Sector-Specific Volatility Patterns in Investment

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Abstract

This paper addresses the question if there are differences between time patterns in the volatility of investment across different industrial sectors. A competitive partial-equilibrium model with quadratic adjustment costs in investment and a GARCH demand shock is developed to predict aggregate investment in a sector. It is shown that under the assumptions made in the model, the GARCH property is inherited by the aggregate investment process in the rational-expectations equilibrium. The equation for investment from the model is estimated on quarterly time series from six industrial sectors in the UK. As conjectured, GARCH effects play an important role in some sectors but are not significant in others. Astonishingly, the volatility patterns are in general very different across sectors. This suggests that sector-specific factors are more important in determining investment volatility than the macroeconomic environment.

1 Introduction

There has been considerable interest in the volatility of aggregate investment in connection with the volatility of macroeconomic aggregates in recent years. For example, there is a literature on the so-called *Great Moderation*, the large and significant decline in macroeconomic volatility in the U.S. since the middle of the 1980s.[1] In the context of emerging markets, it has been investigated if the maturity of a country's financial sector is linked to the volatility of investment growth, consumption and gross domestic product (Denizer, Iyigun & Owen 2000). Furthermore, there is a large body of literature on the investment inflows into countries that experience periods of large exchange rate volatility. Byrne & Davis (2002) is an example for this.

However, there has been almost no interest in the volatility of investment *below* the aggregate level. This paper attempts to fill this gap and tackle the question if there are differences between the time patterns in volatility of investment across different sectors. On an intuitive level, it is quite reasonable to hypothesize that investment volatility should be time-varying in sectors that experience times of rapid changes in investment incentives with spells of consolidation and relative calm inbetween. The changes in investment incentives could be brought about by changes in products and production technologies, which tend to come in bulks in some industries. These effects should be relatively weak in consolidated sectors, however. In these sectors, one would expect volatility in investment to be constant.

The paper develops a theoretical linear-quadratic model with perfect competition in the final-goods market and quadratic adjustment costs in the investment technology. The model features a stochastic process which shifts production cost and the demand for the final good. The process is assumed to be autoregressive of order one. Furthermore, it can exhibit time-varying conditional variance, which is assumed to have a GARCH(1,1) structure. The model yields a simple linear equation for aggregate investment in equilibrium. I show that the AR(1) and the GARCH(1,1) property are inherited by the aggregate investment process and estimate the equation directly on aggregate investment data for six industrial sectors in the UK[2]: Chemicals;

[1] Campbell (2004), for example, studies if macroeconomic uncertainty and the predictability of macroeconomic variables has an effect on the volatility of macroeconomic aggregates.

[2] A simple operation in the lag-operator is needed to eliminate the sector's aggregate capital stock from the estimation equation.

Engineering; Food, Drink and Tobacco; Fuels; Metals; Textiles and Leather.

Indeed, the evidence suggests that the demand shifter has distinctive GARCH properties for three of the six sectors. In the three other sectors, the hypothesis of a constant conditional variance cannot be rejected. Surprisingly, there is little evidence for an autoregressive structure in the process of the demand shifter. As for the time patterns of volatility, it is interesting to notice that the path of conditional variance over time does not display any similarities across sectors. Sector-specific factors seem to be far more important in determining conditional volatility of investment than the general macroeconomic climate.

The paper is organized as follows: Section 2 describes the data used in the estimations. In section 3, the theoretical partial-equilibrium model is developed. Section 4 presents the estimation results. In section 5, I recover the process for the demand shifter in the six sectors which is implied by the estimation results and its conditional variance. Section 6 concludes.

2 Data

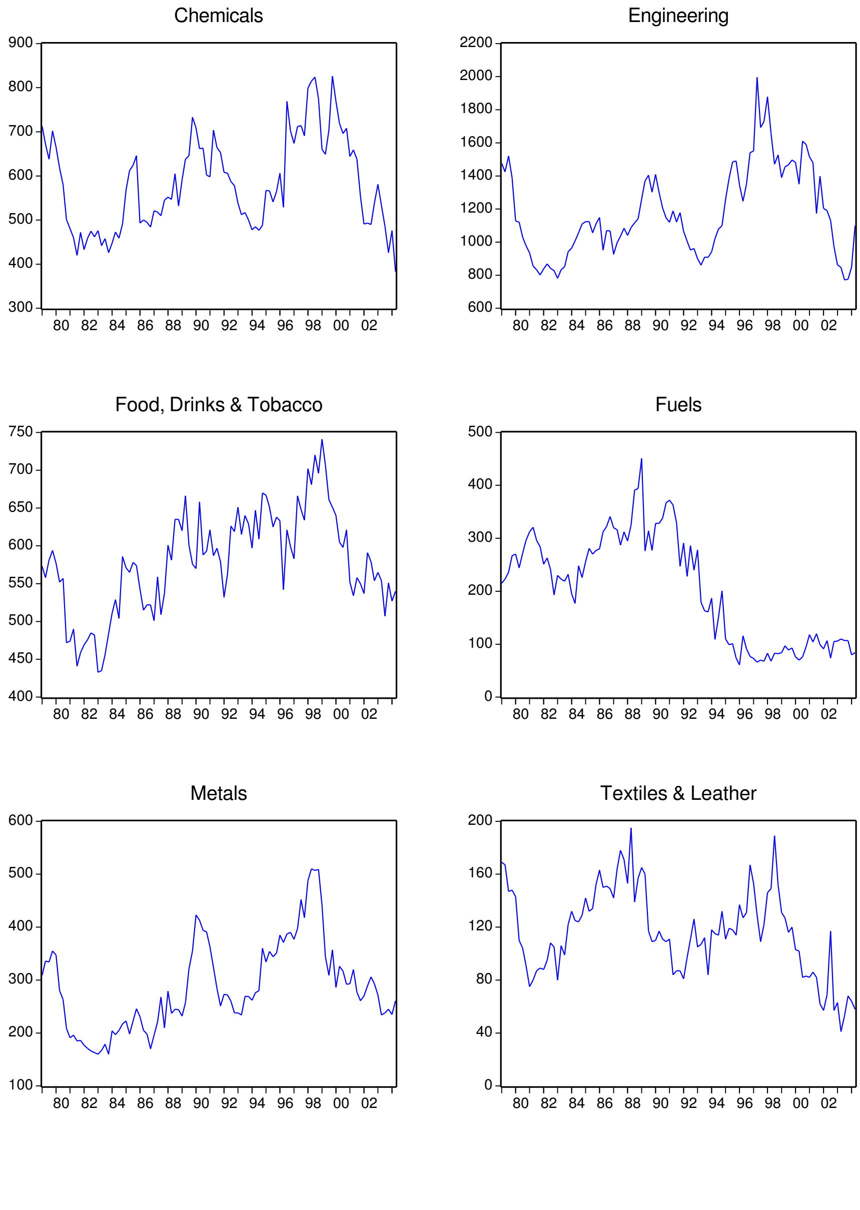
The data set that that is used for estimating the model is taken from the UK's Office for National Statistics' website. It consists of six quarterly time series for aggregate business investment by industry at 2001 prices in millions of British Pounds. The six industries are: Chemicals; Engineering; Food, Drink and Tobacco; Fuels; Metals; Textiles and Leather. The data set ranges from the first quarter of 1979 to the second quarter of 2004. The time series are seasonally adjusted and measure investment by total capital expenditure in the sector. The time series are plotted in figure 1.

Visual inspection suggests that the series can be assumed to be stationary over the sample period. Hence, no effort will be made to include a time trend in the theoretical model.

3 The Model

I consider a partial-equilibrium model to derive a relationship between investment in a sector and the expected demand and cost shocks. The model is similar in spirit to the ones studied by Lucas & Prescott (1971) and Sargent (1987). In those models, firms face adjustment costs when they change

Figure 1: Investment Data



output over time. In the model considered here, however, there will be an adjustment cost for investment in the capital stock.

There are n firms in a competitive market for a single consumption good. Each firm i produces y_t^i at time t with the linear technology

$$y_t^i = k_t^i, \tag{1}$$

where k_t^i is the capital stock of firm i at time t.[3] Aggregate quantities are denoted by upper-case letters:

$$Y_t = \sum_{i=1}^{n} y_t^i \tag{2}$$

$$K_t = \sum_{i=1}^{n} k_t^i \tag{3}$$

The decision variable for the firm is investment i_t.[4] I assume the following standard form of depreciation:

$$k_{t+1} = \delta k_t + i_t, \tag{4}$$

where $0 < \delta < 1$.

Aggregate demand in the market for the final good in the sector is determined by the following linear equation:

$$p_t = A_0 - A_1 Y_t + \nu_t \tag{5}$$

ν_t is a demand shifter that follows an AR(1) process:

$$\nu_{t+1} = \rho \nu_t + \varepsilon_{t+1}, \tag{6}$$

where $-1 < \rho < 1$ such that the process is stationary. The AR(1) structure allows for persistence in the shocks to the demand curve. I assume that the innovation ε_{t+1} is a martingale difference sequence which follows a GARCH(1,1) process. This means:

$$h_t := \mathrm{Var}_t(\varepsilon_{t+1}) = \mathrm{E}_t[\varepsilon_{t+1}^2]$$
$$h_{t+1} = \omega + \alpha \varepsilon_{t+1}^2 + \beta h_t,$$

[3]There is no loss in generality by assuming that the linear relationship is one-to-one.

[4]I drop the subscript i when it is clear that the respective variable refers to the representative firm.

where $\omega > 0$, $\beta > 0$ and $\alpha + \beta > 0$. This process captures the idea that the shifts in the demand curve might be greater in more turbulent times and smaller in calm times. Turbulent times might be induced by shifts in preferences, e.g. in industries where certain products might become "in" or "out" following certain trends. In the beginning of a fad for a good, demand might behave more erratically than normally.

Alternatively, technological innovations might bring about changes in the nature of some goods or create entirely new goods in a sector. These changes in the nature of the good are not explicitly modeled here, but the representative good should be thought of as encompassing all products that are offered by the sector under consideration. Later on –when the optimization problem of the firm is considered– I will show that also changes in the production technology and expectations about *future* demand and *future* productivity fit into this framework.

Now the stage is set to consider the decision problem of the firm. The firm chooses investment i_t at time t observing the current demand shock ν_t, its own capital stock k_t and the industry-wide capacity K_t. There are quadratic costs of investment, which can be interpreted as convex adjustment costs in the capital stock. Convex adjustment costs capture the idea that gradual increases in the capital stock have a lower per-unit cost than big, abrupt changes. The firm is risk-neutral and discounts future profits at a constant rate R^{-1}. Its maximization problem is

$$\max_{\{i_t\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} R^{-t}(p_t k_t - 0.5 g i_t)$$

$$\text{s. t. } k_{t+1} = \delta k_t + i_t$$

This formulation assumes that the marginal cost of a unit of investment is a time-invariant linear function in i_t with slope g.

Also, it is important to notice that this formulation of the firm's problem includes possibly time-varying costs c_t . This can be seen if p_t is thought of as a market price minus a time-t production cost that is uniform across firms:

$$p_t = p_t^S - c_t,$$

where p_t^S is the price paid for the final good on the market. Hence, the demand shifter ν_t also includes shocks to production costs, which might be induced by technological progress in production, shocks to factor costs or other changes related to production.

It is assumed that any particular firm is so small that its decisions' impact on aggregates is negligible. If this holds, then forecasts for future prices will be based solely upon the aggregate state variables K_t and ν_t. Expectations are required to be rational, i.e. the law of motion for K_t as*perceived* by the representative firm has to be the same as the *actual* law of motion that results from the decisions of the n firms *given the perceived law of motion.* Assume for now that the perceived law of motion for capital is linear in the relevant state variables:

$$K_{t+1} = H_0 + H_1 K_t + H_2 \nu_t \tag{7}$$

Using the production function (1), we can write the Bellman equation for this problem as follows:

$$v(k_t, K_t, \nu_t) = \max_{i_t}\{(A_0 - A_1 a K_t + \nu_t) a k_t - 0.5 g i_t^2 + R^{-1}\,\mathrm{E}_t\left[v(k_{t+1}, K_{t+1}, \nu_{t+1})\right]\}, \tag{8}$$

where the maximization is subject to the law of motion for capital (4), the stochastic process for the demand shifter (6) and the perceived law of motion for the aggregate capital stock (7). As to be seen very soon, the GARCH state variable h_t need not be included in the set of state variables since the firm is risk-neutral. Applying the Benveniste-Scheinkman Theorem, we get the following first-order condition:

$$g i_t = R^{-1}\,\mathrm{E}_t[A_0 - A_1 K_{t+1} + \nu_{t+1}], \tag{9}$$

where the expectation E_t is taken regarding the stochastic process ν_t.

It is worthwhile to pause at this moment to have a look at this marginal condition. The left-hand side is the marginal cost of one unit of investment in t; the more the firm increases investment, the higher the cost of a marginal unit. On the right-hand side, we have the marginal benefit of investing in one more unit of capital today. By increasing the capital stock by one unit, the company is able to sell one more unit of output at the price given by the market in $t + 1$. This price depends negatively on aggregate supply in the next period –which is determined by the aggregate capital stock K_{t+1}– and positively on the expected value of the demand shifter ν_{t+1}.

It is noteworthy that the firm cares only about the *expectation* of the demand shifter in $t + 1$, not its current *level*. Hence the process ν_t can also be thought of as a process of *expected profitability* in a sector that is common knowledge among all firms. This is in some sense a more satisfactory notion of ν_t than the narrow one introduced before, since we would expect investment to respond more strongly to future expected profitability than to a current, transient shock.

Furthermore it is important to notice that the firm is not concerned about the future variance of demand since it is risk-neutral – only the expectation of ν_t enters the first-order condition. Hence the current conditional variance h_t need not be included in the set of state variables for the firm's problem.

Solving (9) for the optimal level of investment yields:

$$i_t = \frac{(A_0 - A_1 H_0) - A_1 H_1 K_t + (\rho - A_1 H_2)\nu_t}{Rg} \tag{10}$$

Setting the number of firms n equal to one and using equations (4) and (9) yields the following equation for the actual law of motion of the capital stock given the perceived law of motion (7):

$$K_{t+1} = \frac{(A_0 - A_1 H_0)}{Rg} + \left(\delta - \frac{A_1 H_1}{Rg}\right) K_t + \frac{\rho - A_1 H_2}{Rg}\nu_t$$

In the rational-expectations equilibrium, this actual law of motion has to be equal to the perceived law of motion (7). Setting $\frac{(A_0 - A_1 H_0)}{Rg} = H_0$, $\delta - \frac{A_1 H_1}{Rg} = H_1$ and $\frac{\rho - A_1 H_2}{Rg} = H_2$, one can back out the following values for H_0, H_1 and H_2 in the rational-expectations equilibrium:

$$H_0 = \frac{A_0}{A_1 + Rg}$$
$$H_1 = \frac{\delta g}{g + A_1/R}$$
$$H_2 = \frac{\rho}{A_1 + Rg}$$

Using the equation for optimal investment (10) one can obtain the process for aggregate investment:

$$I_t = \frac{A_0}{A_1 + Rg} - \frac{\delta}{A_1 + Rg} K_t + \frac{\rho}{A_1 + Rg}\nu_t$$

Thus, aggregate investment is a linear function of the current capital stock and the current demand shifter. To simplify notation, write

$$I_t = \gamma_0 + \gamma_1 K_t + \gamma_2 \nu_t. \tag{11}$$

4 Estimation

Since there are no data available on the aggregate capital stock in the respective sectors, some work on equation (11) has to be done to obtain a tractable estimation equation. From the law of motion for capital (4) and the assumption that δ is bound between zero and one, capital can be expressed as a distributed lag of investment:

$$K_t = (1 - \delta L)^{-1} I_{t-1}, \tag{12}$$

where L is the lag-operator: $Lz_t = z_{t-1}$. Now, pre-multiply equation (11) by $(1 - \delta L)^{-1}$ and use (12) to obtain:

$$I_t = (1 - \delta)\gamma_0 + (\delta + \gamma_1) I_{t-1} + (1 - \delta L)\gamma_2 \nu_t \tag{13}$$

As for the innovation of the process, we have now applied an MA(1) filter to an AR(1) process, which results in an ARMA(1,1) process. Explicitly, define the innovation of the new process by

$$\zeta_t := (1 - \delta L)\gamma_2 \nu_t. \tag{14}$$

Then, multiply out using the law of motion for ν_t from equation (6):

$$\zeta_t = \gamma_2(\rho\nu_{t-1} + \varepsilon_t - \delta\rho\nu_{t-2} - \delta\varepsilon_{t-1})$$

Collecting the two terms containing lags of ν_t and observing that $\zeta_{t-1} = (1 - \delta L)\gamma_2 \nu_{t-1}$ this amounts to:

$$\zeta_t = \rho\zeta_{t-1} + \gamma_2\varepsilon_t - \gamma_2\delta\varepsilon_{t-1}$$

Hence we can estimate the following equation as an autoregression with one lag where the disturbance follows an ARMA(1,1) process with GARCH(1,1) innovations. This can be done using standard procedures. The estimation equation is

$$I_t = \tilde{\gamma_0} + \tilde{\gamma_1} I_{t-1} + \zeta_t. \tag{15}$$

Since γ_2 is only a scaling parameter for the variance of ε_t, δ will be identified by the MA(1) parameter times -1. Furthermore, ρ (the persistence parameter in the AR(1) process for the demand shifter) is identified by the AR(1) parameter for the residual ζ_t. The characteristics of the GARCH process ε_t can be gleaned from the estimates for ω, α and β in the GARCH equation. The other parameters in the model –the marginal adjustment cost g, the discount rate R^{-1} and the parameters A_0 and A_1 in the underlying demand curve– are not identified. However, this is not necessarily a drawback given that the primary purpose of the estimation exercise is to learn something about the behavior of the hidden process for the demand shifter.

Table 1: Results of Estimation

	Chemicals	Engineering	Food	Fuels	Metals	Textiles
constant	45.49	104.02	42.37	1.8311	24.94	11.25
	(38.35)	(7090.38)	(34.39)	(3.2937)	(2598.56)	(6.1605)
I_{t-1}	0.9153*	0.9151	0.9259*	0.9809*	0.9295	0.8946*
	(0.0642)	(5.7888)	(0.0596)	(0.0168)	(7.3562)	(0.0493)
AR(1)	0.2016	0.9142	-0.1671	0.5354*	0.9289	-0.1563
	(0.4803)	(5.7954)	(0.4806)	(0.1778)	(7.3819)	(0.7221)
MA(1)	-0.3786	-0.9581*	-0.1469	-0.7389*	-0.9837*	-0.0202
	(0.5159)	(0.0763)	(0.5143)	(0.1355)	(0.0486)	(0.7517)
ω	1280.45	777.66	156.75	34.25	455.77	172.42
	(1770.28)	(477.90)	(117.82)	(46.48)	(198.43)	(304.72)
α	-0.0525	0.2460*	-0.0944*	0.3443*	0.4289	0.0994
	(0.0838)	(0.02920)	(0.0386)	(0.1293)	(0.2537)	(0.1176)
β	0.5770	0.7253*	0.9678*	0.6784*	0.1221	0.3243
	(0.5985)	(0.0859)	(0.0677)	(0.0861)	(0.2520)	(1.0375)

Standard errors are given in parenthesis. Coefficients which are significantly different from zero on the 5-percent level are marked with * if the respective null hypothesis is appropriate.

The estimates for the six sectors are given in table 1. It is reassuring that for all six sectors the MA(1) coefficient is negative but greater than -1, which yields estimates for the depreciation rate δ that are in the sensible range from zero to one. However, some of the estimates are not as close to -1 as one might expect given that the period under consideration is only one quarter.

Contrary to the initial conjecture, the AR(1) parameter is not very large in four of the six sectors – only in the sectors Engineering and Metals, the estimates of ρ are close to one. In the Textiles sector, the estimated ρ is even negative; its standard deviation of the estimate is quite big, however. Only in the Fuels sector do we obtain a coefficient significantly different from zero.

As for the GARCH component of the innovation, the three sectors Engineering, Food and Fuels show evidence for varying conditional variances. For the Engineering sector, this could be expected. The other two sectors, however, are not obvious candidates for this category. In the remaining three sectors (Chemicals, Metals and Textiles), there is little evidence for a GARCH structure in the innovations to the residual process. For the old-fashioned Metals sector, this comes as little surprise. One might expect the Chemicals sector, however, in the GARCH category.

5 Recovering the Demand Shifter

It would be desirable to recover the demand shifter ν_t from the residual ζ_t in order to have a closer look at the investment incentives that the firms were facing at any point in time. However, since it is not possible to identify γ_2, we cannot get our hands on ν_t itself. But it is possible to find a tractable expression for a *multiple* of it. Use equation (14) to obtain:

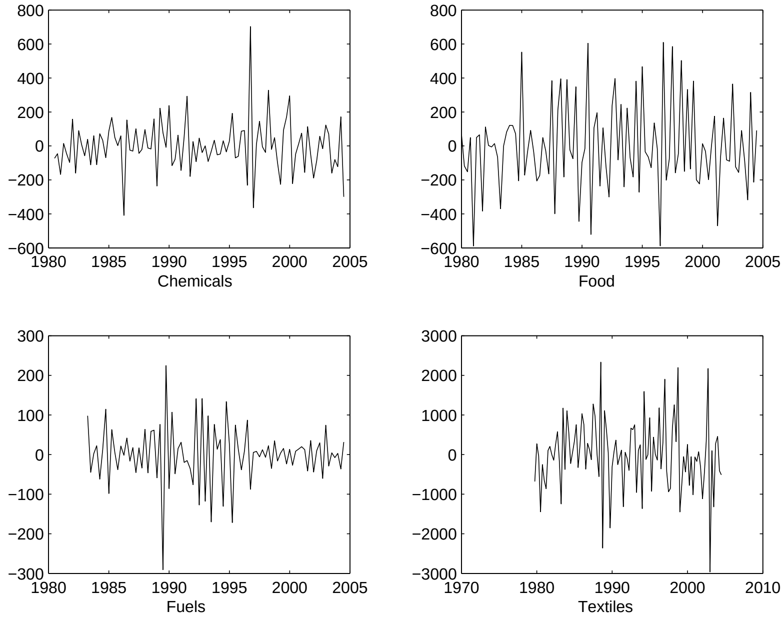
$$\tilde{\nu}_t := \gamma_2 \nu_t = (1 - \delta L)^{-1} \zeta_t = \sum_{j=0}^{\infty} \delta^j \zeta_{t-j}$$

Figure 2 shows the results for these calculations for the four sectors where δ is small enough to allow for a precise calculation of ν_t. The processes are very erratic and it is hardly possible to identify prolonged spells of either above- or below-average investment incentives. This is not surprising given that the AR(1) parameters are rather small for the plotted series. However, some of them exhibit telltale signs of autoregressive conditional heteroskedasticity. To get a clearer picture of these volatility patterns, figure 3 plots the conditional standard deviations implied by the GARCH estimates for the six sectors.[5]

It is quite striking how little the volatility patterns resemble each other across sectors. Volatility peaks and bottoms out at quite different points in time. This implies that for the sectors under consideration, sector-specific factors are far more important in determining conditional volatility than the general macroeconomic climate in the UK.

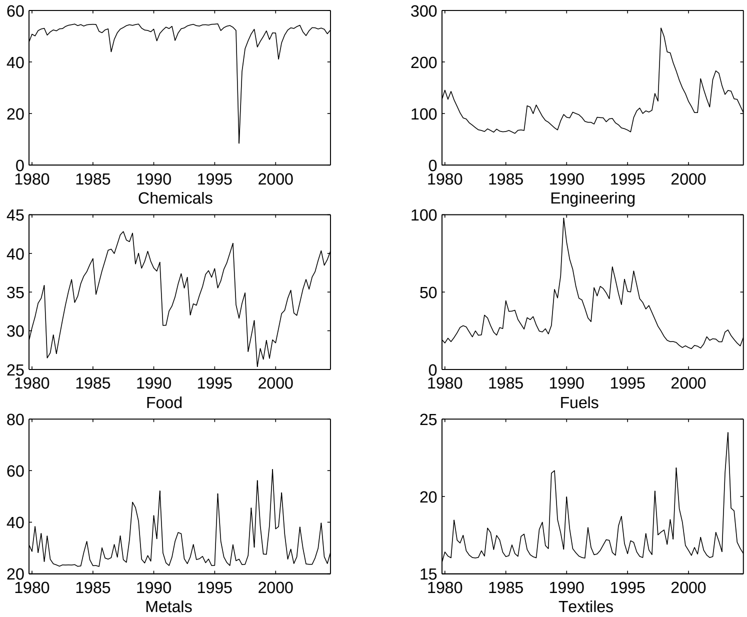
[5] The innovation for $\tilde{\nu}_t$ is the same as for ζ_t; hence the conditional-variance process for $\tilde{\nu}_t$ can be taken directly from the estimations for the GARCH process of ζ_t performed in section 4.

Figure 2: Demand Shifter



Points are only shown when the unknown parts of ν_t account for less than 1 percent of its unconditional variance. Since the series for the sectors Engineering and Metals have very large AR-coefficients, this point is not reached within the sample period and the graphs are omitted.

Figure 3: Conditional Standard Deviation



In the three sectors where the GARCH pattern is significant, the graphs show the following: In the Engineering sector, volatility of the demand shifter peaked in the late 1990s and was relatively low over the rest of the sample period. For the Food, Drink and Tobacco industry, the second half of the 1980s, the years around 1995 and the years 2002-2004 have exhibited most uncertainty. In the Fuels sector, finally, there was a large volatility spike in 1990 (probably linked to the Gulf War following Iraq's invasion of Kuwait) and a less pronounced period of elevated volatility in the years from 1993 to 1995.

6 Conclusions

In this paper, a competitive partial-equilibrium model with quadratic adjustment costs in investment and a stochastic demand shifter has been developed to predict aggregate investment in a sector. The demand shifter is assumed to have an AR(1), GARCH(1,1) structure. It is shown that under the assumptions made in the model, these properties are inherited by the process for aggregate investment in the rational-expectations equilibrium.

The model's predictions are tested for aggregate investment data from six sectors in the UK. The results can be summarized by three stylized facts: First, GARCH patterns play an important role in three of the six sectors. These differences seem to stem from the fact that demand and production costs are more volatile in some periods and less in others. Second, evidence for an autoregressive component in the demand shifter in the sectors under consideration is a lot harder to find than evidence for the GARCH structure. This is very surprising since the premises of the theoretical model support the idea of an AR(1) structure in the demand shifter. Third, the time paths of conditional variance in the six sectors do not resemble each other at all. This suggests that sector-specific factors are far more important in determining the conditional volatility of investment in a sector than general macroeconomic conditions.

However, some caveats are appropriate. The assumptions that each firm is of negligible size and that the nature of the investment is smooth and continuous might be violated in some of the sectors under consideration. Evidence suggests that investment is a lot more bulky than in the stylized standard model. If this bulkiness comes together with a small number of firms, than big investment projects of large competitors have a sizable impact on the aggregate time series. This might jeopardize the estimation exercise.

Furthermore, some of the depreciation rates implied by the estimates seem to be too high for quarterly data. This problem could be addressed by applying Bayesian estimation techniques that set a tight prior on the MA(1) parameter in the estimation. This procedure would likely obtain parameter estimates that are more in line with the spirit of the theoretical model developed in the first half of the paper.

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