

# A Re-Examination of the "Underground Economy" in the United States

A Comment on Tanzi

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THE PROCEDURES EMPLOYED by Vito Tanzi in a series of papers (1982, 1983, 1984) purport to estimate the size and growth of the "underground economy" in the United States. The motivation for this critique is the concern that Tanzi's methodology for measuring the "underground economy" is seriously flawed both conceptually and empirically.

Since the popular term "underground economy" is difficult to define precisely, the term "unreported income" is used to refer to the difference between the amount of income that ought to be reported to the tax authority under full compliance with the tax code and the amount actually reported.<sup>1</sup> Acharya (1984) has criticized Tanzi's failure to state clearly precisely what he sets out to measure, and finds fault with several of the assumptions that underlie his procedure. However, Acharya's critique neither ex-

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<sup>1</sup> The concept of unreported income should not be confused with the concept of unrecorded income. *Unrecorded* income refers to the amount of income that is inadvertently omitted from national income and product account (NIPA) measures of aggregate economic activity. To the extent that NIPA measures depend on tax source data that are biased downward as a result of unreported income, an increase in the latter can also increase the former. The relationship between the two measures is, however highly complex, as is indicated by recent efforts on the part of the U.S. Department of Commerce, Bureau of Economic Analysis (1985) to revise NIPA statistics in order to take account of discovered misreporting in tax source information.

amines Tanzi's empirical results in detail nor presents an acceptable alternative procedure.

The present comment specifies a general currency ratio model (GCR) for estimating the size and growth of unreported income.<sup>2</sup> The model helps to identify Tanzi's contribution to the literature and makes explicit the shortcomings of his particular approach. The GCR model is general enough to permit derivations of special cases that include earlier attempts to estimate unreported income. One of the special cases is a corrected version of the approach employed by Tanzi. Alternative estimates derived from special cases of the GCR model indicate that Tanzi's published empirical results grossly underestimate both the size and the growth path of unreported income.

The identification of the specific errors in Tanzi's method, and the presentation of an alternative framework, will, it is hoped, encourage a re-examination of other studies based on his approach. Despite the inherent complexity of attempting to estimate a phenomenon whose *raison d'être* is to defy detection, the framework for analysis presented in this paper is intended to encourage further research on what is increasingly recognized as a significant area of investigation. Noncompliance with tax laws has important implications for macroeconomic analysis, fiscal policy, and the reliability of macroeconomic information systems.

## A General Currency Ratio Model

### *Model Specification*

One method for obtaining estimates of unrecorded income relies on variations in the ratio of currency to demand deposits. The specific assumptions required to implement the estimation of unrecorded income by means of a currency ratio method are clarified by reference to a particular model. The GCR model presented below is sufficiently general to encompass all previous currency ratio methods (including a correct variant of Tanzi's method) as special cases.

Let

$C$  = actual currency stock

$D$  = actual stock of checkable deposits

<sup>2</sup> This model was first discussed in Feige (1980).

- $Y_o$  = reported income<sup>3</sup>  
 $u$  = subscript to denote the unreported sector  
 $o$  = subscript to denote the "official" or reported sector  
 $k_o$  = the ratio of currency to checkable deposits in the reported sector  
 $k_u$  = the ratio of currency to checkable deposits in the unreported sector  
 $v_u$  = unreported sector income velocity  
 $v_o$  = reported sector income velocity

The general currency ratio model contains the following specifications:

$$C = C_u + C_o \quad (1)$$

$$D = D_u + D_o \quad (2)$$

$$k_o = \frac{C_o}{D_o} \quad (3)$$

$$k_u = \frac{C_u}{D_u} \quad (4)$$

$$v_o = \frac{Y_o}{C_o + D_o} \quad (5)$$

$$v_u = \frac{Y_u}{C_u + D_u} \quad (6)$$

$$\beta = \frac{v_o}{v_u} \quad (7)$$

Equations (1) and (2) decompose the actual stocks of currency and checkable deposits<sup>4</sup> into their unreported and reported com-

ponents. Equations (3) and (4) are definitions of the terms  $k_o$  and  $k_u$ , which can be specified as constants or stable functions of other variables. Similarly, equations (5) and (6) define income velocity in the two sectors. To solve the model for  $Y_u$ , equation (6) is evaluated in terms of the model's observable variables, namely  $C$ ,  $D$ , and  $Y_o$ . Repeated substitution and rearrangement of terms yields the general solution for  $Y_u$  as

$$Y_u = \frac{1}{\beta} \cdot Y_o \cdot \frac{(k_u + 1) \cdot (C - k_o D)}{(k_o + 1) \cdot (k_u D - C)}, \quad (8)$$

which expresses unreported income as a function of the observable variables  $Y_o$ ,  $C$ , and  $D$  and three parameters or functions,  $\beta$ ,  $k_u$ , and  $k_o$ .<sup>5</sup>

The simplest variants of the general  $C/D$  method employ the following restrictive assumptions:<sup>6</sup>

- (1) Currency is the exclusive medium of exchange for unreported transactions ( $D_u \rightarrow 0$ ,  $k_u \rightarrow \infty$ ).
- (2) The amount of unreported income produced by a dollar of currency transacted in the unreported sector is the same as the amount of reported income produced by a dollar of currency transacted in the reported sector ( $\beta = 1$ ).
- (3) The ratio of currency to checkable deposits remains constant except for changes induced by the growth of unreported income ( $k_{ot} = k_o$  for all  $t$ ). Imposing these restrictions on the GCR model yields a simpler form of equation (8), namely,

$$Y_u = Y_o \cdot \left\{ \frac{C_o - k_o D}{(k_o + 1) D} \right\}. \quad (8a)$$

Assumptions (1) and (2) are implicit in Tanzi's approach and have been questioned by Feige (1980) and Acharya (1984). Tanzi's

directly employs the ratio of  $(C/M2)$ . The M2 measure includes time deposits that do not function as a final medium of exchange. Tanzi's procedure is internally inconsistent: he estimates the demand for money in terms of the  $(C/M2)$  ratio, but later he applies the M1 income velocity to determine "underground" income. Travelers checks and credit cards are sometimes considered to be media of exchange; however, since the purchase of travelers checks or the settlement of credit card accounts requires the use of currency or checkable deposits, the inclusion of these assets would amount to double counting.

<sup>5</sup> Four of the five critiques mentioned in Acharya's (1984) comment on Tanzi's work refer to the appropriate specification of the parameters  $\beta$ ,  $k_u$ , and  $k_o$ . Specifically, Acharya questions Tanzi's assumptions that (1)  $\beta$  is a constant and equal to unity; (2)  $k_u \rightarrow \infty$ ; and (3)  $k_o$  can be represented as a simple function of tax rates.

<sup>6</sup> These are the assumptions employed by Cagan (1958) and Gutmann (1977).

<sup>3</sup> Since the object of the analysis is to estimate the amount of income that is not properly reported to the tax authority, reported income refers to the amount of income actually reported to the tax authority, namely, "adjusted gross income." Alternatively, when  $Y_o$  is chosen to be a recorded NIPA income measure such as measured gross national product (GNP) (the variable employed by Tanzi), the GCR model must be adapted in order to produce an estimate of unrecorded income, namely the amount of income that ought to be recorded in NIPA income measures but is in fact excluded. The implementation of estimates of unrecorded income is empirically complex and is beyond the scope of the present paper. Acharya (1984) has criticized Tanzi's inappropriate use of measured GNP in his analysis. This same error is found in Gutmann (1977).

<sup>4</sup> The appropriate definition of the assets that constitute the money supply must be limited to those assets that function as a final medium of exchange. In practice, currency, demand deposits, and other checkable deposits comprise the final exchange media in the United States. Tanzi's estimation procedure incor-

sole contribution to the literature is his effort to relax assumption (3), namely, to consider  $k_o$  as a stable function rather than as a constant over time.

#### *Econometric Specifications of $k_o$*

The simplest versions of the GCR model assumed that the benchmark estimate of  $k_o$  was a constant rather than a function of other economic variables. In the spirit of Cagan's (1958) original investigation of the currency ratio,  $k_o$  is specified as a function to be estimated by econometric means. The advantage of this approach is that it takes explicit account of those economic factors that are believed to have affected observed variations in the currency ratio over time.  $k_o$  varies over time as a function of the behavioral determinants of the currency ratio in the reported sector of the economy. Since  $k_o$  is itself unobserved, it is necessary to derive an expression for  $k_o$  in terms of observed variables. Maintaining Tanzi's assumption that demand deposits are never used for the payment of unreported incomes ( $D_u \rightarrow 0$ ), it follows that the observed  $C/D$  ratio is defined as

$$\frac{C}{D} = \frac{C_o}{D_o} + \frac{C_u}{D_o} = k_o + \frac{C_u}{D_o} \quad (9)$$

Tanzi's papers suggest that the ratio  $k_o$  can be approximated by a function such as

$$k_o = f_1(y, r, ws) \quad (10)$$

and

$$\frac{C_u}{D_o} = f_2(\tau), \quad (11)$$

where ( $y$ ) is an appropriate measure of income, ( $r$ ) is an interest rate, ( $ws$ ) is the income share of wages and salaries, and ( $\tau$ ) is an appropriately defined tax rate that represents the incentives for tax evasion. Since the left-hand side ratios in equations (10) and (11) are unobservable, an estimate of  $k_o$  must be obtained from a regression of the observed  $C/D$  ratio as shown in equation (9). Thus,

$$\frac{C}{D} = f_1(y, r, ws) + f_2(\tau). \quad (12)$$

Equation (12) reveals that the observed currency ratio is the sum of two functions. Tanzi's choice of a multiplicative (log linear)

functional form to estimate the currency ratio therefore violates the additive specification implied by equation (12).<sup>7</sup>

Tanzi's empirical implementation of the simple  $C/D$  model is further flawed by his choice of variables and his method for estimating  $k_o$ . Tanzi specifies the currency ratio as  $C/M2$  rather than  $C/D$ , but later he employs the income velocity of  $M1$  to calculate  $Y_u$ . Tanzi does not explain this inconsistent treatment. Acharya (1984, p. 744) has correctly objected to Tanzi's use of measured gross national product as a proxy for  $y$ , but acknowledges that the appropriate income variable "is not easy to construct—and it is not offered by Tanzi" (p. 744). The closest approximation to the desired income variable is the income series produced by the U.S. Department of Commerce, Bureau of Economic Analysis that estimates the amount of total adjusted gross income (AGI) implied by independent NIPA income measures.<sup>8</sup> This latter income estimate is employed in the results presented below.

The choice of an appropriate tax rate ( $\tau$ ) to represent incentives for misreporting income is also of importance. Tanzi's (1983) results are based on two inadequate tax rate proxies. The first is a "weighted average tax rate on interest income" which does not reflect tax incentives on other income sources, and the second is the "ratio of personal income taxes to personal income net of transfers" which reflects the ex post average tax rate rather than the average effective marginal tax rate. Barro and Sahasakul (1983) have constructed time series estimates of the "average effective marginal tax rate," and these are employed in the present paper.<sup>9</sup>

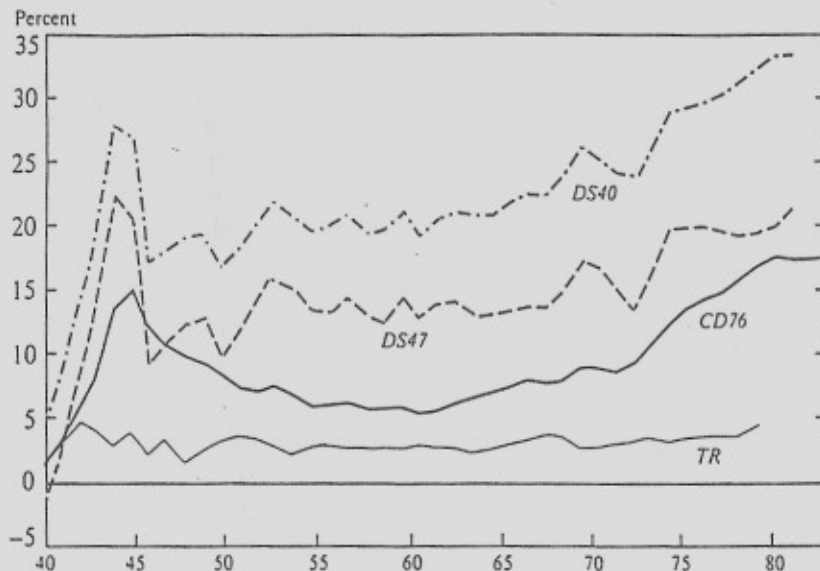
To derive the predicted time path of  $k_o$ , it is necessary to estimate equation (12) and then to obtain the dynamic forecast of  $k_o$  by setting  $f_2(\tau) = 0$ . The estimating equation is assumed to be linear and the disturbance term is modeled as a first-order ARMA process. The reported results are derived by estimating equation (12) for the periods 1940–80 and 1947–80.

<sup>7</sup>This appears to be the point suggested by Acharya (1984, p. 744) who notes, "either the estimate of the underground economy is wholly additive . . . or . . . is logically flawed."

<sup>8</sup>The Bureau's estimate of total AGI (Park, 1981) is based on reported personal income (PI), which is adjusted by reconciliation items reflecting items of income included in PI but excluded from AGI and items of income included in AGI but excluded from PI.

<sup>9</sup>The author's appendix (see n. 1 above) reveals that this tax rate is significantly different in both level and temporal pattern than the tax rates employed by Tanzi.

Figure 1. Ratio of Unreported to Reported Income: Alternate Specifications



Both periods abstract from the effects of the Great Depression.<sup>10</sup> The shorter estimation period also excludes the period of price, wage, and interest controls that obtained during World War II. The reported results for the period 1940–46 and for 1981–82 are derived from dynamic aftcasts and forecasts from the estimated equation.<sup>11</sup> The forecast values of  $k_o$  are then substituted into equation (8a) in order to obtain a conceptually consistent estimate of  $Y_u/Y_o$ .

Figure 1 presents the estimates of  $Y_u/Y_o$  that result from a dynamic simulation (DS40, DS47) of the GCR model allowing for a variable  $k_o$ . Tanzi's reported results (TR) and the results from the simple C/D specification (CD76) are included for comparison.<sup>12</sup> Figure 1 reveals that the variable  $k_o$  specifications (DS40, DS47) yield higher estimates of the percentage of adjusted gross income

<sup>10</sup> Tanzi estimates the model over the period 1930–80. Exclusion of the depression years is justified on the grounds that the C/D ratio was primarily affected by bank failures during this period.

<sup>11</sup> The estimated equations are available in the author's appendix.

<sup>12</sup> The CD76 results represent the fixed  $k_o$  assumption and are obtained from equation (8a) employing as a benchmark for  $k_o$  the IRS estimate of unreported income for 1976 (IRS, 1983).

that is unreported than those from the fixed  $k_o$  specification. This result is expected, since many economists had predicted the emergence of a "cashless society" implying a declining temporal pattern for  $k_o$  ratio rather than constancy of  $k_o$  over time. Figure 1 reveals that Tanzi's reported results are well below those obtained by the alternative procedures and that they display a very different temporal growth path than those obtained from a corrected version of the variable  $k_o$  specifications.

It is also possible to relax the two remaining restrictions ( $k_u \rightarrow \infty$ ;  $\beta = 1$ ) imposed by Tanzi and the simple (C/D) method, namely, that currency is the exclusive medium of exchange in the underground economy, and that the income velocities of the recorded and unrecorded sectors are identical.

Despite Tanzi's assertion that "in the United States, where the Internal Revenue Service can at any time get a statement of all checks written and cashed by any individual, I have difficulty in believing that many of these transactions occur through the banking system," there is, in fact, considerable evidence to suggest that payments of unreported incomes are by no means confined to the use of currency. For example, a study by the U.S. Internal Revenue Service—IRS—(1979, p. 13) on unreported income found that "the unreported income problem extends beyond incomes paid in currency." The IRS suggests that between one fourth and one third of unreported income did not represent proceeds from currency transactions. A more recent survey (conducted by the University of Michigan's Institute for Social Research and commissioned by the IRS)<sup>13</sup> of "informal" markets found that only 50 percent of informal purchases of home repairs and catering services were paid with cash, and that approximately 25–30 percent of purchases of other informal services were made by non-cash payments. These findings suggest that perhaps 75 percent of all unreported income payments are made with cash, thus  $k_u \approx 3$ .

Alternative estimates of  $Y_u/Y_o$  can then be obtained by employing the general solution of the GCR model presented in equation (8) with  $k_u = 3$ .<sup>14</sup> It is also possible to examine the sensitivity of the final estimates to variations in the parameter  $\beta$  describing the ratio of income velocities in the two sectors. Tanzi assumes equality between the two velocities ( $\beta = 1$ ). To the extent that unreported income is largely derived from the service sector, which involves

<sup>13</sup> Smith (1982; 1985).

<sup>14</sup> For simplicity of exposition, we retain the assumption that  $k_o$  is a constant determined by the 1976 IRS benchmark estimate (IRS, 1983).

fewer intermediate transactions than does the nonservice sector,  $\beta < 1$ . On the other hand, a lower propensity to consume unreported income would imply that  $\beta > 1$ . The former assumption produces higher estimates of unreported income, whereas the latter assumption reduces estimated unreported income.

Figure 2 presents estimates of  $Y_u/Y_o$  given the general currency ratio model with the specifications

$$\begin{aligned} k_u &= 3 \\ k_{ot} &= k_o = k_{1976} \\ \beta &= 1; \beta = 1.1; \beta = 0.9 \end{aligned}$$

Relaxation of the assumption that currency is the exclusive medium of exchange for transacting unreported income increases the estimates of unreported income to levels that approach the range estimated by the variable  $k_o$  specifications. Departures from the  $\beta = 1$  assumptions impart inverse proportional changes to the estimated amount of unreported income.

#### A Summary of the Sensitivity Analysis Findings

The foregoing sensitivity analysis of the consequences of relaxing the most restrictive forms of the GCR model reveals that

Figure 2. Ratio of Unreported to Reported Income:  $K_u = 3$  (Alternate  $\beta$ )

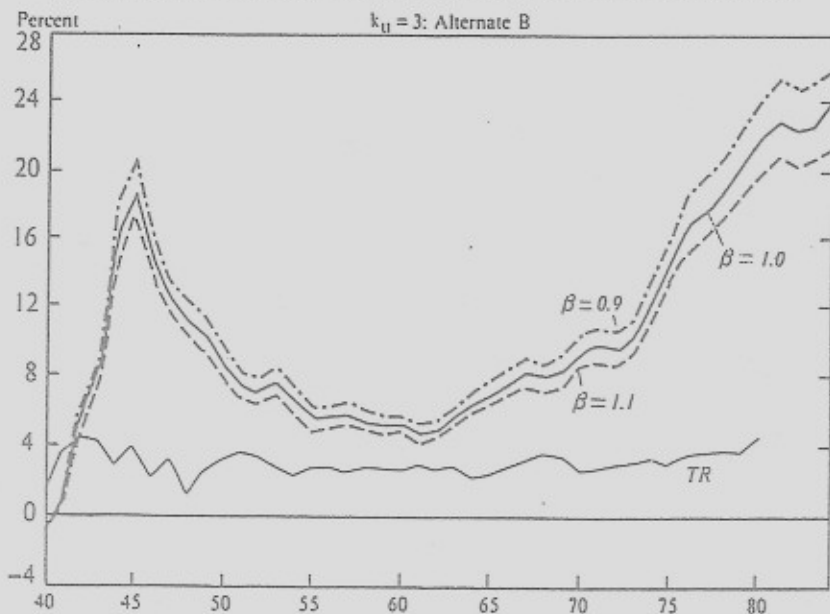
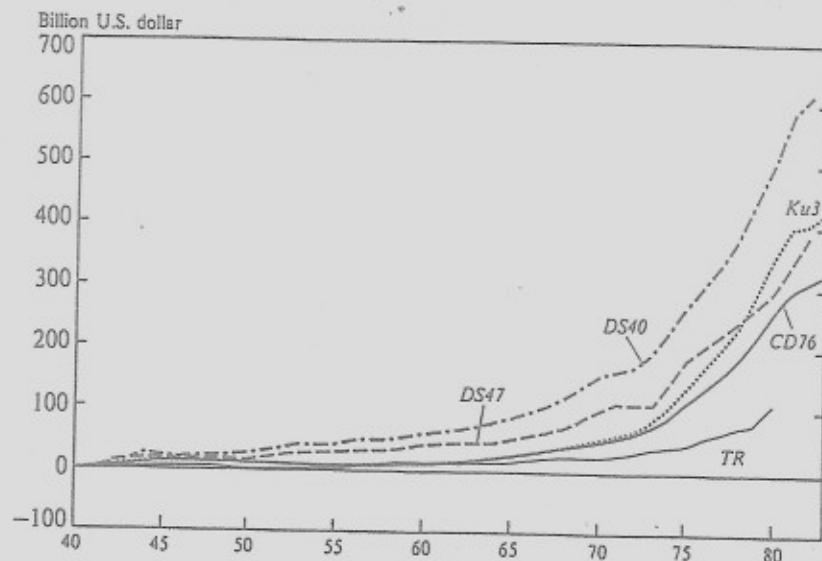


Figure 3. Estimated Unreported Income: Alternate Estimates



estimates of the size and growth of unreported income significantly increase when a functional specification of  $k_o$  replaces the assumption of a constant  $k_o$  and when checks are permitted to function as a means of payment for unreported income.

Figure 3 summarizes the alternate estimates of unreported income. The figure reveals that Tanzi's reported estimate (TR) falls well below those obtained by either the simple C/D method (CD76) or by the version of the GCR model that permits checks to be used for unreported transactions [ $K_u 3$ ].<sup>15</sup> The versions of the GCR model that permit a variable  $k_o$  specification (DS47 and DS40), and are therefore closest to the spirit of Tanzi's approach, produce the largest estimates of unreported income. The analysis therefore suggests that earlier estimates of the unduly restrictive special cases of the GCR model, like that of Gutmann (1977), produced significant underestimates of unreported income. Moreover, Tanzi's (1982, 1983) published estimates of unrecorded income are several hundred billion dollars below those obtained from a corrected specification incorporating the spirit of his approach.

<sup>15</sup>The figures reported for ( $K_u 3$ ) are based on an assumed value of unity for  $\beta$ .

The author's appendix presents efforts to replicate Tanzi's reported results and attempts to determine the key factors that lead Tanzi to such gross underestimates of unreported income. The replication effort reveals that the major sources of underestimation are Tanzi's inappropriate use of static rather than dynamic forecasts, and his incorrect specification of the currency ratio as being multiplicative rather than additive.<sup>16</sup>

The replication efforts indicate that Tanzi's erroneous results are due to his inappropriate use of static forecasting methods, his misspecification of the appropriate definition and functional form for the currency ratio, his incorrect choice of income and tax rate variables, and the particular time period he chose for analysis.

### Estimates of Revenue Losses Due to Noncompliance

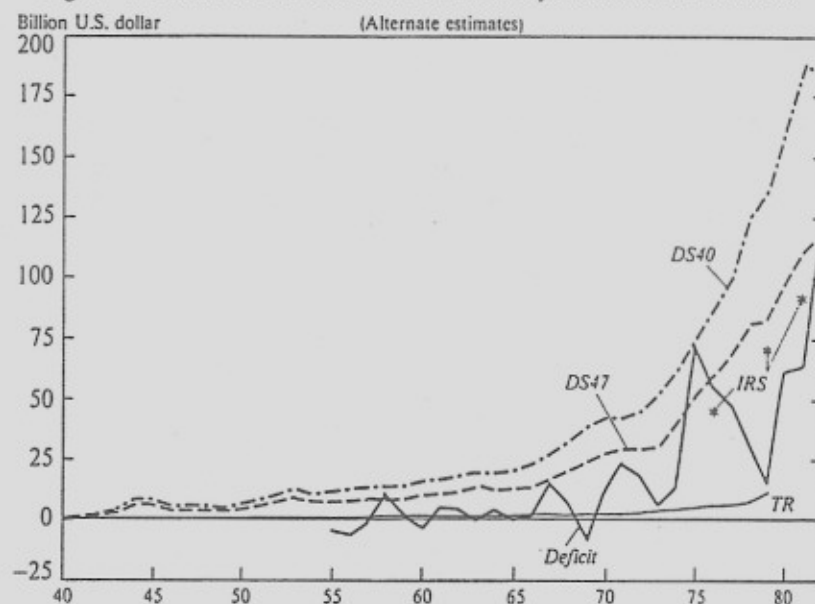
In light of the current concern with the problem of growing deficits, it is instructive to examine the implications of the aforementioned estimates of unreported income for the loss of tax revenues that result from underreporting of income on tax returns. A rough estimate of the implied tax losses are calculated by applying the recent Barro and Sahasakul (1983) estimates of the average marginal tax rate on federal income taxes to the estimated amounts of unreported income derived from several of the special cases of the GCR model.<sup>17</sup>

Figure 4 displays various estimates of amounts of federal income tax revenue losses owing to unreported income. The figure displays Tanzi's (1983) published estimates (TR), the estimates produced by the IRS (1983), and the estimates derived from the variable  $k_o$  specification of the GCR model (DS47 and DS40). Figure 4 includes as a reference point the actual budget deficit.

<sup>16</sup>The author's appendix also includes a formal proof to demonstrate that Tanzi's reliance on static forecasting procedures produces a systematic downward bias in his estimates of "underground" income. Dynamic forecasts are shown to yield unbiased estimates of the counterfactual case that Tanzi seeks to estimate. When his procedure is replicated using correct dynamic forecasting methods, with his original data and misspecified functional form, his original model is shown to produce estimates of the "underground" economy that by 1980 exceed his reported estimates by \$288 billion.

<sup>17</sup>The estimated tax losses would be considerably higher if account were taken of the loss of social security contributions that result from unreported income and the fact that unreported income would be subject to higher marginal tax rates as a result of the progressivity in the tax schedules.

Figure 4. Federal Income Revenue Loss and Deficit: Alternate Estimates



The actual budget has been in deficit since 1970. Both the IRS estimates and the variable  $k_o$  in the GCR model estimates suggest that full compliance with existing tax codes would have produced sizable budget surpluses for most of the decade. Once again, this finding contradicts the results published by Tanzi.

### Summary and Conclusions

The foregoing analysis has sought to demonstrate that Tanzi's procedure for estimating the "underground economy" is seriously flawed in both concept and execution. Moreover, his characterization of his own erroneous results are inconsistent with recent independent estimates of unrecorded income. Tanzi's contribution to the literature lies in his effort to relax a particularly restrictive assumption that has been implicitly imposed in several earlier efforts to estimate the size of unreported income by employing a currency ratio methodology. When this insight is correctly incorporated in the currency ratio model, it produces estimates of unreported income and tax revenue losses that are several times larger than those reported by Tanzi.