Inflation and Output Dynamics with State-Dependent Frequency of Price Changes

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Abstract

This paper extends Calvo's (1983) time-dependent pricing model to incorporate state-dependent features in pricing, while preserving tractability. The pricing scheme delivers a generalized New Keynesian Phillips curve with an explicit role for the frequency of price revisions. The model's novel feature shows that inflation responds to movements of relative prices and to endogenous fluctuations in the average frequency of price adjustment. The model offers, therefore, a microfounded rationale for systematic deviations in the inflation-marginal cost relation predicted by the new Keynesian Phillips curve. As a byproduct, the model determines endogenously the short-run slope of the Phillips curve. Simulations predict weaker responses of output and stronger responses of inflation to technology, preference and monetary shocks than those of a close time-dependent model. (JEL E31)

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I. Introduction

Calvo's (1983) pricing model assumes that nominal individual prices are not revised in every period and that firms do not change prices synchronously. Instead, firms revise prices when they receive a random signal with constant probability over time and equal for all price-setters.

At the firm level Calvo's pricing assumptions imply that price setters cannot respond to large shocks in the economy between price revisions. This was first pointed out by Caplin and Leahy (1991). Further, Romer (1990) and Golosov and Lucas (2003) pointed out that, at the aggregate level, the frequency of price revisions is constant and thus it does not respond to the state of the economy. State-dependent pricing models such as Caplin and Leahy (1991, 1997), Caplin and Spulber (1987) or Dotsey, King and Wolman (1999) concentrate in relaxing the first aforementioned implication. In the light of Woodford (2003), this paper focuses on the second.

Woodford (2003, Ch.3) criticizes the existing literature on state-dependent pricing based on the argument that firms do not evaluate their pricing plans continously; they mostly reconsider prices at a particular date of the year because there are significant costs in the information gathering process (Zbaraki et al. [2000]). In contrast, state-dependent models as those mentioned above assume that firms evaluate their pricing policy in every period and set a new pricing policy only if they find it convenient. However, such assumption is less appealing if the main cost in the pricing process is the cost of learning the state of the economy. As discussed in Blanchard and Fisher (1989, p. 413), if the cost of new prices is only the cost of learning the state of the economy, then the pricing rule must be time dependent.

On the other hand, it is appealing to think that even if firms set the date for price revisions in calendar time due to costs of monitoring the economy, once the date for evaluation of prices arrives, firms should consider the state of the economy in their new prices and in their planned future dates for evaluation of prices. Moreover if firms have the ability to choose between more versus less frequent future price evaluations, the aggregate frequency of price changes can vary with the state of the economy.

This paper proposes a one-sector framework that incorporates state-dependent fluctuations in the average frequency of price revisions by combining time-dependent and state-dependent features in the firms' pricing scheme. Following Calvo (1983), the model assumes that firms change prices when they receive a random signal with constant probability over time. However, different from time-dependent models, firms are allowed to choose a higher (exogenously given) probability of price revisions. Price-setters must pay a lump-sum cost to benefit from faster price revisions. As in Dotsey, King and Wolman (1999) this lump-sum cost is random. An entrepreneur speeds up the expected frequency of price changes if the cost of doing so is compensated by the change in the value of the firm.

The proposed pricing scheme delivers a generalized New Keynesian Phillips curve with an explicit role for the frequency of price revisions in the inflation-output relation. In the Phillips curve of the model, as in time-dependent models with two sectors with different degrees of nominal rigidities (Carlstrom, Fuerst and Ghironi [2002]), current inflation responds to movements of relative prices. However, different from time-dependent models, current inflation also reacts to endogenous fluctuations of the average frequency of price adjustments.

The new terms in the Phillips curve would look, to someone using the standard Calvo (1983) pricing model, like exogenous cost-push shocks disturbing the relation between inflation and marginal cost. Here however, the additional terms are not exogenous but endogenous variables that respond systematically to exogenous shocks. The model provides, therefore, a novel way to interpret what these cost-push shocks might be.

As byproduct, the short-run slope of the Phillips curve in the space of current inflation and marginal cost is endogenously determined by the steady state of the economy. More flexible prices—endogenously induced by lowering the random lump-sum cost incurred to change prices more often—lead to a steeper Phillips curve.

I use the model to study the dynamics of output, inflation, the interest rate and the frequency of price changes in the presence of exogenous shocks. The responses of output to technology shocks, preference shocks, and shocks to the Taylor rule are weaker than those predicted by a closely related time-dependent model. On the other hand, responses of inflation are stronger. Simulations also show that inflation and the frequency of price changes move in the same direction after preference and monetary shocks, and in opposite directions after technology shocks. This is due to procyclical movements of the frequency of price changes that are inherited from procyclicality of profits around a steady-state with zero inflation. This result is in line with the common wisdom that the frequency of price changes is positive correlated with the inflation rate, evidence which is in Cecchetti (1986).

The rest of the paper is organized as follows. Section 2 presents the dynamic, stochastic, general equilibrium model. Section 3 presents the log-linear version of the model and discusses the new features of the Phillips curve. Section 4 calibrates the model and presents impulse responses. Section 5 concludes.

II. The model

The economy is populated by a representative household, a continuum of monopolistic firms indexed by $z \in [0,1]$, a monetary authority, and a fiscal authority.

A. *The household*

The household's period utility function at t is

$$U(C_t, M_t/P_t, N_t) \equiv \frac{\varphi_{d,t}}{(1-\Gamma)(1-\gamma)} \left[C_t^{1-\gamma} + (M_t/P_t)^{1-\gamma}\right]^{1-\Gamma} + \kappa \varphi_{d,t}^{\iota} \frac{(1-N_t)^{1-\zeta}}{1-\zeta}$$

where $\Gamma \geq 0$, $\gamma > 0$, $\kappa > 0$, $\iota > 0$, and $\zeta \geq 0$. $C_t \equiv \left[\int_0^1 [c_t(z)]^{(\theta-1)/\theta} dz\right]^{\theta/(\theta-1)}$, with $\theta > 1$, is the Dixit-Stiglitz aggregator of consumption over varieties of goods $c_t(z)$. M_t denotes nominal cash balances, P_t is the price index and N_t is time allocated to labor, with the total endowment of time per period normalized to one. $\varphi_{d,t}$ is a preference shock that follows a stationary stochastic process.

The budget constraint is

$$M_{t-1} + A_t + B_{t-1} + W_t N_t + \Delta_t \geq \int_0^1 p_t(z)\, c_t(z) \mathrm{d}z + B_t/(1+r_t) + M_t.$$

The sources of funds are nominal cash balances left available in period $t-1$, M_{t-1}, nominal transfers A_t received from the monetary authority, nominal bonds maturing at period t, B_{t-1}, income from working a fraction N_t of the endowed time at a nominal wage rate W_t, and lump-sum transfers equal to the nominal profits from the monopolistic firms, denoted by Δ_t[1]. The uses of funds consist of consumption of the good $c_t(z)$ purchased at the nominal price $p_t(z)$ for $z \in [0,1]$, bonds purchased

[1]Later it will become clear that this transfers come from two sources. After tax profits from firms and government revenues from taxes on profits. Thus the total transfer equals to the before-taxes-profits, that is $\Delta_t = \int_0^1 \Delta_t(z)\mathrm{d}z$, where $\Delta_t(z)$ denotes before-taxes-profits of firm z.

at t with nominal value of $B_t/(1 + r_t)$, where r_t is the net nominal interest rate between t and $t + 1$, and the money balances M_t carried into $t + 1$.

The household chooses C_t, M_t/P_t, N_t, and B_t/P_t to maximize

$$\sum_{i=0}^{\infty} \beta^i E_t U\left(C_{t+i},\, M_{t+i}/P_{t+i},\, N_{t+i}\right)$$

subject to the budget constraint. Expenditure minimization yields the demand for the variety $c_t(z)$:

(1) $$c_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} C_t\,,$$

where

(2) $$P_t \equiv \left[\int_0^1 \left[p_t(z)\right]^{1-\theta} \mathrm{d}z\right]^{\frac{1}{1-\theta}}$$

is the utility-based price index.

Let χ_t denote the Lagrange multiplier associated to the budget constraint, the first-order conditions for C_t, M_t/P_t, N_t, and B_t/P_t, respectively, imply:

(3) $$\varphi_{d,t}\, \left[C_t^{1-\gamma} + (M_t/P_t)^{1-\gamma}\right]^{-\Gamma} C_t^{-\gamma} = \chi_t,$$

(4) $$\varphi_{d,t}\, \left[C_t^{1-\gamma} + (M_t/P_t)^{1-\gamma}\right]^{-\Gamma} (M_t/P_t)^{-\gamma} = \chi_t - \beta E_t \frac{\chi_{t+1}}{\Pi_{t+1}}\,,$$

(5) $$\kappa \varphi_{d,t}^{\iota}\, (1 - N_t)^{-\zeta} = \chi_t w_t,$$

and

$$\chi_t = \beta E_t \frac{\chi_{t+1}\left[1+r_t\right]}{\Pi_{t+1}}, \tag{6}$$

where $\Pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate, and $w_t \equiv W_t / P_t$ is the real wage.

B. *The Firms*

In every period $t = 0, 1, 2, ...$, each firm produces a distinct perishable good indexed with the same index of the producing firm.

The pricing scheme

Extending Calvo's (1983) pricing, I assume that the continuum of firms is formed by two sets of monopolistic firms, $L \equiv \{z \mid z \in [0, \overline{\mu}]\}$ and $H \equiv \{z \mid z \in (\overline{\mu}, 1]\}$. The firms in set L revise prices with probability $(1-\alpha_L)$ in each period, while the firms in set H revise prices with probability $(1-\alpha_H)$ in every period, with $(1-\alpha_H) > (1-\alpha_L)$.

Once a firm z in L receives the random signal of price revisions, following Dotsey, King and Wolman (1999), it also observes the realization of a random lump-sum cost $\xi \geq 0$ with cumulative density function $G(\cdot)$. Different from Dotsey, King and Wolman (1999), ξ measures the random cost, in units of output, that the firm has to pay in order to increase its probability of price revisions from $(1-\alpha_L)$ to $(1-\alpha_H)$.[2] If the firm does not pay the random cost, it is subject to the lower probability of price revisions, but it can set a new price without cost. Note that as in Calvo's pricing, I assume that the physical cost of changing prices is zero.

[2] In Dotsey, King and Wolman (1999), firms evaluate in every period if it is convenient to change prices or keep the same price, given the physical cost of changing prices which they assume random.

A firm $z \in L$ that pays the random cost at t will be subject to a probability of price revisions $(1 - \alpha_H)$ until it receives a new random signal, say at $t + i$. Then, the monopolistic firm will choose at $t + i$ either to pay the random cost again and keep the higher probability of price revisions, or not to pay the random cost and set its probability of price adjustment equal to $(1 - \alpha_L)$. Note that firms in the set H have no incentive to choose a lower probability of price revisions because they can adjust prices with a higher probability without incurring the random cost.

Value of the firm

To save notation, define the subindex $j \in \{H,\, L\}$. The value of z at t can be described using four recursions, two of them associated to its value at t, $D_{0j,t}$, given that z is setting a new price subjet to the probability$(1 - \alpha_j)$. The other two recursions are associated to the value of z at $t + i$, $D_{1j,t+i}$, with $i = 1, 2, 3...$, subject to $(1 - \alpha_j)$, given that z has not changed its price since t. The four recursions account for the possibility of acting under two different probabilities of price revisions and the two possibilities of being allowed to change prices or not. These recursions are described in what follows.

a) $z \in L$

Consider first the maximization problem for a firm $z \in L$ receiving the random signal to revise prices at t. The firm decides a new price and its probability of price revisions. The decision is based on the value of the firm under each probability of price adjustment.

Let $I_{t+1}(1)$ be the indicator function equal to 1 if $z \in L$ chooses $(1 - \alpha_H)$ and zero otherwise. Let $\lambda_{t+1} \equiv \Pr[I_{t+1}(1) = 1]$ be the probability of z choosing $(1 - \alpha_H)$ at $t + 1$. Also let $d\,(p_{j,t}(z), \cdot)$ be the real profits at t for the firm z, given the price $p_{j,t}(z)$. Moreover, assume that profits are levied at a tax rate $\tau_j \geq 0$ for firms acting under the probability of price revisions $(1-\alpha_j)$.

Note that he model allows for, but does not require, differentiated tax rates. As argued below, for the case of an economy with zero steady-state inflation, it will be useful to introduce a tax on the profits of firms acting under $(1-\alpha_L)$. In particular, I will assume $\tau_L > 0$ and $\tau_H = 0$. This will be the the only role for the fiscal policy in the model.

The real value at t of the firm $z \in L$ acting subject to the probability $(1-\alpha_j)$ that receives the random signal of price revision, gross of the random cost, is given by the recursion

$$
\begin{aligned}
D_{0j,t}\left(S_t\right) = \max_{p_{j,t}(z)} \Big\{ & (1-\tau_j) d\left(p_{j,t}(z),\, S_t\right) \\
& + \beta\, \alpha_j\, E_t\, \frac{\chi_{t+1}}{\chi_t} D_{1j,t+1}\left(p_{j,t}(z\,),\, S_{t+1}\right) \\
& + \beta\, \left(1-\alpha_j\right)\, E_t\, \frac{\chi_{t+1}}{\chi_t} \lambda_{t+1}\, \left[D_{0H,t+1}\left(S_{t+1}\right) - \Xi_{t+1}\right] \\
& + \beta\, \left(1-\alpha_j\right)\, E_t\, \frac{\chi_{t+1}}{\chi_t} \left(1-\lambda_{t+1}\right)\, D_{0L,t+1}\left(S_{t+1}\right) \Big\}\,,
\end{aligned} \tag{7}
$$

where S_t is a vector of variables describing the state of the economy at t, $\beta \frac{\chi_{t+1}}{\chi_t}$ is the stochastic discount factor, and $E_t\, \Xi_{t+1}$, defined below, is the expected random cost conditional on choosing $(1-\alpha_H)$ at $t+1$ with probability λ_{t+1}.

The recursion (7) has a straightforward interpretation. For example, set $j = H$. Then, it follows from (7) that the value of the firm $z \in L$ at t acting subject to $(1-\alpha_H)$, $D_{0H,t}(\cdot)$, equals the after-tax-profits $(1-\tau_H)d\left(p_{j,t}(z), \cdot\right)$ plus the discounted expected value of the firm at $t+1$. The last three lines in (7) describe the expected value of the firm at $t+1$ under the three possible circumstances.

First, with probability α_H the firm is not allowed to change its price. Thus it is not allowed to choose a different probability of price adjustment. In that case, the value of the firm at $t+1$ is $D_{1H,t+1}(\cdot)$.

Second, with probability $(1-\alpha_H)$ the firm receives the random signal of price revision—which is strictly time dependent—and, with expected probability $E_t(1-\alpha_H)\lambda_{t+1}$, the firm decides to pay the random cost with conditional expected value $E_t\Xi_{t+1}$. In that case, the expected value of the firm is $E_t[D_{0H,t+1}-\Xi_{t+1}]$.

Finally with probability $(1-\alpha_H)$ the firm is allowed to revise its price, and with expected probability $E_t(1-\lambda_{t+1})$ the firm decides not to pay the random cost. Therefore it will be subject to the probability of price changes $(1-\alpha_L)$. In that case, the expected value of the firm is $E_t D_{0L,t+1}(\cdot)$.

Following the same principle, the value of the firm at $t+i$, for $i=1,2,3,...$, acting under $(1-\alpha_j)$, if it does not receive the signal of price revisions since t, is

$$
\begin{aligned}
D_{1j,t+i}\left(S_{t+i}\right) &= (1-\tau_j)d\left(p_{j,t}(z),\, S_{t+i}\right) \\
&+ \beta\,\alpha_j\,E_t\,\frac{\chi_{t+1+i}}{\chi_{t+i}} D_{1j,t+1+i}\left(p_{j,t}(z)\,,\,S_{t+1+i}\right) \\
&+ \beta\,\left(1-\alpha_j\right)\,E_t\,\frac{\chi_{t+1+i}}{\chi_{t+i}}\lambda_{t+1+i}\,\left[D_{0H,t+1+i}\left(S_{t+1+i}\right)-\Xi_{t+1+i}\right] \\
&+ \beta\,\left(1-\alpha_j\right)\,E_t\,\frac{\chi_{t+1+i}}{\chi_{t+i}}\left(1-\lambda_{t+1+i}\right)\,D_{0L,t+1+i}\left(S_{t+1+i}\right)\,.
\end{aligned}
\tag{8}
$$

Note that the maximization operator is not present in (8) because the firm cannot revise prices; the only decision made is input demand, which is implicit in the definition of $d(\cdot)$.

b) $z\in H$

Now consider the value of a firm $z\in H$ receiving the random signal to change prices at t. Since the firms in H can change prices with high probability without incurring the random cost, they choose $(1-\alpha_H)$ with probability one.

The value of $z \in H$ is

$$
\begin{aligned}
\overline{D}_{0H,t}\,(S_t) = & \max_{p_{H,t}(z)} \Big\{(1-\tau_H) d\,(p_{H,t}\,(z),\, S_t) \\
& + \beta\,\alpha_H\,E_t\,\frac{\chi_{t+1}}{\chi_t}\overline{D}_{1H,t+1}\,(p_{H,t}(z'\,),\, S_{t+1}) \\
& + \beta\,(1-\alpha_H)\,\,E_t\,\frac{\chi_{t+1}}{\chi_t}\overline{D}_{0H,t+1}\,(S_{t+1})\Big\}\,,
\end{aligned} \tag{9}
$$

where

$$
\begin{aligned}
\overline{D}_{1H,t+i}\,(S_{t+i}) = & \,(1-\tau_H) d\,(p_{H,t}(z),\, S_{t+i}) \\
& + \beta\,\alpha_H\,E_t\,\frac{\chi_{t+1+i}}{\chi_{t+i}}\overline{D}_{1H,t+1+i}\,(p_{H,t}\,(z'\,),\, S_{t+1+i}) \\
& + \beta\,(1-\alpha_H)\,\,E_t\,\frac{\chi_{t+1+i}}{\chi_{t+i}}\,\overline{D}_{0H,t+1+i}\,(S_{t+1+i})\,\,.
\end{aligned} \tag{10}
$$

for $i = 1, 2, 3, ...$ The interpretation of the recursions (9) and (10) is similar to those described above.

Probability of switching from low to high expected frequency of price changes

A firm $z \in L$ receiving the random signal of price revisions at t chooses the high probability of price revisions if and only if the value of the firm at t under $(1-\alpha_H)$ exceeds the value of the firm at t under $(1-\alpha_L)$ by at least the lump-sum random cost associated, that is, if and only if

$$
D_{0H,t} - D_{0L,t} \geq \xi\,. \tag{11}
$$

Before observing the realization of ξ, the probability of z choosing $(1-\alpha_H)$ is $\Pr\left[D_{0H,t} - D_{0L,t} \geq \xi\right] = G\left(D_{0H,t} - D_{0L,t}\right)$.

As argued by Dotsey, King and Wolman (1999), the continuity of $G(\cdot)$ and the fact that there is la large number of firms imply that the fraction of firms that chooses $(1-\alpha_H)$, conditional on receiving the random signal of price revisions, is $\lambda_t = G(D_{0H,t} - D_{0L,t})$. Moreover, letting $g(\cdot)$ denote the density function of ξ, the conditional expected random cost at t is $\Xi_t \equiv 1/G(D_{0H,t} - D_{0L,t}) \cdot \int_0^{[D_{0H,t}-D_{0L,t}]} xg(x)\mathrm{d}x$.

For parameterization purposes, assume $g(\xi) \equiv \begin{cases} b\cdot\exp(-b\cdot\xi); & \xi \geq 0 \\ 0; & \xi < 0 \end{cases}$.[3] Thus, the probability of z choosing $(1-\alpha_H)$ and the conditional expected random cost of doing so are, respectively:[4]

$$\lambda_t = 1 - \exp\left(-b\left[D_{0H,t} - D_{0L,t}\right]\right) \tag{12}$$

and

$$E_t\Xi_{t+1} = E_t\frac{1}{\lambda_{t+1}}\left[\frac{1}{b} - \left[1/b + D_{0H,t+1} - D_{0L,t+1}\right]\cdot\exp\left(-b\,\left[D_{0H,t+1} - D_{0L,t+1}\right]\right)\right]. \tag{13}$$

Optimal new prices

Firm z acting under $(1-\alpha_j)$ maximizes its expected present value by choosing the price $p_{j,t}(z)$ charged at t subject to the pricing scheme described above, the demand

[3] Different from Dotsey, King and Wolman (1999) or Burstein (2002), I do not need to impose an upper bound for the random variable ξ. This is because firms have the option of not paying the random cost and still change prices with a lower frequency.

[4] Note that the expected random cost is conditional on ξ satisfying $[D_{0H,t} - D_{0L,t}] \geq \xi \geq 0$. Otherwise, according to (11), the firm chooses not to pay the random cost. To obtain equation (13) compute $1/G(D_{0H,t} - D_{0L,t}) \cdot \int_0^{[D_{0H,t}-D_{0L,t}]} x\,g(x)\mathrm{d}x$, forward the resulting expression one period and take the expected value. Note that the term $1/\lambda_{t+1}$ in (13) is part of the conditional distribution, i.e., $g(\xi|\xi<\xi_0) = g(\xi)/G(\xi_0)$.

for the good z, and the technology

$$y_t(z) = \varphi_{T,t} N_t(z), \tag{14}$$

where $y_t(z)$ is the total output produced by the firm, $N_t(z)$ is the amount of labor employed by the firm z, and $\varphi_{T,t}$ is a productivity shock that follows a stationary stochastic process. $y_t(z)$ has two components: output produced to satisfy consumer demand $y_{c,t}(z)$ and output required in pricing activities by firms incurring the random lump-sum cost, $y_{p,t}(z)$, i.e., $y_t(z) \equiv y_{c,t}(z) + y_{p,t}(z)$.

Constant returns to scale imply that the total cost of production required to meet consumer demand can be written as $\psi_t y_{c,t}(z)$, where ψ_t is the real marginal cost implied by optimal input demand[5]. This, together with the market clearing condition $c_t(z) = y_{c,t}(z)$ and equation (1) yields the profit function gross of the random lump-sum cost as

$$d(p_{j,t},\, S_t) = \left[\frac{p_{j,t}(z)}{P_t} - \psi_t\right] \left(\frac{p_{j,t}(z)}{P_t}\right)^{-\theta} C_t\,. \tag{15}$$

According to (7), the first-order condition for the optimal new price of firm $z \in L$ acting subject to $(1-\alpha_j)$ is

$$0 = (1-\tau_j)\frac{\partial d(p_{j,t}(z),\, S_t)}{\partial p_{j,t}(z)} + \beta\alpha_j\, E_t\, \frac{\chi_{t+1}}{\chi_t} \frac{\partial D_{1j,t+1}(p_{j,t}(z),\, S_{t+1})}{\partial p_{j,t}(z)}\,, \tag{16}$$

where, from equation (8)

$$\frac{\partial D_{1j,t+i}(p_{j,t}(z),\, S_{t+i})}{\partial p_{j,t}(z)} = (1-\tau_j)\frac{\partial d(p_{j,t}(z),\, S_{t+i})}{\partial p_{j,t}(z)} + \beta\alpha_j\, E_t\, \frac{\chi_{t+1+i}}{\chi_{t+i}} \frac{\partial D_{1j,t+1+i}(p_{j,t}(z),\, S_{t+1+i})}{\partial p_{j,t}(z)}\,. \tag{17}$$

[5] Marginal cost is not firm specific because labor is freely mobile and $\varphi_{T,t}$ is common across firms.

for $i = 1, 2, 3, ...$

For firm $z \in H$, equation (9) implies the first-order condition

$$0 = (1-\tau_H)\frac{\partial d(p_{H,t}(z),\ S_t)}{\partial p_{H,t}(z)} + \beta\alpha_H\, E_t\, \frac{\chi_{t+1}}{\chi_t}\frac{\partial \overline{D}_{1H,t+1}(p_{H,t}(z),\ S_{t+1})}{\partial p_{H,t}(z)}\,, \tag{18}$$

where, from equation (10)

$$\frac{\partial \overline{D}_{1H,t+i}(p_{H,t}(z),\ S_{t+i})}{\partial p_{H,t}(z)} = (1-\tau_H)\frac{\partial d(p_{H,t}(z),\ S_{t+i})}{\partial p_{H,t}(z)} + \beta\alpha_H\, E_t\, \frac{\chi_{t+1+i}}{\chi_{t+i}}\frac{\partial \overline{D}_{1H,t+1+i}(p_{H,t}(z),\ S_{t+1+i})}{\partial p_{H,t}(z)} \tag{19}$$

for $i = 1, 2, 3, ...$

Using (15) to obtain $\partial d(\cdot)/\, \partial p_{j,t}(z)$, substituting (17) into (16) recursively, and substituting (19) into (18) recursively, the optimal new price set at t by *any* firm under $(1-\alpha_j)$ is

$$p^*_{j,t} = \frac{\theta}{\theta-1}\frac{E_t\, \sum_{i=0}^{\infty}(\beta\alpha_j)^i\, \frac{\chi_{t+i}}{\chi_t}\, \psi_{t+i}\, (P_{t+i})^{\theta}C_{t+i}}{E_t\, \sum_{i=0}^{\infty}(\beta\alpha_j)^i\, \frac{\chi_{t+i}}{\chi_t}\, (P_{t+i})^{\theta-1}C_{t+i}}\,, \tag{20}$$

where I dropped the firm-subindex z because the new price $p^*_{j,t}$ is common for all firms subject to the probability $(1-\alpha_j)$.

Dynamics of average frequency of price changes

Recall that firms $z \in L$ can revise prices with probability $(1-\alpha_L)$ without cost, but a firm $z \in L$ can choose the higher probability of price revisions $(1-\alpha_H)$ if it pays the random lump-sum cost ξ. Let V_t be the mass of firms $z \in L$ that are subject to the probability $(1-\alpha_H)$ at time t. The mass V_t accounts for all firms $z \in L$ that in their last price revision before and up to t chose $(1-\alpha_H)$ and have not revised prices since then. Note that the mass of firms $z \in L$ choosing $(1-\alpha_H)$ at t is given by the difference $V_t - V_{t-1}$.

Let μ_t be the mass of firms acting subject to $(1 - \alpha_L)$ at t—recall that no $z \in H$ chooses $(1-\alpha_L)$ in any period. Given the initial conditions μ_0 and V_0, the dynamics of V_t and μ_t can be described with the recursions

$$
\begin{aligned}
V_t &= V_{t-1} + \lambda_t(1 - \alpha_L)\,\mu_{t-1} - (1 - \lambda_t)(1 - \alpha_H)V_{t-1}\,, \\
\mu_t &= \overline{\mu} - V_t\,, \\
\mu_0 &= \mu, \text{ and } V_0 = V\,.
\end{aligned}
\tag{21}
$$

The first recursion in (21) implies that the net mass of firms $z \in L$ choosing $(1-\alpha_H)$ at t, $V_t - V_{t-1}$, equals the mass of firms that decided to switch from $(1-\alpha_L)$ to $(1 - \alpha_H)$ at the beginning of the period, minus the mass of firms switching back from $(1 - \alpha_H)$ to $(1 - \alpha_L)$. Thus, the second term in the first equation states that, at time t, a fraction λ_t of the mass receiving (at the beginning of t) the random signal of pricing-plan revisions with low probability, $(1-\alpha_L)\mu_{t-1}$, will choose $(1-\alpha_H)$—i.e. they will pay the random cost. The third term states that a fraction $(1 - \lambda_t)$ of $z \in L$ under $(1 - \alpha_H)$ decides not to pay the random cost and switches back to $(1 - \alpha_L)$, i.e., $(1 - \lambda_t)(1 - \alpha_H)V_{t-1}$ choose $(1 - \alpha_L)$.

The second equation in (21) holds because the mass of firms in L, $\overline{\mu}$, is constant, so that $V_t + \mu_t = \overline{\mu}$ for all $t = 0, 1, 2....$ The initial conditions are endogenously determined by the steady state of the economy (see Appendix A).

Assuming that one period represents a quarter, it follows that, in average, firms in the economy change prices

$$
F_t \equiv (1 - \alpha_L)\mu_t + (1 - \alpha_H)(1 - \mu_t) \tag{22}
$$

times per quarter. Note that, although the expected frequency of price revisions can take only two values at firm level, the average frequency of price revisions at the aggregate level is a double-bounded continuous function of μ_t, with upper and lower bounds $(1-\alpha_H)$ and $(1-\alpha_L)\overline{\mu}+(1-\alpha_H)(1-\overline{\mu})$, respectively.

The price level

To make explicit the effects of firms changing their probability of price revisions on the price level, it is convenient to rewrite the price index (2), in terms of the price sub-indexes $P_{L,t}$ and $P_{H,t}$ as follows

$$\text{(23)} \qquad P_t \equiv \left[\int_0^1 [p_t(z)]^{1-\theta}\,\mathrm{d}z\right]^{\frac{1}{1-\theta}} \equiv \left[\delta_t\, P_{L,t}^{1-\theta} + (1-\delta_t\,)P_{H,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \;,$$

where

$$P_{L,t} \equiv \left[\frac{1}{\delta_t}\int_0^{\mu_t} [p_t(s)]^{1-\theta}\,\mathrm{d}s\right]^{\frac{1}{1-\theta}} \text{ and } P_{H,t} \equiv \left[\frac{1}{1-\delta_t}\int_{\mu_t}^1 [p_t(s)]^{1-\theta}\,\mathrm{d}s\right]^{\frac{1}{1-\theta}} \;.$$

With the proper selection of the index $s \in [0,1]$, the integral in the sub-index $P_{j,t}$ aggregates prices of firms subject to the probability $(1-\alpha_j)$. Note that the choice of the weight $\delta_t \in (0,1)$ does not affect the price index definition nor its dynamics. If $\delta_t = \mu_t$, the price sub-indexes $P_{L,t}$ and $P_{H,t}$ are the consumer price sub-index of the baskets of goods produced by the firms $s \in [0,\, \mu_t]$ and $s \in (\mu_t,\, 1]$, respectively. However, it is convenient to define the sub-indexes $P_{j,t}$ with δ_t equal to the steady-state value of μ_t in order to make explicit the effect of the average frequency of price changes in the Phillips curve. Thus, I assume $\delta_t = \mu$ in what follows.

Recursions for price sub-indexes

As in the standard Calvo (1983)-Yun (1996) setup, the dynamics of the price sub-indexes can be described using a simple recursion. Given that the probability of not changing prices for each firm under $(1-\alpha_j)$ in every period is equal to α_j, the price subindex $P_{j,t}$ will contain a fraction α_j of the prices prevailing in the previous period. Moreover, since all firms setting a new price at t under $(1-\alpha_j)$ will choose the same price $p^*_{j,t}$, we can express the price sub-indexes at t as

(24) $$P_{L,t}^{(1-\theta)} = \alpha_L \, P_{L,t-1}^{(1-\theta)} + \frac{1}{\mu} \left[(1-\alpha_L)\mu_{t-1} - (V_t - V_{t-1})\right] \left(p^*_{L,t}\right)^{(1-\theta)} ,$$

and

(25)
$$P_{H,t}^{(1-\theta)} = \alpha_H \, P_{H,t-1}^{(1-\theta)} + \frac{1}{1-\mu} \left[(1-\alpha_H)(1-\mu_{t-1}) + (V_t - V_{t-1})\right] \left(p^*_{H,t}\right)^{(1-\theta)} .$$

In (24), the mass of firms setting the new price $p^*_{L,t}$ is expressed as the mass of firms that had the opportunity to revise prices at the beginning of the period t, $(1-\alpha_L)\mu_{t-1}$, minus the net mass of those that decided to choose $(1-\alpha_H)$ at t, $(V_t - V_{t-1})$. Similarly, in (25), the mass of firms setting the new price $p^*_{H,t}$ is expressed as the mass of firms under the high probability that received the random signal of price changes at the beginning of the period, $(1-\alpha_H)(1-\mu_{t-1})$, plus the net mass of firms $z \in L$ choosing $(1-\alpha_H)$ at t, $(V_t - V_{t-1})$. Equations (23)-(25) describe the evolution of the price index.

Finally, to close the model, we must specify the monetary policy. I do this in the log-linear version of the model, which facilitates the study of its dynamic properties.

III. Log-linearized Economy

I analyze the dynamics of the model in its log-linear version. I denote by $\widehat{x}_t \equiv dx_t/x$ the percentage (logarithmic) deviation of the variable x_t from its steady-state value, which is written without the time subscript.

The steady state around which the analysis is centered assumes zero steady-state inflation and is calibrated to have a constant mass of firms $z \in L$ under $(1-\alpha_H)$, thus, given an exogenous shock, the average frequency of price revisions can increase or decrease. However, in a zero steady-state inflation economy there is no natural incentive inducing firms $z \in L$ to pay the random cost and choose $(1-\alpha_H)$. Hence, it is convenient to introduce differentiated tax rates that generate such incentive. As mentioned before, I assume $\tau_L > 0$ and $\tau_H = 0$.[6] Moreover, I assume that tax revenues are rebated to consumers in a lump-sum fashion.

Monetary Policy

To close the model, assume that the central bank follows a modified Taylor (1993) rule

(26) $$\widehat{r}_t = \sigma_r \widehat{r}_{t-1} + \sigma_\pi \widehat{\Pi}_{t-1} + \sigma_y \widehat{Y}_{c,t-1} + \varepsilon_{r,t}\,.$$

where $\sigma_r \geq 0$, $\sigma_\pi > 0$ and $\sigma_y \geq 0$ are parameters chosen by the central bank. In particular, $\sigma_r > 0$ implies that the central bank adjusts gradually the nominal interest rate in response to fluctuations in inflation and output. $\varepsilon_{r,t}$ is an i.i.d. shock with standard deviation Φ_r.

[6]We can show that in the presence of positive steady-state inflation, the incentive for a steady-state fraction of $z \in L$ to choose $(1-\alpha_H)$ arises naturally and is increasing in the steady-state value of inflation. Moreover, constant tax rates on profits do not affect either the firm's pricing decisions or the dynamics of the economy in other form than the indicated.

A Generalized Phillips Curve

The Phillips curve of the model is obtained from equations (20)-(25). As shown in Appendix B, defining the ratio of price sub-indexes (24) and (25) as $T_t \equiv P_{L,t}/P_{H,t}$, the model yields the Phillips curve

$$\begin{aligned}\widehat{\Pi}_t &= \beta E_t \widehat{\Pi}_{t+1} + [\mu a_L + (1-\mu) a_H]\, \widehat{\psi}_t \\ &\qquad + \mu(1-\mu)(a_H - a_L)\widehat{T}_t - \frac{1}{\theta-1} F\, [f - \beta]\, \widehat{F}_t\,,\end{aligned} \tag{27}$$

where all the coefficients are positive, with $a_L \equiv (1-\alpha_L)(1-\beta\alpha_L)/\alpha_L$, $a_H \equiv (1-\alpha_H)(1-\beta\alpha_H)/\alpha_H$, and $f \equiv (\alpha_H^{-1} - \alpha_L^{-1})/(\alpha_L - \alpha_H)$.

Equation (27) generalizes the New Keynesian Phillips curve in the sense that it makes explicit the role of fluctuations in the average frequency of price changes. In (27), as in the textbook version of Calvo's (1983) model, inflation is forward looking and responds to fluctuations in marginal cost. Moreover, in the Phillips curve (27), as in Carlstrom, Fuerst, and Ghironi (2002), the ratio of price sub-indexes affects current inflation.[7] Appendix C shows that T_t is governed by the second-order difference equation

(28)

$$\begin{aligned}&\beta E_t \widehat{T}_{t+1} - \tau_1 \widehat{T}_t + \widehat{T}_{t-1} = \\ &\qquad (a_H - a_L)\widehat{\psi}_t + \frac{V}{\theta-1}\frac{1}{\mu(1-\mu)} E_t\left[\beta \widehat{V}_{t+1} - \tau_2 \mu(1-\mu)\widehat{V}_t + \widehat{V}_{t-1}\right]\end{aligned}$$

where $\tau_1 \equiv [1+\beta+(1-\mu)a_L + \mu a_H]$ and $\tau_2 \equiv \frac{1}{\mu}(\beta\alpha_L + \alpha_L^{-1}) + \frac{1}{1-\mu}(\beta\alpha_H + \alpha_H^{-1})$.

[7] Carlstrom, Fuerst, and Ghironi (2002) investigate the determinacy properties of a two-sector model with different degrees of nominal rigidity. In Carlstrom, Fuerst, and Ghironi (2002) T_t represents the ratio of price sub-indexes for the corresponding sub-baskets. Here, as mentioned above, T_t does not represent the ratio of price sub-indexes, since the weights in the price sub-indexes are fixed (μ, $1-\mu$), while the mass of firms forming the sub-baskets is allowed to change (μ_t, $1-\mu_t$).

Finally, different from time-dependent models, endogenous frequency of price adjustments affects the dynamics of current inflation in the Phillips curve in two forms. First, the short-run slope of the Phillips curve—in the (Π_t,ψ_t) space—is endogenously determined by the steady-state mass μ, which also determines the steady-state of the average frequency of price revisions in equation (22). Figure 1 shows that, by lowering the unconditional mean of the random cost ($E[\xi] = 1/b$), we obtain higher values of the steady-state frequency of price adjustments, which in turn are associated to steeper slopes of the Phillips curve in the (Π_t,ψ_t) space.[8] That is, more flexible prices induced by lower adjustment costs lead to a steeper Phillips curve.[9]

Second, from the price sub-indexes (24) and (25) it is clear that the dynamic evolution of the mass of firms setting new prices at different intervals of time, play a role in shaping the evolution of the price level. $\widehat{F}_t$ appears in the Phillips curve to account for the evolution of such mass of firms, which—using (21) and (22)—can be expressed in terms of the average frequency of price revisions.

Log-linear versions of the mass of firms accelerating price changes (21) and the average frequency of price revisions (22) yield

(29) $$\widehat{V}_t = v_1 \widehat{V}_{t-1} + v_2 \, \widehat{\lambda}_t \, ,$$

[8]Sheshinski and Weiss (1993) show that an inverse relation between the cost of price adjustment and the frequency of price adjustment holds in a state-dependent deterministic model. Golosov and Lucas (2003) find such relation in a stochastic state-dependent model.

[9]Using Romer's (1990) setup, Bakhshi, Burriel-Llombart and Khan (2002) analyze the short-run slope of the Phillips curve in a model with positive steady-state inflation and (ad hoc) endogenous price flexibility. They find a positive relation between steady-state inflation and the slope of the Phillips curve.

and

$$\widehat{F}_t = (\alpha_L - \alpha_H)\frac{V}{F}\widehat{V}_t, \tag{30}$$

where $v_1 \equiv [1-(1-\alpha_L)\lambda-(1-\alpha_H)(1-\lambda)]$, thus $v_1 \in (0,1)$), and $v_2 \equiv \lambda[(1-\alpha_H)+\mu(1-\alpha_L)/V]$.

From (29) and (30), we can see that any shock perturbing $\widehat{\lambda}_t$ leads to persistent changes in the frequency of price changes $\widehat{F}_t$. The persistence of the frequency of price changes—measured by v_1—is a consequence of the time-dependent feature of the model, i.e., because firms are not allowed to vary the probability of price adjustments in every period.

Appendix C shows that log-linear versions of equations (7) and (8) together with the definition of λ_t in (12) yield the following forward-looking equation for the probability of choosing $(1-\alpha_H)$ as opposed to $(1-\alpha_L)$:

$$\widehat{\lambda}_t = \beta v_1 E_t \widehat{\lambda}_{t+1} + (\tau_L - \tau_H)d\widehat{d}_t + v_3 E_t\left(\widehat{\chi}_{t+1} - \widehat{\chi}_t\right) \tag{31}$$

where $v_3 \equiv \frac{1-\lambda}{\lambda} b\left[D_H - D_L - (\tau_L - \tau_H)d\right]$. Log-linearizing the profit function (15), we get

$$\widehat{d}_t = \widehat{C}_t - (\theta-1)\widehat{\psi}_t\,. \tag{32}$$

Equations (29) and (30) illustrate that the driving force behind fluctuations in the frequency of price changes F_t is the probability of choosing faster price revisions, λ_t. Equation (31) shows that such probability is determined by the string of current and future profit-differentials between firm setting prices under each probability, $(\tau_L - \tau_H)d\widehat{d}_t$, and the effect of the discount factor. Hence, if the effect of profit-differentials dominates in (31), we expect the frequency of price changes to co-move with profits.

The Rest of the Model

Households. Log-linearizing the Euler equation for consumption (3) and the Fisher equation (6), we obtain

(33) $$\widehat{\chi}_t = \left[(\gamma-1)\nu C^{1-\gamma} - \gamma\right] \widehat{C}_t + \nu m^{1-\gamma}[\gamma-1]\widehat{m}_t + \widehat{\varphi}_{d,t}$$

and

(34) $$\widehat{\chi}_t = E_t \left[\widehat{\chi}_{t+1} + \widehat{r}_t - \widehat{\Pi}_{t+1}\right],$$

where $\nu \equiv \Gamma\left[C^{1-\gamma} + m^{1-\gamma}\right]^{-1}$. Using the approximation $1/(1+r_t) \approx 1 - r_t$, equations (4), (6) and (3) deliver the money demand

(35) $$\widehat{m}_t \approx \widehat{C}_t - \frac{1}{\gamma}\widehat{r}_t,$$

with unit elasticity in consumption and interest rate elasticity $1/\gamma$.

Firms. The production technology (14) implies that in the aggregate $Y_t = A\varphi_{T,t} N_t$, where $Y_t \equiv Y_{c,t} + Y_{p,t}$, $Y_{c,t} \equiv \int_0^1 y_{c,t}(z)\mathrm{d}z$, $Y_{p,t} \equiv \int_0^1 y_{p,t}(z)\mathrm{d}z$ and $N_t \equiv \int_0^1 N_t(z)\mathrm{d}z$. Thus, the following condition holds

(36) $$Y_c\widehat{Y}_{c,t} = Y\left[\widehat{N}_t + \widehat{\varphi}_{T,t}\right] - Y_p\widehat{Y}_{p,t}$$

Since the real wage is not firm-specific, the real marginal cost is $\psi_t = w_t/\,\varphi_{T,t}$.

Accordingly,

$$\widehat{\psi}_t = \widehat{w}_t - \widehat{\varphi}_{T,t}\,. \tag{37}$$

Log-linearizing equation (13) yields the log-linear version of the conditional expected lump-sum cost as

$$\widehat{\Xi}_t = \frac{1}{\Xi}\left[D_H - D_L - \Xi\right]\widehat{\lambda}_t\,. \tag{38}$$

The total output in pricing activities, $Y_{p,t}$, is calculated by multiplying the conditional average random cost (13) times the total mass of firms paying the random cost, that is, $Y_{p,t} = \lambda_t\left[(1-\alpha_H)V_{t-1} + (1-\alpha_L)\mu_{t-1}\right]\Xi_t$. Using $\widehat{\mu}_t = -V/\mu\widehat{V}_t$ from (21) we obtain

$$\widehat{Y}_{p,t} = \frac{\Xi V\lambda}{Y_p}(\alpha_L - \alpha_H)\widehat{V}_{t-1} + \widehat{\lambda}_t + \widehat{\Xi}_t\,. \tag{39}$$

Market clearing conditions. Following Yun (1996), I define the alternative price index $\overline{P}_t^{-\theta} \equiv \int_0^1 p_t(z)^{-\theta}\mathrm{d}z$. Aggregating the goods market clearing condition, $y_c(z) = c(z)$, yields the following relation between the linear aggregator and the Dixit-Stiglitz aggregator for consumption: $Y_{c,t} = \left[\frac{\overline{P}_t}{P_t}\right]^{-\theta} C_t$. That condition can be approximated[10] by

$$\widehat{Y}_{c,t} \approx \widehat{C}_t\,. \tag{40}$$

Appendix D shows the remaining set of equations used to account for the exact equilibrium condition.

[10]The fact that (40) is a good approximation for the log-linear version of the (exact) market clearing condition is also found by Dotsey, King and Wolman (1997).

Combining (14) with the first-order condition for labor supply (5), we obtain equilibrium wage

$$\widehat{w}_t = \frac{\zeta N}{1-N}\frac{1}{Y}\left[Y_c\widehat{Y}_{c,t} + Y_p\widehat{Y}_{p,t}\right] - \frac{\zeta N}{1-N}\widehat{\varphi}_{T,t} - \widehat{\chi}_t + \iota\widehat{\varphi}_{d,t}\,. \tag{41}$$

Exogenous shocks. To complete the description of the log-linear model we must specify two exogenous processes. The productivity shock $\widehat{\varphi}_{T,t}$ and the preference shock $\widehat{\varphi}_{d,t}$ follow autoregressive processes of order one:

$$\widehat{\varphi}_{T,t} = \rho_T\,\widehat{\varphi}_{T,t-1} + \varepsilon_{T,t}\,, \tag{42}$$

and

$$\widehat{\varphi}_{d,t} = \rho_d\,\widehat{\varphi}_{d,t-1} + \varepsilon_{d,t}\,, \tag{43}$$

where $0 \leq \rho_T < 1$ and $0 \leq \rho_d < 1$. $\varepsilon_{T,t}$ and$\varepsilon_{d,t}$ are i.i.d. shocks with zero mean and standard deviations Φ_T and Φ_d, respectively.

IV. Calibration and Impulse Responses

Calibration

Equations (26)-(41) form a system of sixteen equations in sixteen endogenous variables: $\widehat{Y}_{c,t}$, $\widehat{\Pi}_t$, $\widehat{r}_t$, $\widehat{F}_t$, $\widehat{V}_t$, $\widehat{T}_t$, $\widehat{\psi}_t$, $\widehat{C}_t$, $\widehat{N}_t$, $\widehat{w}_t$, $\widehat{\lambda}_t$, $\widehat{Y}_{p,t}$, $\widehat{d}_t$, $\widehat{\Xi}_t$, $\widehat{m}_t$ and $\widehat{\chi}_t$. The model also includes three exogenous disturbances, a shock to the Taylor rule, a productivity shock (42) and a preference shock (43).

I use parameter values from the literature. The stylized fact of procyclical profits is a key issue for calibration (Rotemberg and Woodford [1999] and Christiano, Eichenbaum and Evans [1996]). Although profit variation plays no role in most monetary models of the business cycle, in this model, firm's decisions about speeding up future changes in prices are based on the value of the firm, which in turn is mainly determined by the string of current and expected future profits. Thus, the evolution of profits directly influences the dynamics of the average frequency of price revisions.

Christiano, Eichenbaum and Evans (1996) discuss how the standard new Keynesian model requires a high value of the firm's markup in order to produce procyclical movements in profits. This property is inherited by our model. Here, I do not attempt to find a remedy, but I impose a high markup for the monopolistic firms and an infinite elasticity of labor supply to generate procyclicality of profits.[11]

I choose the parameters shaping preferences as follows. The discount factor ($\beta = 0.99$) implies a rate of return of 4.1 percent annually. $\Gamma = 1$ corresponds to a non-separable logarithmic utility in consumption and real money balances. From (35), the inverse of γ is the interest rate elasticity of money demand. Thus $1/\gamma = 0.118$ is in line with empirical estimates in Ireland (1997b). The price elasticity of demand for the final good in equation (1) implies a steady-state markup of 50 percent ($\theta = 3$) above marginal cost. In steady-state, households allocate one third of the endowed time to labor ($N = 1/3$) and their labor supply is infinite-elastic ($\zeta = 0$). I set ι to $1/2$, so the preference shock $\varphi_{d,t}$ has qualitatively the same effect on inflation and output as McCallum and Nelson's (1997) IS shock. However, this reduces the volatility of marginal cost in the presence of preference shocks.

The parameters of the pricing mechanism are chosen to stay close to the standard time-dependent model. $(1 - \alpha_L) = 1/5$ implies that firms under the low

[11] Rotemberg and Woodford (1999) propose some remedies to Christiano, Eichenbaum and Evans' critique.

frequency of price changes revise prices once every five quarters on average; $(1-\alpha_H) = 1/3$ implies that, under the high frequency of price adjustments, firms set new prices prices once every three quarters on average. The size of the set of firms that endogenously choose their frequency of price changes is $\overline{\mu} = 0.99$. These values imply an upper and lower bound on fluctuations of the average frequency of price revisions of $(1-\alpha_H) = .333$ and $(1-\alpha_L)\overline{\mu} + (1-\alpha_H)(1-\overline{\mu}) = 0.2133$, respectively.

Note that in the model, the probabilities of price adjustment $(1-\alpha_L)$ and $(1-\alpha_H)$ represent two possibilities that one firm can adopt as part of its optimal pricing policy. Values for the frequency of price chances in that range are common in the literature. Note that this approach is different from the two-sector model with different degrees of nominal rigidity of Carlstrom, Fuerst and Ghironi (2002) or Bils and Klenow (2004) which capture intersectoral heterogeneity in nominal rigidities.

The parameter b in the distribution of the random cost $G(\cdot)$ is chosen so that the unconditional mean of ξ is the same as in Dotsey, King an Wolman (1999), i.e. $E[\xi] = 1/b = 0.006$. Golosov and Lucas' (2003) calibration implies that the random lump-sum cost of price revisions is about 1.9 percent of profits. According to our calibration, the (unconditional) expected cost represents 1.3 percent of profits. The values of differentiated tax rates on profits, $\tau_L = 0.005$ and $\tau_H = 0$, allow for the average frequency of price changes to increase by 17 percent or decrease by 23 percent with respect to its steady state ($F = 0.28$), without hitting the upper or lower bounds.

The parameter values for the Taylor rule are in line with Ireland's (2002) estimates for the post-1980 U. S. economy: $\sigma_r = 0.5541$, $\sigma_\pi = 0.5751$, $\sigma_y = 0$. Finally, I calibrate the exogenous shocks (42) and (43) as follows: $\rho_T = .956$, $\rho_d = .892$, $\Phi_T = .007$, $\Phi_d = .035$ and $\Phi_r = .0025$.

Parameter values are consistent with a unique rationale expectation equilibrium. In particular, monetary policy responds to inflation aggressively enough to ensure determinacy.

Impulse Responses

To analyze the effects of the new features of the model, I calculate the impulse responses for the three exogenous shocks—preference shock, technology shock and shock to the Taylor rule—using the techniques described in Uhlig (1999). I compare the results with a closely related time-dependent model. In this model, the central bank follows the Taylor rule (26), households have the same preferences and confront the same budget constraint as those described above, but firms follow a pricing scheme as in Calvo (1983) with probability of changing prices $(1-\alpha)$. Accordingly, the Phillips curve of the model is $\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\widehat{\psi}_t$.

The first-order condition for the representative household's problem imply the Euler equation (33), the Fisher equation (34) and the money demand (35). The aggregate production of output is $\widehat{Y}_{c,t} = \widehat{N}_t + \widehat{\varphi}_{T,t}$ and marginal cost is is given by (37). Equilibrium in the goods market yields equation (40) and equilibrium wage is $\widehat{w}_t = \frac{\zeta N}{1-N}\left[\widehat{Y}_{c,t} - \widehat{\varphi}_{T,t}\right] - \widehat{\chi}_t + \iota\widehat{\varphi}_{d,t}$. Finally, productivity and preference shocks follow the processes (42) and (43), respectively. That forms a system of nine equations in nine endogenous variables ($Y_{c,t}$, Π_t, r_t, ψ_t, C_t, w_t, N_t, m_t and χ_t) plus three exogenous shocks ($\varphi_{T,t}$, $\varphi_{d,t}$ and $\varphi_{r,t}$) and the corresponding assumptions. The new parameter is α so that I set $(1-\alpha) = (1-\alpha_L)$. All other parameter values are the same in both models.

Figure 2, shows the responses of interest rate, inflation, output and the average frequency of price changes to a positive, one standard deviation preference shock. The response of inflation is stronger in the model of this paper, while the response of output is weaker than those in the time-dependent benchmark. The same property holds also for inflation and output responses to productivity shocks and shocks to the Taylor rule, as shown in Figures 3 and 4. For monetary expansions this result is found also by Dotsey, King and Wolman (1997).

Figures 2, 3, and 4 also show that for small shocks the dynamics of output and inflation in the model with elements of state-dependent pricing is well approximated by the time-dependent model. That conclusion is also found by Dotsey, King and Wolman (1997), Burstein (2002) or Klenow and Kryvtsov (2004).

Moreover, Figures 2, 3 and 4 show that the frequency of price changes is procyclical. This result follows from the procyclicality of profits. Under zero steady-state inflation, the difference in the value of firms adjusting prices faster versus those adjusting slower is proportional to profits—common for both type of firms. Thus, more firms are willing to cover the costs of additional price revisions in booms, causing upward fluctuations in the average frequency of price changes. Furthermore, procyclical movements in the average frequency of price revisions imply that inflation and the average frequency of price changes move in the same direction after preference shocks or shocks to the Taylor rule, but they move in opposite directions after technology shocks. This result is in line with the conventional wisdom that the frequency of price changes is positive correlated with the inflation rate. For example, such relation is *assumed* in Bakhshi, Burriel-Llombart and Khan (2002). Moreover, evidence of that correlation is found by Cecchetti (1986) and suggested in Zbaraki et al. (2000).

Figure 5 shows the evolution of the relative price $T_t \equiv P_{L,t}/P_{H,t}$. Figures 2 to 5 show that the impulse responses of the terms T_t and F_t in the Phillips curve (27) are persistent. As menitioned in the introduction, those terms can be identified as cost-push shocks by someone using the standard Calvo (1983) or Rotemberg (1982) pricing model. For example, Ireland (2004), using data for the U.S. economy in the postwar period, finds evidence of systematic deviations in the inflation-output relation predicted by a model with Rotemberg (1982) pricing. In Ireland's (2004) model the cost-push shock is characterized as exogenous stochastic disturbances in the degree of monopolistic power that follow an autorregresive process of order one. Consistently with the prediction of our model, Ireland finds that such shocks are very persistent (with a correlation coefficient of 0.9672) [12].

V. Conclusions

This paper introduced elements of state-dependent pricing in a tractable fashion in a dynamic, stochastic, general equilibrium monetary model. The pricing scheme proposed represents a natural extension of Calvo's (1983) pricing which generates endogenous movements in the average frequency of price revisions. The incorporation of time-dependent and state-dependent features allows to the model to escape Woodford's (2003) critique of state-dependent models discussed in the introduction and, by the same token, preserves the tractability of time-dependent models.

The pricing mechanism delivers a generalized New Keynesian Phillips curve in the sense that it makes explicit the role of relative prices and the frequency price revisions as additional endogenous variables that affect the inflation-output trade-off. The model offers, therefore, a microfounded rationale for systematic deviations in the inflation-output relation predicted by the new Keynesian Phillips curve, i.e. cost-push shocks. Different from Steinnson (2003) or Ireland (2004), who microfound cost-push shocks as exogenous stochastic disturbances to the elasticity of substitution between goods, here, such deviations arise endogenously.

[12] Moreover, Ireland (2004) finds that cost-push shocks are more relevant than technology shocks in explaining the behavior of inflation, output and interest rates.

The model predicts that exogenous shocks would have persistent effects in both terms, relative prices and the frequency price revisions. This prediction is in line with Ireland's (2004) estimates who finds that, in the postwar period for the U.S. economy, cost-push shocks are highly persistent.

Additionaly, I see this as a basic setup suited to tackle questions for which endogenous price flexibility is central in a dynamic, stochastic, general equilibrium framework. For example, we know since Ireland (1997) that we can explain the empirical evidence on disinflationary programs implemented in high and moderate inflation economies found by Gordon (1982) and Sargent (1982) by allowing for endogenous speed of price adjustments. Moreover, Calvo, Celasun and Kumhof (2003) show that the frequency of price adjustments (exogenously given in their model) plays an important role in measuring welfare costs of disinflation programs. This suggests that elements of state-dependent pricing are a desirable feature in models of disinflation programs.

Appendix A: The steady state

Relative prices and marginal cost

From the optimal new price (20), in steady state—assuming zero steady-state inflation—I obtain

$$\frac{p_j^*}{P} = \frac{\theta}{\theta - 1}\psi \,.$$

The price sub-indexes (24) and (25), in steady state imply

$$P_j = p_j^*.$$

The price index (23) yields

$$1 = \mu \left(\frac{P_L}{P}\right)^{1-\theta} + (1-\mu)\left(\frac{P_H}{P}\right)^{1-\theta},$$

using the last three equations we obtain the steady-state marginal cost:

$$1 = \mu \left(\frac{\theta}{\theta - 1}\psi\right)^{1-\theta} + (1-\mu)\left(\frac{\theta}{\theta - 1}\psi\right)^{1-\theta} \Longrightarrow \psi = \frac{\theta - 1}{\theta}\,. \tag{A-1}$$

Note that equation (A-1) also holds in the Calvo (1983)–Yun (1996) setup.

Value of the firm, output, costs in pricing activities

From equations (7) and (8), note that $D_{0j} = D_{1j}$ in steady state. Let D_j denote the steady-state value of the firm under $(1 - \alpha_j)$. Then, conditions (12) and (13) in steady state become

$$\lambda = 1 - \exp\left(-b\left[D_H - D_L\right]\right) \tag{A-2}$$

and

$$\text{(A-3)} \qquad \Xi = \frac{1}{\lambda}\left[\frac{1}{b} - (1/b + D_H - D_L)\exp\left(-b\left[D_H - D_L\right]\right)\right].$$

Next, from the profit function (15), imposing the goods market clearing condition $C = Y_c$, steady-state profits become $d = \frac{1}{\theta}Y_c$. Using this and steady-state versions of (7) for $j = H$ and $j = L$, I obtain

$$\text{(A-4)} \qquad D_H = \frac{\Omega_H}{\Omega}\frac{1}{\theta}Y_c - \frac{\beta\lambda(1-\alpha_H)(1-\beta\alpha_L)}{\Omega}\Xi$$

and

$$\text{(A-5)} \qquad D_L = \frac{\Omega_L}{\Omega}\frac{1}{\theta}Y_c - \frac{\beta\lambda(1-\alpha_L)(1-\beta\alpha_H)}{\Omega}\Xi\,,$$

where $\Omega \equiv (1-\beta)\left\{1-\beta\left[\alpha_H + \lambda(\alpha_L - \alpha_H)\right]\right\}$, $\Omega_H \equiv (1-\beta\lambda\alpha_L)\cdot(1-\tau_H) - \beta(1-\lambda)\cdot[(\tau_L-\tau_H)+\alpha_H(1-\tau_L)]$, and $\Omega_L \equiv [1-\beta(1-\lambda)\alpha_H]\cdot(1-\tau_L) - \beta\lambda\cdot[\alpha_L(1-\tau_H)-(\tau_L-\tau_H)]$.[13]

I choose the parameter κ such that the steady-state labor effort is a convenient level N ($N = 1/3$, see below), thus steady-state output is

$$\text{(A-6)} \qquad Y_c + Y_p = AN.$$

Output employed in pricing activities is calculated by multiplying the conditional average random cost (in units of output) times the total mass of firms paying

[13]Note that, if $\alpha_H = \alpha_L = \alpha$ and $\tau_H = \tau_L = 0$, then (A-4) and (A-5) imply $D_H = D_L = \frac{1}{1-\beta}d - \frac{\beta\lambda(1-\alpha)}{\beta}\Xi$. Moreover, from (A-2) it follows that $\lambda = 0$, so that $D_H = D_L = \frac{1}{1-\beta}d$.

the random cost. Accordingly, in steady state:

(A-7)

$$Y_p = [(1-\alpha_H)V + (1-\alpha_L)\mu] \cdot \left[\frac{1}{b} - \left(D_H - D_L + \frac{1}{b}\right) \exp\left(-b\left[D_H - D_L\right]\right)\right] .$$

The steady-state mass of firms accelerating price revisions is given by equation (21) as

(A-8)
$$V = \frac{\lambda(1-\alpha_L)\overline{\mu}}{(1-\alpha_H)(1-\lambda)+\lambda(1-\alpha_L)} .$$

Equations (A-2)–(A-8) constitute a nonlinear system of seven equations in seven variables: λ, D_H, D_L, Ξ, Y_p, Y_c, and V. Its solution yields the steady-state levels of those variables.

Other variables

The first-order conditions (3), (4) and (6) yield the steady-state real money balances and Lagrange multiplier as:

(A-9)
$$m = Y_c\,(1-\beta)^{-1/\gamma}$$

and

(A-10)
$$\chi = \left[Y_c^{(1-\gamma)} + m^{(1-\gamma)}\right]^{-\Gamma} Y_c^{-\gamma} ,$$

respectively, where $m \equiv M/P$.

The steady-state real wage is determined from the relation

(A-11)
$$\psi = \frac{1}{w} .$$

I choose the value of κ (in the first-order condition for labor supply, 5) so that

(A-12) $$\kappa\,(1-N)^{-\zeta} = \chi w\,,$$

is satisfied for $N = 1/3$.

Using $\mu = \overline{\mu} - V$ to find the steady-state value of μ, the steady-state average frequency of price revisions is given by equation (22)

(A-13) $$F = (1-\alpha_L)\mu + (1-\alpha_H)(1-\mu)\,.$$

Appendix B: Deriving The Phillips Curve

Log-linearizing the second equation in (21) we obtain

(B-1) $$\widehat{\mu}_t = -V/\mu\widehat{V}_t\,.$$

Log-linearizing the price index (23) yields

(B-2) $$\widehat{P}_t = \mu\widehat{P}_{L,t} + (1-\mu)\widehat{P}_{H,t}\,.$$

From (B-2), defining $\Pi_{j,t} \equiv \frac{P_{j,t}}{P_{j,t-1}}$,

(B-3) $$\widehat{\Pi}_t = \mu\widehat{\Pi}_{L,t} + (1-\mu)\widehat{\Pi}_{H,t}\,.$$

The log-linear version of equation (20) can be written as

(B-4) $$\widehat{p}^*_{j,t} = (1-\beta\alpha_j)(\widehat{P}_t + \widehat{\psi}_t) + E_t\,\beta\alpha_j\widehat{p}^*_{j,t+1}\,.$$

Using (B-1), the log-linear versions of equations (24) and (25) are

(B-5) $$\widehat{P}_{L,t} = \alpha_L \widehat{P}_{L,t-1} + (1-\alpha_L)\widehat{p}^*_{L,t} + \tfrac{1}{\theta-1}\tfrac{V}{\mu}\left[\widehat{V}_t - \alpha_L \widehat{V}_{t-1}\right]$$

(B-6) $$\widehat{P}_{H,t} = \alpha_H \widehat{P}_{H,t-1} + (1-\alpha_H)\widehat{p}^*_{H,t} - \tfrac{1}{\theta-1}\tfrac{V}{1-\mu}\left[\widehat{V}_t - \alpha_H \widehat{V}_{t-1}\right] .$$

Next, let $R_{j,t} \equiv \frac{P_{j,t}}{P_t}$ and recall $T_t \equiv \frac{P_{L,t}}{P_{H,t}}$. Thus, from (B-2), we have

(B-7) $$\widehat{R}_{L,t} = (1-\mu)\widehat{T}_t \;\text{ and }\; \widehat{R}_{H,t} = -\mu \widehat{T}_t \, .$$

Forwarding (B-5) and solving for $\widehat{p}^*_{L,t+1}$, I obtain

$$(1-\alpha_L)\widehat{p}^*_{L,t+1} = \widehat{\Pi}_{L,t+1} + (1-\alpha_L)\widehat{P}_{L,t} - \frac{1}{\theta-1}\frac{V}{\mu}(\widehat{V}_{t+1} - \alpha_L \widehat{V}_t) \, .$$

Substituting the last equation into the right-hand side of (B-4) for $j = L$, substituting the resulting equation into (B-5), and rearranging yields

(B-8)
$$\widehat{\Pi}_{L,t} = \beta E_t \widehat{\Pi}_{L,t+1} + \frac{(1-\alpha_L)(1-\beta\alpha_L)}{\alpha_L}\left[\widehat{\psi}_t - \widehat{R}_{L,t}\right] \\ - \beta \frac{1}{\theta-1}\frac{V}{\mu} E_t\left[\widehat{V}_{t+1} - \alpha_L \widehat{V}_t\right] + \frac{1}{\alpha_L}\frac{1}{\theta-1}\frac{V}{\mu}\left[\widehat{V}_t - \alpha_L \widehat{V}_{t-1}\right] \, .$$

Forwarding (B-6) and solving for $\widehat{p}^*_{H,t+1}$, I obtain

$$(1-\alpha_H)\widehat{p}^*_{H,t+1} = \widehat{\Pi}_{H,t+1} + (1-\alpha_H)\widehat{P}_{H,t} + \frac{1}{\theta-1}\frac{V}{1-\mu}(\widehat{V}_{t+1} - \alpha_H \widehat{V}_t).$$

Substituting the last equation into the right-hand side of (B-4) for $j = H$, substituting the resulting equation into (B-6), and rearranging yields

(B-9)

$$\widehat{\Pi}_{H,t} = \beta E_t \widehat{\Pi}_{H,t+1} + \frac{(1-\alpha_H)(1-\beta\alpha_H)}{\alpha_H}\left[\widehat{\psi}_t - \widehat{R}_{H,t}\right] + \beta\frac{1}{\theta-1}\frac{V}{1-\mu}E_t\left[\widehat{V}_{t+1} - \alpha_H\widehat{V}_t\right] - \frac{1}{\alpha_H}\frac{1}{\theta-1}\frac{V}{1-\mu}\left[\widehat{V}_t - \alpha_H\widehat{V}_{t-1}\right].$$

Next, multiplying (B-8) times μ and (B-9) times $(1-\mu)$, substituting the resulting equations into (B-3) and using (B-7) yields

(B-10)

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + [\mu\,(1-\alpha_L)(1-\beta\alpha_L)/\,\alpha_L + (1-\mu)\,(1-\alpha_H)(1-\beta\alpha_H)/\,\alpha_H]\,\widehat{\psi}_t + \mu(1-\mu)\,[(1-\alpha_H)(1-\beta\alpha_H)/\,\alpha_H - (1-\alpha_L)(1-\beta\alpha_L)/\,\alpha_L]\,\widehat{T}_t - \frac{1}{\theta-1}V\left[\left(\alpha_H^{-1} - \alpha_L^{-1}\right) - \beta(\alpha_L - \alpha_H)\right]\widehat{V}_t$$

Finally, using (30), and the definitions of a_L, a_H and f in the text, we obtain the Phillips curve equation (27).

Appendix C: Difference equations for T_t and λ_t

The second-order difference equation for T_t, (28), is obtained as follows. Rewrite (B-5) as

(C-1)

$$\beta E_t R_{L,t+1} - [1+\beta+(1-\alpha_L)(1-\beta\alpha_L)/\,\alpha_L]\,R_{L,t} + R_{L,t-1} = \Pi_t - \beta E_t \Pi_{t+1} - [(1-\alpha_L)(1-\beta\alpha_L)/\,\alpha_L]\,\psi_t - \beta\frac{1}{\theta-1}\frac{V}{\mu}E_t\left[\widehat{V}_{t+1} - \alpha_L\widehat{V}_t\right] + \frac{1}{\alpha_L}\frac{1}{\theta-1}\frac{V}{\mu}\left[\widehat{V}_t - \alpha_L\widehat{V}_{t-1}\right].$$

Similarly, rewrite (B-6) as

(C-2)

$$\beta E_t R_{H,t+1} - [1+\beta+(1-\alpha_H)(1-\beta\alpha_H)/\,\alpha_H]\, R_{H,t} + R_{H,t-1} = \Pi_t - \beta E_t \Pi_{t+1}$$
$$-[(1-\alpha_H)(1-\beta\alpha_H)/\,\alpha_H]\,\psi_t + \beta\frac{1}{\theta-1}\frac{V}{1-\mu}E_t\left[\widehat{V}_{t+1}-\alpha_H\widehat{V}_t\right] - \frac{1}{\alpha_H}\frac{1}{\theta-1}\frac{V}{1-\mu}\left[\widehat{V}_t-\alpha_H\widehat{V}_{t-1}\right].$$

Use (B-7) to express (C-1) and (C-2) in terms of T_t, subtract (C-2) from (C-1), collect common terms, and use the definitions of τ_1, τ_2 a_H and a_L in the text to obtain (28).

To derive equation (31), log-linearize equation (12) to obtain

(C-3)
$$\widehat{\lambda}_t = \frac{1-\lambda}{\lambda} b\left[D_H\widehat{D}_{H,t} - D_L\widehat{D}_{L,t}\right].$$

Using the result $D_{0H} = D_{1H}$ in Appendix A, log-linearizing (7) and (8)—evaluated at the optimum price—and comparing the resulting equations, note that the value of the firm satisfies $\widehat{D}_{0j,t} = \widehat{D}_{1j,t}$. Thus, denote with $\widehat{D}_{j,t}$ the value of the firm acting under $(1-\alpha_j)$. Equation (7) for $j=H$ implies

(C-4)

$$D_H\widehat{D}_{H,t} = (1-\tau_H)d\,\widehat{d}_t + [\beta\alpha_H + \beta\lambda(1-\alpha_H)]\,D_H E_t\widehat{D}_{H,t+1}$$
$$+\beta(1-\alpha_H)(1-\lambda)D_L E_t\widehat{D}_{L,t+1} + [D_H-(1-\tau_H)d]\,E_t\left(\widehat{\chi}_{t+1}-\widehat{\chi}_t\right)$$

and for $j=L$ it implies

(C-5)

$$D_L\widehat{D}_{L,t} = (1-\tau_L)d\,\widehat{d}_t + \beta\lambda(1-\alpha_L)D_H E_t\widehat{D}_{H,t+1}$$
$$+[\beta-\beta\lambda(1-\alpha_L)]\,D_L E_t\widehat{D}_{L,t+1} + [D_L-(1-\tau_L)d]\,E_t\left(\widehat{\chi}_{t+1}-\widehat{\chi}_t\right).$$

Subtract $\beta(1-\alpha_H)(1-\lambda)D_H E_t \widehat{D}_{H,t+1}$ from both sides of equation (C-4) to obtain

$$D_H \widehat{D}_{H,t} - \beta(1-\alpha_H)(1-\lambda)D_H E_t \widehat{D}_{H,t+1} = (1-\tau_H)d\,\widehat{d}_t + [\beta\alpha_H + \beta\lambda(1-\alpha_H)]\,D_H E_t \widehat{D}_{H,t+1}$$
$$+\,\beta(1-\alpha_H)(1-\lambda)E_t\left[D_L \widehat{D}_{L,t+1} - D_H \widehat{D}_{H,t+1}\right] + [D_H - (1-\tau_H)d]\,E_t\left(\widehat{\chi}_{t+1} - \widehat{\chi}_t\right).$$

Use (C-3) in the last expression and simplify to find

(C-6)

$$D_H \widehat{D}_{H,t} = \beta D_H E_t \widehat{D}_{H,t+1} + (1-\tau_H)d\,\widehat{d}_t - \beta\frac{\lambda}{1-\lambda}\frac{1}{b}(1-\lambda)(1-\alpha_H)E_t\widehat{\lambda}_{t+1}$$
$$+\,[D_H - (1-\tau_H)d]\,E_t\left(\widehat{\chi}_{t+1} - \widehat{\chi}_t\right)$$

Subtract $\beta\lambda(1-\alpha_L)D_L E_t \widehat{D}_{L,t+1}$ from both sides of equation (C-5), use (C-3), and simplify to obtain

(C-7)
$$D_L \widehat{D}_{L,t} = \beta D_L E_t \widehat{D}_{L,t+1} + (1-\tau_L)d\,\widehat{d}_t + \beta\frac{\lambda}{1-\lambda}\frac{1}{b}\lambda(1-\alpha_L)E_t\widehat{\lambda}_{t+1}$$
$$+\,[D_L - (1-\tau_L)d]\,E_t\left(\widehat{\chi}_{t+1} - \widehat{\chi}_t\right)$$

Next, subtract (C-7) from (C-6) and use (C-3) to obtain equation (31).

Appendix D: Aggregate goods market clearing

Using Yun's (1996) alternative price index, $\overline{P}_t^{(-\theta)} \equiv \int_0^1 [p_t(z)]^{-\theta}\,\mathrm{d}z$, the goods market clearing condition is

$$Y_{c,t} = \left(\frac{\overline{P}_t}{P_t}\right)^{-\theta} C_t\,.$$

Note that the alternative price index can be written as

$$\overline{P}_t^{(-\theta)} = \mu \overline{P}_{L,t}^{(-\theta)} + (1-\mu)\overline{P}_{H,t}^{(-\theta)},$$

where $\overline{P}_{L,t}^{(-\theta)} \equiv \frac{1}{\mu}\int_0^{\mu_t} p_t(s)^{-\theta}\mathrm{d}s$ and $\overline{P}_t^{(-\theta)} \equiv \frac{1}{1-\mu}\int_{\mu_t}^1 p_t(s)^{-\theta}\mathrm{d}s$. Thus,

(D-1) $$\overline{P}_{L,t}^{(-\theta)} = \alpha_L \overline{P}_{L,t-1}^{(-\theta)} + \frac{1}{\mu}\left[(1-\alpha_L)\mu_{t-1} - (V_t - V_{t-1})\right]\left[p_{L,t}^*\right]^{(-\theta)}$$

and

(D-2)
$$\overline{P}_{H,t}^{(-\theta)} = \alpha_H \overline{P}_{H,t-1}^{(-\theta)} + \frac{1}{1-\mu}\left[(1-\alpha_H)(1-\mu_{t-1}) + (V_t - V_{t-1})\right]\left[p_{H,t}^*\right]^{(-\theta)} .$$

Next, define the ratios of price indexes $\overline{R}_t \equiv \overline{P}_t / P_t$, $\overline{R}_{L,t} \equiv \overline{P}_{L,t} / P_{L,t}$ and $\overline{R}_{H,t} \equiv \overline{P}_{H,t} / P_{H,t}$. The log-linear versions of the conditions above are[14]:

(D-3) $$\widehat{Y}_{c,t} = \widehat{C}_t - \theta \widehat{\overline{R}}_t,$$

(D-4) $$\widehat{\overline{R}}_t = \mu \widehat{\overline{R}}_{L,t} + (1-\mu)\widehat{\overline{R}}_{H,t},$$

[14]To obtain (D-5) and (D-6), note that log-linearizing (D-1) and (D-2) we obtain:

$$\widehat{\overline{P}}_{L,t} = \alpha_L \widehat{\overline{P}}_{L,t-1} + (1-\alpha_L)\widehat{p}_{L,t}^* + \frac{V}{\mu}\frac{1}{\theta}(\widehat{V}_t - \alpha_L \widehat{V}_{t-1})$$

and

$$\widehat{\overline{P}}_{H,t} = \alpha_H \widehat{\overline{P}}_{H,t-1} + (1-\alpha_H)\widehat{p}_{H,t}^* - \frac{V}{1-\mu}\frac{1}{\theta}(\widehat{V}_t - \alpha_H \widehat{V}_{t-1}).$$

Next, subtract (B-5) and (B-6) from the equations above to obtain $\widehat{\overline{R}}_{L,t}$ and $\widehat{\overline{R}}_{H,t}$.

(D-5) $$\widehat{\overline{R}}_{L,t} = \alpha_L \widehat{\overline{R}}_{L,t-1} - \frac{V}{\mu} \frac{1}{\theta(\theta-1)} \left[\widehat{V}_t - \alpha_L \widehat{V}_{t-1} \right]$$

and

(D-6) $$\widehat{\overline{R}}_{H,t} = \alpha_H \widehat{\overline{R}}_{H,t-1} + \frac{V}{\mu} \frac{1}{\theta(\theta-1)} \left[\widehat{V}_t - \alpha_H \widehat{V}_{t-1} \right] .$$

To calculate impulse responses, I replace the approximation (40) in the text with (D-3) and include equations (D-4) to (D-6).

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Figure 1.

Slope of Phillips Curve in the (Π_t, Ψ_t) Space

(varying the unconditional mean of the random cost $E[\xi] = 1/b$)

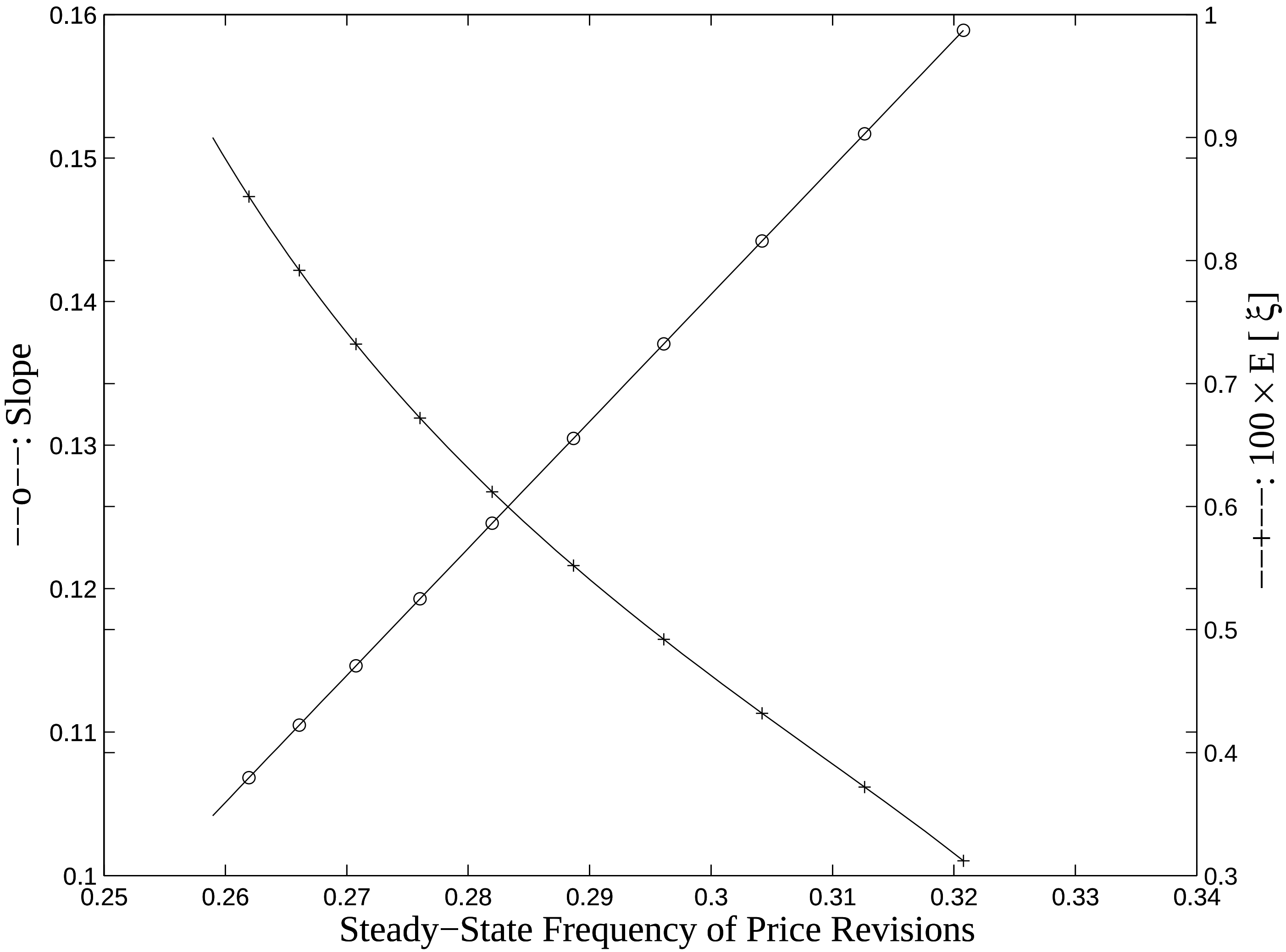
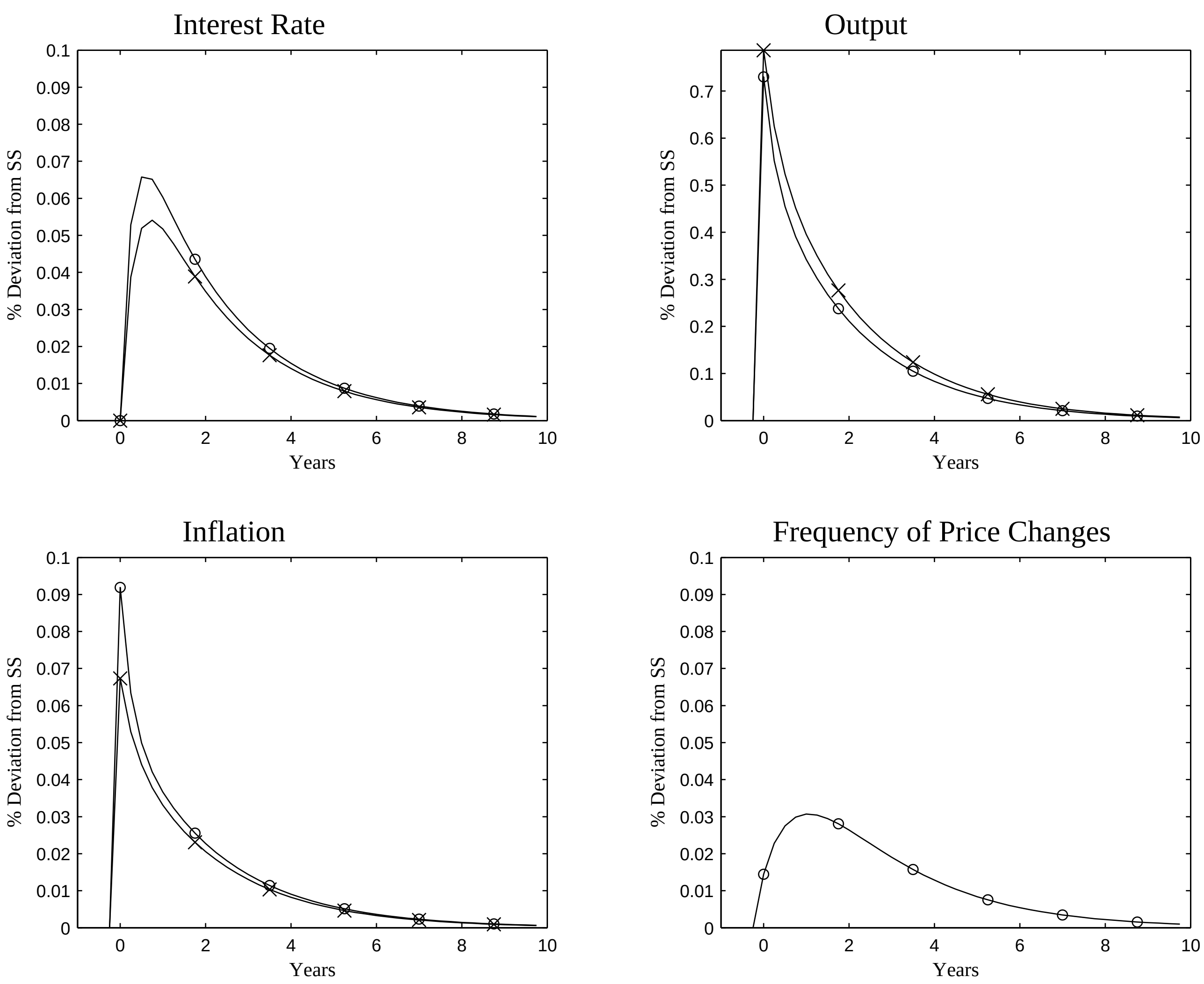
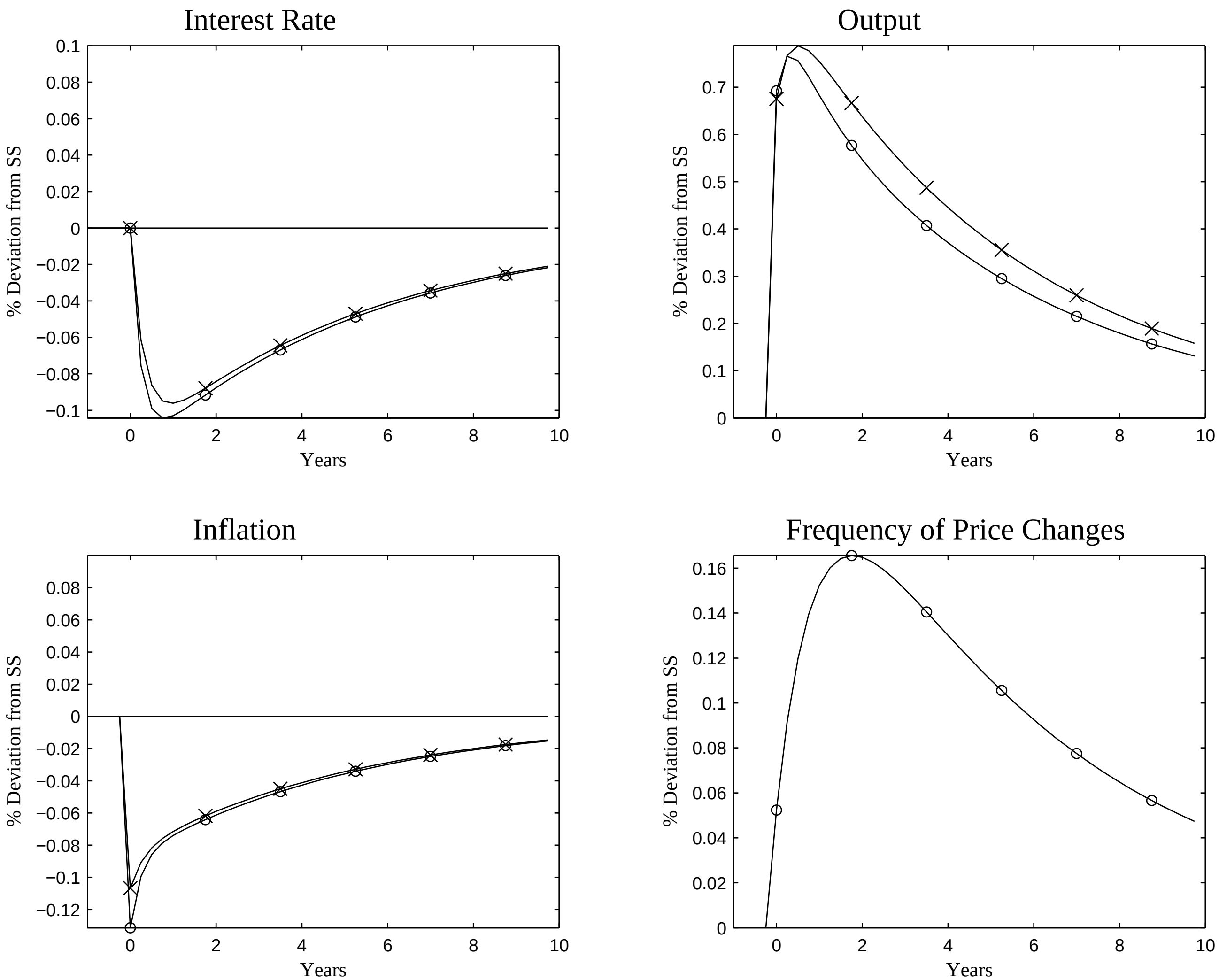


Figure 2. Response to a Preference Shock



$-o-$ Model with State $-$ Dependent Frequency of Price Changes, $- \times -$ Time $-$ Dependent Model

Figure 3. Response to a Productivity Shock



−o − Model with State − Dependent Frequency of Price Changes, − × − Time − Dependent Model

Figure 4. Response to a Expansionary Taylor Rule Shock

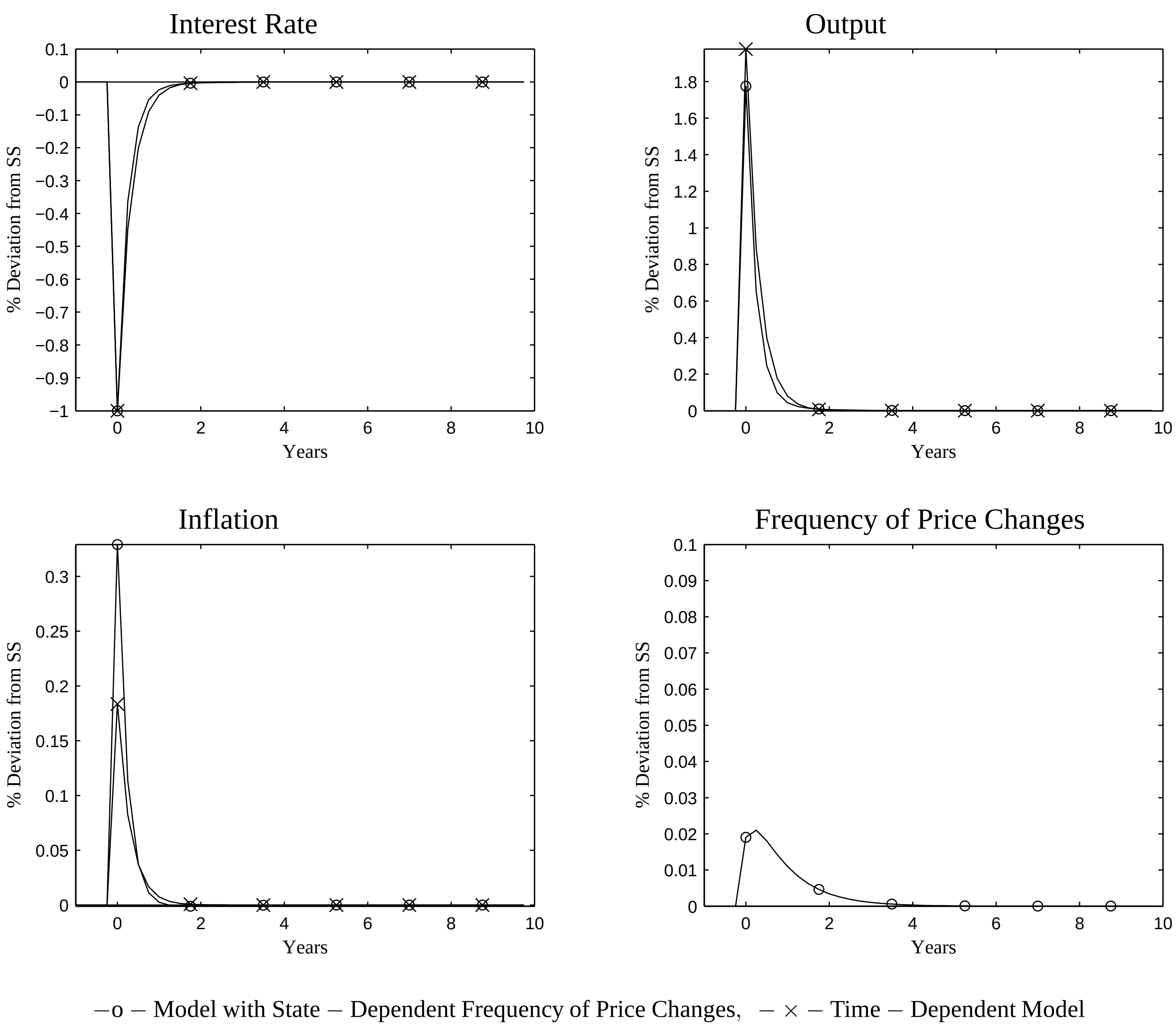
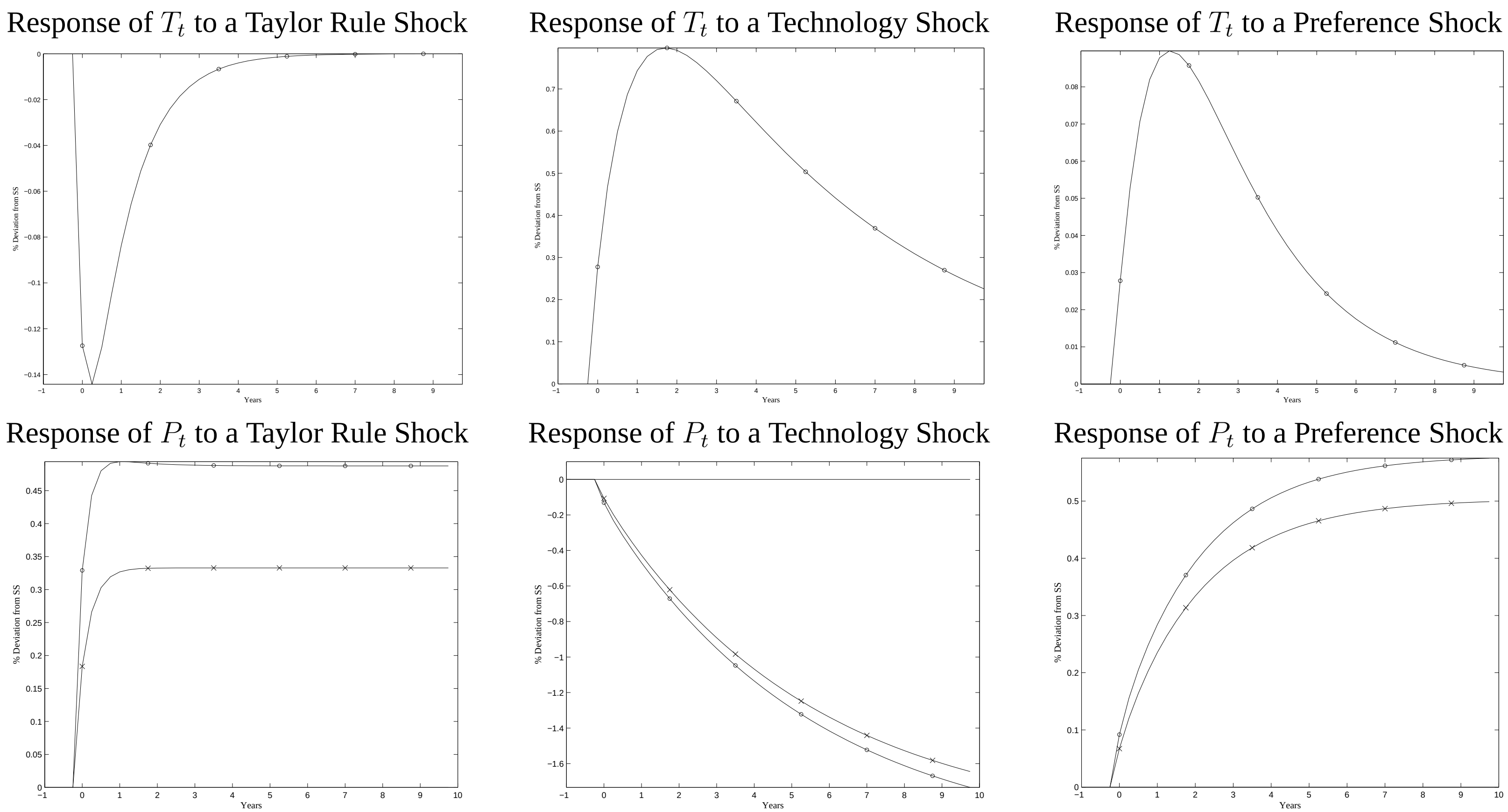


Figure 5. Response of the Relative Price T_t and the Price Index P_t



–o–Model with State-Dependent Frequency of Price Changes,–×– Time-Dependent Model