Investment Cycles

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Abstract

It is common amonst macroeconomists to view aggregate investment fluctuations as a rational response to fluctuating incentives, driven by exogenous movements in total factor productivity. However, this approach raises a number of questions. Why treat investments in physical capital as endogenous, while treating those in intangible capital as exogenous? Relatedly, why would productivity changes exhibit such strong co–movement across diverse sectors of the economy, and why are the short–run, empirical relationships between aggregate investment and measures of investment incentives, such as Tobin's Q, so weak? We address these and other related issues using a model of "implementation cycles" that incorporates physical capital. In doing so, we demonstrate the crucial role played by endogenous innovation and incomplete contracting, inherent to the process of creative destruction.

Key Words: Inflexibility of installed capital, Tobin's Q, recessions, endogenous cycles and growth
JEL: E0, E3, O3, O4

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1 Introduction

Fluctuations in the aggregate investment rate are a central feature of the business cycle. As Figure 1 illustrates, the rate of U.S. investment in fixed, non–residential assets displays regular, and recurring patterns of activity over time.[1] According to Keynes (1936), investment fluctuations played a central *causal* role in driving business cycles. He argued that the co–movement of investment across diverse sectors of the economy was exogenously driven by a kind of mass psychology which he termed “animal spirits”.[2] More recently, economists have attempted to understand movements in aggregate investment as an optimal response to measurable incentives. In the canonical Real Business Cycle (RBC) model, for example, fluctuations in aggregate investment are driven by exogenous fluctuations in total factor productivity (TFP) that change the incentives to produce investment goods relative to consumption goods. However, this increasingly standard approach raises a number of conceptual and quantitative questions.

First, why treat investment in tangible, physical capital as an endogenous response to incentives, while implicitly treating shifts in the production function as exogenous? Many of these shifts result from costly investments in intangible capital, which seem just as likely to respond endogenously to incentives. For example, re–organization of firms may require a costly re–allocation of managerial effort that will only take place if the anticipated returns are sufficiently high. Over the past 15 years there have been considerable advances in understanding the potential and actual role of endogenous innovation on long run growth, but this has had relatively little influence on business cycle research.[3]

Second, why do these apparent shifts in TFP take place in such a clustered fashion across diverse sectors of the economy? Assuming from the outset that TFP movements affect all sectors symmetrically seems no better on a conceptual level than directly assuming that investment co–moves across sectors because of animal spirits.[4] One possibility is that these shifts are the result of general purpose technologies (GPTs) which affect all sectors. However, there is little evidence supporting this idea at business cycle frequencies.[5] As Lucas (1981) reasons, while productivity

[1]The investment rate fell during all post–war, NBER–dated recessions (the shaded regions in Figure 1) and typically rose during expansions. The only exceptions were around 1967 and 1987, which saw large declines in investment but aggregate slowdowns that were not dated as NBER recessions.

[2]One modern incarnation of this idea is to model animal spirits as purely exogenous, but self–fulfilling changes in expectations (see e.g. Farmer and Guo 1994). In this case, investment is optimal but the aggregate incentives are stochastically driven by “sunspots”.

[3]One clue to the potential importance of viewing technology shifts as, at least partially, endogenous comes from the work of Hall (1988) who finds that the Solow residual is significantly correlated with factors that do not seem likely to have a direct impact on technology.

[4]The RBC literature generally takes this clustering of productivity improvements as given, and focuses on the propagation mechanism.

[5]We discuss this literature in more detail in the following section.

shocks may be important at the firm level, it is not immediately obvious why they would be important for economy–wide aggregate output fluctuations.

Thirdly, if investment really is optimally determined, why is the short–term empirical relationship between aggregate investment and contemporaneous measures of investment incentives apparently so weak? In particular, while there is some evidence of a long run relationship, neither micro nor macro level empirical work has generally found a significant short–run relationship between investment and Tobin's Q — the ratio of the equity value of firms, to the book value of the capital stock.[6] As is well known, one cannot infer from this that investment is sub–optimal because Tobin's Q need not reflect the *marginal* incentives to invest,[7] and because equity values are likely to include the values of intangible, as well as tangible, capital.[8] But then the question arises as to what kind of relationship we should expect to observe between investment and *measurable* proxies of financial incentives over the business cycle.

Figure 1 is suggestive of some kind of relationship. Figure 1(a) shows the investment rate and Tobin's Q for the US between 1953 and 2003. Figure 1(b) shows the rate of change in the four–quarter moving average of each time series.[9] In general there appears to be a lead–lag relationship, with the investment rate most highly correlated with the value of Tobin's Q three to four quarters earlier. It is this observation that has led some investigators to specify an exogenous "time–to–plan" period in their quantitative analyses (see Section 2). However, the relationship is more complex than this. In particular, Tobin's Q appears to lead investment especially during the latter part of expansions and recessions, with Q falling several quarters before investment declines and rising several quarters prior to expansions.[10] During the early phases of expansions and recessions the two series exhibit a contemporaneous correlation.

A final conceptual issue is whether it is reasonable to view investment declines, and hence recessions, as being driven by bad technology outcomes? The recent RBC literature has demonstrated that, in the presence of capital and labor market rigidities, it is not necessary to have *negative* shocks to TFP in order to generate downturns in output (see King and Rebelo, 2000). However, the traditional Keynesian view that recessions largely result from sharp declines in aggregate investment demand, driving production below capacity, seems consistent with the beliefs of policy–makers and many in the private sector. The implications of such views seem worthwhile to at least explore in a formal framework.

[6] First suggested by Tobin (1968) and Brainard and Tobin (1969). See Cabellero, 1999 for a recent survey.

[7] As shown by Abel (1979) and Hayahsi (1982), when there are adjustment costs, marginal and average Q need not be equal.

[8] See Hall (2001b).

[9] Similar figures appear in Cabellero (1999).

[10] There are a number of rationalizations of this behavior in the literature. We discuss these in the next section.

Figure 1 (a): Levels

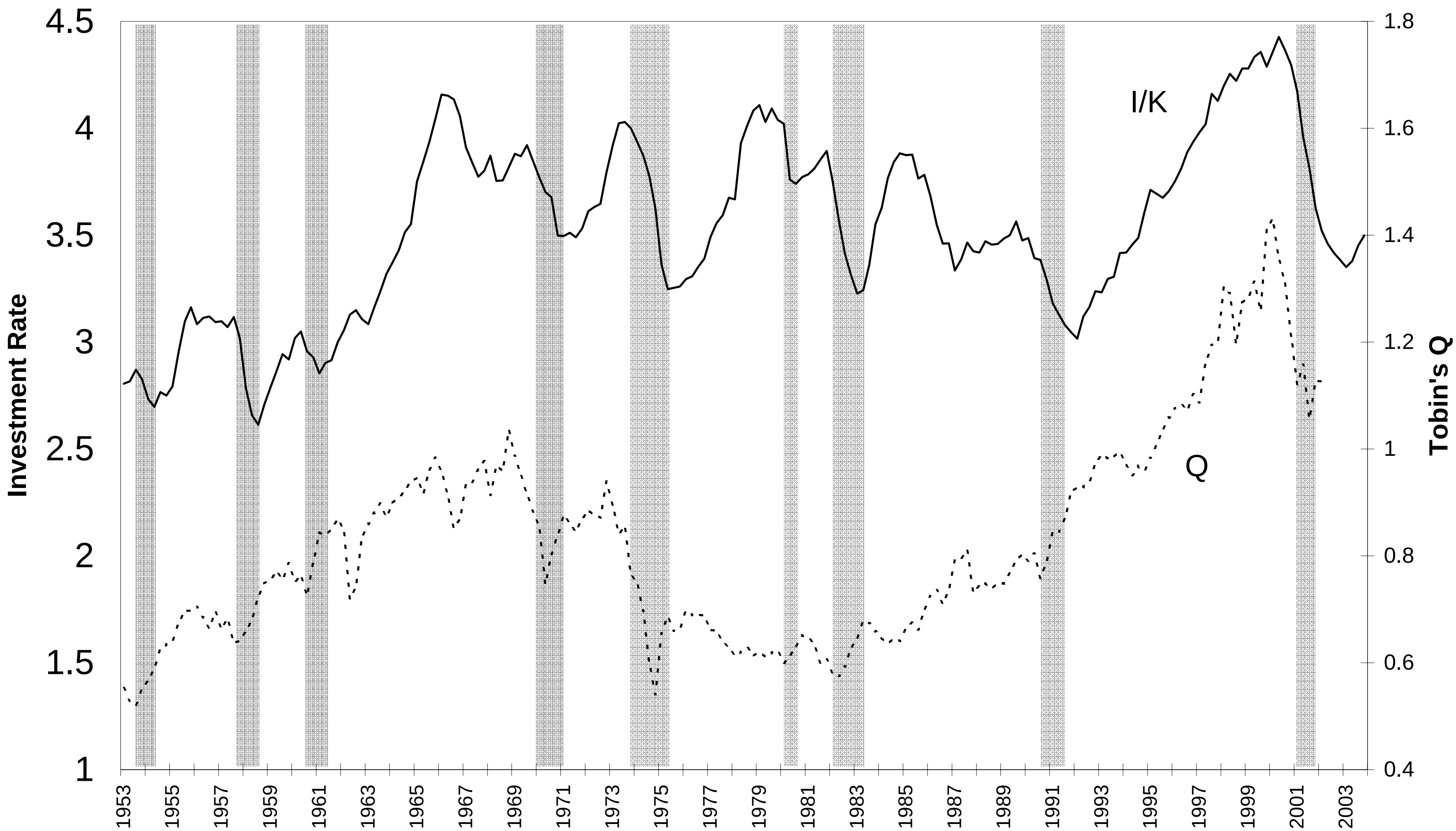
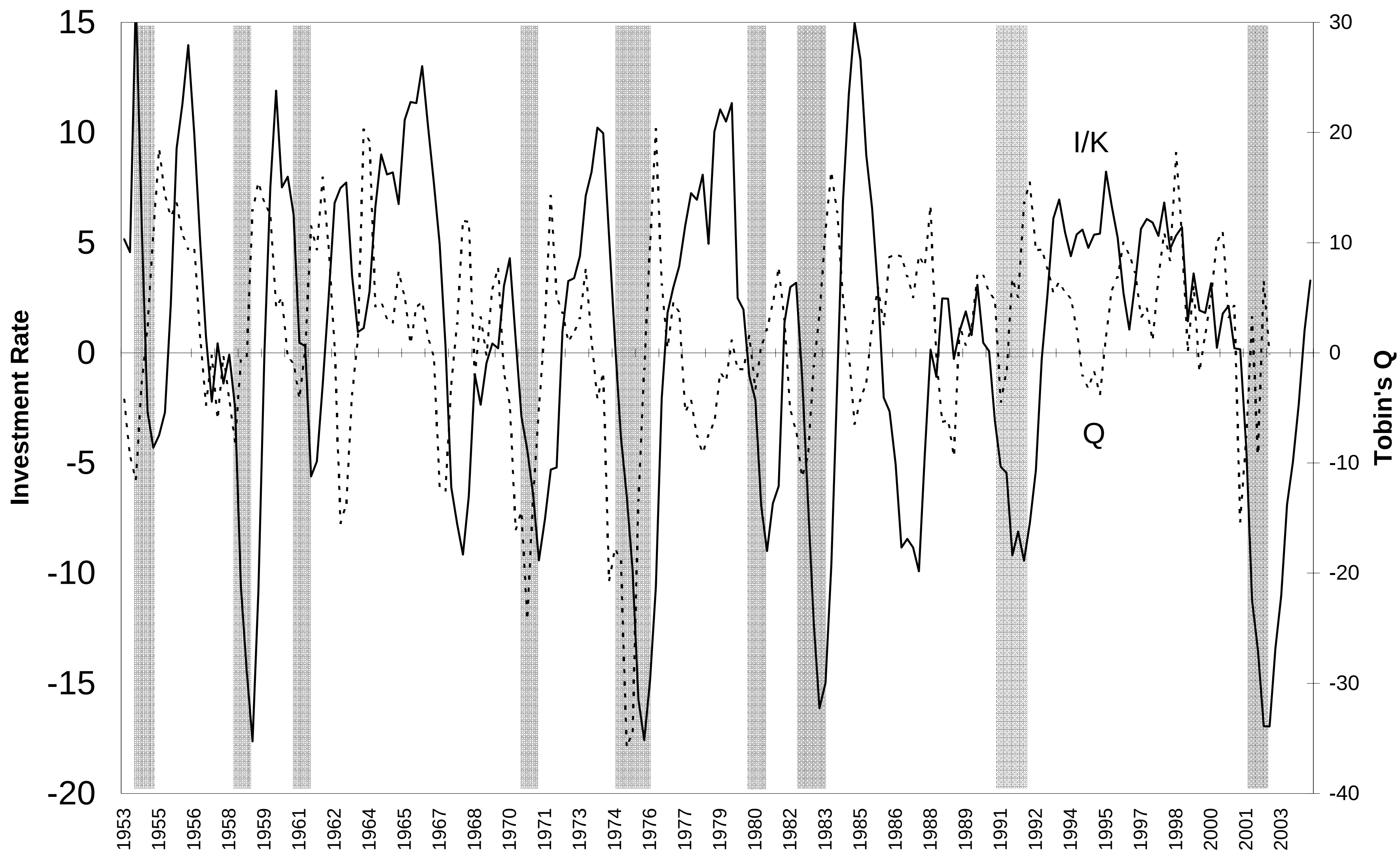


Figure 1 (b): % Changes



In this article, we take the view that in order to understand the relationships between aggregate investment, productivity and the stockmarket over the business cycle, one must deal head-on with the source and timing of productivity fluctuations, the reasons for sectoral co–movement, and the apparent delay in investment in response to incentives. Our starting point for thinking about these issues is Shleifer's (1986) model of "implementation cycles". He shows that in the presence of imperfect competition, the implementation of a productivity improvement by one firm may increase the demand for anothers' products by raising aggregate demand. This induces innovators, who anticipate short-lived profits due to imitation, to delay the implementation of their own innovation until others implement, thereby generating self–enforcing booms in aggregate activity. Though capable of generating both co–movement and delay in implementation, Shleifer's model cannot, however, serve as a framework for understanding investment cycles. This is because the sectoral co–movement that he establishes is not robust to the introduction of capital or, in fact, *any* storable commodity. Anticipating a boom, producers would produce early when wages are low, store the output, and sell in the boom when prices are high, thus undermining the cycle.[11]

Francois and Lloyd-Ellis (2003) show how a similar process of endogenous clustering can arise due to the process of "creative destruction" familiar from Schumpeterian endogenous growth models. Like imitation, potential obsolescence limits incumbency and provides incentives to cluster implementation. However, in their framework, where productive resources are needed to generate new innovations, allowing for the possibility of storage does not rule out clustering, and in fact yields a *unique* cyclical equilibrium.[12] Moreover, because this costly innovation tends to be bunched just before a boom, it causes a downturn in aggregate output (even if the measure of GDP includes this investment).[13] Because of its ability to accommodate storage, this framework is more promising as a vehicle for understanding investment. However, the addition of physical capital which is completely flexible after being installed still undermines their cyclical equilibrium.

Full flexibility of installed physical capital is, however, at odds with recent evidence on investment behavior. In particular, there is considerable direct evidence that many types of physical investment are not reversible and feature inflexible characteristics once installed (see Ramey and Shapiro, 2001, Kasahara, 2002). Doms and Dunne (1993) also document the considerable "lumpiness" of plant level investments, while Cabellero and Engel (1998) demonstrate the high skewness

[11] Shleifer's model is also subject to a number of other criticisms including the fact that there are no downturns and that there is a continuum of multiple cyclical equilibria, making the predictions of the model rather imprecise. Moreover, while the timing of implementation booms is endogenous, the innovations themselves arise exogenously.

[12] Shleifer (1986) assumes that innovations arise exogenously. When innovation is endogenous, growth is intimately related to the business cycle.

[13] Our interpretation of innovation is not R&D, but rather labour–intensive activities such as re–organization or the development of new ideas. While capital–intensive R&D is often found to be mildly procyclical, Francois and Lloyd–Ellis (2003), discuss evidence that other kinds of innovative activities are counter–cyclical.

and kurtosis observed in aggregate investment data.[14] Moreover, the variation in "shiftwork" over the business cycle (see Bresnahan and Ramey 1994, Hammermesh 1989 and Mayshar and Solon, 1993) is consistent with some degree of inflexibility in factor proportions, since it implies that capital is being used less intensively during recessions than is optimal ex ante.

Here we introduce physical capital into the framework developed by Francois and Lloyd-Ellis (2003) in order to understand the business–cycle relationships between investment, productivity and the stockmarket.[15] We model production in a way which captures, as simply as possible, the inflexibility of installed capital relative to uninstalled capital. Specifically, we assume that, once installed, capital becomes irreversible, lumpy and sector–specific. Moreover, we assume that while the ratio of utilized capital to labor hours can be increased as output expands and more capital is added, it cannot be adjusted as output contracts and no new capital is being added.[16] Instead we assume that capital utilization is variable, so that contractions are associated with under–utilization. Our assumptions are similar in effect to the assumption of "putty–clay" production technology.[17]

Along the equilibrium growth path that we study, expansions are triggered by the implementation of accumulated productivity improvements. These improvements arrive stochastically across sectors during the recession, gradually increasing firm values, so that Tobin's Q starts to rise prior to the boom. However, since firms optimally choose to delay implementation, investment lags behind the increase in Q. During the expansion, capital is accumulated continuously and smoothly. At its end, the economy enters a recessionary phase where capital ceases to be accumulated. As demand falls, the fact that the ratio of utilized capital to labor hours is fixed implies that producers continuously reduce capital utilization throughout the recession. This anticipated decline in demand causes Tobin's Q to fall during the preceding investment boom — thus Q leads investment into the recession too.

Although our focus here is on the nature of investment cycles, our results are delivered in a framework where the economy's aggregate fluctuations arise endogenously. There are no simple

[14] A related literature emphasizes the role of plant level investment irreversibilities. However, recent work (see Veracierto, 2002 and Thomas 2002,) in the RBC tradition has found that the effects of such irreversibilities at the aggregate level are virtually non–existent.

[15] Using the simpler model of Shleifer (1986) as a vehicle for this analysis will not work, even with inflexible capital. Storage of *any* kind undermines the clustering of activity there. The endogenous innovation, present in Francois and Lloyd-Ellis (2003), is a necessary part of the equilibrium.

[16] To fix ideas, consider the example of a car manufacturer. As the demand for cars expands, it can add new equipment to a given workforce working at maximum capacity, thereby raising the capital–labour ratio and increasing labour productivity. However, as output contracts the manufacturer retains the installed capital (due to irreversibility), but uses it less intensively and reduces the number of shifts in proportion, so that the ratio of utilized capital to labour hours remains fixed. The lumpiness assumption implies that the manufacturer cannot rent out the capital to another car manfacturer during breaks between shifts.

[17] We discuss the relationship to that literature in Section 2.

causal relationships between the variables of interest studied here, instead all of these are general equilibrium implications arising from the growth process. Expansions are “neoclassical”, supply–side phenomena which directly raise both *potential* output, through the delayed implementation of productivity improvements, and *actual* output through increased production labor, re–utilization of installed capital and subsequent capital accumulation. Recessions are “Keynesian” demand–side contractions during which actual output falls below its potential, investment slows, and some capital resources are left under–utilized. These reductions in aggregate demand are an equilibrium response to the anticipated future expansion, as effort shifts into long–run growth promoting activities, and out of current production.[18]

A key feature of our model is that the owners of physical capital and the owners of intangible capital are distinct entities (e.g. banks and entrepreneurs). In our baseline model, we allow capitalists to offer a sequence of future prices per unit of *utilized* capital that they can *commit* to ex ante. During expansions, threat of entry from replacement capitalists induces the incumbent capitalist to offer a capital price sequence whose present value is just sufficient to cover the cost of the capital. However, during downturns, the competition faced by incumbent capitalists is diminished and, if they could, they would raise their price above the competitive level that they had originally offered. By assuming that incumbent capitalist are committed to prices offered before the downturn occurs, we effectively rule out such opportunistic behavior. In an extended version of the model (Section 9) we show that the same outcomes can be supported through endogenous, incomplete contracts.

The paper proceeds as follows: Section 2 discusses the relationship between this paper and others in the literature. Section 3 sets up the basic model and Section 4 posits and describes the cyclical growth path. Section 5 develops the implications for the movement of key aggregates and prices through the posited cycle. Section 6 sets up the key existence conditions and Section 7 characterizes the stationary cyclical growth path. Section 8 explores the main implications of the cycle and Section 9 shows that our results are robust to allowing a greater range of contracting possibilities. Section 10 concludes. All proofs are in the Appendix.

2 Relationship to the Previous Literature

A standard way to think about the relationship between investment and Tobin’s Q (common in the RBC literature) is to abstract from issues regarding intangibles, but to assume that capital is subject to quadratic costs of adjustment. The prediction of such a model is that investment should exhibit a positive contemporaneous relationship with Tobin’s Q. Adding additional constraints

[18] Here we are assuming that all labour is skilled and is mobile across sectors. As we discuss in our conclusion, introducing unskilled labour with putty–clay production would also result in unemployment during recesions.

such as a "time to build" assumption helps to smooth out the response of investment to measured incentives, but this alone cannot capture the observed delay of 3 to 4 quarters. In order to capture the lead–lag relationship discussed above, Christiano and Todd (1996), Bernanke, Gertler and Gilchrist (1999) and Christiano and Vigfusson (2001) also introduce a notion of "time to plan" — a fixed time period between the date when the decision to invest more (less) is made and the date when the actual funds are allocated. Although this "does the job" in some sense, the assumption is somewhat ad hoc. Our approach offers an alternative rationalization that endogenizes the delay as a result of strategic timing decisions.

A second common approach to thinking about the relationship between Tobin's Q and investment emphasizes the role of intangibles in affecting the economy–wide value of firms. Hall (2001b), Hobijn and Jovanovic(2001) and Laitner and Stolyarov (2003), for example, all emphasize the long run implications of the IT revolution, the anticipation of which is dated to the early 1970's. The idea is that the stock market moved immediately with the arrival of the information, but investment was delayed until the 1990s. Laitner and Stolyarov (2003) emphasize the capital and knowledge obsolescence caused by the arrival of such a general purpose technology (GPT). However, while their framework is applicable to long–term cycles, there is little evidence supporting the arrival of GPTs at business cycle frequencies (see Jovanovic and Lach, 1998, and Andolfatto and Macdonald, 1998).[19]

Our model incorporates the role of *both* knowledge capital and adjustment costs in determining the relationship between investment and Tobin's Q over the business cycle. With endogenous innovation, of course, a component of firm values must reflect the returns to intangible investments. In addition, our assumptions regarding capital can be viewed as reflecting a form of asymmetric adjustment costs (see also Cabellero, 1999). When expanding, capital adjustment is unimpeded but, once installed, capital is prohibitively costly to adjust and cannot be converted into a consumption good, nor into another capital good with different capital-labor intensity. In fact, our assumptions regarding the ex post inflexibility of capital are similar to those of the "putty-clay" model (Johansen, 1959), except that here capital is not vintage–specific and is infinitely lived.[20] As in the putty-clay model, however, the irreversibility we assume implies a tight connection between changes in demand and changes in employment and capacity utilization. Our assumption that investments are lumpy, in that they cannot be partly dismantled and used elsewhere is also consistent with micro evidence (see Cooper, Haltiwanger and Power, 1999).

Most previous work on endogenous cycles and growth has been restricted to single sector

[19] Indeed, Laitner and Stolyarov (2003) cite evidence suggesting there have only been seven major technological innovations of this kind identified in the last 200 years.

[20] Fuss (1977) and Gilchrist and Williams (2000) present evidence supporting a putty-clay view of capital.

settings.[21] These works cannot be translated readily into a multi-sector setting because they include no force generating co-movement. One exception is the model developed by Matsuyama (1999, 2001)[22] who, like Shleifer (1986), emphasizes the role of short–lived monopoly power due to exogenous imitation. The cycles that arise in his model do not depend on delay to generate cyclical behavior, and are thus robust to capital accumulation through the cycle. However, Matsuyama's framework is more suited to understanding longer–term movements in the *nature* of growth, rather than business cycle fluctuations.[23] In particular, there is no phase of his cycle that could be called a recession: production and consumption never decline, and capital is always fully utilized.[24]

3 The Model

3.1 Assumptions

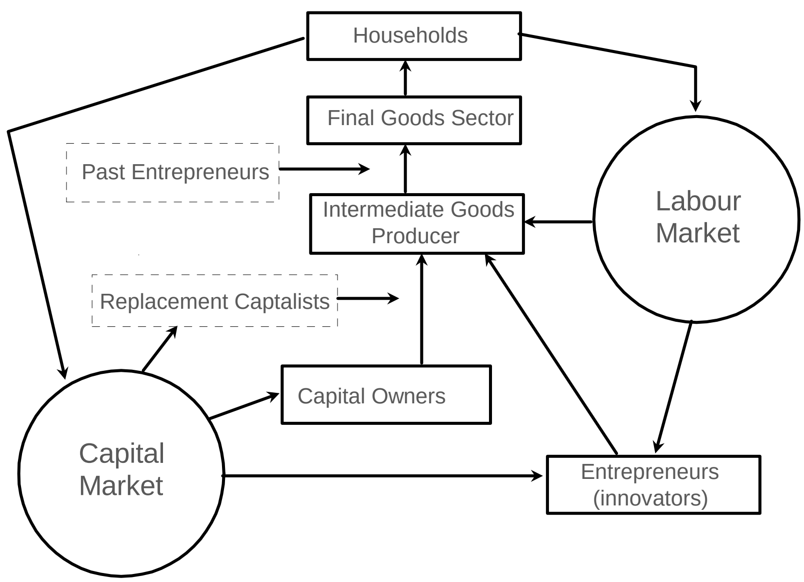


Figure 1: Structure of the Economy

The structure of the economy is illustrated in Figure 1. There is no aggregate uncertainty. Time is continuous and indexed by $t \geq 0$. The economy is closed and there is no government

[21] Jovanovic and Rob (1990), Cheng and Dinopolous (1992), Helpman and Trajtenberg (1998), Li (2000), Aghion and Howitt (1998), Evans, Honkephoja and Romer (1998) and Walde (2001).

[22] Another exception is Francois and Shi (1999), but that model inherits the lack of robustness to storage in Shleifer (1986).

[23] Indeed, Matsuyama is careful to apply his model to longer term issues such as the US productivty slowdown.

[24] Morover the innovative process is capital intensive, suggesting R&D plays a central role.

sector. The representative household has isoelastic preferences

$$U(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \frac{C(\tau)^{1-\sigma} - 1}{1 - \sigma} d\tau \tag{1}$$

where ρ denotes the rate of time preference and σ represents the inverse of the elasticity of intertemporal substitution. The household maximizes (1) subject to the intertemporal budget constraint

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]} C(\tau) d\tau \leq S(t) + \int_t^{\infty} e^{-[R(\tau)-R(t)]} w(\tau) d\tau \tag{2}$$

where $w(t)$ denotes wage income, $S(t)$ denotes the household's stock of assets (firm shares and capital) at time t and $R(t)$ denotes the discount factor from time zero to t. The population is normalized to unity and each household is endowed with one unit of labor hours, which it supplies inelastically.

Final output can be used for the production of consumption, $C(t)$, investment, $\dot{K}(t)$, or can be stored at an arbitrarily small flow cost of $\nu > 0$ per unit time. It is produced by competitive firms according to a Cobb–Douglas production function utilizing a continuum of intermediates, x_i, indexed by $i \in [0, 1]$:

$$C(t) + \dot{K}(t) \leq Y(t) = \exp\left(\int_0^1 \ln x_i(t) di \right). \tag{3}$$

For simplicity we also assume that there is no physical depreciation. Intermediates are completely used up in production, but can be produced and stored for later use. Incumbent intermediate producers must therefore decide whether to sell now, or store and sell later at the flow storage cost ν.

Output of intermediate i depends upon the state of technology in sector i, $A_i(t)$, the stock of installed capital, $K_i(t)$, the variable rate at which that capital is utilized, $\lambda_i(t) \leq 1$, and labor hours, $L_i(t)$, according to the following production technology:

$$x_i(t) = \begin{cases} K_i(t)^{\alpha} \left[A_i(t) L_i(t)\right]^{1-\alpha} & \text{if } x_i(t) \geq x_i(z) \\ K_i(z)^{\alpha} \left[A_i(t) L_i(z)\right]^{1-\alpha} \min\left[\lambda_i(t),\ \frac{L_i(t)}{L_i(z)}\right] & \text{if } x_i(t) < x_i(z). \end{cases} \tag{4}$$

Here

$$z = \arg\max_{s<t} \{x_i(s)\} \tag{5}$$

is the date at which the last increment to capital was installed and $\{K_i(z), L_i(z)\}$ is the capital–labor combination chosen at that date. Labor hours are perfectly mobile across sectors but, installed capital, $K_i(t)$, is sector–specific, irreversible and non-divisible, so that any part of it that is not utilized cannot be used elsewhere (i.e. $\dot{K}_i \geq 0$). We denote the level of utilized capital by $K_i^u(t) = \lambda(t) K_i(t)$.

An implication of this structure is that during an expansion, when new capital is being built, a firm's ability to substitute between capital and labour is represented by Cobb–Douglas production isoquants (curved in Figure 2). However during a contraction, when the firm produces below capacity, its production possibilities are represented by Leontief production isoquants whose kink points lie along a ray from the origin to the chosen point on the full-capacity isoquant. In such a situation, the installed capital will optimally be used less intensively in proportion to the labor hours allocated to production. One interpretation of this is that there are fewer shifts.

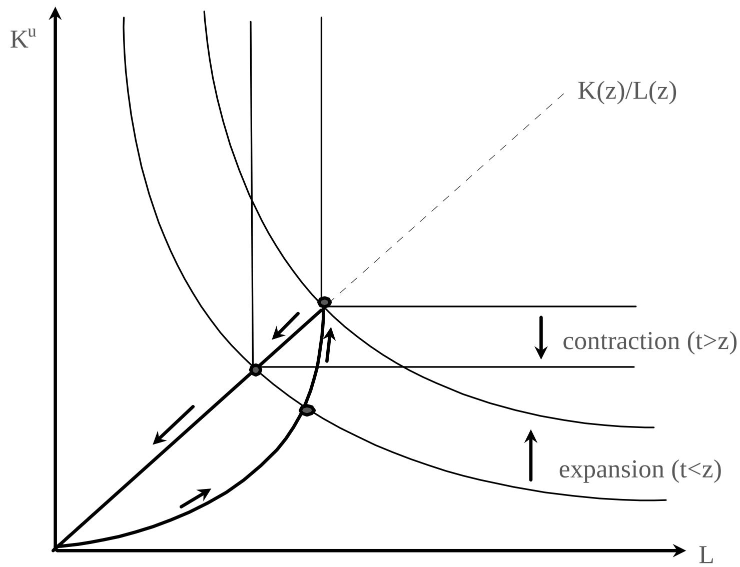


Figure 2: Implications of Inflexiblity of Installed Capital

3.1.1 Innovation

The innovation process is exactly as in the quality–ladder model of Grossman and Helpman (1991). Competitive entrepreneurs in each sector allocate labor effort to innovation, and finance this by selling equity shares to households. The probability of success in instant t is $\delta H_i(t)$, where δ is a parameter, and H_i represents the labor hours allocated to innovation in sector i. At each date, entrepreneurs decide whether or not to allocate labor hours to innovation, and if they do so, how much. The aggregate labor hours allocated to innovation is given by $H(t) = \int_0^1 H_i(t)dt$.

New ideas and innovations dominate old ones by a factor e^{γ}. Successful entrepreneurs must choose whether or not to implement their innovation immediately or delay implementation until a later date.[25] Once they implement, the associated knowledge becomes publicly available, and

[25] We adopt a broad interpretation of innovation. Recently, Comin (2002) has estimated that the contribution of

can be built upon by rivals. However, prior to implementation, the knowledge is privately held by the entrepreneur.[26] We let the indicator function $Z_i(t)$ take on the value 1 if there exists a successful innovation in sector i which has not yet been implemented, and 0 otherwise. The set of instants in which new ideas are implemented in sector i is denoted by Ω_i. We let $V_i^I(t)$ denote the expected present value of profits from implementing a success at time t, and $V_i^D(t)$ denote that of delaying implementation from time t until the most profitable future date.

3.1.2 The Market for Fixed Capital

Entrepreneurs cannot simply "sell" their idea to capital owners, but must be involved in its implementation. We assume entrepreneurs have insufficient wealth to purchase the capital stock needed to implement, and hence must effectively rent it from capital owners (e.g. banks). In the basic version of our model, we assume that the capitalist is able to offer a rental price sequence *per unit of utilized capital* into the indefinite future $\{q_i(\tau)\}_{\tau=t}^{\infty}$. We assume that the price sequence represents a binding commitment that cannot be adjusted ex post. Under such a price sequence the present value of the capitalist's net income in sector i is therefore:

$$V_i^K(t) = \int_t^{\infty} e^{-[R(\tau)-R(t)]} \left[q_i(\tau)\,\lambda_i(\tau) K_i(\tau) - \dot{K}_i(\tau) \right] d\tau. \tag{6}$$

Since capital is sector specific, the price sequence that is offered in equilibrium is determined by the possibility of a "replacement capitalist" building an alternative capital stock to displace that of the current capital owner within the sector (see Figure 1). If the threat of entry were always sufficient to induce competitive pricing,[27] there would be no need for long–term price commitments. In the cyclical equilibrium that we study, however, the threat of entry is sufficient to lead to competitive pricing *only* during expansions. During contractions, replacement capitalists have reduced incentives for entry which, in the absence of price commitments, would allow incumbent capitalists to price gouge. Anticipating this, entrepreneurs demand binding price commitments from capital suppliers before entering the recession. A capital owner unwilling to provide such a commitment will be passed over in favor of a replacement capitalist who is.

It may seem unusual to assume that capital owners charge a rental price per unit of utilized rather than installed capital. This assumption simplifies the exposition considerably, by allowing

measured R&D to productivity growth in the US is less that 1/2 of 1%. As he notes, a larger contribution is likely to come from unpatented managerial and organizational innovations.

[26] Even for the case of intellectual property, Cohen, Nelson and Walsh (2000) show that firms make extensive use of secrecy in protecting productivity improvements. Secrecy likely plays a more prominent role for entrepreneurial innovations, which are the key here.

[27] In order to maintain competition in capital supply it will be assumed that, in the event of a competing capital stock being built, ties in tended prices are always broken in favour of the entrant. Due to storage costs, entry of replacement capital will imply scrapping of the pre-existing stock.

us to decentralize the decisions of entrepreneurs and capital owners. In Section 9, we show that the equilibrium price sequences can be replicated as part of an endogenous, incomplete contracting equilibrium, in which contracts optimally specify the rental price of *installed* capital and the utilization rate.

3.2 Definition of Equilibrium

Given initial state variables[28] $\{A_i(0),\, Z_i(0),\, K_i\,(0)\}_{i=0}^{1}$, an equilibrium for this economy consists of:

(1) sequences $\left\{\hat{p}_t(t),\ \widehat{q}_i(t),\ \hat{\lambda}_i(t),\, \hat{x}_i(t).\ \widehat{K}_i(t),\ \widehat{L}_i(t),\ \widehat{H}_i(t),\ \widehat{A}_i(t),\ \widehat{Z}_i(t),\ \widehat{V}_i^I\,(t)\,,\ \widehat{V}_i^D\,(t)\,,\ \widehat{V}_i^K\,(t)\right\}_{t\in[0,\infty)}$ for each intermediate sector i, and

(2) economy wide sequences $\left\{\widehat{Y}(t),\ \widehat{R}\,(t)\,,\ \widehat{w}\,(t)\,,\ \widehat{C}\,(t)\,,\ \widehat{S}\,(t)\right\}_{t\in[0,\infty)}$

which satisfy the following conditions:

• Households allocate consumption over time to maximize equation (1) subject to the budget constraint, equation (2). The first–order conditions of the household's optimization require that

$$\widehat{C}(t)^{\sigma} = \widehat{C}(\tau)^{\sigma} e^{\widehat{R}(t)-\widehat{R}(\tau)-\rho(t-\tau)} \qquad \forall\ t,\tau, \tag{7}$$

and that the transversality condition holds

$$\lim_{\tau\to\infty} e^{-\widehat{R}(\tau)}\widehat{S}(\tau) = 0 \tag{8}$$

• Final goods producers choose intermediates, x_i, to minimize costs given prices p_i, subject to (3). The derived demand for intermediate i is

$$x_i^d(t) = \frac{Y(t)}{p_i(t)}. \tag{9}$$

• Intermediate goods producers choose combinations of utilized capital $K_i^u(t)$, and labour hours, $L_i(t)$ to minimize costs given factor prices, subject to (4):

$$K_i^u(t) = \frac{x_i(t)}{A_i^{1-\alpha}(t)}\left[\left(\frac{\alpha}{1-\alpha}\right)\frac{w(t)}{q_i(t)}\right]^{1-\alpha} \quad \text{and} \quad L_i(t) = \frac{x_i(t)}{A_i(t)^{1-\alpha}}\left[\left(\frac{1-\alpha}{\alpha}\right)\frac{q_i(t)}{w(t)}\right]^{\alpha}. \tag{10}$$

• The unit elasticity of demand for intermediates implies that limit pricing at the unit cost of the previous incumbent is optimal. It follows that

$$p_i(t) = \frac{q_i\,(t)^{\alpha}\,w\,(t)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}e^{-(1-\alpha)\gamma}A_i^{1-\alpha}(t)} \qquad \forall t \tag{11}$$

[28]Without loss of generality, we assume no stored output at time 0.

The resulting instantaneous profit earned in each sector is given by

$$\pi(t) = (1 - e^{-(1-\alpha)\gamma})Y(t). \tag{12}$$

• Capital owners buy final output in the form of new capital if and only if $K_i^u(t) = K_i(t)$ and $\dot{K}_i^u(t) > 0$ and rent it to intermediate producers.

• Labor markets clear:

$$\int_0^1 \widehat{L}_i(t)di + \widehat{H}(t) = 1 \tag{13}$$

• Arbitrage trading in financial markets implies that, for all assets that are held in strictly positive amounts by households, the rate of return between time t and time s must equal $\frac{\widehat{R}(s)-\widehat{R}(t)}{s-t}$.

• Free entry into innovation — entrepreneurs select the sector in which they innovate so as to maximize the expected present value of the innovation, and

$$\delta \max[\widehat{V}_i^D(t), \widehat{V}_i^I(t)] \leq \widehat{w}(t), \quad \widehat{H}_i(t) \geq 0 \qquad \text{with at least one equality.} \tag{14}$$

• At instants where there is implementation, entrepreneurs with innovations must prefer to implement rather than delay until a later date

$$\widehat{V}_i^I(t) \geq \widehat{V}_i^D(t) \quad \forall\, t \in \widehat{\Omega}_i. \tag{15}$$

• At instants where there is no implementation, either there must be no innovations available to implement, or entrepreneurs with innovations must prefer to delay rather than implement:

$$\begin{aligned} \text{Either } \widehat{Z}_i(t) &= 0, \\ \text{or if } \widehat{Z}_i(t) &= 1, \ \widehat{V}_i^I(t) \leq \widehat{V}_i^D(t) \quad \forall\, t \notin \widehat{\Omega}_i. \end{aligned} \tag{16}$$

• Free entry into final output production.

• Free entry of replacement capital: $\widehat{V}_i^K(t) \leq \widehat{K}_i(t)$, where $\widehat{V}_i^K$ is the value of capital determined under the value maximizing sequence of price commitments.

4 The Posited Cyclical Growth Path

In this section, we informally posit a cyclical equilibrium growth path and the behavior of agents in the economy. We then detail the implications for investment, consumption and innovation.[29] In Section 5, we formally derive the implications of this behavior over each phase of the cycle, and Section 6 then demonstrates the consistency of the posited behavior of entrepreneurs and capitalists in an equilibrium steady state, and derives the conditions for existence.

[29] There is a second equilibrium balanced growth path along which growth is constant and innovations are always implemented immediately. We chracterize this "standard" growth path in Appendix B.

Figures 3 and 4 depict the movement of key variables during the cycle. Cycles are indexed by the subscript v, and feature a consistently recurring pattern through their phases. The vth cycle features three distinct phases:

• The **expansion** is triggered by a productivity boom at time T_{v-1} and continues through subsequent capital accumulation, leading to continued growth in output, consumption and wages. Over this phase interest rates fall and investment declines. Since its productivity in manufacturing is high, no labor is allocated to innovation. As capital accumulates the returns to physical investment decline, while the return to innovation grows as the subsequent boom approaches. Eventually innovation and reorganization re-commence, drawing labor hours from production. Due to the rigidities of installed capital, the marginal product of capital drops to zero.

• The **contraction** thus starts with a collapse in fixed capital formation at time T_v^E. Intermediate producers experience a reduction in aggregate demand and cut back on the labour hours they employ in production. This labour effort is optimally re-allocated to relatively labor–intensive innovation and re–organization. Successful entrepreneurs find it optimal to delay implementation until the boom. Due to the rigidity of installed capital, labor's departure from production implies that capital is not fully utilized. Through the downturn the economy continues to contract through declining consumption expenditure, capital utilization falls and innovation and reorganization continue to increase.

• The **boom** occurs at an endogenously determined date, T_v, when the value of implementing stored innovations first exceeds the value of delaying their implementation. At that point, successful entrepreneurs implement, starting the upswing once again. The returns to production rise above those of innovation, drawing labor back into production. Returns to capital also rise with the new more productive technologies, so that capital accumulation recommences and the cycle begins again.

4.1 Consumption

Since the discount factor jumps up at the boom, consumption exhibits a discontinuity during implementation periods. The optimal evolution of consumption from the beginning of one cycle to the beginning of the next is given by the difference equation

$$\sigma \ln \frac{C_0(T_v)}{C_0(T_{v-1})} = R_0(T_v) - R_0(T_{v-1}) - \rho\left(T_v - T_{v-1}\right). \tag{17}$$

where the 0 subscript is used to denote values of variables the instant after the implementation boom.[30] Note that a sufficient condition for the boundedness of the consumer's optimization

[30]Formally, for any variable $X(\cdot)$, we define $X(t) = \lim_{\tau \to t^-} X(\tau)$ and $X_0(t) = \lim_{\tau \to t^+} X(\tau)$.

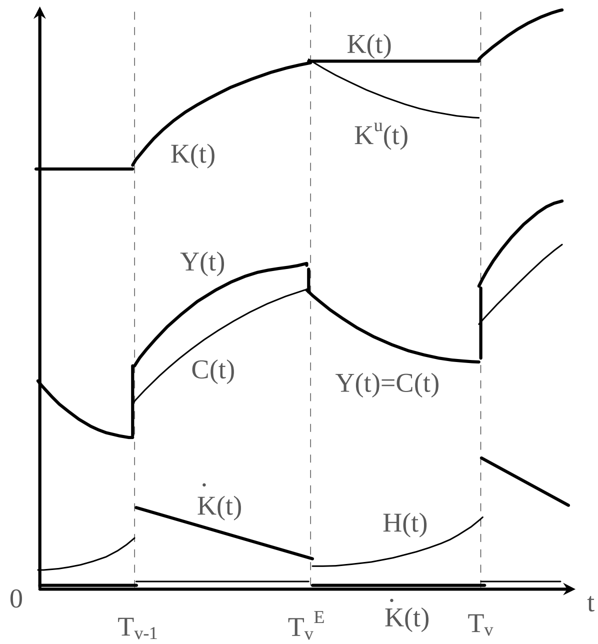


Figure 3: Evolution of Aggregates over the Cycle

problem is that $\ln \frac{C_0(T_v)}{C_0(T_{v-1})} < R(T_v) - R(T_{v-1})$ for all v, or that

$$\frac{(1-\sigma)}{T_v - T_{v-1}} \ln \frac{C_0(T_v)}{C_0(T_{v-1})} < \rho \quad \forall\ v. \tag{18}$$

The discount factor used to discount from some time t during the cycle to the beginning of the next cycle is given by

$$\beta(t) = R_0(T_v) - R(t) = R_0(T_v) - R_0(T_{v-1}) - \int_{T_{v-1}}^{t} r(s)ds. \tag{19}$$

4.2 Innovation and Implementation

Let $P_i(s)$ denote the probability that, since time T_{v-1}, no entrepreneurial success has been made in sector i by time s. It follows that the probability of there being no entrepreneurial success by time T_v conditional on there having been none by time t, is given by $P_i(T_v)/P_i(t)$. Hence, the value of an incumbent firm in a sector where no entrepreneurial success has occurred by time t during the vth cycle can be expressed as

$$V_i^I(t) = \int_t^{T_v} e^{-\int_t^\tau r(s)ds} \pi_i(\tau) d\tau + \frac{P_i(T_v)}{P_i(t)} e^{-\beta(t)} V_{0,i}^I(T_v). \tag{20}$$

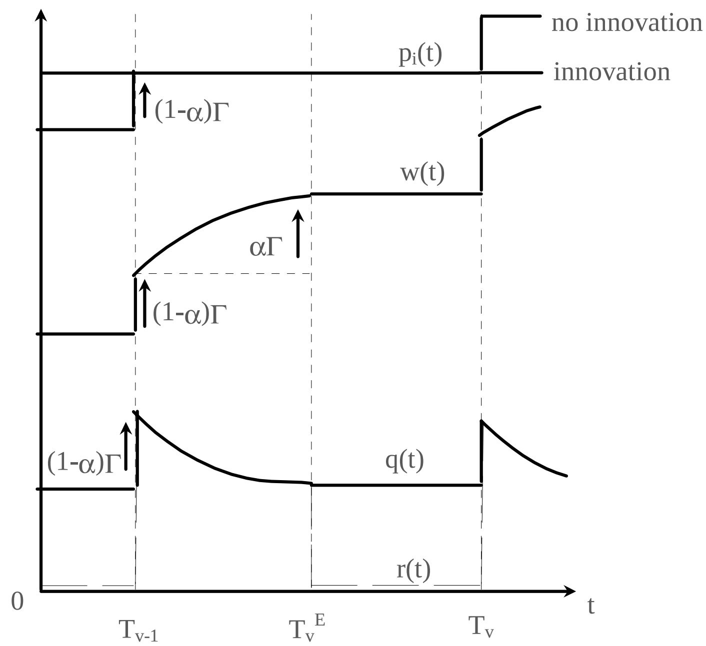


Figure 4: Evolution of Prices over the Cycle

The first term here represents the discounted profit stream that accrues to the entrepreneur with certainty during the current cycle, and the second term is the expected discounted value of being an incumbent thereafter.

Lemma 1 *In a cyclical equilibrium, successful entrepreneurs can credibly signal a success immediately and all entrepreneurship in their sector will stop until the next round of implementation.*

Unsuccessful entrepreneurs have no incentive to falsely announce success. As a result, an entrepreneur's signal is credible, and other entrepreneurs will exert their efforts in sectors where they have a better chance of becoming the dominant entrepreneur.

In the cyclical equilibrium, entrepreneurs' conjectures ensure no more entrepreneurship in a sector once a signal of success has been received, until after the next implementation. The expected value of an entrepreneurial success occurring at some time $t \in \left(T_v^E, T_v\right)$ but whose implementation is delayed until time T_v is thus:

$$V_i^D(t) = e^{-\beta(t)} V_{0,i}^I(T_v). \tag{21}$$

In the cyclical equilibrium, such delay is optimal; i.e. $V_i^D(t) > V_i^I(t)$ throughout the contraction. Successful entrepreneurs are happier to forego immediate profits and delay implementation until the boom in order to ensure a longer reign of incumbency. Since no implementation occurs during the cycle, by delaying, the entrepreneur is assured of incumbency until at least T_{v+1}. Incumbency beyond that time depends on the probability that there has not been another entrepreneurial success in that sector up until then.[31]

The symmetry of sectors implies that entrepreneurial effort is allocated evenly over all sectors that have not yet experienced a success within the cycle. This clearly depends on some sectors not having already received an entrepreneurial innovation, an equilibrium condition that will be imposed subsequently (see Section 6). Thus the probability of not being displaced at the next implementation is

$$P_i(T_v) = \exp\left(-\int_{T_v^E}^{T_v} \widetilde{H}_i(\tau)d\tau\right), \tag{22}$$

where $\widetilde{H}_i(\tau)$ denotes the quantity of labor that would be allocated to entrepreneurship if no entrepreneurial success had occurred prior to time τ in sector i. The amount of entrepreneurship varies over the cycle, but at the beginning of each cycle all industries are symmetric with respect to this probability: $P_i(T_v) = P(T_v)\ \forall i$.

5 The Three Phases of the Cycle

5.1 The (Neoclassical) Expansion

During the expansion the economy's dynamics are essentially identical to those of the Ramsey model with no technological change:[32]

Proposition 1 *: During the expansion, capital and consumption evolve according to:*

$$\frac{\dot{C}(t)}{C(t)} = \frac{\alpha e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}K(t)^{\alpha-1} - \rho}{\sigma} \tag{23}$$

$$\dot{K}(t) = \overline{A}_{v-1}^{1-\alpha}K(t)^{\alpha} - C(t). \tag{24}$$

To understand this, observe that between implementation periods, consumption satisfies the familiar differential equation:

$$\frac{\dot{C}\,(t)}{C\,(t)} = \frac{r\,(t) - \rho}{\sigma}, \tag{25}$$

where $r\,(t) = \dot{R}\,(t)$. As long as utilized capital is anticipated to grow, capitalists never acquire more capital than is needed for production, so that $K_i^u(t) = K_i(t)$. The existence of potential

[31] A signal of further entrepreneurial success submitted by an incumbent is not credible in equilibrium because incumbents have incentive to lie to protect their profit stream. No such incentive exists for entrants since, without a success, profits are zero. Note also that the reason for delay here differs from Shleifer (1986) where the length of incumbency is exogenously given.

[32] Note that, unlike the Ramsey model, the rate of return on savings is not equal to the marginal product of capital, but rather is a fraction $e^{-(1-\alpha)\gamma}$ of it. This reflects the entrepreneurial share of this marginal product accruing as a monopoly rent.

replacement capitalists implies that capital owners cannot earn excess returns on marginal units, so that $q_i(t) = q(t) = r(t)$. It follows that all firms choose the same capital–labour ratio and capital is rented at a competitive price equal to its marginal product net of profits to the entrepreneur:

$$r(t) = q(t) = \alpha e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} K(t)^{\alpha-1}. \tag{26}$$

where

$$\overline{A}_{v-1} = \exp\left(\int_0^1 \ln A_i(T_{v-1}) di\right).$$

denotes the state of technology in use during the vth cycle. With all labour allocated to production, it also follows that aggregate final output can be expressed as

$$Y(t) = \overline{A}_{v-1}^{1-\alpha} K(t)^{\alpha}. \tag{27}$$

With technology fixed during this phase, the price of capital declines and the wage rises in proportion to the capital stock. Since the next implementation boom is some time away, the present value of engaging in entrepreneurship initially falls below the wage, $\delta V^D(t) < w(t)$, so that no labor is allocated to innovation or re–organization. During the expansion, $\delta V^D(t)$, grows at the rate of interest and eventually equals $w(t)$.[33] At this point, if all workers were to remain in production, returns to entrepreneurship would strictly dominate those in production. As a result, labor hours are re–allocated from production and into innovation, which triggers the contractionary phase.

5.2 The (Keynesian) Contraction

As labour is withdrawn from production, the ratio of utilized capital to labour hours and technology are both fixed. Consequently, the marginal product of capital remains constant. A further implication is that, through the contraction, the wage remains constant:

Lemma 2 *: The wage for $t \in [T_v^E, T_v]$ is constant and determined by the level of technology and the capital–labor ratio chosen at the last peak, $K(T_v^E)$:*[34]

$$w(t) = \bar{w}_v = (1-\alpha) e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} K(T_v^E)^{\alpha}. \tag{28}$$

Since there is free entry into entrepreneurship, $w(t) = \delta V^D(t)$, and so the value of entrepreneurship, $\delta V^D(t)$, is also constant. Since the time until implementation for a successful entrepreneur is falling and there is no stream of profits (because implementation is delayed), the

[33] We derive conditions which ensure this is the case subsequently.

[34] Since the labour force is normalized to unity, the capital–labour ratio equals the capital stock during the expansion.

instantaneous interest rate necessarily equals zero. If it were not, entrepreneurial activity would be delayed to the instant before the boom. Therefore:

$$r(t) = \frac{\dot{V}^D(t)}{V^D(t)} = \frac{\dot{w}(t)}{w(t)} = 0. \tag{29}$$

Note that this zero interest rate is consistent with the fact there is now excess (under–utilized) capital in the economy. Since marginal returns to new capital in this phase are zero, physical investment ceases and the only investment is that in innovation, undertaken by entrepreneurs.

Lemma 3 *: At T_v^E, investment in physical capital falls discretely to zero and entrepreneurship jumps discretely to $H_0(T_v^E) > 0$.*

A switch like this across types of investment is also a feature of the models of Matsuyama (1999, 2001) and Walde (2002). However, here factor intensity differences between innovation and production lead to a protracted recession.

Although investment falls discretely at $t = T_v^E$, consumption is constant across the transition between phases because the discount factor does not change discretely. With the fixed ratio of utilized capital to labor hours, the decline in output due to the fall in investment demand is proportional to the fraction of labor hours withdrawn from production. It follows that the fraction of labor hours allocated to entrepreneurship at the start of the downturn, $H_v = H_0(T_v^E)$, equals the rate of investment at the peak of the expansion:

$$H_v = \frac{\dot{K}\left(T_v^E\right)}{Y\left(T_v^E\right)} = 1 - \frac{C\left(T_v^E\right)}{\overline{A}_{v-1}^{1-\alpha} K(T_v^E)^{\alpha}}. \tag{30}$$

Although consumption cannot fall discretely at T_v^E, the zero interest rate implies that consumption must be declining after T_v^E,[35]

$$\frac{\dot{C}(t)}{C(t)} = -\frac{\rho}{\sigma}, \tag{31}$$

as resources flow out of production and into entrepreneurship.

Since $Y(t) = C(t)$, the growth rate in the hours allocated to production is also given by (31) and so aggregate entrepreneurship at time t is given by

$$H(t) = 1 - (1 - H_v)\, e^{-\frac{\rho}{\sigma}[t - T_v^E]}. \tag{32}$$

Note that due to the fixed capital-labor ratio, as labor leaves current production, capital utilization falls in the same proportion. It follows that the capital utilization rate specified in (10) is given by

$$\lambda(t) = (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}. \tag{33}$$

[35] Although $r = 0$, strict preference for zero storage results from the arbitrarily small storage costs.

5.2.1 The Rental Price of Capital during Downturns

In the absence of a capital price commitment, intermediate producers would be vulnerable to an increasing rental price through the downturn. This is because replacement capital owners are better off waiting until the boom, when capital will be relatively cheap, rather than displacing the incumbent immediately and earning the rental rate for a relatively short time. To see this formally, observe that, in order to forestall entry by a competing capitalist, the incumbent capitalist is constrained to offer a price sequence, $q(\tau)$ and induced capital utilization, $\lambda(\tau)$ which satisfies

$$V^K (K(t), t) \leq K(t), \tag{34}$$

where $V^K (K(t), \tau)$ denotes the value of the installed capital at time τ. During the downturn $r(t) = 0$ and $\dot{K}(\tau) = 0$, so that for $t \in \left[T_v^E, T_v\right]$, the condition can be expressed as

$$\int_t^{T_v} q(\tau) \lambda(\tau) K(T_v^E) dt + e^{-\beta(T_v)} V^K \left(K(T_v^E), T_v\right) \leq K(T_v^E). \tag{35}$$

However competition from potential replacement capitalists at the beginning of the next cycle ensures that $V^K \left(K(T_v^E), T_v\right) = K(T_v^E)$. Dividing by $K(T_v^E)$ and re–arranging, using (33), yields a necessary restriction to forestall entry during the downturn:

$$(1 - H_v) \int_t^{T_v} q(\tau) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau \leq 1 - e^{-\beta(T_v)}. \tag{36}$$

The right hand side of this expression is constant throughout the downturn, but the left–hand side decreases through the downturn for a given sequence $\{q(\tau)\}_{\tau=t}^{T_v}$. It follows that, in the absence of a binding price commitment, the capitalist could raise $q(\tau)$ above what had previously been offered and still satisfy (36). Anticipating the potential for such price gouging, entrepreneurs demand the commitment before T_v^E, while the cost of replacement capital is still low. Any such price commitment must satisfy (36) which will bind at $t = T_v^E$:

Lemma 4 *Any price commitment $q_i^c(\tau)$ signed at some date $t \in [T_{v-1}, T_v^E)$ must satisfy:*

$$\int_{T_v^E}^{T_v} q^c(\tau) (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau = 1 - e^{-\beta(T_v)} \tag{37}$$

There are a number of price sequences $q^c(\tau)$ that could satisfy this condition, however the average level of prices satisfying it through $t \in [T_{v-1}, T_v^E)$ is unique. Let this average in the *vth* cycle be

$$\overline{q}_v \equiv \frac{\int_{T_v^E}^{T_v} q^c(\tau) (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau}{\int_{T_v^E}^{T_v} (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau}. \tag{38}$$

Using (37), and integrating the denominator through the downturn this implies:

$$\overline{q}_v = \frac{1 - e^{-\beta(T_v)}}{(1 - H_v)\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)}, \tag{39}$$

where $\Delta_v^E = T_v - T_v^E$. The capitalist could never be made better off by committing to a price sequence that varies through the recession instead of the constant price $\overline{q}_v$.[36]

Given $\overline{w}_v$ and $\overline{q}_v$, entrepreneurs choose a cost–minimizing ratio of utilized capital to labor hours. Since this is constant through downturn, it follows that the installed capital–labour ratio at the peak must satisfy $\frac{K(T_v^E)}{L(T_v^E)} = \left(\frac{\alpha}{1-\alpha}\right)\frac{\overline{w}_v}{\overline{q}_v}$. Given the constant wage from (28) it then directly follows that:

Proposition 2 *Cost minimization ensures that capital is installed only up to the point at which the marginal return to capital is equal to its average rental price:*

$$q(T_v^E) = \alpha e^{-(1-\alpha)\gamma}\overline{A}_{v-1}^{1-\alpha}K(T_v^E)^{\alpha-1} = \overline{q}, \tag{40}$$

where $\overline{A}_{v-1} = \exp\left(\int_0^1 \ln A_i(T_{v-1})di\right)$ *denotes the average level of technology in use in the vth cycle.*

Equating (39) and (40), substituting for $1 - H_v$ using (30), it follows that the capital–consumption ratio at the height of the expansion can be expressed as:

$$\frac{K\left(T_v^E\right)}{C\left(T_v^E\right)} = \frac{\alpha e^{-(1-\alpha)\gamma}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)}{1 - e^{-(1-\alpha)\Gamma_v}}. \tag{41}$$

Note that, since there is no depreciation and no capital is accumulated through the recession, $K\left(T_v^E\right)$ is also the capital stock at the beginning of the next boom

5.2.2 Does GDP really contract?

During this phase, $Y(t)$ is not equal to real GDP because it does not include the contribution of entrepreneurial inputs. This can easily be corrected as follows:

$$\begin{aligned} \text{GDP} &= Y(t) + \bar{w}_v H(t) \\ &= \pi(t) + \bar{w}_v\left(1 - H(t)\right) + \bar{q}_v K^u(t) + \bar{w}_v H(t) \\ &= (1 - e^{-(1-\alpha)\gamma})Y(t) + \bar{q}_v \lambda(t) K\left(T_v^E\right) + \bar{w}_v. \end{aligned}$$

[36] Under any variable price sequence that averages to $\overline{q}_v$, entrepreneurs would have incentive to increase demand for capital when $q(\tau) < \overline{q}_v$, store that output not needed to meet the demand of the final goods sector, and correspondingly reduce production, and demand over those τ such that $q(\tau) > \overline{q}_v$. By substituting capital demand to times when the price is low, returns to the capital owner would fall.

Clearly, GDP contracts through this phase, because both profits and payments to capital owners, $\lambda(t)\bar{q}_v$, fall. Thus, the recession is not a result of mis–measurement, or because innovative inputs are not being counted. The reason is that, due to imperfect competition, wages are less than the marginal product of labour. As labour hours are transferred to innovative activities, the marginal cost in terms of output exceeds the marginal benefit of innovation. In effect, the transfer of labour imposes a negative externality on intermediate producers and capital owners.

5.3 The (Schumpeterian) Boom

We denote the improvement in aggregate productivity during implementation, $e^{(1-\alpha)\Gamma_v}$, where

$$\Gamma_v = \ln\left[\overline{A}_v/\overline{A}_{v-1}\right]. \tag{42}$$

Productivity growth at the boom is given by $\Gamma_v = (1 - P(T_v))\gamma$, where $P(T_v)$ is defined by (22). Substituting in the allocation of labor to entrepreneurship through the downturn given by (32) and letting

$$\Delta_v^E = T_v - T_v^E, \tag{43}$$

yields the following implication:

Proposition 3 *: In an equilibrium where there is positive entrepreneurship only over the interval $(T_v^E, T_v]$, the growth in productivity during the subsequent boom is given by*

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma(1 - H_v)\left(\frac{1 - e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right). \tag{44}$$

Over the boom, the asset market must simultaneously ensure that entrepreneurs holding innovations are willing to implement immediately (and no earlier) and that, for households, holding equity in firms (weakly) dominates holding claims on alternative assets (particularly stored intermediates). The following Proposition demonstrates that these conditions imply that during the boom the discount factor must equal productivity growth:[37]

Proposition 4 *Asset market clearing at the boom requires that*

$$\beta(T_v) = (1 - \alpha)\Gamma_v. \tag{45}$$

[37] Shleifer's (1986) model featured multiple expectations–driven steady state cycles. Such multiplicity cannot occur here because, unlike Shleifer, the possibility of storage that we allow forces a tight relationship between Γ_v and Δ_v^E as depicted in Proposition 3. Since Γ_v, Δ_v^E pairs must satisfy this restriction as well, in general, multiple solutions cannot be found.

During the boom, $\beta(T_v)$ equals the growth in firm values and wages grow in proportion to productivity. Since, just before the boom, $\delta V^I(T_v) = w(T_v)$, a corollary is that

$$\delta V_0^I(T_v) = w_0(T_v) = (1-\alpha)e^{-(1-\alpha)\gamma}\overline{A}_v^{1-\alpha}K(T_v^E)^\alpha. \tag{46}$$

The growth in output at the boom exceeds the growth in productivity for two reasons: first labor is re–allocated back into production, and second the previously under–utilized capital is now being used productively. Since just before the boom, both inputs are a fraction $(1-H_v)e^{-\frac{\rho}{\sigma}\Delta_v^E}$ of their peak levels, output growth through the boom is given by

$$\begin{aligned}\Delta \ln Y(T_v) &= (1-\alpha)\Gamma_v + (1-\alpha)\Delta \ln L + \alpha\Delta \ln K^u \\ &= (1-\alpha)\Gamma_v + \frac{\rho}{\sigma}\Delta_v^E - \ln(1-H_v) \end{aligned} \tag{47}$$

It follows directly from Proposition 4 that growth in output exceeds the discount factor across the boom. Since profits are proportional to output, this explains why firms are willing to delay implementation during the downturn.

The boom in output can be decomposed into a boom in consumption and investment. From the Euler equation, we can compute consumption growth across the boom:

$$\Delta \ln C(T_v) = \frac{(1-\alpha)}{\sigma}\Gamma_v. \tag{48}$$

Notice that whether the growth in consumption exceeds the growth in productivity at the boom, depends on the value of σ. In particular, if $\sigma < 1$, consumption growth must exceed aggregate productivity growth. Finally, since in the instant prior to the boom $C(T_v) = Y(T_v)$, it follows that the investment rate at the boom jumps to

$$\frac{\dot{K}_0(T_v)}{Y_0(T_v)} = 1 - (1-H_v)e^{\left(\frac{1-\sigma}{\sigma}\right)(1-\alpha)\Gamma_v - \frac{\rho}{\sigma}\Delta_v^E}. \tag{49}$$

6 Optimal Behavior During the Cycle

Optimal entrepreneurial behavior imposes the following requirements on our hypothesized equilibrium cycle:

• Successful entrepreneurs at time $t = T_v$ must prefer to implement immediately, rather than delay implementation until later in the cycle or the beginning of the next cycle:

$$V_0^I(T_v) > V_0^D(T_v). \tag{E1}$$

• Entrepreneurs who successfully innovate during the downturn must prefer to wait until the beginning of the next cycle rather than implement earlier and sell at the limit price:

$$V^I(t) < V^D(t) \qquad \forall\, t \in (T_v^E, T_v) \tag{E2}$$

• No entrepreneur wants to innovate during the slowdown of the cycle. Since in this phase of the cycle $\delta V^D(t) < w(t)$, this condition requires that

$$\delta V^I(t) < w(t) \qquad \forall\, t \in (0, T_v^E) \tag{E3}$$

• Finally, in constructing the equilibrium above, we have implicitly imposed the requirement that the downturn is not long enough that all sectors innovate:

$$P(T_v) > 0. \tag{E4}$$

Taken together conditions (E1) through (E4) are restrictions on entrepreneurial behavior that must be satisfied for the cyclical growth path we have posited to be an equilibrium.

Figure 5 illustrates the evolution of the value functions and wages through the cycle. Following the boom at T_{v-1}, δV^I remains above δV^D for a while so that if there were any new innovations, immediate implementation would dominate delay. However, over this phase, the relative value of labor in production, w, exceeds returns to entrepreneurship, so that no entrepreneurial successes are available to implement. Throughout this expansionary phase, investment occurs so that the wage continues to rise. At the same time, V^D also rises as the next implementation period draws closer. Throughout this phase V^I declines as the duration of guaranteed positive profits falls.

The end of the expansion corresponds to the commencement of entrepreneurship — when the increasing value of delayed implementation eventually meets the opportunity cost of labor in production, $w = \delta V^D$. Since, during the contraction, interest rates are zero, V^D remains constant so that the wage must also be constant. Initially, V^I continues to fall, but eventually rises again as the probability of remaining the incumbent at the boom, given that an entrepreneurial success has not arrived in one's sector, increases. This increase in V^I is the force that will eventually trigger the next boom that ends the recession. It occurs when V^I just exceeds V^D and entrepreneurs implement stored entrepreneurial successes, leading to an increase in productivity, a jump in demand, movement of labor back to production, and full capacity utilization.

7 The Stationary Cyclical Growth Path

To allow a stationary representation, we normalize all aggregates by dividing by $\bar{A}_{v-1}$ and denote the result with lower case variables. First recall from Proposition 1, that the dynamics of the

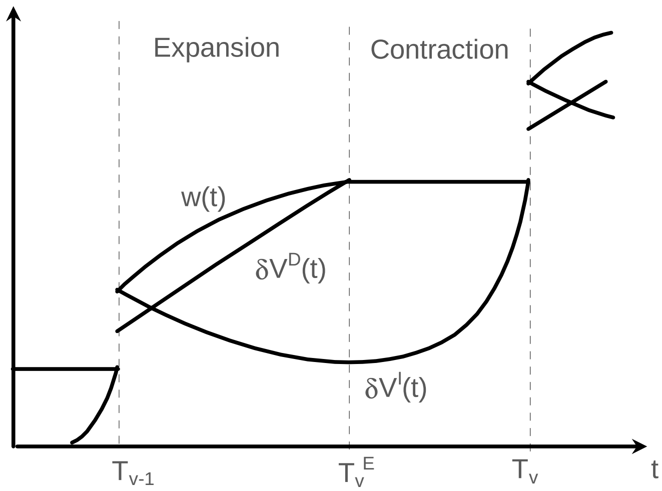


Figure 5: Existence

economy during the expansion are analogous to those in the Ramsey model without technological change. Let $c_v = c(T_v^E)$ and $k_v = k(T_v^E)$ denote the normalized values of consumption and capital at the peak of the vth expansion. Given initial values $c_0(T_{v-1})$ and $k_0(T_{v-1})$, and an expansion length Δ_v^X, it is possible to summarize the expansion as follows:

$$c_v = f(c_0(T_{v-1}), k_0(T_{v-1}), \Delta_v^X) \quad (50)$$

$$k_v = g(c_0(T_{v-1}), k_0(T_{v-1}), \Delta_v^X), \quad (51)$$

where $f(\cdot)$ and $g(\cdot)$ are well–defined functions. Since capital accumulation stops in the recession, and $\overline{A}$ rises by $e^{\Gamma_{v-1}}$, it follows that

$$k_0(T_{v-1}) = e^{-\Gamma_{v-1}} k_{v-1}. \quad (52)$$

From (31), consumption declines by a factor $e^{-\frac{\rho}{\sigma}\Delta_{v-1}^E}$ in the recession. When combined with its increase at the boom, from (48), this yields

$$c_0(T_{v-1}) = e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1} - \frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}. \quad (53)$$

Substituting for $c_0(T_{v-1})$ and $k_0(T_{v-1})$ in (50) and (51) then yields

$$c_v = f(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1} - \frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}, e^{-\Gamma_{v-1}} k_{v-1}, \Delta_v^X) \quad (54)$$

$$k_v = g(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma_{v-1} - \frac{\rho}{\sigma}\Delta_{v-1}^E} c_{v-1}, e^{-\Gamma_{v-1}} k_{v-1}, \Delta_v^X). \quad (55)$$

Imposing stationarity, so that $\Gamma_v = \Gamma$, $k_v = k$, $c_v = c$, $\Delta_v^E = \Delta^E$ and $\Delta_v^X = \Delta^X$ for all v, on the system described by (44), (41), (54), (55) and (46) yields a system of five equations in the five unknowns that summarize the stationary cycle:

Proposition 5 *The stationary cycle* $(\Gamma, k, c, \Delta^E, \Delta^X)$ *satisfies the following system of equations:*

$$\Gamma = \delta\gamma\Delta^E - \delta\gamma\frac{c}{k^\alpha}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right) \quad (56)$$

$$\frac{c}{k} = \frac{1-e^{-(1-\alpha)\Gamma}}{\alpha e^{-(1-\alpha)\gamma}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta^E}}{\rho/\sigma}\right)} \quad (57)$$

$$c = f(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma-\frac{\rho}{\sigma}\Delta^E}c, e^{-\Gamma}k, \Delta^X) \quad (58)$$

$$k = g(e^{\left(\frac{1-\alpha}{\sigma}-1\right)\Gamma-\frac{\rho}{\sigma}\Delta^E}c, e^{-\Gamma}k, \Delta^X) \quad (59)$$

$$\delta v_0^I(c, k, \Gamma, \Delta^E, \Delta^X) = (1-\alpha)e^{-(1-\alpha)\gamma}e^{-\alpha\Gamma}k^\alpha \quad (60)$$

where $v_0^I = V_0^I(T_v)/\bar{A}_v$.

The stationary cycle can be understood heuristically from the phase diagram in Figure 6. Here the process of capital accumulation in the expansionary phase, $t \in \left(T_{v-1}, T_v^E\right)$, within a steady state cycle is depicted.

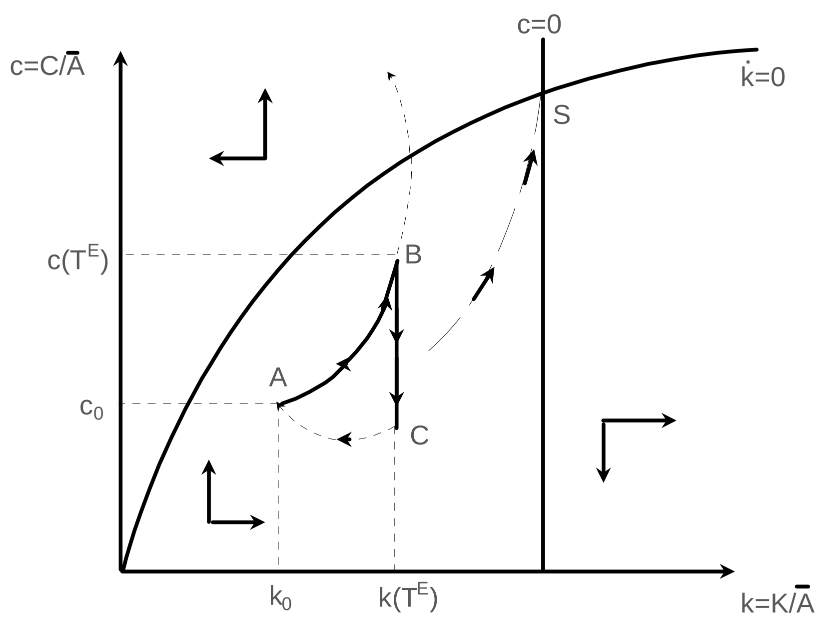


Figure 6: Phase Diagram

The economy does not evolve along the standard stable trajectory of the Ramsey model terminating at the steady state, S. Instead, the evolution of the cycling economy during the expansion is depicted by the path between A and B in the figure. Capital is accumulated starting at the point k_0 corresponding to point A in the diagram, according to (23) and (24). The point

k_0 denotes the inherited capital stock at the boom. Accumulation ends at $k\left(T^E\right)$, at which point investment stops until the next cycle. Note that if allowed to continue along such a path the economy would eventually violate transversality, but capital accumulation stops and consumption declines so that the economy evolves from B to C through the downturn. During this phase, the dynamics of the economy are no longer dictated by the Ramsey phase diagram. When this phase ends, implementation of stored productivity improvements occurs at the next boom, and $\bar{A}$ increases, so that k fall discretely. If $\sigma < 1$, consumption grows by more than productivity at the boom, so that c rises discretely. The boom is therefore depicted by the dotted arrow back to point A. At this point, investment in the expansionary phase recommences for the next cycle. The connection between the two phases of the cycle arises due to the allocation of resources to entrepreneurship. This allocation of resources will be reflected in the size of the increment to $\bar{A}$, Γ.

7.1 A Numerical Example

We numerically solve the model for various combinations of parameters and check the existence conditions (E1)–(E4). We choose parameters to fall within reasonable bounds of known values, and present a baseline case given in Table 1:

Table 1: Baseline Parameters

Parameter	Value
α	0.22
γ	0.13546
σ	0.79
ρ	0.02
δ	1.39

The parameters α and γ were chosen so as to obtain a labor share of 0.7, a capital share of 0.2 and a profit share of 0.1. These values correspond approximately to those estimated by Atkeson and Kehoe (2002). The value of γ corresponds to a markup rate of around 15%. The intertemporal elasticity of substitution $\frac{1}{\sigma}$ is slightly high, but in Appendix C, we provide simulation results for various values below, including $\sigma = 1$. Given $\sigma = 0.79$, we calibrated δ and ρ so as to match a long–run annual growth rate of 2.2% and an average risk–free real interest rate of 3.8%, values which correspond to annual data for the post–war US. The baseline case above yields a cycle length of a little less than 4 years, $H_v = .2044$, and $k_v = 7.668$. In this, and all simulations we have computed, steady state values are unique.[38]

[38] Francois and Lloyd-Ellis (2003) explicitly establish uniqueness of the stationary cycle when capital accumulation is not allowed. It seems likely that the introduction of capital would not lead to an additional stationary cycle

8 Implications

8.1 Tobin's Q and Investment

Tobin's Q is typically measured as the ratio of the value of firms to the book value of their capital stock. In our model Tobin's Q is given by

$$Q(t) = \frac{V^K(t) + \Pi(t)}{K(t)}, \tag{61}$$

where $\Pi(t)$ denotes the stock market value of the intangible capital tied up in firms, and recall that $V^K(t)$ is the market value of their physical capital. Figure 7 illustrates the evolution of Tobin's Q, its tangible component (V^K/K) and aggregate investment over the cycle.

During an expansion $V^K(t) = K(t)$ and, the value of intangible capital is equal to the value of incumbent firms: $\Pi(t) = V^I(t)$. It follows that

$$Q(t) = 1 + \frac{V^I(t)}{K(t)} \qquad \forall t \in (T_{v-1}, T_v^E). \tag{62}$$

Since $V^I(t)$ declines and $K(t)$ grows during the expansion, $Q(t)$ must decline.

In the downturn, the value of the physical capital stock declines below the capital stock, so that

$$V^K(t) = \left[\bar{q}\int_t^{T_v} \lambda(\tau)d\tau + e^{-\beta(T_v)}\right] K(T_v^E) < K(T_v^E). \tag{63}$$

Also some sectors experience innovations, so there exist terminal firms who are certain to be made obsolete at the next round of innovation. At any point in time the measure of sectors in which no innovation has occurred is $P(t)$, therefore the total value of firms on the stockmarket is given by

$$\Pi(t) = (1 - P(t))[V^T(t) + V^D(t)] + P(t)V^I(t), \tag{64}$$

where $V^T(t)$ denotes the value of "terminal" firms who are certain to be made obsolete during the next wave of implementation. The value of these firms can be written as

$$V^T(t) = V^I(t) - \frac{P(T_v)}{P(t)} V^D(t). \tag{65}$$

Substituting into (64) yields

$$\hat{\Pi}(t) = V^I(t) + (1 - P(t))\left[1 - \frac{P(T_v)}{P(t)} V^D(t)\right]. \tag{66}$$

here, but we have not been able to establish this analytically.

Through the downturn, the value of intangible capital initially falls and then rises again as the economy approaches the next boom.[39] Immediately prior to the boom $P(t) = P(T_v)$, so that again $\Pi(T_v) = V^I(T_v)$. The value of Q during the downturn is thus given by

$$Q(t) = \bar{q}\int_t^{T_v} \lambda(\tau)d\tau + e^{-\beta(T_v)} + \frac{\hat{\Pi}(t)}{K(T_v^E)} \qquad \forall\ t \in [T_v^E, T_v). \tag{67}$$

During the contraction, then, $Q(t)$ initially declines as $K(t)$ remains unchanged and the decline in $V^k(t)$ dominates. However, eventually the growth in the value of intangible capital, $\hat{\Pi}(t)$, starts to dominate as we approach the boom, so that $Q(t)$ rises in anticipation. At the boom, since the book value of capital remains unchanged, but the market value of physical capital grows by a factor $e^{(1-\alpha)\Gamma_v}$, Tobin's Q rises rapidly.

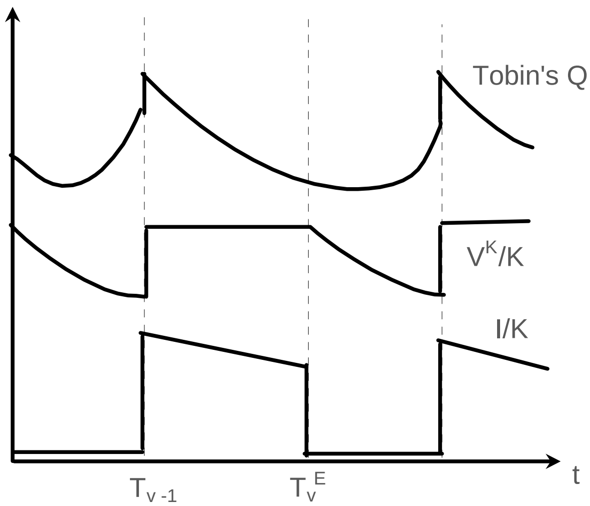


Figure 7: Tobin's Q and Investment

The qualitative behavior of Tobin's Q in our model thus accords quite well with its aggregate counterpart in US data. As illustrated in Figure 1, Tobin's Q tends to reach a peak prior to the peak of expansions and then reaches a minimum before the end of the NBER–dated recessions. The most rapid periods of growth in Tobin's Q therefore start to occur before the end of recessions and continue through the subsequent boom just as they do in our stationary cycle.

8.2 Additional Results

Although, our focus is on investment and the stockmarket, our model also *endogenously* generates behavior of a number of key variables that are qualitatively consistent with the facts:

[39] This cyclical anticipation of future profits implicit in aggregate stock prices accords well with the findings of Hall (2001).

• Output growth is characterized by a three–stage process — Output grows rapidly at the boom, at a lower rate during the subsequent expansion before turning negative. This is characterization is consistent with time–series evidence provided by non–linear econometric models of output (e.g. Dahl and Gonzalez–Rivera, 2003).

• Labour productivity is strongly pro–cyclical — During expansions, all labor is used in production and capital is fully utilized, so that labor productivity rises though capital accumulation. In contractions, labor is reallocated to innovative activities, capital utilization falls, and output declines. If utilized capital and labor were correctly measured, labour productivity should remain constant through the recession. If the re–allocation of labor were not fully measured, then it would appear that labor is being hoarded (see Fay and Medoff 1985),[40] and measured labour productivity would fall. This is consistent with the evidence of Fernald and Basu (1999), for example. Even if labour re–allocations were correctly accounted for, measured productivity would still fall since capital utilization is typically not well measured.

• Wages rise less than output during booms and expansions, and do not fall during contractions — as a result, wages are inherently less procyclical than output, which again is consistent with most evidence for the US. Since there are no aggregate employment fluctuations in our model, one must be careful in interpreting this implication. We discuss extensions of the model that would allow for unemployment in our conclusion.

• Labor and capital *inputs* into consumption and investment sectors are both pro–cyclical — the canonical RBC model implies that inputs into consumption are countercyclical (e.g. Christiano and Fisher 1995). The allocation of labor to consumption good production can be inferred from equation (48). As long as $\sigma < 1$ consumption growth exceeds productivity growth so that the allocation of labor and capital to consumption must rise at the boom. The reason labor in both consumption and investment good production can rise is because of the endogenous shutting down of entrepreneurship at the boom. This mechanism is similar to that generated by introducing "homework" in Benhabib, Rogerson and Wright (1991).

• The term spread is small during expansions and high midway through contractions — we define the term spread as the difference between a 30 year annualized interest rate and a 3 month interest rate. The cycle analyzed here exhibits a low value of the term spread through the expansion, and a high value in the recession. The highest value occurs at the start of the recession then, towards the end of the recession it tracks down as the three month rate starts to include the increased

[40] Entrepreneurship is, at best, likely to be only partially measured in the data, since much of it involves activities that will raise long-term firm profits but have little directly recorded output value contemporaneously.

discount over the boom. Similarly, at the start of the expansion the term spread is at its lowest point, thus again providing a leading indication of the imminent contraction. Estrella and Mishkin (1996) argue that the term spread is a superior predictor over other leading indicators at leads from 2 to 4 quarters.

9 Endogenous Incomplete Contracting

In describing the cyclical equilibrium above we have assumed that capital owners can offer long term commitments with respect to the rental price per unit of utilized capital. While it simplifies the exposition, this assumption is problematic for two reasons. First, one would normally expect the rental price to be per unit of *installed* capital. Secondly, during the recession such price commitments are clearly ex–post inefficient — if a more productive technology has arrived in a sector, would it not be Pareto improving to break the commitment so as to induce early entry by new innovators who might fully utilize the capital? In this section, we relax the assumption of price–commitment and instead only allow agents to write long term, enforceable supply contracts. We show that the behavior described above can be derived as the equilibrium outcome of constrained–efficient contracting between agents. A key incompleteness in the contracting environment arises as a natural consequence of the process of creative destruction — capital owners cannot write contingent contracts regarding innovations that do not exist yet.

As with exogenous price commitments, prior to the peak of each cycle, capital owners compete by offering long term contracts. However, since entrepreneurs lose their productive advantage when displaced by superior producers, they cannot make unconditional promises to purchase capital into the indefinite future. All such contracts are thus contingent upon the entrepreneur continuing production. Note further that the existence of an exclusive contract over the supply of capital in sector i, places the intermediate producer in that sector in a strong market position relative to its rivals. In order to prevent ex–post price gouging by the intermediate goods producer, the final goods producer will also demand a contract over the supply of intermediate goods. Such a contract will also be written prior to the peak, when there is still effective competition from the past incumbent.[41]

Intermediate Supply Contracts written at t specify future output, $x_i(\tau)$, and prices, $p_i(\tau)$, for all τ up to a chosen contract termination date, T_i^X. Thus, such a contract is a tuple: $\left\{\{x_i(\tau), p_i(\tau)\}_{\tau\in\left[t,T_i^X\right]},\ T_i^X\right\}$. Since the productive advantage of an intermediate producer lasts only until a superior technology is implemented in that sector, contracts allow the termination of

[41] Though conceptually feasible, contracts written over the supply of labor and final output are redundant in the equilibria we study and will not be considered further.

agreements before T_i^X if shutting down production. Otherwise, the parties can break contracts only by mutual agreement. The value of the arrangement to final goods producers is denoted $V^Y(t)$.

Lemma 5 *Given a sequence of input prices* $w(t)$, $q(t)$ *for* $t \in [T_{v-1}, T_v)$, *an intermediate supply contract specifying price and quantity sequences satisfying (11) and (9) for* $t \in [T_{v-1}, T_v)$ *maximizes the present value of the intermediate producers profits* $\max\left[V_i^I(t), V_i^D(t)\right]$ *subject to* $V^Y(t) \geq 0$.

Capital Supply Contracts specify future binding levels of installed capital, $K_i(\tau)$, and an effective price for each unit of installed capital, $\lambda_i(\tau)q_i(\tau)$, for all τ up to a chosen contract termination date, T_i^K. Thus a contract signed at time t is a tuple $\left\{\{K_i(\tau), \lambda_i(\tau)q_i(\tau)\}_{\tau\in\left[t,T_i^K\right]}, \; T_i^K\right\}$. Contracts can be altered under the same conditions as in intermediate supply contracts. In equilibrium, capital supply contracts written at t must be undominated:

$$\max\left[\widehat{V}_i^I(t), \widehat{V}_i^D(t)\right] + \widehat{V}_i^K(t) \geq \max\left[V_i^I(t), V_i^D(t)\right] + V_i^K(t).$$

The existence conditions (E1) through (E4) take as given that entrepreneurs do not produce in excess of current demand and store their output until the boom. Provided that the incumbent entrepreneur does not terminate the capital supply contract, (45) ensures that storage across the boom is not optimal. However, since the capital utilization and rental price sequences that we have derived previously imply that the capital stock is being under–utilized, it is possible that just before the boom, a rival entrepreneur who has successfully innovated may be able to "buy out" the contract and utilize all the capital, meeting the current demand for output and storing the remainder until the boom.

This rival would not benefit from taking over the capital contract of the incumbent under identical terms. From (45), producing output and storing it until the boom is not optimal if he must pay a constant amount $\bar{q}$ for capital. Moreover, under (E2) implementation and sale before the boom is not optimal. However, the rival may be willing to take over the use rights if able to pay $\overline{q}_v$ for the amount $K(t)$ in the incumbent's contract, utilize extra units of idle capital at some price $\widetilde{q} < \overline{q}_v$, and store. Clearly any $\widetilde{q} > 0$ for the excess units would be amenable to the capitalist. The most the rival will be willing to pay per period for the current capital is $\overline{q}_v K(T_v^E)$, since $e^{-\beta(T_v)}q_0(T_v) = \overline{q}_v$. To buy out the contract, the rival must compensate the incumbent for the loss of profits sustained for the remainder of the cycle and must offer the capitalist at least the payment he is currently receiving, $\overline{q}_v(1-H_v)e^{-\frac{\rho}{\sigma}(t-T_v^E)}K(T_v^E)$ per period. It follows that such a contract buy–out will not a be mutually acceptable at time t if

$$\int_t^{T_v} \pi(\tau)d\tau + \int_t^{T_v} \overline{q}(1 - H_v)e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}K(T_v^E)d\tau \geq \int_t^{T_v} \overline{q}K(T_v^E)d\tau. \tag{68}$$

The following proposition provides a sufficient condition for this to hold throughout the downturn:

Proposition 6 *Provided that*

$$(1 - (1 - \alpha)e^{-(1-\alpha)\gamma})(1 - H_v)e^{-\frac{\rho}{\sigma}\Delta_v^E} > \alpha e^{-(1-\alpha)\gamma} \tag{E5}$$

capital supply contracts specifying price and quantity sequences given by (69), (74), (33) and (40) are undominated.

In effect, condition (E5) explains how it is possible for there to be under–utilized capital during a recession even though there exist rivals who could potentially use the capital stock more profitably. The reason is that the capital stock is "lumpy", so that the rival cannot use part of it while the incumbent continues to produce. For this reason, the rival must compensate the incumbent for his profit loss and this "endogenous" fixed cost is too large for entry to be profitable under recessionary demand conditions. Entry does not become profitable until the boom. There, demand is high and entry costs low because the previous incumbent's profits do not need to be compensated — they have already been destroyed by the implementation of a superior production process. In our numerical simulations, we find that (E5) is rarely binding, so it does not appear to be a strong requirement for existence of the cyclical equilibrium.

In the absence of exogenous price commitment, the cyclical equilibrium is supported by limitations on the contracting environment. The critical, and we think realistic, assumption is that only future prices and quantities can be contracted ex ante. Allowing for a richer set of contracting possibilities would overturn this result. For example, if the new incumbent entrepreneur (who arrives probabilistically in the downturn) could somehow be party to the contract prior to time T_v^E, then full utilization of the capital through the downturn could also be contracted ex ante. Such a rich contracting environment, however, seems to require unrealistically complex and difficult to observe details to be enforceable between the parties. Thus endogenous under–utilization, which corresponds to that observed in actual business cycles, arises here due to natural limitations in contracting.

10 Concluding Remarks

This article shows how the qualitative relationship between investment and Tobin's Q over the US business cycle, arises quite naturally in a model of endogenous cyclical growth. During recessions, entrepreneurs delay the implementation of innovations, whose present value is reflected in stock market values, until demand is high and the expected length of incumbency is maximized. Once these innovations are implemented, the marginal product of capital jumps, inducing a prolonged investment boom. During this period of high investment, the incentives to innovate are low and subsequent booms are far away, so that the market value of existing firms starts to decline. This decline anticipates the subsequent crash in investment, as resources are shifted out of current production and into longer term activities.

In order to study these cycles in the rate of fixed capital formation, we have extended the existing literature on endogenous implementation cycles. The extension is non–trivial because the introduction of physical capital into models like that of Shleifer (1986) undermines the existence of cycles by allowing agents to store. In the presence of costly, endogenous innovation, however, a unique cyclical equilibrium emerges which is robust to storage (and therefore the introduction of capital).

Our model also generates movements in other aggregates over the cycle which are qualitatively similar, in some respects, to those observed in US data. It should be reiterated that these results arise in a framework where both the economy's cyclical behavior and its growth path are fully endogenized. Moreover, the framework we explore has remarkably few degrees of freedom; the model is fully specified by five exogenous parameters: two summarizing household preferences, two underlying the productivity of entrepreneurship, and one pinning down factor shares in production.

We do not claim that the current framework is capable of providing a quantitative account of the business cycle. However, in future work we will build on this parsimonious structure to explore a number of key extensions:

• Aggregate uncertainty and stochastic cycle lengths — The length and other characteristics of actual business cycles, vary from cycle to cycle and look rather different from the deterministic stationary equilibrium cycle described here. Introducing some degree of aggregate uncertainty would help to address this. However, in order to develop such an extension we need to develop a deeper understanding of the local transitional dynamics of the model. It turns out that these dynamics are not as complex as one might expect at first blush. The reason is that the path back to the stationary cycle (at least locally) involves the accumulation of only one factor: either physical capital or intangible. Although a full analysis of these local dynamics is beyond the scope of the current paper, we believe it is feasible.

• Unemployment — A natural way to introduce unemployment into the model is to allow for unskilled labour which cannot be used in entrepreneurship and is not directly substitutable with skilled labor in production. With putty–clay production, the marginal value of this unskilled labor falls to zero during the downturn and some fraction of unskilled workers would become unemployed (just like physical capital). In a competitive labor market, this would drive unskilled wages down to their reservation level. However, in the presence of labor market imperfections, such as efficiency wages and search frictions, the dynamics of unemployment and wages interact with the process of creative destruction in a more complex manner. In further work we explore these dynamics more fully.

• Government policy — The framework developed here (as well as its extensions) provide a natural framework for thinking about counter–cyclical policy. First, the question arises as to whether removing or reducing cycles is a valid policy objective at all. Francois and Lloyd–Ellis (2003) show that switching from the cyclical equilibrium to a corresponding acyclical one would reduce long–run growth but increase welfare. Similar results are likely to carry over to the stationary cycle in the current model. A second issue is that of how to implement a counter–cyclical policy. The recession here is Keynesian in that it is associated with deficient demand, and the government could intervene, for example, by raising demand for goods and services and taxing savings. However, such a policy would effectively channel resources away from innovative activities and may dampen growth. On the other hand the anticipation of higher demand during a downturn might stimulate innovation, so the overall effect is unclear.

Appendix A – Proofs

Proof of Lemma 1 We show: (1) that if a signal of success from a potential entrepreneur is credible, other entrepreneurs stop innovation in that sector; (2) given (1) entrepreneurs have no incentive to falsely claim success.

Part (1): If entrepreneur i's signal of success is credible then all other entrepreneurs believe that i has a productivity advantage which is e^{γ} times better than the existing incumbent. If continuing to innovate in that sector, another entrepreneur will, with positive probability, also develop a productive advantage of e^{γ}. Such an innovation yields expected profit of 0, since, in developing their improvement, they do not observe the non-implemented improvements of others, so that both firms Bertrand compete with the same technology. Returns to attempting innovation in another sector where there has been no signal of success, or from simply working in production, $w(t) > 0$, are thus strictly higher.

Part (2): If success signals are credible, entrepreneurs know that upon success, further innovation in their sector will cease from Part (1) by their sending of a costless signal. They are thus indifferent between falsely signalling success when it has not arrived, and sending no signal. Thus, there exists a signalling equilibrium in which only successful entrepreneurs send a signal of success.■

Proof of Proposition 1: First note that in the absence of uncertainty or adjustment costs, and as long as utilized capital is anticipated to grow, capital owners never acquire more capital than is needed for production, so that

$$K_i^u(t) = K_i(t). \tag{69}$$

Substituting into (6) and differentiating with respect to time yields

$$\dot{V}_i^K(t) = r(t) V_i^K(t) - q_i(t) K_i(t) + \dot{K}_i(t) = \dot{K}_i(t). \tag{70}$$

Since, during this phase, $V_i^k(t) = K_i(t)$, it follows that $q_i(t) = q(t) = r(t)\ \forall\ i$. Combining (**??**) with (9), (10) and (11), it follows that all firms choose the same capital–labour ratio. From the production function we have

$$\ln Y(t) = \int_0^1 \ln \frac{Y(t)}{p_i(t)} di \tag{71}$$

Substituting for $p_i(t)$ using (11) yields

$$0 = \int_0^1 \ln \frac{q(t)^{\alpha} w(t)^{1-\alpha}}{\mu e^{-(1-\alpha)\gamma} A_i^{1-\alpha}(T_{v-1})} di \tag{72}$$

which re-arranges to (73)

$$q(t)^{\alpha} w(t)^{1-\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha} e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha}. \tag{73}$$

Thus, the input price index for $t \in [T_{v-1}, T_v^E]$ is constant and uniquely determined by the level of technology

Through this phase, capital is rented at a competitive price, i.e. its marginal product net of profits to the entrepreneur:

$$r(t) = q(t) = \alpha e^{-(1-\alpha)\gamma} \overline{A}_{v-1}^{1-\alpha} K(t)^{\alpha-1}. \tag{74}$$

Using this in the consumer's Euler equation yields the equations in Proposition 1.■

Proof of Lemma 3: Suppose instead that there exists an intermediate phase in which neither capital is accumulated nor entrepreneurship occurs. Consider the first instant of that phase. Since in the instant prior to that capital was being accumulated, the marginal return to investment in physical capital must exceed ρ. Since the marginal product of capital cannot jump downwards discretely at full capital utilization, there are only two possibilities: either (1) $r(T_v^E) = \rho$ at the start of the intermediate phase or (2) $r(T_v^E) > \rho$ at the start of the intermediate phase. Situation (2) can be ruled out directly since, by assumption, in the intermediate phase there is no entrepreneurship, and so it must be the case that $r > \rho$ and investment will occur. Situation (1) occurs if the marginal return to capital converges continuously to $r = \rho$ along the neoclassical accumulation phase. But this corresponds exactly with the path of accumulation along the stable trajectory of the Ramsey model which does not converge in finite time — this would then imply an infinite length to the capital accumulation phase.■

Proof of Proposition 3: Long–run productivity growth is given by

$$\Gamma_v = (1 - P(T_v))\gamma \tag{75}$$

Integrating (32) over the downturn and substituting for $H(\cdot)$ using (32) yields

$$1 - P(T_v) = \delta \int_{T_v^E}^{T_v} \left(1 - (1 - H_v) e^{-\frac{\rho}{\sigma}[t - T_v^E]}\right) d\tau. \tag{76}$$

Substitution into (75) and integrating gives (44).■

Proof of Lemma 4: If $V_i^k(t) > K_i(t)$ it is feasible for the the builder of a new capital stock in sector i to commit to a price sequence $q_i^c(\tau)$, which would be strictly preferred by the current incumbent producer. A preferred sequence for the leading producer would be one in which prices were no higher than the contracted sequence above, but which had a strictly lower price in at least one instant. This is feasible if $V_i^k(t) > K_i(t)$. Finally, no new capitalist would enter offering a sequence $V_i^k(t) < K_i(t)$, so that any equilibrium price sequence must at least satisfy (37).■

Proof of Proposition 4: For an entrepreneur who is holding an innovation, $V^I(t)$ is the value of implementing immediately. During the boom, for entrepreneurs to prefer to implement immediately, it must be the case that

$$V_0^I(T_v) > V_0^D(T_v). \tag{77}$$

Just prior to the boom, when the probability of displacement is negligible, the value of implementing immediately must equal that of delaying until the boom:

$$\delta V^I(T_v) = \delta V^D(T_v) = w(T_v). \tag{78}$$

From (77), the return to entrepreneurship at the boom is the value of immediate (rather than delayed) incumbency. It follows that free entry into entrepreneurship at the boom requires that

$$\delta V_0^I(T_v) \leq w_0(T_v). \tag{79}$$

The opportunity cost of financing entrepreneurship is the rate of return on shares in incumbent firms in sectors where no innovation has occurred. Just prior to the boom, this is given by the capital gains in sectors where no entrepreneurial successes have occurred;

$$\beta(T_v) = \log\left(\frac{V_0^I(T_v)}{V^I(T_v)}\right). \tag{80}$$

Note that since the short–term interest rate is zero over this phase, $\beta(t) = \beta(T_v)$, $\forall\, t \in (T_v^E, T_v)$. Combined with (78) and (79) it follows that asset market clearing at the boom requires

$$\beta(T_v) \leq \log\left(\frac{w_0(T_v)}{w(T_v)}\right) = (1-\alpha)\Gamma_v. \tag{81}$$

Free entry into entrepreneurship ensures that $\beta(T_v) > (1-\alpha)\Gamma_v$ cannot obtain in equilibrium.

Provided that $\beta(t) > 0$, households will never choose to store final output from within a cycle to the beginning of the next either because it is dominated by the long–run rate of return on claims to future profits. However, the return on stored intermediate output in sectors with no entrepreneurial successes is strictly positive, because of the increase in its price that occurs as a result of the boom. If innovative activities are to be financed at time t, it cannot be the case that households are strictly better off buying claims to stored intermediate goods rather than holding claims to firm profits. In sectors with no entrepreneurial success, incumbent firms could sell such claims, use them to finance greater current production and then store the good to sell at the beginning of the next boom when the price is higher. In this case, since the cost of production is the same whether the good is stored or not, the rate of return on claims to stored intermediates in sector i is $\log p_{i,v+1} / p_{i,v} = (1-\alpha)\Gamma_v$. It follows that the long run rate of return on claims to firm profits an instant prior to the boom must satisfy

$$\beta(T_v) \geq (1-\alpha)\Gamma_v. \tag{82}$$

Free–entry into arbitrage ensures that $\beta(T_v) < (1-\alpha)\Gamma_v$ cannot obtain in equilibrium. Because there is a risk of obsolescence, this condition implies that at any time prior to the boom the expected rate of return on claims to stored intermediates is strictly less than $\beta(t)$. Combining (81) and (82) yields the condition in the statement of the Proposition.■

Proof of Proposition 5: Substituting for $1 - H_v$ in Proposition 3 using (30), we can express the size of the boom as

$$\Gamma_v = \delta\gamma\Delta_v^E - \delta\gamma\frac{c_v}{k_v^\alpha}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right), \tag{83}$$

Propositions 4 and 2 yield

$$\alpha e^{-(1-\alpha)\gamma}\frac{c_v}{k_v} = \frac{1-e^{-(1-\alpha)\Gamma_v}}{\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)}. \tag{84}$$

Finally, asset market clearing over the boom (conditions (78) to (81)) imply:

$$\delta v_0^I(c_v, c_{v-1}, k_v, k_{v-1}, \Gamma_v, \Gamma_{v-1}, \Delta_v^E, \Delta_v^X) = \frac{w_0\left(T_{v-1}\right)}{\bar{A}_{v-1}} = (1-\alpha)e^{-(1-\alpha)\gamma}e^{-\alpha\Gamma_{v-1}}k_{v-1}^\alpha, \tag{85}$$

where

$$v_0^I = V_0^I(T_{v-1})/\bar{A}_{v-1} \tag{86}$$

$$= \int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds}\frac{\pi(\tau)}{\bar{A}_{v-1}}d\tau + e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds}\int_{T_v^E}^{T_v}\frac{\pi(\tau)}{\bar{A}_{v-1}}d\tau + P(T_v)\frac{V_0^I(T_v)}{\bar{A}_v}. \tag{87}$$

$$= (1-e^{-(1-\alpha)\gamma})\left[\int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds}y(\tau)d\tau + e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds}\int_{T_v^E}^{T_v} y(\tau)d\tau\right] + \left(1-\frac{\Gamma_v}{\gamma}\right)\frac{w_0(T_v)}{\delta\bar{A}_v} \tag{88}$$

$$= (1-e^{-(1-\alpha)\gamma})e^{-\alpha\Gamma_{v-1}}k_{v-1}^\alpha\int_{T_{v-1}}^{T_v^E} e^{-\int_{T_{v-1}}^{\tau} r(s)ds}\left(\frac{k(\tau)}{k_0(T_{v-1})}\right)^\alpha d\tau$$

$$+e^{-\int_{T_{v-1}}^{T_v^E} r(s)ds}\left\{(1-e^{-(1-\alpha)\gamma})c_v\delta\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)\right.$$

$$\left.+\left[1-\delta\Delta_v^E+\delta\frac{c_v}{k_v^\alpha}\left(\frac{1-e^{-\frac{\rho}{\sigma}\Delta_v^E}}{\rho/\sigma}\right)\right](1-\alpha)e^{-(1-\alpha)\gamma}k_v^\alpha\right\}. \tag{89}$$

Proof of Lemma 5: Due to the unit elasticity of final producer demand, intermediate producing entrepreneurs wish to set price as high as possible. Thus, contracting a lower price at any instant is not optimal for the leader in i. Offering a $p_i^c(t) > p_i(t)$ in any instant would lead to a bid by the previous incumbent that would be both feasible and preferred by the final good producer. Thus $p_i^c(t)$ is the profit maximizing price and $x_i^c(t) = \frac{Y(t)}{p_i^c(t)}$for all $t \in [T_v, T_{v+1})$.■

Proof of Proposition 6: Condition (68) can be expressed as

$$\begin{aligned} \int_t^{T_v} \pi(\tau) d\tau &\geq \int_t^{T_v} \overline{q} K(T_v^E) \left(1 - (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau \\ (1 - e^{-(1-\alpha)\gamma}) Y(T_v^E) \int_t^{T_v} (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \overline{q} K(T_v^E) \int_t^{T_v} \left(1 - (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau \end{aligned}$$

Since $\overline{q} K(T_v^E) = \alpha e^{-(1-\alpha)\gamma} Y(T_v^E)$, this can be expressed as

$$\begin{aligned} (1 - e^{-(1-\alpha)\gamma}) \int_t^{T_v} (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \alpha e^{-(1-\alpha)\gamma} \int_t^{T_v} \left(1 - (1 - H_v) e^{-\frac{\rho}{\sigma}(\tau - T_v^E)}\right) d\tau \\ (1 - (1 - \alpha) e^{-(1-\alpha)\gamma})(1 - H_v) \int_t^{T_v} e^{-\frac{\rho}{\sigma}(\tau - T_v^E)} d\tau &\geq \alpha e^{-(1-\alpha)\gamma} (T_v - t) \end{aligned}$$

Since this holds with equality at $t = T_v$, a sufficient condition is that the left hand side declines more rapidly with t than the right hand side. That is

$$(1 - (1 - \alpha) e^{-(1-\alpha)\gamma})(1 - H_v) e^{-\frac{\rho}{\sigma}(t - T_v^E)} > \alpha e^{-(1-\alpha)\gamma} \qquad (90)$$

This will hold for all t if holds for $t = T_v$. Hence, condition (50) follows.■

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Appendix B: The Acyclical Growth Path

Proposition 7 *: If*

$$\frac{(1-e^{-(1-\alpha)\gamma})\gamma(1-\sigma)}{1-\alpha e^{-(1-\alpha)\gamma}} < \frac{\rho}{\delta} \tag{91}$$

then there exists an acyclical equilibrium with a constant growth rate given by

$$g^a = \max\left[\frac{[\delta(1-e^{-(1-\alpha)\gamma}) - \rho(1-\alpha)e^{-(1-\alpha)\gamma}]\gamma}{1-\alpha e^{-(1-\alpha)\gamma} - \gamma\left(1-\sigma\right)\left(1-\alpha\right)e^{-(1-\alpha)\gamma}}, 0\right]. \tag{92}$$

Proof: Substituting for x_i and p_i into (10) yields $K_i = K$, and $L_i = L = 1$ for all i with q and w given by:

$$\begin{aligned} q(t) &= \frac{\alpha e^{-(1-\alpha)\gamma} Y(t)}{K(t)} && (93) \\ w(t) &= (1-\alpha)e^{-(1-\alpha)\gamma} Y(t). && (94) \end{aligned}$$

Since $q\left(t\right) = r\left(t\right) > 0$, accumulating capital dominates storage, so that:

$$\dot{K}(t) = Y(t) - C(t), \tag{95}$$

Since all successes are implemented immediately, the aggregate rate of productivity growth is

$$g(t) = \delta\gamma H(t) \tag{96}$$

No–arbitrage implies that

$$r(t) + \delta H(t) = \frac{\pi(t)}{V^I(t)} + \frac{\dot{V}^I(t)}{V^I(t)} \tag{97}$$

Since, innovation occurs in every period, free entry into entrepreneurship implies that

$$\delta V^I(t) = w(t). \tag{98}$$

Along the balanced growth path, all aggregates grow at the rate g. From the Euler equation it follows that

$$r(t) = \rho + \sigma g. \tag{99}$$

Differencing (94) and (98) w.r.t. to time, using these to substitute for $\frac{\dot{V}^I(t)}{V^I(t)}$ in (97), and using (99) to substitute for $r(t)$ and (12) to sustitute for $\pi(t)$, we get

$$\rho + \sigma g + \frac{g}{\gamma} = \frac{\delta(1-e^{-(1-\alpha)\gamma})}{(1-\alpha)e^{-(1-\alpha)\gamma}} + g. \tag{100}$$

Solving for g yields (92).■

Appendix C: Simulation Results

Table 2: Comparative Stationary Cycles

Parameters					$k(T_v^E)$	g	Δ
σ	δ	ρ	α	γ			
.79	1.39	.02	.22	.13546	7.668	2.200	3.92
.78					7.577	2.185	1.69
.80					7.743	2.213	5.87
	1.38				7.778	2.191	5.06
	1.40				7.553	2.207	2.65
		.0197			7.565	2.183	1.46
		.0203			7.74	2.213	6.05
			.213		7.332	2.213	5.61
			.227		8.008	2.185	2.08
				.13446	7.843	2.177	5.29
				.13646	7.491	2.221	2.47
.9	1.593				6.574	2.541	5.29
1	1.861				5.407	2.967	4.12

Table 2 lists the numerical implications for growth, cycle length and terminal values of capital stocks for various combinations of parameter values, including the baseline case. A first thing to note is the extreme sensitivity of cycle length, $\Delta = \Delta^X + \Delta^E$, to changes in parameters. In contrast, the long–run growth rate is much less sensitive to changes in parameters than along the acyclical growth path. Generally, increases in parameters that directly raise the impact of entrepreneurship, δ and γ, increase the growth rate, as in the acyclical steady state. Changes in σ and ρ also have effects similar to those present in the acyclical steady state. Additionally, however, changes in these parameters alter cycle length in ways which counterveil, and sometimes overshadow, the direct effects. For example, increasing σ, lowering inter-temporal substitutability, generally induces lower growth in the acyclical steady state because consumers are less willing to delay consumption to the future. A similar effect is present here. However, as the table shows, this increase also raises cycle length and amplitude, inducing more entrepreneurship and a larger boom. The net effect, as the table shows, is an increase in growth rate for this configuration.

Values of σ closer to 1 do not satisfy our existence conditions given the values of other parameters assumed in the baseline case. However, if we allow δ to rise somewhat, higher values of σ are consistent with the cycle (see the last two rows of Table 2). Intuitively, with higher entrepreneurial productivity, both the size of booms and the average growth rate tend to be higher in equilibrium. As a result, households are willing to delay consumption enough even for low elasticities of intertemporal substitution. As can be seen, the long–run growth rate in such cases tends to be higher and the cycles shorter.