A Positive Theory of Government Debt

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Abstract

All developed countries have government debt, usually a sizeable proportion of output. This paper proposes that governments that cannot commit to future policy choices face a trade-off that explains the level of debt. On the one hand, the government would like to increase debt and delay taxation, so as to reduce current distortions. On the other hand, inflating current prices lowers the real value of nominal debt and so there is a motive to reduce it now. This trade-off generates a level of long-run debt that is interior and independent of initial debt, a feature missing in previous theories. The sign and size of long-run debt will depend on how the incentives to increase and decrease debt play out against each other. The critical determinant is how easy or difficult it is for households to substitute away from goods being taxed by inflation.

Under reasonable assumptions, the model is successful in replicating certain U.S. facts, most importantly its debt-to-output ratio. Permanent changes in fundamentals —other than the degree of substitutability of the good being taxed by inflation— have small effects on long-run debt. This is consistent with the empirical observation that macroeconomic variables cannot alone explain cross-country differences in public debt levels. The model is extended to include stochastic government expenditure. In line with the tax-smoothing argument, the model predicts that governments increase debt when expenditure temporarily rises. However, labor taxes fluctuate significantly.

Keywords: Government debt, time-consistency, Markov-perfect equilibrium.

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1 Introduction

All governments from developed countries have positive debt. In most cases, the size of the debt is substantial in terms of output. For example, the average debt-to-output ratio for developed countries that belong to the OECD[1] was around 50% between 1980 and 2001.

Although one could think of many roles for debt, the focus in this paper will be on government debt as an alternative to current taxation. Existing theories are successful in explaining why debt changes and motivating why it is accumulated. However, none of these models is able to deliver a long-run level of debt that is independent of initial debt, and is neither zero nor equal to some upper bound. Hence, even though we can think of reasons why governments would like to issue debt, we lack a theory of the level of government debt. This is the issue addressed here.

The model proposed has an economy with no capital and no uncertainty. Households value consumption and leisure, and use money carried over from the previous period to buy goods. There is a benevolent government that cannot commit to future policy choices and has no access to lump-sum taxes. Lack of commitment plays an important role, since the government today disagrees with its future self on what to do from tomorrow on. The reason for this is that the government today will take into account future government policies, whereas tomorrow, the government will not account for how its actions affected past decisions ("bygones are bygones"). Hence, if the government today were allowed to also decide what to do from tomorrow on, it would disagree with what it would decide if it were allowed to re-optimize in future periods.

The government trades off current for future distortions. Each period, it decides how much to distort the economy and how much debt to leave for the next period. If the only available instruments were labor taxes and debt, then there is a motive for debt due to "interest rate manipulation", as first suggested by Lucas and Stokey (1983) and recently analyzed by Krusell, Martin and Ríos-Rull (2004). In this case, as long as outstanding debt in non-negative, there is never a motive to decrease debt, i.e., it is never optimal for a government to induce higher private consumption in the future at the expense of lower current consumption.

In this paper, the government can print money and debt is nominal. Printing money increases the price level and this has two effects, one distortive and the other non-distortive. The former is the inflation tax, which basically acts as a tax on consumption. The latter is the reduction in the real value of nominal debt. This second effect is driven by lack of commitment and is similar to a capital levy. An increase in the price level tomorrow will be viewed by the current government as purely distortionary, since it internalizes the reduction in future household wealth. However, a reduction in current wealth is viewed as non-distortionary[2]. The distortionary effect of inflation is what prevents the government from raising prices to infinity so as to reduce the real value of debt to zero. Hence, whether the government wants to increase or decrease debt depends critically on which of the two effects dominates.

A key insight is that as debt increases, so do the gains from reducing it. Thus, how the two effects play out depends on how distortive the inflation tax is. For example, if goods bought with money have close substitutes then the distortion of the inflation tax will be low and hence there

[1] Source: OECD (2004).

[2] For an example of what happens under lack of commitment with capital taxes, which are non-distortionary for the current government, see Martin (2004).

is a large incentive to reduce debt. Note that this argument still applies, whether there are labor taxes or not.

There are then two opposing effects determining the actions of the government, which are driven by lack of commitment. A positive theory of debt should have the incentive to delay taxation dominating for low levels of debt and the incentive to reduce debt dominating for high levels of debt. In this way, one ensures the existence of a steady state level of debt that is neither zero nor a corner solution.

Government policy is characterized using Markov strategies, which are time-consistent by construction. This implies that policy depends only on pay-off relevant variables and so there is no consideration for other mechanisms, such as reputation, which may potentially be relevant. The idea for this to have a model in which one can understand the basic mechanism that delivers an interior long-run level of debt, that is independent of initial debt.

To evaluate the model quantitatively, it is modified to include labor taxes. The model is then calibrated to match selected target statistics from the U.S. economy, most importantly its debt-to-output ratio. Although the model fits the U.S. economy well, it has a hard time delivering high levels of long-run debt, say double or triple those of the U.S. One reason for this is that inflating prices to reduce the real value of debt becomes very rapidly a dominant strategy for a government with debt. This leaves open the question of what else matters for debt: is it self-control, reputation, central bank independence, political economy reasons? The answer is not clear, since any mechanism would have to work such that it only reduces the incentive to inflate prices, i.e., leaving the incentive to delay taxation and increase debt relatively unaltered.

Permanent changes in target statistics have small effects on long-run debt. This is consistent with the empirical observation that macroeconomic variables —such as government expenditure to GDP, taxes and inflation rates— cannot alone explain cross-country differences in public debt levels[3]. The model predicts that differences in debt among countries are explained only by how easy or difficult it is for households to substitute away from goods that are being taxed by inflation. If it is difficult, then the cost for the government of inflating prices today is high and hence the incentives to increase debt and delay taxation are greater. It is not clear whether this alone can satisfactorily explain cross-country differences and more work should be devoted to incorporate other motives for debt into the model or —as mentioned above— features that reduce the incentive to inflate prices.

By considering the case of stochastic government expenditure, it can be shown that the model explains qualitatively well why governments raise their debt when expenditure temporarily increases. This is in agreement with previous theories of debt. The model differs in that labor taxes fluctuate significantly.

Modern theories of government debt started with Barro (1974) and the Ricardian equivalence result. In this case, taxes are lump-sum and hence, the composition of expenditure finance is irrelevant[4]. Assuming a fixed interest rate, Barro (1979) shows that when exogenous government expenditure fluctuates, debt should be used to smooth distortionary taxation. This model also

[3] Alesina and Perotti (1996) propose politico-institutional explanations, such as electoral systems, party structure, government fragmentation and political polarization.

[4] Bassetto and Kocherlakota (2004) extend this result to situations in which the government can freely adjust the timing of (distortionary) tax payments.

explains how government debt increases during wartime, but some have pointed out that it does not explain why debt increased in several countries during peacetime. More importantly, the theory predicts that debt levels are irrelevant for current debt issue, i.e., debt moves randomly, in accordance with government expenditure and income shocks. In this sense, this is a theory of the change in debt, not its level.

The empirical predictions of Barro's model have been contested by a number of studies. For example, Trehan and Walsh (1990) reject the tax-smoothing hypothesis for the U.S. between 1914 and 1986, although they cannot reject it for the post-war period. Bohn (1998) shows that the debt-to-output ratio in the U.S. displays mean-reversion if one controls for wartime spending and cyclical fluctuations. In other words, government policy reacts to changes in the debt-to-income ratio.

In what could be viewed as a micro-foundation for Barro (1979), we have the Ramsey problem of optimal taxation. Lucas and Stokey (1983) show that there is a fundamental time-consistency problem in the conduct of optimal debt policy. Chari, Christiano and Kehoe (1991) show that when the government can commit, debt is used to absorb cyclical shocks and so fluctuates around some stationary value. In the absence of shocks, the government would basically increase debt in the first period and then never again. Thus, stationary long-run debt is not independent of initial debt. Moreover, the commitment assumption implies the solution is time-inconsistent.

In the political economy literature, there is a large number of models with partisan politicians that explain a bias towards debt. Two examples of this are Persson and Svensson (1989) and Alesina and Tabellini (1990). These theories show how disagreement between parties, given lack of commitment, makes them use debt strategically to influence the behavior of future government incumbents. This type of stories are used to explain why certain countries have significantly larger amounts of government debt. A survey of this literature can be found in Persson and Tabellini (1998).

In a recent paper, Krusell, Martin and Ríos-Rull (2004) take the Lucas and Stokey (1983) economy and analyze the equilibrium under lack of commitment. In this case, the government has an incentive to "manipulate interest rates" and thus always wants to delay taxation and increase debt. This means that the maximum level of debt the government can sustain with its tax revenue becomes a binding constraint. As it turn out, this implies that in equilibrium there are infinite but countable steady states. Hence, the implied dynamics looks remarkably similar to the commitment (Ramsey) case: the government increases debt for one or two periods and then leaves it constant. Thus, depending on the initial level of debt, long-run debt can be very low, very high or anything in between.

In a closely related paper, Díaz-Giménez, Giovannetti, Marimón and Teles (2004) analyze the same basic model as the one presented in Section 2. Assuming a particular utility function, they get zero long-run debt and thus conclude that nominal debt is a burden to monetary policy. This interpretation is the result of having focused on a utility function that makes the incentive to inflate prices dominate whenever debt is positive. Section 2 includes a proposition that shows under which conditions long-run debt is negative, zero or positive. As mentioned above, this has to do with how distortive the inflation tax is.

There are some other, more normative theories in which debt plays a different role. For example, Diamond (1965), Aiyagari and McGrattan (1998) and more recently, Shin (2003), propose models in which government debt is used to reduce some dynamic inefficiency. In these models, the role played by government debt is not intrinsic to it, i.e., could be played by other assets.

As a side question, one could also ask what constitutes public debt. As is standard in the literature, this paper takes the position that it is debt held by the public. Others, such as Eisner and Pieper (1984), have argued that all assets and liabilities should be considered. Unfortunately, the valuation of these proves to be difficult, with problems ranging from technical to conceptual[5]. Elmendorf and Mankiw (1998) provide a more detailed discussion on this.

The paper is organized as follows. Section 2 introduces a basic model with the sufficient elements to make a theory of debt. Section 3 adds fiscal policy and calibrates the model to fit certain facts of the U.S. economy. It then provides some comparative statics to test the model and considers the case of stochastic government expenditure. Section 4 concludes.

2 A Basic Model of Government Debt

2.1 The economy

The basic model is similar to the Lucas and Stokey (1983) economy. There is no capital and no uncertainty. There is a benevolent government that has to finance g every period. Output y is linear in labor

$$y_t = n_t,$$

which implies the aggregate resource constraint

$$c_t + g = n_t. \tag{1}$$

The government can finance g by issuing nominal debt or printing money. Hence, it has to satisfy the following period budget constraint

$$\bar{M}_t + \bar{B}_t + \bar{p}_t g = \bar{M}_{t+1} + q_t \bar{B}_{t+1}, \tag{2}$$

where $\bar{M}$ is the aggregate money stock, $\bar{B}$ is the stock of nominal bonds, $\bar{p}$ is the price level and q is the nominal price of bonds (i.e., the inverse of the gross nominal interest rate). For individual levels of money and bonds, use lower case letters.

The economy is populated by a continuum of infinitely lived households that derive utility from consumption and leisure. The present value of lifetime utility is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$$

where $\beta \in (0, 1)$ is the time discount factor and u is strictly increasing, strictly concave and differentiable in both arguments.

[5]Incidentally, the Office of Management and Budget (1996) estimated that government assets were worth roughly the same as its non-debt liabilities in 1995. Moreover, net liabilities seem to have followed debt quite closely since 1975.

There is a cash-in-advance constraint (as in Svensson, 1985)

$$\bar{p}_t c_t \leq \bar{m}_t, \tag{3}$$

so that households have positive money balances in equilibrium.

The government announces its choice for $\bar{M}_{t+1}$ and $\bar{B}_{t+1}$ at the beginning of the period. Next, as is standard in the cash-in-advance literature, the household divides into two agents, a shopper and a producer/seller. There are two subperiods: the goods market operates in the first subperiod and the securities market opens in the second. This timing is important: if the securities market were to open before the goods market, then the problem of the government would be static. To see this, note that prices and allocations at t would depend on $\bar{M}_{t+1}$ instead of $\bar{M}_t$, which would only appear in the government budget constraint together with $\bar{B}_t$. Even though individual money and bond holdings matter for the household, from an aggregate point of view, the composition of the government's nominal liabilities at the beginning of the period would be irrelevant. Hence, the actions of the current government would have no effect on future government decisions[6].

In the first subperiod, the shopper takes the household's money balances and sets out to buy the consumption good from the —only— household that produces the variety he likes. The producer/seller stays at home, works n hours and sells the produced good in exchange for money. Instead of the usual "helicopter drop", the government issues money by buying g from each household. Hence, the government crowds-out shoppers in the goods market.

In the second subperiod, the household becomes one again and the securities market opens. Each household carries bonds acquired in the previous period and money acquired from selling its output in the goods market, and chooses how much money and bonds it wants to carry to the next period. The government either buys or sells bonds, depending on its decision for $\bar{M}_{t+1}$ and $\bar{B}_{t+1}$. Given the supply and demand for money and bonds, the nominal interest rate adjusts to clear the securities market.

This environment implies the following consolidated budget constraint for the household:

$$\bar{p}_t c_t + \bar{m}_{t+1} + q_t \bar{b}_{t+1} = \bar{p}_t n_t + \bar{m}_t + \bar{b}_t. \tag{4}$$

2.2 Solving the competitive equilibrium

Since we will be solving for government policy functions, it is convenient to write the problem of the household recursively. So, what is the aggregate state variable in this economy? Nominal variables, such as money and bonds, can not be state variables nor can we use their real values, since the price level is endogenous. What is going to matter is how much of the nominal assets at the beginning of the period are money. This is so, since that is the only part that can be used to make purchases in the goods market. Hence, the aggregate state variable has to be some measure of the composition of nominal assets, say B/M[7].

Next, we need to make a few transformations[8] (see for example, Cooley and Hansen, 1991). Redefine individual and aggregate nominal variables (except for q) by dividing them by the aggregate

[6]In other words, the problem would not have an aggregate state variable.

[7]Other possible —and equivalent— state variables are: $\frac{\bar{B}}{\bar{B}+\bar{M}}$, $\frac{\bar{M}}{\bar{B}}$ and $\frac{\bar{M}}{\bar{M}+\bar{B}}$.

[8]Some of the usual transformations will not be useful here since for a Markov government, variables dated before t are meaningless. So for example, having $\pi_t = \frac{\bar{p}_t}{\bar{p}_{t-1}}$ in the household budget constraint is of no use.

money stock, i.e., for any nominal variable $\bar{x}$, let

$$x \equiv \frac{\bar{x}}{\bar{M}}.$$

Furthermore, define the money growth rate μ as

$$\mu_t = \frac{\bar{M}_{t+1}}{\bar{M}_t} - 1.$$

Using the above and switching to recursive notation (where primes denote next period variables) we can rewrite the government budget constraint (2) as

$$g = \frac{(1+\mu)(1+qB') - (1+B)}{p}, \tag{5}$$

the household budget constraint (4) as

$$c = n + \frac{m + b - (1+\mu)(m' + qb')}{p} \tag{6}$$

and, after using the above, the cash-in advance constraint (3) as

$$\frac{(1+\mu)(m' + qb') - b}{p} - n \geq 0. \tag{7}$$

Given some government policy $\{B' = \mathcal{B}(B),\ \mu = \psi(B)\}$ that satisfies the government budget constraint (5), the problem of the household can be written as follows

$$v(m,b,B) = \max_{n,m',b'} u\Big(n + \frac{m + b - (1+\mu)(m' + qb')}{p}, 1-n\Big) + \lambda\left[\frac{(1+\mu)(m' + qb') - b}{p} - n\right] + \beta v(m',b',B')$$

Taking the first-order conditions gives

$$\begin{aligned} u_c - u_\ell - \lambda &= 0 \\ -\frac{(1+\mu)(u_c - \lambda)}{p} + \beta v'_m &= 0 \\ -\frac{(1+\mu)(u_c - \lambda)q}{p} + \beta v'_b &= 0. \end{aligned}$$

Apply the envelope theorem

$$\begin{aligned} v_m &= \frac{u_c}{p} \\ v_b &= \frac{u_c - \lambda}{p} \end{aligned}$$

and use

$$1 + \pi \equiv \frac{p'(1+\mu)}{p},$$

where π is the rate of inflation between today and tomorrow, to write the equations determining prices

$$u_\ell = \frac{\beta u_c'}{1+\pi} \tag{8}$$

$$q u_\ell = \frac{\beta u_\ell'}{1+\pi}. \tag{9}$$

Note that p' is really a function of tomorrow's government policy, which in equilibrium is a function of the aggregate state. So let $p' = \mathcal{P}(B')$, where $\mathcal{P}$ indicates the price level induced by government policy in equilibrium.

Putting (8) and (9) together we get a simpler expression for the nominal price of bonds

$$q = \frac{u_\ell'}{u_c'} \tag{10}$$

and the real price of bonds (i.e., the inverse of the gross real interest rate)

$$\frac{1}{1+r} = \frac{\beta u_\ell'}{u_\ell}. \tag{11}$$

In equilibrium, all households make the same choices, so we have $m = M = 1$ and $b = B$. Moreover, the cash-in-advance constraint holds with equality

$$c = \frac{1}{p}. \tag{12}$$

2.3 The problem of the government

Since there is lack of commitment and reputational mechanisms are not operative, the current government sets its policy for the period based on fundamentals, taking as given the policy of future governments and that households will behave competitively. The government is benevolent and thus will choose its policy so as to maximize the discounted utility of the representative household.

Time-inconsistency problems arise when the strategies of future governments are taken into account by the current government when solving for the optimal policy. Since bygones are bygones, future governments will not internalize this, just as the current government does not consider how its policy affected past decisions. Thus, if the current government were to decide all future actions today, it would change its mind in the future about what policy to implement.

Government policy is characterized using Markov strategies, which are time-consistent by construction. Markov strategies depend only on pay-off relevant variables, in this case the state variable, B.

From equations (8) and (10) we can see that the time-inconsistency comes from two sources, the price level and the price of bonds (the latter through the marginal utilities of consumption and leisure tomorrow, which due to (1) and (12) depend on tomorrow's price level). Both variables depend on the price level tomorrow, which in turn depends on tomorrow's government policy, i.e., they depend on $\mathcal{P}(B')$. But the government tomorrow will not take into account that its policy affects prices today and thus the time-inconsistency.

From the aggregate resource constraint (1) and the cash-in-advance constraint (12) we have that if we know p, then we know consumption ($c = 1/p$) and leisure ($\ell = 1 - 1/p - g$). Putting together the first-order conditions of the household (8) and (10), and the government budget constraint (5) gives μ as a function of B, B', p and $\mathcal{P}(B')$

$$\mu = \frac{\beta u_c'(1+B)}{\beta(u_c' + u_\ell' B') - u_\ell \mathcal{P}(B') g} - 1,$$

so that it can be taken out from the government's problem. Moreover, we can now summarize the first-order conditions of the household in a single equation

$$-u_\ell \left(\frac{1+B}{p} + g \right) + \beta \, \frac{u_c' + u_\ell' B'}{\mathcal{P}(B')} = 0. \tag{13}$$

Call the left hand side $\eta(B, B', p, \mathcal{P}(B'))$. Note that (13) has to be satisfied in any competitive equilibrium, i.e., for any equilibrium debt function $\mathcal{B}(B)$, the equilibrium price function $\mathcal{P}(B)$ has to satisfy

$$\eta(B, \mathcal{B}(B), \mathcal{P}(B), \mathcal{P}(\mathcal{B}(B))) = 0.$$

One way to write the problem of the government recursively is to have it choose B' and p, given B and $\mathcal{P}(B')$ and subject to (1), (12) and (13). As mentioned above, the function $\mathcal{P}$ is an equilibrium object that refers to the price induced by the policy adopted by governments. The current government is of course not constrained to satisfy $\mathcal{P}$, but will do so in equilibrium[9].

Given the perception that future governments will induce $\mathcal{P}(B)$, the problem of the current government is

$$\mathcal{V}(B) = \max_{B',p} u\left(\frac{1}{p},\ 1 - \frac{1}{p} - g \right) + \beta\ \mathcal{V}(B')$$

subject to

$$\begin{aligned} \eta(B, B', p, \mathcal{P}(B')) &= 0 \\ u_c - u_\ell &\geq 0. \end{aligned}$$

A Markov-Perfect equilibrium is then a set of functions $\{\mathcal{V}, \mathcal{B}, \mathcal{P}\}$ that solves the above problem. The inequality constraint comes from the first-order condition of the household and has to be satisfied in any competitive equilibrium (otherwise, the nominal interest rate is negative and the household has arbitrage opportunities). The problem of the government can be solved numerically as outlined in the appendix. The presence of $\mathcal{P}(B')$ in the constraint makes the problem atypical. In particular, if we were to take the first-order conditions to the problem of the government, we would have to deal with the derivative of the price function (more on this below). The method proposed in the appendix approximates the equilibrium functions globally.

It is possible to greatly simplify the problem by assuming a suitable functional form for the utility function. This will allow us to build some intuition and get some analytical results. We can then verify numerically that the results hold for more general utility functions.

[9]The government's problem is like solving its best response to the strategies of future governments.

Suppose the utility function is

$$u(c,\ell) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} + \gamma\ell & \text{if} \quad \sigma \neq 1 \\ \\ \log(c) + \gamma\ell & \text{if} \quad \sigma = 1 \end{cases} \tag{14}$$

Since now $u_\ell = \gamma$[10], equation (13) gives a closed-form solution for p as a function of B, B' and $\mathcal{P}(B')$. After some rearrangement we get the following consumption function (which by the cash-in-advance constraint is the inverse of p)

$$\mathcal{C}(B, B', \mathcal{P}(B')) = \frac{\beta(\mathcal{P}(B')^\sigma + \gamma B') - \gamma g \mathcal{P}(B')}{\gamma(1+B)\mathcal{P}(B')}. \tag{15}$$

So, given some $\mathcal{P}(B)$ and assuming the inequality constraint does not bind, the problem of the current government can be simply written as follows

$$\mathcal{V}(B) = \max_{B'} \ u\left(\mathcal{C}(B, B', \mathcal{P}(B')),\ 1 - \mathcal{C}(B, B', \mathcal{P}(B')) - g\right) + \beta\mathcal{V}(B').$$

Taking the first-order condition gives

$$(u_c - u_\ell)\ (\mathcal{C}_{B'} + \mathcal{C}_{p'}\mathcal{P}'_B) + \beta\ \mathcal{V}'_B = 0.$$

Apply the envelope theorem

$$\mathcal{V}_B = (u_c - u_\ell)\ \mathcal{C}_B,$$

and, after updating $\mathcal{V}_B$, get

$$(u_c - u_\ell)\ (\mathcal{C}_{B'} + \mathcal{C}_{p'}\mathcal{P}'_B) + \beta\ (u'_c - u'_\ell)\ \mathcal{C}'_B = 0. \tag{16}$$

This equation is known as the Generalized Euler Equation (GEE). What makes it special is that after applying the envelope theorem, we do not get rid of the derivatives of all choice variables. In particular, the GEE has $\mathcal{P}'_B$, which is the derivative of $\mathcal{P}(B')$ with respect to B'. This makes the problem atypical since now in steady state there is one more unknown than equations. The appearance of $\mathcal{P}'_B$ in the GEE reflects the time-inconsistency problem. The current government takes into account how its actions affect the policy choices of future governments. One would expect this effect to be washed-off by the envelope condition; but this is not the case when there is lack of commitment and time-consistency problems, since governments do not take into account how their actions affect previous governments' actions.

The GEE basically states the intertemporal trade-off faced by the government: equating the marginal effects today and (present value) tomorrow of changing the current debt-to-money ratio. In other words, the GEE shows the trade-off between current and future distortions.

Let's look at the GEE in more detail. The gap between marginal utilities is the size of the distortion created by government policy. This gap is equal to zero only if the government deflates prices such that the nominal interest rate is zero (the Friedman rule). The term $\mathcal{C}_{B'} + \mathcal{C}_{p'}\mathcal{P}'_B$ represents the effect of a change in debt on current private consumption. Note that it includes the

[10] Note that this implies $r = \frac{1}{\beta} - 1$, i.e., the government cannot affect the real interest rate.

effect of a change in tomorrow's price level. The term $\mathcal{C}'_B$ shows the partial effect of a change in debt today on tomorrow's consumption. The GEE only includes this partial effect, since the effect of a change in B' on B'' is taken care of by the envelope condition.

Typically, because the government prints money at a rate higher than the Friedman rule, the gap between marginal utilities, $u_c - u_\ell$, is strictly positive. Hence, a positive effect on private consumption today will be offset by lower consumption tomorrow, and viceversa. For example, if the government decides to increase the debt-to-money ratio, the real price will be lower and hence, consumption will be higher. Tomorrow, since there is more debt, the government has to raise more revenue, which implies a higher distortion, and hence, lower consumption. Thus, the government basically decides whether it increases private consumption today at the expense of lower future consumption or it lowers current consumption to induce higher consumption from tomorrow on.

Printing money increases the price level and this has two effect. One, the inflation tax, is distortive and acts as a consumption tax. The other effect, the reduction in the real value of debt, is non-distortive. The reason for this is that bond holdings at the beginning of the period are inelastically supplied. Thus, the current government views taxing these holdings, i.e., reducing their real value through an increase in the price level, as non-distortive. The non-distortive effect is similar, in this sense, to a capital levy. Note however, that the government will view future increases in prices as purely distortive.

Debt increases or decreases depending on which effect dominates. The key insight here is that as debt increases, so do the gains from reducing it, since the non-distortive effect gains weight. Thus, how the two effects play out against each other depends on how distortive the inflation tax is. If goods bought with money have close substitutes, then the distortion of the inflation tax is low and hence, there is a large incentive to reduce debt. The opposite happens if these goods are difficult to substitute. It is important to point out, that this argument still applies if we add labor taxes.

A positive theory of government debt should have the incentive to delay taxation dominating for low levels of debt and the incentive to reduce debt dominating for high levels of debt. As mentioned above, as debt increases so does the incentive to reduce it. Thus, to deliver a positive —potentially large— long-run level of debt, one would have to insure that the inflation tax is sufficiently distortive. The following proposition establishes the predictions of the model for long-run debt.

Proposition 1 *If the utility function is as in* (14) *then, for a corresponding equilibrium, there exist two steady states that satisfy the GEE* (16). *One is the first-best and the other has* $B^* > 0$ *if* $\sigma > 1$ *or* $B^* = 0$ *if* $\sigma = 1$ *or* $B^* < 0$ *if* $\sigma < 1$.

Proof. In steady state, the GEE simplifies to

$$(u_c - u_\ell)\ (\mathcal{C}_{B'} + \mathcal{C}_{p'}\mathcal{P}_B + \beta\mathcal{C}_B) = 0.$$

Hence, either $u_c = u_\ell$ or $\mathcal{C}_{B'} + \mathcal{C}_{p'}\mathcal{P}_B + \beta\mathcal{C}_B = 0$.

$u_c = u_\ell$ gives the first-best, i.e., the Pareto optimum.

To find the distortionay steady state B^*, first get

$$\begin{aligned}\mathcal{C}_B &= \frac{-1}{(1+B)p}\\ \mathcal{C}_{B'} &= \frac{\beta}{(1+B)p'}\\ \mathcal{C}_p &= \frac{\beta}{(1+B)p'^2}\left(\frac{1-\sigma}{\gamma}p'^{\sigma}+B'\right).\end{aligned}$$

Hence, any steady state —other than the Pareto— has to satisfy

$$\frac{\beta}{(1+B)p}-\frac{\beta\,\mathcal{P}_B}{(1+B)p^2}\left(\frac{1-\sigma}{\gamma}p^{\sigma}+B\right)-\frac{\beta}{(1+B)p}=0,$$

which can be rearranged to

$$\frac{\beta}{(1+B)p}\left[1-\frac{\beta\mathcal{P}_B}{p}\left(\frac{1-\sigma}{\gamma}p^{\sigma}+B\right)-1\right]=0.$$

Since $\frac{\beta}{(1+B)p}\neq 0$, and $\mathcal{P}_B=0$ cannot happen in steady state[11], we have

$$B^*=\frac{\sigma-1}{\gamma}p^{*\sigma},$$

where, from (15), p^* solves

$$1+p^*g-p^{*\sigma}\left[\frac{(1-\sigma)(1-\beta)+\beta}{\gamma}\right]=0. \qquad (17)$$

Given that $p>0$ in any competitive equilibrium, we have $B^*>0$ if $\sigma>1$, $B^*=0$ if $\sigma=1$ or $B^*<0$ if $\sigma<1$. ■

Equation (17) restricts the values of parameters that are compatible with the existence of an equilibrium with a distortionary steady state. Note that $p^*=0$ is never a solution.

The first-best implies $p=\gamma^{\frac{1}{\sigma}}$, $\mu=\beta-1$ and $q=1$ (i.e., the Friedman rule). The debt-to-money ratio at the Pareto optimum is

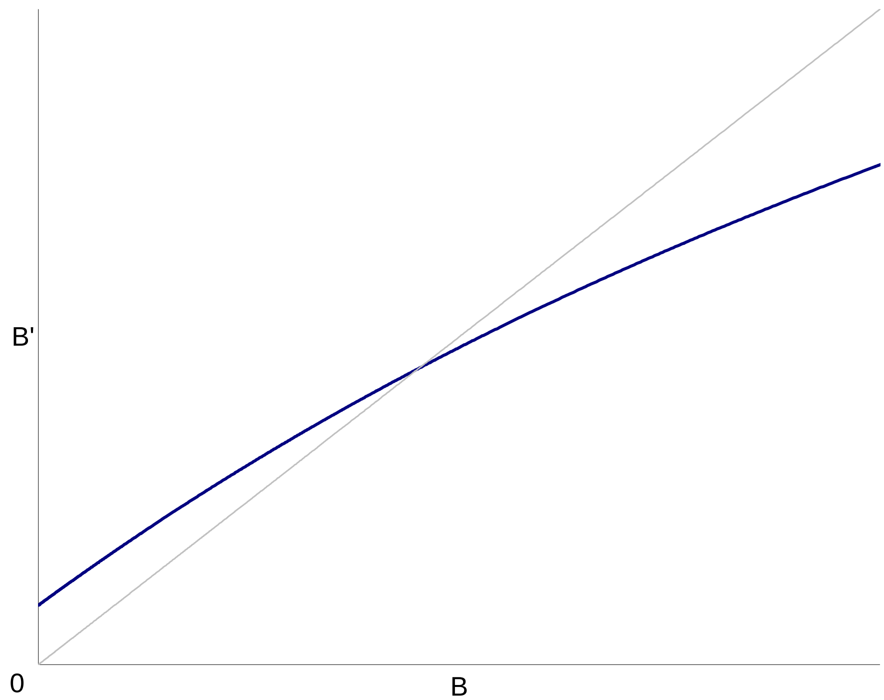
$$B^{PO}=-1-\frac{g\gamma^{\frac{1}{\sigma}}}{1-\beta}$$

which is less than minus one if there is positive government expenditure. So the first-best has negative nominal debt larger —in absolute value— than the money stock. Having large enough positive claims on the private sector allows the government to finance its expenditure, while deflating such that the nominal interest rate is zero. It is possible to verify numerically that the Pareto optimum is an unstable steady state.

On the other hand, we can verify numerically that the second-best steady state is stable. Hence, *Proposition 1* shows that if the utility function is logarithmic in consumption (i.e., $\sigma=1$) then

[11] $\mathcal{P}_B=0$ would imply locally exploding debt, since it implies that whatever the current government does, will have no effect on future governments' policy.

Figure 1: Debt Function $\mathcal{B}(B)$



governments with positive debt will gradually eliminate it. This is the result obtained by Díaz-Giménez, Giovannetti, Marimón and Teles (2004). More importantly, the proposition shows that given a low enough intertemporal elasticity of substitution, the long-run level of government debt will be positive. Figure 1 shows the solution for $\mathcal{B}(B)$. See the appendix for an explanation of how it was computed.

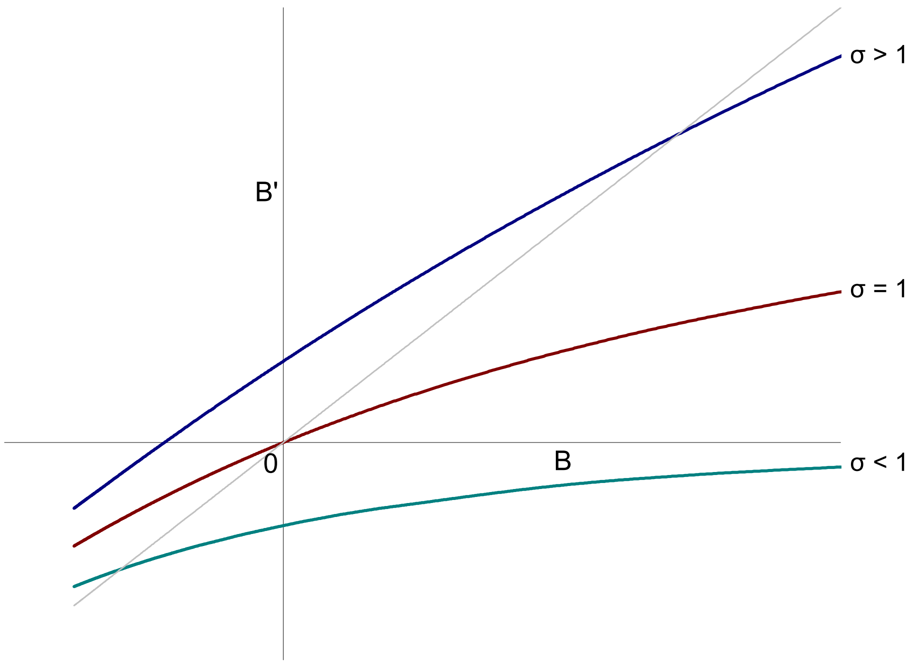
We can also verify numerically that the result of *Proposition 1* holds as long as we assume that consumption and leisure are separable, i.e., it is not necessary to assume linear utility in leisure. Figure 2 shows the case where the utility function is logarithmic in leisure. The three debt functions correspond to different intertemporal elasticities of substitution in consumption. The parameter γ is chosen so that that hours worked —and hence the relative size of government expenditure— are the same in all three steady states.

With lower intertemporal elasticity of substitution than log-utility, current consumption becomes less elastic, making it more costly for the government to reduce current consumption in exchange for higher future consumption, i.e., reducing the incentive to inflate prices today to reduce the real value of debt. Hence, at least for low levels of debt, the government has an incentive to increase it.

Another interesting exercise is to verify numerically what happens to long-run debt when one increases the curvature of leisure. Say

$$u(c, \ell) = \frac{c^{1-\sigma}}{1-\sigma} + \gamma \frac{\ell^{1-\chi}}{1-\chi}.$$

Figure 2: Debt Function for $u(c,\ell) = \frac{c^{1-\sigma}}{1-\sigma} + \gamma \log \ell$



Here, $\chi = 0$ corresponds to the linear case, $\chi = 1$ to the logarithmic case. As χ increases, the long-run level of debt decreases[12]. This is in sharp contrast with what happens as we increase the curvature in consumption. The reason for this is that if households do not like to substitute labor intertemporally, then the motive for delaying taxation diminishes.

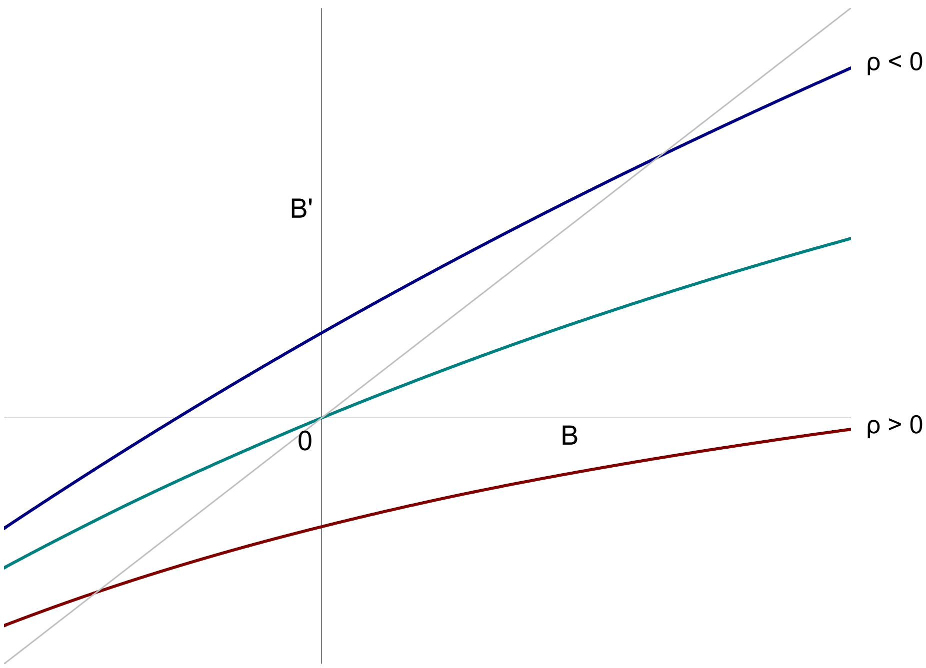
It is important to remember that the factor that enables the model to deliver a positive level of long-run debt, is not the intertemporal elasticity of substitution in consumption *per se*, but rather how distortive inflation is, i.e., how difficult it is for the household to substitute current consumption for another good. In the case of the separable utility function, the relevant trade-off for the household is consumption today versus consumption tomorrow. If the utility function is non-separable and of the constant elasticity of substitution (CES) class, then the critical trade-off will be between current consumption and leisure. The elasticity of intertemporal substitution in consumption still matters, but the effect is small. Say for example that

$$u(c,\ell) = \frac{(\alpha c^{\rho} + (1-\alpha)\ell^{\rho})^{\frac{1-\sigma}{\rho}} - 1}{1-\sigma}.$$

In this case, the critical parameter will be ρ. To get a large, positive level of long-run debt, ρ will have to be negative, i.e., the elasticity of substitution between consumption and leisure has to be less than one, meaning the goods are complements. In this way, the household finds it difficult to substitute consumption for leisure and inflating prices becomes costly, thus reducing the government's motive to decrease the debt. The intertemporal elasticity of substitution still plays a

[12] Moreover, steady state debt approaches zero as χ approaches infinity.

Figure 3: Debt function for $u(c,\ell) = \frac{(\alpha c^{\rho} + (1-\alpha)\ell^{\rho})^{\frac{1-\sigma}{\rho}} - 1}{1-\sigma}$



role, although its effects are second-order. Thus, if σ is larger than one, it is possible to get positive long-run debt even if consumption and leisure are substitutes (i.e., $\rho > 0$). In this case however, debt would be small. Figure 3 shows long-run debt for the cases of $\rho < 0$ and $\rho > 0$, when $\sigma > 1$. It also shows the case of $\rho > 0$, but low enough so that long-run debt is zero. As before, γ is set so that all steady states have the same hours worked.

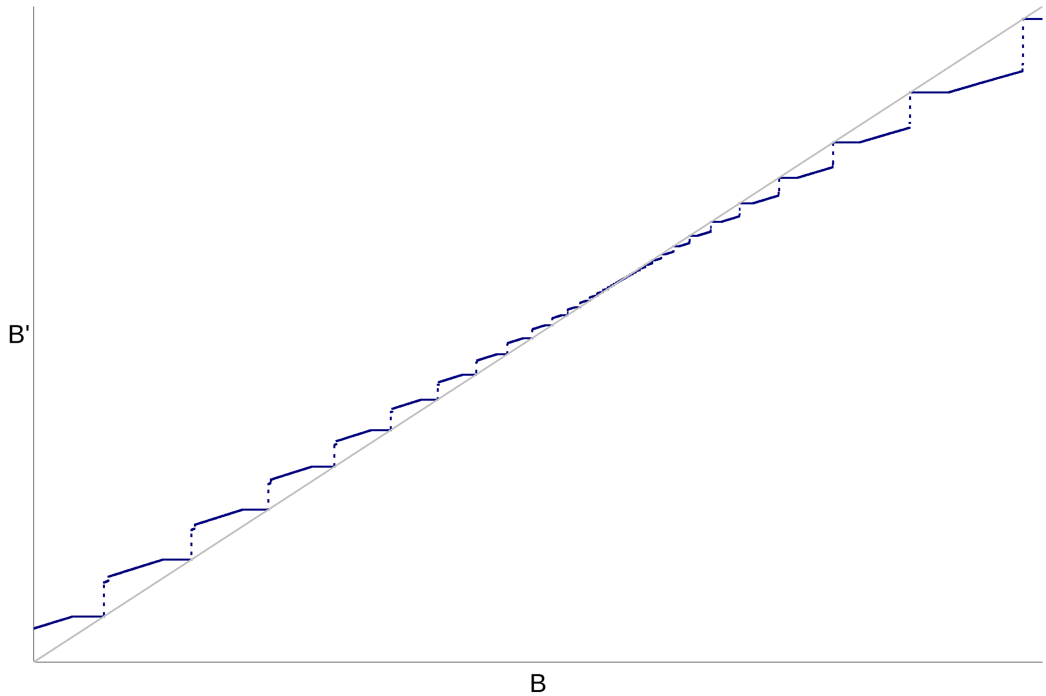
2.4 Multiple equilibria

So far the assumption has been that the solution is differentiable. As it turns out, there is another solution for $\mathcal{B}(B)$ that is discontinuous. This solution solves the government problem but does not satisfy the GEE with equality. Krusell and Smith (2003) and Krusell, Martin and Ríos-Rull (2004) also find co-existence of continuous and discrete solutions (in the case of the latter, only for negative debt). This seems to be a recurring feature in this class of models.

The discrete solution looks like a step function (see Figure 4). For certain neighborhoods of debt levels, the government chooses the same level for tomorrow. For particular levels of debt, the government's decision rule is discontinuous, i.e., it decides to increase or decrease debt suddenly by a large amount. At some intervals, the solution is differentiable and not flat. Here, the GEE is satisfied with equality.

The non-differentiable equilibrium is an artifice of the infinite horizon since it is not the limit

Figure 4: Discrete Debt Function



of finite horizon economies[13]. It is also an artifice of the recursive representation of the model. Basically, the functional representation of the problem of the government has "space to spare", in the sense that we can construct equilibria that are not the limit of finite horizon equilibria. We can use this as a selection device to prefer the differentiable equilibrium over the non-differentiable.

The discount factor plays an important role in how the equilibrium looks like. If β is high enough, then the equilibrium has infinitely many —but countable— steady states. This is the case shown in Figure 4. If β is low enough then we still get a step function, but one that does not touch the 45-degree line except at the smooth equilibrium steady state.

How is this solution found? Start by identifying the long-run level of debt of the differentiable equilibrium, B^*. Next, create a grid with n points of debt, where $x_i, i = 1, \ldots, n$ refers to grid point i. Then let $x_n = B^*$ and choose a lower bound (say $x_1 = 0$).

Since x_n is a steady state, we know the values of $\mathcal{V}(x_n)$ and $\mathcal{P}(x_n)$. Next, pick x_{n-1} and let p^*_{n-1} be the real price level if x_{n-1} was a steady state. Then check which of the following expressions is higher:

$$u\left(\mathcal{C}(x_{n-1}, x_n, \mathcal{P}(x_n)),\ 1 - \mathcal{C}(x_{n-1}, x_n, \mathcal{P}(x_n)) - g\right) + \beta\mathcal{V}(x_n)$$

or

$$u\Big(\mathcal{C}(x_{n-1}, x_{n-1}, p^*_{n-1}),\ 1 - \mathcal{C}(x_{n-1}, x_{n-1}, p^*_{n-1}) - g\Big) + \beta\mathcal{V}(x_n).$$

This will tell whether the government prefers to stay at x_{n-1} or increase debt to x_n. Note

[13]To consider finite horizon versions of the model one would have to assume some appropriate terminal conditions, so that the price level does not go to infinity in the last period.

that because of monotonicity of the solution, it is not necessary to check whether the government wanted to increase debt beyond B^*. Next, we move to x_{n-2} and compare the values of staying at x_{n-2} or increasing debt to x_{n-1} or x_n. Continue this process for all remaining grid points.

After solving to the left of B^*, we proceed to solve to the right of it, using a similar technique. Note that the whole procedure did not involve a single iteration, hence the only numerical errors come from the size of the grid.

To verify that the obtained solution $\{\mathcal{V}, \mathcal{B}, \mathcal{P}\}$ is indeed an equilibrium, we check the one-shot deviation to the solutions found, i.e., for every B, we solve the government's problem given $\mathcal{P}(B')$ and $\mathcal{V}(B')$.

2.5 The commitment case

Let's show that the assumption about whether there is commitment or not, is critical. The argument is that if the government could commit to all its future policy choices then it would change the debt in the first period and then never again. Thus with commitment, the model does not provide any predictions for long-run debt, as it would depend entirely on the initial level of debt.

Say the utility function is as in (14) and with $\sigma = 1$. Combining and rearranging the two first-order conditions of the household, the cash-in-advance constraint and the government budget constraint delivers

$$c = \frac{\frac{\beta}{\gamma} + \beta B' c' - g}{1 + B} \tag{18}$$

and $\mu = \frac{\beta}{\gamma c} - 1$. The Ramsey problem would normally be solved by finding the Ramsey allocation, i.e., finding $\{B\}_{t=0}^{\infty}$ that given B_0 maximizes the discounted utility of the household. This is a bit problematic since the expression for consumption at t —given by equation (18)— is a function of consumption at $t+1$.

As it turns out, a simple transformation will allow the problem to be posted recursively. Let

$$\tilde{B} \equiv Bc$$

and thus

$$c = \frac{\beta}{\gamma} + \beta \tilde{B}' - \tilde{B} - g.$$

The government's problem can then be written as a two-stage problem. The second-stage is a standard dynamic programming problem with $\tilde{B}$ as the state variable and gives the solution for $t = 1, \ldots, \infty$. The first-stage is the problem of choosing variables at time zero.

The second-stage problem is

$$W(\tilde{B}) = \max_{\tilde{B}'} u(c,\ 1 - c - g) + \beta W(\tilde{B}')$$

subject to

$$c = \frac{\beta}{\gamma} + \beta \tilde{B}' - \tilde{B} - g.$$

Taking the first-order condition and applying the envelope theorem gives

$$\beta(c^{-1} - \gamma) - \beta(c'^{-1} - \gamma) = 0,$$

which implies that $c = c'$ for every $\tilde{B}$. Hence, the solution is $\tilde{B}' = \tilde{B}$, which in turn implies $B' = B$, i.e., the government keeps the debt constant.

The first-stage problem is

$$\max_{\tilde{B}_0',\ \tilde{B}_1'} u(c_0,\ 1 - c_0 - g) + \beta W(\tilde{B}_1')$$

subject to

$$\begin{aligned} c_0 &= \frac{\beta}{\gamma} + \beta \tilde{B}_1' - \tilde{B}_0 - g \\ \tilde{B}_0 &= B_0 c_0. \end{aligned}$$

A government with initial positive debt always reduces it in the first period. If σ was larger than 1, then we would have something similar as in the case of the government without commitment: the government increases the debt for low levels of debt and decreases it for high levels. Nevertheless, the solution for the second-stage problem is the same, i.e., the government does not change the level of debt after the initial period.

3 Quantitative Evaluation of the Model

The model outlined in the previous section is useful to show how we can construct a theory of debt, but it is a bit too simple if we are interested in testing it quantitatively to see how good a theory we have. One important omission is fiscal policy. Unfortunately, one cannot just include labor taxes to the simple model. The reason for this is that labor taxes and inflation would distort the same margin, since from the aggregate resource constraint consumption and labor differ only by a constant[14]. A Markov government would tax the good with a higher base (labor in this case) while subsidizing the other. Why? Because in this way it wants to have both taxes behave as one lump-sum: by taxing more than it needs, it can give part of it back to offset the disincentive to work. The problem with this is that in equilibrium, the government will set the labor tax rate as high as possible and the inflation rate as low as possible, so as to minimize the distortion. In the absence of bounds, the money growth rate goes to the Friedman rule. With bounds, we get a corner solution[15].

To prevent the above we need to have government policy distort two different margins. A simple way to do it is to have credit goods, i.e., goods that are not purchased with money. The aggregate resource constraint then becomes

$$c_1 + c_2 + g = n, \tag{19}$$

[14]Making g endogenous does not solve this problem since the household takes it parametrically, i.e., as if it were a constant.

[15]For an example of this in a real economy with labor and capital incomes taxes, see Martin (2004).

where c_1 is consumption of the cash good and c_2 is consumption of the credit good. The introduction of labor taxes and credit goods does not significantly modify the environment of the basic model. The economy is now described by (19) and the following equations

$$\begin{aligned} g &= \tau n + \frac{(1+\mu)(1+qB') - (1+B)}{p} \\ c_1 &\leq \frac{1}{p} \\ c_1 &= (1-\tau)n + \frac{m + b - (1+\mu)(m' + qb')}{p} - c_2, \end{aligned}$$

which are the government budget constraint, the cash-in-advance constraint and the household's budget constraint, respectively.

3.1 The private sector

Given some utility function $u(c_1, c_2, \ell)$ satisfying the usual assumptions, we can solve for the competitive equilibrium. The first-order conditions of the household's problem are

$$(1-\tau)\, u_{c_2} = u_\ell \tag{20}$$

$$u_{c_1} - u_{c_2} = \lambda \tag{21}$$

$$u_{c_2} = \frac{\beta u'_{c_1}}{1+\pi} \tag{22}$$

$$q = \frac{u'_{c_2}}{u'_{c_1}}, \tag{23}$$

where λ is the Lagrange multiplier of the cash-in-advance constraint. In addition, the cash-in-advance constraint binds, and so

$$c_1 = \frac{1}{p}. \tag{24}$$

3.2 The problem of the government

From (19) and (24) we can write

$$c_2 = n - \frac{1}{p} - g.$$

Next, we use (20) to have

$$\tau = 1 - \frac{u_\ell}{u_{c_2}}$$

and (22) to get

$$\mu = \frac{\beta u'_{c_1} p}{u_{c_2} p'} - 1.$$

Hence, we can write the government budget constraint as a function of n, $n' = \mathcal{N}(B')$, p, $p' = \mathcal{P}(B')$, B and B', i.e.,

$$\left(1 - \frac{u_\ell}{u_{c_2}}\right) n + \left[\frac{\beta u'_{c_1}}{u_{c_2} p'} \left(1 + \frac{u'_{c_2}}{u'_{c_1}} B'\right) - \frac{1+B}{p}\right] - g = 0.$$

Call the left hand side of this equation $\eta(B, B', n, p)$. In contrast to Section 2, the notation here has been abbreviated, as n' and p' depend on B'. We also need to include the inequality constraint implied by the household's first-order condition (21), i.e.,

$$u_{c_1} - u_{c_2} \geq 0.$$

Call the left hand side of this equation $\varepsilon(n, p)$.

So, given the perception that future governments will induce $\mathcal{N}$ and $\mathcal{P}$, the problem of the government can be written as

$$\mathcal{V}(B) = \max_{n,p,B'} \; u\left(\frac{1}{p},\; n - \frac{1}{p} - g,\; 1-n\right) + \beta\; \mathcal{V}(B').$$

subject to

$$\begin{aligned} \eta(B, B', n, p) &= 0 \\ \varepsilon(n, p) &\geq 0. \end{aligned}$$

A Markov-perfect equilibrium is a set of functions $\{\mathcal{V}, \mathcal{B}, \mathcal{N}, \mathcal{P}\}$ that solves the above problem. After taking the first-order conditions and applying the envelope theorem we get the following static and dynamic GEEs (assume for now that the inequality constraint does not bind)

$$\begin{aligned} \frac{(u_{c_1} - u_{c_2})}{p^2 \eta_p} - \frac{(u_\ell - u_{c_2})}{\eta_n} &= 0 \\ (u_\ell - u_{c_2})\frac{\eta_{B'}}{\eta_n} + \beta(u'_\ell - u'_{c_2})\frac{\eta'_B}{\eta'_n} &= 0. \end{aligned}$$

If on the other hand, the inequality constraint binds then we have

$$\begin{aligned} u_{c_1} - u_{c_2} &= 0 \\ (u_\ell - u_{c_2})\xi + \beta(u'_\ell - u'_{c_2})\xi' &= 0. \end{aligned}$$

where

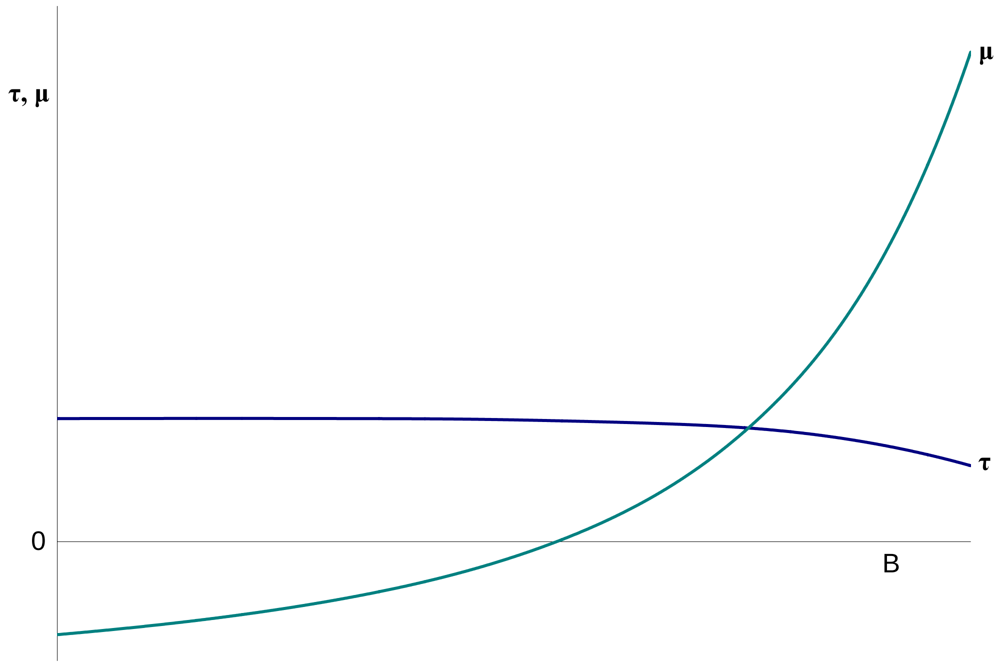
$$\xi = \frac{p^2 \varepsilon_p \; \eta_{B'}}{\eta_p \; \varepsilon_n - p^2 \varepsilon_p \; \eta_n}.$$

Note that $\eta_{B'}$ contains both $\mathcal{N}'_B$ and $\mathcal{P}'_B$ and so this is not a standard recursive problem. To solve it numerically, use the procedure from the appendix.

3.3 A qualitative analysis of the solution

Before attempting a calibration of the model it is useful to analyze it qualitatively. As in the case of the simpler model of the previous section, the typical debt function $\mathcal{B}(B)$ is increasing and has a stable steady state that is not the first-best. What is more interesting is how labor taxes and the money growth rate interact with each other. As Figure 5 shows, when debt is positive, both tax instruments are substitutes: the labor tax is decreasing in debt and the money rate is

Figure 5: Labor Tax and Money Growth Rate



increasing in debt. The reason for this is that the government —instead of distorting all margins a little— taxes the margin with the highest return and tries to minimize the distortion of the other margin. When debt is low, consumption of the cash good and labor are both high, but labor has the biggest tax base. Hence, the government taxes labor heavily and runs a disinflation so as to minimize the cash-in-advance distortion. When debt is high, it is still true that labor is higher than consumption of the cash good, but in this case inflating prices has an added bonus: it decreases the real value of government debt. This incentive makes inflation a more attractive source of funds and so the government inflates prices while it reduces the labor tax significantly (and may even subsidize labor). So, with higher debt there is a shift from labor income taxation to inflation. This policy scheme implies that inflation and the nominal interest rate are increasing functions of debt, while the real interest rate is a decreasing function of debt.

The steady state level of debt depends again on the appropriate elasticity of substitution. Take for example a utility function that is separable and exhibits a constant intertemporal elasticity of substitution in the cash good, say

$$u(c_1, c_2, \ell) = \alpha \frac{c_1^{1-\sigma_1}}{1-\sigma_1} + (1-\alpha)\frac{c_2^{1-\sigma_2}}{1-\sigma_2} + \gamma\ell.$$

In this case, the steady state level of debt (call it B^*) will depend heavily on σ_1. We can verify numerically that, as in the case of the simple model of the previous section, if $\sigma_1 > 1$ then $B^* > 0$, if $\sigma_1 = 1$ then $B^* = 0$ and if $\sigma_1 < 1$ then $B^* < 0$.

If the utility function is non-separable and exhibits a constant elasticity of substitution between the cash and the credit good, as in

$$u(c_1, c_2, \ell) = \frac{\left[(\alpha c_1^\rho + (1-\alpha)c_2^\rho)^{\gamma/\rho}\ \ell^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma}, \tag{25}$$

then the steady state level of debt will depend critically on ρ. In this case, to get a large, positive steady state level of debt, ρ will have to be negative, i.e., cash and credit goods have to be complements. The reason for this is similar to that outlined in the previous section. If the household can easily substitute cash goods for credit goods, then inflation becomes a cheap —in terms of distortions— source of funds for the government and the gains from reducing the debt increase.

3.4 Calibration

The next step is to calibrate the model so that its steady state matches selected statistics of the U.S. economy. The first thing to do is choose a functional form for the utility function, so that we know all the parameters involved in the exercise. The choice is (25), since it makes it easy to match parameters with targets and allows to target inflation and the level of debt separately. This particular functional form exhibits a constant elasticity of substitution between the cash and the credit good and is Cobb-Douglas (and thus unit elastic) between $(\alpha c_1^\rho + (1-\alpha)c_2^\rho)^{1/\rho}$ and leisure. As noted above, the key parameter to determine the size and sign of debt in steady state is ρ.

The parameters to calibrate are then α, β, γ, ρ, σ and g. The selected target statistics are B/py, c_1/c_2, g/y, n, π and R —the nominal interest rate— in the U.S. for the period 1962-2001.

In models with a cash-in-advance constraint, the period length is usually short —either a month or a quarter— to account for the counterfactual implication that the velocity of money in terms of consumption is equal to one. Then we would determine what cash goods are, through a suitable choice of a monetary aggregate. Next, α and ρ would be pinned down by a regression using (23) (see Chari, Christiano and Kehoe, 1991). This is not the approach followed here. Since the inflation tax plays a role in determining long-run debt, it is important to target its tax base. In this case, the most appropriate monetary aggregate seems to be M_1. The problem is that M_1 is larger than quarterly consumption; hence, the period has to be longer, say a year.

By subtracting the definition of money from consumption of nondurable goods and services we get the amount of credit goods consumed in a year. Then we get our target value for c_1/c_2, which is equal to 0.41 for the period 1962-2001.

To target debt, we can use data from the Congressional Budget Office (2004). The series of debt held by public —i.e., excluding holdings by federal agencies— in terms of GDP, averages 36% for the period 1962-2001. This figure includes the holdings of Federal Reserve Banks, which amount to 5% of GDP. Hence, the target for debt over GDP is 31%.

Data from the Federal Reserve System shows that the nominal interest rate for the 1-year constant maturity Treasury Bill, averaged 6.7% annual. Next, use the annual variation of the consumption of nondurable and services deflator as the measure for inflation, which gives an average of 4.3% annual[16]. This implies a target real interest rate of 2.3% annual.

[16] Using the CPI gives a slightly larger figure, 4.5% annual.

The fraction of time devoted to labor is set to 0.3, as is standard in the business cycle literature. The target for government expenditure over GDP is taken out of data from the Congressional Budget Office. Between 1962 and 2001, outlays of the Federal Government averaged 20% of GDP. Note that this figure includes transfers, both to states and households. Using the more traditional measure for g from the national accounts —which consolidates federal and state expenditure, but does not include transfers to the private sector— gives 20% as well.

Now we need to choose parameter values that make the model match the targeted statistics. The discount factor is easy to calibrate. From equations (22) and (23) we have that in steady state

$$r = \frac{1}{\beta} - 1,$$

and so the value for β consistent with a real interest of 2.3% annual is 0.9775.

Given that the target for hours worked is 0.3 and for g/y is 0.2, g has to be equal to 0.06.

The rest of the parameters have to be fine-tuned through successive iterations, although most of them affect primarily only one statistic.

As described above, ρ is the main parameter determining long-run debt. The value of ρ is set to -3.205. Next, from the price of bonds equation (9) we get the following steady state condition

$$\frac{1}{1+R} = \frac{1-\alpha}{\alpha} \left(\frac{c_1}{c_2} \right)^{1-\rho},$$

which sets the value of α to 0.0245.

Finally, set γ to 0.303 to get $n = 0.3$ and σ to 3.53 to match an annual inflation rate of 4.3%.

Table 1 gives a summary of the parameter values chosen for the calibration exercise. Table 2 shows the steady state statistics of the artificial economy.

Table 1: Parameter values

Parameter	α	β	γ	ρ	σ	g
Value	0.0245	0.9775	0.303	-3.205	3.530	0.060

Table 2: Steady state statistics

Statistic	τ	π	R	n	g/y	c_1/c_2	B/py
Value	0.197	0.043	0.067	0.300	0.200	0.410	0.310

It could seem as rather unfortunate that the model relies so heavily on one parameter —ρ— for its predictions on long-run debt. But it is not clear why other parameters should matter. In this sense, data from the OECD shows that countries with similar levels of government debt, have very different fundamentals (i.e., government expenditure, hours worked, inflation rate, etc.). The model is consistent with this observation. Moreover, what really matters is not so much the elasticity of substitution between cash and credit goods, but how easy or difficult it is for the private sector to substitute away from goods being taxed by inflation. More elaborated models may incorporate additional reasons why inflation is costly and reduce the quantitative role of ρ.

The elasticity of substitution between cash and credit goods implied by the benchmark parametrization is around a quarter. This means that cash and credit goods are (strong) complements. One could argue that these two goods are inherently substitutes. This comment has some merit, but one should also point out that the model is not alone in this respect. First, papers in the inflation cost literature —see Aiyagari, Brown and Eckstein (1998) and Erosa and Ventura (2002)— actually assume perfect complementarity between cash and credit goods. Second, consider that the cash-in-advance constraint implies that consumption of cash goods equals real balances. The money-in-utility function literature assumes complementarity between consumption and real balances (for example, Lucas (2000) sets the elasticity of substitution at 0.5). Hence, the implied behavior between money and consumption is somewhat similar.

3.5 Comparative statics

An interesting exercise to learn how the model behaves is to do some comparative statics. In particular, what happens to the long-run level of debt if we change one of the target statistics? This would allow us to understand what the model predicts for countries with different characteristics or for the same country when there is a permanent change in one of the target statistics. The idea is then to change parameters so that only a single target statistics is changed at a time. However, ρ is left unchanged so that we can verify the effect on debt. Table 3 shows a summary of this exercise.

Table 3: Comparative statics

	Bench mark	$\Delta g/y$	Δn	$\Delta c_1/c_2$	$\Delta\pi$	Δr
α	0.025	0.025	0.025	0.006	0.024	0.025
β	0.978	0.978	0.978	0.978	0.978	0.962
γ	0.303	0.303	0.253	0.303	0.303	0.306
ρ	−3.205	−3.205	−3.205	−3.205	−3.205	−3.205
σ	3.530	2.365	3.742	3.528	2.799	3.782
g	0.060	0.080	0.050	0.060	0.060	0.060
τ	0.197	0.264	0.197	0.199	0.201	0.202
π	0.043	0.043	0.043	0.043	0.024	0.043
R	0.067	0.067	0.067	0.067	0.048	0.085
n	0.300	0.300	0.250	0.300	0.300	0.300
g/y	0.200	0.267	0.200	0.200	0.200	0.200
c_1/c_2	0.410	0.410	0.410	0.295	0.410	0.410
B/py	0.310	0.276	0.308	0.297	0.298	0.317

The first thing is to look at what happens if government expenditure were higher. Let g increase to 0.08, so that government expenditure over GDP increases by a third. To keep the 4.3% annual inflation rate, we need to decrease σ accordingly. The result —compared to the benchmark case—

is an increase in income taxes, from 19.7% to 26.4% and a 3.4 percentage point (or 11%) decrease of the debt over GDP ratio. Hence, a country with higher government expenditure would have less long-run debt. That permanent increases in expenditure are financed with taxes rather than debt, is in accordance with the traditional theories of debt. The way it works in this case is as follows. Higher government expenditure with the same fraction of time spent working implies less consumption. This means that the base for the inflation tax is lower. If debt is kept the same —or at least is not reduced significantly— this implies higher labor taxes. The lower long-run level of debt comes from the fact that the higher tax pressure makes pushing the tax burden to the future —i.e., increasing debt— less attractive. This is so since tax distortions are convex in the tax rate.

It is interesting to note that it is not possible to increase too much the steady state debt by decreasing g. If for example g/y is reduced to 1%, then debt over GDP increases to only 35%[17]. Further reductions of expenditure do not contribute much more.

The next column in table 3, involves reducing the fraction of hours worked to one quarter. Since the relative size of the public sector and the inflation rate are left unaltered, this change has almost no effect on debt.

Looking at data from the U.S. since 1962, we can see a steady reduction of c_1/c_2. This can be attributed to continuous innovations in the financial system, which in general allow households to maintain lower stocks of M_1. A reduction of c_1/c_2 reduces the relative importance of the cash good. Hence, the government can inflate prices more since it is hurting the household less. This should imply a decrease of steady state debt. The impact of a change in c_1/c_2 depends on its size. A moderate decrease —say 10%— achieves very little change in long-run debt. Table 3 shows the case where the ratio is reduced to its 1992-2001 average of 29.5%, almost 30% less than benchmark. In this case, the reduction in long-run debt is around 4%.

Another exercise is to verify the effect of a permanent reduction in the inflation rate. To reduce the inflation rate leaving everything else constant, we need to increase the intertemporal elasticity of substitution, i.e., decrease σ. This raises the tax motive since households are less hurt by variations in consumption over time. In effect, the labor tax function shifts up (the money growth rate also shifts up, but only slightly). This decreases the motive for debt and the debt function shifts down. We get then a decrease in steady state inflation and debt, since we are basically moving down along the money growth rate function (which is increasing in debt). Table 3 shows the case of inflation being reduced to its 1992-2001 average of 2.4%, almost half the benchmark value. The reduction in long-run debt is roughly 3%, which amounts to 1% of GDP.

The final column of Table 3 shows the effect of an increase of the real interest rate, from 2.3% annual to 4.0% annual. This is a substantial change, but only delivers only 2.4% increase in long-run debt over GDP. The reason for the increase in steady state debt is that since households care less about the future, the government has more reason to delay taxation.

The comparative statics exercise shows that permanent changes in target statistics would have a hard time explaining substantial changes in long-run debt. It all comes down again on how easy or difficult it is for the private sector to substitute away from the good being taxed by inflation, i.e., on the value of ρ. The key then seems to be finding ways to "flatten" the money growth rate

[17] In this case, σ is left at its benchmark value, since to get a 4.3 annual inflation rate with such a low government expenditure would imply an extremely low elasticity of intertemporal substitution, which is difficult to solve numerically with accuracy.

function, i.e., reduce the incentive for the government to use the inflation tax when debt becomes large.

3.6 Stochastic government expenditure

Countries frequently increase their public debt when government expenditure temporarily increases. Barro's (1979) tax-smoothing argument works well along this dimension, so it is important to test the model on this.

It is relatively simple to add stochastic government expenditure to the model. Say there is a normal level of expenditure g_1 and a high level of expenditure g_2, which happens only sporadically and does not last for too long. If today we are in the normal expenditure state, then tomorrow the government will have to spend g_1 with probability θ_1 and g_2 with probability $1-\theta_1$. Likewise, if today we are in the high expenditure state, then tomorrow the government will have to spend g_1 with probability $1-\theta_2$ and g_2 with probability θ_2.

If today the exogenous state is g_1 then the first-order conditions of the representative household become

$$(1-\tau_1)u^1_{c_2} = u^1_\ell \tag{26}$$

$$u^1_{c_1} - u^1_{c_2} = \lambda_1 \tag{27}$$

$$p_1 = \frac{u^1_{c_2}(1+\mu_1)}{\beta}\left[\theta_1 \frac{p'_1}{u'^1_{c_1}} + (1-\theta_1)\frac{p'_2}{u'^2_{c_1}}\right] \tag{28}$$

$$q_1 = \theta_1 \frac{u'^1_{c_2}}{u'^1_{c_1}} + (1-\theta_1)\frac{u'^2_{c_2}}{u'^2_{c_1}}, \tag{29}$$

where the "1" or "2" subscripts refer to whether a variable corresponds to state g_1 or g_2, respectively. For functions, use a superscript to denote that they are being evaluated at the corresponding state. Of course, there is also a set of corresponding first-order conditions when the state today is g_2.

As before, it is possible to summarize (27), (28) and (29) into a single equation

$$\eta^1(B, B'_1, n_1, p_1) = 0.$$

This function is now a bit more complicated than before, since it includes n'_1, n'_2, p'_1 and p'_2 (which are all functions of B'). The superscript on the η function indicates what probabilities to use (in this case, θ_1 and $1-\theta_1$).

Now we are ready to write the problem of the government. For expositional simplicity, lets ignore (27) as a constraint, although when solving the model one has to be aware that it may be binding. So, given that the exogenous state is g_1 and that future governments will induce $\mathcal{N}^1$ and $\mathcal{P}^1$ if the state is g_1 and $\mathcal{N}^2$ and $\mathcal{P}^2$ if the state is g_2, the problem of the government is

$$\mathcal{V}^1(B) = \max_{n_1, p_1, B'_1} u\left(\frac{1}{p_1},\ n_1 - \frac{1}{p_1} - g_1,\ 1-n_1\right) + \beta\left[\theta_1 \mathcal{V}^1(B'_1) + (1-\theta_1)\mathcal{V}^2(B'_1)\right]$$

subject to

$$\eta^1(B, B'_1, n_1, p_1) = 0.$$

Whereas if the exogenous state is g_2 then the problem of the government is

$$\mathcal{V}^2(B) = \max_{n_2, p_2, B_2'} \; u\left(\frac{1}{p_2},\; n_2 - \frac{1}{p_2} - g_2,\; 1 - n_2\right) + \beta\left[(1-\theta_2)\mathcal{V}^1(B_2') + \theta_2 \mathcal{V}^2(B_2')\right]$$

subject to

$$\eta^2(B, B_2', n_2, p_2) = 0.$$

Hence, a Markov-perfect equilibrium is a set of functions $\{\mathcal{V}^1, \mathcal{V}^2, \mathcal{B}^1, \mathcal{B}^2, \mathcal{N}^1, \mathcal{N}^2, \mathcal{P}^1, \mathcal{P}^2\}$ that solves the above problem.

For the exercise, lets adopt the same parametrization as in the benchmark calibration plus: $g_1 = 0.06$, $g_2 = 0.08$, $\theta_1 = 0.975$ and $\theta_2 = 0.5$. Hence, the state with normal expenditure has an identical parametrization as the benchmark case. The other state has 33% more expenditure (g/y raises from 20% to 27%), a moderate increase, much smaller —for example— than the experienced in the U.S. during World War II. The probabilities imply that on average, the state with high expenditure occurs once every 40 years and has a duration of 2 years. In sum, it is a state that shows a moderate increase in government expenditure, occurs infrequently and is not expected to last long.

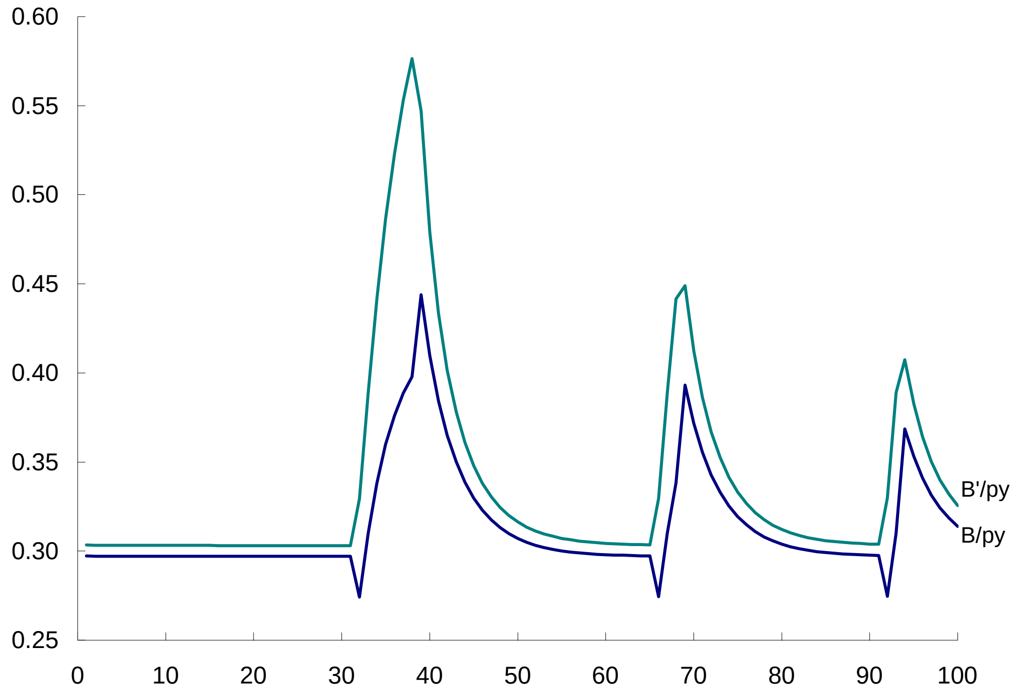
The solution shows that the equilibrium debt function in the high expenditure state ($\mathcal{B}^2$) is always above the one for the normal expenditure state ($\mathcal{B}^1$). The same happens for the tax and money growth rate functions. So, when the economy moves from the low to the high expenditure state, we should expect an increase of B', τ and μ. Interestingly enough, the debt function for the deterministic case lies in between the two debt functions for the stochastic case. When compared to the deterministic case, the government in the normal expenditure state has a lower incentive to have debt, since it internalizes the fact that it will have to distort the economy more when expenditure increases. In this sense, the steady state for g_1 has debt over GDP of almost 30% and an annual inflation rate of around 2%. On the other hand, if the economy stays in the high expenditure state long enough, then debt over GDP climbs to 43%, with an inflation rate of 53% annual. This means that economies with a transitory increase in government expenditure should show a higher debt to GDP ratio. The problem with the model is that it also predicts very high inflation rates. This is due to the fact that the money growth rate function starts to climb very rapidly with high levels of debt.

A simulation of the stochastic model is run for $1,000$ periods. The averages of variables are similar to the steady state statistics of the deterministic case. Figure 6 shows a sample of 100 periods in which the high expenditure state occurs 3 times and lasts 7, 3 and 2 periods, respectively[18]. The figure shows two variables: $\bar{B}/\bar{p}y$ and $\bar{B}'/\bar{p}y$, i.e., beginning and end of period debt over GDP. Regardless of how it is measured, debt to GDP increases more the longer the economy stays in the high expenditure state. It is also clear that debt remains high for several periods after the economy returns to the normal expenditure state. In other words, the expenditure shock has a persistent effect on the level of debt.

The time series for $\bar{B}/\bar{p}y$ shows a decrease for the period in which there is a change from normal to high expenditure. This can also be seen in the data and the reason is that $\bar{B}$ corresponds to

[18] This sample is rather atypical, but was chosen since it shows what happens when the high expenditure state lasts long enough.

Figure 6: Simulated Debt over GDP in a sample of 100 years



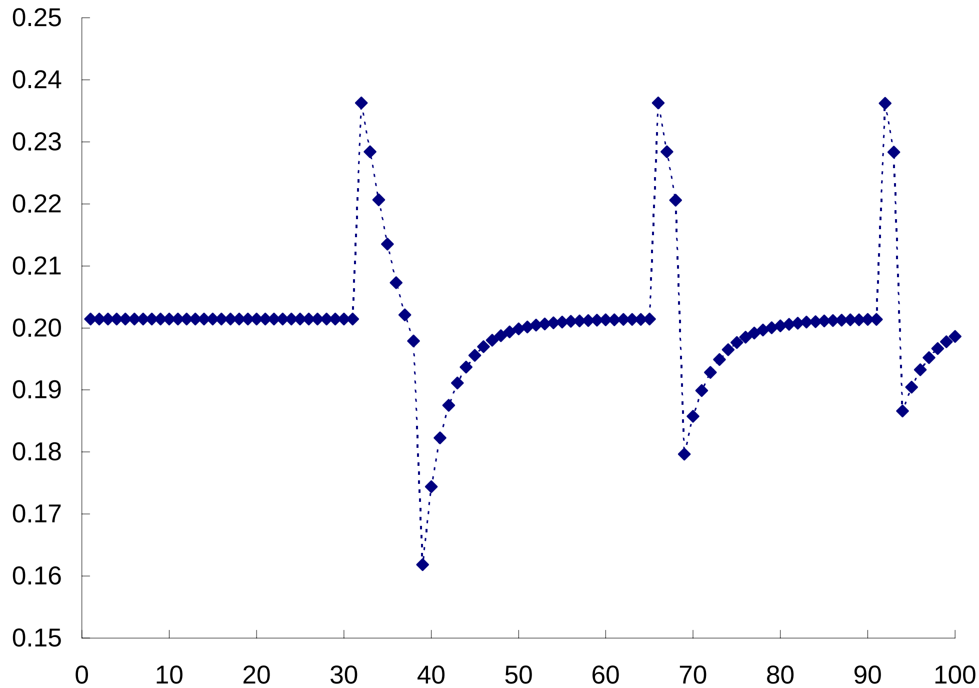
beginning-of-period debt, while prices and hours react to the new state (both increase). If we measured debt over GDP as we do with the data, we would use $\bar{B}'/\bar{p}y$, i.e., end-of-period debt over period GDP. In this case, debt to GDP increases much more, up to almost 60%, and does not show an initial decline. The reason for the large discrepancy between the two series is inflation. This can be seen when the economy is in the normal expenditure state: here the two series differ by only a small amount.

Concurrent with the increase in debt, there is an increase in taxes and a sharp increase in inflation. In the case of this particular sample, labor taxes raise up to 23.6% and inflation increases up to 41.8%. When the economy gets back to the normal expenditure state, taxes and inflation react differently. Labor taxes diminish sharply well below normal expenditure steady state (down to 16.2% after the 7 periods of high expenditure), whereas inflation diminishes gradually towards the normal expenditure steady state. This is just a consequence of labor taxes being decreasing in debt (for a given expenditure state) and the money growth rate being increasing. This dynamics is in sharp contrast with the tax-smoothing argument. Figure 7 shows the evolution of labor taxes for the particular sample.

It is possible to increase debt in the high expenditure state by modifying some of the parameters. In this sense, one would need to do any of the following: lower g_1, raise g_2, raise θ_1 or lower θ_2.

Both lowering g_1 or raising g_2 increases the distance between the debt functions of the two states, i.e., lowers the incentive to issue debt in the normal expenditure state and increases it in the high expenditure state. The effect is small for the former and large for the latter. For example, if g_1 is set to 0.05 —which implies $g/y = 16.7\%$— then the debt to GDP ratio in the long-run for the

Figure 7: Simulated Labor Taxes in a sample of 100 years



high expenditure state increases to 48%. With a lower g_1, the government with high expenditure can increase the debt more since it knows that when expenditure returns to normal, the government will not have to distort as much as before. On the other hand, if we increase g_2 then, since it is only temporary, the government in the high expenditure state prefers to run a higher deficit so as not to distort the economy too much. The effects are similar in magnitude as in the case of lowering g_1, although the effect on debt for the normal expenditure case is more noticeable and inflation in the high expenditure case is a lot higher. Both modifications indicate that if governments in normal times expect high increases of expenditure in the future, then their debt policy will be more conservative so as to lower the distortions necessary to finance the temporarily higher expenditure.

Raising θ_1, which means making the high expenditure state less frequent, increases the debt function in both states, but only slightly. In the case of the normal expenditure state, the closer θ_1 is to 1, the closer the debt function is to the deterministic case in which only the normal expenditure state exists.

More interestingly, lowering θ_2, i.e., lowering the average duration of the high expenditure state, also increases the debt function in both states. The effects on the normal expenditure state are hardly noticeable, but can be really large in the high expenditure state. For example, if we lower θ_2 from 0.5 to 0.05 then long-run debt to GDP for the high expenditure state increases from 43% to 51%. Of course, with such a low probability it would be a rare event that the high expenditure state lasts long enough to get there.

In sum, events that were expected to increase government expenditure more, to be less frequent and last shorter, make governments increase the debt more when the shock arrives. Additionally, the larger is the increase in expenditure, the more conservative is the government's debt policy in normal times.

4 Concluding Remarks

This paper presents a theory of debt that delivers an interior steady state that is independent of initial conditions, and that can replicate certain U.S. data. The key elements are nominal government debt and lack of commitment. The theory also works qualitatively well in explaining how countries react when there is a temporary increase in government expenditure, but in this case, labor fluctuates significantly. Still, the model has a hard time explaining why some countries exhibit persistently very large amounts of debt. The reason is that when debt is large, the government has large incentives to reduce it by inflating prices. Further study should be devoted to think of ways to limit this. Natural candidates are self-control, reputation, central bank independence and political economy reasons.

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Appendix

A Numerical Computation of the Basic Model

To solve the problem of the government in the basic model, define a grid over B and apply the following algorithm.

1. Guess the decision rules: $\mathcal{B}^0(B)$, $\mathcal{P}^0(B)$, $\mathcal{V}^0(B)$.
2. For every B in the grid solve for B' and p given $\mathcal{B}^0(B)$, $\mathcal{P}^0(B)$, $\mathcal{V}^0(B)$, i.e., solve

$$\mathcal{V}^1(B) = \max_{B',p} u\left(\frac{1}{p},\ 1 - \frac{1}{p} - g\right) + \beta\ \mathcal{V}^0(B')$$

subject to

$$\eta(B, B', p, \mathcal{P}^0(B')) = 0.$$

Call the solution $\mathcal{B}^1(B)$ and $\mathcal{P}^1(B)$. Note that this problem simplifies greatly if the utility function is as defined in (14).

3. Check convergence of the decision rules and the value function. If the convergence error is not below the desired tolerance, then update the decision rules and the value function and go back to step 2.

The algorithm assumes the solution is smooth and one can use cubic splines to interpolate the value of functions between grid points.

Instead of using value function iteration, one could use the GEE (16). Remember that this equation contains the derivative of $\mathcal{P}(B)$. If cubic splines are used to interpolate functions, then it is very easy to calculate the derivatives. The algorithm in this case is very similar to the one outlined above.

Since there coexists a discrete solution, one has to be careful when applying this algorithm. On the iteration path, the discrete solution can potentially "leak" and the algorithm will not converge to a smooth solution. To avoid this, use a small number of grid points, say 50[19]. Once we achieve convergence we can verify the precision of the solution by evaluating the GEE over a finer grid, say $1,000$ points.

Another way to solve the model is to forget about iteration and use a projection method. This involves to simultaneously solve the GEE evaluated at all the grid points. To approximate the policy functions one could use Chebyshev polynomials, so that it is straightforward to evaluate their derivatives.

Alternatively, one could be interested in finding local solution only. In that case, one could use the perturbation method proposed by Klein, Krusell and Ríos-Rull (2004).

[19] When applying the algorithm to the model of section 3, the number of grid points has to be even lower: between 10 and 20.