# Does Stationarity Characterize Real GDP Movements? Results from Non-Linear Unit Root Tests.

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**Abstract**

Using non-linear unit root tests this paper investigates non-stationarity of real GDP per capita for seven OECD countries over the period 1900-2000. Non-linear unit root tests are more powerful than traditional ADF statistics in rejecting the null unit root hypothesis. To this end we adopt a first order Fourier approximation that may capture many features of non-linear adjustment. Empirical results show that, contrary to what the linear ADF statistics suggest, stationarity characterizes six out of the seven countries. This finding stands at variance with other recent studies which conclude that movements in real GDP per capita can be characterized as a non-stationary process.

**Keywords:** Unit root tests, non-linear model, real GDP.

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## 1. Introduction

An important feature of business cycle theory is the trend stationarity of real output. This means that shocks have only a transitory impact on real output movements leading the economy towards an equilibrium value. However the stationarity of real output is an unlikely possibility. A substantial number of empirical studies give support to the contention that real output levels are non-stationary: see, for example, Nelson and Plosser (1982), Wasserfallen (1986), Cheung and Chinn (1996) and Rapach (2002). This finding has important implications for business cycle theory as it suggests that real factors such as technology shocks play an important role in economic fluctuations. Within this context, business cycle theory no longer implies stationary fluctuations around a deterministic trend. Finally, the presence of a unit root in output movements has consequences for the way we forecast economic activity and evaluate the role and importance of macroeconomic stabilization programs.

The empirical literature cited above reached the conclusion that real GDP levels are non-stationary by using either univariate unit root statistics (Cheung and Chinn, 1996) or panel unit root tests (Rapach, 2002) along the lines of the augmented Dickey-Fuller (ADF) statistic. The key feature of all these tests is that they work upon the hypothesis that a symmetric adjustment process exists. However, a very recent and expanding empirical literature allows for non-linear dynamics for unit root testing procedures: see for example Caner and Hansen (2000), Shin and Lee (2001) and Kapetanios et al (2003). According to Enders and Granger (1998) all standard linear unit root tests have lower power in the presence of misspecified dynamics. They reviewed many important examples of asymmetric adjustment of economic variables.

The aim of the present note is to provide direct evidence for real GDP stationarity using unit root methods with non-linear dynamics. These methods are more suitable for detecting non-stationarity in the levels of real GDP per capita.

## 2. A non-linear unit root methodology

Enders and Ludlow (1999) and Ludlow and Enders (2000) proposed the following extension to the standard linear AR(1) model:

$$y_t = \alpha(t) y_{t-1} + u_t \qquad (1)$$

where $y_t$ is a stationary random variable, $u_t$ is a white noise disturbance term and $\alpha(t)$ is a deterministic but unknown function of time[1]. They showed that although a sufficiently long Fourier series represents the function $\alpha(t)$ exactly, equivalent results are obtained by considering only a single frequency so that:

$$\alpha(t) = \varphi_o + \varphi_1 \sin\frac{2\pi k}{T}\cdot t + \varphi_2 \cos\frac{2\pi k}{T}\cdot t \qquad (2)$$

where $k$ is an integer in the interval 1 to T/2.

If the actual data generating process is linear, then $\varphi_1 = \varphi_2 = 0$ and the standard Dickey-Fuller regression emerges as a special case. On the other hand, if $\varphi_1 \neq \varphi_2 \neq 0$ then the unit root null hypothesis $|\varphi_o| < 1$ is neither a necessary nor sufficient condition for mean reversion of the $y_t$ series.

The main advantage in using (2) to detect non-stationarity in comparison to other non-linear unit root tests (see Enders and Granger 1998, Caner and Hansen 2000, Kapetanios et al, 2003, and He and Sandberg, 2003) is that we do not need to specify the precise adjustment mechanism or the nature of the asymmetry. All we need to find is the most appropriate values of the coefficients $\varphi_i$ ($i$=0,1,2) and $k$. In this context Enders and Ludlow (1999) proved that a necessary and sufficient condition for mean reversion is that $|\varphi_o| < 1 + \frac{r^2}{4}$ and $< 2$, where $r = \sqrt{\varphi_1^2} + \sqrt{\varphi_2^2}$. Finally, for various values of $\varphi_o$ we can derive the adjustment paths of the $y_t$ series.

Given that equation (2) is not known to the searcher we need to derive parameter estimates for $\varphi_i$, $i$=0,1,2 and $k$ as well as their statistical levels of significance by performing the following steps.

To find the most appropriate value for the integer $k$ we estimate the following regression in first differences using each integer value of $k$ in the interval 1 to T/2.

$$\Delta y_t = \left[\mu + \varphi_1 \sin\frac{2\pi k}{T}\cdot t + \varphi_2 \cos\frac{2\pi k}{T}\cdot t\right] y_{t-1} + e_t \qquad (3)$$

where $e_t$ is a white noise disturbance term.

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[1] Of course $\alpha(t)$ can be also a $p$-th order difference equation, a threshold function or a switching function.

Parameter $\mu$ in equation (3) corresponds to the value $(\varphi_o - 1)$ in equation (2). Reversion requires $|\mu| < 1 + \frac{r^2}{4}$.

The optimal level $k^*$ is chosen so as to minimize the sum of squared errors. The coefficients that correspond to the optimal $k^*$ are denoted by $\mu^*$, $\varphi_1^*$ and $\varphi_2^*$. Based on the estimated coefficients of (3) we can now test the following hypotheses:

- $\mu^* = 0$ (4)
- $\mu^* = \varphi_1^* = \varphi_2^* = 0$ (5)
- $\varphi_1^* = \varphi_2^* = 0$ (6)
- $\mu^* = \frac{(\varphi_1^*)^2}{4} + \frac{(\varphi_2^*)^2}{4}$ (7)

A t-statistic can be used to test the null hypothesis (4) while an F- statistic is necessary for the null hypotheses (5)-(7). Monte Carlo simulations approximate all these empirical distributions.

## 3. Empirical results.

Table 1 reports single country ADF linear tests using annual data on real GDP per capita in logarithmic form for seven OECD countries, namely, Belgium, Denmark, France, Italy, the Netherlands, the USA and the UK over the period 1900-2000. Data description and sources are given in Maddison [2]. The unit root null hypothesis is not rejected at conventional levels of significance for any country except for the USA at the 5% level. This finding is consistent with the real GDP unit root literature.

**[Insert Table 1]**

We now apply the potentially more powerful non-linear unit root statistic. Following standard practice we use the de-meaned and de-trended data by first regressing each series on a constant and a time trend and saving the residuals. Next we replace $y_t$ by the corresponding residual series and estimate equation (3) for each integer value of $k$ in the interval 1 to (101/2). Having identified the optimal $k^*$ we present in Table 2 the values of the test

[2] The data can be downloaded from Maddison web site.

statistics for the hypotheses (4)-(7). To ensure that $e_t$ is white noise we introduced additional terms $\Delta y_{t-L}$ on the right hand side of (3). The number of lags (L) was selected optimally using Schwarz's Bayesian criterion.

**[Insert Table 2]**

The results in Table 2 indicate that, firstly, $\mu^*$ was not statistically significant in any of the countries. This finding is consistent with the ADF results in Table 1 which show acceptance of the unit root hypothesis. It is easy to conclude that real GDP per capita does not enter the model linearly. However, as discussed in Section 2, $\mu^* = 0$ is neither a necessary nor sufficient condition for stationarity. A second finding from Table 2 is that, for all countries apart from the UK, the null hypotheses $\mu^* = \varphi_1^* = \varphi_2^* = 0$ and $\varphi_1^* = \varphi_2^* = 0$ are rejected at conventional levels of statistical significance. This means that, for all countries in Table 2 except for the UK, a stationary process characterizes real GDP per capita movements as business cycle theory predicts. Other things being equal, all shocks have temporary effects on output levels which subsequently revert to their equilibrium values. Thirdly, the non-linear restriction $\mu^* = \frac{(\varphi_1^*)^2}{4} + \frac{(\varphi_2^*)^2}{4}$ is strongly rejected for all countries apart from Italy and the Netherlands, casting doubt on the reliability of the results based on the t and F-statistics. However, Enders and Ludlow (1999), using power functions, concluded that when $\mu^*$ is near –0.1 and $^*$is near 0.27 the power of the Dickey-Fuller, $\mu^* = 0$ and $\mu^* = \frac{(\varphi_1^*)^2}{4} + \frac{(\varphi_2^*)^2}{4}$ tests are all very low. In these circumstances the F-tests for $\mu^* = \varphi_1^* = \varphi_2^* = 0$ and $\varphi_1^* = \varphi_2^* = 0$ are both quite powerful. In our case the $\mu^*$ values are –0.09 for Belgium, -0.11 for Denmark, -0.13 for France, -0.07 for the UK and -0.08 for the USA. The corresponding $^*$values are 0.30, 0.33, 0.34, 0.18 and 0.28 respectively. It is obvious that all estimates of $\mu^*$ and $^*$ are close to the values proposed by Enders and Ludlow (1999), leaving little doubt about the stationarity properties of our data set. The exception is the estimated $^*$ value for the UK which is far from the value 0.27. In this case results based on ADF regression are more powerful compared to those derived from a non-linear unit root model. Finally, we can derive the various adjustment paths towards the equilibrium of the real GDP per capita based on the values of $\mu^*$ and $^*$. Given that $\left|\mu^*\right| < r^*$ and $\left|\mu^*\right| + r^* < 1$ for all countries examined, the real GDP per capita in every country exhibits periods of exploding behavior although the overall process reverts towards the equilibrium level. This behavior is described in Figure 1 within the region $ebc$. Therefore we can conclude that in all cases apart from the UK, we can overturn the non-stationary conclusion of the linear ADF test by applying non-linear unit root statistics.

**[Insert Figure 1]**

## 4. Concluding remarks.

Using models that do not assume a linear adjustment, this paper investigates international real GDP per capita stationarity for seven OECD countries over the period 1900-2000. Standard linear ADF statistics show that the data are basically non-stationary for all these countries apart from the USA. In contrast, when we adopt a non-linear model which has higher power than a standard univariate unit root statistic to reject a false null hypothesis of unit root behavior, the empirical evidence suggests that real GDP per capita is well characterized by a non-linear mean reverting process which exhibits periods of exploding behavior. This might offer an alternative explanation for the difficulty researchers have encountered in rejecting the unit root hypothesis for real GDP per capita.

## References


Caner, M., and B.E. Hansen, 2001, Threshold autoregression with a unit root, *Ecometrica* 69, 1565-1596.

Cheung, Y. -W. and Chinn, M.D., 1996, “Deterministic, stochastic, and segmented trends in aggregate output: A cross-country analysis”, *Oxford Economic Papers* 48, 134-162.

Enders, W. and Granger C.W.J., 1998, Unit root tests and asymmetric adjustment with an example using the term structure of interest rates, *Journal of Business of Economic and Statistics* 16, 304-312.

Enders, W. and Ludlow, J., 1999, “Non-linear decay: Tests for an attractor using Fourier approximation”, Working paper, Department of Economics, Iowa State University.

He, C. and Sandberg, R., 2003, A Dickey-Fuller type of test against nonlinear globally stationary dynamic models, Working Paper, Department of Economic Statistics, Stockholm School of Economics.

Kapetanios, G, Shin, Y. and Snell, A., 2003, Testing for a unit root in the non-linear STAR framework, *Journal of Econometrics* 112, 359-379.

Ludlow, J. and Enders, W., 2000, “Estimating non linear ARMA models using Fourier coefficients”, *International Journal of Forecasting* 16, 333-347.

Nelson, C.R. and Plosser, C.I., 1982, “Trends and random walks in macro-economic time series” *Journal of Monetary Economics*, 10, 139-162.

Pantula,S.G., Gonzalez-Farias, G. and Fuller, W.A., 1994, A comparison of unit root test criteria, *Journal of Business of Economics and Statistics*, 12, 449-459.

Rapach, D.E., 2002, “Are real GDP levels nonstationary? Evidence from panel data sets”, *Southern Economic Journal* 68, 73-495.

Shin, D.W. and Lee, O., 2001, Test for asymmetry in possibly nonstationary time series data, *Journal of Business and Economic Statistics* 19, 233-244.

Wasserfallen, W., 1986, “Non-stationarities in macro-economic-time series- further evidence and implications”, *Canadian Journal of Economics* 19, 498-510.

**Table 1. Linear ADF unit root tests**

| *Country* | *ADF Statistic* | *L* |
|---|---|---|
| Belgium | **-1.87 [0.72]** | 3 |
| Denmark | **1.80 [0.73]** | 2 |
| France | **-2.09 [0.56]** | 5 |
| Italy | **-1.64 [0.82]** | 3 |
| Netherlands | **-2.25 [0.49]** | 4 |
| UK | **-2.09 [0.59]** | 3 |
| USA | -3.55** [0.03] | 3 |

**NOTES**: *ADF is the augmented Dickey-Fuller t-test for a unit with a constant and trend. The optimal lag structure* $(L)$ *for the ADF regression was selected via the Pantula et al. principle (1994).* $p$ *-values are reported in brackets. Boldface values denote sampling evidence in favour of unit roots. An (**) indicates statistical significance at 5% statistical level.*

**Table 2. Non-linear unit root tests**

| *Country* | *Hypothesis Testing* | | | | *L* |
|---|---|---|---|---|---|
| | $\mu^* = 0$ t-statistic | $\mu^* = \varphi_1^* = \varphi_2^* = 0$ F-Statistic | $\varphi_1^* = \varphi_2^* = 0$ F-Statistic | $\mu^* = \frac{(\varphi_1^*)^2}{4} + \frac{(\varphi_2^*)^2}{4}$ F-Statistic | |
| Belgium | -2.17 | 9.87** | 14.80*** | 7.50 | 1 |
| Denmark | -2.17 | 9.39** | 14.09*** | 6.71 | 1 |
| France | -2.52 | 17.41*** | 26.12*** | 11.48 | 1 |
| Italy | -1.17 | 22.18*** | 32.27*** | 16.73** | 1 |
| Netherlan. | -1.41 | 30.73*** | 46.09*** | 20.70*** | 1 |
| UK | -1.87 | 4.26 | 6.39 | 3.13 | 1 |
| USA | -1.78 | 7.45* | 11.18*** | 5.57 | 1 |
| | | | | | |
| Critical Values | | | | | |
| 10% | -3.21 | 7.14 | 6.53 | 11.59 | |
| 5% | -3.58 | 8.03 | 7.33 | 14.25 | |
| 1% | -4.28 | 9.95 | 9.30 | 19.55 | |

**NOTES**: *The number of optimal lags* $(L)$ *was selected optimally using Schwarz's Bayesian criterion. (***), (**) and (*) denote rejection of the null hypothesis at 1%, 5% and 10% statistical level respectively.*

Figure 1

The Range of Reversion