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Effects of the supply-side channel on stabilisation properties of policy rules

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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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Effects of the supply-side channel on stabilisation properties of policy rules

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Abstract

In this paper we introduce an application of the supply-side channel for fiscal policy to the basic New Keynesian model. We use a proportional tax rate instead of lump sum tax and introduce the distortions of a tax wedge. We derive a closed economy forward-looking model with government consumption and no capital. Households' labour supply decisions are endogenised. Monetary policy is conducted by a Taylor-type interest rate rule and fiscal policy follows a simple debt rule. We analyse the stability of the model when fiscal policy has both demand and supply-side effects and compare results with the standard case of only demand effects. We show that taking supply-side effects into account restricts the fiscal policy parameter range consistent with the dynamic stability of the economy. We also argue that allowing fiscal policy to affect both supply and demand results in more persistent inflation as well as output responses to shocks, than without the supply-side channel. We also discuss the different monetary and fiscal policy regimes and their implication on the stability of inflation and output.

Key words: inflation, fiscal and monetary policy, stabilisation

JEL classification numbers: E52, E31, E63

Tarjontakanavan vaikutukset talouspolitiikan sääntöjen vakausominaisuuksiin

Suomen Pankin keskustelualoitteita 34/2003

Jukka Railavo
Tutkimusosasto

Tiivistelmä

Tässä tutkimuksessa arvioidaan finanssipolitiikan tarjontakanavan vaikutuksia uuskeynesiläisessä mallissa. Tutkimuksessa on käytetty suhteellista verotusta kiinteän könttäsummaveron sijasta, mikä tuo tarjontaan verokiilan aiheuttamia vääristymiä. Tutkimuksessa johdetaan eteenpäin katsova suljetun talouden malli, jossa ei oteta huomioon pääomaa, mutta joka sisältää julkisen sektorin. Kotitalous voi päättää itse työntarjonnasta. Rahapolitiikkaa hoidetaan Taylorin säännön avulla, ja finanssipolitiikka seuraa yksinkertaista velkasääntöä. Tutkimuksessa analysoidaan mallin vakautta, kun finanssipolitiikalla on sekä kysyntä- että tarjontavaikutuksia. Työssä osoitetaan, että tarjontavaikutusten huomioon ottaminen rajoittaa finanssipolitiikan parametrien vaihteluväliä, joka on sopusoinnussa talouden dynaamisen tasapainon kanssa. Tutkimuksessa väitetään, että kun finanssipolitiikan sallitaan vaikuttaa myös tarjontaan kysynnän lisäksi, on sokkien seurauksena pysyvämpiä inflaatio- ja tuotantovaikutuksia kuin ilman tarjontakanavaa. Lisäksi tutkimuksessa vertaillaan eri raha- ja finanssipolitiikan yhdistelmiä ja niiden vaikutusta inflaation ja tuotannon vakauteen.

Avainsanat: inflaatio, finanssi- ja rahapolitiikka, vakaus

JEL-luokittelu: E52, E31, E63

Contents

1 Introduction

The literature on monetary economics has recently emphasised the link between monetary and fiscal policy discovered in Sargent and Wallace's (1981) 'unpleasant monetary arithmetic'. This link is widely examined in fiscal theory of the price level literature, see eg Woodford (1994, 1995, 1996, 2001) and Sims (1994) and also in public finance literature, eg Benhabib, Schmitt-Grohé and Uribe (2001) and Schmitt-Grohé and Uribe (2002). The basic idea is that fiscal policy can have important implications on the price level in the type of models where traditionally prices have been only affected by the monetary policy. In extreme cases fiscal variables can determine the price level independently of monetary policy.

In this paper we formulate a simple closed economy New Keynesian model with endogenous labour supply and no capital. The only form of tax in the model is income tax, which is proportional and has distortionary effects. The use of a distortionary tax changes the output reaction compared to the conventional results is dependent on the labour supply elasticity[1]. We use the Rotemberg (1987) model to introduce price stickiness, which according to Schmitt-Grohé and Uribe (2002) reduces inflation volatility. They show that even a small degree of price stickiness is sufficient to sustain a low inflation tax.

Monetary policy is conducted using a Taylor (1993) -type monetary policy rule and the fiscal policy rule is based on a simple Leeper (1991) -type tax rule responding to the government liabilities. In the literature the form of the interest rate rule is widely discussed. Surveys on monetary policy rules is found in eg Clarida, Galí and Gertler (1999) and McCallum (1999). Leeper -type fiscal policy rules are used in eg Wouters and Dombrecht (2000), Andrés, Ballabriga and Vallés (2002) and Evans and Honkapohja (2002a).

The literature on fiscal policy rules has concentrated on the demand effects of fiscal policy. Benhabib, Schmitt-Grohé and Uribe (2001) show that the specification of demand and supply have crucial effects on stability conditions. Therefore we derive the supply-side to represent not only price stickiness, but also to include the fiscal variables, government spending and taxes. Technical development has also a role in supply-side. We analyse the stability of the model when fiscal policy has both demand and supply-side effects and compare results with the standard case of only demand effects. We show that taking supply-side effects into account restricts the fiscal policy parameter range consistent with the dynamic stability of the economy. We argue that allowing fiscal policy to affect both supply and demand results in more persistent inflation and output responses to demand, supply and monetary policy shocks than without the supply-side channel. We also discuss the different monetary and fiscal policy regimes and their implication on stabilising inflation and output.

The paper has the following structure. Section 2 shows in detail the derivation of the model. We go through the optimisation problem of a household and the cost minimisation problem of a firm. We introduce the

[1] See eg Dotsey (1994), McGrattan (1994) and Ludvigson (1996).

government sector and the policy rules used. Section 3 presents calibration and the stability analysis of the model. We also go through the results of diagnostic and policy simulation. Section 4 concludes and discusses possible further research issues.

2 The model

2.1 The household

We begin by specifying an optimization-based model with no capital. We assume that there exists only one household, which also owns the only firm. We use the money in the utility function type of approach and assume that money yields direct utility by incorporating money balances into the utility function of the agent as in Sidrauski (1967). A typical household seeks to maximise a utility function $v^1\left(c_t, \frac{M_t}{P_t}, L_t\right) + v^2(g_t)$, where utility depends on real consumption c_t, real money balances $\frac{M_t}{P_t}$, leisure L_t and real public consumption g_t. Upper case letters refer to nominal values, whereas lower case letter are values in real terms. Utility of the household depends on both private and public consumption, but they are assumed to be separable. An increase in public consumption increases the level of households utility, but do not affect the marginal utility of the household consumption.

Denoting labour l_t, we can write leisure as $L_t = 1 - l_t$. Also denoting real money balances by $\frac{M_t}{P_t} = m_t$, we can write the utility function as $v^1(c_t, m_t, 1 - l_t) + v^2(g_t)$. Now we are able to write a new utility function $u^1(c_t, m_t, l_t) + v^2(g_t)$, which depends on real consumption, real money balances, labour and government consumption.

The household faces a sequence of budget constraints in nominal terms with two nominal assets, interest bearing bonds B_t and no interest bearing money balances M_t. The nominal interest rate R_t represents the yield on nominal bonds. Household has nominal net income $P_t w_t l_t(1 - \tau_t)$, where w_t is real gross wage and τ_t is tax rate. Household's flow budget constraint is in nominal terms

$$P_t c_t + M_t - M_{t-1} + B_t \leq (1 + R_{t-1}) B_{t-1} + P_t w_t l_t (1 - \tau_t), \tag{2.1}$$

which can be written in real terms[2] as

$$c_t + m_t - (1 - \pi_t) m_{t-1} + b_t \leq (1 + r_{t-1}) b_{t-1} + w_t l_t (1 - \tau_t). \tag{2.2}$$

The household maximises the utility function

$$E_t \sum_{t=0}^{\infty} \delta^t u^1(c_t, m_t, l_t) \tag{2.3}$$

[2] Inflation π_t is defined to be $\frac{P_t - P_{t-1}}{P_t} = \pi_t$, which implies that $1 - \pi_t = \frac{P_{t-1}}{P_t}$. Then nominal interest rate R_t is defined $1 + R_t = (1 + r_t) / (1 - E_t \pi_{t+1})$, where r_t is real interest rate and $E_t \pi_{t+1}$ is the expected inflation rate.

subject to equation 2.2. The household's discount factor is $\delta = \frac{1}{(1+\rho)}$, $\rho > 0$ and E_t is the expectation operator conditional on information available in period t. We assume that the utility function $u^1(c_t, m_t, l_t)$ is continuous, increasing and concave.

The first order conditions with respect to private consumption, real money balances and labour are

$$u_c^1(c_t, m_t, l_t) - \xi_t = 0, \tag{2.4}$$

$$u_m^1(c_t, m_t, l_t) - \xi_t + \delta E_t\left[\xi_{t+1}(1-\pi_{t+1})\right] = 0, \tag{2.5}$$

$$u_l^1(c_t, m_t, l_t) + \xi_t w_t(1-\tau_t) = 0, \tag{2.6}$$

$$\xi_t = \delta E_t \xi_{t+1}(1+r_t), \tag{2.7}$$

where ξ is the Lagrangean multiplier and subscripts denote partial derivatives. The equilibrium also has to satisfy transversality conditions

$$\lim_{t\to\infty} \delta^t m_t = 0 \text{ and } \lim_{t\to\infty} \delta^t b_t = 0. \tag{2.8}$$

First order conditions give us the allocation of income between consumption and money balances. They also tell how much leisure a household is willing to trade for consumption and money. We combine equations 2.4 and 2.7 to yield

$$E_t\left[\frac{u_c^1(c_t)}{u_c^1(c_{t+1})}\right] = (1+r_t)\delta, \tag{2.9}$$

which is the Euler condition for optimal intertemporal allocation of consumption. We combine also equations 2.4, 2.5 and 2.7 and use the definition of nominal interest rate R_t to get

$$u_m^1(c_t, m_t, l_t) = u_c^1(c_t, m_t, l_t)\frac{R_t}{1+R_t}, \tag{2.10}$$

which says that the marginal rate of substitution between money and consumption is equal to the opportunity cost of holding money. The opportunity cost is directly related to the nominal interest rate. A combination of equations 2.4 and 2.6 yields the households labour supply function

$$u_l^1(c_t, m_t, l_t) = -\left[u_c^1(c_t, m_t, l_t) w_t(1-\tau_t)\right], \tag{2.11}$$

which states that the marginal rate of substitution between labour supply and consumption is equal to the real net wage rate.

Now we assume a periodical utility function expressed in form $u^1(c_t, m_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\Gamma m_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\lambda}}{1+\lambda}$. This is a CRRA utility function, where $\sigma \geq 0$[3] is the measure of risk aversion and Γ is a positive constant. $\lambda \geq 0$ is the inverse of the labour supply elasticity. When $\lambda = 0$ preferences are linear in labour and the labour supply elasticity is infinite (Hansen (1985)). Using the periodical utility function, the Euler condition equation 2.9 can be rewritten as

$$c_t^{-\sigma} = E_t c_{t+1}^{-\sigma}(1+r_t)\delta. \tag{2.12}$$

[3] We also assume that $\sigma \neq 1$.

To log-linearise equation 2.12, we first take natural logarithms and rearrange using the approximation $\ln E_t c_{t+1}^{-\sigma} \approx -\sigma E_t \ln c_{t+1}$ to yield

$$\ln c_t = E_t \ln c_{t+1} - \frac{1}{\sigma} \ln(1 + r_t) - \frac{1}{\sigma} \ln \delta. \tag{2.13}$$

The equation 2.13 holds also at steady state $\overline{c}_t$, $\overline{c}_{t+1}$ and $\overline{r}_t$. We denote the steady state values of variables with bar while variables with hat are defined as logarithmic fractional deviations from the steady state values. Subtract the steady state values from equation 2.13

$$\ln c_t - \ln \overline{c}_t = E_t \ln c_{t+1} - \ln \overline{c}_{t+1} - \frac{1}{\sigma} \left[\ln(1 + r_t) - \ln(1 + \overline{r}_t) \right]. \tag{2.14}$$

The logarithmic deviation from steady state is then, for example for consumption, $\widehat{c}_t = \ln \left(\frac{c_t}{\overline{c}_t} \right)$. We also use the approximation $\ln (1 + \widehat{r}_t) \approx \widehat{r}_t$. Now we can write the equation 2.14 in deviations from steady state form

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \frac{1}{\sigma} \widehat{r}_t. \tag{2.15}$$

As in Walsh (2003, Chapter 5), the government purchases the amount g_t of total output in addition to the consumption by households. We use the economy-wide resource constraint to eliminate private consumption $\widehat{c}_t$ from equation 2.15. An economy-wide resource constraint is

$$y_t = c_t + g_t, \tag{2.16}$$

where y_t stands for total output. The resource constraint is a very strict formulation, because the increase in government consumption replaces private consumption one-for-one, which causes crowding out of private consumption. Log-linearisation of the equation 2.16 around steady state yields[4]

$$\widehat{y}_t = \frac{\overline{c}_t}{\overline{y}_t} \widehat{c}_t + \frac{\overline{g}_t}{\overline{y}_t} \widehat{g}_t. \tag{2.17}$$

Next we move equation 2.17 one period forward, take expectations and solve with respect to $E_t \widehat{c}_{t+1}$, and we get

$$E_t \widehat{c}_{t+1} = \frac{\overline{y}_{t+1}}{\overline{c}_{t+1}} E_t \widehat{y}_{t+1} - \frac{\overline{g}_{t+1}}{\overline{c}_{t+1}} E_t \widehat{g}_{t+1}. \tag{2.18}$$

Now we substitute the updated resource constraint equation 2.18 into Euler equation 2.15 and we are able to eliminate the expected consumption. Euler equation is now in form

$$\widehat{c}_t = \frac{\overline{y}_{t+1}}{\overline{c}_{t+1}} E_t \widehat{y}_{t+1} - \frac{\overline{g}_{t+1}}{\overline{c}_{t+1}} E_t \widehat{g}_{t+1} - \frac{1}{\sigma} \widehat{r}_t. \tag{2.19}$$

Next, we substitute the Euler equation 2.19 back into the resource constraint equation 2.17 and rearrange to yield

$$\widehat{y}_t = \frac{\overline{c}_t}{\overline{c}_{t+1}} \frac{\overline{y}_{t+1}}{\overline{y}_t} E_t \widehat{y}_{t+1} + \frac{\overline{g}_t}{\overline{y}_t} \left[\widehat{g}_t - \frac{\overline{c}_t}{\overline{c}_{t+1}} \frac{\overline{g}_{t+1}}{\overline{g}_t} E_t \widehat{g}_{t+1} \right] - \frac{\overline{c}_t}{\overline{y}_t} \frac{1}{\sigma} \widehat{r}_t. \tag{2.20}$$

[4] For details, see eg Uhlig (1999).

Since $\frac{\overline{c}_t}{\overline{c}_{t+1}} = \frac{\overline{y}_t}{\overline{y}_{t+1}} = \frac{\overline{g}_t}{\overline{g}_{t+1}}$ in the steady state, we obtain the IS curve in form

$$\widehat{y}_t = E_t \widehat{y}_{t+1} + \frac{\overline{g}_t}{\overline{y}_t} \left[\widehat{g}_t - E_t \widehat{g}_{t+1} \right] - \frac{\overline{c}_t}{\overline{y}_t} \frac{1}{\sigma} \widehat{r}_t, \tag{2.21}$$

which is the commonly used form of output equation in monetary policy literature (see eg Woodford, 1999). The aggregate demand equation 2.21 includes no lagged variable affecting the output today, but expectations of future are crucial. Woodford (1999) emphasises the theoretical and empirical importance of the forward-looking elements for output. Monetary policy affects aggregate demand only to the extent that it affects the deviations of the real rate from $\overline{r}$. Moreover, monetary policy affects aggregate demand through the expected future real interest rate instead of nominal interest rates. This has a dynamic link to the expected future inflation. In order to write the aggregate demand equation in levels, we use the notation $\widehat{c}_t = \ln\left(\frac{c_t}{\overline{c}_t}\right)$ and write equation 2.21 in form

$$\begin{aligned} \ln y_t = {} & E_t \ln y_{t+1} + \left[\ln \overline{y}_t - \ln \overline{y}_{t+1} \right] + \frac{\overline{g}_t}{\overline{y}_t} \left[\ln g_t - E_t \ln g_{t+1} \right] \\ & - \frac{\overline{g}_t}{\overline{y}_t} \left[\ln \overline{g}_t - \ln \overline{g}_{t+1} \right] - \frac{\overline{c}_t}{\overline{y}_t} \frac{1}{\sigma} \left[r_t - \overline{r}_t \right]. \end{aligned} \tag{2.22}$$

Since both the equation 2.13 and the resource constraint hold in the steady state, we use them to write equation 2.22 in a simpler form

$$\ln y_t = E_t \ln y_{t+1} + \frac{\overline{g}_t}{\overline{y}_t} \left[\ln g_t - E_t \ln g_{t+1} \right] - \frac{\overline{c}_t}{\overline{y}_t} \frac{1}{\sigma} r_t - \frac{\overline{c}_t}{\overline{y}_t} \frac{1}{\sigma} \ln \delta. \tag{2.23}$$

This is the (logarithmic) level counter part of the equation 2.21 we derived earlier.

To obtain money demand, we use the assumed periodical utility function $u^1(c_t, m_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\Gamma m_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\lambda}}{1+\lambda}$ to rewrite equation 2.10 in form

$$\Gamma m_t^{-\sigma} = c_t^{-\sigma} \frac{R_t}{1+R_t}. \tag{2.24}$$

Note that $\frac{R_t}{1+R_t} = 1 - \frac{1}{1+R_t}$. Now, we take logarithms of equation 2.24 and subtract steady state values $\overline{m}_t$, $\overline{c}_t$ and $\left(1+\overline{R}_t\right)$ to yield

$$\begin{aligned} \left(\ln m_t - \ln \overline{m}_t\right) = {} & \left(\ln c_t - \ln \overline{c}_t\right) \\ & - \frac{1}{\sigma} \left(\ln(1+R_t) - \ln\left(1+\overline{R}_t\right) \right) + \frac{1}{\sigma} \left(\ln \Gamma - \ln \Gamma \right). \end{aligned} \tag{2.25}$$

Recalling that $\widehat{c}_t = \ln\left(\frac{c_t}{\overline{c}_t}\right)$, equation 2.25 yields a log-linear form[5]

$$\widehat{m}_t = \widehat{c}_t - \frac{1}{\sigma} \widehat{R}_t. \tag{2.26}$$

[5] We approximate the deviation of nominal interest rate from steady state value by $\widehat{R}_t \approx \ln(1+R_t) - \ln\left(1+\overline{R}_t\right)$.

We substitute the economy resource constraint $\widehat{c}_t = \frac{\overline{y}_t}{\overline{c}_t}\widehat{y}_t - \frac{\overline{g}_t}{\overline{c}_t}\widehat{g}_t$ into equation 2.26

$$\widehat{m}_t = \frac{\overline{y}_t}{\overline{c}_t}\widehat{y}_t - \frac{\overline{g}_t}{\overline{c}_t}\widehat{g}_t - \frac{1}{\sigma}\widehat{R}_t, \tag{2.27}$$

which is the log-linear money demand equation. Since money is assumed to pay no interest, the opportunity cost of holding money is the nominal interest rate R_t. If the real interest rate is constant, the opportunity cost is affected by the rate of inflation. If the price level is constant, inflation is equal to zero, the forgone earnings of holding money are determined by the real interest rate. If the price level is rising with a positive inflation rate, the real value of money is declining and the opportunity cost of holding money increases. In equilibrium it is required that money demand equals money supply.

To write the money demand equation 2.27 again in log-levels, we use the notation $\widehat{m} = \ln m_t - \ln \overline{m}_t$ for real variables and $\widehat{R}_t = \ln(1+R_t) - \ln\left(1+\overline{R}_t\right)$ for nominal interest rate and get

$$\begin{aligned}\ln m_t - \ln \overline{m}_t = {} & \frac{\overline{y}_t}{\overline{c}_t}\left[\ln y_t - \ln \overline{y}_t\right] - \frac{\overline{g}_t}{\overline{c}_t}\left[\ln g_t - \ln \overline{g}_t\right] \\ & - \frac{1}{\sigma}\left[\ln(1+R_t) - \ln\left(1+\overline{R}_t\right)\right].\end{aligned} \tag{2.28}$$

We use the steady state of the resource constraint, the steady state of equation 2.25 and approximation of nominal interest rate $R_t \approx \ln(1+R_t)$ to write equation 2.28 in form

$$\ln m_t = \frac{\overline{y}_t}{\overline{c}_t}\ln y_t - \frac{\overline{g}_t}{\overline{c}_t}\ln g_t - \frac{1}{\sigma}R_t + \frac{1}{\sigma}\ln\Gamma, \tag{2.29}$$

which is the money demand equation written in (logarithmic) levels.

Now we rewrite the household's labour supply equation 2.11 by using the same periodical utility function as above $u^1(c_t, m_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{\Gamma m_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\lambda}}{1+\lambda}$. After taking partial derivatives we can write

$$-l_t^{\lambda} = -\left[c_t^{-\sigma} w_t (1-\tau_t)\right]. \tag{2.30}$$

Labour supply can be expressed as

$$l_t^S = c_t^{-\frac{\sigma}{\lambda}} w_t^{\frac{1}{\lambda}} (1-\tau_t)^{\frac{1}{\lambda}}, \tag{2.31}$$

where labour supply depends on consumption and net wage. The wage elasticity of labour supply is $\frac{1}{\lambda}$.

2.2 The firm

Let's look at the behaviour of a cost minimising firm. The firm hires labour, produces and sells products in a monopolistically competitive goods market. The firm produces a single good with labour l_t and pays wages w_t per unit of labour. In each period the firm minimises a cost function.

$$\min w_t l_t \tag{2.32}$$

subject to the production technology

$$y_t = A l_t, \tag{2.33}$$

where A stands for technological development and is defined as $A = \mu e^{\zeta * Time}$. We assume that production technology is a Cobb–Douglas production function without capital and constant returns to scale. We also assume that there is no public production. This means that we assume that private and public sector productivity are equal. The firm does not make any profit. The production function also defines labour demand $l_t^D = \frac{y_t}{A}$. The cost minimisation implies the following real marginal cost

$$\begin{aligned} &\frac{\partial}{\partial y_t}\left[w_t\left(\frac{y_t}{A}\right)\right] \\ &= w_t \frac{1}{A} \equiv mc_t. \end{aligned} \tag{2.34}$$

The equilibrium wage is given by the labour supply equation 2.31 and labour demand equation 2.33. We substitute the equilibrium wage $w_t = c_t^\sigma \left(\frac{y_t}{A}\right)^\lambda (1 - \tau_t)^{-1}$ into the marginal cost equation 2.34:

$$c_t^\sigma \left(\frac{y_t}{A}\right)^\lambda \frac{1}{A}(1-\tau_t)^{-1} = mc_t. \tag{2.35}$$

Taking natural logarithms of equation 2.35 yields

$$\lambda \ln y_t - (1+\lambda)\ln A + \sigma \ln c_t - \ln(1-\tau_t) = \ln mc_t. \tag{2.36}$$

The equation 2.36 holds also in the steady state. In order to log-linearise we subtract the steady state values and write

$$\begin{aligned} &\lambda\left[\ln y_t - \ln \overline{y}_t\right] + \sigma\left[\ln c_t - \ln \overline{c}_t\right] \\ &-\ln(1-\tau_t) + \ln(1-\overline{\tau}_t) = \ln mc_t - \ln \overline{mc}_t. \end{aligned} \tag{2.37}$$

Rewriting the equation 2.37 by using notation $\widehat{y}_t = \ln \frac{y_t}{\overline{y}_t}$ yields

$$\lambda \widehat{y}_t + \sigma \widehat{c}_t - (1 - \widehat{\tau}_t) = \widehat{mc}_t. \tag{2.38}$$

We substitute the resource constraint $\widehat{c}_t = \frac{\overline{y}_t}{\overline{c}_t}\widehat{y}_t - \frac{\overline{g}_t}{\overline{c}_t}\widehat{g}_t$ into equation 2.38 and we get the marginal cost equation written as deviations from the steady state values

$$\left(\sigma \frac{\overline{y}_t}{\overline{c}_t} + \lambda\right)\widehat{y}_t - \sigma \frac{\overline{g}_t}{\overline{c}_t}\widehat{g}_t - (1 - \widehat{\tau}_t) = \widehat{mc}_t. \tag{2.39}$$

A monopolistically competitive firm is free to set any price it wants and to produce any quantity it is able to produce. An inverse supply function measures the price that must prevail for a firm to supply a given amount of output. The supply function gives the profit maximising output at each price. As the outcome of profit maximisation, the real marginal cost is set equal to mark-up in equilibrium[6]. Now we can use the equation 2.39 and the steady state condition to write the supply function as

$$\ln y_t^* = \frac{\sigma \frac{\bar{g}_t}{\bar{c}_t}}{\sigma \frac{\bar{y}_t}{\bar{c}_t} + \lambda} \ln g_t + \frac{1+\lambda}{\sigma \frac{\bar{y}_t}{\bar{c}_t} + \lambda} \ln A + \frac{1}{\sigma \frac{\bar{y}_t}{\bar{c}_t} + \lambda} \ln \left(1 - \tau_t\right), \tag{2.40}$$

where y_t^* is the flexible price level of output which we call potential output. Now the level of potential output in economy is affected by the fiscal variables, government consumption and taxation, together with technology. An increase in government consumption will expand production possibilities. A decrease in taxation will also increase potential output, since the household is more willing to supply more labour. The potential output equation 2.40 holds also in the steady state.

In order to derive the pricing equation of the firm, we use Rotemberg (1982a, 1982b and 1987) pricing and a cost minimising firm's behaviour. In the model we assume that there exist costs to the firm when it changes prices. This assumption will introduce price stickiness and reflect the empirical aspect that individual price setting is lumpy. It is possible to take quadratic approximation of firm's profits around P^*, which is a path of prices a firm would change if there would no be costs of changing prices. Then a forward-looking firm sets prices by minimising a quadratic loss function

$$\frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j \left[\left(\ln P_{t+j} - \ln P_{t+j-1}\right)^2 + a \left(\ln P_{t+j} - \ln P_{t+j}^*\right)^2 \right], \tag{2.41}$$

where $\beta = \frac{1}{(1+r)}, r > 0$ is the discount factor and a is the adjustment cost parameter. The higher the a is, the more costly it is to change prices. The first order condition is

$$\ln P_t = \frac{1}{1+a+\beta} \ln P_{t-1} + \frac{\beta}{1+a+\beta} E_t \ln P_{t+1} + \frac{a}{1+a+\beta} \ln P_t^*. \tag{2.42}$$

The current price level is the weighted average of the past price level, the expected future price level and of P^*. By denoting inflation $\pi_t = \ln p_t - \ln p_{t-1}$, we can rewrite equation 2.42 to yield

$$\pi_t = \beta E_t \pi_{t+1} - a \left(\ln P_t - \ln P_t^*\right). \tag{2.43}$$

Note that the long-run prices $\ln P_t^*$ are determined by the marginal cost, which is in real terms $\ln P_t^* - \ln P_t = mc_t$. Log-linearising this and equation 2.43 we get

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + a \left(\widehat{mc}_t\right).$$

Then using the marginal cost equation 2.39 we can write the Phillips curve in terms of deviations from the steady state

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + a \left[\left(\sigma \frac{\bar{y}_t}{\bar{c}_t} + \lambda \right) \widehat{y}_t - \sigma \frac{\bar{g}_t}{\bar{c}_t} \widehat{g}_t - \left(1 - \widehat{\tau}_t\right) \right]. \tag{2.44}$$

[6] In equilibrium the nominal marginal cost is equal to the nominal price and mark-up. In real terms this is $mc = \frac{1}{\kappa}$, where κ is the mark-up. Taking logs, we can write $\ln mc = -\ln \kappa$. However, the mark-up does not deviate around the steady state in equilibrium. Hence, we can set $\widehat{mc} = 0$.

Rewriting equation 2.44 by using notation $\widehat{y}_t = \ln \frac{y_t}{\overline{y}_t}$ and using the steady-state condition of equation 2.40, we can write the expectations, technology and tax augmented Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + a \left[\left(\sigma \frac{\overline{y}_t}{\overline{c}_t} + \lambda \right) \ln y_t - \sigma \frac{\overline{g}_t}{\overline{c}_t} \ln g_t - (1+\lambda) \ln A - \ln (1-\tau_t) \right], \tag{2.45}$$

which can be written with the help of potential output y_t^* defined by equation 2.40 at time t as

$$\pi_t = \beta E_t \pi_{t+1} + a \left[\left(\sigma \frac{\overline{y}_t}{\overline{c}_t} + \lambda \right) (\ln y_t - \ln y_t^*) \right], \tag{2.46}$$

which is the New Keynesian supply curve. Now, current inflation depends on expected future values of inflation, not on past inflation. The model has a resemblance with Woodford (1999), where he points out that there is an important dynamic link from expectations of the future to the present for both inflation and output. Leong (2002) finds support to the forward looking New Keynesian model from simulation exercises. Unlike Woodford (1999), we treat potential output y_t^* endogenously instead of assuming that it is an exogenous disturbance. As fiscal policy affects potential output, inflation will also react to fiscal policy. Later we compare the results with endogenous potential output with those obtained when potential output is exogenous.

2.3 The government

We construct the intertemporal budget constraint for policy authority, linking debt and policy choices. We write the nominal government flow budget constraint as follows

$$B_t + \tau_t Y_t + M_t - M_{t-1} = (1 + R_{t-1}) B_{t-1} + G_t, \tag{2.47}$$

where B_t denotes government bonds, $\tau_t Y_t$ tax revenue, M_t nominal money balances and G_t nominal government spending. Government policy is characterized by sequences of tax rates on labour, a sequence of total liabilities and a sequence of government consumption. Dividing equation 2.47 by P_t gives

$$b_t + \tau_t y_t + \frac{M_t - M_{t-1}}{P_t} = (1 + R_{t-1}) b_{t-1} \frac{P_{t-1}}{P_t} + g_t. \tag{2.48}$$

Equation 2.48 simplifies when we use the same approximation as before for the real interest rate $(1 + r_t) = (1 + R_t)(1 - E_t \pi_{t+1})$. The real flow budget constraint is

$$b_t + \tau_t y_t + \pi_t m_{t-1} + m_t - m_{t-1} = (1 + r_{t-1}) b_{t-1} + g_t. \tag{2.49}$$

The policy authority balances the budget with new debt (b_t), with tax (τ_t) and seigniorage revenue ($\pi_t m_{t-1} + m_t - m_{t-1}$). The intertemporal government budget constraint is

$$(1+r)\, b_t \leq \sum \left(\frac{1}{1+r}\right)^i (\pi_{t+i} m_{t-1+i} + m_{t+i} - m_{t-1+i} \\ + \tau_{t+i} y_{t+i} - g_{t+i}) \,, \tag{2.50}$$

which states that the maximum level of outstanding debt including interest payments is determined by the discounted sum of seigniorage revenues and surpluses. As mentioned in Pohjola (2002), if the intertemporal budget constraint is not binding, the policy authority can generate tax and seigniorage revenues in excess of its current commitments. If the intertemporal budget constraint is binding, higher debt levels are feasible only through a credible commitment to larger surpluses and seigniorage in the future. Fiscal policy can rely on seigniorage funding to some extent. Schmitt-Grohé and Uribe (2002) show that even a small amount of price stickiness is sufficient to sustain low inflation tax, and therefore the government will rely more heavily on conventional income tax.

2.4 Policy rules

The recent literature on monetary policy has focused on interest rate setting. We assume that the interest rate is set according to the Taylor (1993) rule. Interest rates are set based on the domestic economic conditions, placing a positive weight on inflation and real output. Taylor suggested that an increase in the nominal interest rate should be more than one-for-one in response to inflation. We write the interest rate rule with respect to inflation deviations from the inflation target and output deviations from potential output familiar in literature, eg Clarida, Galí and Gertler (1999) and McCallum (1999). The interest rate rule is then

$$R_t = \pi_t + r^* + \eta_1 (\pi_t - \pi^*) + \eta_2 (\ln y_t - \ln y_t^*) \,, \tag{2.51}$$

where r^* is the real interest rate in the steady state, π^* is the inflation target and y_t^* is potential output at time t defined by equation 2.40. There is interaction between monetary and fiscal policy, since y_t^* is affected by fiscal policy variables. The Taylor principle is $\eta_1 > 0$ and $\eta_2 > 0$. The larger the values η_1 gets, the tighter is monetary policy. In the literature the discussion about the form of the interest rate rule has emphasised the simple, robust rules and stabilisation properties. We use the contemporaneous time interest rate rule, which according to Bullard and Mitra (2002) is stable with a larger range of parameters than a forward looking rule.

Moreover, recent developments in monetary policy literature have emphasized the link between the degrees to which monetary and fiscal policies respond to inflation rate, debt and macroeconomic stability. We focus on fiscal policy, which is conducted following a Leeper (1991) type debt rule for tax rate, where the policy parameter is directly incorporated to the real government debt outstanding. Total government liabilities are $b_{t-1} + m_{t-1}$. We write the fiscal rule with respect to a distortionary tax rate instead of the lump sum taxes used in Leeper (1991)

$$\tau_t = \tau^* + \phi \left(b_{t-1} + m_{t-1}\right)/y_t - \psi, \tag{2.52}$$

where τ^* is the constant tax rate and ψ is a constant parameter, which can be interpreted to be the debt to GDP target. The rules consists of systematic policy responses to economic conditions. Monetary authority responses to inflation are given by the magnitude $(1 + \eta_1)$ while fiscal authority responses to debt are given by the magnitude ϕ.

3 Stabilising properties of the model

3.1 Parameters

The calibration sets the parameter values used in the stability analysis and in the standard simulation of the theoretical model to the level common in the business cycle literature. Rotemberg and Woodford (1998) estimate the risk aversion coefficient $\sigma = 0.157$ and the output coefficient in Phillips curve to .024. Clarida, Galí and Gertler (2000) set $\sigma = 1$ and the output coefficient to 0.3. As shown in Bullard and Mitra (2002) the model remains stable in the case of contemporaneous time inflation in the Taylor rule, when they increase the value of σ from 0.157 to 1 and the output coefficient from 0.024 to 0.3. We set the risk aversion coefficient σ to 0.5 and the interest rate coefficient of the IS curve $\frac{\bar{c}_t}{\bar{y}_t}\frac{1}{\sigma} = 1.5$. We set λ to be 1.5, hence the labour supply elasticity with respect to real wages is then 0.67. The output coefficient in the Phillips curve $a\left(\sigma\frac{\bar{y}_t}{\bar{c}_t} + \lambda\right) = 0.043$, when the adjustment cost parameter a is 0.02 and the ratio of government consumption to output is set to 0.25, so $\frac{\bar{c}_t}{\bar{y}_t} = 0.75$ and $\frac{\bar{y}_t}{\bar{c}_t} = 1.34$. The output coefficient in the Taylor rule η_2 is set to be equal to 0.4, which is smaller than in the original Taylor (1993) rule, representing that monetary authority is less interested in output than suggested by Taylor. The inflation target π^* is 0.02 and the long term real interest rate r^* is 0.03.

The household discount factor δ is 0.98, but the firm's discount rate β is set to be equal to one. The income elasticity of money demand is 1.34 and the interest rate elasticity of money is 2. The ratio of money balances to GDP is set to be equal to 0.12 by setting the coefficient $\Gamma = 0.3$. The constant tax rate τ^* is set equal to 0.25 and the real debt to GDP target $\psi = 0.6$, which is the maximum debt to GDP ratio permitted under the Maastricht Treaty. These parameter values reflect the economic structure of a large economy, such as the euro area.

3.2 Debt rule with supply-side effects

We analyse the stability of the model by using methods by Blanchard and Kahn (1980). When the model is written in the state-space representation, the Blanchard and Kahn requirement is that the number of roots inside a unit circle should be equal to number of non-predetermined variables for a unique solution under rational expectations. Using the terminology in Evans and Honkapohja (2002b), we say that whether under rational expectations the system possesses a unique stationary rational expectations equilibrium (REE), the system is said to be determinate. If the system is indeterminate then multiple stationary solutions, including sunspot solutions, exist.

We consider the system given by output equation 2.21, real money balances equation 2.27, potential output equation 2.40, inflation equation 2.46, government budget constraint equation 2.49, interest rate rule equation 2.51 and debt rule equation 2.52. Government consumption $\widehat{g}_t = \widehat{g}_{t-1}$ is an exogenous process. Defining

$$\widehat{X}_t' = \left[\begin{array}{cc} \widehat{y}_t & \widehat{\pi}_t \end{array}\right], \tag{3.1}$$

$$\widehat{x}_t' = \left[\begin{array}{ccc} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{array}\right],$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables, the reduced form can be written

$$A\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array}\right] = B\left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array}\right], \tag{3.2}$$

which is equal to

$$\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array}\right] = M\left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array}\right], \tag{3.3}$$

where $M = A^{-1}B$. Matrix M is defined by suitable matrices A and B shown in Appendix A.

Matrix M is a 5×5 matrix and has 5 roots. A contemporaneous time version of the Taylor rule in a New Keynesian model, where the reduced form resembles equation 3.3. In other words, the matrix M is attached to the expected future component, has two roots and both roots should be inside the unit circle for determinacy like shown in Bullard and Mitra (2002). Adding the government budget constraint and the fiscal rule does not increase the number of non-predeterminate variables compared with a New Keynesian model. The system given with equation 3.3 has 2 non-predetermined variables, output and inflation $\{\widehat{y}, \widehat{\pi}\}$ and the number of roots of matrix M inside the unit circle is required to be two for determinacy. Figure 1 shows the number of roots of matrix M inside the unit circle and determinate (D), indeterminate (I) and explosive (E) regions in case of debt rule when the Taylor rule parameter η_1 for inflation runs from -1 to 1 and the debt rule parameter ϕ runs from -1 to 2.

As we can see from figure 1, the determinate, stable regions are in the middle of the right hand side and in the upper and the lower left hand side

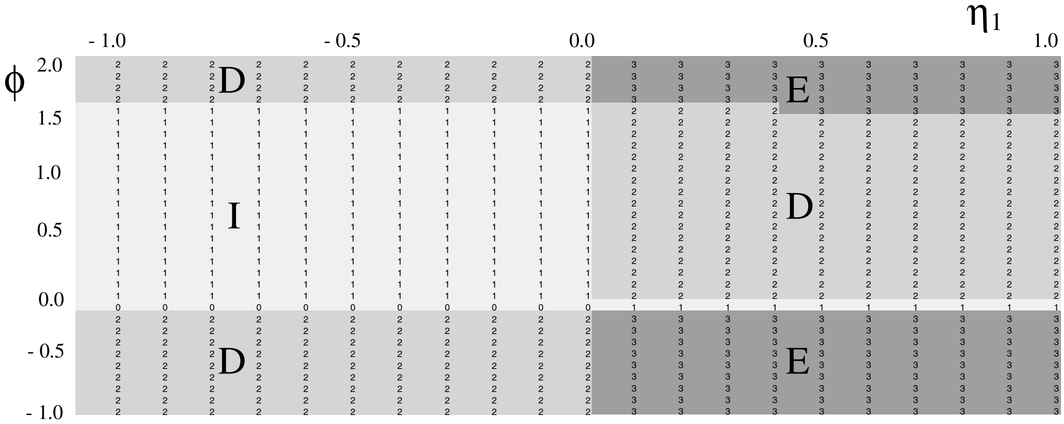

Figure 1: Determinate, indeterminate and explosive regions.

corners. On the right hand side there exists a unique solution with the Taylor rule parameter η_1 larger than zero. According to the Taylor (1993) requirement, the interest rate should react more than one-for-one to inflation, when the fiscal rule reacts to debt positively, but by less than approximately 1.5. Adapting Leeper (1991), an accommodating (active) policy authority is defined as one that is not constraint by budgetary conditions, whereas an aggressive (passive) authority must generate sufficient tax revenues to balance the budget. An aggressive decision rule depends on the current state of government debt, summarised by current and past variables, while an accommodating rule can be formed more freely using past, current or expected future variables. Fiscal policy becomes more aggressive or tighter when the fiscal policy parameter ϕ increases.

Monetary policy can turn from loose[7] to extremely tight while fiscal policy can at the same time change from accommodating to aggressive and stability is still achieved. This conflicts with Leeper (1991) who concludes that both monetary and fiscal policy cannot be active in order to achieve stability. Leith and Wren–Lewis (2000) state that the distinction between monetary policy and fiscal policy dominated regimes depends heavily on assumptions [8]. In addition, Leith and Wren–Lewis (2002) conclude that it is difficut to make the distinction in advance. We observed that the introduction of proportional taxes and endogenous labour supply makes both monetary policy and fiscal policy parameters to appear in the same root. This made the interaction between monetary and fiscal policy much stronger, but less clear than in literature so far, eg Leeper (1991).

There exist limits for fiscal policy in order to obtain stability and active monetary policy at the same time. This means that the Taylor principle holds and thereby monetary policy stabilises the economy, as is widely recognized. Edge and Rudd (2002) conclude that the introduction of distortionary taxation

[7]By loose monetary policy we mean still active monetary policy, that is $\eta_1 > 0$

[8]Leith and Wren-Lewis (2000) allow their model to have deviations from Ricardian equivalence, nominal inertia in price setting, the possibility that government debt is denominated in real terms and feedback from debt disequilibrium to government spending.

increases the lowest value of the inflation parameter in the Taylor rule to have a determinate equilibrium. Positive fiscal policy reactions to changes in debt are reasonable in a sense that the policy authority uses at least some income taxation trying to fulfil the budget constraint and does not fully rely on debt and seigniorage. The area on the right hand side can be considered the most plausible combination of monetary and fiscal policy parameters consistent with the dynamical stability of the economy. In this area both monetary and fiscal policies determine prices.

Other stable areas are found in the regions where the Taylor principle is no longer valid. In the upper left hand corner the fiscal parameter has got large values, which implies extremely tight fiscal policy. In the lower left hand corner, fiscal policy will react negatively to an increase in debt. In both cases monetary policy is accommodating enough for the fiscal policy to ensure stability.

The area with a negative Taylor rule parameter and a fiscal parameter between zero and 1.5 is indeterminate and there is no unique solution. Upper and lower right hand corners display parameter values for regions with explosive solution.

Due to distortionary tax rate the model exhibits non-neutrality and changes in labour supply affect output and inflation. In addition, the labour supply elasticity has an impact on stability properties. Figure 2 shows the number of roots of matrix M inside the unit circle and determinate (D), indeterminate (I) and explosive (E) regions in case of the debt rule when the inverse labour supply parameter λ runs from 0 to 2 and the deficit rule parameter ϕ runs from -1 to 2. The Taylor rule parameter η_1 is kept constant at 0.5. Findings in Dotsey (1994) and Ludvigson (1996) emphasise the importance of elasticity of labour supply as a reaction to expansionary fiscal policy. With inelastic labour supply Dotsey (1994) finds out that a cut in the tax rate even reduces output if the deficit is paid with proportional taxes. Ludvigson (1996) demonstrates that output may be increased if labour supply is elastic even tough the deficit will be financed by increasing the proportional taxes in the future. We see that when labour supply elasticity increases, value of λ decreases, the more easily the economy becomes unstable with respect to the choice of the fiscal rule parameter. It is easier for the fiscal authority to finance spending by using taxes when labour supply is less elastic than with infinitely elastic labour supply.

3.3 Debt rule with exogenous potential output

Stabilising properties of simple fiscal policy rules together with monetary policy rules has been studied in eg Leeper (1991), Andrés, Ballabriga and Vallés (2002) and Evans and Honkapohja (2002a). Traditionally the focus of fiscal policy has been on demand side effects and, hence, the potential output has ben treated as an exogenous shock variable (see eg Woodford, 1999). In order to compare the results with supply-side effects to previous studies, we also study the determinacy of a system with exogenous potential output. We form

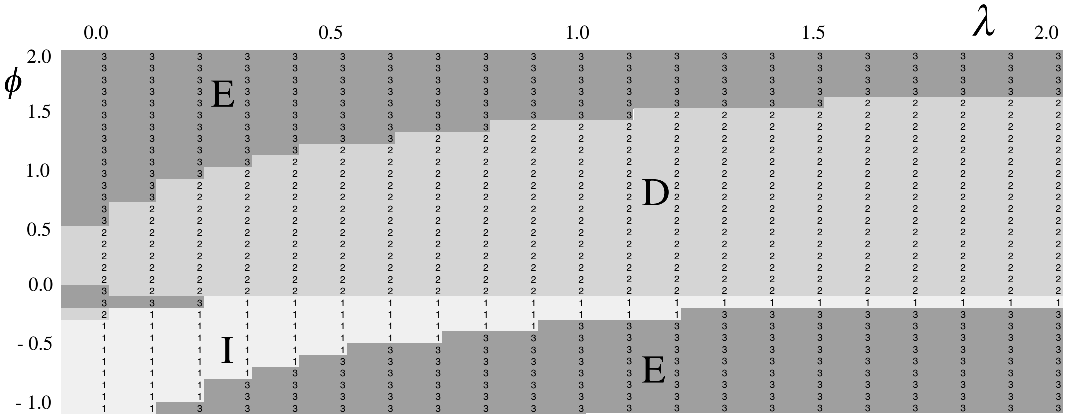

Figure 2: Determinate, indeterminate and explosive regions.

the system similarly as above, but now without the potential output equation 2.40. The system is now given by output equation 2.21, real money balances equation 2.27, inflation equation 2.46, government budget constraint equation 2.49, interest rate rule equations 2.51, debt rule equation 2.52 and exogenous government consumption, $\widehat{g}_t = \widehat{g}_{t-1}$. Defining

$$\widehat{X}_t' = \left[\begin{array}{cc} \widehat{y}_t & \widehat{\pi}_t \end{array} \right], \tag{3.4}$$

$$\widehat{x}_t' = \left[\begin{array}{ccc} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{array} \right],$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables. The reduced form can be written

$$\mathcal{A} \left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = \mathcal{B} \left[\begin{array}{c} E_t \widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right], \tag{3.5}$$

which is equal to

$$\mathcal{M} \left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = \left[\begin{array}{c} E_t \widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right], \tag{3.6}$$

where $\mathcal{M} = \mathcal{A}\mathcal{B}^{-1}$. Matrix $\mathcal{M}$ is defined by suitable matrices $\mathcal{A}$ and $\mathcal{B}$ shown in appendix B. Matrix $\mathcal{M}$ is now associated with the contemporaneous time variables and hence we require the number of roots outside the unit circle to be two for determinacy, since the system has 2 non-predetermined variables, output and inflation $\{\widehat{y}, \widehat{\pi}\}$ Figure 3 shows the number of roots of matrix $\mathcal{M}$ outside the unit circle and the determinate (D), indeterminate (I) and explosive (E) regions in case of debt rule when potential output is exogenous and fiscal policy has no supply effects. The Taylor rule parameter η_1 for inflation runs from -1 to 1 and the deficit rule parameter ϕ runs from -1 to 2.5. Stabilising properties of monetary and fiscal policy change compared to the case with endogenous potential output. The stable region in the middle of the right hand side in figure 3 is now larger than in the case when fiscal policy has output effects shown in figure 1. The fiscal policy can be more aggressive in

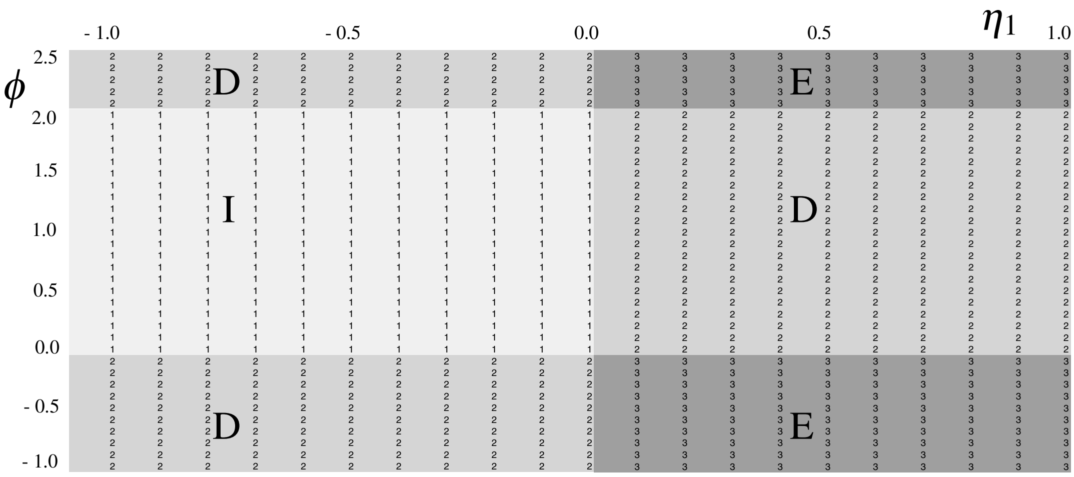

Figure 3: Determinate, indeterminate and explosive regions.

trying to fulfil the budget constraint with tax revenue instead of debt and is still able to stabilise the economy.

Similarly the indeterminate region is also larger than with endogenous potential output. When the supply-side channel is not present, fiscal policy can be more passive and fulfil the government budget constraint, whereby active monetary policy can stabilise the economy. On the other hand, with accommodating monetary policy, negative fiscal policy parameters stabilise the economy identically with the case with supply-side effects. The stable and non-stable regions shown in figure 3 are similar with Evans and Honkapohja (2002a), when the discount factor is set equal to 1.

3.4 Simulation

In this section we discuss further the supply-side effects of fiscal policy. We analyse the case when fiscal policy has both demand and supply effects and compare results with the standard case with only demand effects. Simulation results of models with only demand effects of fiscal policy can be found in eg Wouters and Dombrecht (2000) and Andrés, Ballabriga and Vallés (2002). They use a Leeper-type fiscal policy rule to analyse the interaction between monetary and fiscal policy. Our results have resemblance with Kortelainen (2002) results, when he uses a general equilibrium model for the euro area to analyse the credibility of monetary policy. In his model the supply-side channel is present. In figures 4–9 we show the responses to permanent and temporary demand, supply and monetary policy shocks. The dotted line represents the case with endogenous potential output, when fiscal policy is allowed to have supply-side effects. The solid line represents the exogenous potential output and fiscal policy has no supply-side effects. The Taylor rule parameter value η_1 is 0.5 and the fiscal policy parameter ϕ is 0.1. In figures 10–11 we show the responses with supply-side effects to the temporary demand shock using different policy parameters.

Figure 4 shows the responses in the case of expansionary fiscal policy involving a permanent increase of 1 percent of GDP in public consumption. When we allow fiscal policy to affect potential output through labour supply, we observe that an increase of 1 percent of GDP in public consumption raises potential output by 0.4 percent. Due to the resource constraint, private consumption is displaced by public consumption. In the long run prices and labour supply adjust the economy towards a new equilibrium. The shock shifts resources from the private sector to the public one and there are crowding-out effects of private consumption in the long run. The effect can be empirically too large: our assumption that productivity is equal in the private and public sectors may show too large expansion as a response to an increase in public spending. This lowers the demand for money permanently. As a response to the jump in output, the tax rate increases after the initial drop and shifts to another level. The reaction in the tax rate is quite similar in both cases. With the supply-side channel present, the increased taxes reduce net wages and labour supply of the household and output decreases in the long run. Potential output reacts more strongly than output and inflation and the nominal interest rate decreases, exhibiting non-neutrality of the model due to distortionary taxation and endogenous labour supply. Adding supply-side effects of fiscal policy adds the disinflatory effect in the case of a permanent fiscal shock in the short run and shifts the price level down permanently.

The impacts of a technology shock are shown in figure 5. We increase the technology level term, μ, in the potential output equation by 1 percent and allow then potential output to respond to the changes in other variables. In the exogenous potential output case, the level of potential output is increased by 1 percent. Real output reacts by the same amount as potential output does. As there are positive output effects, the tax rate and the debt to GDP ratio will drop more than in the case with exogenous potential output, since the increase in labour supply allows an additional tax cut increasing output more. As a result, inflation slows down and the nominal interest rate decreases slightly. All this results in an initial reaction of potential output which is twice as large as the initial shock was. In the long run production converges to a new higher equilibrium level. The model exhibits non-neutrality in the case of endogenous potential output in the short run. With exogenous potential output, inflation and the interest rate are left unchanged. The demand for money will pick up following higher household income as labour supply increases and taxes become lower resulting in more private consumption.

Figure 6 shows the effect of an increase in the inflation target, which may be interpreted as a credible relaxation of monetary policy. A permanent shift of 1 percentage point in the inflation target raises actual inflation immediately by 1 percentage point. Output reacts positively as the expectations of monetary policy change. A declining government debt to GDP ratio and hence a declining tax rate allow labour supply to grow and hence potential output and real output to increase in the long run to a higher equilibrium. As a result of a permanent shock, the debt to GDP ratio and the tax rate converge to a new lower equilibrium. The nominal interest rate reacts more strongly in the short run than inflation as a reaction to the positive output gap. In the long run, inflation and the nominal interest rate are linked together by the Fisher equation. Demand for money decreases as the interest rate increases and the opportunity cost of holding money rises. With endogenous potential output, inflation increases initially slightly more than in the case of exogenous potential output, where there is an equal increase in inflation and the interest rate while output remains unchanged.

Figures 7, 8 and 9 show the previous shocks, respectively, when they are temporary, only one period long. The shocks show clearly the impact of the supply-side channel and the non-neurtality effect caused by endogenous labour supply. The initial responses are almost as large in both cases, but the convergency is slower in the case with supply-side channel.

A temporary public consumption shock will increase both output, inflation and the interest rate immediately. The debt to GDP ratio and the tax rate will decrease initially, but jump up and converge to the equilibrium from the positive values. As the tax rate increases, potential output will drop after the initial jump and create a more persistent inflation path. The increase in government spending has inflationary effects. The channel from taxes to supply and inflation causes more permanent effects also on demand. The real output losses are larger than without the supply-side channel of fiscal policy as labour supply reacts to a rise in taxation and output potential decreases.

A 1 percent positive shock to technology will reduce inflation and the interest rate as output increases. The debt to GDP ratio and taxes are reduced. In the long run all variables will converge back to the equilibrium. Now the supply-side channel will again indicate more persistent reactions. In the case of endogenous potential output, the tax rate and the debt to GDP rate decrease more and real output remains above the baseline for a longer time than if potential output were fixed. The gain from lower inflation and price level is also larger.

A raise in the inflation target increases actual inflation only by few tenth of a percentage point initially. The nominal interest rate however drops by some 0.3 percentage point allowing output to increase as the real interest rate decreases. As a result, both output and inflation will increase, since the output gap widens. The debt to GDP ratio and the tax rate decrease in both cases. When the supply-side channel is present, output potential becomes larger. The output gap closes in the long-run and inflation converges to the target level from below. The shock actually decreases actual inflation, as the decrease in taxation increases labour supply and hence wage inflation.

In figure 10 we show how different monetary policy rules respond to the temporary increase of 1 percent of real GDP in public consumption. The solid line represent the Taylor rule with the inflation parameter $\eta_1 = 0.1$, which is referred to as a loose monetary policy rule. The dotted line is a tighter monetary policy rule with $\eta_1 = 0.5$, which is the traditional Taylor rule case. As we see, the tighter rule allows inflation and the interest rate to increase less than the loose rule. The change in the monetary policy rule does not have any effect, or has a very minor effect, on output and on the fiscal variables, debt and the tax rate.

Responses to the change in the fiscal policy rule are shown in figure 11. The solid line represents a tighter fiscal policy rule, with parameter $\phi = 1$, which allows large changes in the tax rate, but does not allow large changes in the debt to GDP ratio. The dotted line is a loose fiscal policy rule, with parameter $\phi = 0.1$. With the tighter fiscal policy rule, inflation and the interest rate react more and return faster to the baseline than with the loose fiscal rule. A tighter fiscal policy has to keep the government budget constraint in balance. The debt to GDP ratio jumps up only for short while. Tight fiscal policy uses more inflation taxes in the short run, but returns faster to the baseline as the debt to GDP ratio is stabilised. The nominal interest rate reacts quite strongly when fiscal policy is tight and also losses in output are larger in the short run, but remain smaller all in all.

4 Conclusions

This paper studied the effect of the supply-side channel on stabilisation policies. Using a closed economy New Keynesian model with endogenous labour supply and no capital, we studied the stability conditions dependent on the interaction between monetary policy based on the interest rate rule and fiscal policy conducted by a tax rule based on government liabilities.

The use of a proportional tax rate together with endogenous labour supply changes the stability conditions for fiscal policy. The use of a distortionary tax rate made the clear distinction between fiscal policy and monetary dominated regimes used in the literature unclear. The stability conditions depend on both the fiscal policy parameter and the monetary policy, ie Taylor rule, parameter simultaneously. Neither fiscal nor monetary policy can alone determine prices.

We showed that it is possible for accommodating (active) fiscal policy together with active monetary policy to stabilise the economy contrary to the common result that if both monetary and fiscal policy authorities act actively, stability is not reached. The introduction of a supply-side channel by endogenous labour supply decisions with a distortionary tax rate restricts the parameter range of fiscal policy consistent with the dynamic stability of the economy compared with the traditional case with only demand effects.

The non-neutrality effects shown in the simulation results are due to endogenous labour supply, distortionary taxes and the government budget constraint. Contrary to the conventional results in New Keynesian models, permanent shocks have both short and long run effects on output and inflation. For instance, a permanent increase in public consumption decreases production in the long run as household labour supply decreases. In the short run public spending has positive effects on output and negative impacts on inflation.

As fiscal policy affects real economy and prices in the model, it is suitable for studies on the combinations of the fiscal variables on which policy reactions should be based. As in the monetary policy rule literature, it would be interesting to compare different fiscal policy rules and to look at how the stabilising conditions change.

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A Appendix. The system with supply-side effects of fiscal policy

The system is given by the output equation A.1, real money balances equation A.2, inflation equation A.3, potential output equation A.5, interest rate rule equation A.6, government budget constraint equation A.7 and debt rule for tax rate equation A.8. Government consumption equation A.4 is an exogenous process around its steady state. We use the log-linearisation techniques[9] in Uhlig (1999) to centre the government budget constraint 2.49 and the tax rule 2.52 around constant steady state, and move them one period forward. We also write the Taylor rule 2.51, potential output 2.40 and the Phillips curve equation 2.46 as deviations from steady state. The system can be written as

$$\widehat{y}_t = E_t\widehat{y}_{t+1} + \frac{\overline{g}_t}{\overline{y}_t}\left[\widehat{g}_t - E_t\widehat{g}_{t+1}\right] - \frac{\overline{c}_t}{\overline{y}_t}\frac{1}{\sigma}\left(\widehat{R}_t - \widehat{\pi}_{t+1}\right) \tag{A.1}$$

$$\widehat{m}_t = \frac{\overline{y}_t}{\overline{c}_t}\widehat{y}_t - \frac{\overline{g}_t}{\overline{c}_t}\widehat{g}_t - \frac{1}{\sigma}\widehat{R}_t \tag{A.2}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + a\left[\left(\sigma\frac{\overline{y}_t}{\overline{c}_t} + \lambda\right)\left(\widehat{y}_t - \widehat{y}_t^*\right)\right] \tag{A.3}$$

$$\widehat{g}_t = \widehat{g}_{t+1} \tag{A.4}$$

$$\widehat{y}_t^* = \frac{\sigma\frac{\overline{g}_t}{\overline{c}_t}}{\sigma\frac{\overline{y}_t}{\overline{c}_t} + \lambda}\widehat{g}_t - \frac{1}{\sigma\frac{\overline{y}_t}{\overline{c}_t} + \lambda}\widehat{\tau}_t \tag{A.5}$$

$$\widehat{R}_t = (1+\eta_1)\,\widehat{\pi}_t + \eta_2\left(\widehat{y}_t - \widehat{y}_t^*\right) \tag{A.6}$$

$$\begin{aligned}
& \left[\left(1 + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\left(\frac{1}{\overline{R} - \overline{\pi}} + 1\right)\right]\widehat{b}_t + \frac{\overline{m}}{\overline{y}}\left(\frac{1}{\overline{\tau}} - \frac{\overline{\pi}}{\overline{\tau}}\right)\widehat{m}_t \\
+ \quad & \left[\left(1 + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\frac{\overline{R}}{\overline{R} - \overline{\pi}}\right]\widehat{R}_t = \\
& \widehat{y}_{t+1} + \left[\left(1 + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\frac{\overline{\pi}}{\overline{R} - \overline{\pi}} + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\right]\widehat{\pi}_{t+1} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\widehat{g}_{t+1} \\
& + \frac{1}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\widehat{m}_{t+1} + \left[\left(1 + \frac{\overline{\pi}}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} - \frac{1}{\overline{\tau}}\frac{\overline{g}}{\overline{y}}\right)\frac{1}{\overline{R} - \overline{\pi}}\right]\widehat{b}_{t+1} + \widehat{\tau}_{t+1}
\end{aligned} \tag{A.7}$$

$$\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\widehat{m}_t + \left(1 - \frac{\overline{\tau^*}}{\overline{\tau}} - \frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}} + \frac{\psi}{\overline{\tau}}\right)\widehat{b}_t = \left(1 + \frac{\psi}{\overline{\tau}} - \frac{\overline{\tau^*}}{\overline{\tau}}\right)\widehat{y}_{t+1} + \widehat{\tau}_{t+1} \tag{A.8}$$

We solve the steady state tax rate by setting the steady state government budget constraint and steady state debt rule equal. Then we write the tax rate to be

$$\overline{\tau} = \left(\frac{\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}}{\overline{R} - \overline{\pi}} + \frac{1}{\phi}\overline{\tau^*} + \frac{\overline{m}}{\overline{y}} - \frac{1}{\phi}\psi\right)\frac{1}{\frac{1}{\phi} - \frac{1}{\overline{R} - \overline{\pi}}}. \tag{A.9}$$

[9] In log-linearisation we use notations $c_t = \overline{c}e^{\widehat{c}_t} \approx \overline{c}\,(1 + \widehat{c}_t)$ and $\tau_t y_t = \overline{\tau}\overline{y}e^{\widehat{\tau}_t + \widehat{y}_t} \approx \overline{\tau}\overline{y}\,(1 + \widehat{\tau}_t + \widehat{y}_t)$. By using the steady state conditions, the coefficients can be eliminated.

After some subtitution we can write the 8 equation system with 5 equations. Then we write the system in the state-space form. Defining

$$\widehat{X}_t' = \left[\begin{array}{cc} \widehat{y}_t & \widehat{\pi}_t \end{array} \right], \tag{A.10}$$

$$\widehat{x}_t' = \left[\begin{array}{ccc} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{array} \right],$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables. The reduced form can be written

$$A \left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = B \left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right], \tag{A.11}$$

which is equal to

$$\left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = M \left[\begin{array}{c} E_t\widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right], \tag{A.12}$$

where $M = A^{-1}B$. The matrices A and B can be written

$$A = \left[\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & 0 & a_{15} \\ a_{21} & a_{22} & a_{23} & 0 & a_{25} \\ 0 & 0 & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{array} \right], B = \left[\begin{array}{ccccc} b_{11} & b_{12} & b_{13} & 0 & 0 \\ 0 & b_{22} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & 0 & 0 & 0 & b_{55} \end{array} \right],$$

where

$$a_{11} = \left[1 + \frac{\overline{c}_t}{\overline{y}_t}\frac{1}{\sigma}\eta_2\right], a_{12} = \left[\frac{\overline{c}_t}{\overline{y}_t}\frac{1}{\sigma}\left(1+\eta_1\right)\right], a_{13} = -\left[\frac{\overline{g}_t}{\overline{y}_t} + \frac{\overline{c}_t}{\overline{y}_t}\frac{1}{\sigma}\eta_2\frac{\sigma\frac{\overline{g}_t}{\overline{c}_t}}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda}\right],$$

$$a_{15} = \left[\frac{\overline{c}_t}{\overline{y}_t}\frac{1}{\sigma}\eta_2\frac{1}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda}\right], a_{21} = \left[-a\left(\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda\right)\right], a_{22} = 1,$$

$$a_{23} = \left[a\left(\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda\right)\frac{\sigma\frac{\overline{g}_t}{\overline{c}_t}}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda}\right], a_{25} = -\left[a\left(\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda\right)\frac{1}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda}\right],$$

$$a_{33} = 1, a_{41} = \left[\frac{\overline{m}}{\overline{y}}\left(\frac{1-\overline{\pi}}{\overline{\tau}}\right)\left(\frac{\overline{y}_t}{\overline{c}_t} - \frac{1}{\sigma}\eta_2\right) + \left(1+\frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\eta_2\right],$$

$$a_{42} = -\left[\frac{\overline{m}}{\overline{y}}\left(\frac{1-\overline{\pi}}{\overline{\tau}}\right)\frac{1}{\sigma}\left(1+\eta_1\right)\right] + \left(1+\frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\left(1+\eta_1\right),$$

$$a_{43} = \left[\frac{\overline{m}}{\overline{y}}\left(\frac{1-\overline{\pi}}{\overline{\tau}}\right)\left(\frac{1}{\sigma}\eta_2\frac{\sigma\frac{\overline{g}_t}{\overline{c}_t}}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda} - \frac{\overline{g}_t}{\overline{c}_t}\right) - \left(1+\frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\eta_2\frac{\sigma\frac{\overline{g}_t}{\overline{c}_t}}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda}\right],$$

$$a_{44} = \left[\left(1+\frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\left(\frac{1}{\overline{R}-\overline{\pi}}+1\right)\right],$$

$$a_{45} = \left[\frac{\overline{m}}{\overline{y}}\left(\frac{1}{\overline{\tau}} - \frac{\overline{\pi}}{\overline{\tau}}\right)\frac{1}{\sigma}\eta_2\frac{1}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda} + \left(1+\frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{R}}{\overline{R}-\overline{\pi}}\eta_2\frac{1}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda}\right],$$

$$a_{51} = \left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\left(\frac{\overline{y}_t}{\overline{c}_t} - \frac{1}{\sigma}\eta_2\right)\right], a_{52} = -\left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}\left(1+\eta_1\right)\right],$$

$$a_{53} = \left[\frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\left(\frac{1}{\sigma}\eta_2\frac{\sigma\frac{\overline{g}_t}{\overline{c}_t}}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda} - \frac{\overline{g}_t}{\overline{c}_t}\right)\right], a_{54} = \left[1 - \frac{1}{\overline{\tau}}\left(\overline{\tau^*} + \psi - \phi\frac{\overline{m}}{\overline{y}}\right)\right],$$

$$a_{55} = \left[1 - \frac{\phi}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}\eta_2\frac{1}{\sigma\frac{\overline{y}_t}{\overline{c}_t}+\lambda}\right], b_{11} = 1, b_{12} = \frac{\overline{c}_t}{\overline{y}_t}\frac{1}{\sigma}, b_{13} = -\frac{\overline{g}_t}{\overline{y}_t}, b_{22} = \beta, b_{33} = 1,$$

$$b_{41} = \left[1 + \frac{1}{\overline{\tau}}\frac{\overline{m}}{\overline{y}}\left(\frac{\overline{y}_t}{\overline{c}_t} - \frac{1}{\sigma}\eta_2\right)\right],$$

$$b_{42} = \left[\left(1+\frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{\overline{\pi}}{\overline{R}-\overline{\pi}} + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{m}}{\overline{y}}\frac{1}{\sigma}\left(1+\eta_1\right)\right)\right],$$

$$b_{43} = \left[\frac{1}{\overline{\tau}} \left(\frac{\overline{m}}{\overline{y}} \left(\frac{1}{\sigma} \eta_2 \frac{\sigma \frac{\overline{g}_t}{\overline{c}_t}}{\sigma \frac{\overline{y}_t}{\overline{c}_t} + \lambda} - \frac{\overline{g}_t}{\overline{c}_t} \right) - \frac{\overline{g}}{\overline{y}} \right) \right],$$

$$b_{44} = \left[\left(1 + \frac{1}{\overline{\tau}} \left(\overline{\pi} \frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}} \right) \right) \frac{1}{\overline{R} - \overline{\pi}} \right],$$

$$b_{45} = \left[1 - \frac{1}{\overline{\tau}} \frac{\overline{m}}{\overline{y}} \frac{1}{\sigma} \eta_2 \frac{1}{\sigma \frac{\overline{y}_t}{\overline{c}_t} + \lambda} \right], b_{51} = \left[1 + \frac{\psi - \overline{\tau^*}}{\overline{\tau}} \right], b_{55} = 1.$$

B Appendix. The system without supply-side effects of fiscal policy

The system is the same as above, but potential output is treated as an exogenous variable and therefore potential output equation A.5 is ignored. After substitutions the reduced system has a state-space representation. Defining

$$\widehat{X}_t' = \left[\begin{array}{cc} \widehat{y}_t & \widehat{\pi}_t \end{array} \right], \tag{B.1}$$

$$\widehat{x}_t' = \left[\begin{array}{ccc} \widehat{g}_t & \widehat{b}_t & \widehat{\tau}_t \end{array} \right],$$

where $\widehat{X}_t'$ is the vector with non-predetermined variables and $\widehat{x}_t'$ is the vector of predetermined variables. The reduced form can be written

$$\mathcal{A} \left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = \mathcal{B} \left[\begin{array}{c} E_t \widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right], \tag{B.2}$$

which is equal to

$$\mathcal{M} \left[\begin{array}{c} \widehat{X}_t \\ \widehat{x}_t \end{array} \right] = \left[\begin{array}{c} E_t \widehat{X}_{t+1} \\ \widehat{x}_{t+1} \end{array} \right], \tag{B.3}$$

where $\mathcal{M} = \mathcal{A}\mathcal{B}^{-1}$. Matrix $\mathcal{M}$ is defined by suitable matrices $\mathcal{A}$ and $\mathcal{B}$ which are written

$$\mathcal{A} = \left[\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 0 \end{array} \right], \mathcal{B} = \left[\begin{array}{ccccc} b_{11} & b_{12} & b_{13} & 0 & 0 \\ 0 & b_{22} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & 0 & 0 & 0 & b_{55} \end{array} \right],$$

where

$$\begin{aligned}
a_{11} &= \left[1 + \tfrac{\overline{c}_t}{\overline{y}_t}\tfrac{1}{\sigma}\eta_2\right], a_{12} = \left[\tfrac{\overline{c}_t}{\overline{y}_t}\tfrac{1}{\sigma}(1+\eta_1)\right], a_{13} = -\tfrac{\overline{g}_t}{\overline{y}_t}, \\
a_{21} &= \left[-a\left(\sigma\tfrac{\overline{y}_t}{\overline{c}_t} + \lambda\right)\right], a_{22} = 1, , a_{33} = 1, \\
a_{41} &= \left[\tfrac{\overline{m}}{\overline{y}}\left(\tfrac{1-\overline{\pi}}{\overline{\tau}}\right)\left(\tfrac{\overline{y}_t}{\overline{c}_t} - \tfrac{1}{\sigma}\eta_2\right)\right], a_{42} = \left[\tfrac{\overline{m}}{\overline{y}}\left(\tfrac{1-\overline{\pi}}{\overline{\tau}}\right)\tfrac{1}{\sigma}(1+\eta_1)\right], \\
a_{43} &= \left[\tfrac{\overline{m}}{\overline{y}}\left(\tfrac{1-\overline{\pi}}{\overline{\tau}}\right)\tfrac{\overline{g}_t}{\overline{c}_t}\right], a_{44} = \left[\left(1 + \tfrac{1}{\overline{\tau}}\left(\overline{\pi}\tfrac{\overline{m}}{\overline{y}} - \tfrac{\overline{g}}{\overline{y}}\right)\right)\left(\tfrac{1}{\overline{R}-\overline{\pi}} + 1\right)\right], \\
a_{51} &= \left[\tfrac{\phi}{\overline{\tau}}\tfrac{\overline{m}}{\overline{y}}\left(\tfrac{\overline{y}_t}{\overline{c}_t} - \tfrac{1}{\sigma}\eta_2\right)\right], a_{52} = -\left[\tfrac{\phi}{\overline{\tau}}\tfrac{\overline{m}}{\overline{y}}\tfrac{1}{\sigma}(1+\eta_1)\right], \\
a_{53} &= -\left[\tfrac{\phi}{\overline{\tau}}\tfrac{\overline{m}}{\overline{y}}\tfrac{\overline{g}_t}{\overline{c}_t}\right], a_{54} = \left[1 - \tfrac{1}{\overline{\tau}}\left(\overline{\tau^*} + \psi - \phi\tfrac{\overline{m}}{\overline{y}}\right)\right], \\
b_{11} &= 1, b_{12} = \tfrac{\overline{c}_t}{\overline{y}_t}\tfrac{1}{\sigma}, b_{13} = -\tfrac{\overline{g}_t}{\overline{y}_t}, b_{22} = \beta, b_{33} = 1 \\
b_{41} &= \left[1 + \tfrac{1}{\overline{\tau}}\tfrac{\overline{m}}{\overline{y}}\left(\tfrac{\overline{y}_t}{\overline{c}_t} - \tfrac{1}{\sigma}\eta_2\right) - \left(1 + \tfrac{1}{\overline{\tau}}\left(\overline{\pi}\tfrac{\overline{m}}{\overline{y}} - \tfrac{\overline{g}}{\overline{y}}\right)\right)\tfrac{\overline{R}}{\overline{R}-\overline{\pi}}\eta_2\right], \\
b_{42} &= \left[\left(1 + \tfrac{1}{\overline{\tau}}\left(\overline{\pi}\tfrac{\overline{m}}{\overline{y}} - \tfrac{\overline{g}}{\overline{y}}\right)\right)\tfrac{\overline{\pi}}{\overline{R}-\overline{\pi}} + \tfrac{1}{\overline{\tau}}\left(\overline{\pi}\tfrac{\overline{m}}{\overline{y}} - \tfrac{\overline{m}}{\overline{y}}\tfrac{1}{\sigma}(1+\eta_1)\right)\right. \\
&\left. - \left(1 + \tfrac{1}{\overline{\tau}}\left(\overline{\pi}\tfrac{\overline{m}}{\overline{y}} - \tfrac{\overline{g}}{\overline{y}}\right)\right)\tfrac{\overline{R}}{\overline{R}-\overline{\pi}}(1+\eta_1)\right], b_{43} = -\left[\tfrac{1}{\overline{\tau}}\left(\tfrac{\overline{m}}{\overline{y}}\tfrac{\overline{g}_t}{\overline{c}_t} + \tfrac{\overline{g}}{\overline{y}}\right)\right],
\end{aligned}$$

$$b_{44} = \left[\left(1 + \frac{1}{\overline{\tau}}\left(\overline{\pi}\frac{\overline{m}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\right)\frac{1}{\overline{R}-\overline{\pi}}\right], b_{45} = 1, b_{51} = \left[1 + \frac{\psi - \overline{\tau^*}}{\overline{\tau}}\right], b_{55} = 1$$

Matrix $\mathcal{A}$ is singular, so it has no inverse, therefore we define matrix $\mathcal{M} = \mathcal{A}\mathcal{B}^{-1}$, which is a 5×5 matrix and has five roots. Now the roots of matrix $\mathcal{M}$ are inverse of the roots of matrix M defined in Appendix A. To avoid singularity of matrix $\mathcal{A}$ we could have substituted tax rule into the system. Then the matrix $\mathcal{M}$ would have been 4×4 matrix. The conclusions, however, would remain the same.

C Appendix. The model

C.1 The dynamic model

- 1. IS (Euler equation)

$\ln y_t = \ln y_{t+1} + \frac{\overline{g}}{\overline{y}} \left(\ln g_t - \ln g_{t+1} \right) - \frac{\overline{c}}{\overline{y}} \frac{1}{\sigma} \left(R_t - \pi_{t+1} \right) - \frac{\overline{c}}{\overline{y}} \frac{1}{\sigma} \ln \delta$

- 2. LM

$\ln m_t = \frac{\overline{y}}{\overline{c}} \ln y_t - \frac{\overline{g}}{\overline{c}} \ln g_t - \frac{1}{\sigma} R_t + \frac{1}{\sigma} \ln \Gamma$

- 3. Phillips curve

$\pi_t = \beta \pi_{t+1} + a \left[\left(\sigma \frac{\overline{y}}{\overline{c}} + \lambda \right) \left(\ln y_t - \ln y_t^* \right) \right]$

- 4. Potential output (supply function)

$\ln y_t^* = \frac{1}{\left(\sigma \frac{\overline{y}}{\overline{c}} + \lambda \right)} \left[\sigma \frac{\overline{g}}{\overline{c}} \ln g_t + (1 + \lambda) \left(\ln \mu + \zeta * Time_t \right) \right.$
$\left. + \ln \left(1 - \tau_t \right) \right]$

- 5. Budget constraint

$b_t = \left(1 + R_{t-1} - \pi_t \right) * b_{t-1} - \tau_t y_t + \left(1 - \pi_t \right) m_{t-1} - m_t + g_t$

- 6. Taylor rule

$R_t = \pi_t + r^* + \eta_1 \left(\pi_t - \pi^* \right) + \eta_2 \left(\ln y_t - \ln y_t^* \right)$

- 7. Tax rule

$\tau_t = \tau^* + \phi \left(b_{t-1} + m_{t-1} \right) / y_t - \psi$

- 8. Definition

$\ln g_t = \ln g_{t-1} + \theta$

- 9. Definition

$\ln P_t = \ln P_{t-1} + \pi_t$

- 10. Long run growth rate

$\theta = \frac{(1+\lambda)\zeta}{\sigma + \lambda}$

C.2 The steady state model

- 1. IS (Euler equation)

$\overline{R} = \sigma\theta + \overline{\pi} - \ln\delta$

- 2. LM

$\ln\overline{m} = \frac{\overline{y}}{\overline{c}}\ln\overline{y} - \frac{\overline{g}}{\overline{c}}\ln\overline{g} - \frac{1}{\sigma}\overline{R} + \frac{1}{\sigma}\ln\Gamma$

- 3. Phillips curve

$\overline{y} = \overline{y}^{*} + \frac{(1-\beta)\overline{\pi}}{\left(\sigma\frac{\overline{y}}{\overline{c}}+\lambda\right)}$

- 4. Potential output (supply function)

$\ln\overline{y}^{*} = \frac{1}{\left(\sigma\frac{\overline{y}}{\overline{c}}+\lambda\right)}\left[\sigma\frac{\overline{g}}{\overline{c}}\ln\overline{g} + (1+\lambda)\left(\ln\mu + \zeta * Time\right)\right.$
$\left. - \ln\left(1-\overline{\tau}\right)\right]$

- 5. Budget constraint

$\overline{b} = \frac{\overline{g}-\overline{\tau}\overline{y}-\overline{m}\left[1-(1-\overline{\pi})\frac{1}{\exp\theta}\right]}{1-\left(1+\overline{R}-\overline{\pi}\right)\frac{1}{\exp\theta}}$

- 6. Taylor rule

$\overline{\pi} = \pi^{*} + \frac{\sigma\theta - r^{*} + \ln\delta}{\eta_1}$

- 7. Tax rule

$\overline{\tau} = \tau^{*} + \frac{\phi}{\overline{y}}\frac{\left(\overline{b}+\overline{m}\right)}{\exp\theta} - \psi$

- 8. Definition

$\ln\overline{g}_t = \ln\overline{g}_{t-1} + \theta$

- 9. Definition

$\ln\overline{P}_t = \ln\overline{P}_{t-1} + \overline{\pi}$

- 10. Long run growth rate

$\theta = \frac{(1+\lambda)\zeta}{\sigma+\lambda}$

Figure 4: Permanent increase of 1% of real GDP in public consumption, deviations from baseline.

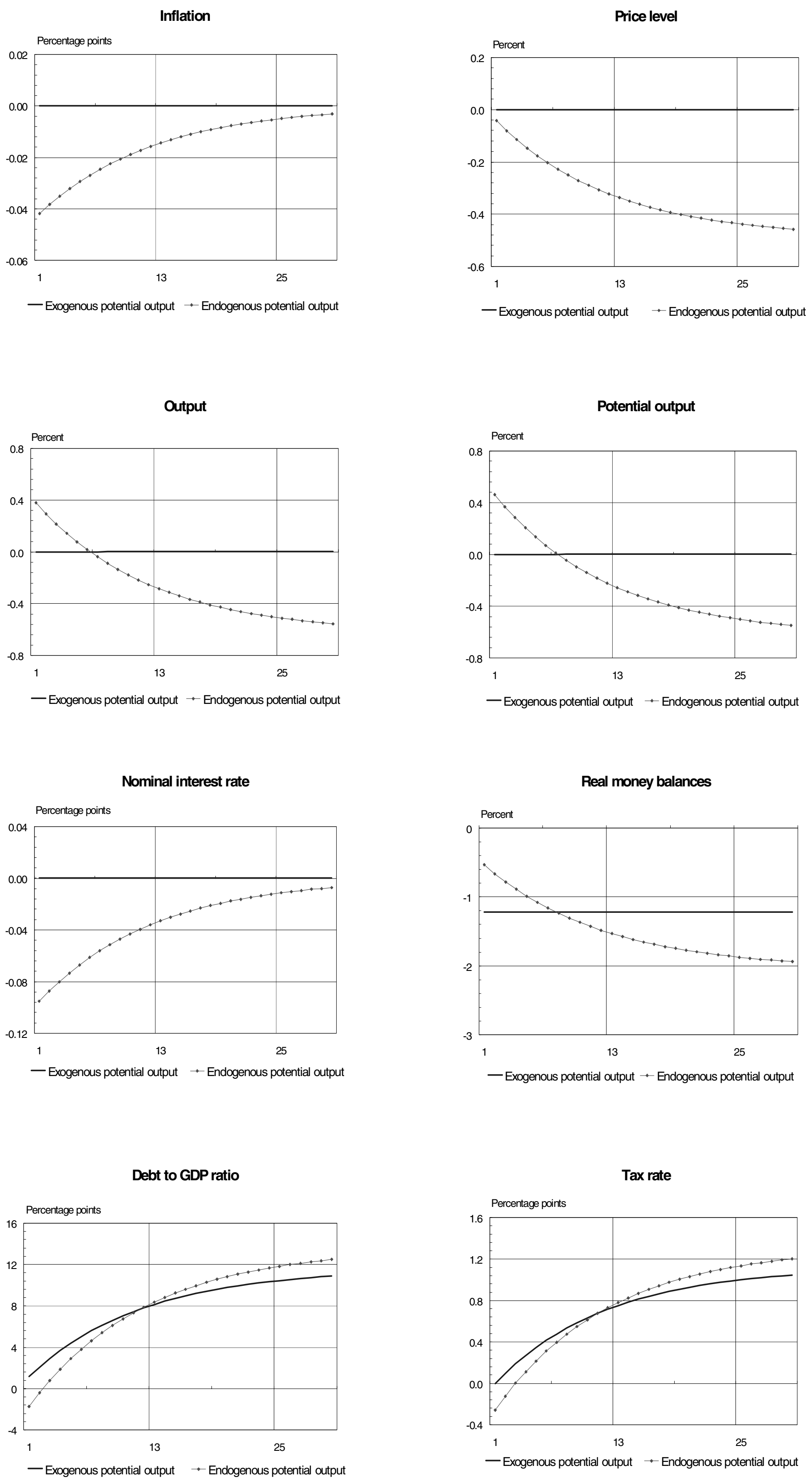


Figure 5: Permanent 1 percent increase in technology, deviations from baseline.

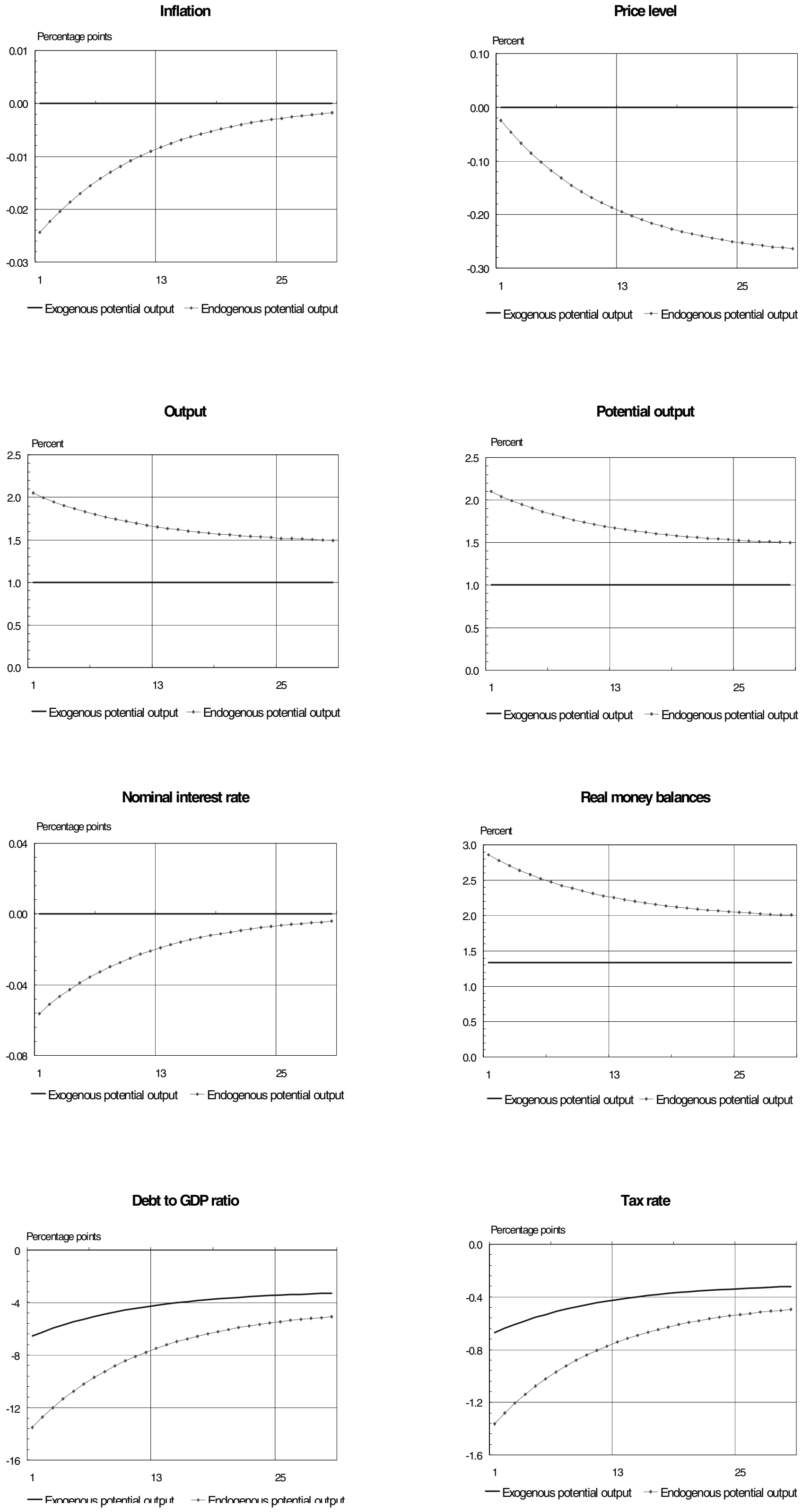


Figure 6: Permanent 1 percentage point increase in inflation target, deviatios from baseline.

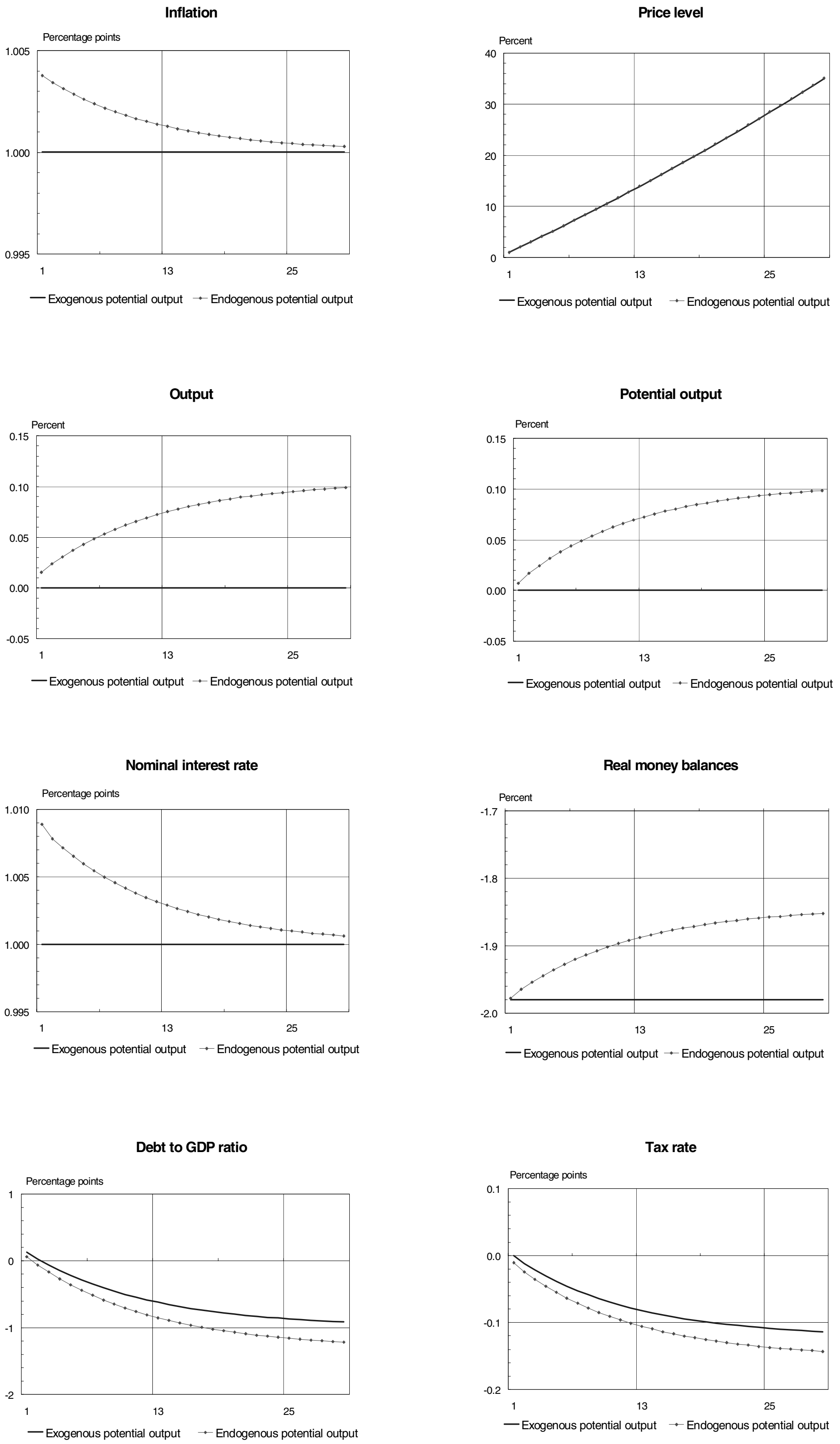


Figure 7: Temporary (one period) increase of 1% of real GDP in public consumption, deviations from baseline.

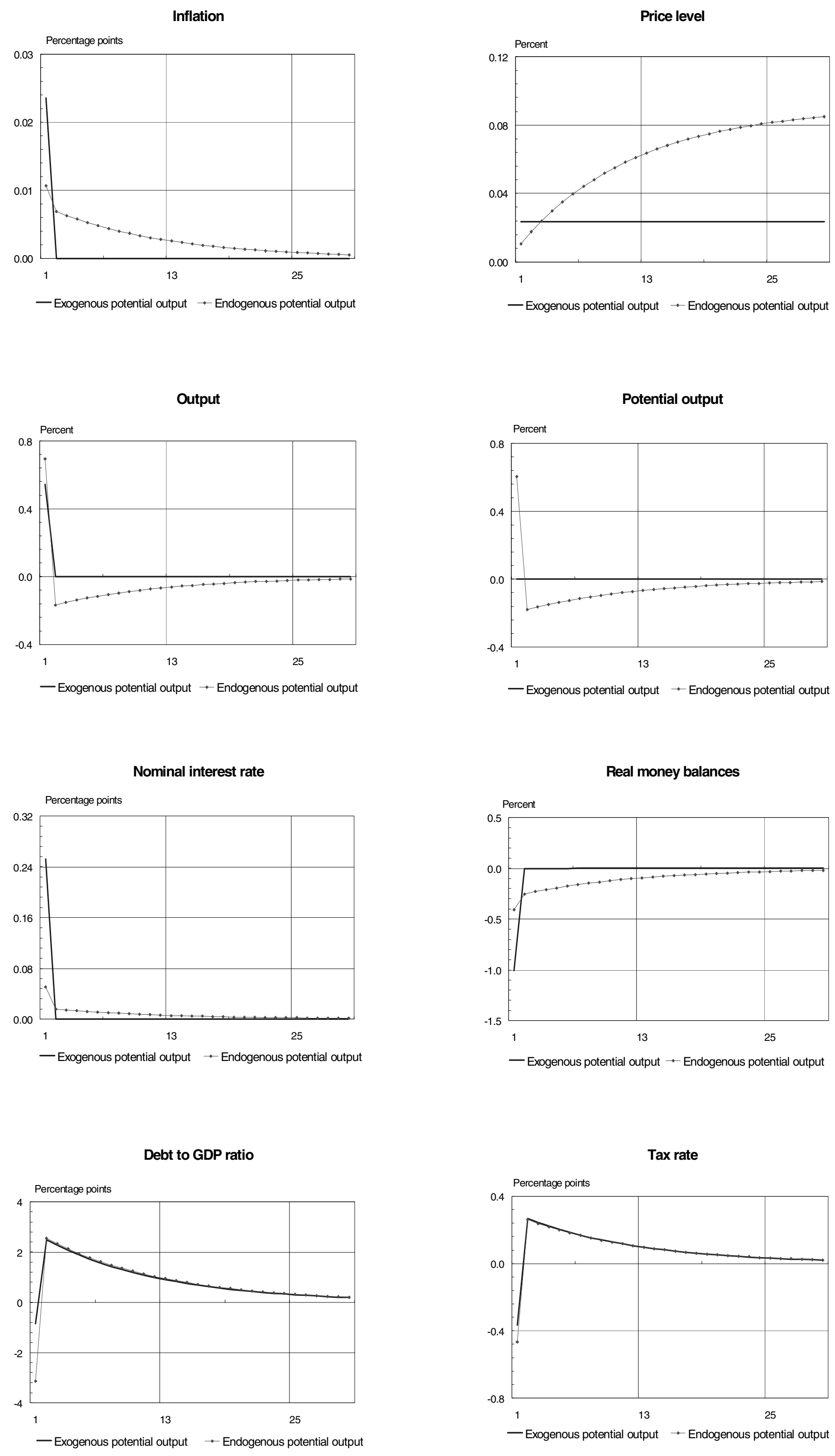


Figure 8: Temporary (one period) 1% increase in technology, deviations from baseline.

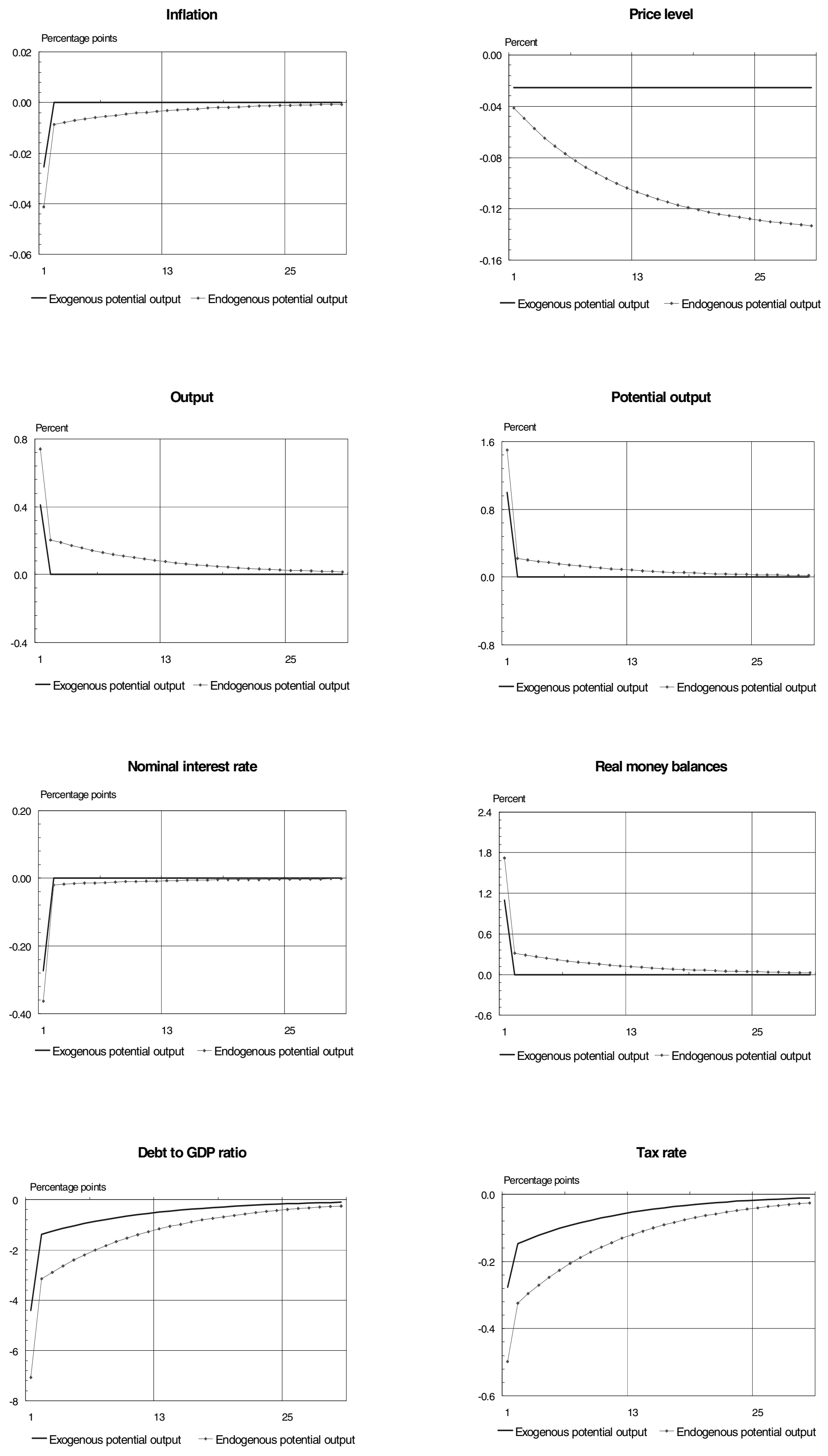


Figure 9: Temporary (one period) 1 percentage point increase in inflation target, deviations from baseline.

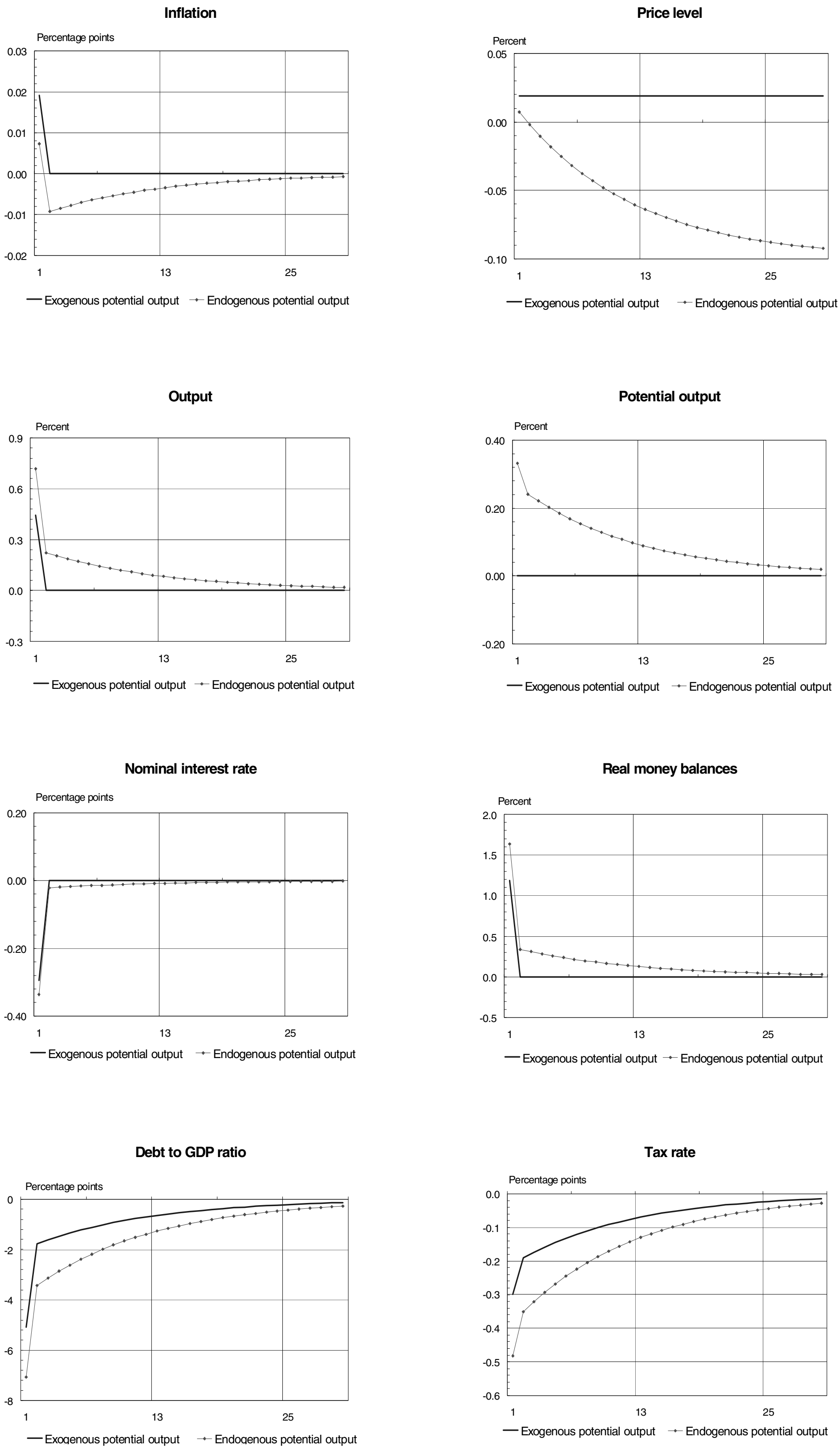


Figure 10: Temporary (one period) increase of 1 % of real GDP in public consumption, deviations from baseline.

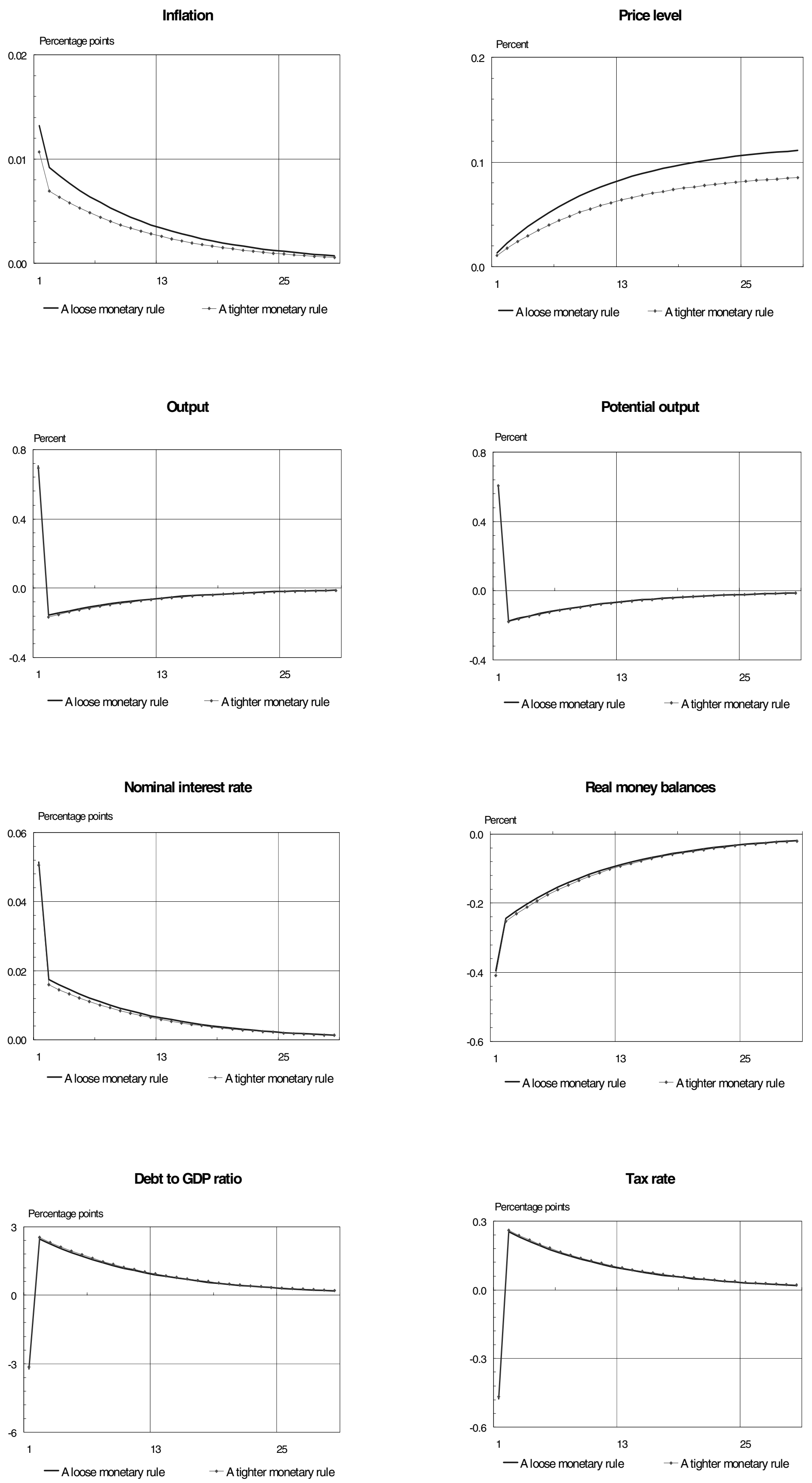
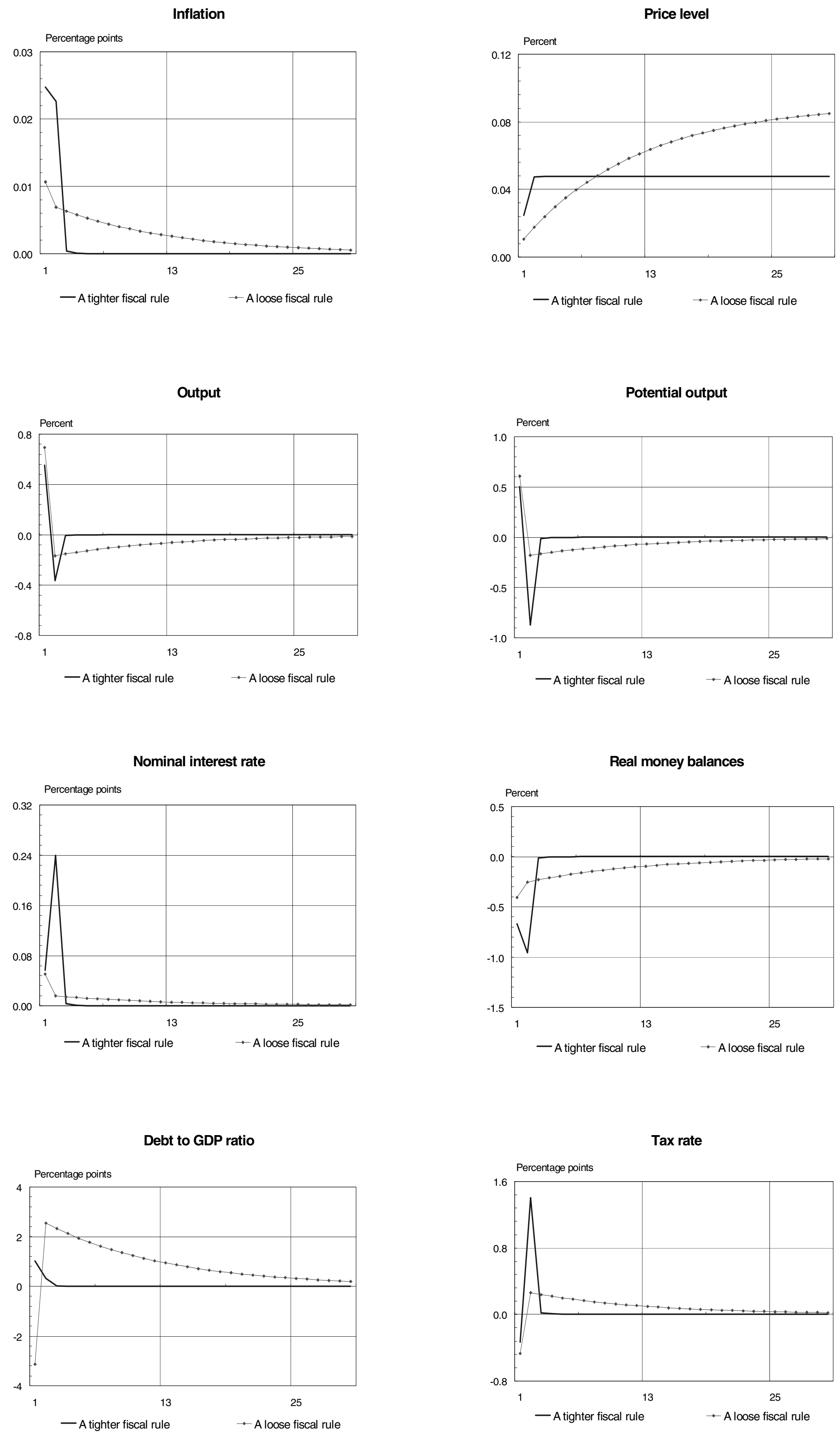


Figure 11: Temporary (one period) increase of 1 % of real GDP in public consumption, deviations from baseline.



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