Optimal Monetary Policy under Heterogeneous Expectations

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Abstract

Monetary policy has an important role in the determination of the inflation rate and the output gap time trajectories. Monetary authorities should choose the nominal interest rate time path that best serves the goals of price stability (primarily) and output growth (as a consequence of the first). In this paper it is presented a framework under which an optimal interest rate rule is computed, and this rule is found to be stabilizing. The stability result is true for a homogeneous expectations scenario, where all individuals believe that inflation converges to a long run low level. Introducing expectations heterogeneity under a bounded rationality – discrete choice setup, this result continues to hold, but now we cannot exclude periods of strong price instability that, nevertheless, do not tend to persist for long periods of time.

Keywords: Optimal monetary policy; Price stability; Inflation targeting; Heterogeneous expectations; Bounded rationality; Discrete choice.

JEL classification: E43, E52.

I. INTRODUCTION

Central banks, like the European Central Bank or the American Federal Reserve, are the guardians of price stability. Monetary policy has, as primary objective, the control of inflationary pressures. Alongside with this central goal, monetary policy makers can also aim to stimulate growth or fine-tune the economy, but this goal should not jeopardize price stability, the ultimate and central policy target. As Svensson and Woodford (2003) clearly put it, “In recent years, many central banks have adopted ‘inflation targeting’ frameworks for the conduct of monetary policy. These have proven in a number of countries to be effective means of first lowering inflation and then maintaining both low and stable inflation and inflation expectations, without negative consequences for the output gap. Thus, the new approach to monetary policy has been judged quite successful, as far as its consequences for the average level of inflation and the output gap are concerned.” (page 1).

To control inflation, central banks have a high degree of independence relatively to political power. Independent monetary policy implies that the central banks can choose their targets (as referred, mainly inflation but possibly also output and employment) and also the instrument to attain the wanted goals. The instrument of monetary policy is commonly the short-run market interest rate that is set for money transactions between banks.

Setting nominal interest rates to influence the inflation rate is a complex activity. It is necessary some degree of discretion to respond to changes in the economic environment, but without a somehow fixed rule it is not possible for the policy makers to gain credibility and therefore to build a reputation that helps to maintain inflation expectations low. The rules versus discretion debate is a crucial debate in monetary policy, that has gained special relevance with the work of Kydland and Prescott (1977) and Barro and Gordon (1983). These authors have emphasized that monetary policy effectiveness is directly dependent on central bank's credibility.

The question of credibility arises because we have a two way system: on one hand it is true that the central bank responds to private sector changes in expected inflation through the use of its monetary policy instrument but, on the other hand, the private sector behavior is also dependent on how the course of monetary policy is perceived by the economic agents and on expectations that are formed about future monetary policy. Having this idea in mind, the technical literature emphasizes that expectations of low inflation should not be used by central banks to adopt and pursue output oriented expansionary policies. If central banks act in this way, the public will not in fact expect low inflation what implies a final result of high inflation without significant output gains.

This is indeed a game between the central bank and the private economy that does not lead to an optimal outcome; if the policy maker announces that inflation will equal some low value and the private economy sets its expectations accordingly, the policy maker can deviate from the policy once expectations are formed in order to gain in terms of output, that is, reneging on the commitment raises social welfare. The problem is that private sector expectations will not be maintained and as a result monetary policy decisions give place to inefficiently high inflation. Therefore, under discretion there is an inflationary bias and we can classify monetary policy as being dynamically inconsistent. Total discretionarity should be, in this way, avoided by monetary authorities.

Because a commitment makes monetary policy credible, rules tend to give better results in controlling inflation growth. Nevertheless, the commitment should not be totally binding. It has to be somehow flexible to face eventual unexpected circumstances and to account for errors that might arise when the central bank makes the evaluation of economic conditions and forms its own expectations about the economy's expectations.[2] Furthermore, the commitment may not come in the form of a specific value or time path for the nominal interest rate; for instance, Rogoff (1985) talks about delegation: if the central bank is known to be especially averse to inflation and it is common knowledge that it acts in an independent way, this can be sufficient to solve the dynamic inconsistency problem because, as referred, the main issue is credibility. If this credibility comes from reputation or from a more or less binding rule this is not the most relevant.

Rules that set interest rate time paths in some initial moment are essentially of two types:

(1) optimal rules, that are obtained by solving an intertemporal optimization model where the central bank takes a policy problem constrained by some given conditions about the functioning of the economic system, and,

(2) Taylor rules or non-optimal interest rate rules, which link the interest rate to expected inflation and expected output gap. Taylor rules are non optimal in the sense that the interest rate time path is not derived from an optimal control setup, but as it is known from the literature, they may have practical advantages, namely in the sense that steady state stability can be assured, what is not always true for optimal rules.

The issue of stability is a central one. The framework to adopt in this paper will consider an optimal interest rate rule that, for a set of parameters with reasonable values, is stabilizing and thus Taylor rules will not be in the centre of our concern. Nevertheless, Taylor rules stability is a widely discussed matter;[3] a commonly accepted remark about monetary policy is that a particular class of Taylor rules is stabilizing, but not all Taylor rules. Taylor rules may be of two kinds: (1) passive interest rate rules (for a one point increase in expected inflation, the interest rate is risen by less than one point); (2) active interest rate rules (for a one point increase in expected inflation, the central bank rises the interest rate by more than one point). Following Taylor (1993), active interest rate rules are stabilizing while passive ones are not. In a simple rational expectations framework this has been taken along the last decade as an acceptable result, but, for some authors, a more sophisticated setup implies the necessity to review it. Bernanke and Woodford (1997) show that indeterminacy and multiple stationary rational expectations equilibria might arise; Bullard and Mitra (2002) and Evans and Honkapohja (2003) state that the indeterminacy result may exist when expectations are not fully rational but result from a learning process; Benhabib, Schmitt-Grohé and Uribe (2001*a*, 2001*b*, 2001*c*) argue that active interest rate rules are only locally stable, that is, if the initial state of the economy is not in the vicinity of the steady state, then the system will converge to a liquidity trap, that is, to a state in which the nominal interest rate is near zero and inflation is eventually negative. Depending on the specification of the model, active monetary policy may result also in multiple equilibria or even chaotic dynamics. The important result according to these authors is that a local analysis (in the vicinity of the steady state) might wrongly lead to the conclusion that active monetary policy is stabilizing when it is not – in fact, under the mentioned reasoning, models that accurately describe monetary policy concerns generally lead to global indeterminacy.

The adoption of different types of rules is a subject that has been widely studied in the past few years, in theoretical grounds [see, e.g., Christiano and Gust (1999), Giannoni and Woodford (2002), Kerr and King (1996), Rotemberg and Woodford (1999), Rudebusch and Svensson (1999), Svensson (1999, 2002)] and also from an empirical point of view [Clarida, Galí and Gertler (1998), Judd and Rudebusch (1998) and King (1997), among others]. Although these studies encounter different results concerning the notion of an optimal monetary policy and of a stabilizing monetary policy, they have important points in common: they all understand that monetary policy should be forward-looking, in particular based in expectations about future inflation; it is also generally accepted that monetary policy should be guided by the understanding that there is a short-run trade-off between price stability and real economic activity (a kind of Phillips curve relation); furthermore, the transmission mechanism of monetary policy is commonly accepted to be linked with the influence of the real interest rate over investment and output. It is under a framework that takes these features into account that we will study in the following sections optimal monetary policy.

As mentioned, in this kind of analysis of the impact of monetary policy, the formation of expectations is a fundamental issue. One important point, discussed in Honkapohja and Mitra (2003) has to do with the matching of private and central bank expectations. The argument is that if the private institutions acquire knowledge about how the central bank sets its decisions according to forecasts of such private institutions, then these institutions might change their forecasts strategically in order to influence monetary policy decisions. In this scenario, the central bank would have to consider internal forecasts that could deviate from private economy expectations. Conventional monetary policy models avoid this kind of consideration by assuming that the private economy does not have these strategic capabilities.

Another crucial point about expectations relates to how they are formed. The trivial analysis, that looks at the formation of optimal rules and to the stability of optimal and non-optimal rules, considers rational expectations. Recent macroeconomic literature points to other ways of forming expectations. An important strand of literature at this level is the learning approach developed by Sargent (1993, 1999), Marimon (1997) and Evans and Honkapohja (1999, 2001). Under the learning approach, the expectations of the agents are adjusted over time as new data becomes available. Parameter updating is made through standard econometric estimation procedures. Other expectation formation rules include genetic algorithms / computational intelligence [Arifovic (1994, 1998)], eductive learning, that is, learning through a mental process of reasoning [Guesnerie (1992, 2002)], and discrete choice models [Brock and Hommes (1997, 1998)]. Discrete choice has been used predominantly in financial markets to explain how heterogeneity of expectations might lead to asset prices time paths that are erratic, impossible to predict and that can deviate for long periods of time from the fundamental solution.

The analysis to undertake in the following sections is concerned with the themes discussed previously in this introduction, namely monetary policy, interest rate rules and expectations. The following remarks will guide our discussion:

First – it is important to assess optimality and stability of monetary rules;

Second – not all individuals form expectations about future events in the same way. Given this evidence, the rational expectations framework is replaced by a discrete choice setup, where individuals choose to form expectations according to different rules.[4] There are heterogeneous expectations and these are guided by a bounded rationality mechanism that is given by discrete choice theory.

In this way, our main goal is to use the expectation formation setup proposed by Brock and Hommes (1997, 1998), Gaunersdorfer, Hommes and Wagener (2003) and Diks and Van der Weide (2003), among others, which has leaded to the influential ‘rational routes to randomness’ literature on financial asset pricing, and apply it to the formation of expectations about inflation. With inflation expectations formed in this way, we intend to study optimal interest rate rules stability and find out if there are important differences relatively to the rational expectations benchmark.

The manuscript is organized as follows. The second section formalizes a monetary policy model. The model is the two equation system proposed by Clarida, Galí and Gertler (1999) and Woodford (1999, 2003), which should be viewed as a characterization of the short-run conditions governing the functioning of the economic system. The system contains an IS curve (an equation that relates the output gap inversely to the real interest rate) and a Phillips curve (an equation that relates inflation positively with the output gap). The two referred equations are the constraints of a policy design problem, which consists on the setting of the interest rate time path that best serves the policy goal, that is mainly price stability. In the third section, optimality and stability of monetary policy are addressed; in this section we restrict the analysis to homogeneous expectations, and expectations are formed in a fundamentalist way: economic agents believe that inflation and output will converge to some known long run value. The fourth section introduces heterogeneous expectations and explains the discrete choice / adaptive learning monetary policy problem, while section five characterizes the conditions for monetary policy stability under our setup. The most interesting point in this characterization is that stability is found, but this is a new kind of stability: interest rate and inflation paths are not constant or fully predictable, although there is a reversion to the mean mechanism that avoids the values of the variables to depart permanently or for long periods of time from a steady state trend. Finally, section six discusses the most relevant conclusions.

II. THE BASELINE MODEL

We begin by setting up a framework to the analysis of monetary policy. This framework is adapted directly from Clarida, Galí and Gertler (1999), but has its roots on the staggered nominal price setting setup with monopolistically competitive firms due to Fischer (1977), Taylor (1980), Calvo (1983) and Yun (1996).

The reduced form of the model is a two equation system composed by an IS curve and a Phillips curve. The equations are, respectively,

$$x_t = -\varphi.[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + g_t \tag{1}$$

$$\pi_t = \lambda.x_t + \beta.E_t\pi_{t+1} + u_t \tag{2}$$

The endogenous variables of the system (1)-(2) are the output gap (x_t) and period t inflation rate (π_t). The output gap is defined by $x_t=y_t-z_t$, where y_t is the deviation of output from a deterministic long run trend (in logs) and z_t is the natural level of output (also in logs). Note that z_t is the level of output that would arise if wages and prices were perfectly flexible; it represents potential output. Variable i_t is the nominal interest rate, the instrument of monetary policy and consequently the control variable of the policy problem.

The parameters (all positive values) are,

φ: interest elasticity. This parameter reflects intertemporal substitution of consumption, since it establishes a negative effect of the real interest rate on current output.

λ: output-inflation elasticity. This establishes a relation between output and prices growth; hence, the higher the value of the parameter the more sensitive are prices due to output changes. A low λ indicates price rigidity.

β: discount factor. This represents the degree of sensitivity relating expected inflation with current inflation. Note that ½<β<1, and the higher is β the lower is the discount rate.

Expected values assume a central place in the equations: $E_t\pi_{t+1}$ and E_tx_{t+1} are expected inflation rate and expected output gap respectively (these values are expected in period t for period t+1). These are private sector expectations that the central bank considers to base its monetary policy decisions. In the next section, where optimality and stability of monetary policy are discussed, a rule for the formation of expectations has to be considered. We will assume that economic agents predict that inflation and output gap will converge for some long run values π* and x*. The rates of convergence will be v, $w \in (0,1)$ and expectations will be based on period t-1 observations. Therefore,

$$E_t x_{t+1} = x^* + w.(x_{t-1} - x^*) \tag{3}$$

$$E_t \pi_{t+1} = \pi^* + v.(\pi_{t-1} - \pi^*) \tag{4}$$

Finally, g_t and u_t are disturbance terms. We define these variables as AR(1) processes,

$$g_t = \mu.g_{t-1} + \hat{g}_t \tag{5}$$

$$u_t = \rho.u_{t-1} + \hat{u}_t \tag{6}$$

where $0 \leq \mu, \rho \leq 1$. According to Honkapohja and Mitra (2003), g_t represents shocks to government purchases and / or potential output; u_t represents any cost push shocks to marginal costs other than those entering through x_t. Variables $\hat{g}_t$ and $\hat{u}_t$ are i.i.d. random variables with zero mean and variances σ_g^2 and σ_u^2, respectively.

The two equations, (1) and (2), are constraints for the central bank in setting monetary policy. The central bank maximizes a policy function subject to constraints

(1) and (2). The objective function is a welfare measure that guides policy choices; two concerns are in order: price stability and output deviations, and thus the policy function is

$$V_t = -\frac{1}{2}.E_t.\left\{\sum_{j=t}^{\infty} \beta^j.\left[\alpha.x_{t+j}^2 + \pi_{t+j}^2\right]\right\} \tag{7}$$

Parameter α reflects the relative weight on output deviations. Because the main goal of the central bank is to promote price stability, α is certainly less than one, and it is probably near zero in practice for monetary authorities in developed economies.

The policy problem is, then, the following: the central bank chooses a time path for the instrument or control variable i_t to engineer time paths of the target variables x_t and π_t that maximize the objective function (7) given the two constraints on behavior (1) and (2).

A first step in the quest for an optimal interest rate time path should be the replacement in (1) and (2) of expectations variables for the rules under which expectations are formed [(3) and (4)]. Some computation transforms (1) and (2) in (8) and (9),

$$\Delta x_t = -\varphi.i_t + v.\varphi.\pi_{t-1} - (1-w).x_{t-1} + (1-v).\varphi.\pi^* + (1-w).x^* + g_t \tag{8}$$

$$\begin{aligned}\Delta \pi_t = -\varphi.\lambda.i_t - \left[1 - v.(\beta + \varphi.\lambda)\right].\pi_{t-1} + w.\lambda.x_{t-1} + \\ (1-v).(\beta+\varphi.\lambda).\pi^* + (1-w).\lambda.x^* + \lambda.g_t + u_t\end{aligned} \tag{9}$$

where $\Delta x_t \equiv x_t - x_{t-1}$ and $\Delta \pi_t \equiv \pi_t - \pi_{t-1}$.

In the following section the policy problem will be solved. We will distinguish between discretion and commitment and find, for both cases optimality conditions and optimal interest rate paths. Giving then attention solely to the commitment case we will address the stability concern. One finds that for reasonable parameter values the optimal interest rate rule is stabilizing: there is a convergence to the steady state locus independently of the initial point. Note once again that these results are true for expectations given by (3) and (4).

III. Optimal Monetary Policy. The Stability Concern

As stated in the introduction, the problem in the previous section may be solved under discretion or under commitment. Discretion means that the problem is solved in each time moment by the central bank, and so we have a sequence of static optimization problems instead of a dynamic intertemporal problem. In the case of discretion, $E_t\pi_{t+1}$ and E_tx_{t+1} are taken as given by the central bank, and thus (3) and (4) are ignored. Setting up a Lagrangean function and finding an optimality condition, we have,

$$x_t = -\frac{\lambda}{\alpha}.\pi_t \tag{10}$$

Equation (10) is an equation presented in Clarida, Galí and Gertler (1999) that simply says that under optimal conditions there is a trade-off between inflation and the output gap. Relation (10) will hold for a specific interest rate that is optimal for the given maximization problem. This interest rate can be revealed by replacing (10) and (2) in (1),

$$i_t = \left[1 + \frac{\lambda.\beta}{\varphi.(\lambda^2 + \alpha)}\right].E_t\pi_{t+1} + \frac{1}{\varphi}.E_t x_{t+1} + \frac{1}{\varphi}.g_t + \frac{\lambda}{\varphi.(\lambda^2 + \alpha)}.u_t \tag{11}$$

Interest rate (11) is the optimal interest rate under discretion. It is the interest rate that maximizes the value of the objective function. Note that, as it is intuitively true, high expected inflation and output gap imply the choice of a higher interest rate.[5]

If the central bank is concerned only in controlling inflation, α=0, the interest rate optimal value will simplify to

$$i_t = \left(1 + \frac{\beta}{\varphi.\lambda}\right).E_t\pi_{t+1} + \frac{1}{\varphi}.E_t x_{t+1} + \frac{1}{\varphi}.g_t + \frac{1}{\varphi.\lambda}.u_t \tag{12}$$

Under discretion the stability issue is not a concern, because the monetary authority can change the interest rate in each time moment and thus avoid that the inflation rate and the output gap depart substantially from long run values; the problem with this kind of procedure is the one identified in the introduction: the dynamic inconsistency problem.

Let us turn now to commitment. Under commitment we have an intertemporal problem: the central bank chooses today the optimal interest rate for all future time moments. What is lost in terms of discretionarity is gained in terms of reputation and thus expectations predictability. The issue of stability now arises in the sense that convergence to the steady state may not be guaranteed in the initial moment. The optimal control problem is, we recall, the maximization of (7) subject to (8) and (9), with x_0, π_0 given.

To solve the optimal control problem we set up a Hamiltonian function (a dynamic Lagrangean function) and determine the optimality conditions. The procedure is presented in detail in an appendix in the end of the text. From the optimal solution of the model we derive the optimal interest rate rule; this is:

$$\begin{aligned} i_t = \frac{\beta}{\varphi.\eta}.\Bigg\{ & \left[\frac{v.\varphi.\eta}{\beta} + v.(\beta+\lambda) - 1\right].\pi_{t-1} + \left(\alpha + \frac{w.\eta}{\beta}\right).x_{t-1} + (1-v).\left(\beta + \lambda + \frac{\varphi.\eta}{\beta}\right).\pi^* + \\ & \frac{(1-w).\eta}{\beta}.x^* + \frac{\eta}{\beta}.g_t + \left(1 + \frac{\lambda}{\beta}\right).u_t \Bigg\} \end{aligned} \tag{13}$$

with $\eta \equiv \beta.(\lambda - \alpha) + \lambda^2 + \alpha.(1 - v.\beta^2) > 0$. Equation (13) is the interest rate the central bank chooses in an initial moment for the future. This interest rate is chosen according to last period observed inflation rate and output gap, to long run expected inflation rate and output gap and to the disturbance terms. It is relevant to notice that the higher last period output gap and the higher the long run fundamental expected values, the larger will be the value of the nominal interest rate set by monetary authorities. The same is presumably true for last period inflation rate but this is not an unambiguous result – for some combination of parameter values a higher inflation rate in t-1 may lead to a lower interest rate; looking at (13) we conclude that i_t varies positively with π_{t-1} if the velocity of convergence of inflation to the correspondent long run value is not too fast (high v), that is, the following condition must be satisfied,

$$v > \frac{\beta}{\varphi.\eta + \beta.(\beta + \lambda)} \tag{14}$$

Given the optimal interest rate rule, (13), we are interested in knowing if this rule is stable, that is, if the time paths of the inflation rate and of the output gap tend for their long run steady values, independently from where the initial point (x_0,π_0) is located. We will show that, under reasonable values for parameters, stability is observed and, thus, in our framework, where expectations imply the belief in a convergence process to the steady state, optimal monetary policy is synonymous of the best possible achievable outcome.

To study the stability of optimal policy, we have to replace interest rate (13) in the system of equations (8) and (9). Once again, the derivation is made in the appendix in the end of the paper. The obtained system is, under matricial notation,

$$\begin{bmatrix}\Delta x_t \\ \Delta \pi_t\end{bmatrix} = \begin{bmatrix} -\left(1+\dfrac{\alpha.\beta}{\eta}\right) & \dfrac{\beta}{\eta}.[1-v.(\beta+\lambda)] \\ -\dfrac{\alpha.\beta.\lambda}{\eta} & \dfrac{\beta.\lambda}{\eta}.[1-v.(\beta+\lambda)]+v.\beta-1 \end{bmatrix} . \begin{bmatrix} x_{t-1} \\ \pi_{t-1}\end{bmatrix} + \begin{bmatrix} -(1-v).\beta.\dfrac{\beta+\lambda}{\eta} \\ (1-v).\beta.\left[1-\lambda.\dfrac{\beta+\lambda}{\eta}\right]\end{bmatrix} .\pi^* + \begin{bmatrix} -\dfrac{\beta+\lambda}{\eta} \\ 1-\lambda.\dfrac{\beta+\lambda}{\eta}\end{bmatrix} .u_t \tag{15}$$

Stability is analyzed through the first squared matrix of system (15). Let this be matrix *J*. Stability requires that the eigenvalues of *J* must be, both, located in the interval (-2,0). To compute these eigenvaules we take reasonable values for the several parameters. We rely on Benigno and López-Salido (2002) and recall the constraints upon parameters to choose the set $\{v, \varphi, \beta, \lambda, \alpha\}$={0.75, 0.5, 0.98, 0.75, 0.1}.[6]

With these values, system (15) becomes

$$\begin{bmatrix}\Delta x_t \\ \Delta \pi_t\end{bmatrix} = \begin{bmatrix} -1.0798 & -0.2375 \\ -0.0599 & -0.4431 \end{bmatrix} . \begin{bmatrix} x_{t-1} \\ \pi_{t-1}\end{bmatrix} + \begin{bmatrix} -0.3453 \\ -0.0140 \end{bmatrix} .\pi^* + \begin{bmatrix} -1.4094 \\ -0.0570 \end{bmatrix} .u_t \tag{16}$$

Solving for the steady state, we find that long run values depend on the disturbance term: $(x^*,\pi^*)=(-1.0575.u_t;\ 0.0499.u_t)$. Relatively to the eigenvalues, these are ε_1=-1.1014 and ε_2=-0.4215. Hence, stability is guaranteed for the chosen set of parameter values – independently of the initial levels of inflation and output deviation from its trend, the imposition of an interest rate path for all future moments that results from an optimal choice leads to the accomplishment of a long run steady state with low inflation and a low deviation of output from trend values.

To illustrate the stability result of the optimal rule we draw the time path of the inflation rate and the time path of the output gap. To proceed with this representation we have to specify the evolution of u_t. Let ρ=0.9 and σ_u=0.001. Figures 1 and 2 respect to the steady state paths of inflation rate and output gap. As shown, there is no tendency for these paths to deviate from a constant mean and the only fluctuations that are observed are the ones that result from the disturbance variable u_t.

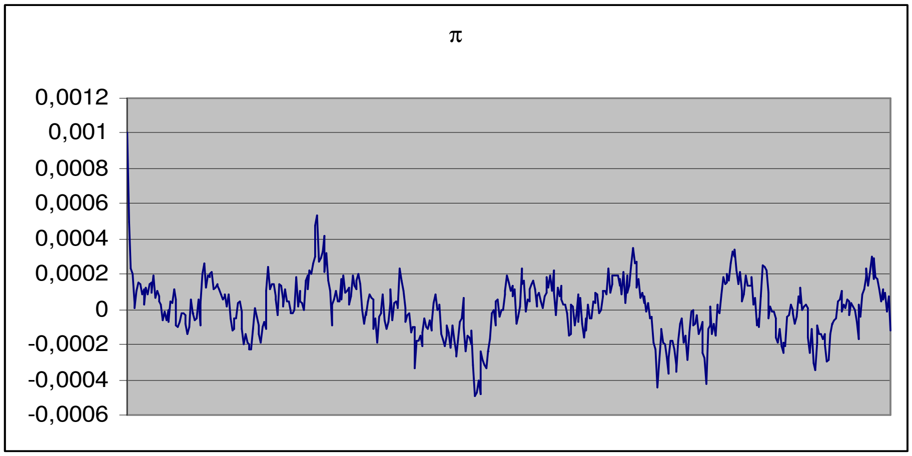

Figure 1 – Inflation rate time path under an optimal monetary policy rule

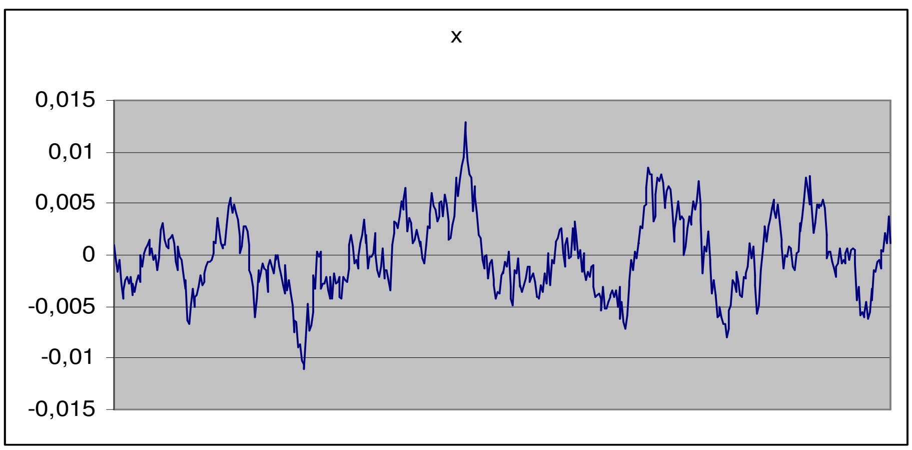

Figure 2 – Output gap time path under an optimal monetary policy rule

IV. HETEROGENEOUS INFLATION EXPECTATIONS

The previous section looked at monetary policy under a setup where the central bank knows that individuals know that the inflation rate and the output gap tend to constant (plus a disturbance) steady state values. In this section, we continue to consider the optimal interest rate rule (13), that is, we consider that the monetary authority believes that private expectations are formed under the convergence assumption, but now in reality only a fraction of the economic agents will form their expectations in this way. The alternative expectations formation setting is one in which individuals are trend followers – they will predict inflation according to past variations in this rate. Individuals can alternate between expectation formation rules according to the performance of such rules. In this section we present in detail this setup, and section V will illustrate numerically how the new assumption about expectations can lead to significant changes in steady state results.

The monetary authority uses the model in the previous section to choose the interest rate rule. This is (13), which under our numerical example corresponds to (now we consider also a value for the other convergence parameter: w=0.75),

$$i_t = 1.225.\pi_{t-1} + 1.6597.x_{t-1} + 0.9406.\pi^* + 0.3069.x^* + 2.g_t + 2.8188.u_t \quad (17)$$

The difference relatively to last section's setup is that we no longer consider that expectations about inflation are homogeneous.[7] Part of the individuals on the economy believe that next period inflation will be given by a rule like (4), but other agents will be trend followers, that is, they will adjust their expectations according to a rule where previous inflation changes are taken into account. The rules that the two groups of individuals assume concerning inflation expectations are

$$E_{1t}\pi_{t+1} = \pi^* + v.(\pi_{t-1} - \pi^*) \quad (18)$$

$$E_{2t}\pi_{t+1} = \pi_{t-1} + m.(\pi_{t-1} - \pi_{t-2}) \quad (19)$$

Equation (18) is equal to (4), but now only for a group of agents of type h=1, while (19) is a trend following expectation rule that individuals of type h=2 believe to best represent the true inflation path over time. Parameter m>0 translates the importance of past inflation changes in the formation of inflation expectations.

If the private sector economic agents were fully rational they would choose between (18) and (19) in order to get the best possible result in terms of accomplished or expected benefits (measured in terms of output and price stability). The main assumption in this heterogeneous expectations setup is that individuals are not fully rational. They obey to a bounded rationality process, in which they change behavior in face of changes in their outcome, but where this process is not immediate and definitive. The bounded rationality assumption is linked with discrete choice theory, as developed in Manski and McFadden (1981) and Anderson, de Palma and Thisse (1993), which involves the following mechanism.

Assume that n_{1t} is the share of individuals that follow the expectations formation rule (18) and that, consequently, 1- n_{1t} is the share of individuals that in a given time moment *t* choose the other rule, (19). They will change their behavior according to (20).

$$n_{1t} = \frac{e^{-b.U_{1t}}}{e^{-b.U_{1t}} + e^{-b.U_{2t}}} \tag{20}$$

Parameter *b* is the intensity of choice. This positive parameter reflects the degree of rationality in the choice. If *b* is close to zero, individuals prefer to stay with their present choice even if this performs worse than the other; a high *b* represents a high degree of rationality, where the obtained results determine in a more straightforward way the choice of the best strategy.

Variable U_{ht}, h=1,2, is a fitness function or performance measure. It represents the way in which previous expectations have performed in terms of the individuals objective function. We assume $U_{ht}=\chi.f_{ht}+\zeta.U_{ht-1}$, h=1,2, $0<\zeta<1$, $\chi>0$ and $f_{ht} = -\left(\frac{E_{ht-1}\pi_t - \pi_{t-1}}{\pi_{t-1}}\right)^2$. Function *f* measures the deviation of the expected inflation rate relatively to the observed inflation rate in the last period; this value is measured as a rate and it is presented in such a way to give always a negative value (thus, the closer to zero is f the better is the forecast made by the given type of agents concerning future inflation.

Having described the way in which the private sector sets up its expectations and how individuals may change the expectations formation rule, we can now integrate this setup into the two equation monetary model [(1) and (2)]. In this system, the nominal interest rate is the optimal one and expectations about future inflation are given by a weighted average of different expectations, i.e.,

$$E_t\pi_{t+1} = n_{1t}.E_{1t}\pi_{t+1} + (1-n_{1t}).E_{2t}\pi_{t+1} \tag{21}$$

To further develop the model we recover the numerical example. Meanwhile some new parameters have been presented. The following values are adopted: $\{m, b, \chi, \zeta\}$=$\{0.95, 1, 0.01, 0.9\}$. In section V we discuss the results of the following set of equations:

- i_t: given by (17);
- $\Delta x_t = -0.5.[i_t - E_t\pi_{t+1}] - 0.25.(x_{t-1} - x^*) + g_t$;
- $\pi_t = -0.375.i_t + 1.355.E_t\pi_{t+1} + 0.5625.x_{t-1} + 0.1875.x^* + 0.75.g_t + u_t$;
- $E_t\pi_{t+1}$: given by (21);
- $E_{1t}\pi_{t+1}$ and $E_{2t}\pi_{t+1}$: given by (18) and (19);
- n_{ht}, U_{ht}, f_{ht}: as defined in this section.

As before, we assume ρ=0.9 and σ_u=0.001; also, now we consider μ=0.9 and σ_g=0.001.

Anticipating the results to evidence in the next section, the introduction of a new expectations formation rule for inflation and the consideration of a boundedly rational scenario will lead to a long run path for inflation that is not unstable (the inflation rate does not tend to infinity, neither a multiple equilibria result is present) but that also is not stable in the sense of section III. There, the inflation rate could be found around a steady state value according to the Markov process underlying the term u_t. In this new framework, we will find an erratic pattern, where periods of high inflation and periods of low inflation alternate and where it is impossible to predict future inflation having as reference some initial point. The main result is, then, that in an economy with heterogeneous expectations about inflation, the adoption of an optimal fundamentalist monetary policy rule leads to an inflation time path that is impossible to predict, where volatility clustering may exist and where periods of high and low inflation might have all types of lengths. The good news are that optimal policy is not non stabilizing in the sense of stimulating explosive inflation paths.

V. Stability under Heterogeneous Expectations

Under the values assumed for parameters in the last section and taking $x_0=\pi_0=0.001$, we encounter an inflation time path that can assume multiple forms; an example is presented in figure 3.

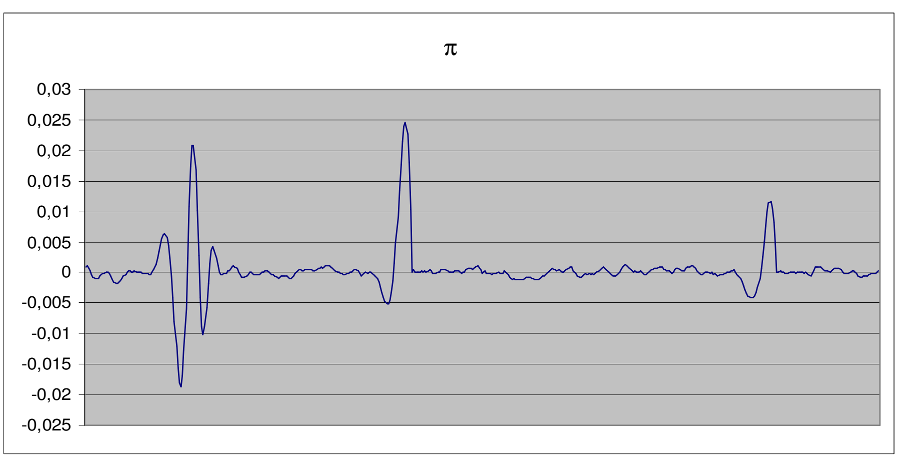

Figure 3 – Inflation rate time path under a heterogeneous expectations setup.

The time path of inflation, under the heterogeneous expectations assumption, is very sensitive to changes in parameter values and in initial values of variables. However, the pattern will be always something like in figure 3. There will be relatively long periods where inflation gravitates along a steady state value (in this case, like in figure 1, this value is zero), but the heterogeneity assumption introduces a second feature: in some moments of time, which initially are impossible to predict, periods of high or low inflation (or both) might arise; stability then returns and is maintained for some time interval.

We conclude that if the central bank acts as if individuals were rational (thinking that inflation tends to a long run constant value), and thus chooses a nominal interest rate accordingly, then the existence of a group of economic agents that are trend followers or chartists implies that stability continues to hold but short periods of intense price instability are unavoidable.

To understand how sensitive the time path of prices growth in figure 3 is to changes in parameters and variables initial values, take the following examples:

a) Alternatively to π_0=0.001, consider π_0=0.002 (the other values in our numerical example continue to be the same),

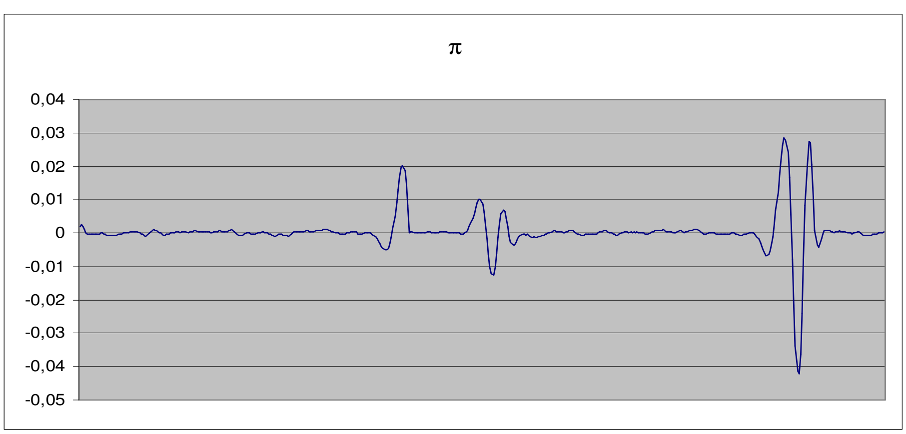

Figure 4 – Inflation rate time path under a heterogeneous expectations setup, with a change in the initial value of inflation.

From figure 4 we regard a same kind of pattern as for the initial example, but the time moments where important fluctuations occur have visibly changed.

b) Other change that can be made relates to the trend followers parameter *m*. Assume a slight positive change in the value of this parameter: *m*=1.05. Figure 5 represents the new inflation rate, that should be compared with our benchmark case in figure 3.

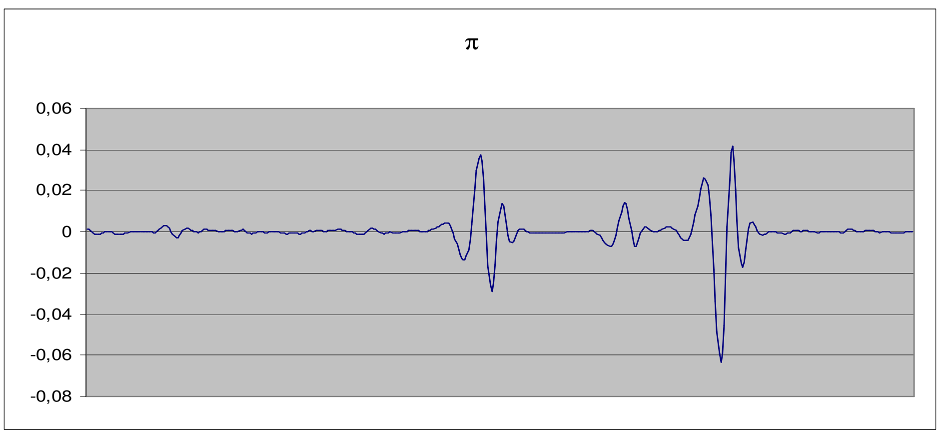

Figure 5 – Inflation rate time path under a heterogeneous expectations setup, with a change in the value of parameter *m*.

Once again, the notorious fact is that significant changes from the steady state result occur at unpredictable points in time.

c) Finally, suppose a change in the value of the intensity of choice parameter. We have stated that this parameter is a measure of the degree of rationality underlying the change from a poor performance expectation rule to the better performance one. Assuming *b*=1.025, the degree of rationality will rise relatively to the considered benchmark case, so individuals will change their expectation rule about future inflation with a higher frequency. The result is a new inflation rate trajectory, presented in figure 6. This figure reveals qualitatively no significant changes.

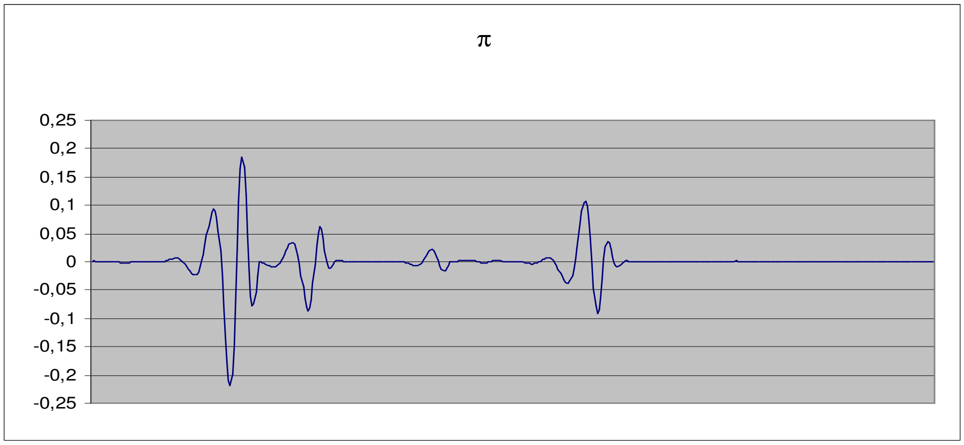

Figure 6 – Inflation rate time path under a heterogeneous expectations setup, with a change in the intensity of choice.

The most notorious change in figure 6 relatively to figure 3 is that the departures of the inflation rate from the steady state result are now more intense, reaching, in one case, 20%.

The three previous modifications relatively to our initial example imply two relevant conclusions:

- the introduction of expectations heterogeneity means that short periods of strong price instability can happen, even under an optimal monetary policy;

- strong price movements can occur at any time moment and thus are impossible to predict (not only the time they happen but also the length they have); nevertheless, optimal monetary policy guarantees, under the model's assumptions, a predictable pattern of stability that alternates with the unpredictable time limited deviations from the steady state.

VI. CONCLUSIONS

In the last few years, monetary policy has been studied under a framework that characterizes short run economic conditions. This setup is composed by an IS curve and by a Phillips curve. This system considers as endogenous variables the inflation rate and a measure of the difference between effective and potential output. These two variables are state variables, that is, they depend on private sector economic conditions and they cannot be determinated by any public authority.

Nevertheless, monetary authorities can influence the results concerning the time trajectories of such two variables through the manipulation of the variable that is usually used as the monetary policy instrument: the nominal interest rate. Though, there is an important problem, that relates to the dynamic inconsistency of monetary policy, which implies that central banks should commit primarily with price stability.

In this way, we can consider an optimal control problem for the monetary authority, which consists in maximizing an objective function, where a low inflation rate is the central concern but where output considerations may also be present. This problem leads to an optimal interest rate rule, that is, to a time path for the interest rate in the future that is the one that best contributes to attain the policy goal.

Under our framework, and given a set of reasonable parameter values, the optimal interest rate rule is stabilizing, meaning that for any initial values of the inflation rate and of the output gap a steady state result is always obtained. This steady state result is not a pair of constant values, but close; oscillations in the long run values of these variables are simply the result of a disturbance term that follows a Markov process.

The stability result is also conditional on the way expectations about future inflation and output gap are formed. Our assumption was that individuals are fundamentalists, that is, they believe that both variables will evolve to a steady state value given some velocity of convergence parameters.

The question we have asked then was does the optimal interest rate rule continues to be stabilizing if not all the agents in the economy have fundamentalist expectations? The answer is yes, but there are important mutations that have to be highlighted.

The heterogeneous expectations setup has considered the same policy problem as in the homogeneous case and it was assumed that the central bank made its policy choice considering that economic agents are all fundamentalists, thus the considered interest rate rule was the same as before. But in this second stage we have separated agents between fundamentalists and chartists; this second group forms expectations about inflation giving attention to the past history of inflation changes and ignoring the convergence to the steady state mechanism. Individuals could change the way they formed their expectations, but such happened in a bounded rationality way, that is, changes to the best performance expectation rule are here not immediate and definitive.

Putting together all the ingredients of the heterogeneous expectations scenario we have found, using the same numerical example as in the homogeneous case, that stability continuous to hold, but a new feature is present; in some unexpected moments of time price stability disappears and large fluctuations in the inflation rate are observed. Nevertheless, these deviations from the steady state result do not tend to persist and the stability result is rapidly recovered.

APPENDIX – FIRST ORDER CONDITIONS, THE DETERMINATION OF THE OPTIMAL INTEREST RATE AND THE IS – PHILLIPS CURVE OPTIMAL SYSTEM

To the policy problem in sections II and III corresponds, in a *t*-1 moment, the following Hamiltonian function,

$$\begin{aligned}\aleph_{t-1} = & -\frac{1}{2}.\left[\alpha.x_{t-1}^2 + \pi_{t-1}^2\right] + \\ & q_{t-1}(x).\left[-\varphi.i_t + v.\varphi.\pi_{t-1} - (1-w).x_{t-1} + (1-v).\varphi.\pi^* + (1-w).x^* + g_t\right] + \\ & q_{t-1}(\pi).\left\{-\varphi.\lambda.i_t - \left[1 - v.(\beta+\varphi.\lambda)\right].\pi_{t-1} + w.\lambda.x_{t-1} + (1-v).(\beta+\varphi.\lambda).\pi^* \right. \\ & \left. + (1-w).\lambda.x^* + \lambda.g_t + u_t\right\}\end{aligned} \qquad \text{(A1)}$$

with $q_{t-1}(x)$ and $q_{t-1}(\pi)$ shadow-prices of x_{t-1} and π_{t-1} respectively.

The first order conditions are:

$$\lim_{t\to\infty} q_t(x).\beta^t.x_t = 0 \qquad \text{(A2)}$$

$$\lim_{t\to\infty} q_t(\pi).\beta^t.\pi_t = 0 \qquad \text{(A3)}$$

$$\aleph_i = 0 \Rightarrow q_{t-1}(x) = -\lambda.q_{t-1}(\pi) \qquad \text{(A4)}$$

$$\Delta q_{t-1}(x) = \frac{1}{\beta}.q_{t-1}(x) + \alpha.x_{t-1} \qquad \text{(A5)}$$

$$\Delta q_{t-1}(\pi) = \left(\frac{1 - v.\beta^2}{\beta}\right).q_{t-1}(\pi) + \pi_{t-1} \qquad \text{(A6)}$$

Conditions (A2) and (A3) are transversality conditions.

To find the optimal nominal interest rate, we begin by differentiating the optimality condition (A4), obtaining

$$\Delta q_{t-1}(x) = -\lambda.\Delta q_{t-1}(\pi) \qquad \text{(A7)}$$

what implies, given (A5) and (A6),

$$\alpha.x_{t-1} - \pi_{t-1} = \frac{1 - v.\beta^2 + \lambda}{\beta}.q_{t-1}(\pi) \qquad \text{(A8)}$$

Considering first differences of (A8) and the equation (A6), we get the relation

$$\alpha.\Delta x_{t-1} - \Delta\pi_{t-1} = \frac{1-v.\beta^2+\lambda}{\beta}.\left[\left(\frac{1-v.\beta^2}{\beta}\right).q_{t-1}(\pi)+\pi_{t-1}\right] \tag{A9}$$

Using (A8) to eliminate the shadow-price from expression (A9),

$$\alpha.\Delta x_{t-1} - \Delta\pi_{t-1} = \frac{1-v.\beta^2}{\beta}.\alpha.x_{t-1} + \frac{\lambda}{\beta}.\pi_{t-1} \tag{A10}$$

Considering relation (A10) in period t and replacing in this Δx_t and $\Delta\pi_t$ by the correspondent IS and Phillips curve expressions, (8) and (9), we find, after a somehow heavy computation the interest rate rule presented in expression (13).

To find the IS – Phillips curve system under conditions of optimality we just have to replace (13) in (8) and (9), getting, after some calculus,

$$\Delta x_t = \frac{\beta}{\eta}.[1-v.(\beta+\lambda)].\pi_{t-1} - \left(1+\alpha.\frac{\beta}{\eta}\right).x_{t-1} - \frac{(1-v).\beta.(\beta+\lambda)}{\eta}.\pi^* - \frac{\beta+\lambda}{\eta}.u_t \tag{A11}$$

$$\begin{aligned}\Delta\pi_t = &\left\{\frac{\beta.\lambda}{\eta}.[1-v.(\beta+\lambda)]+v.\beta-1\right\}.\pi_{t-1} - \frac{\alpha.\beta.\lambda}{\eta}.x_{t-1} + \\ &(1-v).\beta.\left(1-\lambda.\frac{\beta+\lambda}{\eta}\right).\pi^* + \left(1-\lambda.\frac{\beta+\lambda}{\eta}\right).u_t\end{aligned} \tag{A12}$$

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[2] Clarida, Galí and Gertler (1999) emphasize in this respect that "In practice, no major central bank makes any kind of binding commitment over the course of its future monetary policy." (page 1671).

[3] This happens because implementing optimal policy is not easy; it is not clear which is the true economic model and the perception that the central bank has about how expectations are formed is not perfect. Does, in practice, central banks compromise with well definable rules rather than with an abstract concept of optimality that is hard to implement.

[4] Basically, following Brock and Hommes (1998), we will consider that economic agents follow one of two expectation formation rules: (1) a fundamentalist rule, under which individuals expect economic variables to converge through time to a long run fundamental value, and (2) a trend rule, which implies that individuals form expectations according to the recent history regarding economic variables evolution.

[5] If it is expected a positive output gap (GDP above its trend value), then the central bank should raise interest rates; if inflation expectations are above their target, then interest rates also should increase.

[6] Note that we do not consider for now a value for the convergence velocity parameter w because the optimal interest rate rule makes this parameter to disappear from the IS – Phillips curve system.

[7] For simplicity, we continue to assume that expectations about next period output gap are homogeneous and given by (3).