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Abstract

This paper applies the robust control approach to a simple positive theory of monetary policy, when the central bank's model of the economy is subject to misspecifications. It is shown that a central bank should react more aggressively to supply shocks when the model misspecifications grow larger. Moreover, the model misspecifications aggravate the inflation bias and a trade-off between output stabilisation and inflation worsens when the uncertainty surrounding the central bank's model increases. This implies that the larger the model misspecifications are, the more inflation-averse the central bank should be.

Key words: risk-sensitivity, robust control theory, monetary policy, Brainard conservatism, model uncertainty

JEL classification numbers: E58

Optimaalinen rahapolitiikka ja robusti säätöteoria

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Tutkimusosasto

Tiivistelmä

Tässä tutkimuksessa sovelletaan robustia säätöteoriaa optimaalisen rahapolitiikan suunnitteluun tapauksessa, jossa keskuspankin käyttämä talouden malli on epävarma. Robustissa säätöteoriassa ohjaussäännön tarkkuus pyritään pitämään hyvänä mallinnusvirheistä huolimatta. Rahapolitiikan uskottavuusmallilla tehdyt laskelmat osoittavat, että rahapolitiikan tulisi reagoida aktiivisemmin talouteen tuleviin tarjontasokkeihin, kun keskuspankin malli on epätarkka. Lisäksi inflaatioharha kasvaa ja tuotannonkuilun ja inflaation välinen yhteys tulee epäedullisemmaksi malliepävarmuuden lisääntyessä. Tämä johtaa siihen, että keskuspankin optimaalinen inflaationsietokyky pienenee malliepävarmuuden kasvaessa.

Avainsanat: robusti säätöteoria, rahapolitiikka, Brainard konservatiivisuus, malliepävarmuus

JEL-luokittelu: E58

Contents

1 Introduction

Most of the stochastic optimization problems can be formulated in the form of minimization of an expected value function over some feasible set of parameters and probability distribution. In particular, in the Bayesian approach the policy maker subjectively assigns *a priori* probabilities to the corresponding arguments of the optimization problem and then solves the expected value problem. This requires that the statistical properties of the underlying model are known exactly.

This is also the standard procedure in Economics with Rational Expectations. Uncertainty is generally modelled according to some stochastic process whose properties are known or can at least be learned from the historical data. If agents don't know the model exactly, it is common to assume that they at least know it up to additive Gaussian random disturbances. In the standard linear quadratic case, the principle of certainty equivalence says that they can effectively ignore such uncertainty and act as if they knew for certain.

The reality is of course much more complex. The data generating processes of many economic variables can never be known exactly. Measurement errors in the data, misspecifications in the formulation of the model and in the stochastic processes of the disturbances are all too well known for econometrician and the policymakers too. Detecting the true law of motion of the economy, but at the same time maintaining the tractability of the model is difficult even if statistical decision theory and computers have made it much easier than, say, a decade ago. In the practical policymaking context, such complexity means that a certain degree of subjectivity enters into the actual decision making when deciding upon optimal policy.

Rapidly developing literature on robustness, or "uncertainty aversion" considers policy makers' decision problem when the true model is not known exactly. In this robustness literature, concern for model uncertainty is imputed into the actual decision making problem of the agents. The agents have a reference model that reflects their best estimate of the believed law of motion of the economy. However, they want to make decisions that are robust to potential specification errors surrounding the model. This literature builds heavily on the so-called robust control approach which was developed among engineers as a response to shortcomings of the Bayesian approach in the 1980s. Namely, solutions to the expected value problem by standard optimal control methods deliver the best average performance if the underlying model is in fact accurate. However, performance of such optimal rules degrade rapidly if they are applied to an incorrect model and can deliver unwanted instabilities in the dynamic setting.

The basic idea in this robust control approach is to search for a "safe" control sequence, which secures at least a minimum performance under all potential realizations of uncertainty. Zames (1981) recognized that the goal of guaranteeing such a minimum performance in the presence of model uncertainty could be formalized simply by switching the norms in the context of optimal control applications to engineering. His idea of analyzing traditional control problems in the $\mathcal{H}_\infty$ -norm, rather than the standard sum-of-squares

$\mathcal{H}^2$ -norm, sparked a revolution in control theory. $\mathcal{H}_\infty$ - control is based on a deterministic, worst-case analysis of disturbance process. Disturbance process is assumed to be ambiguous up to the degree that its statistical properties are not known. This resulted in the so-called *robust decision theory.* This may sound overly pessimistic view of the world for academics, but for the policymakers such ambiguity is all too well known. In a relatively recent ECB Press Congress[1], W. Duisenberg was asked about the uncertainty surrounding the outlook of growth. W. Duisenberg's answer to the question reveals the type of uncertainty robust control is applied for

"I cannot quantify the uncertainty over a period of time. []...How great the risk are to that assessment. We live in a time of extreme uncertainty at the moment, and I really cannot quantify it"

Later on, it has been discovered how the robust decision theory translates into the stochastic optimization problems under risk-sensitive preferences in the linear quadratic case (Whittle 1981) and Wald's (1950) min-max type of behavior. Starting from the basic principles, Gilboa and Schmeidler (1989) has proved that an uncertainty aversion implies preference ordering which corresponds to Wald's (1950) min-max type of decision rules. Since Hansen and Sargent (2002) started to adapt these methods to economics, robust control methods have begun to attract increasing research interest among economists[2].

This paper adapts robust control perspective to the familiar positive theory of monetary policy of Kydland and Prescott (1979) and Barro and Gordon (1983). In contrast to usual dynamic Linear Quadratic Regulator problems analyzed for instance in Hansen and Sargent (2002), this paper discusses robust control in the simple static context. This may aid understanding the basic intuition behind this new approach to uncertainty[3].

In addition to the usual result according to which concern for unstructured model uncertainty implies more aggressive policy responses, this paper also shows that robustness of a robust rule is contingent on inflation bias and that a trade-off between output stabilization and inflation bias gets worse when uncertainty surrounding the central bank's model increases. This implies that the more uncertain the central bank's model is, the more inflation averse the central bank should be.

The rest of the paper is organized as follows. Section 2 discusses the basic ideas behind robust control and reviews the remarkable similarity between

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[1] http://www.ecb.int/key/sp020912.htm

[2] Min-max behavior has also been supported by many experimental studies. These experimental studies stem from the Allais and Ellsberg paradoxes. In particular, the Ellsberg paradox demonstrates that when information on "likelihoods" of possible events is uncertain, agent's preferences across alternatives cannot be described by ordinary probability measure. He suggested that people prefer acts with a known probability. That is, they take confidence in estimates of subjective probability into account when making choices. Such a pattern is inconsistent with a sure-thing principle of subjective expected utility. The sure-thing principle assumes that a state with a consequence common to all acts is irrelevant in determining preference between the acts. Gilboa and Schmeidler (1989) show that the Ellsberg paradox can be explained if the "sure-thing" principle is relaxed.

[3] For applications of robust control to monetary economics see for instance Hansen and Sargent (2002), Tetlow and von zur Muhlen (2000), Muhlen (1982), Onatski (1999, 2000), Onatski and Stock (2002), Onatski and Williams (2002), Giannoni (2002), Kasa (2002a, 2002b), Kilponen and Salmon (2001), Jääskelä and Kilponen (2001).

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robust control and risk sensitivity. The third section applies the risk sensitivity criteria to the Barro-Gordon type of policy model and familiarizes the reader with the concepts and solution methodology involved in robust control. The fourth section discusses inflation bias and optimal inflation aversion of the central banker in the context of robust control. Section five concludes.

2 Theoretical preliminaries

Basar and Bernhard (1995) provide a dynamic game interpretation of robust decision theory with two participants; the policymaker who attempts to find stabilizing policy rules by minimizing the loss and a malevolent nature which is seeking the disturbances that maximize the policymaker's loss. It is perhaps easiest to start from the following linear system with two inputs (v, u) and two outputs z and y.

$$z(t) = G_{11}v(t) + G_{12}u(t)$$

$$\tag{2.1}$$

$$y(t) = G_{21}v(t) + G_{22}u(t)$$

where $u(t) \in R^m$, $v(t) \in R^l$, $y(t) \in R^p$, $z(t) \in R^q$. G_{ij} are causal operators. As usual in the control problem, we want to design a (feedback) policy rule $u(t) = Ky(t)$, such that $u(t)$ stabilizes the system. In particular, under this control law we can write down a linear system from v to $z : z = \mathcal{T}(\mathcal{K})v$ where

$$\mathcal{T}(K) = G_{11} + G_{12}(I - KG_{22})^{-1}KG_{21} \tag{2.2}$$

is a transfer function. Robust decision problem is then to find an internally stable stabilizing controller K such that

$$\|\mathcal{T}(K)\|_\infty \le \gamma \tag{2.3}$$

where $\|.\|_\infty$ denotes $\mathcal{H}_\infty$ norm and where $\gamma > 0$ provides an upper bound of the norm. Then it can be shown that the inequality

$$\|\mathcal{T}(K)\|_\infty \le \gamma \tag{2.4}$$

is equivalent to

$$\sup_{\|v\|_2 \in L^2} \|y(t)\|_2^2 - \gamma^2 \|v(t)\|_2^2 \le 0 \tag{2.5}$$

where $\|y(t)\|_2$ and $\|v(t)\|_2$ denote Euclidian vector norms (L^2-norms) of real valued vectors y and v.Feedback coefficients K can then be find by solving the following problem:

$$\inf_K \sup_{\|v\|_2 \in L^2} \|y(t)\|_2^2 - \gamma^2 \|v(t)\|_2^2 \le 0 \tag{2.6}$$

and where sup denotes supremum and inf denotes infimum. Solution to above problem implies that we want to limit the rate of growth of $\|y(t)\|_2^2$ as

disturbances $\|v(t)\|_2^2$ grow larger. This is because above problem is equivalent to

$$\inf_K \sup_{\|v\|_2 \neq 0} \frac{\|y(t)\|_2^2}{\|v(t)\|_2^2} \leq \gamma^2 \tag{2.7}$$

In this problem, γ is often called an attenuation factor which provides a way of shaping the "robustness" profile of the rule, ie the degree at which its performance deteriorates as the disturbances grow larger. Finally, assuming that inf and sup exist, we can express the robust decision problem as a familiar min max problem

$$\begin{aligned} &\min_u \max_v \ J_\gamma(u, \nu) \\ &s.t. \\ &(2.1) \end{aligned} \tag{2.8}$$

where objective function is given by

$$J_\gamma(u, \nu) = \|y(t)\|_2^2 - \gamma^2 \|v(t)\|_2^2 \tag{2.9}$$

Novelty of this is that we can apply standard Nash equilibrium concept to the solution of game described in (2.8) and (2.9). That is, the optimal feedback policy rule (u^*) and the most destabilizing deterministic input (v^*) is a solution to

$$\begin{aligned} u^* &\in \ \arg\min_u \{J_\gamma(u, \nu)\} \\ v^* &\in \ \arg\max_v \{J_\gamma(u, \nu)\} \end{aligned}$$

In particular, solution pair (u^*, v^*) is a saddle point when it fulfills the following inequalities

$$J_\gamma(u^*, v) \leq J_\gamma(u^*, v^*) \leq J(u, v^*), \ \forall \ v \in \mathcal{V}, \ u \in \mathcal{U} \tag{2.10}$$

In other words $J_\gamma(u, v)$ has a minimum with respect to u and maximum with respect to v at the point (u^*, v^*). All that has changed with respect to the ordinary optimization problem is that the unobserved disturbance input v is introduced into the objective function directly. v is treated simply as another control term which is penalized by a factor $\gamma^2 > 0$[4]. This constraints the maximizing choice of deterministic disturbance vector v. The policymaker plays now a mind game against the nature, or in the language of Whittle (1999), against "a Phantom other" who disagrees with the policymaker: While the policymaker wants to minimize J_γ by choosing some u^* at given v, the Phantom wants J_γ to find v^* which maximizes J_γ at any given u. The policy rule that results from this policymaker-phantom game, therefore, is equivalent to a min-max policy rule in pure strategies.

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[4]In practice, nothing prevents $\gamma^2 < 0$. In this case the policymaker would be a risk-lover, or in the language of Whittle (1981) optimistic. The policymaker would believe that nature plays into the "same pocket" as the policymaker. So, far, the monetary policy literature has concentrated on the risk-averse case, although it might be equally interesting to study situations where the policymaker is "optimistic".

Currently, one of the most disputed issue on usefulness of robust control to economics is related to choice of γ, which is essentially a free parameter in the model: the policymaker can choose the parameter γ, which then determines subjectively the ambiguity surrounding the model at hand. In fact, if we let $\gamma^* \equiv \inf(\gamma)$, then γ^* is the minimum value of $\|\mathcal{T}(\mathcal{K})\|_\infty$ that can be obtained. So the decision rule that obtains this infimum is known as $\mathcal{H}_\infty$ optimal rule, which maximizes the robustness of the rule. In most of cases, however, it is sufficient to design a decision rule where $\gamma > \gamma^*$. In engineering, γ is can be calibrated such that the controlled system fulfills an ex ante set tolerance bounds. However, in economics such tolerance bounds are not so easy to detect and decide upon ex ante. Anderson, Hansen and Sargent (2000) suggest to use detection error probabilities to calibrate γ from the past data. Basic aim in this approach is to design robust rules that provide robustness against those candidate models that are difficult to distinguish from the reference models using statistical discrimination theory. Detection error probabilities provide a way of measuring the distance between the reference model and the candidate models. Kasa (2002b) provides a preliminary discussion on information theoretic approach to robust control and suggests that rational inattention approach discussed in Sims (2001) may provide an alternative way of constraining the uncertainty. Furthermore, Onatski and Williams (2002) suggest the Set Membership identification methods due to Milanese and Taragna (2001) and Model Error Modelling of Ljung (1999). This paper largely abstracts from this discussion, however.

3   Application

3.1   Preliminaries of the model

In this section, we apply the basic ideas robust control to a very familiar positive theory of monetary policy originally developed in Kydland and Prescott (1977) and Barro and Gordon (1983).

Let the economy evolve[5] according to

$$x = \alpha(\pi - \pi^e) + \epsilon + v \tag{3.1}$$

which is the celebrated Lucas Supply function (or expectations augmented Phillips curve), but with an additional deterministic disturbance component v, which introduces ambiguity of the model. x is the output gap, α is the slope of the Phillips curve and ϵ is a Gaussian type of supply shock with zero mean and some variance σ_ϵ^2. In this simple model α can also be interpreted as a policy multiplier, similarly to the original Brainard (1967) paper on monetary policy and uncertainty.

The two additive disturbance terms v and ϵ are now treated fundamentally differently. Whilst ϵ is assumed to be a Gaussian error term with a prior

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[5]Notice that this is a "time-less" game so I have droped the time subscripts for convenience

known stochastic properties, the policymaker is not able to assign any a prior probability distribution to v. The uncertainty introduced by v is Knightian, in the sense that v is totally ambiguous. Svensson (2003), for instance, refers to term like v as "unavoidable judgement term" often present in the practical policymaking context. However, he makes an assumption that unconditional expectation of v is zero, and therefore it does not affect on optimal decisions at all. In the spirit of Hansen and Sargent (2002) and more in the spirit of robust control, v represents model misspecification errors. According to this interpretation, different values of v represent model perturbations in the sense that the policymaker does not know exactly the position of the Phillips Curve in $(x, (\pi - \pi^e))$ space[6]. The model with $v = 0$ represents the reference model, while the models with $v \neq 0$, represent candidate models surrounding the reference model.[7]

In order to hedge against this ambiguity, the policy maker makes a particular subjective assessment of v, based on the robust control approach. That is, the policymaker hedges against the loss from the worst-case model. In other words, the policymaker chooses the worst case $v \neq 0$ at any given π and then designs corresponding policy rule π which minimizes the loss at given v. In order to introduce such subjective assesment of v into the decision making problem, we replace the standard quadratic loss function by an "uncertainty averse" quadratic loss function.

Let us start from the standard linear quadratic loss function given by

$$L = [(x - \hat{x})^2 + \lambda(\pi - \hat{\pi})^2] \tag{3.2}$$

where x and π are output and inflation, $\hat{x}$ are $\hat{\pi}$ are output and inflation targets of the policymaker and finally $\lambda > 0$ is the inflation aversion parameter of the central banker as in the conventional monetary policy models. Expectations operator can be dropped since the disturbance process is deterministic in robust control.

In this model, the rate of inflation is set by the central bank and the level of output is determined by an Expectations Augmented Phillips Curve, by the central bank's chosen rate of inflation and the expected inflation rate of the private sector. The key element of this stylized model is the hypothesis that the output level $\hat{x}$ targeted by the policymaker(s) is above the level that would be determined by the market without policy intervention: discretionary policy making leads to an inflation bias that arises from monetary policy aimed at raising output above its equilibrium level (Kydland and Prescott (1977), Calvo (1978), Barro and Gordon (1983)). In the first part of the analysis, however, I assume away this conventional inflationary bias problem and focus on robust control. This can be done by simply assuming that $\hat{x} = 0$. Inflation bias

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[6]Independently on this paper, Hansen and Sargent (2002) study the similar Phillips curve example in Chapter 5 of their manuscript.

[7]In the literature, there is an ungoing debate on the link between robust control and Knightian uncertainty (due to Knight (1921)), concerning the extension of Gilboa and Schmeidlers's axioms to an intertemporal setting. See for instance Epstein and Schneider (2001) and Hansen, Sargent, Turmuhambetova and William (2001).

problem will be dealt with in the latter half of this paper. For simplicity, I assume that the inflation target ($\hat{\pi}$) is zero.

A loss function with uncertainty aversion can then be written as

$$J_\theta = [x^2 + \lambda\pi^2] - \theta v^2 \tag{3.3}$$

where $\theta = \gamma^2 > 0$. The design of a robust policy rule now becomes a min-max problem, or in the language of Whittle (1981), an extremisation where optimal level of inflation is found by minimizing J_θ, with v being chosen to maximize J_θ subject to linear constraint (3.1). More formally, we seek a solution to the following problem

$$\begin{aligned}
\underset{\pi,v}{ext}\ & J_\theta \\
s.t.\ & \\
x\ =\ & \alpha(\pi - \pi^e) + \epsilon + \nu
\end{aligned} \tag{3.4}$$

where "ext" is the extremisation operator.

Indeed, all that has changed with respect to the ordinary optimization problem is that there is an unobserved endogenous component v: v is simply another control term which is penalized by factor θ, which restrains the maximizing choice of v. It is perhaps already clear from (3.3) that letting $\theta \to \infty$ brings us back to the ordinary certainty equivalent case. This corresponds to subjective assessment that v is on average zero. In contrast, small values of θ means that the policymaker fears his model is very inaccurate, believing that v can take values that are far from zero. Thinking of v as representing unstructured model misspecification errors, permits interpretation of the resulting min-max policy rule as *robust* to additive model uncertainty arising from v. In due course, we will see that θ is always bounded from below. In other words, θ need to be large enough for the evil nature's problem to be concave. This implies a restriction on the size of uncertainty, $|v|$. Choosing θ arbitrarily close to the neighborhood of this bound is what is typically done in the $\mathcal{H}_\infty$ control problem.

3.2   Discretion

Solution to the simple extremisation problem given in (3.4) delivers a robust policy rule and a worst-case shock implicitly in the (v, π) plane

$$\pi(v)\ =\ \alpha\frac{\alpha\pi^e - \epsilon - v}{\lambda + \alpha^2} \tag{3.5a}$$

$$v(\pi)\ =\ -\frac{\alpha(\pi - \pi^e) + \epsilon}{1 - \theta} \tag{3.5b}$$

From (3.5b) and (3.3) we see that for the evil nature's problem to be well defined and concave, we must restrict $\theta > 1$. If $\theta \le 1$, it would be "optimal" for nature to choose $v = \infty$. This would violate the boundedness assumption of robust control. Consequently, $\theta = 1$ represents a lower bound for θ, or neurotic -breakdown point in the language of Whittle (1999):

"[]...marks a point at which the optimizer is so pessimistic that his apprehension of uncertainties completely overrides the assurance given by known statistical behavior. It is not stretching matters to term this point of "neurotic breakdown" [Whittle, 1999, p. 6]."

Choosing θ equal to $\theta = 1$, delivers the "maximally robust policy rule". Such a policy rule is also optimal in the sense that it stabilizes the economy subject to the largest possible perturbations, in this case v, from the certain model where $v = 0$. This is exactly a motivation in the $\mathcal{H}_\infty$ optimal control problem. Notice also, that ϵ is already known at the time of the decision made by the policymaker and the nature.

The private sector need to form rational expectations of inflation, π^e. Applying expectations operator on (3.5a)–(3.5b) it is easy to show inflation expectations are zero at equilibrium. Consequently, setting $\pi^e = 0$ in the above equations and solving delivers reduced form expressions for the worst-case shock v^* and for inflation π^*

$$v^* = \frac{\lambda}{\lambda(\theta - 1) + \alpha^2\theta}\epsilon \tag{3.6}$$

$$\pi^* = -\frac{\theta\alpha}{\lambda(\theta - 1) + \alpha^2\theta}\epsilon \tag{3.7}$$

Using above equations we can calculate the equilibrium output gap (x^*) in the worst-case as

$$x^* = \frac{\lambda\theta}{\lambda(\theta - 1) + \alpha^2\theta}\epsilon \tag{3.8}$$

Furthermore notice that

$$\pi^{c.e.} = \lim_{\theta \to \infty} \pi^* = -\frac{\alpha}{\lambda + \alpha^2}\epsilon \tag{3.9}$$

where $c.e.$ denotes certainty equivalent. That is, when the robustness parameter θ approaches infinity, it is possible to recover the certainty equivalent rule from the extremisation problem.

On the contrary, the more uncertainty averse the policymaker is – *ceteris paribus* –, more "aggressive" he becomes. In this simple model, this shows up as a desire to stabilize inflation more, at any given λ. This tendency becomes even more pronounced as θ becomes smaller. This can be shown easily. First, remember that $\lim_{\theta \to \infty} \pi^* = \pi^{c.e.}$ and then notice from (3.7) that

$$\left|\frac{\partial^2\pi^*}{\partial\epsilon\partial\theta}\right| < 0 \tag{3.10}$$

This is a typical result obtained in many different contexts of robust control theory and in many applications of robust control to monetary policy[8]. It is in sharp contrast to a familiar Brainard (1967) result, according to which policymaker should act more cautiously under policy multiplier uncertainty when compared to certainty equivalent rule.

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[8] Onatski and Stock (2000), Tetlow and von Muhlen (2000), Kilponen and Salmon (2001), Jääskelä and Kilponen (2001), Hansen and Sargent (2002).

On the other hand, as the central banker becomes more inflation conservative ((λ) becomes bigger) and $\theta > 1$, it becomes harder to distinguish whether the central bank's reactions are driven by its concern for model uncertainty or not. At the limit where $\lambda \to \infty$, inflation conservativeness completely over-rides the risk-sensitivity: the central banker's reactions towards supply shocks are driven purely by his concern for inflation stabilization. This reflects also a difficult of determining θ parameter ex post from the data. Finally, reason why v can be interpreted as characterizing "the model misspecification errors", becomes clear by considering the worst-case model at any given π and π^e. Substituting (3.5b) into (3.1) delivers

$$x^w(\theta) = \frac{\theta}{\theta - 1} \alpha (\pi - \pi^e) + \frac{\theta}{\theta - 1} \epsilon \qquad (3.11)$$

In (3.11) the single parameter θ parameterizes the model uncertainty at any given policy instrument π. A specific value of θ pins down the worst-case model, against which the policymaker seeks for the loss minimizing policy rule. In this sense, $x^w(\theta)$ characterizes the worst prior of the policymaker.

3.3 Robustness

The robust policy rule is designed to perform reasonably well across a range of alternative models, but it has not been designed to be optimal relative to any particular model. As we have seen in the previous section, apprehension about a larger degree of uncertainty requires that the policymaker acts more aggressively. This brings better stability performance, but it does not come without costs. Roughly speaking, increasing robustness means that the policymaker needs to pay an insurance (or robustness) premium when compared to the certainty equivalent rule: concern for robustness brings more aggressive policy responses and generates more volatile movements in the policy instrument, namely inflation, yet it performs better as the model misspecifications become more serious.

In order to highlight this crucial property of the robust rule, it is useful to compare the robust rules and certainty equivalent rule under different range of model misspecifications $|v|$. Effectively, this can be done by evaluating the average (or expected) loss $E[(x^2 + \lambda \pi^2)]$ at different realizations of uncertainty, given that the policymaker has committed to a particular robust policy rule, given by (3.7).

Figure (1) plots an average loss of optimal certainty equivalent rule and robust rule with $|v|$ varying along the ordinate axis and vertical axis providing the average loss. In the figure (1), solid line corresponds to optimal (certainty equivalent) rule while dashed line corresponds to robust rule with $\theta = 1$. As expected, certainty equivalent rule generates the lowest expected loss for the central bank, when there is no uncertainty $|v| \to 0$. However, when the model misspecification error becomes larger, the performance of optimal rule deteriorates more rapidly than that of the robust rule. Finally, as the model misspecification errors becomes large enough, the robust rule outperforms the certainty equivalent rule.
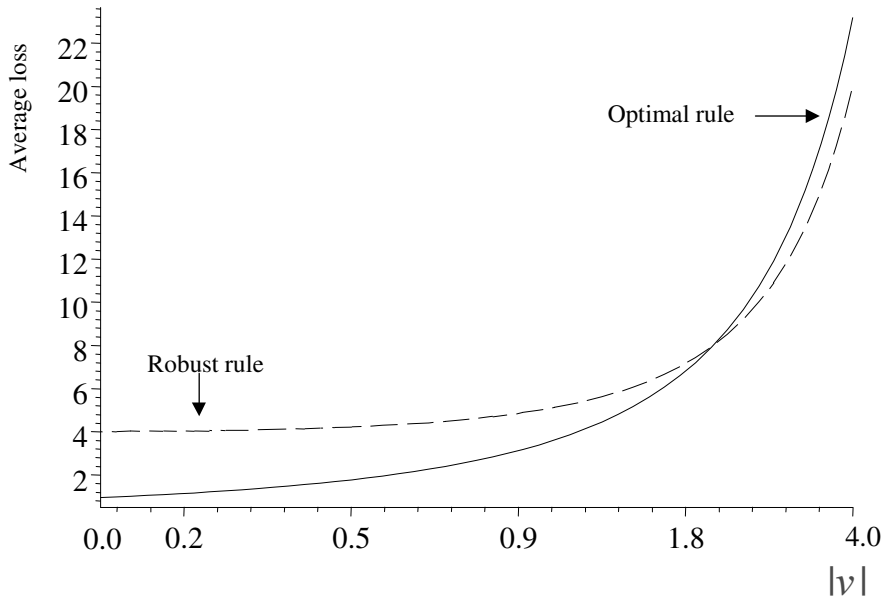
Figure 1: Average loss under different rules and model misspecification errors. Solid line corresponds to certainty equivalent rule and dashed line corresponds to maximally robust rule with $\theta = 1$.

Under this particular parameter constellation ($\lambda = 1$, $\alpha = .5$ and $\sigma_\epsilon = 1$) cut-off point is around $|v| = 2$. This cut-off point depends crucially upon structural parameters of the model, assumed variance of the supply shock and also inflation aversion of the central bank (see (3.6)). Most importantly, however, it depends upon the degree at which the policymaker actually wants to hedge against the uncertainty. In the above picture, we have assumed that the policymaker is overly pessimistic and has set θ at the breakdown point of 1. While such extreme pessimism might in some cases be desirable, it is unlikely that the policymaker would be ready to sustain so large performance deviations from optimal certainty equivalent rule. It is more likely, that the policymaker wants to find a right balance between robustness properties of the rule and average performance as the time evolves, either through learning or otherwise actively monitoring its historical record and adjusting the robustness properties of the rule accordingly. In this simple example, this boils down in choosing appropriate θ which in turn determines the robustness profile, or the rate of growth at which the performance of the rule deteriorates as disturbances grow larger, of the desired rule.

4 Discretion and inflation bias

4.1 Preliminaries

In the previous section, we have seen how easy it is to devise a rational behavior for the risk-sensitive central banker or in other words, when the central banker fears his model is misspecified. Instead of minimizing a conventional linear quadratic loss function, the central banker should hedge against model uncertainty by solving the maximisation problem with respect to worst-case model. Linear quadratic optimization theory still applies and technically speaking, the decision problem is no more difficult than solving a "mental Nash game" between the two actors with opposite interests. Although it may not be possible to distinguish between inflation conservativeness and uncertainty aversion in this simple model, we have seen that the when the central bank takes the model uncertainty into account in its decision problem, it should act more aggressively.

In this section, we turn to analyze the very same game, but introduce the inflation bias into the model. The easiest way to do this is to re-consider a loss function

$$J'_\theta = \frac{1}{2}[(x - \widehat{x})^2 + \lambda\pi^2 - \theta v^2] \tag{4.1}$$

where $\widehat{x} > 0$ is a source of the inflation bias. The policymaker's target level of output $(\widehat{x})$ is on average unattainable high. Therefore, after the rational expectations are formed, the policymaker would like to inflict surprise inflation.

The current monetary policy literature of course discredits[9] much of such analysis, arguing that output target which is somehow away from the natural rest point of the economy is unrealistic. At the same time, however, the literature following Kydland and Prescott's (1977) and further debate followed Barro and Gordon (1983) and Rogoff (1985) have actually re-shaped the whole central bank institution. Or, as Blinder (1997) puts it,

> "[...] the noun "central banker" practically cries out for the adjective "conservative" [Blinder, 1997, pp. 14]".

Moreover, introducing a potential conflict between the private sector and the policymaker is the easiest way to introduce explicit role for expectations and relate the discussion to the analysis of Rogoff's inflation conservative central bank. Formally, the central banker solvers now the following problem:

$$\underset{\pi,v}{ext} \ \frac{1}{2}[(x - \widehat{x})^2 + \lambda\pi^2)) - \theta v^2]$$
$$s.t. \tag{4.2}$$
$$x = \alpha(\pi - \pi^e) + \epsilon + v$$

The first order conditions of this game are

$$\frac{\partial J'_\theta}{\partial \pi} = (\alpha(\pi - \pi^e) + \epsilon + v - \widehat{x})\alpha + \lambda\pi = 0 \tag{4.3}$$

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[9]See for instance Blinder (1997).

$$\frac{\partial J'_\theta}{\partial v} = \alpha\left(\pi - \pi^e\right) + \epsilon + v - \widehat{x} - \theta v = 0 \tag{4.4}$$

Taking rational expectations over (4.3) and (4.4) and solving for $\pi^e(\theta)$, delivers after straightforward manipulations rationally expected inflation

$$\pi^e(\theta) = \frac{\alpha}{\lambda}\frac{\theta}{\theta - 1}\widehat{x} \tag{4.5}$$

As in the earlier analysis, in order for robust control problem to be bounded, it is necessary to restrict $\theta \geq 1$. Then, when $\widehat{x} > 0$, expected inflation is positive. An over-ambitious output target generates an incentive to inflict surprise inflation. This incentive is correctly anticipated by the rational agents, who expect that inflation will deviate on average from zero. In this robust control setting, however, this deviation depends upon θ and therefore implicitly on uncertainty $|v|$.

From the private sector's point of view, the evil agent's choice can be interpreted as pessimistic distortion to the output target of the central bank. The private sector then forms its expectations according to the worst-prior. Uncertainty, originally introduced into the form of "shock uncertainty", implies ambiguity of all the exogenous variables of the model (and this is taken into account by the private sector when forming its expectations). As noted by Onatski and Williams (2002), there is a potential inconsistency between robustness to shock uncertainty and the robustness to parametric uncertainty. This means that policy that is designed to be robust with respect to "shock uncertainty" does not mean it is robust to parametric uncertainty too. This is due to the fact that the robustness to parametric uncertainty cannot be judged ex ante, but it depends often on complicated way on the specific policy that is actually analyzed.

Nevertheless, given (4.5), we can easily solve the equilibrium. Inserting inflation expectations into the first order conditions and solving the resulting two order system[10] delivers the worst-case inflation, output gap and the worst-case shock as a function of over ambitious output target $\widehat{x}$ and the exogenous supply shock ϵ:

$$\pi^*_{tc} = \frac{\alpha}{\lambda}\frac{\theta}{\theta - 1}\widehat{x} - \frac{\alpha\theta}{\alpha^2\theta + \lambda(\theta - 1)}\epsilon \tag{4.8}$$

$$x^*_{tc} = \frac{1}{1 - \theta}\widehat{x} + \frac{\lambda\theta}{\alpha^2\theta + \lambda(\theta - 1)}\epsilon \tag{4.9}$$

$$v^*_{tc} = \frac{1}{1 - \theta}\widehat{x} + \frac{\lambda}{\alpha^2\theta + \lambda(\theta - 1)}\epsilon \tag{4.10}$$

The fact that the rational agents hedge against the worst-case prior puts the central bank into a difficult position. Evil agent generates worst case

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[10] The two order system reads at given $\pi^e(\theta)$ reads

$$\left(\alpha\left(\pi - \frac{\alpha}{\lambda}\frac{\theta}{\theta - 1}\widehat{x}\right) + \epsilon + v - \widehat{x}\right)\alpha + \lambda\pi = 0 \tag{4.6}$$

$$\alpha\left(\pi - \frac{\alpha}{\lambda}\frac{\theta}{\theta - 1}\widehat{x}\right) + \epsilon + v - \widehat{x} - \theta v = 0 \tag{4.7}$$

perturbation both for the mean and for induced variance of additive shock (4.10). The best the central bank can do is to generate inflation in order to avoid permanently negative output gap. Inflation bias problem is in other words magnified as the pessimism towards the central bank's model increases.

The model's worst-case analysis reflects in an important way the potential problem of designing policy rules that are robust to "unstructured" uncertainty, in particular in the context where private sector's expectations play a role. Ambiguity which is introduced into the output target of the central bank through unstructured shock uncertainty, is simply detrimental for the whole economy when uncertainty is large. This becomes apparent when repeating the robustness analysis in the previous section.

Namely, if over-ambitious output target $\hat{x}$ is far enough from the natural rest point of the economy initially, resulting inflation bias can lead in the situation where certainty equivalent rule out-performs the robust rule, no matter of the size of model misspecification errors $|v|$. This seems rather surprising, given that we were explicitly interested in finding a robust rule, which performs reasonably well across a range of different models.

However, this result can most easily be understood by comparing the inflation biases and output gap in the two cases:

$$E\left(\pi_{tc}^* - \pi_{tc}^{c.e.}\right) = \frac{\alpha}{\lambda}\frac{1}{\theta-1}\widehat{x} \tag{4.11}$$

For $\theta > 1$, we notice that as $\theta \to 1$ and therefore model misspecifications become larger, the difference between the inflation biases in the two cases increases and in fact, approaches infinity. This difference is also larger the bigger $\widehat{x}$ is.

As for the output gap, remember first that in the conventional case, output gap is on average on its target value and it is only inflation which is subject to "bias". Again, this can be seen easily by taking a limit of the expected value of (4.9) with respect to θ. When the central bank's model is subject to misspecifications, however, there is an additional bias to the worst-case output gap. This bias can be written as

$$E\left(x_{tc}^* - \widehat{x}\right) = E\left(x_{tc}^* - x_{tc.}^{c.e.}\right) = \frac{\theta}{1-\theta}\widehat{x} \tag{4.12}$$

The output gap bias (4.12) depends upon θ and $\widehat{x}$ only. Given our restriction that $\theta > 1$, (4.12) suggests that in the worst-case equilibrium output is on average lower than its target value. Consequently, the worst-case equilibria in this game is characterized by too low output and too high inflation on average at the same time. That is, a problem of inflation bias becomes more pronounced as the central bank's model becomes more uncertain – *ceteris paribus* – and the rational agents correctly anticipate this. If the central bank's model is very inaccurate, inflation bias becomes so high that it outweighs a gain from robustness[11].

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[11]Otherwise, robustness properties of robust rule are similar to the case without inflation bias when over-ambituous output target is low enough. As the model misspefication error becomes larger, the performance of certainty equivalent rule deteriorates more rapidly than the robust rule.

In this simple example, this problem originates from the fact that we have only one free parameter to adjust for the uncertainty in the mean and induced variance of the disturbance process. This can be seen directly from (4.10). One way of avoiding this problem, is to use a scaling parameter, say ρ, which would allow to scale uncertainty in the mean of the disturbance process. For instance, by writing $J_\theta = \frac{1}{2}(x - \rho\hat{x})^2 + \lambda\pi^2) - \theta v^2$ and solving the problem as usual, we would get $v_{tc}^* = \frac{1}{1-\theta}\rho\hat{x} + \frac{\lambda}{\alpha^2\theta+\lambda(\theta-1)}\epsilon$. Then, by choosing ρ appropriately, it would be possible to adjust for the uncertainty surrounding the mean $\left(\frac{1}{1-\theta}\rho\hat{x}\right)$ and induced variance $\left(\frac{\lambda}{\alpha^2\theta+\lambda(\theta-1)}\right)^2\sigma_\epsilon^2$ of the disturbance process independently.

Finally, making the central bank more inflation conservative, ie increasing the inflation stabilization weight (λ), decreases the absolute value of the inflation bias$\left(\frac{\alpha}{\lambda}\frac{\theta}{\theta-1}\hat{x}\right)$, but generates a more volatile output. The next section discusses how this trade-off could be exploited, bringing us finally back to Rogoff's (1985) seminal article.

4.2   Delegation once more

In a very influential article Rogoff (1985) suggested that by choosing some particular preferences for the central banker, social welfare could be improved with respect to discretionary equilibrium. Rogoff's main point was that an optimally inflation averse policy would find the (socially) best balance between the output stabilization and prices in response to supply shocks. Appointing optimally inflation averse central banker would partially offset the inflation bias, but with an expense of generating more volatile output. I will next re-frame the Rogoff's analysis in the context of risk sensitivity.

Consider a society with the following risk-sensitive welfare criteria

$$J_{\theta_s}^S = \frac{1}{2}E\left((x - \hat{x})^2 + \lambda_s\pi^2\right) - \theta_s v^2 \tag{4.13}$$

where λ_s and θ_s represent the inflation aversion and risk sensitivity of the society respectively. The idea, as in Rogoff (1985), is to use (4.13) and evaluate the welfare loss in a delegation game, where the central bank's preference parameters can differ from those of society. Formally, the delegation problem in this risk-sensitive environment is to solve following extremisation problem

$$\underset{\lambda_b,\theta_b}{ext}\ J_{\theta_s}^S$$
$$s.t.$$
$$\pi_{tc}^* = \frac{\alpha}{\lambda_b}\frac{\theta_b}{\theta_b - 1}\hat{x} - \frac{\alpha\theta_b}{\alpha^2\theta_b + \lambda(\theta_b - 1)}\epsilon \tag{4.14}$$
$$x_{tc}^* - \hat{x} = \frac{\theta_b}{1 - \theta_b}\hat{x} + \frac{\lambda_b\theta_b}{\alpha^2\theta_b + \lambda_b(\theta_b - 1)}\epsilon$$
$$v_{tc}^* = \frac{1}{1 - \theta_b}\hat{x} + \frac{\lambda_b}{\alpha^2\theta_b + \lambda_b(\theta_b - 1)}\epsilon$$

λ_s	θ							
	∞(c.e.)	10	5	2	1.5	1.2	1.01	1
.5	1.3	1.4	1.4	1.6	1.9	2.5	13.1	∞
1	2.3	2.4	2.4	2.7	3.0	3.7	16.8	∞
1.5	3.4	3.4	3.4	3.7	4.0	4.9	19.6	∞
2	4.4	4.4	4.5	4.7	5.03	5.9	21.9	∞

Table 1: The risk sensitivity and the optimal degree of inflation Conservatism

Substituting constraints into the society's welfare criteria, then, delivers a rather complicated extremisation problem

$$\underset{\lambda_b,\theta_b}{ext} \left(\left(\lambda_b^2 + \lambda_s \alpha^2 \right) \theta_b^2 - \theta_s \lambda_b^2 \right) \left(\frac{(\widehat{x})^2}{\lambda_b^2 \left(\theta_b - 1 \right)^2} + \frac{\sigma_\epsilon^2}{\left(\theta_b \left(\alpha^2 + \lambda_b \right) - \lambda_b \right)^2} \right) \quad (4.15)$$

We need to separate two cases. First is the case where $\theta_s = 0$ and the second one where θ_s takes a positive value. In the first case, the society would have no concern for model misspecification errors and the society would simply seek for parameter stipulation where $J_{\theta_s=0}^S$ would be minimized. It is easy to show that society would be best of by choosing $\theta_b = \theta_s = 0$ and, as in the ordinary Rogoff's article, it would be optimal to choose $\lambda_b > \lambda_s$. The society would be better of by appointing more conservative central banker. It is clearly the second case, which is more interesting.

In this context, I will study only a special case where both the society and the central bank "agree" to have some $\theta_b = \theta_s \in \,]1,\infty]$ and see whether the model misspeficiations have some implications regarding the "optimal inflation aversion" of the central bank. Notice that $\theta_b = \theta_s = \theta$ effectively transforms the min-max problem into that of minimizing the loss function $J_{\theta_s}^S$ with respect to λ_b at given θ.

Unfortunately, analytical derivation of this is cumbersome. However, the simulations show that as the central bank's model becomes more inaccurate, the optimal degree of inflation conservatism of the central bank increases. Eventually, as the θ approaches the breakdown point of $\theta = 1$, the optimal degree of conservatism becomes infinite! These results are depicted numerically in table 1.

Table 1 shows how the optimal degree of conservatism changes as the preferences regarding inflation and model uncertainty changes. The column labeled $\infty(c.e.)$ shows the optimal degree of conservatism in the ordinary case without model misspecifications. The column inf (θ) shows the optimal degree of conservatism when θ_s is arbitrarily close to 1.

Moving horizontally from left to right shows how the optimal degree of conservatism changes at some given level of inflation conservativeness of the society. Moving vertically downwards shows how the optimal degree of conservatism changes at some given level of model uncertainty. The results clearly show that regardless of the inflation aversion of the society(λ_s), more inaccurate the model is, the more conservative the central bank should be.

This result is easily understood by noticing that inflation bias is decreasing function of θ and λ_b as shown in (4.8). The trade-off between output

stabilization and inflation bias gets worse as the model misspefications grow larger. Therefore, from the society's welfare perspective, it becomes necessary to appoint even more conservative central banker. This very same result might suggest , in fact, that gains from solving the inflation bias problem when the model uncertainty is present, might be even higher than within a certain environment[12].

5   Conclusions

In this paper a concern for model misspecifications has been introduced into the central bank's objective function. It has been shown how this "uncertainty aversion" relates to the robust control theory, developed in the engineering literature in the 1980s. As usual in the robust control literature, the paper shows that the model uncertainty implies more aggressive policy responses. In the absence of inflation bias, it has also been shown that such a policy rule's performance deteriorates slower when model misspecifications grow larger and robust rule thus performs better under large model misspefications. This result, however, is contingent on the size of the inflation bias, when the model includes overambitious output target for the central bank. Another important observation is that putting too much weight on robustness properties of the rule delivers policy rules that are overly reactive and do not perform well in terms of average performance. Consequently, when applying the robust control to the actual policymaking context, one should bear in mind that there is a trade-off between robustness properties of the policy rules and average performance.

Finally, model uncertainty has some interesting implications for the optimal inflation conservatism of the central bank, too. Namely, by re-framing Rogoff's (1985) analysis, it has been shown that the more uncertain the central bank's model is, the more conservative the central should be. This is due to the fact that the trade-off between output stabilization and inflation bias gets worse when the model uncertainty increases.

Undoubtedly, several interesting topics within this conventional framework have been left unexplored. One interesting direction for research would be to explore further the inflation bias problem in the context of misspecified model. The fact that delegation of monetary policy to risk-averse, but inflation conservative central banker partially solves the inflation bias problem, might suggest that Svensson's (1997) inflation targeting regime would completely solve the problem of inflation bias in this environment, too. Furthermore, several interesting issues arise by altering the Lucas Supply Function. For instance, as shown by Roberts (1995), most of the New-Keynesian models suggest that the Lucas Supply function is misspecified and that forward-looking behavior of the agents should be included in the aggregate supply function. This implies that the policymaker cannot take inflation expectations as predetermined since the aggregate supply function contains expectations of

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[12]Similar conclusions are obtained by Kasa (2002a) in a much more complicated dynamic game.

future inflation. In fact, many of the more practical applications of robust control theory to monetary policy rely on the New-Keynesian type of models.

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