MODELING ECONOMIC FLUCTUATION IN SUB-SAHARAN AFRICA
A VECTOR AUTOREGRESSIVE APPROACH

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1.0 RESEARCH PROBLEM

For most of the twentieth century, especially since the great depression, most macroeconomists have looked upon the sharp fluctuations in output and unemployment as prima facie evidence of major market imperfections and explored what these imperfections may be (Blanchard and fisher, 1990). In the last 15 years, however, some have argued that this is a misguided research strategy, since of research belongs basically can be explained without invoking imperfections. This line of research belongs basically to the real business cycle theory, which proceeds from the assumption that there are large random fluctuations in the rate of technological change (Mankiw, 1990 and Charnvitayapong et al 1995). Because these fluctuations in technology lead to fluctuations in relative prices, individuals rationally alter their labor supply and consumption. Thus the theory explains recessions as periods of technological regress, that is, declines in society’s technological ability and assumes that monetary policy is irrelevant for economic fluctuations.

In fact, a controversial result of some current research on this real business cycles is the claim that a common stochastic trend—the cumulative effect of permanent shocks to productivity underlies the bulk of economic fluctuations (king et al, 1991). If confirmed, this will imply that many forces have been relatively unimportant over historical business cycles, including the monetary and fiscal policy shocks in traditional macroeconomics analysis.

This research study therefore intends to show that the hypothesis of a common stochastic productivity has a set of econometric implications that allows us to test for its presence, measure its importance, and extract estimates of its realized value. Moreover, we intend to know how much of the cyclical variation in the research data can be attributed to innovations in the common stochastic trends and whether innovations associated with nominal variables explain important cyclical movements in the real variables.
Building on a venerable economic tradition, it is hoped that the estimated relationships will separate the regular response of policy to the economy from the response of the economy to policy; and therefore producing a more accurate measure of the effects of the policy changes and shocks (Sims et al, 1996). In this way, the proposed model will integrate policy behaviour variously with broad macroeconomic aggregates to provide a fuller understanding of the factors underlying the bulk of economic fluctuations in sub-Saharan Africa. Unfortunately, these economies has remained stagnated, deteriorated and distorted for the past decades (Nwaobi, 1993b,)

2.0 OBJECTIVES OF STUDY
The fundamental aim of this study is to build a specific structural long run model that incorporate the important features of the national economies under study. This objective requires us to carry out the following steps:

1. To construct new classical based vector auto regression models for selected African Economics

2. To examine whether business cycles are mainly the result of permanent shocks to productivity.

3. To identify the role of monetary and fiscal policy during economic fluctuations, using impulse response analysis and persistence profiles (i.e., by considering the effect of variable-specific and system-wide shocks).

4. And finally to compute multivariate dynamic forecasts as well as performing a sensitivity analysis.
3.0 THEORETICAL AND METHODOLOGICAL FRAMEWORK

In a real-business-cycle model with permanent productivity shocks, output $Y_t$ is produced via a constant-returns-to-scale Cobb-Douglas production function:

$$Y_t = \lambda_t k_t^{1-\theta} N_t^\theta$$  (1.1)

where $k_t$ is the capital stock and $N_t$ represents labor input. Total factor productivity, $\lambda_t$, follows a logarithmic random walk:

$$\log (\lambda_t) = \mu_{\lambda} + \log (\lambda_{t-1}) + \xi_t$$  (1.2)

where the innovations, $(\xi_t)$, are independent and identically distributed with a mean of 0 and a variance of $\sigma^2$. The parameter $\mu_{\lambda}$ represents the average rate of growth in productivity; $\xi_t$ represents deviations of actual growth from this average.

Within the basic neoclassical model with deterministic trends, it is familiar that per capita consumption, investment, and output all grow at the rate $\mu_{\lambda}/\theta$ in steady state. The common deterministic trend implies that the great ratios of investment and consumption to output are constant along the steady-state growth path. When uncertainty is added, realizations of $\xi$ change the forecast of trend productivity equally at all future dates:

$$E_t \log(\lambda_{t+s}) = E_{t-1}(\lambda_{t+s}) + \xi$$

A positive productivity shock raises the expected long-run growth path: there is a common stochastic trend in the logarithms of consumption, investment, and output. The stochastic trend is $\log ((\lambda_t)/\theta)$, and its growth rate is $(\mu_{\lambda} + \xi)/\theta$, the analogue of the deterministic models common growth rate.
restriction, $\mu/\theta$. With common stochastic trends, the ratios $C_t/Y_t$ and $I_t/Y_t$ become stationary stochastic processes.

Thus, let $X_t$ be a vector of the logarithms of output, consumption, and investment at date $t$, denoted by $Y_t$, $C_t$, and $I_t$. Each component of $X_t$ is expected to be integrated of order one because of the random-walk nature of productivity; yet, the balanced growth implication of the theory implies that the difference between any two elements of $X_t$ is integrated of order zero. However, in systems that incorporate both real nominal variables, additional co integrating relations may plausibly arise. The first is money demand relation:

$$M_t - P_t = \beta_y Y_t - \beta_R R_t + V_t$$  \hspace{1cm} (1.3)

where $M_t - P_t$ is the logarithm of real balances, $R_t$ is the nominal interest rate and $V_t$ is the money-demand disturbance. The other co integrating relation is the fisher relation:

$$R_t = r_t + E_t \Delta P_{t+1}$$  \hspace{1cm} (1.4)

Where $r_t$ is the ex ante real rate of interest, $P_t$ is the logarithm of the price level, and $E_t \Delta P_{t+1}$ denotes the expected rate of inflation between $t$ and $t+1$. If real balances, output, and interest rates are I (1), while the money – demand disturbance in (1.3) is I (0), then real balances, output, and nominal interest rate are co integrated. If the real rate is I (0) and the inflation rate is I (1), then (1.4) implies that nominal interest rates and inflation are co integrated.
To identify trends and their impulse response functions, let $X_t$ denote an $n \times 1$ vector of time series. The individual series are assumed to be $I(1)$ and to have the Wold representation:

$$\Delta X_t = \mu + C(L)\epsilon_t$$  \hspace{1cm} (1.5)

where $\epsilon_t$ is the vector of one-step-head linear forecast errors in $X_t$ given information on lagged values of $X$. The $\epsilon$’s are serially uncorrelated with a mean of zero and covariance matrix $\Sigma_{\epsilon}$. To consider the set of structural relations that led to (1.5), we specify the following model:

$$\Delta X_t = \mu + \Gamma(L)\eta_t$$  \hspace{1cm} (1.6)

where $\eta_t$ is an $n \times 1$ vector of serially uncorrelated structural disturbances with a mean of zero and a covariance matrix $\Sigma_{\eta}$. The reduced form of (1.6) will be of the form (1.5) with $\epsilon_t = \Gamma_o \eta_t$ and $C(L) = \Gamma(L) \Gamma_o^{-1}$. Here, identification is achieved through two sets of restrictions. First, the co-integration restrictions impose constraints on the matrix of long-run multipliers $\Gamma(1)$ in (1.6). This identifies the permanent components. Second, the innovations in the permanent components are assumed to be uncorrelated with the innovations to the remaining transitory components. This identifies the dynamic response of the economic variables to the permanent innovations.

And given a three-variable model with $X_t = (y_t, C_t, i_t)'$. Because there are two co-integrating vectors, there is only one permanent innovation, the balanced-growth innovation $\eta_t'$. This shock corresponds to $\xi_t$ in the neoclassical model. The other two shocks, $\eta_{t2}$ and $\eta_{t3}$, have only transitory effects on $X_t$. Thus, the first identification restriction (the balanced-growth co-integrating vectors) implies that the matrix of long run multipliers is

$$\Gamma(1) = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$  \hspace{1cm} (1.7)
Where the values of the coefficients in the first column of $\Gamma(1)$ are normalized to I to fix the scale of $\eta_t^1$. Equation (1.7) serves to identify the balanced-growth shock as the common long-run component in $X_t$, since the innovation in the long run forecast of $X_t$ is $(1 1 1)' \eta_t^1 = C (1) \epsilon_t$ which can be calculated directly from the reduced form (1.5). The second restriction, that $\eta_t^1$ is uncorrelated with $\eta_t^2$ and $\eta_t^3$ is used to determine the dynamic effects of $\eta_t^1$ on $X_t$, that is to identify the first column of $\Gamma(L)$.

To introduce nominal shocks, the above three-variable real model is augmented by real balances, nominal interest rates, and inflation. The resulting six-variable model has three common stochastic trends, and this makes identification more complicated. The following restrictions are used to identify the model. First, as in the model with $k=1$, we assume that $\eta_t^1$ and $\eta_t^2$ are uncorrelated. Second, the permanent shocks, $\eta_t^1$, are assumed to be mutually uncorrelated. Third, $A$ is assumed to be lower triangular, which permits writing $A = A \Pi$, where $A$ is a matrix with no unknown parameters and $\Pi$ is a $K \times K$ lower triangular matrix. $A$ can be chosen in a way that associates each shock with a familiar economic mechanism: the first disturbance is interpreted as a balanced-growth shock, the second is a long-run neutral inflation shock and the third is a permanent real-interest-rate shock. This constrained reduced form is estimated as a VAR with error-correction terms (i.e., a vector error-correction model (VECM)).

However, it is important that the analysis of co integration be accompanied by some estimate of the speed with which the economy or the market under consideration return to their equilibrium state, once shocked. (Pesaranan and shin, 1996) Such an analysis would be particularly valuable in cases where there are two or more co integrating relation characterizing equilibrium, possibly in different markets, Where we will be able to estimate the relative adjustment speeds of different markets to words their respective
equilibrium. Basically, there are four possible methods that can be used to characterize and estimate the time profile of the effect of shocks on one more cointegrating relations. All these approaches are based on explicit relationship that we drive between the cointegrating relation and the current and lagged values of the shocks in the convergence to equilibrium.

The obvious method has been the impulse response approach (Sims1980) used to estimate the time profile of the effect of ‘particular’ shocks on the cointegrating relations. But such an analysis is subject to the major criticism that the estimate d impulse response function are not unique and depend on the way the shocks in underling VAR model are orthogonalized. An alternative approach is to consider the resultant time profile of the effect not a system-wide shock on the cointegrating relation .the resultant time profile will be unique and do not require the prior orthogonalization of the shocks.

Following Lee and Pesaran (1993), we measure the impact of system-wide shock on the cointegrating relation by their ‘persistence profile, defined as the scaled difference between the condition variances of the n-step and the (n-1)-step-ahead forecast, and viewed as a function of n, the forecast horizon. This measure readily capture the essential difference that exists between cointegrated and noncointegrating relation, and provide unique time profile of the effect of shocks to the cointegrating relation, in the case of relation between I (1) variable that are not co integrated, the effect of a shock persists forever, while in the case of coin grated relation the impact of shock will be transitory and eventually disappear as the economy returns to its steady trend its long run equilibrium

Specifically, we assume the following augmented vector autoregressive Model:

$$Z_t = a_o + a_1 t + \Phi Z_{t-1} + \varnothing W_t + U_t \quad t = 1,2,\ldots,n \quad (1.8)$$

$$= A'g_t + U_t$$
Where $Z_t$ is an $m \times 1$ vector of jointly determined dependent variable and $W_t$ is a $q \times 1$ vector of deterministic or exogenous variable. The statistical framework for the cointegrating VAR is the following general vector error correction mode (VECM)

$$Y_t = a_{oy} + a_{1y}t - \Pi_y Z_{t-1} + \Gamma_{iy} Y_{t-1} + \Theta_y W_t + \varepsilon_t, \quad t = 1, 2, \ldots, n$$

(1.9)

where $Z_t = (Y_t, X'_t)$, 'Y' is an $M_y \times 1$ vector of jointly determined (endogenous) I (1) variables.

$X_t = M_x \times 1$ vector of exogenous I (1) variables.

$X_t = a_{ox} + \Gamma_{ix} Z_{t-i} + \Theta_x W_t + V_t$

(1.10)

$W_t = q \times 1$ vector of exogenous/deterministic 1 (0) variables, excluding the intercepts and/or trends.

However, the disturbance vectors $\varepsilon_t$ and $v_t$ satisfy the following assumptions:

$$Ut = \begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix} \sim iid (0, \Sigma) \quad (1.11)$$

Where $\Sigma$ is a symmetric positive definite matrix and the disturbances in the combined mode $U_t$ are distributed independently of $W_t$.

$$E(U_t/W_t) = 0 \quad (1.12)$$

The intercept and the trend coefficients $a_{oy}$ and $a_{1y}$ are $M_y \times 1$ vector: $\Pi_y$ is the long–run multiplier matrices of order $M_y \times M$, where $M = M_x + M_y$: $\Gamma_{1y}, \Gamma_{2y}, \ldots, \Gamma_{r-1, y}$ are $M_y \times M$

Coefficient matrices capturing the short–run dynamic effect: and $\Theta_y$ is the $M_y \times q$ matrix of coefficient on the I (0) exogenous variable.

Indeed, (1.9) allow for a sub-system approach in which the $M_x$ vector of random variable, $X_t$ is the forcing variable, or common stochastic trend in the sense that the error correction term do not enter in the sub-system for $X_t$. Thus our co integrating analysis allows for contemporaneous and short–term
feedbacks from \( Y_t \) to \( X_t \) but require that no such feedback are possible in the 'long–run forcing variable of the system. The co integrating VAR analysis concerned with the estimation of (1.9) when the rank of the long-run multiplier matrix, \( \Pi \), could at most be equal to \( M_y \). Therefore, rank deficiency of \( \Pi \) can be represented as

\[ H_r : \text{Rank} (\Pi_y) = r < M_y \]

Which follows that \( \Pi_y = \alpha_y \hat{\alpha} \) where \( \alpha \) and are \( M_y \times r \) and \( M \times r \) matrices, each with full column rank, \( r \). In the case where \( \Pi_y \) is rank deficient, we have \( Y_t \sim I(1) \), \( Y_t \sim I(0) \), and \( \hat{\alpha}'Z_t \sim 1(0) \). The \( r \times 1 \) trend stationary relations, \( \hat{\alpha}'Z_t \), are referred to as the cointegrating relation, and characterize the long run equilibrium (steady state) of the VECM (1.9)

It is however, important to recognize that in the case where the vecm (1.9), contains deterministic trends (i.e. \( a_{iy} = 0 \)), in general there will also be a linear trend in the cointegrating relations. Thus, combining the equation systems (1.9) and (1.10), we have

\[
P^{-1}Z_t = a_0 + a_1 t - Z_{t-1} + \Pi Z_{t-1}n + \Gamma_i Z_{t-i} + \varnothing W_t + \Gamma_i Z_{t-i} + \varnothing W_t + \Gamma_i \]

(1.13) for \( t = 1, 2, \ldots, n \), where

\[
Z_t = \begin{bmatrix} Y_t \\ X_t \end{bmatrix} \quad U_t = \begin{bmatrix} U_{yt} \\ V_t \end{bmatrix} \quad a_0 = a_{oy} \quad a_1 = a_{ly} \\
0 \quad 0 \quad \Gamma_i = \begin{bmatrix} \Gamma_{iy} \\ \Gamma_{ix} \end{bmatrix} \quad \varnothing = \begin{bmatrix} \varnothing_{y} \\ \varnothing_{x} \end{bmatrix}
\]

\( \Pi = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Gamma_i = \begin{bmatrix} 0 \end{bmatrix} \quad \varnothing = \begin{bmatrix} \varnothing \end{bmatrix} \)

which is the vector error correction form of (1.8). In the case where \( \Pi \) is rank deficient, the solution of (1.13) involves common stochastic trends, and is given by
\[ Z_t = Z_0 + b_0 t + b_1 \{ t (t + 1)/2 \} + C(1) S_t + C^* (L) (h_t - h_o) \quad (1.14) \]

Where

\[ h_t = \mathcal{O} W_t + U_t \quad (1.15) \]

\[ S_t = U_i, \quad t = 1, 2, \ldots \quad (1.16) \]

\[ b_0 = C(1)a_0 + C^* (1)a_1 \quad (1.17) \]

\[ b_1 = C(1)a_1 \quad (1.18) \]

\[ C(L) = C(1) + (1-L)C^*(L) \quad (1.19) \]

\[ C^*(L) = \sum_{i=0}^{\infty} C_i^* L^i \]

where \( L \) is the one period lag operator and the \( M \times M \) matrices, \( C_i^* \), are obtained recursively from

\[ C_i^* = C_{i-1}^* \Phi_i + \ldots + C_{i-p}^* \Phi_p \quad i = 1, 2, \ldots \quad (1.20) \]

With \( C^{*0} = I_m - C(1) \quad C_i^* = 0, \ i < 0, \) and

\[ \prod C(1) = 0 = C(1) \prod \quad (1.21) \]

The matrices \( \Phi_1, \Phi_2, \ldots, \Phi_p \) are the coefficient matrices in the VAR form of (1.13), and in term of \( \prod, \Gamma_2, \ldots \) and \( \Gamma_{p-1} \) are given by

\[ \Phi_1 = \Gamma_m - \prod + \Gamma \]

\[ \Phi_i = \Gamma_i - \Gamma_{i-1} \quad i = 2, 3, \ldots, p-1 \]

\[ \Phi_p = -\Gamma_{p-1} \]

From solution (1.14) it is clear that, in general, \( Z_t \) will contain a quadratic trend. When \( a_i \neq 0 \), the quadratic trend disappears only if \( C(1) a_i = 0 \), otherwise the number of independent quadratic trend terms in the solution of \( Z_t \) will be equal to the rank of \( C(1) \). And hence depends on the number of cointegrating relations. Since rank \( \{ C(1)\} = m-r \) and without some restrictions on the trend coefficients, \( a_i, \prod_r \) the solution (1.14) has the unsatisfactory property that the nature of the of the trend in \( Z_t \) varies with the assumed number of the cointegrating relations. This outcome can be avoided by restricting the trend coefficients namely, by setting \( a_i = \Pi_r \) and using (1.18) and (1.12), we have
\[ b_1 = C(1)a_i = c(1) \prod_r = 0 \]

and the VECM in (1.13) becomes

\[ Z_t = a_o - \prod (Z_{t-1} - r_i) + \Gamma_i \quad Z_{t-i} + \omega W_i + U_t \quad (1.22) \]

Using (1.14), the cointegrating relations, \( \hat{a}'Z_t \), can also be derived in terms of the shocks \( U_{t-i}, i = 0,1,2, \ldots \), and the current and past values of \( 1(0) \) exogenous values.

Pre-multiplying (1.14) by \( \hat{a} \) and bearing in mind the cointegrating restrictions \( \hat{a}'C(1) = 0 \), we obtain

\[ \hat{A}'Z_t = \hat{a}'Z_o + (\hat{a}'b_o) + \hat{a}'C*(L) (h_t - h_o) \quad (1.23) \]

Using (1.17) we also have

\[ \hat{a}'b_o = \hat{a}'C*(1)a_1 \quad (1.24) \]

And hence when \( a_i \neq 0 \), the cointegrating relations, \( \hat{a}'Z_t \) in general, contain deterministic trends; which do not disappear even if \( a_i \) is restricted. Indeed, when \( a_i = \text{IIR} \), the coefficients of the deterministic trend in the cointegrating relations are given by

\[ \hat{a}'b_o = \hat{a}'C* (1) \prod \]

But using this result in (1.23), we have

\[ \hat{a}'Z_t = \hat{a}'Z_o + (\hat{a}'r)t + \hat{a}'C*(L) (h_t - h_o) \quad (1.25) \]

A test of whether the cointegrating relations are trended can be carried out by testing the following restrictions:

\[ \hat{a}'r = 0 \quad (1.26) \]

Referred to as co-trending restrictions.

The computation of the impulse response function for the cointegrating VAR model can be based on the VECM (1.13), which combines the equation systems for \( Y_t \) and \( X_t \) given by (1.9) and (1.10), respectively. The solution of the combined model is given by (1.14) and the orthogonalised impulse response function of the effect of a unit shock to the \( i \)th variable at time \( t \) in (1.13) on the \( j \)th variable at time \( t + N \) is given by
\[ \text{OI}_{ijN} = e'_j (C(1) + C_{Ni}^*) Te_i \] (1.27)

Where \( T \) is a lower triangular matrix such that \( T = TT' \), \( e_i \) is the selection vector and \( C(I) \) and \( C_{Ni}^* \) are defined by relations (1.19) to (1.21). This implies that the effects of shocks on individual variable in a cointegrating VAR model do not die out and persist forever.

An alternative approach would be to consider the effect of system-wide shocks or variable-specific shocks on the cointegrating relations, \( \hat{\alpha}'Z_t \) rather on the individual variables in the model. The effect of shocks on cointegrating relations is bound to die out, and their time profile contains useful information on the speed of convergence of the model to its cointegrating (or equilibrium) relations. Consider first the time profile of the effect of a unit shock to the variable in \( Z_t \) on the \( j \)th cointegrating relation, namely \( \hat{\alpha}'_j Z_t \). Using (1.23), we have

\[ \text{OI}_i(\hat{\alpha}'_j Z_t N) = \hat{\alpha}'_j A_N Te_i \] (1.27)

For \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, r, N = 0, 1, 2, \ldots \) which give the responses of a unit change in the \( i \)th orthogonalized shock (= \( \sqrt{\sigma_{ii}} \)) on the \( j \)th cointegrating relation \( \hat{\alpha}'_j Z_t \). The corresponding generalized impulse responses are given by

\[ \text{GI}_i(\hat{\alpha}'_j Z_t N) = \frac{\hat{\alpha}'_j A_N e_i}{\sqrt{\sigma_{ii}}} \] (1.28)

For \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, r, \) and \( N = 0, 1, 2, \ldots \)

But given the ambiguities that surround the impulse response analysis with respect to variable specific shocks, we consider the effect of system-wide shocks on cointegrating relations. Such a time profile, referred to as the persistence profile of the effect of system-wide shocks on the \( j \)th cointegrating relationship is given by

\[ h((\hat{\alpha}'_j Z_t N) = \hat{\alpha}'_j Z_t N A' NB_i \] (1.29)

For \( j = 1, 2, \ldots, r, \) and \( N = 0, 1, 2, \ldots \)
The value of this profile is equal to unity on impact, but should tend to zero as $N \rightarrow \infty$ if $\hat{\alpha}^j$ is indeed a cointegrating vector. The persistence profile, $h(\hat{\alpha}^jZ_tN)$ viewed as a function of $N$ provides important information on the speed with which the effect of system wide shocks on the cointegrating relation, $\hat{\alpha}^jZ_t$, disappears even though these shocks generally have lasting impacts on the individual variables in $Z_t$.

The variables to be used are defined as follows:

- **C** = Logarithm of per capita real consumption expenditures
- **I** = Logarithm of per capita gross private domestic fixed investment
- **Y** = Logarithm of per capita “private” gross national product defined as total gross national product less real total government purchases of goods and services.
- **M** = Logarithm of per capita M2 (currency, demand deposits, and saving deposits)
- **P** = Logarithms of the price level measured by the implicit price deflator for our measure of private GNP.
- **R** = The interest rate measured as a treasury bills rate.

**4.0 DATA SOURCES AND COLLECTION**

The major sources of data for the study will be the published and unpublished data from the various countries ministries, and agencies. Other supplementary sources of data will include the statistical publications of the World Bank, United Nations and the International Monetary Fund. Efforts will be made to collect quarterly data on the variables for the study. We shall also search the various intranets extranets and Internet websites (http, www, gopher, telnet, etc) accordingly.
5.0 RESEARCH RESULTS AND DISSEMINATION

Our research result is expected to contribute to public policy making for the various countries studied. This is in addition to contributing to existing knowledge and future research in economics. Our research output will be published accordingly and disseminated to the various professional economists and policy makers in the region. We also intend to present and discuss our research findings before the various academic economists (especially, during the African Economic Research Consortium Network meetings).

BIBLIOGRAPHY


