A Note on the McGrattan and Prescott (2003) Adjustments and the Equity Premium Puzzle

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May 25, 2003

1 Average Equity Premium Puzzle

Mehra and Prescott (1985) consider the equilibrium asset pricing condition

$$0 = \delta E_t \left[\frac{u_c(c_{t+s})}{u_c(c_t)} (r^e_{t,t+s} - r^d_{t,t+s}) \right], \tag{1}$$

where $u(c_t)$ is the period utility function of a household defined over consumption c_t at time t, δ is the household's discount factor, and $r^e_{t,t+s}$ and $r^d_{t,t+s}$ are realized returns on an equity portfolio and on debt, respectively, between t and $t+s$. They use a short holding period for debt and proxy its return with the 90-day U.S. Treasury bill yield. For the equity portfolio, they use the S&P 500 stocks. The average difference in the annual returns over the period 1889-1978 turns out to be 6.2%. They show this mean excess return to be too large to be justified with the standard growth model and call it the 'equity premium puzzle'.

This finding by Mehra and Prescott (1985) has generated a large body of research aiming to explore various aspects of the puzzle both in domestic and also in international asset markets. As a result, there are now three related puzzles: the 'average equity premium puzzle', the 'low risk-free rate puzzle', and the 'excess return volatility puzzle'. To explain the puzzle, economists have taken several directions, among them, different preference orderings and different market structures.[1]

Recently, McGrattan and Prescott (2003) argue that some of the choices made by Mehra and Prescott (1985) need to be revised. In particular, they suggest that i) the T-Bill rate not be used as the 'risk-free' rate since most

[1] See Abel (1990), Campbell and Cochrane (1999), Epstein and Zin (1989), Constantinides and Duffie (1996), among others.

households hold long-term debt in their portfolios instead of short-term government paper, ii) the costs of holding diversified equity portfolios have to be accounted for, iii) taxes on dividends should be deducted from equity portfolio returns, and, iv) the equilibrium condition (1) did not hold during WWII and the Korean War as the government imposed restrictions on production, consumer credit, and the financial intermediaries.

Using the long-term high-grade bonds (and municipal bonds) as the 'risk-free' instrument, and making adjustments ii) and iii) to equity returns (and abstracting from the regulation-laced sub-period 1935-1960) makes the average excess real return less than one percent. In other words, the 'average equity premium puzzle' is no longer a puzzle.

This note uses the measurements proposed by McGrattan and Prescott (2003) and examines various aspects of the equity premium puzzle in detail, including the related low risk-free rate and excess volatility puzzles. First, Hansen and Singleton (1982) GMM tests are performed to test the overidentifying restrictions of the above Euler equation using standard and proposed data sets. Second, following Kocherlakota (1996), pricing errors based on the Euler equations for the two assets are studied. Finally, Shiller (1982) and Hansen and Jagannathan (1991) stochastic volatility bounds are calculated and compared with a consumption-based asset pricing model with power utility.

I find support for two of McGrattan and Prescott (2003) statements. Their adjustments solve the 'mean' equity premium puzzle and come close to solving the 'low risk-free rate puzzle'. However, the excess volatility of equity returns and low correlation of returns with aggregate consumption growth leave other, and possibly more interesting, features of the 'equity premium' puzzle in tact.

2 Alternative Measurements

The standard measurements used in the study of the equity premium puzzle consist of the real returns on the S&P 500 index and the 90-day Treasury Bills. The proposed measurements are an adjusted S&P 500 return series and a long-term debt series. The former takes the standard (real) S&P 500 series and subtracts from it the taxes on dividends and a measure of diversification costs.[2] The latter is high-grade municipal bond yields from 1890 to 1934, and high-grade Moody's Aaa Corporate bond yields from 1935 to 2002.

Table 1 presents the means and standard deviations of these series over the full sample of 1890-2002.

[2] The equity portfolio is a collection of NYSE stocks from 1889 to 1925, the S&P 90 index from 1926 to 1956, and the S&P 500 index from 1957 to 2002. CPI inflation is subtracted from nominal returns to obtain real returns. See McGrattan and Prescott (2003) for more details and sources.

Table 1: Means and Standard Errors of Measurements				
	S&P 500	Adj. S&P 500	T-Bill Rate	Long-term Debt Rate
mean	8.30%	5.08%	1.32%	2.90%
std	20.44%	20.37%	5.10%	3.25%

Note that the adjusted equity premium, the difference between adjusted S&P 500 return and the long-term debt return, is 2.18% (compared with the standard 6.98% over the period 1890 and 2002). However, this is still higher than the 'less than one percentage point' argued by McGrattan and Prescott (2003). There are two reasons for this variation. First, McGrattan and Prescott (2003) mention other items not accounted for in the computation of the after-tax return on an equity portfolio, such as capital income taxes, unmeasured diversification costs (brokerage fees), and possibly higher pre-1980 diversification costs, to suggest an average adjusted return 'below' 5%. Second, they ignore the subperiod 1935-1960 during which the government imposed constraints on households and financial intermediaries, holding bond yields unusually low, making the resulting long-term debt yield about 4%, hence an adjusted equity premium of about 0.67%.

In this note, I use both standard and adjusted measurements over the entire sample period of 1890-2002. Furthermore, I combine standard and alternative measurements in looking at various aspects of the equity premium puzzle to disentangle which adjustment makes the most difference.

3 GMM Tests

Assume that period utility function is given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of relative risk aversion. Hansen and Singleton (1983) tests the consumption-based asset pricing model by using the orthogonality conditions from the Euler equations:

$$\begin{aligned} 1 &= \delta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} r^e_{t,t+1} \right], \\ 1 &= \delta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} r^d_{t,t+1} \right]. \end{aligned}$$

The four instruments used are a constant, lagged values of consumption growth, and lagged returns on the equity portfolio and debt. Table 2 reports GMM estimates of the parameters and the overall test of the model.

Table 2: Parameter Estimates				
Data Used	δ	γ	J	$pr(\chi^2(6) > 0)$
S&P 500 and T-Bills	1.017	2.250*	29.03	0.0001
	(0.026)	(1.244)		
Adj. S&P 500 and T-Bills	1.034	2.902**	14.09	0.0287
	(0.027)	(1.286)		
S&P 500 and L-T Debt	1.058***	5.406***	21.17	0.0017
	(0.020)	(1.021)		
Adj. S&P 500 and L-T Debt	1.050***	4.527***	8.59	0.1978
	(0.019)	(0.821)		
Ignore 1935-1960	1.011	3.446***	4.50	0.6094
	(0.024)	(1.185)		

*** indicates significance at 1%, ** at 5%, and * at 10%; 2-stage GMM.

Using the standard measurements for asset returns, the parameter estimates are similar to those in the literature. The subjective discount factor is above unity, and the risk aversion coefficient is around 2. When the adjusted equity returns are used instead of the S&P 500 returns, the risk aversion coefficient gets near 3 and is now statistically significant at 5% level. The use of long-term debt yields (instead of T-Bill yields) and S&P 500 returns produces an estimate of δ that significantly exceeds unity and a risk aversion coefficient of about 5. The overidentifying restrictions in all three cases are easily rejected.

Adjusted equity returns and the long-term debt yields produce similar parameter estimates. However, in this case, the overidentifying restrictions of the Euler equations cannot be rejected. This conclusion is strengthened if I use the adjusted stock returns and long term debt yields over a restricted sample that leaves out the 1935-1960 period from the full sample 1890-2002 on the ground that government regulations kept the interest rates artificially low.

In the following sections, I will explore other aspects of the equity premium puzzle using the proposed measurements.

4 Pricing Errors

Kocherlakota (1996) studies the pricing errors computed from

$$e^{e-d}_{t+1} = \left[\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}(r^e_{t,t+1} - r^d_{t,t+1})\right], \tag{2}$$

$$e^{d}_{t+1} = \delta\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} r^d_{t,t+1}\right]. \tag{3}$$

If the stochastic discount factor from standard theory, the intertemporal marginal rate of substitution in consumption, were to price these two assets correctly, the pricing errors should be zero on average. Table 3 presents the findings.

Table 3: Equity Premium Puzzle: mean of $\{e_{t+1}^{e-d}\}$

	S&P500/T-B		Adj. S&P500/T-B		S&P 500/Debt		Adj. S&P500/Debt	
γ	$\overline{e}$	t-stat	$\overline{e}$	t-stat	$\overline{e}$	t-stat	$\overline{e}$	t-stat
0	0.0698	3.73	0.0376	2.03	0.0540	2.88	0.0218	1.17
1	0.0682	3.67	0.0365	1.98	0.0527	2.82	0.0211	1.14
2	0.0667	3.60	0.0355	1.93	0.0516	2.77	0.0204	1.10
3	0.0653	3.52	0.0346	1.88	0.0504	2.71	0.0197	1.07
4	0.0639	3.44	0.0336	1.83	0.0494	2.66	0.0191	1.04
5	0.0626	3.36	0.0327	1.77	0.0485	2.60	0.0186	1.00
6	0.0614	3.27	0.0319	1.71	0.0476	2.53	0.0181	0.97
7	0.0602	3.18	0.0310	1.65	0.0468	2.47	0.0176	0.93
8	0.0591	3.08	0.0302	1.58	0.0460	2.40	0.0172	0.90
9	0.0580	2.98	0.0294	1.52	0.0453	2.33	0.0167	0.87
10	0.0570	2.87	0.0287	1.45	0.0447	2.26	0.0164	0.83

Columns 3 and 4 of Table 3 confirm Kocherlakota's findings. Using the standard measurements yields statistically positive pricing errors from equation (2) for risk aversion coefficients up to 10 (and higher). However, using adjusted returns and the standard short-rate now yields pricing errors that are not different from zero on average for most 'reasonable' risk aversion coefficients. When the proposed long-term debt is used as the risk-free rate together with the standard equity returns, pricing errors are still large and statistically significant on average. Finally, when the long-term debt yields are combined with adjusted S&P 500 returns, then for all risk aversion coefficients in the table, pricing errors are statistically zero.[3] McGrattan and Prescott's suggestion appears to have resolved the 'mean' equity premium puzzle.

How about the 'low risk-free rate' puzzle? Table 4 shows average pricing errors computed from equation 3).

[3] Computing the t-tests using the shortened sample 1890-1934 combined with 1961-2002 produces essentially the same results.

Table 4: Low Risk-Free Rate Puzzle: mean of $\{e^d_{t+1}\}$						
	T-Bill Yield		L-T Debt Yield		L-T Debt Yield, no war	
γ	$\overline{e}$	t-stat	$\overline{e}$	t-stat	$\overline{e}$	t-stat
0	0.0031	0.65	0.0187	6.18	0.0276	12.75
1	−0.0134	−2.28	0.0019	0.42	0.0115	2.61
2	−0.0287	−3.53	−0.0137	−1.96	−0.0031	−0.39
3	−0.0426	−3.96	−0.0280	−2.86	−0.0163	−1.41
4	−0.0554	−4.04	−0.0410	−3.24	−0.0281	−1.84
5	−0.0669	−4.03	−0.0529	−3.38	−0.0386	−2.03
6	−0.0772	−3.92	−0.0636	−3.40	−0.0477	−2.08
7	−0.0864	−3.77	−0.0731	−3.35	−0.0555	−2.06
8	−0.0943	−3.60	0.0814	−3.25	−0.0619	−2.00
9	−0.1011	−3.40	−0.0886	−3.11	−0.0671	−1.90
10	−0.1067	−3.20	−0.0946	−2.96	−0.0709	−1.78

Using the T-Bill rate produces results that are similar to Kocherlakota's findings. The minor differences are due to the fact that I am using a longer sample. How much does the use of the long-term debt yield help? The average pricing error is statistically zero only for the log utility case, but otherwise the 'low' risk-free rate puzzle survives. If the sub-period 1935-1960, when the government restrictions were in place, is ignored, then values of γ between 2 and 4 seem to deliver zero mean pricing errors using equation (3) as a reasonable pricing equation.

Overall, the new measurements appear to solve the 'mean' equity premium puzzle, but make the risk-free rate consistent with standard theory only for a few values of the risk aversion coefficient.

McGrattan and Prescott (2003) argue that the excess volatility of the equity returns is still a puzzle. In the next section, I replicate some of the analyses of Campbell (2002) in combining the volatility of the equity returns (and their correlation with consumption growth) in addressing the success of standard theory in rationalizing the asset returns.

5 Volatility Bounds

It seems that the mean pricing error over the sample agrees with McGrattan and Prescott's interpretation. Would a similar conclusion go through after taking into account the volatility of the adjusted equity premium and the correlation of consumption growth and the adjusted excess returns? To examine the implications of the equilibrium asset pricing equation described in the previous sections, I will now follow Campbell (2002) who assumes that the stochastic discount factor $M_{t+1} = \delta(C_{t+1}/C_t)^{-\gamma}$ is conditionally lognormal, and derives the following equation to illustrate the equity premium puzzle

$$E_t[r^e_{t+1} - r^d_{t+1}] + \frac{\sigma^2_{er_m}}{2} = \gamma\sigma_{er_m,\Delta c}, \tag{4}$$

where r^e_{t+1} and r^d_{t+1} are log (gross) real returns on equities and the risk-free instrument, $\sigma^2_{er_m}$ is the variance of the excess return on equities, $\sigma_{er_m,\Delta c}$ is the covariance of excess returns with aggregate consumption growth.[4] Implications of this condition are studied in Table 5.

Table 5: The Equity Premium Puzzle

	$\overline{aer_e}$	σ_{er_m}	$\sigma(m)$	$\sigma_{\Delta c}$	$corr_{er_m,\Delta c}$	$cov_{er_m,\Delta c}$	$RRA(1)$	$RRA(2)$
S&P500/T-B	6.77	19.48	34.73	3.21	0.0917	5.74	117.79	10.80
Adj. S&P500/T-B	3.73	18.82	18.66	3.21	0.0910	5.85	63.75	5.80
S&P500/Debt	5.18	19.65	26.37	3.21	0.0906	5.72	90.56	8.20
Adj. S&P500/Debt	2.15	20.19	10.66	3.21	0.0898	5.83	36.94	3.32
Ignore 1935-1960	0.67	19.86	3.39	3.46	0.1148	7.89	8.53	0.98

The first two column of Table 5 give the sample averages and standard errors of excess returns (equity premiums) using four alternative measurements. The third column is the ratio (times 100) of the first column to the second column, the Sharpe ratio, or the volatility bound for the stochastic discount factor. Columns four, five, and six report the standard error of consumption growth, and the correlation and covariance of consumption growth with excess returns. The last two columns are the implied relative risk aversion coefficients computed using equation (4). Column seven uses the calculated correlation coefficient whereas column eight assumes a correlation coefficient of unity. According to the first row (using standard measurements), the equity premium puzzle exists because of both smoothness of consumption growth and low correlation between excess returns and consumption growth. However, using either the long-term debt yield or adjusted returns to calculate the excess returns suggests that the low correlation is primarily responsible for the difficulty of standard theory to account for the equity premium. With the proposed measurements, a risk aversion coefficient of about 3 would suffice to satisfy the 'new' equity premium, if the consumption growth and excess returns were perfectly correlated. Given the empirically low correlation, however, even the new measurements fail to revive standard theory. Hence, this particular implication of the equity premium puzzle survives.

The last row uses the adjusted measurements and ignores the period 1935-1960 which McGrattan and Prescott (2003) argue as a period with government regulation on financial life that has kept the interest rates artificially low. This would give the McGrattan and Prescott (2003) the best chance at addressing the excess volatility puzzle. There are two outcomes that go in the right direction to help explain the excess volatility puzzle. First, the mean excess return decreases to 0.67, and second, the correlation of excess returns with consumption growth increases to 0.1148. As a result, a risk aversion coefficient of 8.53 is now consistent with the asset pricing implication given in equation (4). However, it is not clear if this value is a plausible coefficient from the standpoint of standard

[4] See Campbell (2002) for details. Tables 5 and 6 replicate the analyses in his Tables 4 and 5.

theory. Assuming perfect correlation between consumption growth and excess return delivers a more plausible coefficient of relative risk aversion, 0.98, as the last column indicates.

To study the low risk-free rate puzzle, Campbell (2002) uses

$$Er^d_{t+1} = -\log\delta + \gamma g - \frac{\gamma^2\sigma^2_{\Delta c}}{2}, \tag{5}$$

where g is the mean growth rate of consumption.

Table 6: The Riskfree Rate Puzzle

	$\overline{r_f}$	$\overline{\Delta c}$	$\sigma(\Delta c)$	$RRA(1)$	$TPR(1)$	$RRA(2)$	$TPR(2)$
S&P500/T-B	1.20	1.79	3.21	117.79	15,770.99	10.80	−11.46
Adj. S&P500/T-B	1.20	1.79	3.21	63.75	163.24	5.80	−7.21
S&P500/Debt	2.85	1.79	3.21	90.56	1302.13	8.20	−8.09
Adj. S&P500/Debt	2.85	1.79	3.21	36.94	7.28	3.32	−2.54
Ignore 1935-1960	3.77	1.71	3.46	8.53	−6.29	0.98	2.11

The first three columns in Table 6 show the average risk-free rate, mean consumption growth rate, and the standard deviation of consumption growth. Using these and the implied risk aversion coefficients from Table 5 in equation (5) produces the implied time preference rates in the two columns labeled $TPR(1)$ and $TPR(2)$. Risk aversion coefficients in $RRA(1)$ yield positive but implausible time preference rates, except for the proposed measurements with a 7.28% rate. Risk aversion coefficients in the $RRA(2)$ column all produce negative rates of time preferences. The last row yields a slightly negative rate of time preference, −2.54%.

When I ignore the regulation period 1935-1960, the time discount rates consistent with the asset pricing condition (5) become −6.29% and 2.11%, respectively. It appears that the adjusted measurements do provide a solution to the risk-free rate puzzle, to the extent that these time preference are considered plausible.

6 Hansen and Jagannathan Volatility Bounds

Lastly, I will describe the Hansen-Jagannathan volatility bounds computed from the adjusted measurements. Figure 1 presents a scatter plot of (gross) annual asset returns, both standard (T-Bills and S&P 500) and adjusted (long-term debt and adjusted S&P 500) over the sample 1890-2002. Note how the overall central tendency has shifted to the right and slightly down.

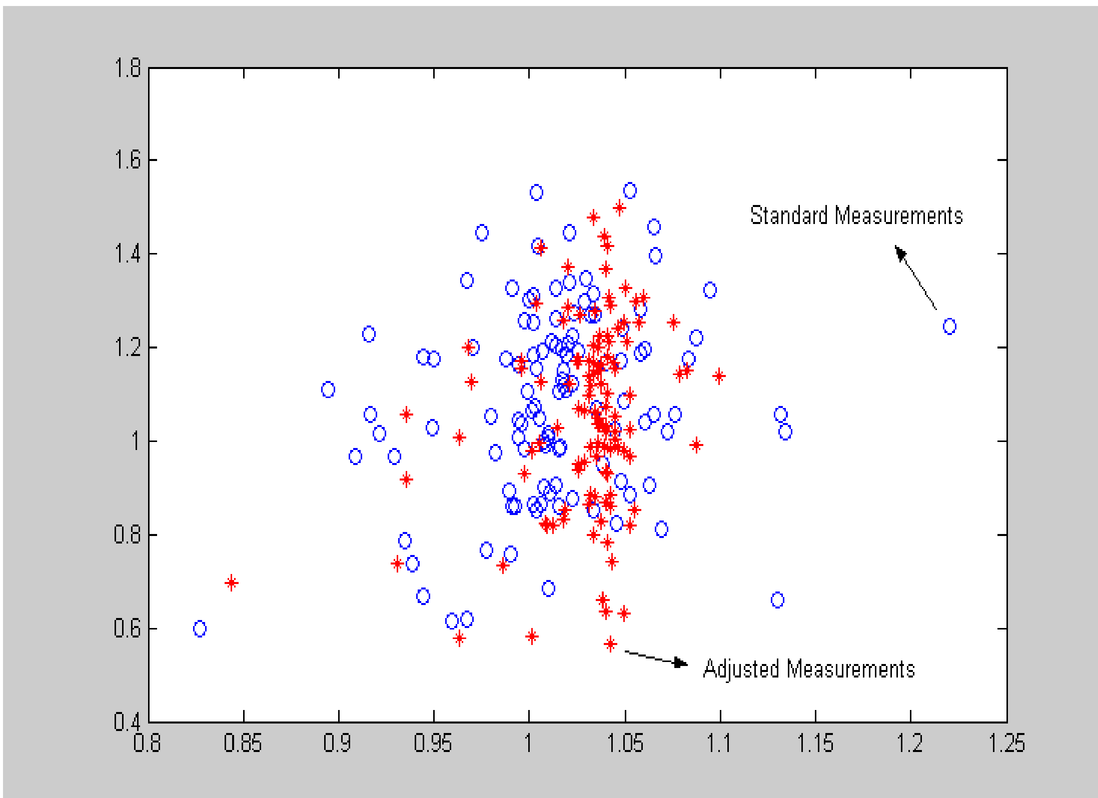
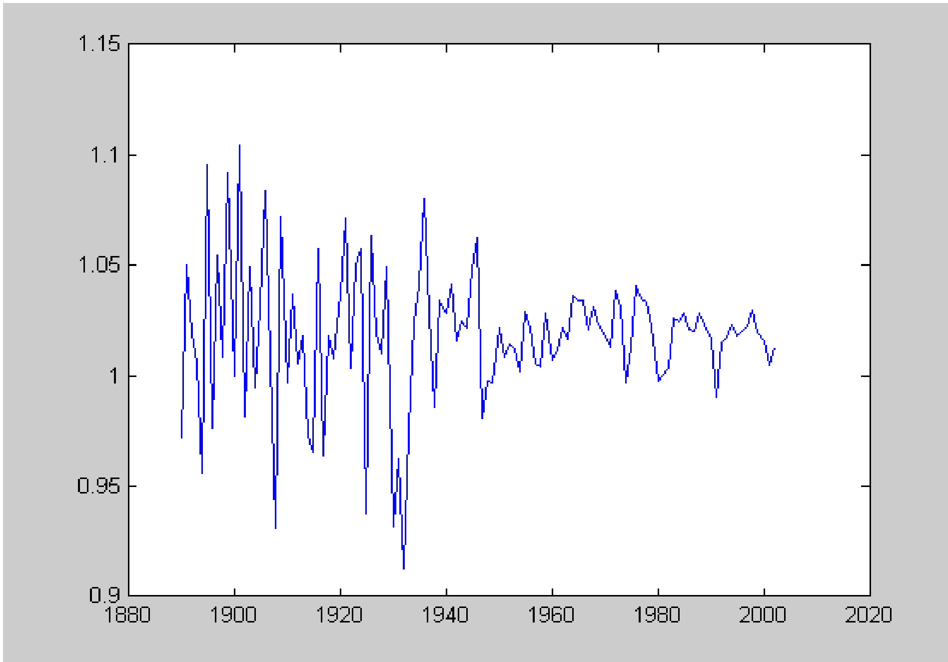


Figure 2 plots the (gross) rate of growth of consumption of nondurables and services.



These data are used to generate the Hansen-Jagannathan volatility bounds in Figure 3 below. Following Ljungqvist and Sargent (2000), the straight line bounds are produced using the restriction on excess returns

$$[var(x'b)]^{0.5} \leq \sigma(m),$$

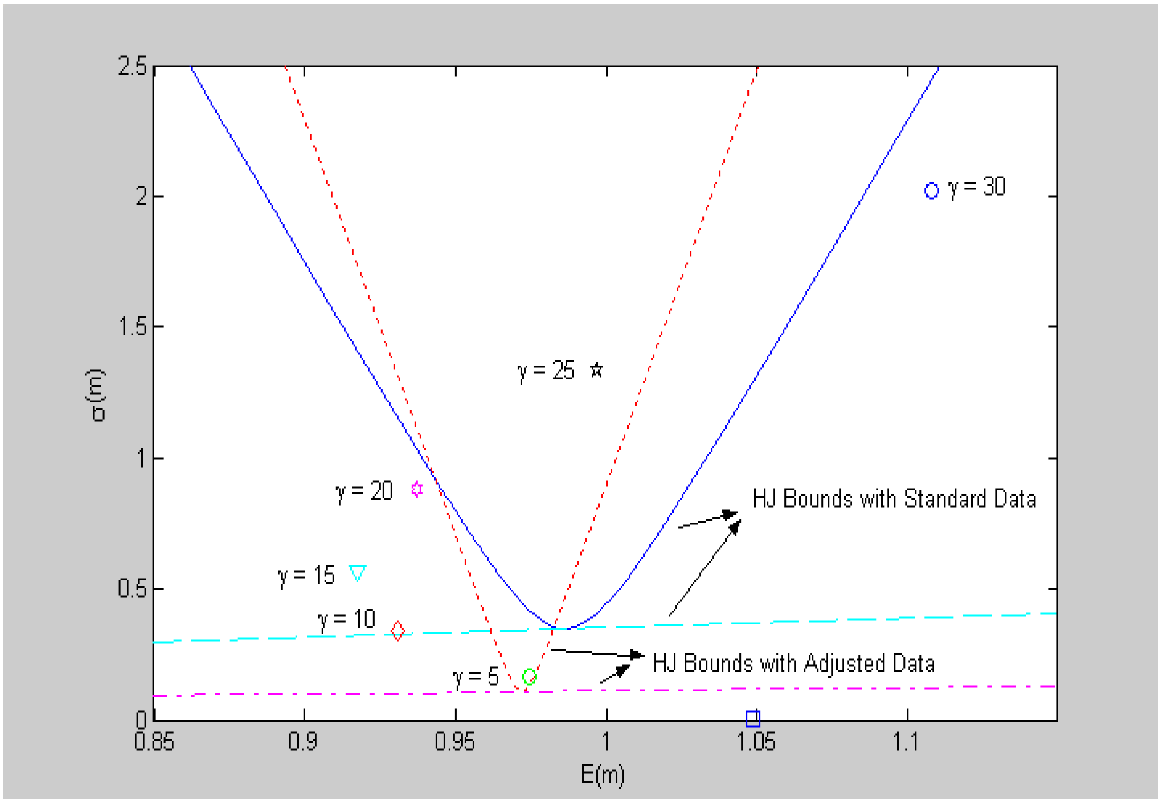
where

$$b = [cov(x,x)]^{-1}[q - E(m)E(x)],$$

x is the excess return, $q = E(mx)$ and m is a stochastic discount factor whose mean can be approximated by $1/E(r_t^d)$. The parabolas are computed using

$$\begin{aligned} q &= \mathbf{1}, \\ b &= [cov(x,x)]^{-1}[\mathbf{1} - E(m)E(x)] \\ \sqrt{b' cov(x,x) b} &\leq \sigma(m), \end{aligned}$$

where x is a 2 by 1 vector of returns on equity and debt.



With standard measurements, it takes an implausibly high risk aversion coefficient like 25 for standard theory to satisfy the volatility bounds. After the McGrattan and Prescott adjustments, a risk aversion coefficient of 5 is sufficient to get within the bounds.

7 Concluding Remarks

McGrattan and Prescott (2003) argue that using an adjusted set of measurements eliminates the average equity premium puzzle. Most of the implications of standard theory considered here agree. The low risk-free rate puzzle, and the excess volatility puzzle remain. Standard theory cannot account for the excess volatility of equity returns relative to that of consumption growth and the low correlation of excess returns with aggregate consumption growth.

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