

***The Third of Working papers for Researches about Development Power (DP)***

Version 1.0

The Completed Time: 2004-11-14

## **The Soft Engine for Economic Growth in a Long-Time: The Economic Development Power, Conversion and Conservation for economic Energy**

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**Abstract** Based on the theories of Development Power<sup>[18]-[20]</sup> and Partial Distribution<sup>[21]-[24]</sup>, and combining with the actual economic development, this paper discusses the main ways how the Development Power (DP) is accumulated or released, establishes the analytic models of DP and the model of relation between DP and productivity; classifies economic development into the three states in energy—the state of normal energy, strong energy and super energy; gives the method of calculating the critical time of strong energy and super energy and the approach to describe the evolving process of these three states; puts forward the laws of conversion and conservation between mass and energy in economic developing. According to that DP is accumulated and released one after another, this paper points out, a sign that accumulation of DP is completed is the most of economic factors are in order, and the sign that release of DP is completed is the most of economic factors are in chaos, and also gives the ways to calculate the most possible time that the DP starts accumulating or releasing. Finally, the author make a comprehensive empirical analysis on all of results in this paper by means of the US GDP data from 1940 to 2003, the outcomes are satisfied.

The conclusions in this paper make clear that the economic Development Power, namely economic development energy, is the engine for the economic growth.

**Key words** Development Power (DP), Partial Distribution, economic growth, energy states of economic development, conversion and conservation on economic energy, analytic model

## **0 Introduction**

Up to now, there are many of the important studies in the economic theory, such as R. E. Lucas's business cycle theory [1981], Prescott's real business cycle theory [1982], P. M. Romer's new growth theory [1986], and D. Kahneman's prospect theory [1979], etc. These theories has availably propeled forward the economic development of world. But, there are many economic problems which have not yet been solved thoroughly. Such as,

- What is the reason that cause the cycle in economic development?
- Which is the the dominant causation that arose the fluctuation of economic market
- Whether there are the mass and the energy movement in the processe of conomic developmenting or not ? how can we describe and measure them if there are mass and the energy movement ?
- There is the difference between the different countries in the rate of economic growth and the level of

productivity, what is the reasons of it ?

In order to solve the problems mentioned above, the concept of Development Power (DP) has been given and discussed first in the references [18-20]. The theory of DP indicates:

- The productivity is the visible behavior ability for mankind to improve the society and nature, and the DP is the invisible behavior motivity for mankind to improve the society and nature. In the economic society, DP is the behavior motivity to develop economy, and we also call it the economic DP (EDP for short). EDP is also the energy of economic development.
- The productivity is the hardware of economic development; and EDP is the software of economic development. Both of them are of symmetry, and the most basic factors in economic development. The movement of EDP will influence the level of productivity, and vice verse.
- EDP is just the economic developing energy. The accumulation and release of EDP bring on the fluctuations of it, namely the fluctuations of economic developing energy (EDE). Just the movement of EDP (or EDE) results in the fluctuation of economic productivity, so it is the motive force of economy.
- The EDP is scaled by economic development energy, and measured by fluctuation rate of level of productivity. That large or small in EDP (or EDE) will decides that the potentials of economic growth is large or small, and the duration of economic growth is long or short.
- If we describe the basic level of productivity with Gross Nation Product (GDP), then the fluctuation rate of productivity could describes EDP. At this time, we could measure EDP by means of variance of productivity level. And assume that the actual level of productivity follows the Partial Distribution<sup>[21]-[23]</sup>.

It is worth to say, the fluctuation is generally a description of risk in mainstream of the economic doctrine, but it is the developing energy in DP theory.

The following discussion will do not distinguish DP, EDP and EDE. Based on the theory of DP and combing with real economic developing, this paper will:

- discusses the main way and basic model of DP in its accumulation and release.
- advances three kind of energy state of economic growth, i.e., the state of normal energy, strong energy and super energy, and gives the approaches to calculate the critical times of strong energy and super energy.
- puts forward the laws of conversion and conservation between mass and energy in economic developing according to the model of relation between DP and productivity.
- points out the sign that accumulation of DP is completed is the most of economic factors are in order, and the sign that release of DP is completed is the most of economic factors are in chaos, and gives the ways to calculate the most possible time that the DP starts accumulating or releasing.
- indicates the reason why the fluctuation of economic production and market occur. The basic reason is there are many violent bifurcations between the productivity and DP, such as DP is too small to support the economic growth, or DP is so larger to reverse the original trend of economic recession quickly.
- explains that the real reason of the economic differences is the differences in their ability of DP accumulating and releasing between developed countries (or regions) and undeveloped countries (or regions). Only the DP is the durative motivity for economic growth.

Finally, we shall make a comprehensive empirical analysis on all of results in this paper by means of the US GDP data from 1940 to 2003.

## 1 The Main Approaches for DP Accumulating and Releasing

The references [18]-[20] indicate: The productivity is the visible ability for mankind to push forward economy, and the DP is the invisible motive force for mankind to push forward economy. DP includes the invisible economic factors such as policy, science and technology, education, knowledge, management, law, culture, idea, etc. so we could say that DP is the soft engine for economic development. The way of DP accumulating or releasing is complicated and diverse. Here, we give a brief description about the basic ways for DP accumulating and releasing, in order to emphasize that DP is very important and of ubiquity in national economic developments.

### **1.1 The main approaches for accumulating DP**

1) The positive innovation to national economic system. The national economic system plays an enormously important function to control the economic development. This function is radically produced from the system development power (SDP). Making a continuous innovation on economic system, and insuring the economic system being reasonable and efficient, will allow SDP to be accumulated continuously, and enable the different branches of economic society to be coordinated one another always, so that the economy could keep on growing.

2) Perfecting continuously the economic policies and economic laws. The perfection of economic policies and laws can urge, control and guide the economy to develop in right direction. This kind of function of economic policies and laws is the policy development power (PDP). Therefore, we could acquire the PDP to accelerate economic growth if renewing and perfecting the national policy and laws system continuously.

3) Establishing effectively the innovating mechanism in science and technology. The developing vitality of economy comes from many of new economic products going into operation, and new economic products are the results of the application of various new science and technology. Just the kind of force for science and technology to impel economic development is the development power of science and technology, STDP for short. So we must establish the innovating mechanism in science and technology, promote science the technical to progress, and translate the results of new science and technology to new economic products, namely accumulate the STDP continuously, in order to get the inexhaustible force of science and technology for economic development.

4) Developing completely the system of economic education. The good education system and the practice environment in economy can make peoples who are engaged in the economic activities learning the necessary knowledge, and get the worthy experience. Those peoples can use their knowledge and the experience in economy to promote the national economic development. So the knowledge and experience are a kind of force, is called the knowledge development power (KDP). Therefore, establishing completely a system of economic education is very important for improving the national economic environment, accumulating the KDP effectively, and providing a good foundation for the economic development.

5) Standardizing pattern and optimizing structure in the economic management. Standardizing management in every level of economic departments plays an important and non-replaceable role in promoting economy to develop orderly and effectively. We could regard this function of Standardizing management as the management development power (MDP). Therefore, the MDP will be accumulated effectively if we make a positive innovation in standardizing the economic management and in optimizing the hierarchical structure of economic departments, so that the developing force in management will be obtained to push forward the economic progress.

6) Designing and achieving the large scale and synthetic projects. Designing and achieving a large scale and

synthetic project usually affect the situation of national construction and the economic development as a whole. Just the force of large scale project push economy forward is a project development power (PDP), e.g. both the success in application to hold the Olympic Game'2008 and the successes in launching and returning of spaceflight airship with person at 2003 are all the better ways to accumulate *PDP* for Chinese economic development.

Just the process of launching the full of items mentioned above completely is the accumulating process for national economy DP.

### 1.2 The main approaches for releasing DP

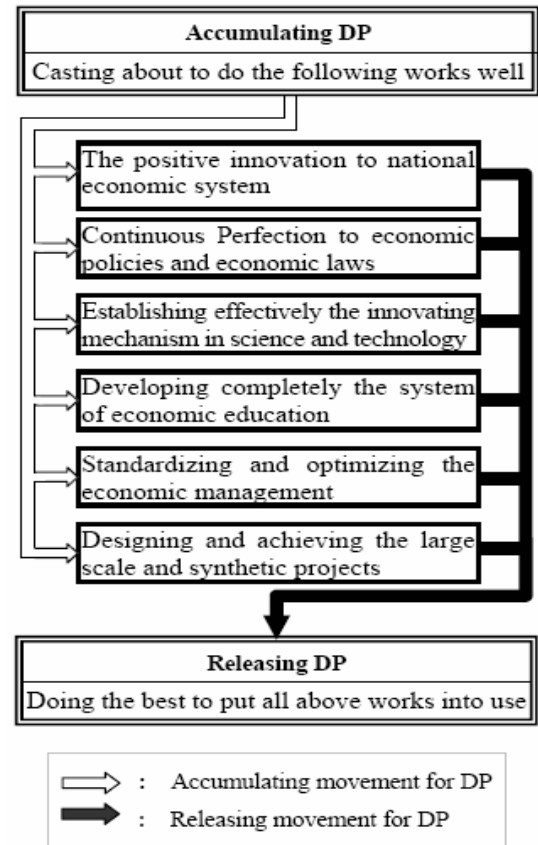
After all of the works mentioned in 1.1 have been done, we have the new or improved systems of economy, new economic policies and laws, new innovating mechanism in science and technology, new or improved system of economic education, standard and optimal management in economy, and other things in that DP are accumulated. At this time, systems and environments of economy are better and ordered, economy starts developing in a good process, and the DP which had been accumulated starts releasing. In the releasing process of DP, the economy will have a stable and order development until DP is released sufficiently, and then a new process of DP accumulating starts.

### 1.3 A brief summary

In figure 1, we describe the approaches and process of accumulating and releasing DP.

To say in short, the DP is accumulated by the group innovations, and the DP is released by the applications of the group innovations. The DP movements of accumulation and release are shown as the fluctuations of economic energy, and those fluctuations influence the economic growth or economic recession.

So we see that DP should be the engine for economic growth.



**Figure 1** The ways and process of accumulating and releasing DP. DP is the behavior motivity and the soft engine to develop economy, it is the invisible force, and include policy, science and technology, education, knowledge, management, systems and laws, culture, idea, etc.

## 2 The Analytic Models of DP and Productivity

### 2.1 Notations and descriptions of models

If note:

$\mu$ —the basic level of economy, namely a measuring index for basic productivity, its real value is generally the production value (e.g. GDP, Gross Domestic Product), the basic productivity for short,  $\mu \geq 0$ . Because the economic growth is essentially the increase for level of productivity, and the economic recession is

essentially the decline for level of productivity, we do not distinguish the level of economy from productivity in the following discussion, i.e. regard them as the same.

$\sigma$ —the fluctuation range of the basic productivity, i.e., the standard variance of the basic the basic productivity level,  $\sigma > 0$ .  $\sigma$  can describe the absolute energy of economic development.

$\nu = \sigma/\mu$ —the fluctuation rate of the basic productivity, is a measuring index of DP (Economic Development Power).  $\nu$  can describe the developing energy of economy, generally  $0 < \nu < 1$ . In the following discussion, we do not distinguish DP from the developing energy of economy, i.e. regard them as the same.

$X$ —the actual productivity.  $X$  is a non-negative random variable. According to assumption in [18]-[20], the actual productivity follows the distribution of density as

$$f(x) = \begin{cases} e^{-\frac{(x-\mu)^2}{2\nu^2}} / \int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\nu^2}} dx & x \geq 0 \\ 0 & x < 0 \end{cases}$$

At this time, we call  $X$  follow the partial distribution <sup>[21]-[25]</sup>, and denote:  $X \in P(\mu, \nu^2)$ . Thus, we have

**Theorem 1** For any  $x \in [0, \infty]$ , the following equations come approximately into existence

$$1) \int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\nu^2}} dx = \sqrt{\frac{\pi}{2}} \nu (\sqrt{1 - e^{-\frac{2}{\pi}(\frac{\mu}{\nu})^2}} + 1).$$

$$2) \int_0^x e^{-\frac{(x-\mu)^2}{2\nu^2}} dx = \sqrt{\frac{\pi}{2}} \nu \times (\sqrt{1 - e^{-\frac{2}{\pi}(\frac{\mu}{\nu})^2}} + \text{sgn}(x - \mu) \sqrt{1 - e^{-\frac{2}{\pi}(\frac{x-\mu}{\nu})^2}}).$$

$$\text{where, } \text{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}.$$

If  $\mu, \nu, X$  are all time-variant, and noting respectively as  $\mu(t), \nu(t), X(t)$ , thus we have a partial process  $X(t) \in P(\mu(t), \nu^2(t))$ . Especially, if  $X(t) \in P(\mu(t), \nu^2(t))$ , we call  $X$  follow a DF process <sup>[26]</sup>.

## 2.2 Some basic results

According to assumptions in references [18]-[20], if the actual productivity  $X \in P(\mu, \nu^2)$ , then we have <sup>[21]-[25]</sup>

1) The average value of actual productivity:

$$E(X) = \mu + \sqrt{\frac{2}{\pi}} \nu \frac{e^{-\frac{\mu^2}{2\nu^2}}}{\sqrt{1 - e^{-\frac{2}{\pi}(\frac{\mu}{\nu})^2}} + 1}$$

where,  $R(X) = \sqrt{\frac{2}{\pi}} \nu \frac{e^{-\frac{\mu^2}{2\nu^2}}}{\sqrt{1 - e^{-\frac{2}{\pi}(\frac{\mu}{\nu})^2}} + 1}$  can expresses that the average increment of actual level of productivity is

exceeding the basic level of it.

2) The average of real DP (development energy):

$$\nu(X) = \sqrt{D(X)}$$

where,  $D(X) = \nu^2 + E(X)[\mu - E(X)]$ .

Because  $R(X) > 0$ , we have  $E(X) > \mu$ , and  $D(X) < \nu^2$ .  $E(X) > \mu$  means the average value of real productivity is

larger than the basic level of productivity, this also means the real productivity is generally in a growing trend; and  $D(X) < v^2$  means DP usually is releasing continuously until it reaches a lower level, at this time DP will release again after it has been accumulated.

### 3 The Analytic Model of DP

#### 3.1 The basic model of DP

Generally speaking, the differential ratio of  $v$  (value of DP) to  $\mu$  (value of productivity) should have something to do with current state of DP, i.e.,

$$\frac{dv}{d\mu} = g(v)$$

where,  $g(v)$  is a continuous function. If productivity is of growth continuously,  $g(v) > 0$  means DP is accumulating,  $g(v) < 0$  means DP is releasing, and  $g(v) = 0$  means DP is stable, and if productivity is of recession continuously, the meanings is reverse.

Specially, if  $g(v) = \gamma v$ ,  $\gamma$  is a constant. Then we obtain

$$v = \alpha e^{\gamma \mu} \quad (1)$$

where, the constant  $\alpha > 0$ .

If DP and productivity are time-variant, from equation (1), we know  $v_0 = \alpha e^{\gamma \mu_0}$ . so the equation (1) can be expressed as

$$v(t) = v_0 e^{\gamma[\mu(t) - \mu_0]} \quad (2)$$

where,  $v_0$  and  $\mu_0$  are the original values of DP and productivity respectively; the constant  $\gamma$ , is called the characteristic index of DP, could describes whether DP is larger or smaller, and has the following meaning:

1) In the process of economic growth, i.e. the productivity  $\mu(t)$  is increasing by degree,

$$\gamma = \begin{cases} > 0, \text{ DP is accumulating} \\ = 0, \text{ DP is stable} \\ < 0, \text{ DP is releasing} \end{cases}$$

2) In the process of economic recession, i.e. the productivity  $\mu(t)$  is of descending

$$\gamma = \begin{cases} > 0, \text{ DP is releasing} \\ = 0, \text{ DP is stable} \\ < 0, \text{ DP is accumulating} \end{cases}$$

According to model (2), if  $v_0 = v(0) > 0$ , we obtain  $\mu(t) = \mu_0 + \frac{1}{\gamma} \ln \left( \frac{v(t)}{v_0} \right)$ . This equation expresses how DP,

i.e. economic developing energy  $v(t)$ , influences productivity  $\mu(t)$ , so we could think that DP is the engine for economic growth.

#### 3.2 The time-variant models of DP and productivity

The equation (2) can also be expressed as

$$v = \frac{v_0 e^{-\gamma\mu_0}}{\delta^2} \left( \pm \delta e^{\frac{\gamma\mu}{2}} \right)^2 \quad (3)$$

where, the meaning of “+” is increasing in productivity growth, and “-” is economic recession;  $\delta(>0)$  is a constant. Let

$$\frac{d\mu}{dt} = \pm \delta e^{\frac{\gamma\mu}{2}} \quad (4)$$

Then, we obtain

$$\mu(t) = -\frac{2}{\gamma} \ln \left( e^{-\frac{\gamma\mu_0}{2}} \mp \frac{\delta\gamma}{2} t \right) \quad (5)$$

where,  $\delta$  is called the parameter in scale, and the equation (4) could be fitted furthest with the economic reality by use of parameter  $\delta$ .

**3.2.1 The time-variant models of DP and productivity in economic growth.** From (3) and (5), and we can express productivity  $\mu(t)$  for economic growth as

$$\mu(t) = -\frac{2}{\gamma} \ln \left( e^{-\frac{\gamma\mu_0}{2}} - \frac{\delta\gamma}{2} t \right) \quad (6)$$

and DP  $v(t)$  can be expressed as

$$v(t) = v_0 e^{-\gamma\mu} \left( e^{-\frac{\gamma\mu_0}{2}} - \frac{\delta\gamma}{2} t \right)^{-2} \quad (7)$$

In the expression (6) and (7),  $\gamma>0$  and  $0 \leq t < \frac{2}{\delta\gamma} e^{-\frac{\gamma\mu_0}{2}}$  are adapt to the economic growth with DP

accumulating; and  $\gamma<0$  and  $t>0$  are adapt to the economic growth with DP releasing. Then, the expression (7) can be expressed as

$$v(t) = v_0 \left( 1 - \frac{\delta\gamma}{2} t e^{\frac{\gamma\mu_0}{2}} \right)^{-2}.$$

**3.2.2 The time-variant models of DP and productivity in economic recession.** Similarly, from (3) and (5), productivity  $\mu(t)$  can be expressed for economic recession as

$$\mu(t) = -\frac{2}{\gamma} \ln \left( e^{-\frac{\gamma\mu_0}{2}} + \frac{\delta\gamma}{2} t \right) \quad (8)$$

and DP  $v(t)$  can be expressed as

$$v(t) = v_0 e^{-\frac{\gamma\mu_0}{2}} \left( e^{-\frac{\gamma\mu_0}{2}} + \frac{\delta\gamma}{2} t \right)^{-2} \quad (9)$$

In the expression (8) and (9),  $\gamma < 0$  and  $0 \leq t < \frac{2}{\delta|\gamma|} (e^{\frac{\gamma\mu_0}{2}} - 1)$  are adapt to the economic recession with DP

accumulating; and  $\gamma > 0$  and  $0 \leq t < \frac{2}{\delta\gamma} (1 - e^{-\frac{\gamma\mu_0}{2}})$  are adapt to the economic growth with DP releasing. Then,

the expression (9) can be expressed as

$$v(t) = v_0 \left( 1 + \frac{\delta\gamma}{2} t e^{\frac{\gamma\mu_0}{2}} \right)^{-2}.$$

**3.2.3 The approach of calculating the parameter  $\delta$ .** For the process of economic growth, and according to (6),

we have  $\mu(1) = -\frac{2}{\gamma} \ln(e^{-\frac{\gamma\mu_0}{2}} - \frac{\delta\gamma}{2})$ . The parameter  $\delta$  can be estimated as the following formula (10):

$$\delta = \frac{2}{\gamma} (e^{-\frac{\gamma\mu_0}{2}} - e^{-\frac{\gamma\mu_1}{2}}) \quad (10)$$

Similarly, for the process of economic recession, and according to (8), we have  $\mu(1) = -\frac{2}{\gamma} \ln(e^{-\frac{\gamma\mu_0}{2}} + \frac{\delta\gamma}{2})$ .

The parameter  $\delta$  can be estimated as the following formula (11):

$$\delta = \frac{2}{\gamma} (e^{-\frac{\gamma\mu_1}{2}} - e^{-\frac{\gamma\mu_0}{2}}) \quad (11)$$

Of course, there are some other approaches for estimating  $\delta$ .

## 4 The Analysis for Economic Development Energy and Its Models

### 4.1 The developing energy analysis in economic growth

In the process of economic growth with DP accumulating, i.e.  $\mu(t)$  is increasing and  $r > 0$ , if let

$v(t) = v_0 \left( 1 - \frac{\delta\gamma}{2} t e^{\frac{\gamma\mu_0}{2}} \right)^{-2} = 1$ , we have

$$t_\varepsilon = \frac{2}{\delta\gamma} e^{-\frac{\gamma\mu_0}{2}} (1 - \sqrt{v_0}) \quad (12)$$

Thus, when  $t > t_\varepsilon$ ,  $v(t) > 1$ .

And let  $e^{-\frac{\gamma\mu_0}{2}} - \frac{\delta\gamma}{2} t = 0$ , we obtain

$$t_\tau = \frac{2}{\delta\gamma} e^{\frac{\gamma\mu_s}{2}} \quad (13)$$

Thus, when  $t = t_\tau$ ,  $v(t) = \infty$ .

**Definition 1** Suppose that economy is growing, and DP is accumulating.  $t_e$  and  $t_\tau$  are respectively determined by (12) and (13), then

1) When  $0 < t < t_e$ , and at this time,  $0 < v(t) < 1$ , we call the state of economy the normal state in energy, and the normal state of economy for short.

2) When  $t_e \leq t < t_\tau$ , and at this time,  $1 < v(t) < \infty$ , i.e.  $DP > 1$ , we call the state of economy the strong state in energy, and the strong state of economy for short.  $t_e$  is the critical time of strong state.

3) When  $t = t_\tau$ , and at this time,  $v(t) = \infty$ , i.e.  $DP = \infty$ , we call the state of economy the super state in energy, and the super state of economy for short.  $t_\tau$  is the critical time of super state.

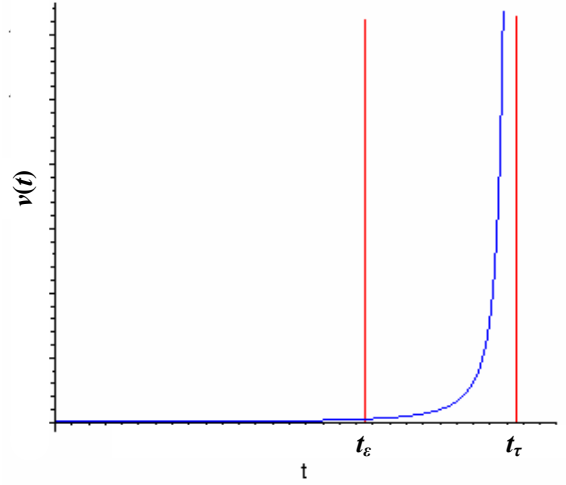
The evolving process of the normal state, the strong state and the super state of economy is shown in figure 2.

Based on definition 1, what we need to explain are:

1) For the strong state in economy,  $DP v(t) \geq 1$ . In fact, that circumstance could be happened. It does not mean that DP (i.e., developing energy) of one industry is larger than 1, where the production value of “one industry” is computed as a whole; it means that DP of one industry (we call it underlying DP) diffuse to many of other industry, and make their DP (we call the derivative DP) accumulate respectively, so that the sum of all the underlying DP and the derivative DP is larger than 1. For example, the development of the computer industry makes the computers being used widely, and the usage of computer causes other industries to develop, such the industries as manufacturing control, weather analysis, spaceflight engineering, scientific simulation, etc. As for manufacturing control, the application of computers can bring about an advance in technique ability of manufacturing control, i.e. DP of manufacturing control being accumulated efficiently, so that the productivity of manufacturing control is propelled forward; and if the products of manufacturing control based on computer are applied to producing process, and achieve a good actual results, thus DP of manufacturing control starts releasing. Further more, the development of manufacturing control will propel the its downstream industries forward, such as spin and weave, paper making, steel industry, machine and electronics, etc. On the other hand, the higher request to computer from manufacturing control will accumulate the DP of computer industry. All of that are the diffusing process of DP, and can be described by figure 3.

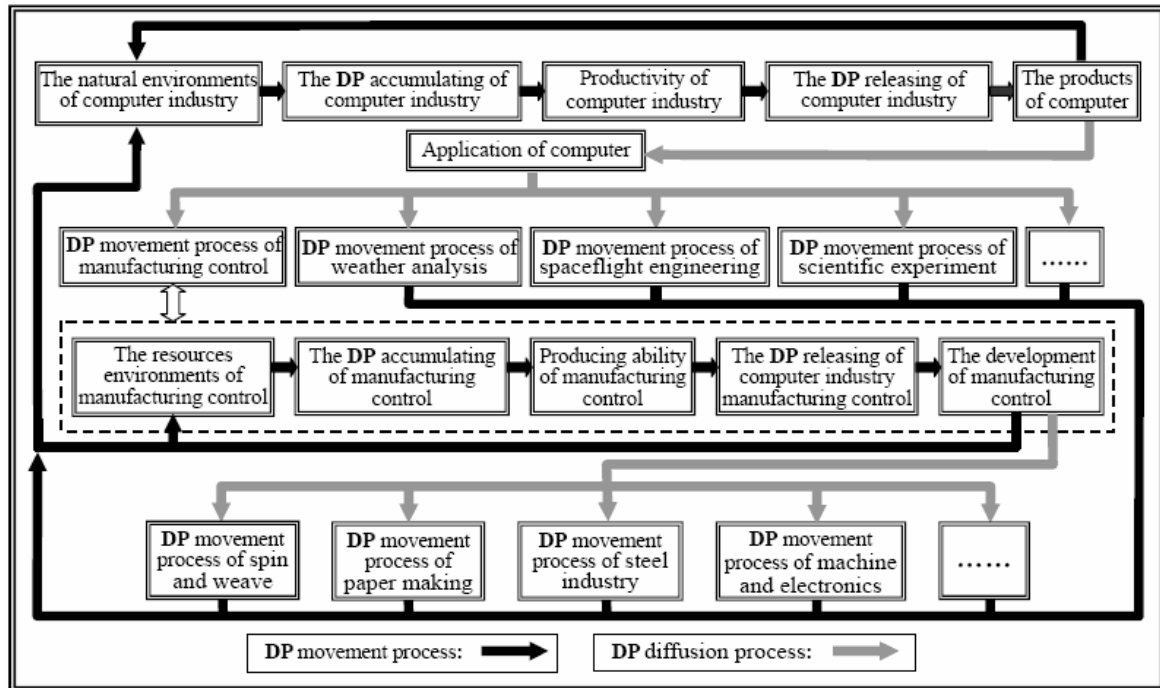
2) For the super state in economy,  $DP v(t) = \infty$ . That is an extreme case of strong state in economy; the sum of underlying DP and its derivative DPs is so large that we can not evaluate DP of the industry by a finite way. We have a similar explanation to  $\mu(t) = \infty$  (productivity level is of infinity) under the super state in economy.

$DP v(t) = \infty$  means, according to the definition of  $v(t)$  in section 2.1, that the variance of the productivity



**Figure 2** The evolving process of the normal state, the strong state and the super state of economy. When  $0 < t < t_e$ , then  $DP v(t) \in (0, 1)$ , the economic state is normal state in energy. When  $t_e \leq t < t_\tau$ , then  $DP v(t) \in (1, \infty)$ , the economic state is strong state in energy.  $t_e$  is the critical time. When  $t = t_\tau$ , then,  $DP v(t) = \infty$ , the economic state is super state in energy,  $t_\tau$  is the critical time of super state

level is of infinity. As we know, the variance of Levy distribution (its characteristic function is  $\hat{l}_\alpha(a, k) = e^{-a|k|^\alpha}$ ) is of infinity, i.e.  $\langle x^2 \rangle = \infty (0 < \alpha < 2)$ . So it might be adaptive to describe the behavior of super state in economy by Levy distribution; if like this, the super state in economic energy might be in accordance with the theory of fractal economy<sup>[28]</sup> in some degree.



**Figure 3** DP movement and diffusion process in computer industry. DP of computer industry, as an underlying DP, could be diffused to other industries, and the DP of those other industries might be accumulated along with underlying DP accumulating. The strong or super state in economy might occur after because the underlying DP and its derivative DP were accumulated.

3) For the economic growth with DP accumulating, in general, the DP  $v(t) < 1$ ; if  $t$ , the durative time of this economic process, is larger than  $t_e$ , the critical time of strong state, i.e.  $t > t_e$  and  $v(t) > 1$ , the diffusion of DP may occur, the industrial economy presents a strong state in energy; and after the industrial economy has come into strong state in energy, the process of economic growth with DP accumulating has been lasted out further and  $t$ , the durative time of this economic process, is equal to  $t_e$ , the critical time of super state, i.e.  $t = t_e$  and  $v(t) = \infty$ , the industrial economy may presents a super state in energy, i.e. a fractals. Is the super state of economy a final state?

4) It is very difficult to keep both productivity and DP growing continuously in a long term. In generally, DP started releasing before the time of its accumulating came to  $t_e$  (the critical time of strong energy in economy). So the industrial economy usually is developed on the normal state in energy. Of course, we do not expel some parts of industry may come into the strong state in energy. On the strong state in energy, it is more difficult for both productivity and DP still to keep on growing. Only a very little of industries will come into the super state in energy, and this shall be confirmed in the hinder empirical researches.

If we regard the force of price rising or falling as the result of energy movement for prices in security

market, there is DP (the energy of price fluctuation) in security market also. There must be energy flow in the diffusion process in which the price fluctuation of one or some securities (stocks, stocks indexes, underlying products, derivatives, etc.) might cause the price fluctuation of other securities. So the strong state or super state in securities might occur. If like this, the super state in security market can be described with the fractal structure in some time.

If the productivity is growing and DP is releasing, i.e.  $\mu(t)$  is increasing and  $\gamma < 0$ , we have

**Proposition 1** In the process of economic growth with DP releasing, both the super state and the strong state in economy can not occur.

*Prove.* Owing to  $\gamma < 0$  (DP is releasing) and  $\delta > 0$ , then  $1 - \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} > 0$ , and there is not the strange point  $t_r$  which make DP  $v(t)$  in expression (7) be equal to  $\infty$ , namely the super state does not occur. If let

$$v(t) = v_0 \left( 1 - \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \right)^{-2} \geq 1. \text{ We have } \left( 1 - \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \right)^2 \leq v_0, \text{ i.e. } -\sqrt{v_0} \leq 1 - \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \leq \sqrt{v_0}. \text{ Owing to } v_0 < 1 \text{ and}$$

$\gamma < 0$ , the inequality  $1 - \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \leq \sqrt{v_0}$  can not come into existence, thus  $v(t) < 1$ .

The result follows.

#### 4.2 The developing energy analysis in economic recession

Here, we give some propositions about developing energy in economic recession (i.e.  $\mu(t)$  is decreasing,) as following.

**Proposition 2** The super state in economy does not occur in the economic recession with DP accumulating.

*Prove.* When  $\gamma > 0$ , the equation  $1 + \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} = 0$  is not existent; and when  $\gamma < 0$ ,  $1 + \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} = 0$  can be expressed as

$$t = \frac{2}{\delta|\gamma|}e^{-\frac{\gamma\mu_0}{2}}, \text{ this is contradicting with } 0 \leq t < \frac{2}{\delta|\gamma|}(e^{\frac{\gamma\mu_0}{2}} - 1) \text{ in expressions (8) and (9), the existent conditions}$$

of productivity (8) and DP (9).

The results follow.

**Proposition 3** The strong state in economy does not occur in the economic recession with DP releasing (i.e.  $\gamma > 0$ ).

*Prove.* If we suppose  $v(t) = v_0 \left( 1 + \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \right)^{-2} \geq 1$ , then  $\left( 1 + \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \right)^2 \leq v_0$ , i.e.,  $-\sqrt{v_0} \leq 1 + \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \leq \sqrt{v_0}$ .

As we know, the inequality  $1 + \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \leq \sqrt{v_0}$  is not existent when  $v_0 < 1$  and  $\gamma > 0$ , so it must be

$$v(t) = v_0 \left( 1 + \frac{\delta\gamma}{2}te^{\frac{\gamma\mu_0}{2}} \right)^{-2} < 1$$

The result follows.

**Proposition 4** In the economic recession with DP accumulating (i.e.  $\gamma < 0$ ) the super state in economy is not

existent, and the strong state in economy may be existent when  $\sqrt{v_0} e^{-\frac{\gamma\mu_0}{2}} > 1$ .

*Prove.* If we suppose  $e^{-\frac{\gamma\mu_0}{2}} + \frac{\delta\gamma}{2}t = 0$ , then  $t_r = \frac{2}{\delta|\gamma|} e^{-\frac{\gamma\mu_0}{2}}$ , this is contradicting with  $0 \leq t < \frac{2}{\delta|\gamma|} (e^{-\frac{\gamma\mu_0}{2}} - 1)$  in expressions (8) and (9), the existent conditions of productivity (8) and DP (9). So the super state in economy is not existent.

Let  $v(t) = v_0 \left(1 + \frac{\delta\gamma}{2} t e^{-\frac{\gamma\mu_0}{2}}\right)^{-2} = 1$ , then  $t_e = \frac{2}{\delta|\gamma|} e^{-\frac{\gamma\mu_0}{2}} (1 - \sqrt{v_0})$ , and compared with  $0 \leq t < \frac{2}{\delta|\gamma|} (e^{-\frac{\gamma\mu_0}{2}} - 1)$  in

expressions (8) and (9), the strong state in economy might be existent only if  $\sqrt{v_0} e^{-\frac{\gamma\mu_0}{2}} > 1$ .

The results follow.

What the proposition 4 means that  $t_e = \frac{2}{\delta|\gamma|} e^{-\frac{\gamma\mu_0}{2}} (1 - \sqrt{v_0})$  is the critical time for some industry in the economic recession with DP accumulating. the economy is at the strong state when  $\sqrt{v_0} e^{-\frac{\gamma\mu_0}{2}} > 1$  and  $t_e \leq t \leq \frac{2}{\delta|\gamma|} (e^{-\frac{\gamma\mu_0}{2}} - 1)$ , at this time, we could think that the industry can not wither away, and further more, we could think that the developing energy of other related industries has swarmed into this industry, and this industry will have a larger increasing space for growth.  $t = \frac{2}{\delta|\gamma|} (e^{-\frac{\gamma\mu_0}{2}} - 1)$  is the limit time for the economic recession with DP accumulating. if the economy of industry has not grown yet when the time reaches the limit time  $t = \frac{2}{\delta|\gamma|} (e^{-\frac{\gamma\mu_0}{2}} - 1)$  in the economic recession with DP accumulating, the industry might wither away gradually.

### 4.3 Analysis for the mass of productivity and economic energy

In a certain period of time, some economic assets may influence substantially the developing course of economic growth or recession, these assets play a more important and magistral role to increase the production value of economy on this period of time, so we have

**Definition 2** The all the assets which play a magistral and a key role to increase the production value of economy (i.e., productivity) are called the high-quality assets of economy.

For example, in the past of several decades, the computer industry plays a magistral and a key role in economic process, the computer and the technique and product of computer have the direct and indirect key function to globe economic growth. So all of the computer assets are the high-quality assets which play a magistral and a key role to increase the production value of computer industry and other industries; and for example again, traffic plays the direct and key role to develop many industries, So all of the traffic assets are the high-quality assets which play a magistral and a key role to increase the production value of traffic industry and other industries. In fact, the high-quality assets are the component parts of productivity, and the most important and key parts of productivity. Although these high-quality assets may be different at different period of economic development, it is not doubtable that they are existent. The productivity would lose their hypostatic

contents if there are not these high-quality assets. In another hand, there are a lot of high-quality assets in economic society means and using of them means that economy is growing or will grow. in general, the larger the quantity of high-quality assets is, the more the energy of economic growth is, and the higher the rate of economic growth is, and vice versa.

Based on the above discussions, we give the definitions about the mass of productivity and the growing speed of the mass of productivity as follow

**Definition 3** The mass of economic productivity is the quantity of high-quality assets in economy. The mass of industrial productivity is the quantity of high-quality assets in the industry.

**Definition 4** The speed of productivity growth means the increasing speed of high-quality assets.

Therefore, if we regard  $m = v_0 e^{-\gamma \mu_0}$  in expression (6) as the mass of productivity, and regard

$v = \left( e^{\frac{\gamma \mu_0}{2}} - \frac{\delta \gamma}{2} t \right)^{-1}$  ( $0 \leq t < \frac{2}{\gamma \delta} e^{\frac{\gamma \mu_0}{2}}$ ) as the variety of speed in productivity, the expression (7) is just the basic

model of relation between mass and energy in economy

$$v(t) = m v^2 \quad (14)$$

where,  $m = v_0 e^{-\gamma \mu_0}$ ,  $v = \left( e^{\frac{\gamma \mu_0}{2}} - \frac{\delta \gamma}{2} t \right)^{-1}$  ( $0 \leq t < \frac{2}{\gamma \delta} e^{\frac{\gamma \mu_0}{2}}$ ) for economic growth, and  $m = v_0 e^{-\gamma \mu_0}$ ,

$v = \left( e^{\frac{\gamma \mu_0}{2}} + \frac{\delta \gamma}{2} t \right)^{-1}$  ( $t > 0$ ) for economic recession.

The equation (14) is similar to the model of conversion between mass and energy in physics. based on expression (14), if speed  $v$  is invariant, thus, the larger the mass  $m$  is, the larger the DP(economic energy)  $v(t)$  is; in another hand, if mass  $m$  is invariant, thus, the speed  $v$  is larger the, the larger the DP(economic energy)  $v(t)$  is; and vice versa. So we can say that the model (14) is able to describe the energy movement in economic process.

#### 4.4 The analysis for conversion and conservation of economic energy

As we know, a cycle of DP movement includes two basic stages, the first stage is of DP accumulating, and second stage is of DP releasing. Suppose that economy is growing under the normal state in economy. According to expression (2), we have the model of relation between DP (economic energy) and productivity at first stage as following

$$v_1(t) = m_1 e^{\gamma_1 \mu_1(t)}$$

where,  $m_1 = v_{10} e^{-\gamma_1 \mu_{10}}$  is the mass of productivity at the start of first stage,  $\gamma_1 > 0$ ,  $\mu_{10}$  is the original value of productivity at first stage,  $v_{10}$  is the original value of DP at first stage.

After the DP is sufficiently accumulated, DP movement comes into the second stage, i.e. DP starts releasing. Also according to expression (2), we have the model of relation between DP (economic energy) and productivity at second stage as following

$$v_2(t) = m_2 e^{\gamma_2 \mu_2(t)}$$

where,  $m_2 = v_{20} e^{-\gamma_2 \mu_{20}}$  is the mass of productivity at the start of second stage,  $\gamma_2 < 0$ ,  $\mu_{20}$  is the original value of productivity at second stage,  $v_{20}$  is the original value of DP at second stage.

If DP ends accumulating and starts releasing at time  $t$ , we should have  $\mu_1(t) = \mu_2(t) = \mu(t)$ , and the equation of energy conservation as:  $v_1(t) = v_2(t)$ , namely  $m_1 e^{\gamma_1 \mu(t)} = m_2 e^{\gamma_2 \mu(t)}$ . So we obtain the formula of energy conversion as

$$m_2 = m_1 e^{(\gamma_1 - \gamma_2) \mu(t)} \quad (15)$$

The expression (15) has described the conversion and the conservation between mass and energy in economy. Based on expression (15), we see  $\gamma_1 > 0$ ,  $\gamma_2 < 0$ , and  $\mu(t) > 0$ , thus  $m_2 > m_1$ . So we could obtain the following conclusions:

- 1)  $m_2 > m_1$  indicates that  $m_1$  (the productivity mass) at the beginning of DP accumulating or the end of DP releasing is smaller than  $m_2$  at the end of DP accumulating or the beginning of DP releasing. This means that accumulating DP is substantively to accumulate energy in the way to increase the mass of productivity, and releasing DP is substantively to release energy in the way to decrease the mass of productivity. So the result of DP accumulating is that the productivity mass (high-quality of assets in economy) increases substantially, and the result of DP releasing is that the productivity mass decreases substantially.
- 2) At the beginning of DP accumulating, the productivity mass is smaller. Just the productivity mass is smaller, then DP needs to accumulate and has the space to accumulate; and at the beginning of DP releasing, the productivity mass is larger. Just the productivity mass is larger, then DP is able to be converted and released in energy.
- 3) If let  $m_2$ ,  $\gamma_2$  and  $\mu(t)$  be invariant in expression (15), the smaller the mass  $m_1$  is, the larger  $\gamma_1$  is, namely the quicker the speed of DP accumulating is. Similarly, If let  $m_1$ ,  $\gamma_1$  and  $\mu(t)$  be invariant in expression (15), the larger the mass  $m_2$  is, the smaller  $\gamma_1$  is, namely the quicker the speed of DP releasing is. These means that the mass and speed can be converted one to another.
- 4) The amount of high-quality assets, i.e. the mass of productivity, is large means that the energies in all of parts and departments in economy or industrial economy are in order, and the total economic energy is higher at this time. The amount of high-quality assets is small means that the energies in all of parts and departments in economy or industrial economy are in chaos, and the total economic energy is lower at this time.
- 5) Based on the discussion above, we also know that the foundation of national economy development is DP. Even though a nation gets a large amount of advanced equipments, the economic energy contained in these equipments will release completely if this nation can not accumulate continuously DP as needed. So we see that the real and basic reason of the differences existed in economic growth for different country or region is the differences in their ability of DP accumulating and releasing. Only the DP is the durative motivity for economic growth. In general, the backward nations or regions have a weak ability to accumulate their DP for economic growth, then they can not enable DP, which has been released, to be accumulated sufficiently again, so that their economy can not grow steadily. Therefore, how DP is accumulated and released efficiently is a true difficulty.
- 6) DP will support an economic growth. If DP is rapidly accumulated and exhausted in a short time, the

economic growth loses its DP (i.e. energy), the decreases of economic production or market sale may occur. In another hand, if economy is in a short recession and DP is rapidly accumulated due to some reasons, the economic growth may occur. So we could say that the basic reason of fluctuation in economy is there are many violent bifurcations between the productivity and DP is the reason why the fluctuation of economic production and market occur.

The conclusion 5) and 6) are especially important.

## 5 The Analysis for Fluctuations of Economic Energy

The accumulation of DP means the economic energy is increasing, and the release of DP means the economic energy is decreasing. Here, we call that economic energy increase and decrease in a fixed range the fluctuations of DP (i.e. economic energy).

Suppose that the developing energy of economy or an industrial economy is fluctuated around a fixed non-zero value. If the fixed value is equal to zero, we think that economy or the industrial economy is closed to death. If the value is smaller, the fluctuations are called at a lower level; if the value is larger, the fluctuations are called at a higher level. A new accumulation for DP may start when the economic energy is at lower level, and a new release for DP may start when the economic energy is at higher level.

Thus, at lower level, what time is the accumulation for DP starts? Or the process of DP accumulating is most easy to start? And at higher level, what time is the release for DP starts? Or the process of DP releasing is most easy to start? We shall solve these questions as follow.

Let  $Y(t)$ , DP of an industrial economy, is fluctuated around  $v$  (a benchmark energy). If the assumptions in [18]-[20] come into existence,  $Y(t)$  follows the  $DF$  process<sup>[26]</sup>, i.e.  $X(t) \in P(v, (kv)^2t)$ ,  $0 < k \leq 1$ ,  $k$  is called a scale coefficient of energy fluctuating. If  $Y(t)$  is fluctuated in the field of  $[v-x, v+x]$  ( $0 < x < v$ ), then denote:

$$u(t) = \int_{v-x}^{v+x} y^2 f_t(y) dy, \text{ where } f_t(y) = \begin{cases} e^{-\frac{(y-v)^2}{2(kv)^2t}} / \int_0^{\infty} e^{-\frac{(y-v)^2}{2(kv)^2t}} dy & y \geq 0 \\ 0 & y < 0 \end{cases}$$

$u(t)$  is the average value function of the square of distance for energy fluctuations, the energy fluctuations function for short in the following discussion.

From theorem 1, we obtain

$$u(t) = \int_{v-x}^{v+x} y^2 e^{-\frac{(y-v)^2}{2(kv)^2t}} dy / \int_0^{\infty} e^{-\frac{(y-v)^2}{2(kv)^2t}} dy$$

$$= 2v \left( \frac{v(1+k^2t) \sqrt{1 - e^{-\frac{2x^2}{\pi(kv)^2t}}} - kx \sqrt{\frac{2t}{\pi}} e^{-\frac{x^2}{2(kv)^2t}}}{1 + \sqrt{1 - e^{-\frac{2x^2}{\pi(kv)^2t}}}} \right) \quad (16)$$

We can see, from the expression (16), energy fluctuations function  $u(t)$  will decline toward zero gradually if the time  $t$  becomes larger and larger, i.e.  $t \rightarrow \infty$ . This make clear that if a industry can not start accumulating DP (i.e. developing energy) at a necessary time in order to make economy growing, it will be closed to death and disappear finally. therefore, the economy or the industrial economy must be accumulated again its energy in some way and come into a new developing cycle if its economic energy is smaller to smaller and always not equal to zero.

Because  $u'_t = \frac{1}{2(kvt)^2} \left( \left( \int_{v-x}^{v+x} y^2 (y-v)^2 f(y) dy - u(x) \int_0^\infty (y-v)^2 f(y) dy \right) \right)$ , let  $u'_t = 0$ , we obtain

$$u(t) \Big|_{u'(t)=0} = \frac{\int_{v-x}^{v+x} y^2 (y-v)^2 e^{-\frac{(y-v)^2}{2(kv)^2 t}} dy}{\int_0^\infty (y-v)^2 e^{-\frac{(y-v)^2}{2(kv)^2 t}} dy}$$

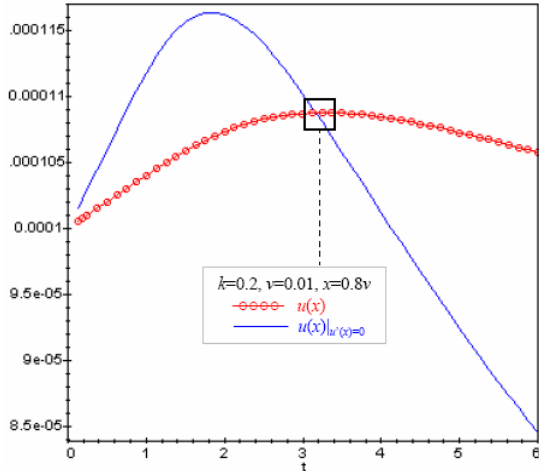
$$= \frac{kv^3 (1 + 3k^2 t) \sqrt{2\pi} (1 - e^{-\frac{2x^2}{\pi(kv\sqrt{t})^2}}) - 2x(x^2 + v^2 + 3(kv\sqrt{t})^2) e^{-\frac{x^2}{2(kv\sqrt{t})^2}}}{\sqrt{\frac{\pi}{2}} kv (1 + \sqrt{1 - e^{-\frac{2}{\pi k^2 t}}}) - v e^{-\frac{1}{2k^2 t}}}$$
(17)

Let (16) equal to (17), we have

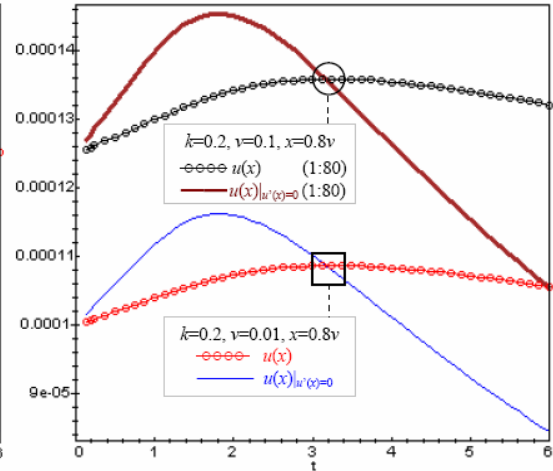
$$u(t) = \frac{\int_{v-x}^{v+x} y^2 e^{-\frac{(y-v)^2}{2(kv)^2 t}} dy}{\int_0^\infty e^{-\frac{(y-v)^2}{2(kv)^2 t}} dy} = \frac{\int_{v-x}^{v+x} y^2 (y-v)^2 e^{-\frac{(y-v)^2}{2(kv)^2 t}} dy}{\int_0^\infty (y-v)^2 e^{-\frac{(y-v)^2}{2(kv)^2 t}} dy} = u(t) \Big|_{u'(t)=0}$$
(18)

**Proposition 5** The energy fluctuations function  $u(t)$  will reaches its maximum value at time  $t=\tau$  determined by equation (18).

Proposition 5 will be approved in figure 4 to figure 7.



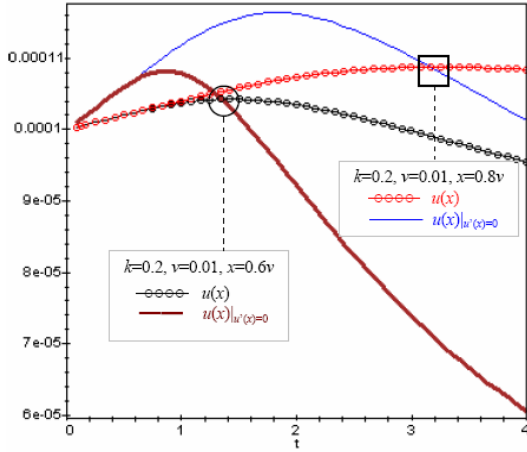
**Figure 4** The maximum value of the energy fluctuation function  $u(t)$ . If the benchmark energy  $v=0.01$ , the scale coefficient of energy fluctuating  $k=0.2$ , scale of energy fluctuating  $x=0.8v$ . thus,  $u(t)$  reaches its maximum value at  $t=\tau$ ,  $t=\tau$ , the intersection of  $u(t)$  and  $u(x)|_{u'(x)=0}$ , is determined by the equation (18).



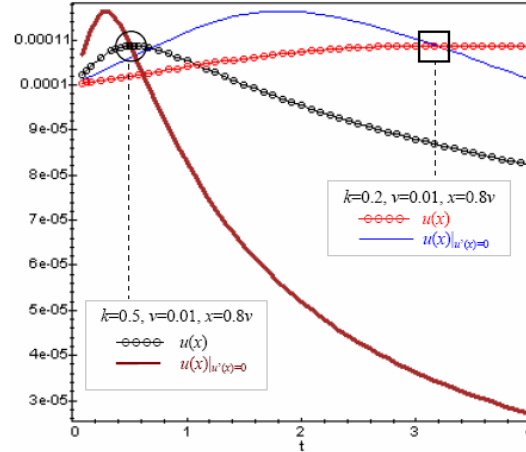
**Figure 5** Comparing the energy fluctuation function  $u(t)$  on different values of benchmark energy. If the scale coefficient of energy fluctuating  $k$  are fixed, the time  $t=\tau$  at which  $u(t)$  reaches its maximum value is not related to the benchmark energy  $v$ .

**Proposition 6** When the economic energy is at the lower level, i.e. the  $v$  is smaller, then  $t=\tau$ , determined by expression (18), is the most possible time at which DP is accumulated again, and the optimal time for DP to accumulate.

**Proposition 7** When the economic energy is at the higher level, i.e. the  $v$  is larger, then  $t=\tau$ , determined by expression (18), is the most possible time at which DP is released, and the optimal time for DP to release.



**Figure 6** Comparing the energy fluctuations function  $u(t)$  on different values of scale of energy fluctuating. If the scale coefficient of energy fluctuating  $k$  is fixed and the scale of energy fluctuating  $x$  changes, both the maximum value of  $u(t)$  and the time  $t=\tau$  at which  $u(t)$  reaches its maximum are all different.



**Figure 7** Comparing the energy fluctuations function  $u(t)$  on different scale coefficients of energy fluctuating. If the scale of energy fluctuating  $x$  is fixed and the scale coefficient of energy fluctuating  $k$  changes, the time  $t=\tau$  at which  $u(t)$  reaches its maximum is different, but the maximum value of  $u(t)$  does not change.

What the quite interest are:

- 1) From the figure 5, we see that If the scale coefficient of energy fluctuating  $k$  are fixed, the time  $t=\tau$  at which  $u(t)$  reaches its maximum value is not related to the benchmark energy  $v$ .
- 2) From the figure 7, we see that If the scale of energy fluctuating  $x$  is fixed and the scale coefficient of energy fluctuating  $k$  changes, the maximum value of  $u(t)$  does not change, but the time  $t=\tau$  at which  $u(t)$  reaches its maximum is different.
- 3) From figure 4 to figure 7, we see that while the economic energy is at the lower level, the stochastic fluctuations becomes stronger, and then becomes weaker after the energy fluctuations function  $u(t)$  reaches its maximum. If the DP does not start accumulating at the maximum of  $u(t)$ , the economic energy of industry will become less and less, and exhaust finally.
- 4) The fluctuations of economic energy will cause that the departments and components of economic society are in order at higher level or in chaos at the lower level, and result in the fluctuation of economic market.
- 5) If the fluctuations of DP maintains at a lower level and DP does not start accumulating all the time, the energy fluctuation function  $u(t)$  will eat and flow around zero, and this fluctuations will get more and more violent along with the process is continued, see also in figure 8 and figure 9.

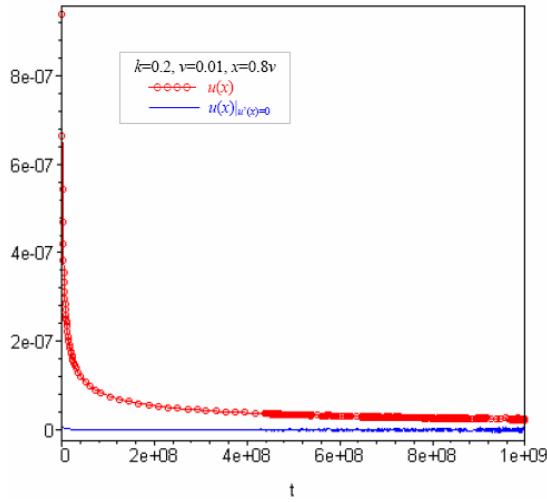
## 6 The Empirical Researches for DP

We could take the national economy as a synthesis industry. Though US economy, productivity  $\mu(t)$ , has been growing approximately from World War II to 2003, DP fluctuates always. There are many different characteristics of DP in this period. In the following empirical researches, we take US GDP (chained) price index (the GDP index for short) in the period of 1940-2003 as the level of productivity, Fiscal Year 2000 = 1.000. Data resource is reference [29]. We have the following notation and expressions:

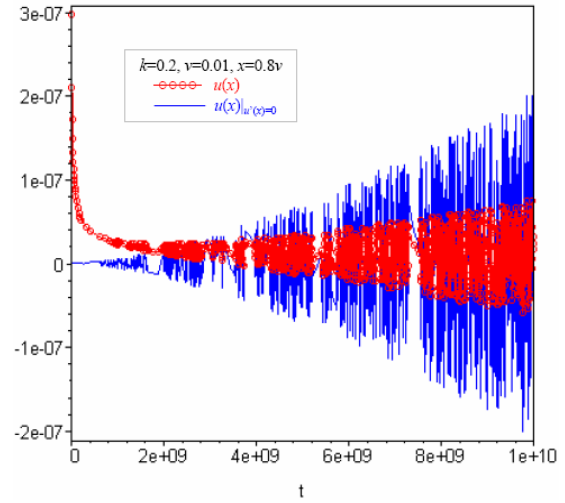
$\mu(t)$ —The basic level of economic productivity at the year  $t$ ,  $t=1940, 1941, \dots, 2003$ , measured by the GDP index.

$v(t)$ —The fluctuating ratio of the productivity level, could measure the development power (DP) for economy.  $v(t)=|\mu(t)-\mu(t-1)|/\mu(t)$ ,  $t=1941, 1942, \dots, 2003$ .

$X(t)$ —The actual level of productivity, a non- negative random variable.  $X(t) \in P(\mu(t), [v(t)]^2)$ .



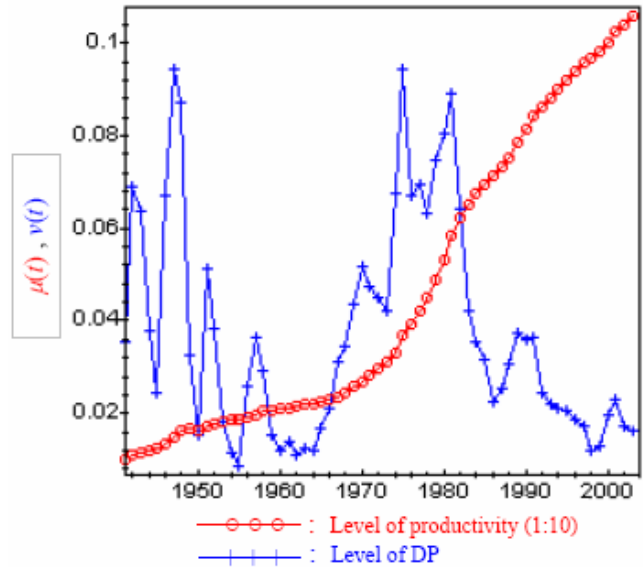
**Figure 8** The varying characteristic the energy fluctuation function  $u(t)$  under the large scale of time(1). If the economic energy is fluctuated at the lower level, and DP does not start accumulating in the field of time  $[0, 1 \times 10^9]$ , thus  $u(t)$  will get smaller and smaller, and close to zero. Here, the benchmark energy  $v=0.01$ , the scale of energy fluctuating  $x=0.8 \times v$ , the scale coefficient of energy fluctuating  $k=0.2$ .



**Figure 9** The varying characteristic the energy fluctuation function  $u(t)$  under the large scale of time(2). If the economic energy is fluctuated at the lower level, and DP does not start accumulating in the field of time  $[0, 1 \times 10^{10}]$ , thus  $u(t)$  will get smaller and smaller, fluctuate round zero, and this fluctuation will get more and more violent. Here, the  $v=0.01$ ,  $x=0.8 \times v$ ,  $k=0.2$ .

The values of  $\mu(t)$  ( $t=1940, 1941, \dots, 2003$ ) and  $v(t)$  ( $t=1941, 1942, \dots, 2003$ ) can be seen in appendix.

The interval of time unit for sampling data GDP is a year, the stability of data is higher, and the difference between GDP price index of one year and that of last year can nicely describe the economic fluctuation, so we adopt the formulas of  $v(t)$  mentioned above. The curves of DP and productivity in US economy are shown in figure 10. In figure 8, the proportion of the real indexes of  $\mu(t)$  to indexes drawn is 1:10. We could see, from figure 10, that the local higher points between 1941 and 2003 go in ascending, descending, ascending again and descending again; and that the local lower points between 1941 and 2003 go in descending, ascending, and descending again. So the DP will release (i.e. the local higher points or the local lower points go in descending) if economic energy is larger to a certain degree, and the DP will accumulate (i.e. the local higher points or the local lower points go in ascending)



**Figure 10** The curves of US economic productivity  $\mu(t)$  (GDP (chained) price index; Fiscal Year 2000 = 1,000.) and DP(Development Power) in the time field from 1941 to 2003. The proportion of the real indexes of  $\mu(t)$  to drawn indexes is 1:10. Though productivity  $\mu(t)$  has been growing approximately, DP fluctuates always.

if economic energy is smaller to a certain degree. Starting from 1980, both the local higher points and the local lower points of DP in US economy are descending.

### 6.1 The empirical research for estimating the DP model

We shall give the estimating analysis for the macrocosmic and microcosmic movement of energy in order to see the characteristics of DP.

**6.1.1 Estimating analysis for the macrocosmic movement of DP.** From figure 10, we know that 1947 and 1975 are the two top values of DP in US economic development. Here, we select the local top values of  $v(t)$  on the years ( $t=1947, 1951, 1957, 1961, 1970, 1975$ ) between 1947 and 1975 as our analytic samples, and give the estimating results by model (1).

1) Between 1947 and 1961, DP  $v_1(t)$  ( $t=1947, 1951, 1957, 1961$ ) gets smaller and smaller, namely DP in economy is released. By use of least square method, we have the following estimated results:

$$\bar{v}_1(t) = 4.460849381e^{-25.87956898\mu(t)} \quad (19)$$

The estimated error is

$$s_1 = \frac{1}{4} \sqrt{\sum_t (v(t) - \bar{v}_1(t))^2} = 0.003105121390, t=1947, 1951, 1957, 1961.$$

Because  $\mu_0 = \mu(1947) = 0.1466, \gamma_1 = -25.87956898$  and  $v_0 = v(1947) = 0.09413369714$ , then the average value of productivity mass in this period of time is  $m_1 = v_0 e^{-\gamma_1 \mu_0} = 4.182485198$ .

2) Between 1961 and 1975, DP  $v_2(t)$  ( $t=1961, 1970, 1975$ ) gets larger and larger, namely DP in economy is accumulated. Also by use of least square method, we have the estimated results as following:

$$\bar{v}_2(t) = 0.001498639279 e^{11.6679130\mu(t)} \quad (20)$$

The estimated error is

$$s_1 = \frac{1}{3} \sqrt{\sum_t (v(t) - \bar{v}_2(t))^2} = 0.007515596277, t=1961, 1970, 1975.$$

Because  $\mu_0 = \mu(1961) = 0.2130, \gamma_2 = 11.6679130$  and  $v_0 = v(1961) = 0.01408450704$ , then the average value of productivity mass in this period of time is  $m_2 = v_0 e^{-\gamma_2 \mu_0} = 0.002546360683$ .

**6.1.2 Estimating analysis for the microcosmic movement of DP.** Here, we take the years ( $t=1986\sim 1998$ ) as our analytic samples for DP  $v(t)$ , and give the estimating results by model (1).

1) From 1986 to 1989, DP  $v_1(t)$  gets larger and larger, namely DP in economy is being accumulated. We have the following estimated results:

$$\bar{v}_1(t) = 0.0001416620460 e^{7.117178613\mu(t)} \quad (21)$$

The estimated error is

$$s_1 = \frac{1}{4} \sqrt{\sum_t (v(t) - \bar{v}_1(t))^2} = 0.00009877721708, t=1986\sim 1989.$$

Because  $\mu_0=\mu(1986)=0.7125$ ,  $\gamma_1=7.117178613$  and  $v_0=v(1986)=0.02273684211$ , then the average value of productivity mass in this period of time is  $m_1=v_0e^{-\gamma_1\mu_0}=0.0001427010848$ .

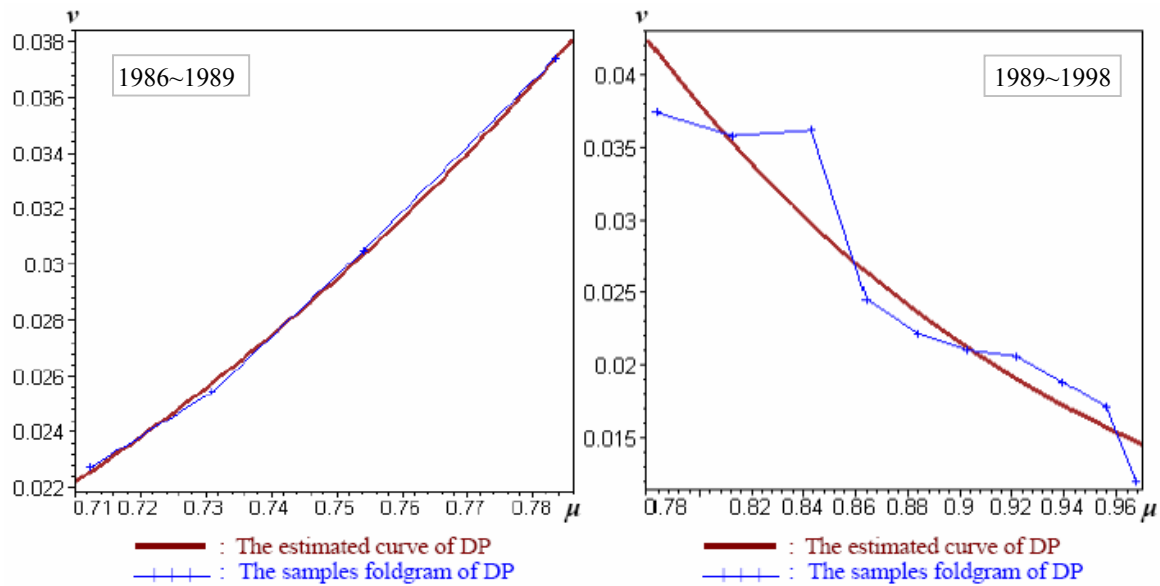
2) From 1989 to 1998, DP  $v(t)$  gets smaller and smaller, namely DP in economy is being released. We have the following estimated results:

$$\bar{v}_2(t)=3.469717623e^{-5.644970610\mu(t)} \quad (22)$$

The estimated error is

$$s_2 = \frac{1}{10} \sqrt{\sum_t (v(t) - \bar{v}_2(t))^2} = 0.0008927384778, t=1989\sim 1998.$$

Because  $\mu_0=\mu(1989)=0.7834$ ,  $\gamma_2=-5.644970610$  and  $v_0=v(1989)=0.03740107225$ , then the average value of productivity mass in this period of time is  $m_2=v_0e^{-\gamma_2\mu_0}=3.114953022$ . 1989 to 1998



**Figure 11** The estimation and fitness for DP of US economy between 1986 and 1989. In this period of time, DP  $v(t)$  of US economy is accumulated continuously. At the beginning of DP accumulating, the mass of productivity is smaller, i.e.  $m_1=0.0001427010848$ , this indicates that there is a larger space for DP accumulating; and the same time, the quantity of high-quality assets in economy is less means that the various components of economic production are in chaos.

**Figure 12** The estimation and fitness for DP of US economy between 1989 and 1998. In this period of time, DP  $v(t)$  of US economy is released continuously. At the beginning of DP releasing, the mass of productivity is larger, i.e.  $m_2=3.114953022$ , this indicates that there is a larger space for DP releasing; and the same time, the quantity of high-quality assets in economy is larger means that the various components of economic production are in order.

The economic growth with DP accumulating from 1986 to 1989 and the economic growth with DP releasing from 1989 to 1998 are separately shown in figure 11 and figure 12. In these two figures, we have given the estimated curve and samples foldgram of the relation between DP and productivity in order to make a

comparison.

## 6.2 The empirical analysis for the conversion and conservation between mass and energy in economy

**6.2.1 The macrocosmic analysis for mass and energy in economy.** According to section 6.1.1, we know that US economy presents mainly a trend of economic growth with DP releasing at four years 1947,1951,1957, 1961; the mass of economic productivity,  $m_1=4.182485198$ , is larger comparatively. and US economy presents mainly a trend of economic growth with DP accumulating at three years 1961,1970 and 1975; the mass of economic productivity in this period,  $m_2=0.002546360683$ , is less obviously. That means the average mass of economic productivity in period of 1947~1961,  $m_1=4.182485198$ , is sufficiently released in the way of energy, so that the mass of economic productivity at the beginning of period of 1961~1975, a process of DP accumulating, is only  $m_2=0.002546360683$ , quite less.

US economy presents mainly a trend of DP releasing in the period of 1947~1961, from the expression (19), DP (the economic energy) of US is  $v_1=\bar{v}_1(1961)=4.460849381e^{-25.87956898\mu(1961)}=0.01800674262$  at the end of this period; and US economy presents mainly a trend of DP accumulating in the period of 1961~1975, from the expression (20), DP of US is  $v_2=\bar{v}_2(1961)=0.001498639279e^{11.6679130\mu(1961)}=0.01798736013$  at the beginning of this period.

Because the margin of  $v_1$  and  $v_2$ ,  $v_1-v_2=0.00001938249$ , is quite small, we think that the economic energy is conservative. It is a result of estimated error, if say there is a difference between  $v_1$  and  $v_2$ , not the error of energy conservation.

**6.2.2 The microcosmic analysis for mass and energy in economy.** According to section 6.1.2, we could see that US economy experienced a whole process of DP movement from 1986 to 1998, i.e. a stage of DP accumulating ( $t=1986\sim 1989$ ) and a stage of DP releasing ( $t=1989\sim 1998$ ). At the stage of DP accumulating, the average value of productivity mass in the period of 1986~1989,  $m_1=0.0001427010848$ , is less; and at the stage of DP releasing, the average value of productivity mass,  $m_2=3.114953022$ , is larger. This explains that because the average mass of productivity is less in the stage of DP accumulating from 1986 to 1989, there is a larger space for DP (the economic energy) to be accumulated; and because the average mass of productivity is larger in the stage of DP releasing from 1989 to 1998, there is a larger space for DP to be released.

US economy presents a trend of DP accumulating in the period of 1986~1989, from the expression (21), DP (the economic energy) of US is  $v_1=\bar{v}_1(1989)=m_1e^{7.11717861\mu(t)}=0.03765985122$  at the end of this period; and US economy presents mainly a trend of DP accumulating in the period of 1989~1998, from the expression (22), DP of US is  $v_2=\bar{v}_2(1989)=m_2e^{-5.644970610\mu(1989)}=0.03740107226$  at the beginning of this period.

Because the margin of  $v_1$  and  $v_2$ ,  $v_1-v_2=0.00025877896$ , is smaller comparatively, we could think that DP (the energy in economy) is conservative.

**6.2.3 A brief summary.** To sum up, We see the mass of economic productivity is released in the way of energy during the period of DP releasing, and to push the economy forward; and in the process of DP is accumulated, the mass of economic productivity, translated from the energy accumulated continuously, becomes larger and larger. This is just a process of conversion between mass and energy in economy. In another hand, the economic

energy is of conservation in the conversion between mass and energy. If say there is a little difference in conversion of energy, it should be s a result of estimated error, not the error of energy conservation itself.

### 6.3 The empirical analysis for economic formation and its evolution

**6.3.1 Analysis for strong state and super state in US economy between 1986 and 1989.** According to expression (19) and the appendix at end of this paper, we have

$$\bar{v}(t)=0.0001416620460e^{7.117178613\mu(t)}$$

Where,  $\gamma=7.117178613$ ,  $\mu_0=\mu(1986)=0.7125$ ,  $\mu_1=\mu(1987)=0.7311$  and  $v_0=v(1986)=0.02273684211$ .

As we know, US economy is an economic growth with DP accumulating in the period of 1986~1989. By use of formula (10), we have calculated  $\delta=0.001425830365$ . And by formula (12) and (13), the critical times of strong state and super state are calculated separately as following

The critical time of strong state for US economy is  $t_e=13.25925922$ , and the critical time of super state for US economy is  $t_r=15.61359232$ .

Based on the calculating results above, if going on developing for 14 years in the way of 1986~1989, US economy could come into the strong state of economy, and if go on developing for 17 years in the way of 1986~1989, US economy could come into the super state of economy. But, only US economy kept on developing for 4 years in the way of 1986~1989, namely DP of US economy starts releasing after 1989, the difference is very larger.

The curves of the productivity  $\mu(t)$  and DP  $v(t)$  determined separately by expression (6) and (7) are shown in figure 13.

**6.3.2 Analysis for strong state and super state in US economy between 1998 and 2001.** According to expression (19) and the appendix at end of this paper, we have

$$\bar{v}(t)=0.000001878275837e^{9.092183212\mu(t)}$$

Where,  $\gamma=9.092183212$ ,  $\mu_0=\mu(1998)=0.9675$ ,  $\mu_1=\mu(1999)=0.9802$  and  $v_0=v(1998)=0.01198966408$ .

As we know, US economy is an economic growth with DP accumulating in the period of 1998~2001. By use of formula (10), we have calculated  $\delta=0.0001517576503$ . And by formula (12) and (13), the critical times of strong state and super state are calculated separately as following

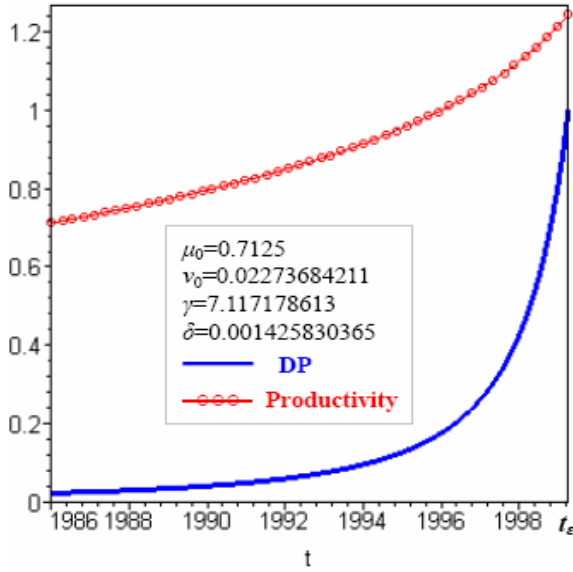
The critical time of strong state for US economy is  $t_e=15.87340453$ , and the critical time of super state for US economy is  $t_r=17.82521825$ .

Based on the calculating results above, if going on developing for 16 years in the way of 1998~2001, US economy could come into the strong state of economy, and if go on developing for 18 years in the way of 1998~2001, US economy could come into the super state of economy. But, only US economy kept on developing for 4 years in the way of 1998~2001, namely DP of US economy starts releasing after 2001, the difference is very larger.

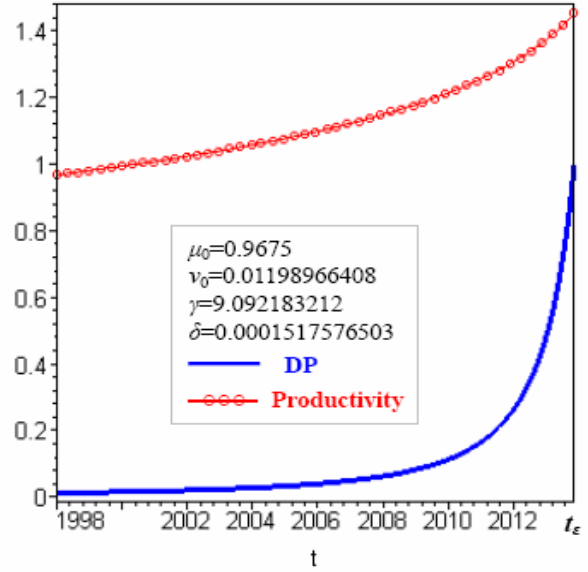
The curves of the productivity  $\mu(t)$  and DP  $v(t)$  determined separately by expression (6) and (7) are shown in figure 14.

### 6.4 The empirical analysis for fluctuations of economic energy

**6.4.1 The fluctuations of economic energy and the optimal time for DP starting accumulating.** From 1960 to 1964, DP of US economy is fluctuated at a lower level. If we let  $v=(v_{1959}+v_{1960})/2=0.01366322432$  and



**Figure 13** Analysis for US economic energy from 1986 to 1989. US economy was growing with DP accumulating during 1986~1989. This process kept on four years only. If this process could keep on fourteen years or more, US economy might come into the strong state in economic energy. In this chart,  $t_\epsilon$  is the critical time of strong state. Again if this process could keep on sixteen years or more, US economy might come into the super state in economic energy.



**Figure 14** Analysis for US economic energy from 1998 to 2001. US economy was growing with DP accumulating during 1998~2001. This process kept on four years only. If this process could keep on sixteen years or more, US economy might come into the strong state in economic energy. In this chart,  $t_\epsilon$  is the critical time of strong state. Again if this process could keep on eighteen years, US economy might come into the super state in economic energy.

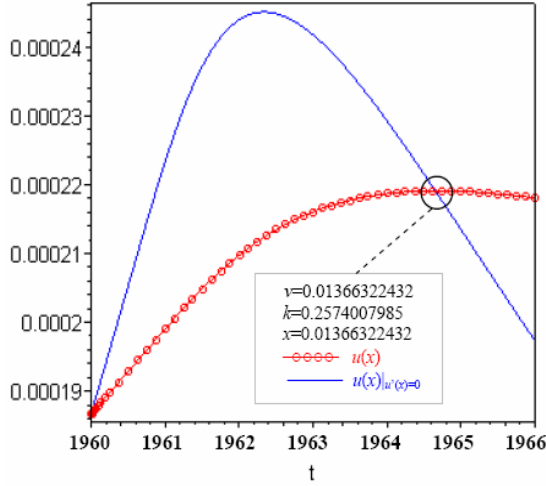
$k=|v_{1959}-v_{1960}|/v=0.257400799$  in the energy fluctuations function (16), and let  $x=v=0.01366322432$  (because the time of which energy fluctuations is at a lower level is generally longer, the fluctuations scale  $x$  should be larger, so we let it be  $x=v \times 100\%$  generally). The evolution process of energy fluctuations function  $u(t)$  is shown in figure 15. According to the formula (18), we find the maximum value of  $u(t)$  which should occur between 1964 and 1965, namely DP of US economy is most possible to start accumulating between 1964 and 1965. Actually, US economy starts a process of growing with DP accumulating from 1965 for six years (1965~1970).

**6.4.2 The fluctuations of economic energy and the optimal time for DP starting releasing.** From 1989 to 1991, DP of US economy is fluctuated at a higher level. If we let  $v=(v_{1988}+v_{1989})/2=0.03395050297$  and  $k=|v_{1988}-v_{1989}|/v=0.2032705835$  in the energy fluctuations function (16), and let  $x=0.75 \times v=0.02546287723$  (because the time of which energy fluctuations is at a higher level is generally shorter, then the fluctuations scale  $x$  should be smaller, so we let it be  $x=v \times 75\%$  generally). The evolution process of energy fluctuations function  $u(t)$  is shown in figure 16. According to the formula (18), we find the maximum value of  $u(t)$  which should occur between 1991 and 1992, namely DP of US economy is most possible to start releasing between 1991 and 1992. Actually, US economy starts a process of growing with DP releasing from 1992 for nine years (1992~2000).

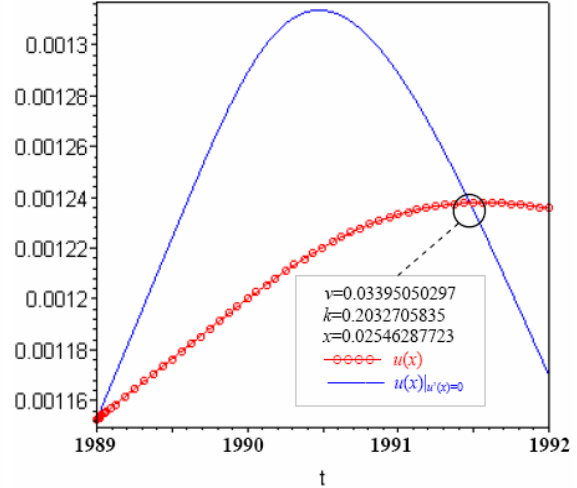
## 7 Conclusions

Based on the theory of Development Power and from the latent characteristics of real economic process,

the works have been done as follow in this paper.



**Figure 15** The fluctuations analysis of US economic energy and the optimal time for DP starting accumulating. DP of US economy is fluctuated at a lower level from 1960 to 1964. Let  $v=0.01366322432$ ;  $k=0.2574007985$ ,  $x=0.01366322432$ . According to the formula (18), we find the maximum value of  $u(t)$  which occurs between 1964 and 1965. Actually, US economy starts a process of growing with DP accumulating from 1965 for six years.



**Figure 16** The fluctuations analysis of US economic energy and the optimal time for DP starting releasing. DP of US economy is fluctuated at a higher level from 1989 to 1992. Let  $v=0.03395050297$ ;  $k=0.2032705835$ ,  $x=0.02546287723$ . According to the formula (18), we find the maximum value of  $u(t)$  which occurs between 1991 and 1992. Actually, US economy starts a process of growing with DP releasing from 1992 for nine years.

First, we have discussed the main ways to accumulate and release the Development Power (DP), established the basic model of relation between productivity (or say the level of economic development) and DP on the Partial Distribution<sup>[21]-[23]</sup>. Thus, we know that there is a symmetrical relation between the economic productivity and the economic DP; and we have illuminated that both of them are not discernible one from another, DP is the latent impetus for productivity growing and the engine for economic growth.

Second, this paper advances three kind of energy state of economic development, i.e., the state of normal energy, strong energy and super energy in economic form, and gives the approaches to calculate the critical times of strong energy and super energy. The DP of economic state of super energy is infinite, means the variance of productivity level is infinite, and in accordance with the basic request in fractal economy. We have indicated that the economic state of super energy will not occur until the DP of an industry (e.g. computer industry) is diffused to other industries, so that DP of those industries has been stirred up; and that the strong state or super state in economic energy will not occur if we take the economy or industry as a whole. The strong state or super state in economic energy may occur only when two or more industries are discussed.

Third, based on the model of relation between DP and productivity, this paper has indicated that there are the laws, conversion and conservation between productivity and DP, in economic field. accumulating DP means deposit the economic energy and convert it into mass of productivity (quantity of high-quality economic assets), and mass of productivity can be reconverted to economic energy in the way of DP releasing to accelerate economic development. This kind of conversion between productivity and DP follows the law of conservation.

Fourth, Based on the laws of conversion and conservation between productivity and DP, this paper has explained that the real reason of the economic differences is the differences in their ability of DP accumulating and releasing between developed countries (or regions) and undeveloped countries (or regions). Only the DP is the durative motivity for economic growth. The differences on the mechanism and environment for DP accumulating and releasing are just the reasons that economic development is unbalance for different countries or regions

Fifth, DP accumulating and DP releasing occur alternately. this paper has pointed out the sign that accumulation of DP is completed is the most of economic factors are in order, and the sign that release of DP is completed is the most of economic factors are in chaos, and gives the ways to calculate the most possible time that the DP starts accumulating or releasing; and has indicated the reason that the fluctuation of economic production and market occurs is there are many violent bifurcations between the productivity and DP, such as DP is too small to support the economic growth, or DP is so larger to reverse the original trend of economic recession quickly.

Finally, author makes the empirical analysis for all of conclusions given in this paper by use of the US GDP data from 1940 to 2003, the perfect results are obtained.

What we need to explain are:

At a discussion for DP, if we want to know the optimal expanding of an industry production and the actual effect of a new technique, see [31] and [32] respectively.

Only the conversion, conservation and fluctuations of single DP are discussed in this paper, the problems about group DP should be discussed aftertime; economic characteristics of some industries which have come into super state in economy should be researched.

## ACKNOWLEDGMENT

The authors would like to thank Prof. William Greene of Economics Stern School of Business NYU and Prof. Paul W. Wilson of Department of Economics University of Texas.

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## Appendix

**US GDP (chained) price index and estimated data of DP**

| <b>Year <math>t</math></b> | <b>Productivity<br/><math>\mu(t)</math></b> | <b>DP<br/><math>v(t)</math></b> | <b>Year <math>t</math></b> | <b>Productivity<br/><math>\mu(t)</math></b> | <b>DP<br/><math>v(t)</math></b> |
|----------------------------|---|---------------------------------|----------------------------|---|---------------------------------|
| 1940                       | 0.0978                                      |                                 | 1972                       | 0.2972                                      | 0.04508748318                   |
| 1941                       | 0.1014                                      | 0.03550295858                   | 1973                       | 0.3103                                      | 0.04221720915                   |
| 1942                       | 0.1089                                      | 0.06887052342                   | 1974                       | 0.3327                                      | 0.06732792305                   |
| 1943                       | 0.1163                                      | 0.06362854686                   | 1975                       | 0.3673                                      | 0.09420092567                   |
| 1944                       | 0.1209                                      | 0.03804797353                   | 1976                       | 0.3938                                      | 0.06729304215                   |
| 1945                       | 0.1239                                      | 0.02421307506                   | 1977                       | 0.4233                                      | 0.06969052681                   |
| 1946                       | 0.1328                                      | 0.06701807229                   | 1978                       | 0.4518                                      | 0.06308100930                   |
| 1947                       | 0.1466                                      | 0.09413369714                   | 1979                       | 0.4882                                      | 0.07455960672                   |
| 1948                       | 0.1606                                      | 0.08717310087                   | 1980                       | 0.5310                                      | 0.08060263653                   |
| 1949                       | 0.1660                                      | 0.03253012048                   | 1981                       | 0.5830                                      | 0.08919382504                   |
| 1950                       | 0.1635                                      | 0.01529051988                   | 1982                       | 0.6229                                      | 0.06405522556                   |
| 1951                       | 0.1723                                      | 0.05107370865                   | 1983                       | 0.6504                                      | 0.04228167282                   |
| 1952                       | 0.1792                                      | 0.03850446429                   | 1984                       | 0.6744                                      | 0.03558718861                   |
| 1953                       | 0.1825                                      | 0.01808219178                   | 1985                       | 0.6963                                      | 0.03145196036                   |
| 1954                       | 0.1846                                      | 0.01137594800                   | 1986                       | 0.7125                                      | 0.02273684211                   |

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|      |        |                |      |        |               |
|------|--------|----------------|------|--------|---------------|
| 1955 | 0.1862 | 0.008592910849 | 1987 | 0.7311 | 0.02544111613 |
| 1956 | 0.1911 | 0.02564102564  | 1988 | 0.7541 | 0.03049993370 |
| 1957 | 0.1983 | 0.03630862330  | 1989 | 0.7834 | 0.03740107225 |
| 1958 | 0.2043 | 0.02936857562  | 1990 | 0.8125 | 0.03581538462 |
| 1959 | 0.2075 | 0.01542168675  | 1991 | 0.8430 | 0.03618030842 |
| 1960 | 0.2100 | 0.01190476190  | 1992 | 0.8642 | 0.02453135848 |
| 1961 | 0.2130 | 0.01408450704  | 1993 | 0.8838 | 0.02217696311 |
| 1962 | 0.2154 | 0.01114206128  | 1994 | 0.9028 | 0.02104563580 |
| 1963 | 0.2181 | 0.01237964237  | 1995 | 0.9218 | 0.02061184639 |
| 1964 | 0.2207 | 0.01178069778  | 1996 | 0.9395 | 0.01883980841 |
| 1965 | 0.2245 | 0.01692650334  | 1997 | 0.9559 | 0.01715660634 |
| 1966 | 0.2293 | 0.02093327519  | 1998 | 0.9675 | 0.01198966408 |
| 1967 | 0.2367 | 0.03126320237  | 1999 | 0.9802 | 0.01295653948 |
| 1968 | 0.2451 | 0.03427172583  | 2000 | 1.0000 | 0.01980000000 |
| 1969 | 0.2563 | 0.04369879048  | 2001 | 1.0234 | 0.02286495994 |
| 1970 | 0.2703 | 0.05179430263  | 2002 | 1.0415 | 0.01737878060 |
| 1971 | 0.2838 | 0.04756871036  | 2003 | 1.0585 | 0.01606046292 |

**Notation:** The data of GDP in this table come from the website: <http://www.whitehouse.gov>

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