Liquidity Trap Prevention and Escape: A Simple Proposition

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Abstract

Liquidity traps occur when the natural nominal interest rate becomes negative. In a model with capital price dynamics explicitly considered, we find that shocks in the future can cause current and lasting liquidity traps. We propose that the central bank can prevent or fix liquidity traps by appending to its inflation-targeting monetary policy with a prioritized promise to defend a lower bound of nominal capital price. (JEL E31, E43, E44, E52, E58, E61, G12)

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1. Introduction

Generally speaking, liquidity traps occur when the economy still has insufficient aggregate demand even when the zero nominal interest bound becomes binding. It is a widely-held view that Japan has already been trapped; and the United States and West Europe are in danger of slipping into one. Motivated by both policy and academic interests, much research has been conducted to examine the implications of low interest rates and deflation in general, and the prevention and cure of liquidity traps in particular—see Buiter (2003); Clouse et al. (2003); Ullersma (2002) for relevant literature reviews.

From a traditional IS-LM perspective, when the zero bound is binding, monetary policy will become ineffective; and fiscal policy needs to play a major role in pulling the economy out of liquidity traps. Yet, many authors point out that monetary policy can still play a major role in fixing liquidity traps, even when the binding zero bound makes it impossible to further reduce the short-term nominal interest rate. For example, the central bank can stimulate aggregate demand through unconventional monetary policy. Another popular notion is that inflation expectations can be "managed" to reduce the risk of a binding zero nominal interest bound, and lower the real interest rate when the bound becomes binding. Also, the exchange rate has been recommended as an instrument to rescue the economy from liquidity traps. Finally, the central bank can break the zero bound through taxing money.

Most of the existing research on liquidity traps uses analytical frameworks that do not explicitly model capital price and hence misses a simple way of escaping from liquidity traps. In this paper we examine how expected capital price depreciation can cause current and lasting liquidity traps, even when the shock that causes the depreciation is in the distant future. Based on the insights provided by our model, we propose that the central bank can prevent or fix liquidity traps by committing to defend a lower bound of nominal capital price, and prioritizing this promise over its commitment to price stability.

The remainder of the paper is organized as follows. Section 2 reviews the major policy recommendations in the literature on fixing liquidity traps. In Section 3 we first diagnose liquidity traps from a capital-price perspective, then based on which propose a simple way out of liquidity traps, and compare it to other propositions in the literature. Finally, section 4 concludes the paper.

2. Propositions on fixing liquidity traps: a brief review

In general, liquidity traps occur when the natural (or equilibrium) nominal interest rate (i.e., the nominal interest rate level that balances aggregate demand and supply) becomes negative. Before reviewing the major propositions in the literature on the way of escaping from liquidity traps, we first provide a brief diagnosis of the occurrence of liquidity traps in an illustrative model.

The demand side of the economy can be modeled by a forward-looking IS curve:

$$y_t^d = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \varepsilon_t^d ; \tag{2.1}$$

whereas the supply side captured by an expectations-augmented Philips curve: [1]

$$y_t^s = \alpha \pi_t - \eta E_t \pi_{t+1} + \varepsilon_t^s . \tag{2.2}$$

[1] y^d and y^s are aggregate demand and supply respectively; y denotes output; i and π are the nominal interest rate and inflation rate respectively; and ε^d and ε^s represent demand and supply shocks respectively. See Woodford (2003) for detail discussion on the microfoundation of the two equations.

According to equation (2.1) and (2.2), the balance between (aggregate) demand and supply can be measured by

$$\Omega_t \equiv y_t^d - y_t^s = E_t y_{t+1} - \sigma i_t - \alpha \pi_t + (\sigma + \eta) E_t \pi_{t+1} + \varepsilon_t^d - \varepsilon_t^s . \tag{2.3}$$

A negative (or positive) Ω_t represents the existence of insufficient (or excessive) aggregate demand; whereas the equilibrium condition is $\Omega_t = 0$.

A major task of stabilization policies is to keep demand and supply in balance (i.e. $\Omega_t = 0$). For monetary policy, the task is usually accomplished by open market operations that keep the nominal interest rate (i_t) at its natural level (i_t^e), which, according to equation (2.3), is determined by

$$i_t^e = \sigma^{-1}\left[E_t y_{t+1} - \alpha \pi_t + (\sigma + \eta) E_t \pi_{t+1} + \varepsilon_t^d - \varepsilon_t^s\right]. \tag{2.4}$$

Both demand and supply shocks (ε_t^d and ε_t^s) can affect i_t^e. As long as the affected natural nominal interest rate remains nonnegative ($i_t^e \geq 0$), the central bank will (in principle) have no problem in keeping the nominal interest rate at its natural level (i.e. $i_t = i_t^e$) so as to maintain the balance between demand and supply (i.e. $\Omega_t = 0$).

However, according to equation (2.4), a negative demand shock ($\varepsilon_t^d < 0$), if sufficiently large ($\left|\varepsilon_t^d\right| > E_t y_{t+1} - \alpha \pi_t + (\sigma + \eta) E_t \pi_{t+1} - \varepsilon_t^s$), can reduce the natural nominal interest rate to negative (i.e. $i_t^e < 0$).[2] On the other hand, due to the durability of money—one dollar of money will always have purchasing power of one dollar—there exists a zero bound on the nominal interest rate (i.e. $i_t \geq 0$). Therefore, in the situation where the

[2] Certainly a positive supply shock ($\varepsilon_t^s > 0$), if sufficiently large, will have a similar effect. To fix idea, in this paper we consider demand shocks only.

natural nominal interest rate becomes negative (i.e. $i_t^e < 0$), the central bank will be unable to reduce the nominal interest rate to its natural level; hence the economy will fall in a liquidity trap, with insufficient aggregate demand (i.e. $\Omega_t < 0$) and the nominal interest rate on its zero bound (i.e., $i_t = 0$).

In summary, with a zero bound on the nominal interest rate, liquidity traps will occur when the natural nominal interest rate becomes negative. Therefore, to escape liquidity traps, either the natural nominal interest rate needs to become nonnegative, or the zero bound needs to be broken. Based on this insight, we review the major policy propositions in the literature on the way of escaping from liquidity traps.

A natural question to ask first is whether policy interventions are necessary; in another word, whether the economy can practically avoid or struggle out of liquidity traps through self-adjustment. According to equation (2.4), a decrease in the current price level (i.e. $\pi_t < 0$) can raise i_t^e and hence help avoiding liquidity traps.[3] However, notwithstanding the downward pressure generated by insufficient demand, price rigidity in the short run makes it unlikely to avoid liquidity traps through immediate and sufficient price reduction. Deflation in the long run could help the economy to adjust out of liquidity traps gradually. However, chronicle deflation can cause deflation expectations (i.e. $E_t\pi_{t+1} < 0$), which will dampen demand and may lead the economy into a "deflation spiral" (Benhabib et al. 2002). In a word, policy interventions are usually necessary to overcome the gravity of liquidity traps.

[3] This works from both demand and supply sides. According to equation (2.2), a decrease in π_t will reduce the aggregate supply y_t^s so as to assuage the problem of insufficient demand. On the other hand, a decrease in current price level could stimulate aggregate demand by increasing the real value of consumers' financial wealth (i.e., the real balance effect). Note that the forward-looking IS curve here (equation 2.1) does not capture this mechanism.

Fiscal policy

A conventional policy recommendation for fixing liquidity traps is expansionary fiscal policies. In essence, fiscal expansions amount to positive demand shocks that could help raising the natural nominal interest rate up above zero. Fiscal authorities can boost sluggish aggregate demand by increasing their own expenditures, or by putting money in consumers' pockets via bond-financed tax cuts or (with the help of monetary authorities) through "money rains".

Notwithstanding generally viewed as a sensible prescription, the desirability and effectiveness of fiscal polices in fixing liquidity traps (or as stabilization policies in general) have been put into question. First, government expenditures intended to stimulate demands are prone to inefficiency. Another concern is Ricardian Equivalence that could limit the impact of fiscal expansion on stimulating demands (Krugman, 1998). By creating purchasing power from the thin air, monetary transfers or tax cuts financed by money printing are likely to increase demand (Goodfriend 2000). Yet, political-economy obstacles aside, this method alone may not be able to accomplish the mission of escaping from liquidity traps, because the magnitude of money rain necessary to flood the economy out of liquidity traps may intimidate even the most aggressive central banks.

Unconventional monetary policy

When the short end of yield curves is touching on the ground zero, the long end can still have some room to fall. Therefore, with the conventional weapon (i.e. the short-term interest rate) running out of ammunition, the central bank still has several unconventional methods to reduce the long-term interest rate.

When the zero bound is binding, monetary expansion through open market purchases cannot further reduce the short-term interest rate, but merely increase the quantity of money held by the private sector. Yet, this “quantity easing” operation could help reducing long-term interest rates through portfolio balance effect (Bernanke and Reinhart 2004). As money and short-term government bonds become perfect substitutes (in terms of the store-of-value function) at a binding zero bound, the portfolio-balance impact of a change in the ratio between them (with a constant total) may not be significant.

The central bank can also influence long-term interest rates via directly purchasing long-term government securities, or even private-sector assets such as stocks and real estate (Clouse et al. 2003; Goodfriend 2000; Meltzer 2001). A downside of this method is its tendency to increase the central bank’s direct involvements in financial business and hence complicate monetary policy implementation. For example, the loading and unloading of private-sector assets on the central bank’s balance sheet will likely involve many governance issues (Buiter, 2003).

Indirectly, the central bank can influence long-term interest rates through a commitment to a lasting low-interest policy, such as the zero-interest policy adopted by Bank of Japan in 2001. If credible, this policy will help reducing long-term interest rates, which can nevertheless remain at a relatively high level due to risk or other premiums.

In summary, should long-term interest rates have room to fall, properly designed unconventional monetary policies will allow the central bank to stimulate aggregate demand through reducing long rates. Nonetheless, if the natural long-term nominal interest rate becomes negative, these unconventional policies will become powerless as well.

Management of inflation expectations

Many authors (e.g. Auerbach and Obstfeld 2003; Eggertsson and Woodford 2003; Krugman 1998) point out that monetary policy can fix liquidity traps through managing inflation expectations. Equation (2.4) indicates a positive relationship between the expected inflation ($E_t\pi_{t+1}$) and the natural nominal interest rate i_t^e.[4] Thus, monetary policy can fix liquidity traps by inducing higher inflation expectations that could raise the negative natural nominal interest rate up above zero. In the same spirit, by targeting a relatively high inflation target, the central bank can reduce the risk of liquidity traps (Buiter, 2003).

While there is not much dispute on the notion that high inflation expectations will help fixing liquidity traps, a consensus has yet to be reached on whether and how central banks can properly "manage" inflation expectations.

One major issue is how central banks can induce inflation expectation in a situation with deflation pressure. Friedman (2003) describes the difficulty in generating inflation expectation in liquidity traps as the conundrum of Tinkerbell's dust. If people believe in central banks' promises on future inflation, inflation will be generated in a self-fulfilling process. Yet, if people form their inflation expectations adaptively based on recent

[4] On the one hand, higher consumers' inflation expectations ($E_t\pi_{t+1}$) will increase aggregate demand by reducing the real interest rate ($i_t - E_t\pi_{t+1}$) and hence put upward pressure on i_t^e —see equation (2.1). On the other hand, a lowered real interest rate tends to reduce aggregate supply through decreasing labor supply. Besides, high expected inflations can also reduce aggregate supply by influence relative prices and hence firms' productions.

experience, central banks' promises on high future inflation will be "spitting in the wind" (Buiter, 2003) and hence have no magic power to jumpstart an inflation process.

More optimistic authors (e.g. Auerbach and Obstfeld 2003; Eggertsson and Woodford 2003; Krugman 1998) argue that, if the central bank can convince consumers of its tolerance to a higher price level in the future (after liquidity traps are over), inflation expectation can be established. Intuitively, once consumers expect a higher price level in period *T*, their optimal reactions will lead to a higher price level in period *T*-1; and then raise the price level in period *T*-2, and so on (Auerbach and Obstfeld 2003). In a word, as long as the central bank can convince people that price will eventually go up sometime in the future, inflation expectation will be generated.

However, many authors (e.g. Adam and Billi 2003; Eggertsson 2003; Krugman 1998) point out that central banks' commitment to high inflation is intrinsically incredible, because it would be optimal for the central bank to renege after the high-inflation promise has helped pulling the economy out of liquidity traps. Auerbach and Obstfeld (2003, p.18), on the other hand, do not share this concern, because their analysis shows that the central bank's "acting against a deflationary trend can achieve a substantial welfare gain while creating only mild and temporary inflation." Although the authors' analysis is based on a special model specification,[5] it does help raising a puzzling question: How could the credibility of a *benevolent* central bank ever become an issue?

[5] Auerbach and Obstfeld's (2003) analysis is based on the "inverse wage Euler equation" (i.e. equation (5) in the paper) that describes wage dynamics over time in their model. Derived solely from consumer's utility maximization, the equation nevertheless contains no choice variables (i.e. consumption and labor), but only wages and interest rates that are taken as parameters by consumers in their utility maximizing calculations. Yet consumers' optimizing behaviors *per se* are not supposed to put any constraints on these parameters. Indeed, the inverse wage Euler equation is a condition for an interior solution to the maximization problem. However, due to the linear specification of labor disutility in the model, corner solutions are more general.

The key to this puzzle is that individual rationality does not necessarily produce collectively rational outcomes. Even though it is in the interest of consumers for the central bank to renege on (rather than sticking to) its high-inflation promise, as long as consumers do not believe in the high-inflation promise, liquidity traps, which are probably the worst outcome, will be resulted. The reason is that individual consumers can hardly realize that their individual spending postponements could aggregately push the economy into a liquidity trap. Or even if they do realize it, the market *per se* provides no coordination mechanisms to orchestrate an altogether increase in spending. In a word, the high-inflation promise is not credible from an *ex ante* point of view, even though the belief in it would be welfare-improving *ex post*.

Even if the central bank can convince consumers of its high-inflation promise, managing inflation expectations will be a challenging task. While excessive inflation (or price) targets could cause runaway inflation, conservative inflation targets will tend to result in actual inflation falling short of the targets. In this respect price-level targeting will perform better than inflation targeting (Eggertsson and Woodford 2003; Wolman 2003). However, price-level targets that systematically overshoot actual prices will have difficulties in establishing their credibility. Consumers' rational expectations will tend to take into account the overshooting, which will increase the complexity of central bank's inflation expectation management.

Exchange rate

Some authors suggest the exchange rate as an instrument to fix liquidity traps (e.g. Coenen and Wieland 2003; McCallum 2000; Svensson 2001, 2003). Recall that downward price rigidity is a key element that prevents the economy from self-adjusting out of liquidity traps. Yet the central bank can effectively reduce domestic prices (in terms of foreign currencies) through exchange rate depreciation, which can stimulate aggregate demand through exports. A problem of this policy is the difficulty in depreciating exchange rate when the zero interest bound is binding (Christiano, 2001). An undervalued currency will tend to attract demands from those who bet on its future appreciation. Under normal situations, these speculative demands can be dampened by reduction in domestic interest rates. Yet a binding zero interest bound will make such adjustment unavailable. Hence the central bank may have to satisfy a large amount of speculative demands on home currency before it can succeed in depreciating the exchange rate. With unlimited ability to supply home currency and strong determination on depreciating its value, the central bank can eventually scare off speculative demands (Svensson 2001, 2003). However, with its beggar-thy-neighbor impacts, exchange rate depreciation intentionally designed to jumpstart sluggish domestic economy is also likely to jumpstart the money-printing machines of trading partners who want to keep their domestic demands intact (Coenen and Wieland 2003; Stevens 2001; Swank 2001).[6] The resulting competitive devaluations can make the depreciation attempt very difficult to succeed. Also, it will cause instability in the foreign exchange market.

[6] For the case of Japan, Svensson (2001) argues that Japan's trading partners may be willing to coordinate a jumpstart mission for future benefits from the imports of a strong Japanese economy. However, there could be numerous reasons to expect unsympathetic trading partners against a tradeoff between current pain and future happiness. For example, policymakers in the trading partners are likely to hesitate on such a tradeoff that may jeopardize their careers. Moreover, trading partners who sacrifice now may not be the ones who get the future benefits. Svensson (2003, p. 34) also argues that currency depreciation tends to be a result of escaping from liquidity trap, directly or indirectly. Yet we share the opinion of Auerbach and Obstfeld (2003) that "incremental market-induced depreciations" would be easier to defend as "a side product of domestically necessary policies".

Svensson (2001, 2003) suggests using a temporary peg (to an undervalued exchange rate) to achieve an upward-sloping price-level target path. Thus, the undervalued exchange rate in Svensson's plan can help jumpstarting the economy through both foreign and domestic demands. However, as just discussed, trading partners' reactions could make the undervalued peg very difficult to sustain. Besides, the peg will force the central bank to raise the nominal interest rate to satisfy the parity between domestic and foreign interest rates. If slow adjustments in inflation are not enough to offset the increase in the nominal interest rate, the resulting increase in the real interest rate will become a force dampening aggregate demand and hence against the intended jumpstart (Swank, 2001).

Gesell's tax

Recall that one necessary condition for liquidity traps to occur is the existence of a zero nominal interest bound. While the above policy propositions focus on unbinding the bound, some authors propose to break the bound through imposing Gesell's tax on money (Buiter and Panigirtzoglou 2003; Goodfriend 2000). To tax money is effectively equivalent to creating inflation directly; hence Gesell's tax can (in principle) fix liquidity traps in a way similar to a rise in inflation expectations.[7] The problem is its feasibility. The administration of Gesell's tax could be challenging and costly (Buiter and Panigirtzoglou, 2003). Moreover, some political-economy obstacles and international complications need to be overcome before such a major change in institution becomes practical (Byrant, 2000).

[7] See Goodfriend (2000) for comparison between inflation tax and Gesell's tax.

3. A simple way to prevent and fix liquidity traps

Should there be no bound on the nominal interest rate, liquidity traps will not happen, because aggregate demand and supply can be balanced at negative nominal interest rates. Negative nominal interest rates imply negative nominal returns to capital.[8] Since earnings can hardly be negative, negative nominal returns to capital can only result from nominal capital price (denoted as *Q*) depreciation.

Therefore, from a *Q*-perspective, liquidity traps will occur in the situation where the current equilibrium nominal capital price (i.e., the nominal capital price level that balances aggregate demand and supply) is not sustainable due to the expectation of low nominal capital price in the future, which could be caused by low future equilibrium nominal capital price, or by financial market's "irrational despair", or both.

In short, according to a *Q*-perspective diagnosis, the root of liquidity traps is the (expectation of) low nominal capital price in the future. Then a straightforward prescription is to keep the future nominal capital price at a high level that can sustain the equilibrium nominal capital price in the present. This is not a difficult task for the central bank whose ability to supply money is (in principle) unlimited. One possible impact of keeping the future nominal capital price high is high future inflation. Yet this is a side effect that the central bank has to tolerate for the sake of preventing or escaping from liquidity traps.

In the following we first examine the occurrence of liquidity traps in a formal model that incorporates capital price dynamics, and then based on which propose a simple way

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[8] For narrative convenience, we use capital as a general term for assets other than money and government bonds.

to prevent or fix liquidity traps. Finally, we compare our proposition to other propositions in the literature.

The Model

At the beginning of period *t*, a representative consumer holds a wealth portfolio that contains three assets: real capital (K_t), one-period nominal government bond (D_t), and money (M_t). Capital *per se* cannot be consumed; yet it provides capital owners with capital earnings (in terms of consumption) at the end of every period. Capital can also be exchanged for consumption or other assets. Bond is a one-period coupon bond denominated in money and issued by government. Money, issued by government, is a non-interest-bearing medium of exchange that provides liquidity services.

The consumer supplies one unit of labor inelastically during period *t* and receives real wage income W_t at the end of which.[9] After paying real lump-sum tax τ_t, the consumer decides period-*t* consumption C_t and wealth portfolio $\{K_{t+1}, D_{t+1}, M_{t+1}\}$ for period *t+1*, subject to the following budget constraint:

$$q_t K_{t+1} + \frac{M_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} = q_t K_t + \frac{M_t}{P_t} + \frac{D_t}{P_t} + R_t K_t + \frac{i_t D_t}{P_t} + W_t - C_t - \tau_t, \qquad (3.1)$$

where P_t and q_t are respectively the price of consumption (in terms of money) and the real price of capital (in terms of consumption) at the end of period *t*; whereas R_t and i_t are respectively the earning rate of capital (K_t) and the interest rate of bond (D_t) during period *t*.

[9] Here we do not consider consumer's decision-making on labor supply for simplicity. Besides, the notion that the problem of insufficient demand can be solved by aggregate supply reduction through more voluntary unemployment is not appealing to us in the analysis of liquidity traps.

Assume perfect substitution between capital and bond; then the period-*t* real interest rate can be defined as

$$r_t \equiv \frac{R_t + q_t - q_{t-1}}{q_{t-1}} = \frac{P_{t-1}(1+i_t) - P_t}{P_t}, \tag{3.2}$$

where the middle term measures the real rate of return to capital; whereas the last term measures the real rate of return to bond.

Using equation (3.2), equation (3.1) can be transformed into

$$A_{t+1} = (1+r_t)A_t - \frac{i_t M_t}{P_t} + W_t - C_t - \tau_t \tag{3.3}$$

where

$$A_t = q_{t-1}K_t + \frac{M_t}{P_{t-1}} + \frac{D_t}{P_{t-1}} \tag{3.4}$$

measures the real value of consumer's wealth portfolio at the end of period *t-1* (or equivalently, the beginning of period *t*).

At the end of period *t*, the consumer's utility maximization problem is given by

$$Max \sum_{v=t}^{\infty} (1+\theta)^{t-v} U(C_v, M_v)$$

subject to

$$A_t = \sum_{v=t}^{\infty} \left(\prod_{u=t}^{v} (1+r_u)^{-1} \right) \left[C_v + \frac{i_v M_v}{P_v} - W_v + \tau_v \right], \tag{3.5}$$

$$\lim_{v\to\infty} \left(\prod_{s=t}^{v} (1+r_t)^{-1} \right) A_v = 0. \tag{3.6}$$

θ is the time-preference parameter. For simplicity, assume log utility function $U = \log C_t + \beta \log(M_t / P_t)$, where β is the liquidity-service parameter. Equation (3.5), derived from equations (3.3) and (3.6), is consumer's lifetime budget constraint. Equation

(3.6) is the transversality condition that prohibits the consumer from accumulating infinite amount of debt.

First order conditions imply

$$C_t = \frac{1+\theta}{1+r_{t+1}} C_{t+1}, \tag{3.7}$$

and

$$\beta C_t = \frac{i_t M_t}{P_t}. \tag{3.8}$$

Substituting equations (3.7) and (3.8) into equation (3.5) and using equation (3.6), we obtain the following lifecycle consumption function (see Mathematical Appendix A.1 for the derivations of equations (3.7)-(3.9)).

$$C_t = \xi\left(A_{t+1} + H_{t+1}\right) = \xi\left(q_t K_{t+1} + \frac{M_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} + H_{t+1}\right). \tag{3.9}$$

Equation (3.9) shows that consumption at the end of period t (period-t consumption in short) is proportional to current wealth. $\xi(=\theta(1+\beta)^{-1})$ is the consumption propensity out of wealth. A_{t+1} is the real non-human wealth at the end of period t; whereas H_{t+1} is the real human wealth at the end of period t, measured by

$$H_{t+1} = \frac{\overline{W} - \tau_{t+1} + H_{t+2}}{1+r_{t+1}} = \sum_{v=t+1}^{\infty}\left(\prod_{u=t+1}^{v}(1+r_u)^{-1}\right)\left[W_v - \tau_v\right]. \tag{3.10}$$

To solve the model analytically, we make the following simplifying yet common assumptions.

Assume capital stock and (unit) capital earning are fixed at $\overline{K}$ and $\overline{R}$ respectively. Fixed capital and labor supplies imply fixed labor income $\overline{W}$. Thus, aggregate supply is fixed at $\overline{Y}$, i.e.,

$$Y_t = \overline{Y} = \overline{R}\overline{K} + \overline{W} \tag{3.11}$$

Assume that the total amount of government-issued assets is constant at $\overline{D}$, i.e.,

$$M_t + D_t = \overline{D}\,. \tag{3.12}$$

Also assume zero government expenditure; and hence the lump sum tax (τ_t) will be used to finance bond interest payments, i.e.,

$$P_t \tau_t = i_t D_t\,. \tag{3.13}$$

The equilibrium and steady state

Denote the equilibrium at the end of period *t* as

$$S_t^e \equiv \left\{ C_t^e,\ P_t^e,\ i_{t+1}^e,\ r_{t+1}^e,\ q_t^e,\ H_{t+1}^e;\ M_{t+1},\ \xi \right\},$$

where C_t^e, P_t^e, i_{t+1}^e, r_{t+1}^e, q_t^e, and H_{t+1}^e are endogenously determined; the money supply M_{t+1} is a policy variable determined by the central bank; and ξ is the consumption propensity parameter.

Consumption is the only component of aggregate demand; thus the goods-market equilibrium condition is given by

$$\overline{Y} = C_t + \varepsilon_t\,, \tag{3.14}$$

where ε_t is a stochastic demand shock with mean $\overline{\varepsilon} = 0$.

The steady state of the economy, denoted as $S \equiv \left\{ C,\ P,\ i,\ r,\ q,\ H;\ M,\ \xi \right\}$, is determined by the simultaneous equations (3.2), (3.8), (3.9), (3.10), (3.13), and (3.14). Note that a symbol without time-subscript represents the steady-state value of the variable. Solving the simultaneous equations gives the steady-state value of each variable as follows (see Mathematics Appendix A.2 for derivations).

$$C = \overline{Y} \tag{3.15}$$

$$P = \frac{\theta}{\beta\overline{Y}} M \tag{3.16}$$

$$i = r = \theta \tag{3.17}$$

$$q = \frac{\overline{R}}{\theta} \tag{3.18}$$

$$H = \frac{\overline{W}}{\theta} - \left(\frac{\overline{D}}{P} - \frac{\beta\overline{Y}}{\theta} \right) = \frac{\overline{W}}{\theta} - \left(\frac{\overline{D} - M}{M} \right) \frac{\beta\overline{Y}}{\theta} \tag{3.19}$$

Zero-inflation Monetary Policy

Equations (3.15)-(3.19) indicate money neutrality. Assume that the central bank pursues a zero-inflation policy by setting the money supply (M_{t+1}) to keep the price level (P_t) constant at $\overline{P}$. According to equation (3.2), this zero-inflation monetary policy implies indifference between the nominal and real interest rates:

$$i_t = r_t = \frac{R_t + q_t - q_{t-1}}{q_{t-1}}. \tag{3.20}$$

We will show that, under the zero-inflation monetary policy, a positive demand shock in the future can cause liquidity traps in the present.

Liquidity traps

Suppose a temporary and positive demand shock is expected to happen at the end of period *T*, which can be described as $\varepsilon_T = \varepsilon\overline{Y}$ ($0 < \varepsilon < 1$)—assume that $\varepsilon_t = 0$ for $t \neq T$. We will show how a large ε_T can cause liquidity traps prior to period *T*.

We start from examining the impact of ε_T on the equilibrium at the end of period *T*, denoted as $S_T^e \equiv \{C_T^e,\ M_{T+1}^e, i_{T+1}^e, r_{T+1}^e, q_T^e, H_{T+1}^e;\ \overline{P}, \xi\}$. Note that the central bank's zero-inflation commitment endogenizes the money supply.

Given fixed aggregate supply ($\overline{Y}$), monetary tightening is needed to dampen the inflation pressure caused by the positive demand shock (ε_T). The tightening will increase the interest rate ($i_{T+1}^e > i$),[10] reduce the capital price ($q_T^e < q$) as well as the human wealth ($H_{T+1}^e < H$). According to equation (3.9), the reduction in q_T^e and H_{T+1}^e will reduce consumption ($C_T^e < C$) so as to accommodate the demand shock ε_T.[11] Specifically, we have the following proposition.

Proposition 1. *Given the demand shock* $\varepsilon_T = \varepsilon\overline{Y} > 0$*, the equilibrium real capital price at the end of period T is lower than its steady-state value (*$q_T^e = \dfrac{(1-\varepsilon)\overline{R}}{\theta} < q$*); whereas the equilibrium interest rate during period T+1 is higher (*$i_{T+1}^e = \dfrac{\theta + \varepsilon}{1-\varepsilon} > i$*).*

Corollary 1. *The greater the shock is, the lower the* q_T^e *will be (*$\partial q_T^e / \partial \varepsilon < 0$*).*

Proof: See Mathematics Appendix A.3.

[10] For narrative convenience, when the real and nominal interest rates are equal under zero inflation, we simply use "the interest rate" without specifying nominal or real; and in notation we use the symbol of the nominal interest rate (i_t).

[11] The "wealth effect" on consumption of interest-rate-induced changes in q_T and H_{T+1} implicitly captures the interest-rate effect on consumption through the substitution effect. See Cai (2003) for discussion on monetary policy transmission from a wealth-effect perspective.

According to equation (3.20), a lower q_T^e will put downward pressure on q_{T-1}. A decrease in the interest rate i_T^e can help releasing the pressure and keep q_{T-1} at its equilibrium level (q_{T-1}^e). However, with a large demand shock ε_T, q_T^e could be reduced to such an extent that q_{T-1}^e is unsustainable even when the zero interest bound is binding. Then a liquidity trap will occur in period *T*-1 with insufficient demand and the interest rate bounded at zero.

Letting $\alpha \equiv \overline{R}\overline{K}\overline{Y}^{-1}$ denotes the capital share, we have the following proposition.

Proposition 2. *Given the demand shock* $\varepsilon_T = \varepsilon\overline{Y}$*, the period-T equilibrium interest rate is less than* $\theta - \dfrac{\varepsilon\alpha}{1+\beta}$*, i.e.,* $i_T^e < \theta - \dfrac{\varepsilon\alpha}{1+\beta}$.

Corollary 2. *If* $\varepsilon > \dfrac{\theta(1+\beta)}{\alpha}$*, then* $i_T^e < 0$.

Proof: See Mathematics Appendix A.4.

According to Corollary 2, with stable consumption price, a positive demand shock in period *T*, if sufficiently enough ($\varepsilon > \theta(1+\beta)\alpha^{-1}$), will result in a negative natural nominal interest rate and hence a liquidity trap in period *T*-1.

The liquidity trap in period *T*-1 implies that the nominal interest rate during period *T* is bound at zero (i.e., $i_T = 0$). Thus, according to equation (3.20), q_{T-1} is equal to $q_T^e + \overline{R}$, higher than q_T^e. However, q_{T-1} may not be high enough to sustain the equilibrium capital price at the end of period *T*-2 (q_{T-2}^e). Therefore, period *T*-2 could also be in the liquidity trap. So could be period *T*-3, and so on. In general, we have the following proposition.

Proposition 3. *Given the demand shock* $\varepsilon_T = \varepsilon \bar{Y}$ *, if* $\varepsilon > \frac{\theta(n+\beta)}{\alpha}$ *, then* $i^e_{T-n+1} < 0$.

Proof: See Mathematics Appendix A.5.

Summary

The expectation of a positive demand shock in the future could push the economy into a liquidity trap prior to the shock's arrival. The greater the shock is, the longer the liquidity trap will be.[12] The trigger of the liquidity trap is a low capital price in the future, which is needed to release the inflation pressure caused by the demand shock. However, the low future capital price could result in periods of capital price undervaluation (relative to its equilibrium level) and hence periods of insufficient demand.

A simple way to avoid or escape from liquidity traps

As capital price undervaluation is responsible for keeping the economy in liquidity traps, a natural way out is to support the capital price at its equilibrium level. With unlimited ability to supply money, the central bank is capable of supporting the nominal capital price at any level. Therefore, the central bank can make a credible commitment to defend a lower bound of nominal capital price, which will prevent (or fix) capital price undervaluation, and hence help avoiding (or escaping) liquidity traps. As the defended

[12] Since a well-defined ε cannot exceed unity, the "retroactive" process of the liquidity trap will eventually stop at some point *m* where q_{T-m} is high enough to sustain q^e_{T-m-1}. In another word, $\forall \varepsilon_T$, $\exists m < \infty : i^e_{T-m+1} < 0$ and $i^e_{T-m} > 0$. In short, the liquidity trap will have a beginning.

lower nominal capital price bound could be incompatible with the current level of consumption price, inflation will be a potential side effect of avoiding or fixing liquidity traps in this way.

Specifically, for the demand-shock-induced liquidity traps discussed in the above, we have the following proposition.

Proposition 4. *The demand-shock-induced liquidity traps can be prevented or fixed by a lower nominal capital price bound at* $Q^L = (1+\theta)^{-1}\overline{P}q$.

To prove this proposition, we start from examining the impacts of a period-*T* demand shock on the equilibrium values of real variables.

Given $\varepsilon_T = \varepsilon\overline{Y}$ and $\varepsilon_{t\neq T} = 0$, equation (3.14) implies that the equilibrium consumption will be $C_T^e = (1-\varepsilon)\overline{Y}$ and $C_{t\neq T}^e = \overline{Y}$.

Thus, according to equation (3.7), the equilibrium real interest rate will be $r_{t\in(-\infty,T-1]}^e = \theta$, $r_T^e = \theta - \varepsilon - \varepsilon\theta$, $r_{T+1}^e = (\theta+\varepsilon)(1-\varepsilon)^{-1}$, and $r_{t\in[T+2,\infty)}^e = \theta$.

Then, according to equation (3.20), the equilibrium real capital price will be $q \le q_{t\in(-\infty,T-2]}^e < q_{T-1}^e$, $q_{T-1}^e = (1+\theta-\varepsilon)(1+\theta)^{-1}(1-\varepsilon)^{-1}q$, $q_T^e = (1-\varepsilon)q < q$, and $q_{t\in[T+1,\infty)}^e = q$.[13]

Recall that liquidity traps happen when the natural nominal interest rate becomes negative (i.e. $i_t^e < 0$). Since all but period-*T* equilibrium real interest rate are positive,

---

[13] q_t^e stays at its steady-state level (q) initially. At the time when the demand shock ε_T is expected, q_t^e will discretely rise above q, and then keeps appreciating till the end of period *T*-1. q_t^e will fall below q at the end of period *T*, then rises back to q at the end of period *T*+1 and stay there afterwards.

given no deflation, $i_t^e < 0$ could only happen in period *T*. Thus, if a situation of $i_T^e < 0$ can be prevented (or fixed), liquidity traps will be avoided (or escaped).

Using equation (3.2), the period-*T* equilibrium nominal interest rate is given by

$$i_T^e = \left(1 + \frac{\overline{R}}{q_T^e}\right)\frac{Q_T^e}{Q_{T-1}^e} - 1, \tag{3.21}$$

where $Q_T^e = P_T^e q_T^e$ represents the equilibrium nominal capital price at the end of period *T*. Equation (3.21) implies that $\partial i_T^e / \partial Q_T^e > 0$. Thus, it is not difficult to verify that, given $P_{T-1} = \overline{P}$, that $Q_T^e \geq Q^L$ will be a sufficient condition for $i_T^e \geq 0$. Put plainly, with the period-*T* nominal capital price being lower bounded at Q^L, the equilibrium nominal interest rate will not fall below zero; therefore, liquidity traps will not happen.

In summary, we have the following proposition.

Proposition 5. *While pure zero-inflation-targeting monetary policy will allow liquidity traps to happen when the period-T demand shock* ε_T *is greater than* $\theta(1+\theta)^{-1}\overline{Y}$*, zero-inflation-targeting monetary policy with a prioritized commitment to defending the lower nominal capital price bound* Q^L *will prevent (or fix) the liquidity traps. The cost of the avoidance (or escape) is a permanent increase in price level from* $\overline{P}$ *(before period T) to* $\overline{P}' = (1+\theta-\varepsilon-\varepsilon\theta)^{-1}\overline{P}$ *from period T onwards.*

Proof: Since $r_T^e = \theta - \varepsilon - \varepsilon\theta$, if $\varepsilon_T > \theta(1+\theta)^{-1}\overline{Y}$, then $r_T^e < 0$; and hence a liquidity trap will occur. When $Q_T^e = \overline{P}' q_T^e$ (where $q_T^e = (1-\varepsilon)q$) is at its zero bound $Q^L = (1+\theta)^{-1}\overline{P}q$, it is not difficult to verify that $\overline{P}' = (1+\theta-\varepsilon-\varepsilon\theta)^{-1}\overline{P}$.

Discussion

The above analysis from a *Q*-perspective diagnoses liquidity traps as resulting from capital price undervaluation that could be caused by the expectation of low capital price in the future. Accordingly, we propose that the central bank can prevent or cure liquidity traps by appending to its inflation-targeting policy with a prioritized commitment to defending a lower bound of nominal capital price. In the following we compare this proposition to its existing counterparts in the literature.

To extract the economy out of liquidity traps, our proposition suggests increasing current capital price. This is similar to the suggestion of monetary transfer in that both of them serve the purpose of making consumers wealthier. Liquidity traps under the Q-perspective are also "poverty traps" where capital price undervaluation makes consumers less wealthy than they should have been could the nominal interest rate be negative. As opposed to monetary transfer that creates more money as concrete purchasing power, increases in capital price can create potential purchasing power through capital revaluation.

Our proposition involves monetary "intervention" to increase current capital price, which is in the same spirit as some unconventional monetary policy suggestions. The novelty of our proposition is to induce higher capital price through setting a lower bound for nominal capital price. If current capital price undervaluation is due to expected low capital price in the future, direct open-market purchases may force the central bank to acquire a large amount of capital before achieving the goal of increasing its price; whereas a credible promise to defend the lower bound could effectively save the central bank from directly involving in the capital market.[14] The zero-interest policy can help sustaining a certain level of capital price, which could nevertheless still fall short of the equilibrium level because of risk or other premiums. In contrast, by promising to defend a properly-chosen lower nominal capital price bound, the central bank can effectively raise capital price to its equilibrium level.

Expectation management is one of the key elements of our proposition. As opposed to the suggestion of managing inflation expectation, our suggestion is to manage the expectation on capital price. In the above model, if the central bank can generate the expectation of a permanent increase in the price level from $\overline{P}$ to $\overline{P}'$ at the end of period T, the result will be the same as our proposition. Yet, the difference is in what the central bank needs to do. According to our proposition, what the central bank needs to do is to promise to defend the lower bound Q^L; then it has no need to worry about the timing and magnitude of the shock ε_T. In contrast, inflation expectation management will require the central bank to inform consumers when the price will rise and by how much—note that $\overline{P}' = (1+\theta-\varepsilon-\varepsilon\theta)^{-1}\overline{P}$ implies that the magnitude of necessary inflation is positively related to the magnitude of the demand shock.

The central bank's credibility is as essential for our proposition to work as it is for the propositions of inflation expectation management. In light of the fact that inflation tends to be a necessary cost of avoiding or fixing liquidity traps (from an ex ante point of view), we have no concern over central bank's willingness to tolerate it. Moreover, the

[14] Open-market purchases may be initially necessary to make a convincing case for the promise. After the credibility of the lower bound is established, central bank's direct involvement in the capital market can be avoided.

central bank is not likely to break its promises because credibility is something too valuable to lose.

Our proposition is (in rationale) similar to the exchange-rate proposition (in general) in that both involve monetary influence on asset prices. While the exchange-rate proposition suggests increasing the price of foreign currency in terms of home currency so as to stimulate foreign demands, our proposition suggests increasing the price of capital in terms of home currency so as to stimulate domestic demands.

Our proposition is also similar to Svensson's plan in that both involve using asset market to induce a desirable outcome in goods market. Yet, while Svensson's plan will have negative impacts on aggregate demand if consumers do not cooperatively expect inflations, the increase in capital price in our proposition will be stimulating irrespective of consumers' inflation expectations. Even if consumers do not believe in the central bank's commitment to defending a lower nominal capital price bound, our proposition will be merely ineffective but not counterproductive.

Finally, our proposition is conceptually similar to the proposition of Gesell's tax. While the essence of taxing money is to make it as unattractive as other assets in terms of the real rate of return, the essence of defending a lower nominal capital price bound is to make other assets as attractive as money in terms of the nominal rate of return. These two different ways will lead to the same result that money will no longer be so attractive as to become a liquidity trap.

Conclusion

Examining liquidity traps in a model that explicitly considers capital price dynamics, we find that a large positive aggregate demand shock in the future can cause long-lasting liquidity traps prior to the arrival of the shock. We propose that the central bank can prevent or fix such liquidity traps by committing to defending a lower bound of nominal capital price. The rationale of our proposition is to use the lower bound to prevent the natural nominal interest rate from falling below zero. Thus our proposition is in principle applicable to liquidity traps caused by any reason.

Considering the fact that severe capital market meltdowns are the immediate causes for Japan being in a liquidity trap and the United States on the verge of one, our proposition appears to be a promising way to prevent or fix liquidity traps. Therefore, further research on its practical feasibility is warrant.

Mathematics Appendix

A.1 The Derivations of Equations (3.7)-(3.9)

Some first-order conditions of the utility maximization problem are $C_t = \lambda^{-1}(1+r_t)$, $C_{t+1}(1+\theta) = \lambda^{-1}(1+r_t)(1+r_{t+1})$, and $M_t\beta^{-1} = (\lambda i_t)^{-1}(1+r_t)P_t$; based on which equations (3.7) and (3.8) are not difficult to derive.

Using equations (3.5)-(3.8), we have the following derivations.

$$
\begin{aligned}
& A_t + \sum_{v=t}^{\infty}\left(\prod_{u=t}^{v}(1+r_u)^{-1}\right)[W_v - T_v] = \sum_{v=t}^{\infty}\left(\prod_{u=t}^{v}(1+r_u)^{-1}\right)\left[C_v + \frac{i_v M_v}{P_v}\right] \\
& = (1+\beta)\sum_{v=t}^{\infty}\left(\prod_{u=t}^{v}(1+r_u)^{-1}\right)C_v \\
& = (1+\beta)\frac{C_t}{1+r_t}\left(1 + \frac{1}{(1+\theta)} + \frac{1}{(1+\theta)^2} + \cdots\right) \\
& = \frac{(1+\beta)(1+\theta)}{\theta}\frac{C_t}{1+r_t}
\end{aligned}
,
$$

which can be rearranged into

$$
C_t = \frac{\theta}{(1+\theta)(1+\beta)}\left[(1+r_t)A_t + (1+r_t)H_t\right],
$$

which, together with equations (3.4) and (3.10), can be further transformed into

$$
\begin{aligned}
C_t &= \frac{\theta}{(1+\theta)(1+\beta)}\left[(1+r_t)A_t + (1+r_t)H_t\right] \\
&= \frac{\theta}{(1+\theta)(1+\beta)}\left[A_{t+1} + \frac{i_t M_t}{P_t} - W_t + C_t + T_t + W_t - T_t + H_{t+1}\right], \\
&= \frac{\theta}{(1+\theta)(1+\beta)}\left[q_t K_{t+1} + \frac{M_{t+1}}{P_t} + \frac{D_{t+1}}{P_t} + \frac{i_t M_t}{P_t} + C_t + H_{t+1}\right]
\end{aligned}
$$

which, together equation (3.8) and some rearrangement, will give equation (3.9).

A.2 The Derivations of the Steady State

According to equation (3.2), the steady-state nominal and real interest rates are equal and given by

$$i = r = \frac{\overline{R}}{q}, \tag{A.2.1}$$

which, together with equation (3.10), gives the steady-state human wealth

$$H = \frac{\overline{W} - \tau}{r} \tag{A.2.2}$$

According to equations (3.8), (3.13), and (3.14), the steady-state money supply, tax, and consumption are given by $M = P\beta C i^{-1}$, $\tau = iDP^{-1}$, and $C = \overline{Y}$ respectively, which, together with equations (3.9), (A.2.1), and (A.2.2), will allow us to solve the steady-state interest rate as $i = r = \theta$. Then, the steady-state values for other variables are straightforward to derive.

A.3 The Proof of Proposition 1

Since the demand shock $\varepsilon_T = \varepsilon\overline{Y} > 0$ is temporary, the economy will be in the steady state again from period T+1 on. Thus, according to equations (3.9), (3.19), and (3.20), we have

$$C_T = \frac{\theta}{1+\beta}\left(\frac{\overline{R}+q}{1+i_{T+1}}\overline{K} + \frac{\overline{D}}{\overline{P}} + \frac{\overline{W} - i_{T+1}\dfrac{D_{T+1}}{\overline{P}} + \dfrac{\overline{W}}{\theta} - \left(\dfrac{\overline{D}}{\overline{P}} - \dfrac{\beta\overline{Y}}{\theta}\right)}{1+i_{T+1}}\right),$$

which, together with equations (3.8), (3.11), and (3.14), gives

$$
\begin{aligned}
(1+i_{T+1}^e)(1-\varepsilon)\overline{Y} &= \frac{\theta}{1+\beta}\left((\overline{R}+q)\overline{K}+\frac{(1+i_{T+1}^e)\overline{D}}{\overline{P}}+\overline{W}-\frac{i_{T+1}^e D_{T+1}}{\overline{P}}+\frac{\overline{W}}{\theta}-\left(\frac{\overline{D}}{\overline{P}}-\frac{\beta\overline{Y}}{\theta}\right)\right) \\
&= \frac{\theta}{1+\beta}\left((\overline{R}+q)\overline{K}+\overline{W}+i_{T+1}^e\frac{M_{T+1}}{\overline{P}}+\frac{\overline{W}}{\theta}+\frac{\beta\overline{Y}}{\theta}\right) \\
&= \frac{\theta}{1+\beta}\left(\overline{R}\overline{K}+\frac{\overline{R}\overline{K}}{\theta}+\overline{W}+\beta\overline{Y}+\frac{\overline{W}}{\theta}+\frac{\beta\overline{Y}}{\theta}\right) \\
&= \frac{\theta}{1+\beta}\left(\overline{Y}+\beta\overline{Y}+\frac{\overline{Y}}{\theta}+\frac{\beta\overline{Y}}{\theta}\right)
\end{aligned},
$$

with which i_{T+1} can be solved as

$$
i_{T+1}^e = \frac{\theta+\varepsilon}{1-\varepsilon},
$$

which, together with equation (3.20), gives

$$
q_T^e = \frac{(1-\varepsilon)\overline{R}}{\theta}.
$$

A.4 The Proof of Proposition 2

According to equation (3.10),

$$
H_{T+1} = \frac{\overline{W}-\frac{i_{T+1}D_{T+1}}{\overline{P}}+H}{1+i_{T+1}} = \frac{\overline{W}-\frac{i_{T+1}\overline{D}}{\overline{P}}+\frac{i_{T+1}M_{T+1}}{\overline{P}}+H}{1+i_{t+1}},
$$

which, together with $i_{T+1} < i$ and $i_{T+1}M_{T+1} = iM$, implies

$$
H_{T+1} < \frac{\overline{W}-\frac{i\overline{D}}{\overline{P}}+\frac{iM}{\overline{P}}+H}{1+i} = H
$$

Therefore, according to equation (3.9),

$$
C_{T-1} < \frac{\theta}{1+\beta}\left(q_{T-1}\overline{K}+\frac{\overline{D}}{\overline{P}}+\frac{\overline{W}-i_T\frac{D_T}{\overline{P}}+H}{1+i_T}\right),
$$

which, together with equations (3.19) and (3.20), gives

$$
\begin{aligned}
(1+i_T^e)\overline{Y} &< \frac{\theta}{1+\beta}\left(\left[\overline{R}+\frac{(1-\varepsilon)\overline{R}}{\theta})\right]\overline{K}+(1+i_T^e)\frac{\overline{D}}{P}+\overline{W}-i_T^e\frac{D_T}{P}+\frac{\overline{W}}{\theta}-\left(\frac{\overline{D}}{P}-\frac{\beta\overline{Y}}{\theta}\right)\right) \\
&= \frac{\theta}{1+\beta}\left(\overline{Y}+\left[\frac{(1-\varepsilon)\overline{R}}{\theta})\right]\overline{K}+\beta\overline{Y}+\frac{\beta\overline{Y}}{\theta}+\frac{\overline{W}}{\theta}\right) \qquad , \\
&= \frac{\theta}{1+\beta}\left(\overline{Y}+\beta\overline{Y}+\frac{\overline{Y}}{\theta}+\frac{\beta\overline{Y}}{\theta}-\frac{\varepsilon}{\theta}\overline{R}\overline{K}\right)
\end{aligned}
$$

according to which the natural nominal interest rate

$$
i_T^e < \theta - \frac{\varepsilon\alpha}{1+\beta}.
$$

A.5 The Proof of Proposition 3

Suppose the economy is in liquidity traps from period T-n+1 to period T-1; then, $q_{T-n+1} = q_T + (n-1)\overline{R}$ and $i_{t\in[T-n+2,T]} = 0$. Therefore, similar to the last proof, we have

$$
C_{T-n} < \frac{\theta}{1+\beta}\left(q_{T-n}\overline{K}+\frac{\overline{D}}{\overline{P}}+\frac{\overline{W}-i_{T-n+1}\dfrac{D_{T-n+1}}{\overline{P}}+(n-1)\overline{W}+H}{1+i_{T-n+1}}\right),
$$

which implies that

$$
\begin{aligned}
(1+i_{T-n+1}^e)\overline{Y} &< \frac{\theta}{1+\beta}\left(\left[\overline{R}+q_{T-n+1}\right]\overline{K}+(1+i_{T-n+1}^e)\frac{\overline{D}}{\overline{P}}+\overline{W}-i_{T-n+1}^e\frac{D_{T-n+1}}{\overline{P}}+(n-1)\overline{W}+H\right) \\
&= \frac{\theta}{1+\beta}\left(n\overline{R}\overline{K}+i_{T-n+1}^e\frac{M_{T-n+1}}{\overline{P}}+n\overline{W}+\frac{\overline{Y}}{\theta}+\frac{\beta\overline{Y}}{\theta}-\frac{\varepsilon\overline{R}\overline{K}}{\theta}\right) \qquad , \\
&= \frac{\theta}{1+\beta}\left(n\overline{Y}+\beta\overline{Y}+\frac{\overline{Y}}{\theta}+\frac{\beta\overline{Y}}{\theta}-\frac{\varepsilon\overline{R}\overline{K}}{\theta}\right)
\end{aligned}
$$

which implies that $i_{T-n+1}^e < \left[\theta(n+\beta)-\varepsilon\alpha\right](1+\beta)^{-1}$. Therefore, if $\varepsilon > \dfrac{\theta(n+\beta)}{\alpha}$, then $i_{T-n+1}^e < 0$.

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