exact information, it was assumed that the average currency quality standard maintained in the Netherlands was 25% higher than that which obtained in the United States.

In order to compare the physical characteristics of the paper used to produce currency in the two nations, the study relies on test results from Schopper machines that both nations employ to conduct fold-endurance tests on their currency issue. The aim is first, to determine the functional relationship between estimates of G*, and quantitative test scores (X) obtained by fold-endurance test procedures in the United States, and then to infer the appropri-

[21] Feige, E. (1986).

[22] The sorting machines used in the United States and the Netherlands are made by different manufacturers. On the basis of interviews with technical experts in currency – sorting technology, it was determined that differences in the calibration settings on currency – sorting equipment produce a currency quality standard in the Netherlands that is approximately 20–25% higher than that maintained in the United States.

ate G* for the Netherlands from fold test scores on Dutch currency.[23] Thus;

$$G^* = f(X) \tag{4.11}$$

and $f(X)$ is estimated to be the square root function displayed in Fig. 1.[24] Estimates of G* for the Netherlands were then obtained from information on (X) for Dutch currency notes. G* for the Netherlands is conservatively estimated to be 80. Taking account of the finding that the currency quality standard of the Netherlands is 25% higher than that of the United States implies that $G^* \approx 60$ for the Netherlands.[25]

Given a point estimate of G*, the final estimation of the time path of the velocity of currency requires an econometric specification of equation (4.8). We shall assume that the velocity of currency of the ith denomination depends upon a non-linear time trend (t); the opportunity cost of holding a unit of the ith denomination ($OC_i(t)$) and the convenience ($CO_i(t)$) of the ith denomination as a store of value. The use of a non-linear time trend reflects Fisher's insight that payment habits are rooted in institutional considerations and thus change gradually over time. The opportunity-cost variable is included to reflect the fact that as the opportunity cost of holding currency increases, individuals will attempt to economize on outstanding currency balances and meet transactions needs by increasing the number of turnovers of the smaller

[23] (X) is the fold-test score obtained on the Schopper machine under given test conditions. Test conditions are determined by temperature ($T = 23°$ C) and humidity ($H = 50\%$). Under these test conditions, U.S. currency notes currently score ($X = 4000$) on cross direction fold endurance tests. In the Netherlands, scores of ($X = 1500$) where obtained under test conditions of ($T = 23°$ C) and ($H = 65\%$) and ($X>900$) under test conditions identical to those used in the U.S. Since the higher humidity test conditions employed in some of the tests in the Netherlands impart greater flexibility to the paper, and thus would result in a somewhat higher (X) score, it was estimated that comparable (X) scores for the Netherlands under identical test conditions would yield $X \approx 1200$. I am indebted to L.A. Wolfe and J. Mercer of the Department of the Treasury, Bureau of Engraving and Printing for providing test scores and technical details on the production and testing of currency notes. Comparable data for the Netherlands are from the Unpublished Reports of the Statistics Committee of the Banknote Printers Association.

[24] β_1 is estimated by OLS from the three available data points on G* and X from U.S. data. The end points are: (0,0) and the current measure of $G^* \approx 155$ and $X = 4000$ described above. The third set of coordinates was obtained from the (1962) study by E.B. Randall Jr. and J. Mandel that concluded that the 1957 introduction of dry intaglio notes increased the lifetime of notes by 30% when compared with the pre-1957 wet intaglio notes. Schopper test scores (X) for the pre-1957 notes where obtained from the Bureau of Engraving and Printing, and a time series for G* was constructed by assuming that the new notes with longer lifetimes were introduced into circulation during the period 1957–1964.

[25] This estimate of $G^* \approx 60$ is considerably larger than that employed by the Netherlands Central Bank. (Boeschoten and Fase (1984). The Bank obtained its estimate by arbitrarily assuming that the 'cash loop' was two. Multiplying this cash loop by an estimate of withdrawals of cash from bank and giro institutions yields an estimate of cash payments and hence an estimate of $V_c \cdot G^* \approx 34$, in turn, is derived from equation (4.10).

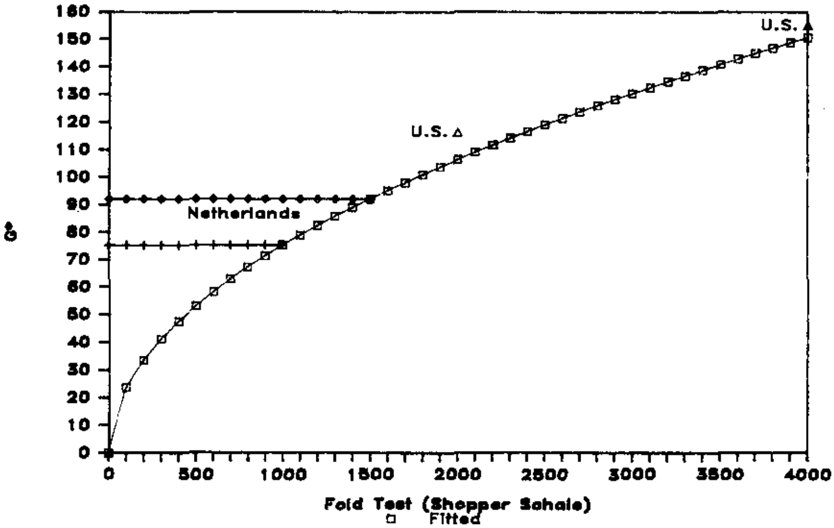

Fig. 4.1. Relationship between fold tests and G*.

average-currency balances. The opportunity cost is represented by:

$$OC_i(t) = 100 \cdot \left\{ \frac{PI_t}{PI_{t-1}} - 1 \right\} \cdot \frac{D_i}{PI_t} \tag{4.12}$$

where,
PI = Price index.
Di = Nominal value of denomination i.
The convenience of the ith denomination as a store of value is assumed to vary directly with its nominal value (D_i) and may be influenced by the availability of other convenient stores of value ($A_i(t)$). In the Netherlands, it is widely believed that some unreported taxable income is hoarded in the form of interest-yielding bearer certificates of deposits issued by banks and giro institutions. In 1976, government officials publicly suggested that banks and giro institution might have to register bearer certificate sales, a policy that would increase the attractiveness of large denomination currency notes for hoarding. A dummy variable (A) is included in the regression for 1000 guilder notes to capture the short-term increase in payment velocity that might be expected as individuals substitute bonds for cash. The longer-run effect would be the opposite, namely to make 1000 guilder notes more attractive for hoards with lower turnover. Finally the regressions include a dummy variable (S) to capture the unusual movements in redemptions and new issues during periods where new-series notes are introduced or old series are prematurely withdrawn.

The foregoing considerations are expressed by letting:

$$Vc_i(t) = \exp\{D_i + \alpha_{i1}T(t) + \alpha_{i2}T^2(t) + + \alpha_{i3}S(t) + \\ + \alpha_{i4}A_i(t) + \alpha_{i5}OC_i(t)\}. \quad (4.13)$$

Imposing the constraint that:[26]

$$\frac{\dot{G}^*(t)}{G^*(t)} = 0$$

and combining (4.9) with (4.13) yields:

$$\log\{T_i(t)\} = \log\{\Pi_i(t) - \frac{\dot{\delta}(t)}{\delta(t)} \cdot P^*_i(t)\} - \log\{\delta(t)\} \quad (4.14)$$

where:

$$T_i(t) = G^*(t)[\{R_i(t) - I_i(t)\}\ \gamma_i(t)]/C_i(t)$$

$$\Pi_i(t) = \text{the rhs of } (4.13).$$

Taking a first-order Taylor's expansion about $\Pi_i(t)$;

$$\log\{T_i(t)\} \approx \log\{\Pi_i(t)\} - \mu_i(t) \quad (4.15)$$

where:

$$\mu_i(t) = \left\{\frac{\frac{\dot{\delta}^*(t)}{\delta(t)} \cdot P^*_i(t)}{\Pi_i(t) \cdot C_i(t)} + \log\{\delta(t)\}\right\}$$

Given the assumptions (4.7b) and (4.7c):

$$E\ \mu_i(t) = 0,$$

and therefore (4.15) can be estimated by ordinary least squares.

The estimation results are reported in Tables 4.1 – 4.4. The most significant effect in the regressions is the constant term, confirming Fisher's notion that payment habits are deeply entrenched and are denomination-specific. The 1000-guilder equation reveals the expected short-term positive effect on velocity of 1000-guilder notes from the announcement that bearer bonds might require registration. The opportunity-cost variable is not significant in any of the equations, a result, explicable in terms of the relatively modest inflation experience of the Netherlands.[27]

[26] This constraint will only be violated in those years where changes in the physical characteristics of the currency issue are introduced. When such changes occur, the constraint is readily maintained by eliminating those specific years from the final estimation procedure.

[27] Manski and Goldin (1987) found that in the high-inflation environment of Israel, the opportunity cost variables significantly contributed to the explanation of the demand for currency.

Table 4.1. 1960–1981 dependent variable: C_{1000}.

Variable	Coefficient	Std. error	t-stat.	2-Tail sig.
D_{1000}	3.0221113	0.4458889	6.7777231	0.000
T	0.0732943	0.0535845	1.3678268	0.190
T^2	−0.0027040	0.0013165	−2.0539010	0.057
S	−0.2331478	0.0884705	−2.6353167	0.018
A_{1000}	0.2688564	0.1017147	2.6432408	0.018
OC_{1000}	−0.0306460	0.1129781	−0.2712562	0.790

R-squared	0.832516	Mean of dependent var	3.119706
Adjusted R-squared	0.780177	S.D. of dependent var	0.314522
S.E. of regression	0.147465	Sum of squared resid	0.347933
Durbin-Watson stat	2.064380	F-statistic	15.90625

Table 4.2. 1960–1981 dependent variable: C_{100}.

Variable	Coefficient	Std. error	t-stat.	2-Tail sig.
D_{100}	2.4802826	0.2468559	10.047490	0.000
T	0.0117898	0.0338228	0.3485743	0.732
T^2	0.0003144	0.0008339	0.3770760	0.711
S	−0.4891320	0.0560489	−8.7268861	0.000
OC_{100}	0.0693581	0.0711516	0.9747931	0.343

R-squared	0.870966	Mean of dependent var	3.039160
Adjusted R-squared	0.840605	S.D. of dependent var	0.234650
S.E. of regression	0.093682	Sum of squared resid	0.149198
Durbin-Watson stat	1.315872	F-statistic	28.68712

Table 4.3. 1960–1981 dependent variable: C_{25}.

Variable	Coefficient	Std. error	t-stat.	2-Tail sig.
C	2.7350857	0.2811110	9.7295574	0.000
T	0.0344256	0.0346537	0.9934169	0.334
T^2	0.0003141	0.0008543	0.3677150	0.718
S	−0.2719165	0.0574308	−4.7346795	0.000
OC_{25}	0.0577513	0.0728097	0.7931818	0.439

R-squared	0.924168	Mean of dependent var	3.681889
Adjusted R-squared	0.906325	S.D. of dependent var	0.313631
S.E. of regression	0.095991	Sum of squared resid	0.156642
Durbin-Watson stat	2.046867	F-statistic	51.79504

Table 4.4. 1960–1981 dependent variable: C_{10}.

Variable	Coefficient	Std. error	T-stat.	2-Tail sig.
D_{10}	3.4804807	0.2069485	16.818099	0.000
T	− 0.0059402	0.0226697	− 0.2620320	0.796
T^2	0.0009004	0.0005590	1.6106490	0.126
S	0.0879227	0.0375682	2.3403481	0.032
OC_{10}	0.0798636	0.0476787	1.6750374	0.112

R-squared	0.925874	Mean of dependent var	3.820583
Adjusted R-squared	0.908432	S.D. of dependent var	0.207537
S.E. of regression	0.062801	Sum of squared resid	0.067048
Durbin-Watson stat	1.204886	F-statistic	53.08443

Given the forecast values of V_c, it is now possible to utilize the foregoing framework to gain further insights into the role of currency payments in the Netherlands economy.

5. Empirical results

5.1 Denomination-specific results

On the basis of the estimated equations presented in Tables 4.1–4.4 it is now

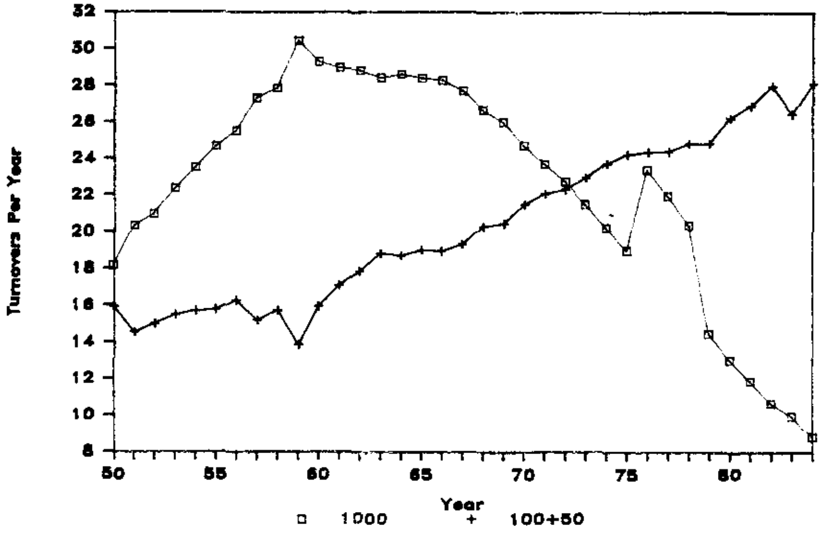

Fig. 5.1. Estimated currency velocity, large denominations.

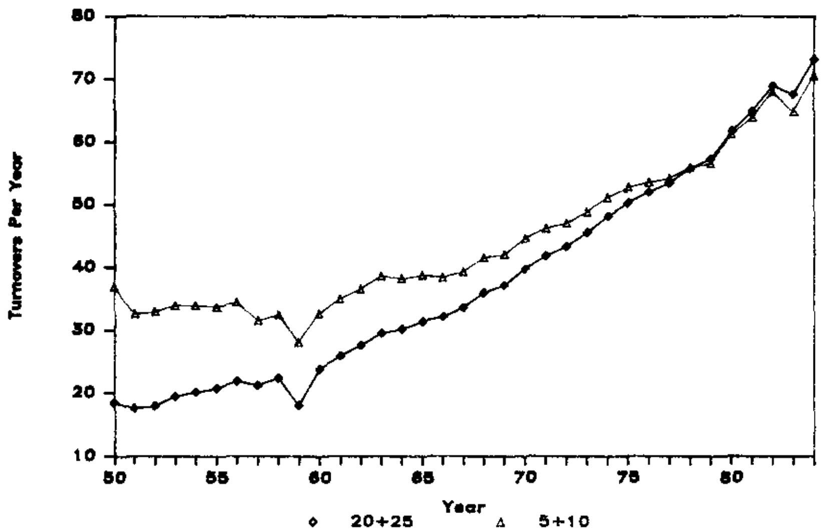

Fig. 5.2. Estimated currency velocity, small denominations.

possible to obtain the forecast values of the velocity of currency for each denomination over time. Figures 5.1 and 5.2 display estimates of currency velocity for large and small denominations respectively.

In general, one would expect to find that the velocity of currency varies inversely with the size of denomination. This expectation is confirmed for all denominations with the exception that the velocity of 1000 guilder notes appears to have exceeded that of 100 guilder notes during the period 1950–1972. By 1984, the average yearly turnover of 1000 guilder notes was 9 turnovers per year compared with 28.5 for 100 guilder notes and more than 70 turnovers for the smaller denomination notes. Over time, the velocity of currency for all notes other than 1000 guilders appeared to increase. The time path of the velocity of 1000-guilder notes suggests that during the decade of the 1950's, these large-denomination notes were widely used as a means of payment, however, during the past twenty five years, the 1000 guilder note was increasingly hoarded.

The average lifetime of notes in circulation, is expected to vary directly with denomination. As displayed in Figures 5.3–5.6 by 1984, 1000-guilders notes had an average lifetime of almost seven years in circulation, whereas, 100 guilder notes circulated 2.1 years and smaller denomination notes remained in circulation for approximately 10 months before being redeemed.

Figure 5.7 displays the estimated average holding period for each denomination. By 1984, small-denomination notes were held for approximately five days

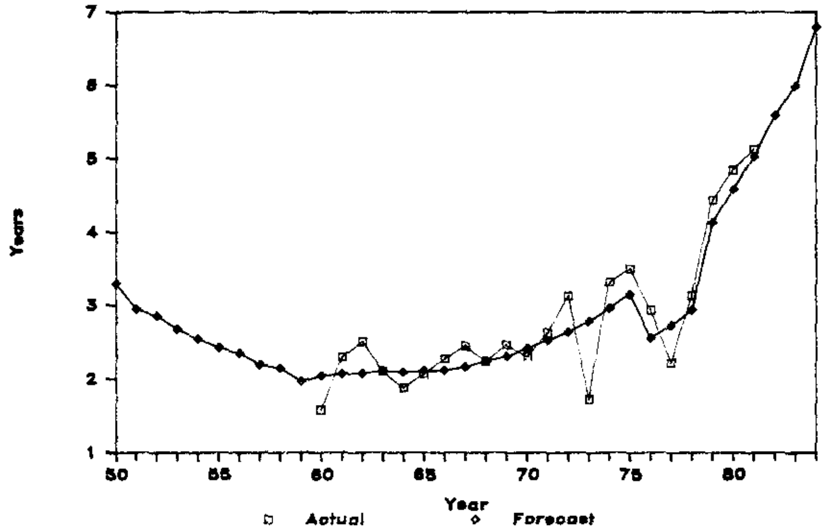

Fig. 5.3. Average lifetime of currency, 1000 guilder denomination.

before being spent, whereas, 100 guilder-notes were held approximately 13 days, and 1000-guilder notes were held approximately 42 days before being spent.

Given the differences in the inter-temporal and denomination-specific holding periods and velocities, it is apparent that the volume of payments per-

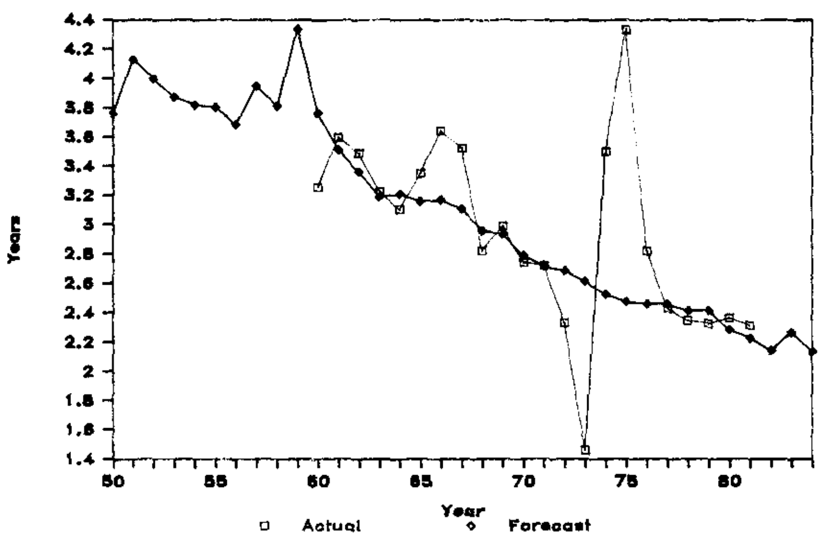

Fig. 5.4. Average lifetime of currency, 50 + 100 guilder denominations.

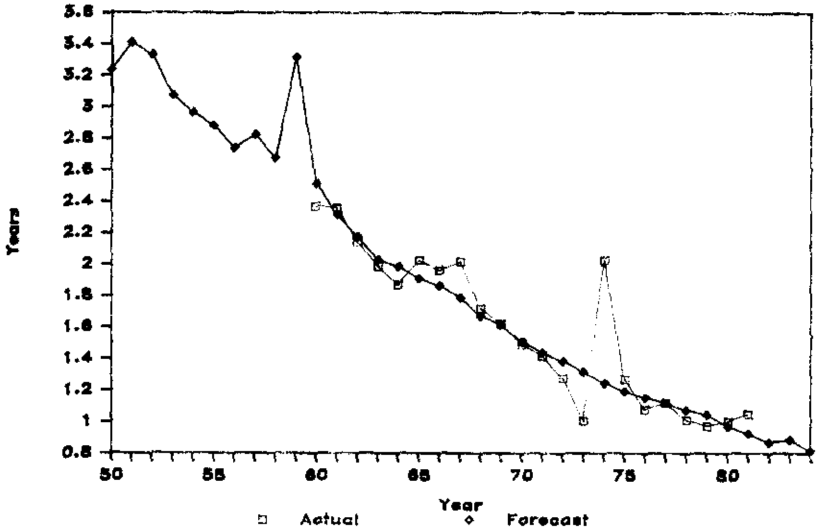

Fig. 5.5. Average lifetime of currency, 20 + 25 guilder denominations.

formed by different denominations of currency over time was affected not only by the amount of each denomination in circulation, but more importantly, by the intensity with which each denomination was used as a means of payment. Given the monetarist emphasis on money 'supply' in juxtaposition to the Fisherian emphasis on the work money does, it is instructive to examine each

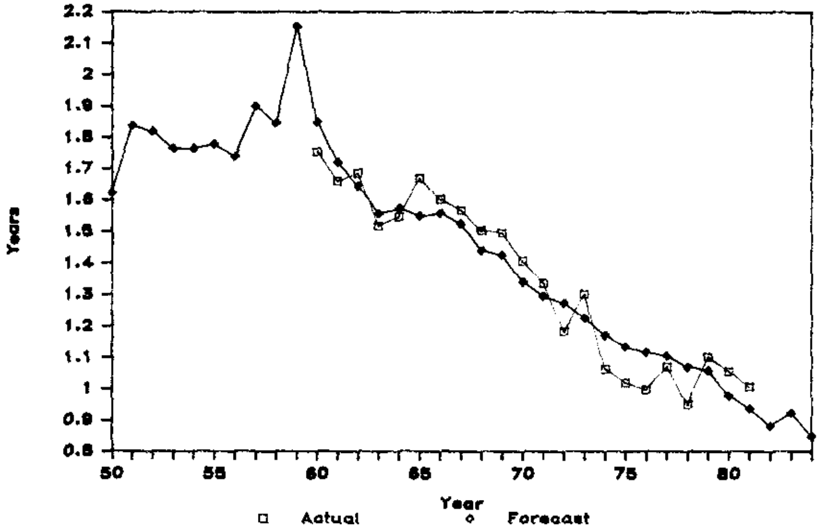

Fig. 5.6. Average lifetime of currency, 10 + 5 guilder denominations.

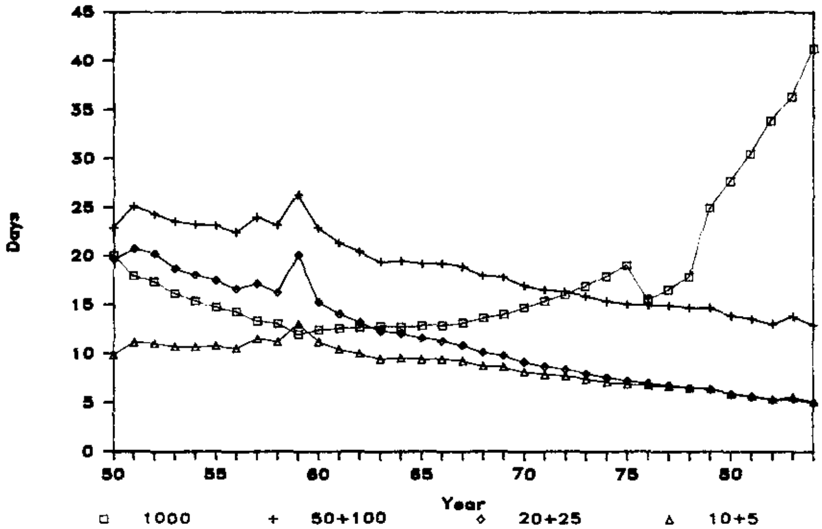

Fig. 5.7. Estimated average holding period, by denomination.

denomination's share of the total currency stock outstanding in comparison to each denomination's share of currency payments effected.

Figures 5.8–5.11 display the share of each denomination as a percent of the value of the currency stock as well as the share of each denomination as a percent of total cash payments.

As revealed by Figures 5.8–5.11 the share of each denomination as a percent of the currency stock is a poor indicator of the payments effected by each denomination. Thus, whereas 1000-guilder notes now make up almost 38% of the total value of the Netherlands currency stock, they account for less than 14% of total currency payments. Conversely, the smaller denomination notes, account for approximately 10% of the nation's currency supply but account for more than 30% of all currency payments.

5.2 Aggregate estimates of currency velocity and cash payments

Given the aforementioned denomination-specific estimates of currency velocity, it is now possible to obtain estimates of the average velocity of currency and the total volume of cash payments in the Netherlands. These estimates can in turn be compared with estimates of currency velocity and aggregate cash payments obtained by the Netherlands Central Bank by means of the simple Fisher cash-loop method.

The final estimates of the denomination-weighted average velocity of cur-

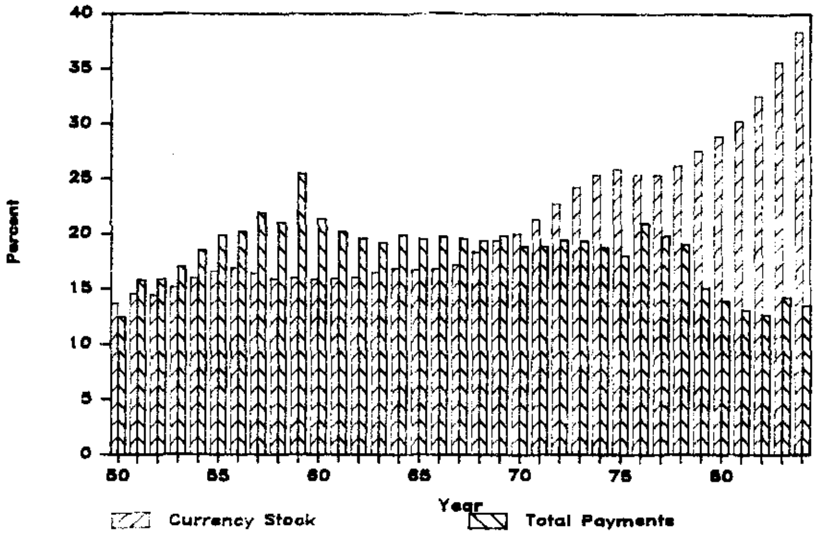

Fig. 5.8. Share of currency stock and payments, 1000 guilder denomination.

rency are displayed in Figure 5.12 as well as a recalculation of the Netherlands Bank's estimates (labeled B-F(1) and B-F(2)).[28]

The new estimates imply that the average unit of currency turns over approximately 25 times per year as compared with the estimate of approximately 15 turnovers per year calculated by Cramer (1981a) and by the Netherlands Bank, who obtained their estimates by assuming a cash loop length of two. According to the new estimates, the average unit of currency is held for approximately two weeks before being spent, compared with an average holding period of 3.5 weeks implied by the Netherlands Bank's estimates.

Figure 5.13 displays a comparison between the estimated average lifetime of currency calculated in the present study, and that employed by the Netherlands Bank's study. The figure reveals a relatively close correspondence between the two calculations except for those periods during which a new series of notes was issued, or an old series notes was withdrawn.[29] Over the period studied, the average currency note remains in circulation for approximately 2.8 years before being deemed unfit for further circulation.

Figure 5.14 displays the final estimates of G^* and $G = G^*\delta$. Recall, that the effective total number of payments that the average bill will sustain through-

[28] Series (B-F(1)) are based on G = 34.3 and (B-F(2)) are based on G = 37. Boeschoten and Fase, (1984), p 45.

[29] It will be recalled that the present analysis adjusts for these aberrations with the dummy variable (S), whereas the Bank's estimates employ the actual observed values of redemptions and issues to calculate average lifetime.

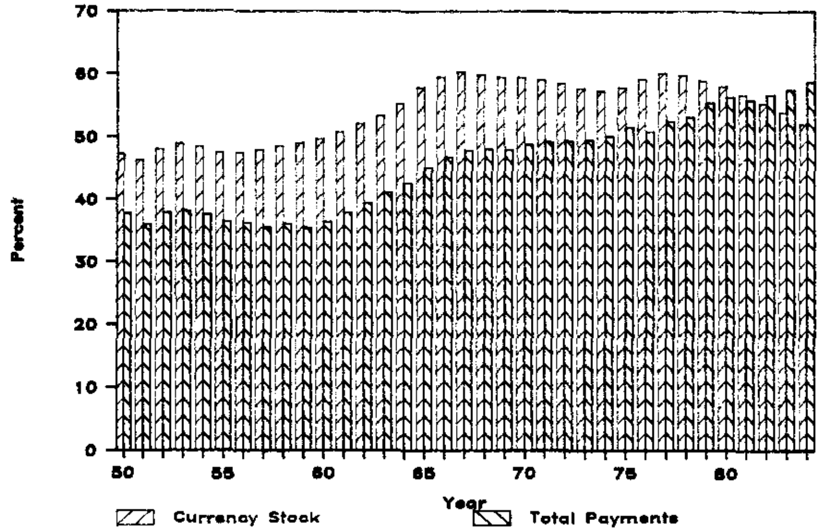

Fig. 5.9. Share of currency stock and payments, 50 and 100 guilder denominations.

out its lifetime (G(t)) depends upon *both* its physical characteristics and departures from the particular quality-fitness standard maintained by the monetary authority. Whereas the physical characteristics of currency remained constant over the period of study, fluctuations in the quality-fitness standard induce changes in the effective number of lifetime payments (G).

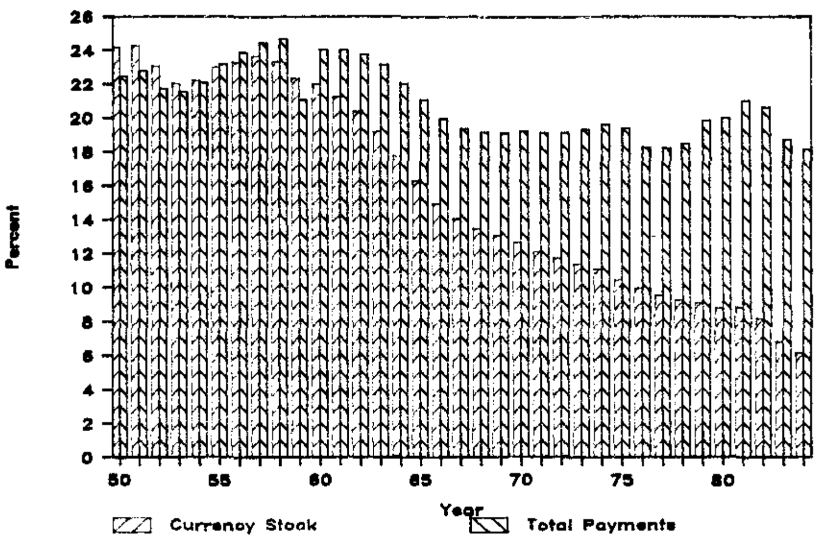

Fig. 5.10. Share of currency stock and payments, 20 and 25 guilder denominations.

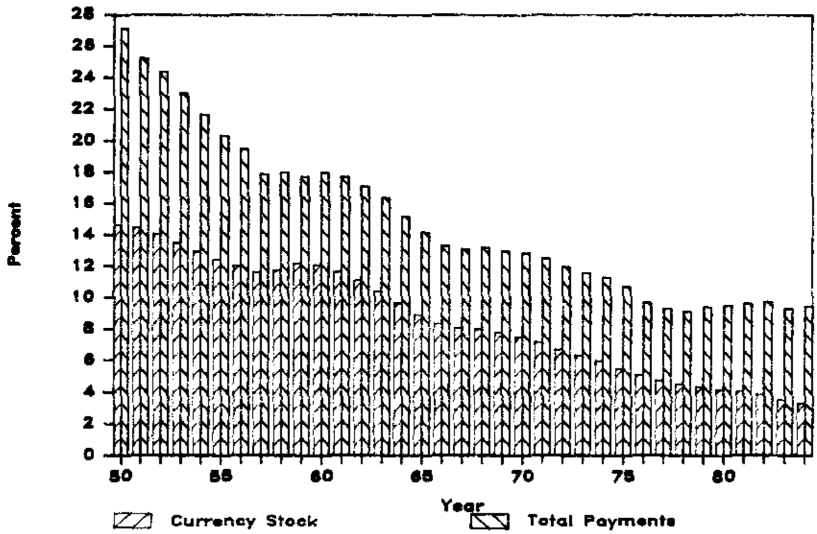

Fig. 5.11. Share of currency stock and payments, 10 and 5 guilder denominations.

The major difference between the findings of the present study and those obtained by Cramer (1981a) and the Netherlands Bank (1984) result from the treatment of the 'cash loop'. In the present study, the value of the 'cash loop' is derived from estimates of total cash payments, whereas in the earlier studies, estimates of cash payments are derived from an *assumed* value for the cash

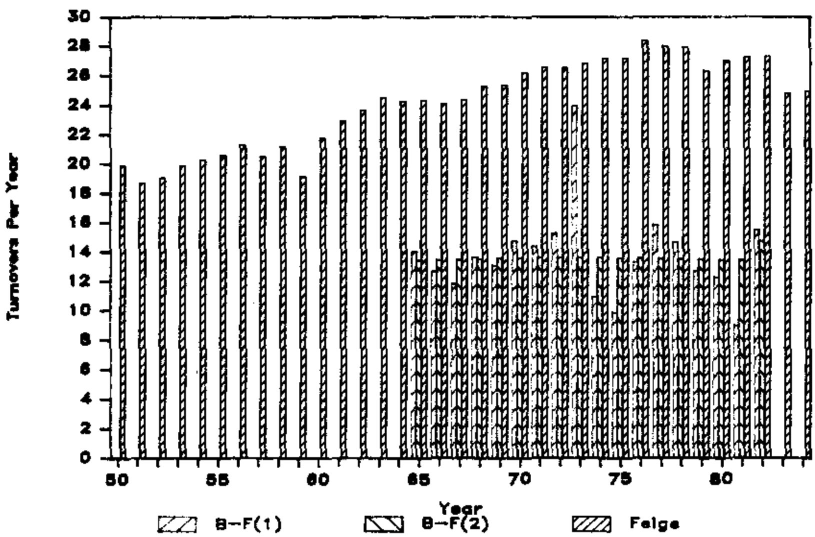

Fig. 5.12. Estimated average currency velocity, weighted by denomination share.

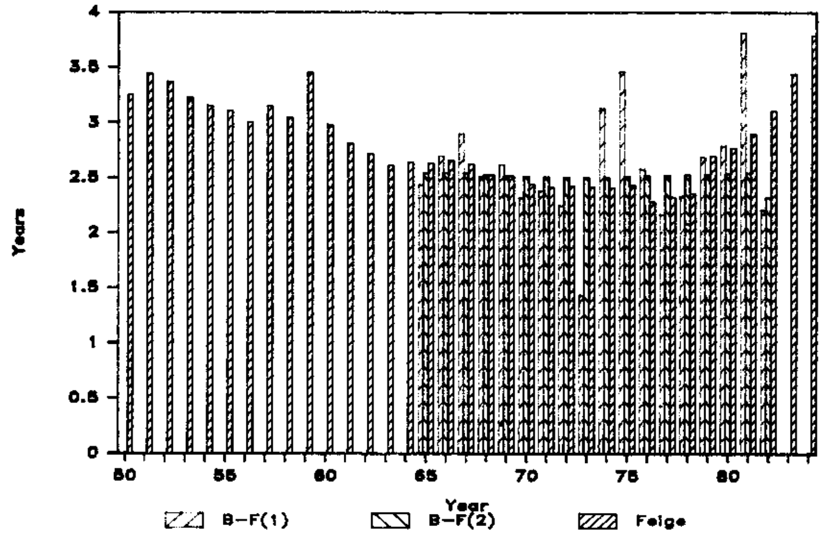

Fig. 5.13. Estimated average lifetimes, weighted by denomination share

loop. Figure 5.15 displays the difference between the estimated 'cash loop' from the present study, and, the assumed 'cash loop' in the earlier studies.

It appears from Figure 5.15 that the cash loop in the Netherlands is approximately equal to four, suggesting that currency is used to make four payments before being returned to the banks. The size of the cash loop may be explained

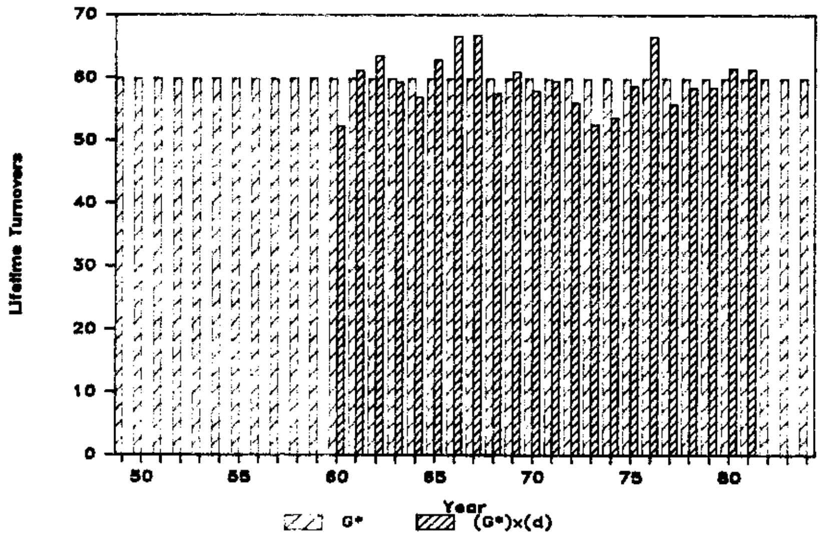

Fig. 5.14. Estimated physical and effective G, total lifetime turnovers.

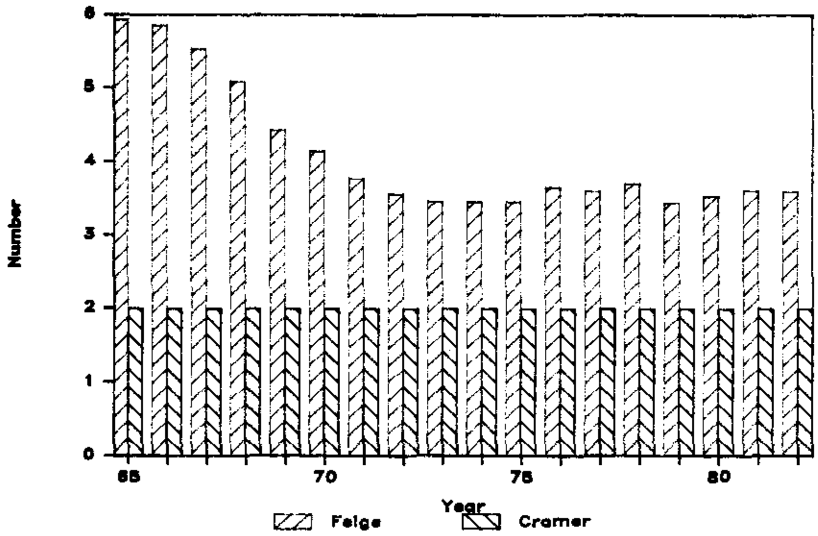

Fig. 5.15. Estimated and assumed cash loop.

by several factors. First, whenever currency is returned to the banking system, an audit trail is produced, since banks in the Netherlands make specific paper entries of all cash deposits and cash withdrawals. Thus, any firm or individual engaged in cash transactions that are not fully reported to the tax authorities has a strong incentive to recirculate the cash directly, rather than deposit it in the banking system. Furthermore, the Netherlands fiscal authority attempts to collect VAT taxes on final sales of goods and services. Individuals who effect transactions in cash, can frequently obtain merchandise and services without being charged the VAT tax. Sellers of merchandise and services can attract customers by agreeing to this practice, but in turn must 'skim' these cash payments from reported final sales. This in turn provides an incentive to hire workers who are willing to receive cash wages. Such workers can thus avoid income taxes, while firms can reduce their cost figures appropriately to match their reduced level of reported sales. Firms thus have a dual incentive to deal in cash. They can avoid employment taxes by paying 'off the books' workers in cash, and they can increase their competitive advantage relative to compliant firms by reducing final prices by the amount of the avoided VAT taxes. In some cases, firms have been known to collect VAT taxes on sales effected with currency, underreport their final sales and pocket the VAT tax collected. With VAT rates in the neighborhood of 18%, such incentives appear substantial.

One might also note that the cash loop is likely to be higher in countries with relatively low rates of theft, since one of the incentives to return currency to banks is security. The *Social and Cultural Report* (1980) indicates that convic-

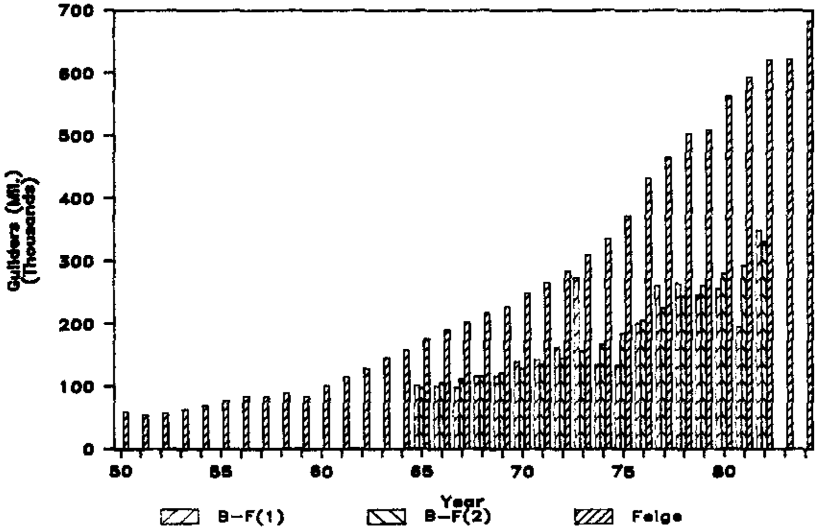

Fig. 5.16. Estimated currency payments

tions for offenses relating to property are less than 200 per 100,000 of the population aged 12–79 and, 'More than half of all recorded cases of simple theft relate to the theft of a vehicle' (p. 166). Thus, it appears that the security incentive to return currency to banks is unlikely to be compelling in the Netherlands.

Figures 5.16–5.19 provide information on the estimated volume of total cash

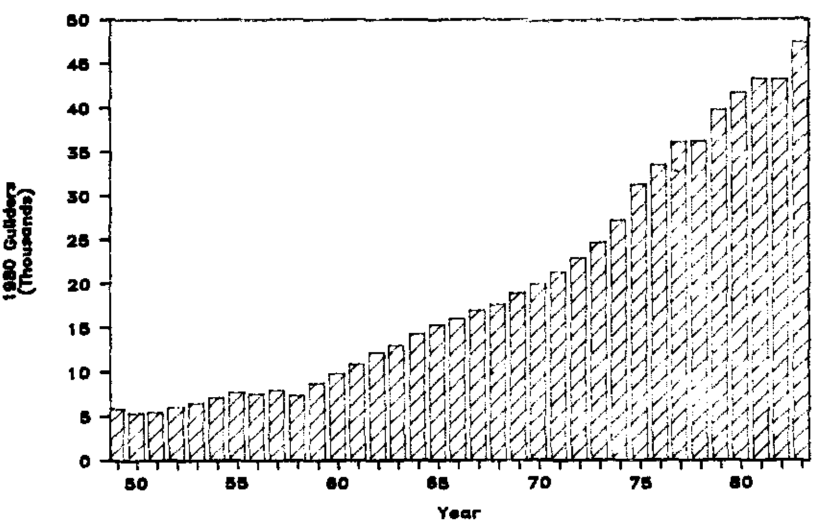

Fig. 5.17. Real per capita cash payments, 1980 guilders.

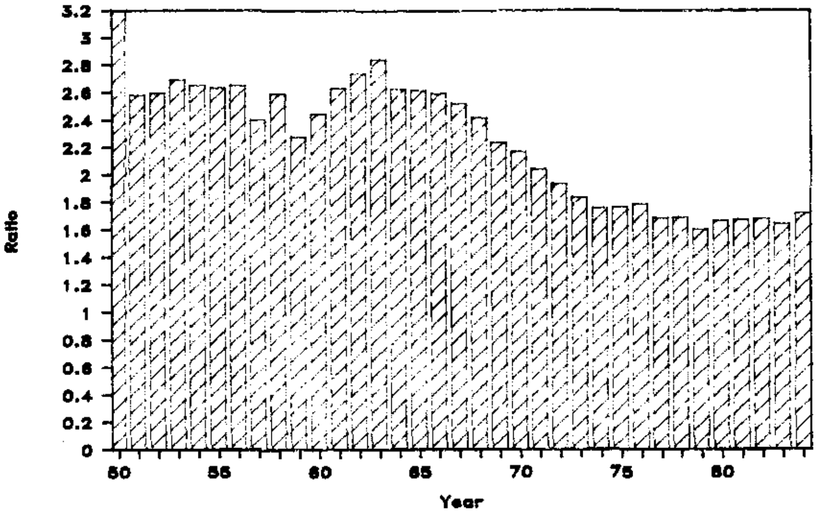

Fig. 5.18. Cash payments per guilder of GDP.

payments in the Netherlands. Figures 5.16 and 5.17 respectively reveal that cash payments have increased steadily throughout the period of estimation in both nominal and real per capita terms. Figure 5.16 reveals that the nominal estimates of cash payments of the present study are more than 1.7 times higher than those estimated by the Netherlands Bank (labeled (B-F(1) and (B-F(2)). As displayed in Figure 5.17 per capita cash payments exceeded 40,000 guilders in 1980. Of this total, approximately Fl. 5000 of payments were made with 1000-guilder notes, Fl. 25,000 of payments were effected with 50 and 100 guilder notes and the remaining Fl. 10,000 were effected with small denomination notes. As displayed in Figure 5.18 cash payments amounted to more than 1.7 times recorded GDP.[30]

In order to gauge the plausibility of these estimates, it is necessary to recall that cash payments represent all transactions that are effected by the use of currency. Such payments would include the sum of: final purchases of goods and services; factor-payment transactions; intermediate goods and services purchased; transfer payments; purchases of existing assets, both real and financial; purchases of newly created financial assets; purchases of foreign goods and services and purchases of foreign assets. On the basis of estimates of total payments made with checks and giro transfers[31] it is possible to estimate

[30] The foregoing calculations assume that the total stock of guilders in circulation are held by residents of the Netherlands. To the extent that some fraction of the Netherlands currency supply is held by residents of other countries, the estimates of total cash payments in the Netherlands will be overstated.

[31] Boeschoten and Fase (1984) pp. 62–63.

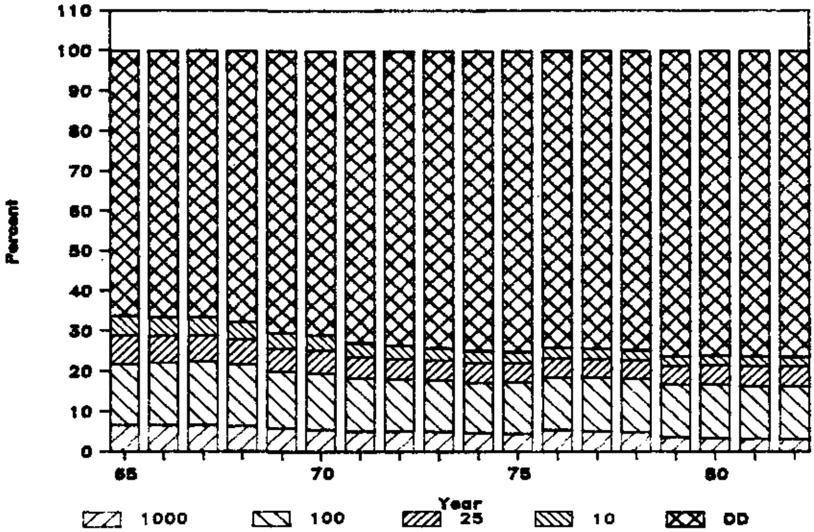

Fig. 5.19. Shares of payments.

the share of total payments (MV) made up by currency payments.

Figure 5.19 reveals that currency payment's share of estimated total payments (MV) declined from 33.7% in 1965 to 23.6% in 1982. The corresponding currency share of the money supply (currency + demand deposits and bank and giro institutions) declined from 45.7% in 1965 to 34% in 1982 (Figure

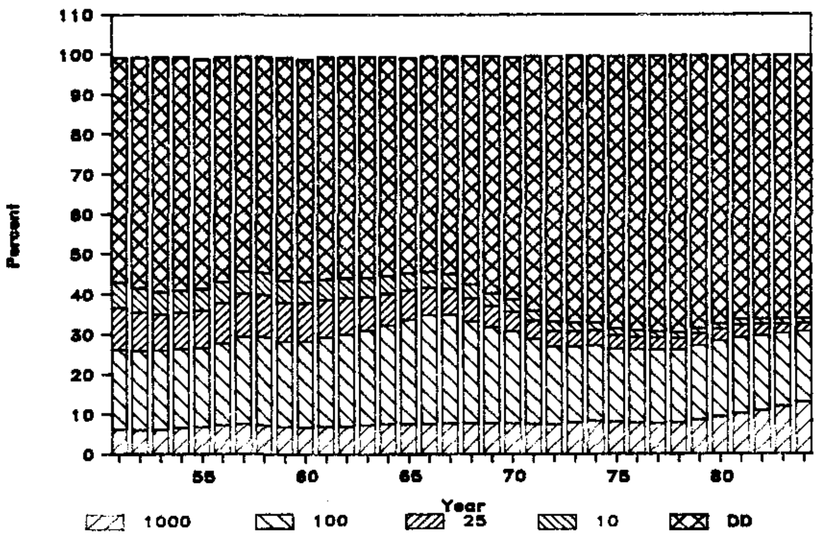

Fig. 5.20. Shares of the money supply.

5.20). In short, the role of currency in the monetary system has declined over time as measured by both stock and payment shares. Nevertheless, we are far from the 'cashless' society that many economists predicted. Moreover, gauging the role of currency and its components by stock measures, gives as inflated picture of the true role of currency in the payments mechanism.

5.3 The relationship between total payments and total transactions

In light of Fisher's claim that knowledge of the velocity of currency and cash payments could open 'a new realm in monetary statistics', it seems useful to explore some of the possible applications of the foregoing estimates of cash payments.

From the perspective of macroeconomic accounting, the measurement of cash payments makes possible the independent measurement of total payments (MV), suggesting the possibility that the equation of exchange (MV = PT) can be employed to provide a higher-order accounting identity than the conventional income-expenditure identity (Y = C + I + G), that has served the profession for the past fifty years.[32] Total transactions (PT), can be viewed as an aggregate that encompasses the key statistical entries in all current macroeconomic accounting schemes, in particular, the national-income and product accounts (NIPA); the input-output accounts (IO); the balance-of-payments accounts (BOP), and the flow-of-funds accounts (FOF).[33] Thus, the separate measurement of total monetary payments (MV) provides an independent empirical check on an appropriately aggregated sum of the entries in all of our present accounting systems. The long sought after, but highly elusive, 'integrated system of economic accounts', finds a natural conceptual framework in the flow equality constraint between payments and transactions. From this perspective, a positive difference between MV and PT, has a natural interpretation, namely, the sum of 'unrecorded' transactions and the 'statistical' discrepancy.

The Netherlands Central Bank has compiled an aggregative series for total transactions (PT),[34] that permits a preliminary examination of the question, 'Does MV = PT?'.

As displayed in Figure 5.21, the difference between estimated total payments and estimated total transactions[35] is both large and growing over time.

[32] See Feige, E. (1985a).

[33] In principle, total transactions must also include transfer payments and 'gross' financial transactions. Since past attempts to integrate current accounting systems have focused on balance sheet constraints rather than flow constraints, many financial flows are currently only available on a 'net' basis.

[34] Boeschoten and Fase (1984).

[35] Cash withdrawals from demand deposits are added to recorded transactions since such withdrawals give rise to demand deposit debits that are included in MV.

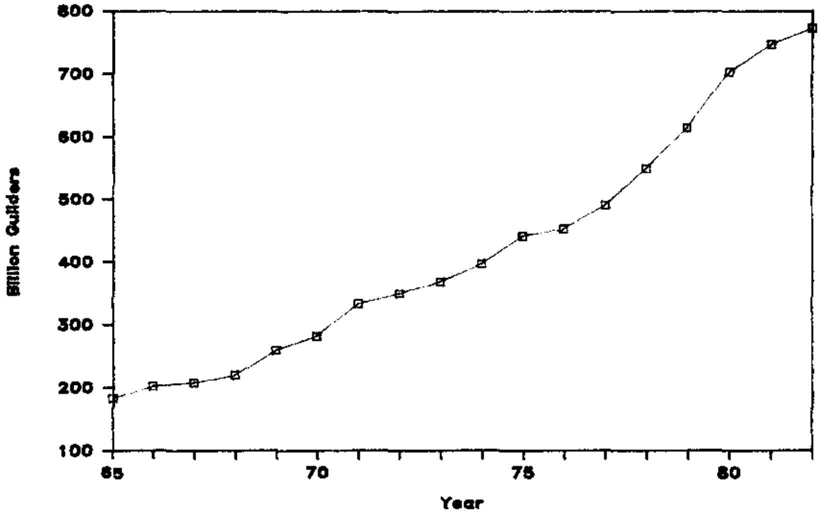

Fig. 5.21. MV–PT.

This sizable discrepancy between estimated payments and estimated transactions suggests that there remains a substantial volume of gross financial transactions that have not been properly accounted for in our current measures of PT. It would therefore be premature to conclude that the observed discrepancy between recorded payments and transactions represents an indicator of the size and growth of 'underground' transactions and hence unrecorded incomes. The problem, however, does not lie with the conceptual framework for estimating 'underground' transactions as suggested by the Netherlands Bank study, but rather with the dearth of empirical information concerning gross financial transactions. The equation-of-exchange constraint on our national accounting systems calls attention to this serious gap in our empirical knowledge. Only after this empirical deficiency is remedied, can we proceed to utilize Fisher's identity to obtain more accurate estimates of underground economic activities.

6. Summary and conclusions

Keynes' biographical essay on Robert Malthus ends with the lament:

> 'If only Malthus, instead of Ricardo, had been the parent stem from which nineteenth-century economics proceeded, what a much wiser and richer place the world would be today'.[36]

[36] Keynes, J.M. (1956), p. 36.

On this 50th anniversary of the publication of Keynes' *General Theory*, one is tempted to ask whether the twentieth-century world of economics would have been a wiser and richer place had its parent stem been Fisher rather than Keynes. Fisher's equation of exchange is clearly a more general conceptual framework than the income-expenditure approach that became the mainstay of Keynesian analysis. Yet, the equation of exchange did not benefit from the conceptual and empirical elaboration enjoyed by the income-expenditure approach. The economics profession's failure to pursue Fisher's payment-transaction approach was at least partly the result of the lacuna created by the inability to estimate the velocity of currency and hence cash payments. This paper has attempted to fill that gap by specifying a theory and a means of estimating currency velocity.

Given the means for measuring currency velocity and cash payments, it is possible to derive total payments estimates and to compare these with independent measures of total transactions. The equation of exchange can then be utilized as a higher-order constraint on the sum of all monetary transactions in the economic system. Once payments and transactions can be independently estimated, the equation of exchange can serve as the conceptual framework for integrating the monetary payments mechanism with an integrated system of currently available transactions accounts.

The ability to measure denomination-specific currency velocities also frees monetary economics from its long-standing tradition of measuring monetary stocks as the nominal sum of the existing media of exchange. Aggregate quantities of goods and services have always been measured by index numbers that take account of the relative value of different goods and services as measured by their market prices. Given estimates of the velocities of the different media of exchange, it now becomes possible to appropriately measure aggregate monetary stocks as index numbers that take account of the relative work that different media of exchange perform in the payments system. The appropriate weights for monetary aggregation are the velocities of the different media used for exchange. Given velocity-weighted measures of the 'money supply', and conversely, money-supply-weighted measures of payment velocities, it becomes possible to formally test the underlying economic relationships between monetary aggregates and final, intermediate and financial transactions and prices. In short, Fisher's equation of exchange can provide the conceptual and empirical basis for the development of a general theory of economic transactions.

7. Acknowledgement

The initial research for this paper was undertaken while the author was a

Fellow at the Netherlands Institute for Advanced Studies and subsequently as Cleveringa Professor at the University of Leiden. Financial support from the A.P. Sloan Foundation and the Graduate School of the University of Wisconsin is gratefully acknowledged as is the assistance of Mark Kennett.

Bibliography

Avery R et al. 1985. Cash and transaction account usage by American households. *Board of Governors of the Federal Reserve System.*

Boeschoten WC and Fase MMG. 1984. *Betalingsverkeer en officieuze economie in Nederland 1965–1982.* De Nederlandsche Bank n.v., monetaire monografieeen, 1.

Cramer JS. 1981a. The work money does: the transactions velocity of circulation of money in the Netherlands, 1950–1978. *European Economic Review,* 15: 307–326.

Cramer JS. 1981b. The volume of transactions and of payments in the United Kingdom 1968–1977. *Oxford Economic Papers,* N.S. 33: 234–255.

Cramer JS. 1986. The volume of transactions and the circulation of money in the United States 1950–1979. The Journal of Business and Economic Statistics, 4: 225–223.

Feige EL. 1979. How big is the irregular economy? *Challenge,* 22, November/December, 5–13.

Feige EL. 1980. *A new perspective on macroeconomic phenomena. The theory and measurement of the unobserved sector of the United States: causes, consequences and implications. Presented at the American Economics Association Meetings, Denver.*

Feige EL. 1985a. *The Swedish payments system and the underground economy.* Stockholm: Industrial Institute for Economic and Social Research.

Feige EL. 1985b. *The equation of exchange and national accounting systems.* International Conference on Income and Wealth, Noordwijk.

Feige EL. 1986. *Currency velocity and cash payments in the United States 1986.*

Fisher I. 1909. A new method of estimating the velocity of circulation of money. *Journal of the Royal Statistical Society,* 72.

Fisher I. 1911. *The purchasing power of money.* London: Macmillan.

Friedman M. 1956. The quantity theory of money – a restatement. *Studies in the Quantity Theory of Money,* Chicago: the University of Chicago Press.

Keynes JM. 1930. *A treatise on money.* London: Macmillan.

Keynes JM. 1936. *The general theory of employment, interest, and money.* London: Macmillan.

Keynes JM. 1956. *Essays and sketches in biography.* Meridian Books.

Laurent R. 1970. *Currency transfers by denomination.* Ph. D. Thesis, Chicago: University of Chicago Press.

Manski C and Goldin E. 1987. *The denomination-specific demand for currency in a high-inflation setting: the Israeli experience.* This volume.

Randall EB Jr and Mandel J. 1962. A statistical comparison of the wearing characteristics of two types of dollar notes. *Materials Research and Standards,* 2: 17–20.

Selden R. 1956. Monetary velocity in the United States, in: *Studies in the quantity theory of money,* edited by M Friedman. Chicago: University of Chicago Press.

Social and Cultural Report. 1980. Rijswijk: Social and Cultural Planning Office.

Spindt P et al. 1985. *A micro-analytic view of the payments mechanism: some preliminary results from the household survey of currency and transactions account usage.* Board of Governors of the Federal Reserve System.