A More Realistic Endogenous Time Preference Model
and the Slump in Japan

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Abstract
This paper presents a more realistic endogenous time preference model, incorporating the property that impatience decreases as consumption increases. The model overcomes a serious drawback of the existing model, which needs the assumption of increasing impatience. The new model is applied to the Japanese economy, which has been mired in a persistent slump since the early 1990s, and the hypothesis that a time preference rate shift is the main cause of the slump is explored. The estimated time preference rate clearly shows that an upward time preference shift of about 2% occurred in Japan.

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INTRODUCTION

The view that time preference is an amalgamation of various factors, and is therefore naturally time-varying, has not been popular among economists since Samuelson (1937) introduced a simplified model of a constant time preference rate, which remains dominant even today. But the reason for its dominance is not theoretical plausibility but merely its simplicity and tractability. Hence, if its simplicity disguises the essential nature of economic phenomena, time-variability of time preference deserves consideration.

The concept of time-varying time preference has a long history, dating back to the era of Böhm-Bawerk (1889) and Fisher (1930). Among recent works, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant time preference rates by nature, and that economic and social factors affect the formation of time preference. This means that many shocks can affect and change time preference throughout life. This is not merely a theoretical possibility; it can actually be observed. For example, Parkin (1988) examined the business cycles in the U.S. explicitly considering time-variability of time preference rate and showed that the rate of time preference was as volatile as technology and leisure preferences in the U.S.

Incorporating endogeneity of time preference, the endogenous time preference model originated by Uzawa (1968) has been used in many analyses. However, this model has not necessarily been seen as a realistic expression of endogeneity of time preference, since it has a serious problem—namely, that the assumption that impatience increases as income or consumption increases is necessary. Without this assumption, the model is unstable. Many empirical researchers conclude that the rate of time preference negatively correlates with permanent income.\(^1\) Hence, the necessity of assuming increasing impatience seems fatal. Exceptionally, Epstein (1987) asserted the plausibility of increasing impatience offering some

\(^1\) See e.g. Lawrance (1991).
counter-arguments, however his view is a minority one, and many economists support arguments in favor of the decreasing time preference rate. Hence, although the Uzawa model attracted attention from some economists such as Epstein and Hynes (1983), Lucas and Stokey (1984), and Obstfeld (1990), analysis focusing on endogeneity of time preference has barely progressed. However, the skepticism engendered by the endogenous time preference model does not necessarily mean that the conjecture that time preference is influenced by future consumption is also false. Rather, this conjecture seems to be widely accepted. The problem is that a desirable endogenous time preference model, where the rate of time preference negatively correlates with income or consumption, has not been presented. Such a model would be more favorable and realistic and provide deeper insights into many economic phenomena.

The theoretical purpose of this paper is to pursue such a realistic endogenous time preference model. The problem of assuming increasing impatience arises in the specification that distant future consumption or utilities have little influence on the factor that forms the rate of time preference. However, this specification seems inadequate as a specification of “the size of utility stream” that, Fisher (1930) asserted, has a considerable influence on the formation of the rate of time preference. Instead, a simple measure—where the entire utilities, from the present to the distant future, are summed up with equal weight—could be a more realistic measure of “the size of utility stream,” and incorporating this measure will make models of endogenous time preference more appropriate. Given the definition of “the size of utility stream” as such, a new, realistic and stable endogenous time preference model with the feature of decreasing impatience can be deduced. Being able to assume decreasing impatience without making an economy unstable is a remarkable result. A stable endogenous time preference model that successfully incorporates the assumption of decreasing impatience may open the road to new approaches.

Subjective interest in this paper is the economic slump in Japan. The Japanese economy, the second largest in the world, has been mired in a persistent slump since the beginning of the
1990s, while other industrialized economies have enjoyed steady growth. This slump arrived on the heels of the high rate of growth that Japan enjoyed during the second half of the 1980s. As Figure 1 shows, the GDP of Japan has been nearly flat since the early 1990s. The average real GDP growth rate was a mere 1.3% annually in the 1990s, compared with 4.0% in the 1980s. The big question is what caused this protracted slump, which is peculiar to Japan among the industrialized economies. Many researchers have tried to answer that, but no consensus has been formed on the main causes behind the slump.

The new endogenous time preference model in this paper predicts that when uncertainty over the future economy increases, the rate of time preference shifts upwards and long-lasting negative impacts will hit the economy because the time preference shift moves the steady state to a lower level. Hence, a time preference shift is a possible cause of the slump in Japan.

Empirical results show that the time preference rate of the Japanese in the second half of the 1980s, when Japan enjoyed the benefits of the so-called bubble economy, was unquestionably lower than it has been since the beginning of the 1990s, and the uncertainty represented by the volatility of equity prices shifted upward exactly in the same period when the time preference shifted. The results strongly support the hypothesis that the time preference rate shifted upward in the late 1980s in Japan triggered by an increase of uncertainty and that it is the principal cause of the protracted slump in Japan.

Of course, this coincidence of movements does not prove the hypothesis that the slump in Japan was caused by an upward time preference shift. Many other possibilities that generate the coincidence of movements need to be examined to assert more definite conclusions. However, the episode of the Japanese economy is so well fitted to the predicted consequence of an upward time preference shift that its implications seem worth pursuing.

This paper is organized as follows. Section I presents a new kind of endogenous time preference model, one that overcomes the serious shortcoming in the existing endogenous time preference model, and then shows that shifts of uncertainty cause the rate of time preference to
shift. In Section II, the hypothesis that a time preference shift induced by an uncertainty shift could be the cause of Japan’s stagnation is presented, and the hypothesis is examined as an application of the new endogenous time preference model. Estimates of the time series of the time preference rate and uncertainty in Japan during the 1980s and the 1990s show their coincidental upward shifts in the late 1980s, and that conforms to the above hypothesis. In Section III, some concluding remarks are offered.

I. A MORE REALISTIC ENDOGENOUS TIME PREFERENCE MODEL

The well-known endogenous time preference model of Uzawa (1968) has attracted considerable attention among economists and has been used in many economic studies. The basic feature of the Uzawa model is:

\[
\theta(u(c)) > 0, \quad \theta' > 0, \quad u' > 0.
\]

The rate of time preference \( \theta \) is an increasing function of present utility \( u(c) \), which is an increasing function of present consumption \( c \). However, the Uzawa model has a serious drawback. The assumption \( \theta' > 0 \) is a necessary condition for stability but is quite controversial and difficult to accept a priori. This unnatural but indispensable assumption makes many economists critical of the endogenous time preference model, so there follows a re-examination of this shortcoming and a new model that overcomes it.

(a) Size effect on impatience

The problem of the assumption \( \theta' > 0 \) arises in the specification that distant future utilities has little influence on the factor that forms the rate of time preference. In this kind of specification, the rate of time preference must be revised every period in accordance with growth of consumption. However, there is no a priori reason why information on distant future
activities should be far less important than information on present and near future activities. How information on present and distant future activities is used for the formation of time preference must be explored in terms of people’s response to information regarding the motif that forms impatience. Concerning this response, Fisher (1930) asserted in Section 6, “The influence of mere size,” that “[I]n general, it may be said that, other things being equal, the smaller the income, the higher the preference for present over future income. It is true of course that a permanently small income implies a keen appreciation of wants as well as of immediate wants. … But it increases the want for immediate income even more than it increases the want for future income.” According to Fisher’s view, a force that influences time preference is a psychological response derived from the perception of the “size of entire income or utility stream.” Hence, it is necessary to probe how people perceive the “size of entire income or utility stream”—namely, how people judge which is bigger among utility streams that have different shapes and continue on infinitely.

Behind the formation of time preference there will be a unique psychological or emotional response to the size of entire utility stream, which is a different kind of response from the mere discounted sum of pleasures derived from consuming goods. Hence, an approach from psychological researches may shed light on the size effect. However, little effort has been directed towards probing the effect that the nature of the size of utility or income stream has on time preference, although a large number of various psychological experiments pertaining to anomalies of the expected utility model with a constant time preference rate have been made.2 Turning to researches in economics, analyses using the endogenous time preference model have so far merely introduced an a priori assumption of endogeneity of time preference without explaining its reasoning in detail.

Hence, Fisher’s insights are still very useful for an examination of the size effect. An important remark in Fisher (1930) quoted above is that the size of infinite utility stream is

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2 See e.g. Frederick, Loewenstein and O’Donoghue (2002).
perceived by anticipating whether the entire utility stream is “permanently” high or low. It may be interpreted that the difference in size among utility streams is perceived by the permanently continuing difference of utilities among different utility streams. If so, the distant future utilities should be taken into account equally with the present utility, otherwise it is impossible to distinguish whether the difference in utilities continues permanently.

Furthermore, it seems likely that anticipating the “permanently” higher utility may enhance the emotion of being guaranteed since guarantee means a “long persisting” secured situation and embodies “continuity,” and it will make people more patient—that is, a strengthened emotion of being guaranteed may generate a positive psychological response and thus decrease the rate of time preference. If a person expects that her future success is guaranteed by continuing to follow her current way of working, she will be motivated and discipline herself, and then be less impatient from the beginning.\(^3\) On the other hand, if a person expects that she will not attain her objectives, she will give up disciplining herself and become more impatient. Thus it seems likely that the size of entire utility stream that affects impatience is closely related with the emotion of being guaranteed. The emotion of being guaranteed is a completely different kind of psychological response from the discounted sum of utilities from consumption. If anything, it is an emotion derived from anticipating a “continuity” of pleasure or utility from the present to the distant future. The key factor of this emotion is “continuity.”

From this point of view, the specification that only current utility influences the formation of time preference, as is the case with the existing endogenous time preference model, may be inadequate as a specification of “the size of utility stream.” Rather, a simple measure of size—where entire utilities from the present to the distant future are summed up with equal weight—could be a measure that satisfies the abovementioned criterion, and incorporating this

\(^3\) Of course, the emotion of being guaranteed may, on the other hand, generate a problem of moral hazard. However, if this feature were strong, the rate of time preference would be observed to be positively correlated with permanent income.
measure will make models of endogenous time preference more appropriate.

(b) Model

Taking the above arguments into account, the “size” of infinite utility stream is defined here as follows.

**Definition:** The “size” of a utility stream \( W \) is

\[
W = \lim_{T \to \infty} \int_0^T w(t) E_t [u(c_t)] dt,
\]

where \( w(t) = \frac{1}{T} \) if \( 0 \leq t \leq T \) otherwise \( w(t) = 0 \), and \( u(c_t) \) [0, \( \infty \)) is utility for consumption \( c_t \) in period \( t \).

\( E_t \) denotes expectations conditional on information available in the period \( t \). \( w(t) \) are weights and have the same value in any period. Thus the weights for evaluation of future utilities are distributed evenly over time, as suggested above. Next, the effect of the size on the formation of the rate of time preference as well as utility and production functions is assumed as follows.

**Assumption:**

(A.1) The aggregated time preference rate function \( \theta(W) \) from \( \mathbb{R} \) to \( \mathbb{R}_+ \) is continuous and continuously differentiable.

(A.2) \( \frac{d\theta}{dW} < 0 \).

(A.3) The utility function \( u(c) \) of the representative consumer from \( \mathbb{R}_+ \) to \( \mathbb{R} \) is continuous for \( c \geq 0 \) and twice continuously differentiable for \( c > 0 \).

(A.4) \( \frac{du}{dc} > 0 \).
The production function $f(x, rs)$ of the representative firm from $\mathbb{R}_+ \times S$ to $\mathbb{R}_+$ is stochastic, continuous for $x \geq 0$, and continuously differentiable for $x > 0$ and $f(0, rs) = 0$, where $S = [\alpha, \beta]$ ($0 < \alpha < \beta < 0$). $x$ is a factor input and $rs$ is random shock in per capita (equal to per worker) terms.

The natural and acceptable feature $\theta' < 0 \ (\frac{d\theta}{dW} < 0)$ is assumed in (A.2). However, for this assumption to be justified, stability needs to be achieved. To see this, it first needs to be shown that the size defined above possesses the following two important features: first, the rate of time preference is determined by the utility at steady state; and second, the rate of time preference is constant unless an unanticipated permanent fundamental shock is given and thus the optimization problem needs to be solved again.

**Proposition 1:** If $\lim_{t \to \infty} E_0(u(c_t)) = E_0(u(c^*))$, where $c^*$ is the consumption at steady state, then

(i) $W = E_0(u(c^*))$,

(ii) The rate of time preference is constant unless a shock that changes the distribution of the steady state consumption and/or utility function is anticipated.

**Proof:**

(i) Since $\lim_{t \to \infty} E_0(u(c_t)) = E_0(u(c^*))$ and $w(t) = \frac{1}{T}$ if $0 \leq t \leq T$ otherwise $w(t) = 0$, then

$$\lim_{T \to \infty} \int_0^T w(t) \left\{ E_0[u(c^*)] - E_0[u(c_t)] \right\} dt = 0.$$  

Moreover,

$$\lim_{T \to \infty} \int_0^T w(t) \left\{ E_0[u(c^*)] - E_0[u(c_t)] \right\} dt = E_0[u(c^*)] - W = 0.$$

Hence, $W = E_0(u(c^*))$.

(ii) It is self-evident by (i). If a shock that changes the distribution of the steady state
consumption and/or utility function is not anticipated, \( W \), which does not depend on \( t \) but on \( c^* \) and \( u \), does not change in any period, thus the rate of time preference is constant.

Hence, the utility at steady state determines the rate of time preference and the rate of time preference continues to be constant until an unanticipated permanent fundamental shock hits the economy. Endogeneity of time preference only matters when such a shock occurs. This feature is intuitively acceptable since it is likely that people set their principles or parameters for behavior while considering the final consequences—that is, the steady state.

The result of constancy yields an important feature—namely, it allows the conventional stochastic general equilibrium model with a constant time preference rate to be applied without any modification, since each optimization in the general equilibrium model is based on conditional expectations and thus, once a constant time preference rate is determined, endogeneity no longer matters. The feature of constancy makes analysis based upon the model in this paper simple and tractable, while endogeneity of time preference is retained.

According to the Proposition 1, the model of endogenous time preference newly specified is expressed as

\[
0(W), \quad W = E_0(u(c^*)), \quad \frac{d\theta}{dW} < 0, \quad \frac{du}{dc} > 0.
\]

This model is deceptively similar to the original Uzawa endogenous time preference model, with the simple replacing of \( c \) by \( c^* \) and \( \theta' > 0 \) by \( \theta' < 0 \). However, the characteristics of the two models are completely different. The most important difference is that the new model (2) with the feature \( \theta' < 0 \) is stable. This is demonstrated as follows.

Here, the following conventional one-good Ramsey model is assumed. The expected utility,
\[ E \int_0^\infty (1 + \theta)^t u(c_t) \, dt \]

is maximized, subject to the constraint,

\[ f(x_t, rs) = c_t + \frac{dx_t}{dt}, \]

where \( x_t, rs, \) and \( c_t \) are in per capita (equal to per worker) terms and the growth rate of population is zero.

In this framework, if a constant time preference rate is given, then the marginal product of capital—that is, the real interest rate in a decentralized economy—converges to the given time preference rate as an economy approaches the steady state. Hence, when a time preference rate is given at a certain value, the corresponding expected steady state consumption is uniquely determined, where the real interest rate equals the given time preference rate. Given that other exogenous parameters are fixed, any predetermined rate of time preference has its unique value of the expected consumption as well as the expected utility at steady state. The expected utility at steady state thus can be expressed as a function of the rate of time preference because there is a one-to-one correspondence between the expected utility at steady state and the rate of time preference. This is a basic consequence of the conventional Ramsey model. Let \( C^* \) be a set of steady state consumptions, given a set of time preference rates \( \Theta \) and with other exogenous parameters fixed. The above function is defined as \( g(\theta) = E_\theta(u(c^*)) : \mathbb{R}^+ \rightarrow \mathbb{R} \) where \( c^* \in C^* \) and \( \theta \in \Theta \), and it is shown in Figure 2.

On the other hand, the rate of time preference \( \theta \) is a continuous function of the steady state consumption \( c^* \) as shown in the model (2): \( \theta(W) \) and \( W = E_0(u(c^*)) \). This is the basic feature of endogeneity of time preference; the expected future utilities have an influence on the formation
of the rate of time preference. Redefine the functions \( \theta(W) \) and \( W = E_0(u(c^*)) \) as a function: 
\[
h^R[E_0(u(c^*))] = \theta: \mathbb{R} \to \mathbb{R}_+. \]
Its reversed function is \( h(\theta) = E_0(u(c^*)): \mathbb{R}_+ \to \mathbb{R} \), and it is shown in Figure 2.

Hence, the equilibrium time preference rate in the model is determined by the point of intersection of two functions—\( g(\theta) \) and \( h(\theta) \), as shown in Figure 2—where both the basic consequence of the conventional Ramsey model and the basic feature of endogeneity of time preference are simultaneously held. The function \( h(\theta) = E_0(u(c^*)) = W \) is a decreasing function of \( \theta \) by the definition (A.2). The function \( g(\theta) = E_0(u(c^*)) = W \) is also a decreasing function of \( \theta \), since the higher rate of time preference induces the lower steady state consumption. Panel (b) and (c) of Figure 2 show two cases of the decreasing function \( g(\theta) \), while panel (a) of Figure 2 shows the ordinarily used permanently constant time preference rate.

The existence of this point of intersection is important for stability. An economy is stable if such a point of intersection exists and the rate of time preference is determined at that point, since, once the rate of time preference corresponding to the intersection, which is constant unless there are unanticipated permanent fundamental shocks, is determined, the model can be treated as a model with a constant time preference rate after that. As was mentioned above, endogeneity of time preference only matters when unanticipated permanent fundamental shocks occur. Conditions for the existence of a point of intersection are as follows.

**Lemma:** \( g(\theta) \) is continuous for \( \theta > 0 \).

**Proof:** As a result of maximization in the model, \( c^*(rs) = f(x^*, rs) \) and \( \theta = f'(x^*, rs) \), where \( x^* \) is the factor input at steady state and \( rs \) is the random shock introduced in the assumption (A.5). Since \( f(x^*, rs) \) and \( f'(x^*, rs) \) are continuous for \( x^* > 0 \) by the assumption (A.5), \( c^*(rs) \) is a continuous function of \( \theta: \mathbb{R}_+ \to \mathbb{R}_+ \) for \( \theta > 0 \). Here, since \( u \) is continuous by the assumption (A.3), thus \( E_0(u(c^*)) = g(\theta) \) is also continuous for \( \theta > 0 \).
**Proposition 2:** Let $D(\theta) = h(\theta) - g(\theta)$. If $\lim_{\theta \to \infty} D(\theta) < 0$ and $\lim_{\theta \to 0} D(\theta) > 0$, or if $\lim_{\theta \to \infty} D(\theta) > 0$ and $\lim_{\theta \to 0} D(\theta) < 0$, then an equilibrium time preference rate such as $D(\theta) = 0$ exists.

**Proof:** $h(\theta)$ is continuous by the assumption (A.1) and $g(\theta)$ is continuous for $\theta > 0$ by the Lemma, thus $D(\theta) = h(\theta) - g(\theta)$ is also continuous for $\theta > 0$. Hence, by the intermediate value theorem, there exists a certain $\theta$ that makes $D(\theta) = 0$. 

Hence the point of intersection of the two functions $g(\theta)$ and $h(\theta)$ can exist, and thus the assumption $\theta' < 0$ adopted in the model does not necessarily cause instability in the model. This is because the tiresome problem of too little influence of the distant future, which inevitably generates the unjustifiable feature $\theta' > 0$, no longer exists. Being able to assume $\theta' < 0$ without making an economy unstable is a remarkable result. Although endogeneity of time preference is intuitively acceptable, so far the necessity of the no-more-compelling assumption $\theta' > 0$ has obstructed further researches in view of endogenous time preference, and many economists have been profoundly skeptical of models having this feature. Hence the stable endogenous time preference model successfully incorporating the assumption $\theta' < 0$ in this paper may open the road to new approaches.

To sum up, as a reasonable result of a re-examination of Fisher’s insights, a model can be deduced that possesses intuitively acceptable features such that, (1) the usually constant rate of time preference shifts occasionally in response to unanticipated permanent fundamental shocks, and (2) higher expected future consumption decreases impatience.

(c) **Effect of uncertainty**

One important feature of the new model is that, in addition to shocks on technology or leisure preference, shocks on uncertainty make the rate of time preference shift. This is not a new finding. Fisher (1930) pointed out that uncertainty, or risk, must naturally have an influence on the rate of time preference and that higher uncertainty tends to raise the rate of time
preference. The influence of uncertainty can be understood as follows.

In general, the uncertainty about $c^*$ can be represented by stochastic dominance of the expected distribution of consumption at steady state in a second degree sense, or the Roschild-Stigliz sense. Given $F(c^*)$, a subjective cumulative distribution function of $c^*(0 \leq a < c^* < b)$, the size $W$ is

\[ W = E_0[u(c^*)] = \int_a^b u(c^*) dF(c^*) \]

Consider two steady state consumptions $c^*_1$ and $c^*_2$. When $u(c^*)$ is increasing and concave in $c^*$, then $E_0(u(c^*_2)) \leq E_0(u(c^*_1))$ if $F(c^*_1)$ second degree stochastically dominates $F(c^*_2)$ with strict inequality for a set of values of $c^*$ with positive probability. If $F(c^*_1)$ stochastically dominates $F(c^*_2)$ in the Roschild-Stigliz sense, then $E_0(u(c^*_2)) \leq E_0(u(c^*_1))$ and the mean of consumption is preserved as well. Thus, if the uncertainty about the steady state consumption increases from $F(c^*_1)$ to $F(c^*_2)$ in these senses, the size of entire utility stream $W = E_0(u(c^*))$ basically decreases.\(^4\)

The effects of the change in uncertainty regarding the steady state consumption can be understood heuristically by Figure 2. When a shock occurs to the subjective cumulative distribution, which makes the uncertainty increase for any $\theta$, all the future endogenous variables should be immediately recalculated. If the utility is increasing and concave, this increase of the uncertainty generally means a shift of the locus $g(\theta) = W$ in Figure 2 downward to the dashed line, since $W = E_0(u(c^*))$ becomes smaller for any $\theta$. In the case of the ordinarily used permanently constant time preference rate shown in panel (a), the increase of the uncertainty

\(^4\) The concept of uncertainty is closely related with the concepts of ambiguity and risk, and precisely speaking they have different meanings. However, here I give the term “uncertainty” the same meaning as that described above; it has roughly the same meaning as risk.
decreases $W$ but has no effect on $\theta$ and thus the rate of time preference does not change. However, if the rate of time preference is sensitive to $W$ as in panel (b), the equilibrium time preference rate increases. On the other hand, if the rate of time preference is much more sensitive to $W$, as in panel (c), the equilibrium time preference rate decreases in reverse. Thus the direction of the effect depends on the relative steepness of the slopes of $g(\theta) = W$ and $h(\theta) = W$.

Hence, in this model, an abrupt shift of time preference influenced by an abrupt change of anticipation of uncertainty is theoretically possible in the stochastic general equilibrium framework. As a result, uncertainty can affect steady states through the channel of shifts of time preference.

Which is more general, panel (b) or panel (c)? That is an empirical question. However, considering the usually accepted notion that panel (a) can be used as an approximation in many cases, the relatively steep slope of $h(\theta) = W$ locus as shown in panel (b) may be more generally observed. Hence, generally an increase in uncertainty may raise the rate of time preference.

On the whole, the model in this paper has characteristics regarding the effects of the size of and uncertainty about anticipated future consumption that are generally the same as those Fisher (1930) asserted; it therefore appears to reflect Fisher’s notion with greater fidelity. That is, the endogenous time preference model in this paper appears to be much more realistic than the existing one.

II. APPLYING THE NEW MODEL TO THE JAPANESE ECONOMY

The new endogenous time preference model in this paper predicts that when uncertainty about the future economy increases, the rate of time preference shifts upwards and then, as a consequence, long-lasting negative impacts will hit the economy because the shift of time preference rate moves the steady state to a lower level. The recent protracted slump of the Japanese economy may be a good example of an upward time preference shift. The Japanese
economy, the second largest in the world, has been mired in a persistent slump since the beginning of the 1990s, while other industrialized economies have enjoyed steady growth in the 1990s; and this decline came on the heels of Japan’s high rate of growth during the second half of the 1980s. At the same time, several surveys have showed that uncertainty and anxiety about the future economy have increased significantly in Japan since the early 1990s. 5 In general, uncertainty is thought to be irrelevant vis-a-vis output fluctuations, since the uncertainty equivalence seems to hold in general.6 However, as shown in Section I, the new endogenous time preference model presents a new mechanism showing the link between uncertainty and real economic activities.7 Hence if coincident increases of the rate of time preference and uncertainty are observed in Japan, it will provide evidence in favor of the hypothesis that the slump in Japan was generated by the mechanism of endogenous time preference explained in this paper.

(a) Existing explanations of the slump in Japan

Although many reasons have been presented to explain Japan’s persistent slump since the early 1990s following the big boom of the 1980s, none seems satisfactory. International shocks such as a sharp rise in oil prices are insufficient because only Japan among the industrialized

5 See e.g. The Bank of Japan, “Questionnaires on consumer sentiment,” Annual report.

6 Even models incorporating precautionary saving cannot reconcile this inference. For example, Gourinchas and Parker (2001) stated that “[t]he recent poor macroeconomic performance of Japan is often blamed on high perceived uncertainty about the future and the associated lower consumption demand. ... In a standard one-good model, precautionary saving need not deliver such an outcome, as higher uncertainty leads to either higher investment or a current-account surplus and, hence, no obvious need for an output decline. The empirical relevance of this explanation should be tested, and its applicability may well be broader than the Japanese slump.”

7 Romer (1990) contended that uncertainty aggravated the Great Depression—that is, the collapse of stock prices in October 1929 generated temporary uncertainty about future income, which led consumers to forgo purchases of durable goods.
economies experienced a severe slump during the 1990s, and no other Japan-specific huge external shocks have been detected. Most explanations attribute the slump to several different domestic factors. Shiller, Kon-Ya, and Tsutsui (1996) admitted that they could not find an unambiguous explanation of the Nikkei stock crash, although they suggested that it was related to changes in expectations. Krugman (1998), McKinnon and Ohno (2000), and Svensson (2001) asserted that Japan was mired in a “liquidity trap.” Bayoumi (1999) and Ramaswamy and Rendu (1999) used VARs to identify the driving force of the slump in the 1990s, and Bayoumi concluded that the principal cause was disruption in financial intermediaries, while Ramaswamy and Rendu concluded it was large negative shocks to investment. Motonishi and Yoshikawa (1999) pointed to weak investment due to low profitability and a credit crunch as the cause of stagnation in the 1990s. Meltzer (2000) suggested that the recession in the early 1990s was induced by a decline in money and the recession in recent years was induced mainly by a fall in real exports. There are various other explanations, depending on which factors are selected and stressed. However, there is no consensus on what caused persistence of the slump. The problem with many of these explanations is that they cannot identify the origin of the shock among deep parameters.

The steady state in Japan likely moved abruptly to a far lower level, which may make all the phenomena consistently explainable. However, what kind of shock could generate such a huge shift of the steady state? The primary suspect among deep parameters may be technology. However, few economists insist that a technology shock is the main source of Japan’s stagnation. Exceptionally, Hayashi and Prescott (2002) attributed the stagnation in Japan to low productivity growth. However, estimation of TFP needs careful scrutiny.\footnote{A problem in Hayashi and Prescott (2002) is that capital utilization is not considered in the estimation of TFP in Japan. The Japanese Ministry of Economy, Trade and Industry asserts that the productivity in Japan is estimated to have grown steadily in the period if the capital utilization rate is properly considered.} Furthermore, a clear theoretical reason why productivity growth fell abruptly and has continued low over 10 years is
not presented in Hayashi and Prescott (2002), and they admit that it remains a puzzle. It is hard to find specific obstacles that have abruptly prevented Japan from utilizing world technologies since the early 1990s, given the fact that Japan had achieved huge success through world technologies prior to 1990. A leisure preference shock is another candidate but this has not been reported, perhaps because, although leisure preference shocks imply that a recession is caused by a strengthened unwillingness of workers to work longer, it seems unlikely that such a situation occurred in Japan, where an unprecedented number of workers were forced to leave their jobs. Explanations based on indeterminacy or multiple-equilibria theories are also possible, though these have not been presented as yet, perhaps because it is difficult to apply those theories practically to the experience of the Japanese economy.

Contrary to the above explanations, stories based on a monetary policy shock such as Meltzer (2000) have drawn attention. Some believe that the dramatic tightening of monetary policy in 1989 to curb the continuing “unreasonable” rise of land prices caused the recession and that the monetary policy following that continued to be too tight. It can be argued whether the unprecedented monetary policy of the Bank of Japan of keeping the short-term nominal interest rate at nearly zero since the mid-90s is still “too tight.” More importantly, a specific monetary transmission mechanism capable of generating such a huge negative impact on real variables continuing over 10 years, despite a desperate monetary policy of near zero interest rates, has not been identified. Without an understanding of this mechanism, a monetary policy shock does not seem to fully explain Japan’s stagnation.

Japan’s stagnation since the early 1990s is evidently still a mystery and remains a challenge for economists. Solving this puzzle requires consideration of a few other deep parameters—such as the rate of time preference and uncertainty—that are not usually considered driving forces of economic fluctuations. The endogenous time preference model newly developed in this paper suggests the possibility that the time preference rate in Japan shifted upward. If such an upward shift can be detected, the situation in the recent Japanese economy may provide evidence in
favor of the mechanism of endogenous time preference explained in this paper.

(b) Characteristics of shifts of time preference

A shift of the rate of time preference seems to have the characteristics needed to explain the stagnant Japanese economy, as it changes the steady state. Even a small change of the rate of time preference will have significant and permanent effects on economic activities and has sufficient power to turn the tide. As shown in Figure 3, the steady state in the phase diagram of the dynamics of the conventional Ramsey model is the point of intersection of the vertical line

\[
\frac{dc_t}{dt} = 0 \quad \text{and the locus} \quad \frac{dk_t}{dt} = 0 ,
\]

where \(c_t\) and \(k_t\) are the per capita consumption and capital, respectively, in the period \(t\). If an economy is at the steady state and then the rate of time preference goes up, the vertical line \(\frac{dc_t}{dt} = 0\) moves to a lower labor-capital ratio, thus production and consumption eventually decrease to the new steady state, and the business is depressed, and vice versa.\(^9\) If there is an upward trend in output owing to positive technology shocks, what otherwise may be a severe output decrease when the rate of time preference rises, may instead show up as a lowered growth rate. This movement coincides with Japan’s abrupt plunging into recession at the beginning of the 1990s and the persistent slump since then, indicating that a shift in the time preference rate may better explain the recent unusual experience of the Japanese economy.

(c) Empirical results

In this subsection, the model presented above is applied to the Japanese economy and the possibility of the time preference shift in Japan is examined. First, the time-series of the time

\[^9\] Certainly, if an economy is not at steady state but on a transition path when time preference shifts, the change of economic course may be different. This is not the case only with time preference shifts but is the same in other shocks that change steady states.
preference rate of the Japanese during this period is estimated, and then, using the estimated series, a break point is checked for in the time preference rate. Second, the uncertainty is estimated and checked for its break point. Finally, the break points of the time preference, uncertainty, and output are examined.

1) Estimation of the time preference rate

As shown in subsection I(b), the framework of the endogenous time preference model in this paper is the conventional Ramsey model and the rate of time preference continues to be constant until an unanticipated permanent fundamental shock hits an economy. Thus, the time preference rate in Japan is estimated using the Euler equation:

\[
\frac{dc_i}{dt} = \sigma (c_i) \left[ r_i - E(p_i) - n_i - \theta_i \right]
\]

(4)

\[
\sigma (c_i) = \frac{u'(c_i)}{u''(c_i)c_i},
\]

(5)

where \( r_i \) is the long-term nominal interest rate, \( n_i \) is the population growth rate, and \( E(p_i) \) is the expected inflation rate. The time series of the time preference rate in Japan is calculated from the following equation, which is simply derived from equation (4), supposing that \( \sigma (c_i) \) is constant.

\[
\theta_i^* = \theta_i + e_i = r_i - E(p_i) - n_i - \frac{\left( \frac{dc_i}{dt} \right)}{c_i \sigma (c_i)},
\]

(6)

where \( \theta_i^* \) is the estimated time series of the time preference rate and \( e_i \) is random noise. The
estimated time series of the time preference rate $\theta^*$, contains the random noise $e_t$; thus it is a kind of “residuals,” as is usually the case with estimated total factor productivity.

The four-quarter inflation rate is the assumed proxy for the expected inflation rate. As for consumer intention regarding the real consumption change, the intention to change consumption at a certain time is assumed to be found in the realized consumption change from that time to a year later.\textsuperscript{10} Although many estimated values of $\sigma$ exist, the simplest value $\sigma = 1$ is used and thus utility is assumed to be logarithmic. The data used for the estimation are quarterly series of the per capita real consumption, GDP deflator, and 10-year government bond yield.\textsuperscript{11}

It is clear from the result shown in Figure 4 that the time preference rate in the second half of the 1980s, when Japan enjoyed the benefits of the so-called bubble-economy, was unquestionably lower than it has been since the early 1990s.\textsuperscript{12} This result strongly supports the hypothesis that the time preference rate shifted upward in the late 1980s. Large temporary fluctuations of the time preference rate are estimated for the periods of the early 1980s and around 1997. The former seems to be influenced largely by the second oil crisis and the following well-known monetary policy regime change, and the latter by the well-known overreaction to the consumption tax hike in 1997. It is likely that some assumptions of this

\textsuperscript{10} This assumption seems acceptable since changes of the consumer’s intention concerning consumption may need some lags to materialize and be accomplished in the following several quarters.

\textsuperscript{11} Data of the real consumption and the GDP deflator are those from the National Accounts released by the Japanese government. Data of population are those released by the Japanese government. Data of the 10-year government bond yield are those published by the Tokyo Stock Exchange. Data of the real consumption in 68SNA are used, since data of the 1980s in 93SNA were estimated by a simple method and are less accurate. GDP deflator data are adjusted corresponding to the effect of introducing the consumption tax in April 1989 and the rate raise in April 1997.

\textsuperscript{12} In some periods, the estimated time preference rates indicate negative, however, this partly reflects a positive trend of the consumption that lowers the scale of the estimated rates of time preference. During the period, the growth rate of the log linear trend of the real consumption is 2.5%; hence, the time preference rate would actually be 2.5% higher than the estimated value in every period and thus be positive throughout the period.
estimation method are temporarily unsatisfied in those periods—that is, the assumption of expectation formation of the inflation rate for the former period and the assumption of the optimal behavior of consumers for the latter period. However, during the period between the mid-1980s and mid-90s, there was fortunately no such big temporary shock that may have invalidated some of the assumptions, and the estimated values are stable during this period except for a presumed shift. Hence, I will concentrate on this period.

That the time preference rate in the 1990s was about 2% higher than that in the second half of the 1980s is apparent from looking at Figure 4. Next, sequential trend break tests were conducted to discover the exact date of the shift. In accordance with Perron (1997) and Ben-David and Pappel (1998), the following form is first regressed for unit root tests.

\[
y_t = \tau + \phi t + \chi DU_t + \xi DT_t + \nu D(T_b) + \eta y_{t-1} + \sum_{j=1}^{q} \kappa_j y_{t-j} + \epsilon_t,
\]

where \( y_t \) is the series concerned, \( \Delta y_t \) is the first difference, and the break dummy variables have the following values when time of the break is \( TB \): \( DU_t = 1 \) if \( t > TB \), 0 otherwise; \( DT_t = t - TB \) if \( t > TB \), 0 otherwise; \( D(T_b) = 1 \) if \( t = TB + 1 \), 0 otherwise. Equation (7) is estimated sequentially from \( TB = 2, \ldots, T-1 \), where \( T \) is the number of the observation. Time of the break is selected by choosing the value \( TB \) for which the absolute value of the t-statistic for \( \eta \) is maximized. Lag length is selected by the procedure suggested by Ng and Perron (1995) and Ben-David and Pappel (1998), such that starting with an upper bound of \( j_{max} \) on \( j \), if the last lag included in equation (7) is significant, then the choice of \( j \) is \( j_{max} \); and if the lag is not significant, then \( q \) is reduced by 1. \( j_{max} \) is initially set at 8 for quarter series and 24 for monthly series. The null hypothesis of a unit root is rejected if the t-statistic for \( \eta \) is greater than the critical values that Perron (1997) provided for this lag length selection method.

Next, trend break tests based on Vogelsang (1997) and Ben-David and Pappel (1998) are performed. The tests are carried out by estimating the following equation:
\[
y_t = \tau + \phi t + \chi DU_t + \zeta DT_t + \sum_{j=1}^{q} \kappa_j y_{t-j} + \epsilon_t
\]

and if a series contains a unit root, by the following equation, as suggested by Vogelsang (1997):

\[
\Delta y_t = \tau + \chi DU_t + \sum_{j=1}^{q} \kappa_j \Delta y_{t-j} + \epsilon_t
\]

These equations are estimated sequentially for each break date. The sup-Wald test, using sequentially estimated Wald-statistics for the null hypothesis \( \chi = \zeta = 0 \), provides the date of the shift.\(^{13}\) This test is applied to the probable trend break during the late 1980s.\(^{14}\)

The results are summarized in the Table. According to the sup-Wald test, the date of the shift was the fourth quarter of 1989.\(^{15}\) During the estimated period, the difference of the average time preference rate between before the trend break and after is 2.3%—that is, the time preference rate shifted up 2.3 percentage points.

**2) Estimation of the uncertainty**

To investigate the shift in the uncertainty, an appropriate proxy of the uncertainty over consumption at steady state is necessary. Stock prices are the obvious first choice. Since stock

\(^{13}\) As for the critical values reported in Vogelsang (1997), the values for only two cases of the trimming parameter \( \lambda^* = 0.01 \) and \( \lambda^* = 0.15 \) are listed. Hence the stricter case of \( \lambda^* = 0.01 \) was applied for the tests in this paper.

\(^{14}\) As for the test period on the probable break in the late 1980s, the start date was set as the first quarter of 1986 and the final date is the fourth quarter of 1995. The reasoning of this choice is that the data of the early 1980s seems highly contaminated by the second oil crisis and the following well-known monetary policy regime change, and the data around 1997 are contaminated by the overreaction to the consumption tax rise, as mentioned above.

\(^{15}\) The null hypothesis of a unit root is rejected at the 1% level. The tests are performed by equation (8) without trend.
prices are formed by an evaluation of the future value of firms that reflects aspects of the future economic environment, volatility of stock prices can be seen as a general measure of future uncertainty. Many empirical researchers used stock market data as an uncertainty proxy. For example, Romer (1990) used stock price volatility as a proxy of the uncertainty about the future income for empirical research on the Great Depression. Thus, this paper uses the volatility of TOPIX (a broad composite stock price index released by the Tokyo Stock Exchange) as the proxy of the uncertainty. Estimation is based on a simple diffusion type model:

\[
\frac{S_{t+1}}{S_t} = \zeta + \varepsilon, \quad \varepsilon \sim (0, \nu^2)
\]

where \(S_t\) is the stock price at the working day \(t\) and \(\zeta\) is a drift. The volatility \(\nu\) for each month is estimated on the assumption that \(\zeta\) and \(\nu\) are constant during a month.

Results are shown in Figure 5. Clearly the volatility of TOPIX since the early 1990s is higher than that before 1990. The average monthly standard deviation of TOPIX in the 1980s is 0.0063, and since the early 1990s it has doubled to 0.0118. This fact suggests plausibility of the hypothesis of this paper that the uncertainty increased in the 1990s and it raised the time preference rate.

To know the exact date of the shift, the same trend break test as in the time preference rate is performed. According to the result of the sup-Wald test, the date of the shift was December 1989, as shown in the Table. This is exactly the same period when the time preference shifted.

\[\text{Campbell and Lettau (1999) noted that multivariate volatility models are notoriously complicated and difficult to estimate, and the choice of a parametric model is less important for describing historical movements in volatility because all models tend to produce historical fitted volatilities that move closely together.}\]

\[\text{17 The null hypothesis of a unit root is rejected at the 1% level. The tests are performed by equation (8) without trend. It is estimated sequentially for each break month with 1% trimming.}\]
This coincidence further strengthens plausibility of the hypothesis that the time preference rate shifted upward triggered by an increase of the uncertainty. The date of the trend break, December 1989, is also coincident with the date of the peak of the unprecedentedly high stock prices that collapsed the following month.

3) Trend break test of the growth rate of the output

As shown in the introduction, the growth rate of the output in Japan since the early 1990s has been very different from the 1980s. To explore the relation among the output, time preference, and uncertainty, the same trend break test is applied to the output (logarithms of the per capita real GDP). According to the results shown in the Table, the date of the trend break in the growth rate of the output was during the third quarter of 1990. This is three quarters after the shifts of the time preference rate and uncertainty. This lag length seems consistent with many observations that leading indicators such as consumer sentiment indices or certain financial variables lead output by a few quarters on average.

(d) Discussion

The finding that the uncertainty and the time preference rate in Japan increased significantly a few quarters before the beginning of the protracted slump in Japan suggests plausibility of the hypothesis that the slump in Japan was generated by the mechanism expressed in the new endogenous time preference model described in this paper. Of course, those results do not mean that the hypothesis is proved. The case of Japan is only one episode, and hence to prove the hypothesis many other episodes need to be examined and compared with each other. Reliability of the estimated time series of the time preference rate is another problem. They are merely “residuals,” thus it will be arguable how to interpret the residuals and whether it really reflects the true time preference rate, in the same way as estimated TFP and technology shocks do.

18 The null hypothesis of a unit root cannot be rejected, thus the tests are performed by equation (9).
However, since the uncertainty and the time preference rate in Japan are estimated to have increased as predicted by the endogenous time preference model in this paper, this possibility should not be ignored and deserves more detailed examination.

Based on the endogenous time preference model in this paper, the following scenario may be possible: in the final days of the big boom of the 1980s, people began to anticipate more uncertainty in the future Japanese economy and as a result an approximately 2% upward shift of the time preference rate occurred during the fourth quarter of 1989. One quarter later, the equity market responded sensitively to the shift and dramatically changed course.\(^{19}\) Next, other economic activities started responding to this new regime with the new time preference, and the real GDP turned the tide during the third quarter of 1990. The uncertainty and the time preference rate remained high even after monetary policy was dramatically changed to usher in unprecedentedly low interest rates, and thus the sluggish economy persisted.

Output fluctuations sharing the same kinds of characteristics as seen in the recent Japanese economy may have occurred before in other countries, and those fluctuations may not be exceptional or irrational episodes, but rather caused by shifts of time preference. The intuitive understanding that an economy is basically governed by the public’s psychology or “animal spirits” has been mentioned often. This paper suggests the possibility that one factor that may influence economic activities is the psychological response that causes the rate of time preference to shift.\(^{20}\)

\(^{19}\) Concerning equity prices in the U.S., Heaton and Lucus (1999), Fama and French (2001), and others have suggested a possibility that the lowered required return—that is, the run-up in equity values—in the 1990s in the U.S. was a reflection of a downward shift of the time discount factor.

\(^{20}\) This argument suggests an answer to an unresolved question of why consumer sentiment indexes lead real economic activities. For example, when Iraq invaded Kuwait in 1990, the consumer sentiment indices in the U.S. dropped sharply and remained low during the Gulf War, and then the U.S. economy took a downturn and was in recession during the period. However, what caused this is still a puzzle. It may be possible to hypothesize that there was an upward shift of the time preference rate triggered by the intensified uncertainty about the future due to the war,
III. CONCLUDING REMARKS

Although the rate of time preference has been seen as an endogenous parameter since the era of Böhm-Bawerk (1889) and Fisher (1930), the existing endogenous time preference model since Uzawa (1968) has not been viewed as an appropriate expression of endogeniety of time preference because it has the serious shortcoming of the assumption $\theta' > 0$. This paper explored a new, intuitively acceptable and desired model of endogenous time preference that overcomes the necessity of the no-more-compelling assumption $\theta'' > 0$. The key element of the new model is the more properly defined “size of utility stream,” by which the rate of time preference is influenced. The “size” is defined in the new model as a simple measure where entire utilities from the present to the distant future are summed up with equal weight. This definition is shown to be realistic and reasonable, and most importantly it leads to a stable endogenous time preference model successfully incorporating the assumption $\theta'' < 0$. It should be stressed that, to the best of my knowledge, the endogenous time preference model presented in this paper is the first model that is both stable and compatible with the plausible and widely accepted conjecture that expected higher future consumption decreases the rate of time preference.

Furthermore, the new model predicts that an increase in uncertainty raises the rate of time preference in general and consequently inflicts negative and protracted impacts on real economic activities. It has often been asserted that in general the certainty equivalence holds and hence uncertainty has little influence on real economic activities, although intuitively future uncertainty appears to have significant influence on them. The model in this paper presents a new view of the causal link between uncertainty and real economic activities.

The subjective interest of this paper is the puzzle of why Japan has been mired in a protracted slump since the early 1990s. The ordinarily supposed driving forces of the business and the shift first came to the surface as the drop of the consumer sentiment indices and then resulted in the recession.
cycle, such as technology shocks, leisure preference shocks, monetary policy shocks, and indeterminacy are not all thought to have sufficient power to fully explain the persistent slump in Japan since the early 1990s. Hence, it is necessary to search for a suspect among the factors that are not usually considered to be driving forces of the business cycle. The rate of time preference and uncertainty are among those few factors.

The new endogenous time preference model in this paper was applied to this puzzle. The estimate shows that a time preference rate shift actually happened in Japan in the form of an upward shift of roughly 2%. The volatility of stock prices that is assumed to be a proxy of the uncertainty about the future economic situation was also estimated to have shifted substantially upward during the same period as the shift of the time preference rate, leading to a shift of the growth rate of the output three quarters later. These results support this paper’s hypothesis on the most probable causes of the economic fluctuation in Japan since the 1980s. The episode of the Japanese economy is so well fitted to the predicted consequence of an upward time preference shift driven by an increase of uncertainty that its implications seem worth pursuing.
References


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Table: Date of shifts

<table>
<thead>
<tr>
<th></th>
<th>mean-Wald</th>
<th>exp-Wald</th>
<th>sup-Wald</th>
<th>Shift date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time preference rate</td>
<td>4.64***</td>
<td>19.48***</td>
<td>45.91***</td>
<td>1989 IVQ</td>
</tr>
<tr>
<td>Volatility of TOPIX(1)</td>
<td>2.60*</td>
<td>2.35**</td>
<td>11.15**</td>
<td>1989 December</td>
</tr>
<tr>
<td>Volatility of TOPIX(2)</td>
<td>3.72**</td>
<td>5.51***</td>
<td>19.00***</td>
<td>1989 December</td>
</tr>
<tr>
<td>Real GDP per capita</td>
<td>3.64**</td>
<td>3.57**</td>
<td>12.43**</td>
<td>1990 IIIQ</td>
</tr>
</tbody>
</table>

Note: 1. ***, ** and * denote respectively statistical significance at 1 %, 5% and 10%. I used critical values in case of $\lambda^*=0.01$ presented in Vogelsang (1997) for more stricter alternative values like: for $P=1$, at 1 % (6.64), at 2.5 % (5.36) and at 5 % (4.42) in mean-Wald, at 1 % (5.24), at 2.5 % (4.18) and at 5 % (3.52) in exp-Wald and at 1 % (19.90), at 2.5 % (17.26) and at 5 % (15.44) in sup-Wald; for $P=0$ case, at 1 % (4.21), at 2.5 % (3.34) and at 5 % (2.66) in mean-Wald, at 1 % (3.63), at 2.5 % (2.80) and at 5 % (2.20) in exp-Wald and at 1 % (14.49), at 2.5 % (12.46) and at 5 % (10.85) in sup-Wald.

2. Estimated by OLS. The test for the time preference is performed during the periods from 1986 IQ to 1995 IVQ. The test for the volatility of TOPIX(1) and (2) are performed during the periods from January 1980 to December 2001 and from June 1949 to December 2001 respectively. The test of the real GDP is performed during the periods from 1980 IQ to 2001 IIIQ.
Figure 2: Effects of the size of utility stream on time preference rate

(a)

W

θ constant

W

g(θ) = W

(b)

W

h(θ) = W

(c)

W

g(θ) = W

h(θ) = W
Figure 3: Effects of a time preference shift

**Old Steady State**

\[ \frac{d\theta}{dt} = 0 \]

**New Steady State**

\[ \frac{d\theta}{dt} = 0 \]

**Old Saddle Path**

**New Saddle Path**

\[ \frac{dk_t}{dt} = 0 \]

**New Steady State**

\[ \frac{dc_t}{dt} = 0 \text{ for new } \theta \]

\[ \frac{dc_t}{dt} = 0 \text{ for old } \theta \]
Figure 4: The estimated time preference rate in Japan

Figure 5: The volatility of the TOPIX composite stock price index