

Endogenous Time-Dependent Rules and Inflation Inertia*

Marco Bonomo

Carlos Carvalho

Graduate School of Economics

Department of Economics

Getulio Vargas Foundation

Princeton University

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Abstract

In this paper we endogenize fixed price time-dependent rules to examine the output effects of monetary disinflation. We derive the optimal rules in and out of inflationary steady states, and develop a methodology to aggregate individual pricing rules which vary through time. Because of strategic complementarities we have to solve both problems simultaneously. This allows us to reassess the output costs of monetary disinflations, including aspects such as the roles of the initial level of inflation, and of the degree of strategic complementarity in price. Finally, we relax the strict assumption of pure time-dependent rules by allowing price setters to reevaluate their rules at the time disinflation is announced.

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1 Introduction

It is largely believed that nominal rigidities have important consequences for the effect of monetary policy. Among several alternatives, the primary dynamic specification of nominal rigidity used to analyze monetary disinflations is a fixed price time-dependent rule, due to Taylor (1979, 1980). In this model each price setter chooses the price that will be fixed during a predetermined period of time¹. Since this rule is usually postulated rather than derived², the time period between adjustments is exogenous. This way of proceeding is clearly inadequate when there are changes in the environment, as is the case when policy rules are changed. When monetary authorities launch a disinflationary program they usually claim that the monetary rule will be changed.

In order to analyze the effect on output of a disinflationary monetary policy in a proper setting, it is necessary to endogenize the fixed price time-dependent rules followed by price setters and aggregate them. This endeavour is straightforward when it is assumed that the economy is in an inflationary steady state. However, when analyzing the cost of disinflation, one is interested in the output effects during the transition between steady states. This requires solving less trivial optimization problems and developing a more general aggregation methodology. Furthermore, since each individual price depends on the aggregate price, both optimization and aggregation problems have to be solved simultaneously.

The derivation of endogenous fixed price time-dependent rules requires understanding the hypotheses that support their optimality. Either the costs of changing prices or of gathering information taken individually would not be enough. The former would generate a rule with fixed prices but which is state-dependent (Sheshinski and Weiss 1977, 1983) while the latter would generate a time-dependent rule with a preset price path rather than a fixed price (Caballero 1989). If we assume the two types of costs are present, then the optimal rule is both time- and state-dependent (Bonomo and Garcia 2001). In order to justify the fixed price time-dependent rule it is necessary to assume that those two kinds of costs are borne together. For example, one cannot choose to incur the cost of information and after the

¹Calvo (1983) introduced a variant of this rule in which adjustment time is stochastic, with a constant hazard rate. This version is widely used nowadays because it is analytically more convenient.

²One exception is Ball, Mankiw and Romer (1988).

optimal price is known decide whether to incur the adjustment cost and change the price. The hypothesis here is that once the single type of cost is incurred, one can get informed and change the price without any extra cost. The assumption is appealing because it rationalizes the fixed price time-dependent rule.

The endogeneity of time-dependent rules has important aggregate effects. Disinflation causes a longer recession with endogenous than with exogenous rules. When agents set new prices during a disinflation, they do it for longer periods of time than before because the loss involved in keeping the price fixed for some period of time will be smaller. The longer periods between adjustments increase the length of the recession, since it takes more time to eliminate the hangover effect of past fixed prices.

Disinflation also tends to cause a deeper recession when evaluated in an endogenous rules setting. This happens as long as money growth is not cut to zero. The reason is that agents with longer horizons set higher prices when faced with lower but still positive inflation.³ Thus, when the endogeneity of rules is taken into consideration, it is not as easy to disinflate as in Ball (1994), who used an exogenous rules setting.

The issue of whether it is easier to disinflate when the initial inflation is high than when it is low becomes more complex, when examined with endogenous rules. If on the one hand contract lengths are shorter when inflation is high (as mentioned by Blanchard, 1997), on the other hand the hangover effect is stronger.⁴ Therefore, the effect of a given disinflation policy when the initial inflation is higher is a more intense but shorter recession.

Endogenous rules have been used recently in order to evaluate monetary policy effects in the context of state-dependent pricing. Caplin and Leahy (1997) derive and aggregate optimal state-dependent pricing rules to investigate the dynamics of output when nominal aggregate demand follows a driftless process. Dotsey et al. (1999) embed endogenous state-dependent rules in a general equilibrium setting to examine the effect of a monetary shock. The issue of disinflation costs with endogenous state-dependent rules is analyzed by Almeida and Bonomo (2002). The results are qualitatively different from those obtained in this work,

³When money growth is cut to zero, endogeneity tends to attenuate the recessive effect. The difference is that in this case agents with longer horizons will face a period with more stable prices at the end of their contracts. Because of discounting, those agents will set prices closer to the optimal (smaller) level.

⁴Price-setters which adjusted a little bit before the announcement set higher relative prices, anticipating that they would be eroded by a higher inflation rate.

illustrating the fact that the type of nominal rigidity is an important modeling choice in macroeconomics. For example, while endogeneity of time-dependent rules increases inflation inertia, endogeneity of state-dependent rules contributes to mitigate it.

Pure state-dependent rules require that price-setters continuously observe all relevant information about state variables, and evaluate the convenience of adjustment (see Bonomo and Garcia, 2001, and Woodford, 2003). This is not an innocuous assumption. In fact, information collection and decision-making costs are often mentioned as more important than adjustment costs (Zbaracki et al., 2000). Thus, it is not surprising that time-dependent rules are considered more realistic. According to Blinder et al. (1998), time-dependent rules are twice as common as state-dependent rules.⁵

On the other hand, price-setters using time-dependent rules ignore important and widely known changes in the environment until their next preset adjustment time. Since this kind of information usually becomes available at no cost and could have an important impact on optimal decisions, it is not reasonable to assume that decision makers will ignore it. This motivated us to relax strict time-dependency by allowing price-setters to re-evaluate their pricing rules at the time disinflation is announced. This is made in order to take into account the new macroeconomic policy, which we assume is a free and widely available information.

The impact of re-evaluation becomes increasingly important for higher initial inflation rates. In comparison with the case of strict time-dependent rules, the model with re-evaluation generates a more abrupt, less deep and longer recession. The recession is more abrupt because reevaluation triggers immediate price adjustments, with price increases outnumbering decreases. The reason for a more attenuated recession is that an important part of the hangover effect is mitigated by the anticipated adjustments of firms with high relative prices. Finally, firms with prices close to their optimal decide to postpone their planned adjustment, extending the recessive impact of the disinflationary policy.⁶

The remaining part of the paper is organized as follows. Section 2 explains our method-

⁵In their interview study they found that nearly 60% of the firms said that they do have periodic price reviews, while 30% said they do not. The remaining firms said that they do have periodic reviews for some products but not for others.

⁶The net impact is recessive since, among firms which decide to postpone adjustments (because their prices are close to their expected optimal levels), firms with higher relative prices outnumber those with lower ones.

ology. We derive and characterize optimal pricing rules under steady state. We also solve for optimal pricing rules during disinflation, and develop a methodology for aggregating them. Section 3 presents our results for pricing rules, and output during disinflation. In section 4, we relax the hypothesis of strict time-dependency by introducing re-evaluation of pricing rules at the time disinflation is announced. The last section concludes.

2 The Model

Our modeling strategy is to build on the static model results of Blanchard and Kiyotaki (1987), and Ball and Romer (1989). Starting from the specification of preferences, endowments and technology, these models derive individual optimal price equations at each moment as a function of aggregate demand (Ball and Romer) or directly as a function of the money supply and price level (Blanchard and Kiyotaki). In order to generate individual uncertainty about the optimal individual price, we add an idiosyncratic shock process to the optimal price equation obtained in those models. These shocks are permanent and thus, together with the money supply process, generate intertemporal links which make the model dynamic.⁷

Our economy is populated by an infinite collection of identical (in all aspects other than the timing of adjustments and realization of idiosyncratic shocks) imperfectly competitive firms indexed in the interval $[0, 1]$. We assume that the optimal level of the individual relative price, in the absence of frictions, is given by:

$$p_i^* - p = \theta y + e_i, \tag{1}$$

where p_i^* is the individual frictionless optimal price, p is the average level of prices, y is aggregate demand and e_i is an idiosyncratic shock to the optimal price level (all variables are in log).⁸ Since firms are identical (although they can have different prices and supply

⁷Having a dynamic macro model with intertemporal consumption and investment decisions would complicate the model without affecting the main insights.

⁸Equation 1 states that the relative optimal price depends on aggregate demand and on shocks specific to the firm. It can be derived from utility maximization in a yeoman farmer economy, as in Ball and Romer (1989).

different quantities), for simplicity we evaluate p at any time t according to:

$$p(t) = \int_0^1 x_i(t) di,$$

where $x_i(t)$ is the price charged by the firm i at time t .

Nominal aggregate demand is given by the quantity of money:

$$y + p = m.$$

Substituting the above equation into equation (1) yields:⁹

$$p_i^* = \theta m + (1 - \theta)p + e_i. \tag{2}$$

If there were no costs to adjust prices and/or obtain information about the frictionless optimal price level, each firm would choose $x_i(t) = p_i^*(t)$ and the resulting aggregate price level would be $p(t) = m(t)$. Thus aggregate output and individual prices would be given by $y(t) = 0$ and $x_i(t) = m(t) + e_i(t)$, respectively.

We assume that the firm can neither observe the stochastic components of p_i^* nor adjust its price based on the known components of p_i^* without paying a lump-sum cost F . On the other hand, to let the price drift away from the optimal entails expected profit losses, which flow at rate $E_{t_0}(x_i(t) - p_i^*(t))^2$, where t_0 is the last time of observation and adjustment and E_{t_0} is the expectation conditioned on the information available at that time.¹⁰ Time is discounted at a constant rate ρ .

Given the stochastic process for the optimal price, each price setter solves for the optimal pricing rule. The cost function after paying the adjustment/information gathering cost at a

⁹This equation can also be derived directly from other specifications, such as Blanchard and Kiyotaki (1987), where real balances enter the utility function.

¹⁰Observe that this form corresponds to a second order Taylor approximation to the expected profit loss for having a price different from the optimal one whenever the second derivative of the profit function is constant.

certain time t_0 , can be written in the following way:

$$V = \min_{\{t_j\}, \{x_i(t_j)\}} E_{t_0} \sum_{j=0}^{\infty} e^{-\rho(t_j - t_0)} \left[\int_0^{t_{j+1} - t_j} e^{-\rho s} (x_i(t_j) - p_i^*(t_j + s))^2 ds + F e^{-\rho(t_{j+1} - t_j)} \right], \quad (3)$$

where t_j is a time of adjustment/information gathering and $x_i(t_j)$ is the price chosen at time t_j .

Next we use this general framework to analyze optimal pricing rules in steady state and during disinflation.

2.1 Steady State

We assume that for each i , e_i follows a driftless Brownian motion with coefficient of diffusion σ and that those individual processes e_i 's are independent of each other. We also assume that the money supply has a deterministic constant rate of growth μ .¹¹ In steady state the aggregate price level will grow at the same rate μ .¹² As a consequence, the frictionless optimal price will be a Brownian motion with a drift given by the rate of the money supply growth:

$$dp_i^* = \mu dt + \sigma dW_i. \quad (4)$$

Given the Markovian nature of the stochastic process for the frictionless optimal price and the lump-sum type of adjustment/information gathering cost, the problem of the firm after paying this cost does not depend either on the specific time when the problem is solved or on the realization of the frictionless optimal price at that time. An adequate state variable for the firm's problem in steady state is the deviation of the individual price x_i from the frictionless optimal level:

$$z_i(t) \equiv x_i(t) - p_i^*(t).$$

¹¹For simplicity we assume that there are no aggregate shocks.

¹²This will be verified below.

Thus, if at t the price deviation is z , the price deviation at $t + s$ is given by:¹³

$$z(t + s) = z + p_i^*(t + s) - p_i^*(t).$$

Hence we can formalize the optimization problem through the following Bellman Equation:

$$V_\mu = \min_{z, \tau} E_t \int_0^\tau [z - (p_i^*(t + s) - p_i^*(t))]^2 e^{-\rho s} ds + F e^{-\rho \tau} + V_\mu e^{-\rho \tau}, \quad (5)$$

where V_μ represents the value function for the steady state problem with money growth rate μ .¹⁴ The first order conditions are:

$$z^* = \frac{\rho}{1 - e^{-\rho \tau^*}} \int_0^{\tau^*} E_t (p_i^*(t + s) - p_i^*(t)) e^{-\rho s} ds; \quad (6)$$

$$\rho(V_\mu + F) = E_t [z^* - (p_i^*(t + \tau^*) - p_i^*(t))]^2, \quad (7)$$

where τ^* and z^* are the optimal contract length and the individual price deviation chosen at the beginning of the contract, respectively. Using the process of the frictionless optimal price (4) in (5), (6) and (7), we arrive at the following equations:

$$V_\mu = \frac{\int_0^{\tau^*} ((z^* - \mu s)^2 + \sigma^2 s) e^{-\rho s} ds + F e^{-\rho \tau^*}}{1 - e^{-\rho \tau^*}}; \quad (8)$$

$$z^* = \mu \left(\frac{1}{\rho} - \frac{e^{-\rho \tau^*}}{1 - e^{-\rho \tau^*}} \tau^* \right); \quad (9)$$

$$\rho(V_\mu + F) = (z^* - \mu \tau^*)^2 + \sigma^2 \tau^*. \quad (10)$$

We can substitute (8) into (10) and then substitute (9) into the resulting expression to

¹³We drop the i subscript for the individual price deviation z , because it is the same for all adjusting firms.

¹⁴The value function in steady state will be the same for all firms, because it depends on the parameters of the stochastic process for p_i^* and not on its realizations.

find the following equation, which defines τ^* implicitly:

$$\begin{aligned} & \rho F + \rho \frac{\int_0^{\tau^*} \left(\left(\mu \left[\frac{1}{\rho} - \frac{e^{-\rho s}}{1 - e^{-\rho \tau^*}} \tau^* \right] - \mu s \right)^2 + \sigma^2 s \right) e^{-\rho s} ds + e^{-\rho \tau^*} F}{1 - e^{-\rho \tau^*}} \\ &= \left[\mu \left[\frac{1}{\rho} - \frac{e^{-\rho \tau^*}}{1 - e^{-\rho \tau^*}} \tau^* \right] - \mu \tau^* \right]^2 + \sigma^2 \tau^*. \end{aligned} \quad (11)$$

Based on the above equation, we can prove the following (the proof is in Appendix A):

Proposition 1 *The optimal contract length in steady state, τ^* , satisfies:*

$$a. \frac{d\tau^*}{d\mu} \mu < 0;^{15} \quad b. \frac{d\tau^*}{d\sigma} < 0; \quad c. \frac{d\tau^*}{d\theta} = 0; \quad d. \frac{d\tau^*}{dF} > 0; \quad e. \frac{d^2\tau^*}{d\mu d\sigma} \mu > 0$$

The optimal contract length has the expected features. It is decreasing in $|\mu|$ and σ since higher inflation or idiosyncratic uncertainty would result in larger quadratic deviations from the frictionless optimal price if τ^* were kept constant (see, for example, Figures 1a and 1b). An increase in F raises the adjustment costs associated with a given contract length, resulting in a higher τ^* . The degree of strategic complementarity, $1 - \theta$, does not affect the choice of τ^* .¹⁶ Finally a higher μ (σ) reduces the sensitivity of τ^* with respect to σ (μ). The optimal contract length tends to increase with ρ essentially because the benefit of postponing adjustment becomes higher.¹⁷

The level of inflation will have aggregate effects even in the steady state. To see this, we first find the aggregate price level using the method of undetermined coefficients (see Appendix B for details):

$$p(t) = \mu t + \mu \left[\frac{1}{\theta \rho} - \frac{\tau^* (1 + e^{\rho \tau^*})}{2\theta (-1 + e^{\rho \tau^*})} \right].$$

Thus, the output level is given by:

$$y(t) = \mu \left[\frac{\tau^* (1 + e^{\rho \tau^*})}{2\theta (-1 + e^{\rho \tau^*})} - \frac{1}{\theta \rho} \right],$$

¹⁵Note that this is equivalent to $\frac{d\tau^*}{d\mu} < 0$ for $\mu > 0$ and $\frac{d\tau^*}{d\mu} > 0$ for $\mu < 0$.

¹⁶This is because it only affects the level of variables $p(t)$ and $p_i^*(t)$, but not their growth rates. This ceases to be true out of the steady state, as will be seen in the next section.

¹⁷This was true for all numerical simulations we performed.

which depends both on the inflation level and on the degree of strategic complementarity. For a positive (negative) inflation, the output level is above (below) the natural level for a frictionless economy.¹⁸ As pointed out by Danziger (1988) in a deterministic state-dependent model, the reason is that discounting induces firms to set prices closer to the optimal at the beginning of the contract, resulting in a lower (higher) aggregate price level. The magnitude of the output level is increasing in the degree of strategic complementarity $(1 - \theta)$, as illustrated in Figure 2.¹⁹ The reason is that, with higher strategic complementarity, each firm's optimal price will be more influenced by the other firms' price deviations, reinforcing the incentive to deviate from the frictionless level.

In our simulations, we follow Ball, Mankiw and Romer (1988) in setting $\sigma = 3\%$. We calibrate F in such a way that with $\mu = 3\%$, $\sigma = 3\%$ and $\rho = 2.5\%$ a year, a firm chooses to collect information and adjust its price once a year. As a result we set $F = 0.000595$. This frequency of adjustments is consistent with the findings of Carlton (1986) and Blinder (1991) that in the American economy the median firm adjusts its price approximately once a year. As a test for that configuration of parameters we can assess whether the adjustment intervals generated for high inflation are plausible. With $\mu = 1$ (annual inflation of 172%) prices are adjusted once every 2 months and with $\mu = 2.5$ (annual inflation of 1120%), the frequency of adjustments increases to once a month. Those implications are consistent with available empirical evidence for high inflation countries, such as Brazil during the 80s (Ferreira 1994).

2.2 Disinflation

To our knowledge, all articles which use time-dependent rules in order to analyze the effect of disinflations have assumed that the pricing rules inherited from the inflationary steady state do not change during disinflation. So, there is no optimization with respect to pricing policies and aggregation is straightforward given the initial distribution of adjustment times.

In this section we relax this simplifying assumption by deriving optimal pricing rules during a generic disinflation path for $m(t)$. This requires solving both an optimization

¹⁸Since $\left[\frac{\tau^*(1+e^{\rho\tau^*})}{2\theta(-1+e^{\rho\tau^*})} - \frac{1}{\theta\rho} \right] > 0$.

¹⁹Since $\frac{dy}{d\theta} = -\frac{y}{\theta}$.

and an aggregation problem. In the absence of strategic complementarities ($\theta = 1$) these problems can be solved separately. Otherwise they must be solved simultaneously. In this case, the optimal rule depends on the expected path for the aggregate price level and the path for the aggregate price results from the aggregation of the individual pricing rules.

In the following subsections we first explain separately the optimal pricing rule problem and our aggregation methodology. In the next section we present the results for specific disinflation paths.

2.2.1 Optimal Pricing Rules

A disinflation is announced at $t = 0$. The problem of a firm adjusting at $t > 0$ can be characterized by the following Bellman equation:

$$V(t) = \min_{x_i(t), \tau(t)} E_t \left[\int_t^{t+\tau(t)} e^{-\rho(s-t)} [x_i(t) - p_i^*(s)]^2 ds \right] + e^{-\rho\tau(t)} F + e^{-\rho\tau(t)} V(t + \tau(t)). \quad (12)$$

The first order conditions are:

$$x_i(t) = \frac{\rho}{1 - e^{-\rho\tau(t)}} \int_t^{t+\tau(t)} E_t p_i^*(s) e^{-\rho(s-t)} ds; \quad (13)$$

$$E_t [x_i(t) - p_i^*(t + \tau(t))]^2 - \rho F - \rho V(t + \tau(t)) + V'(t + \tau(t)) = 0. \quad (14)$$

The problem above can be solved recursively, assuming that after a long time the economy will reach a new steady state. Thus, for t large enough, $V(t) = V_{\mu'}$, where $V_{\mu'}$ is the value function for the new steady state (money growth rate μ').

2.2.2 Aggregation

In most models in the literature the time-dependent rule is exogenous, or the economy is assumed to be in an inflationary steady state (as in Ball, Mankiw and Romer 1988). In those cases a uniform distribution of adjustment times is assumed and aggregation is straightforward: $p(t) = \frac{1}{\tau} \int_0^\tau x(t-s) ds$, where $x(s)$ is the average price of firms which set

prices at s .

With endogenous rules in a changing environment, the contract length changes through time. As a consequence, the distribution of price adjustments will be changing accordingly, and aggregation requires monitoring the evolution of this distribution. We develop a methodology for tracking the evolution of distributions. For simplicity, we assume that the initial distribution is uniform, which is the invariant distribution in the inflationary steady state. However, our methodology could be applied to any initial distribution.

Let $g(\cdot)$ be the function of time which gives the next adjustment time. Then $g(t) = t + \tau(t)$.²⁰ In order to calculate the price level at a time after the announcement, we use the function g to relate the measure of firms which set their actual prices at a specific time u to the measure of firms at times before u that would have their next adjustment at u (those times are $g^{-1}(u)$). Let $Z(t)$ be the correspondence that assigns to t the set of times when the current prices were last adjusted. Formally:

$$Z(t) = \{s : s \leq t \text{ and } g(s) > t\}.$$

Let $g^{-1}(S)$ be the inverse image of the set S under g . Then, $g^{-1}(Z(t))$ is the set of adjustment times for which the next adjustment would be in $Z(t)$. To evaluate the average price at t we need to know the probability measure v of the firms which adjust at subsets of $Z(t)$. We can easily relate this measure to the measure φ in subsets $g^{-1}(Z(t))$, since v is the image measure of φ under g . Then we have:

$$p(t) = \int_{Z(t)} x(s)v(ds) = \int_{g^{-1}(Z(t))} x(g(s))\varphi(ds).$$

We apply the above formula recursively by relating distributions and adjustment time sets during disinflation to distributions and sets at preceding times. We proceed this way until we arrive at a set $g^{-n}(Z(t))$ such that the measure of firms adjusting at the subset of times of this set corresponds to the uniform distribution of the inflationary steady state.

²⁰During credible disinflations g tends to be nondecreasing, since firms tend to choose longer contract lengths. In the case of imperfect credibility, g decreases at the moment the disinflation policy is abandoned (see Bonomo and Carvalho, 2003).

When strategic complementarities are absent, the aggregation and the individual optimal rule problems can be solved separately. Hence, we first solve for the optimal rule and then use the resulting $g(\cdot)$ function to aggregate individual prices as described above.

When there are strategic complementarities, we use an iterative method. We guess a solution for the aggregation problem, i.e. a path for $p(\cdot)$, and solve the optimal rule problem given $p(\cdot)$ to find $g(\cdot)$ and $x(\cdot)$. We then aggregate according to the methodology above to find a new path for $p(\cdot)$. We continue until convergence of both $p(\cdot)$, $g(\cdot)$ and $x(\cdot)$.

3 Disinflation Results

In this section we present both individual and aggregate results for a cold turkey disinflation under perfect credibility.

In this case, the money supply path is given by:

$$\begin{aligned} m(t) &= \mu t, & t < 0; \\ &= \mu' t, & t \geq 0. \end{aligned}$$

We refer to the case of $\mu' = 0$ as “full disinflation,” while $\mu > \mu' > 0$ corresponds to a “partial disinflation.”

When strategic complementarities are absent ($\theta = 1$), the optimization problem for firms which readjust/collect information after the announcement is the same as that under the steady state with the new money growth rate μ' .

When there are strategic complementarities ($\theta < 1$), the problem of firms adjusting after the announcement is no longer equivalent to the steady state problem with the new money growth rate. The optimal price and contract length will depend partly on prices which were set prior to the disinflation announcement. Since the optimal price depends on the aggregate price, the solution requires solving simultaneously for the optimal pricing rule and the aggregate price level.

We start by showing individual results concerning the optimal contract lengths.

3.1 Individual rules

Figure 3 shows the value of $\tau(t)$ chosen by firms before and after the announcement of a full disinflation for several parameter combinations, in the absence of strategic complementarities. For example, if the initial inflation is 10% a year, the money supply stabilization leads to an increase in the time between adjustments from 7.5 months to 14 months. As expected the decrease in the frequency of adjustments is larger when initial inflation is higher as compared to the variance of idiosyncratic shocks.

When there are strategic complementarities, there is a transition phase between steady state values. Our main findings are represented in Table 1. The contract length jumps up immediately after the announcement, decreases slightly for a brief period of time and then increases again, converging to the new steady state level. The gradual increase in contract length occurs because inflation is still decreasing during some time after the announcement.²¹ Since the difference to the new steady state level is always small, the contract length is similar to the one obtained without strategic complementarities. We can conclude that strategic complementarities do not substantially affect optimal contract lengths, although as we will see below, they have important consequences to the dynamics of disinflation.²²

3.2 Aggregate effects

Now we turn to the aggregate results. We examine several cases: full disinflation with no strategic complementarities, and with strategic complementarities, partial disinflation, and disinflation from different initial inflation levels.

3.2.1 Full disinflation with no strategic complementarities

We start with the very particular case in which money growth is reduced to zero and there are no strategic complementarities in price. In this simple case, each firm adjusting after stabilization will set its price equal to the constant money supply,²³ notwithstanding the

²¹The initial decrease is due to the nonlinearities of the model.

²²The solution of the optimization problem involves computing $V'(t + \tau(t))$. When $\theta = 1$, $V'(s) = 0$ for all $s > 0$. With strategic complementarities, $V'(t + \tau(t))$ is of the order of 10^{-5} for all $t > 0$ ($\theta = 0.1$), and we therefore set it equal to zero.

²³Except for idiosyncratic shocks.

contract length. Thus, after all firms have adjusted, the average price will be equal to the money supply. All firms will have adjusted their prices when a time equal to the contract length prevailing during the inflationary steady state has elapsed. Therefore the aggregate effect of disinflation hinges on the prices and contract lengths chosen before the announcement, and the change of contract lengths will have no aggregate effect. As a consequence, starting from a given inflationary steady state in which the contract length is optimal, the effect of disinflation with endogenous rules is identical to that under exogenous rules.

3.2.2 Full disinflation with strategic complementarities

When there are strategic complementarities, the previous equivalence does not hold anymore. The optimal price will not be constant (neglecting the idiosyncratic component) after $t = 0$, being influenced by prices set before $t = 0$. The endogeneity of contracts changes the dynamics of disinflation. Figures 4a and 4b depict results for disinflations starting from $\mu = 0.1$ and $\mu = 1$, respectively. The increase in the contract length causes the recession to last longer than in the case of exogenous rules. On the other hand, longer contracts induce firms to set lower prices, since inflation is declining. As a result, the minimum output level is higher with endogenous rules. Observe that during the recession there are some time intervals in which the output level is constant. The reason is that, after all prices are reset for the first time following the announcement, there is a time interval where no adjustment takes place. Therefore the aggregate price remains constant during this interval. Its duration corresponds to the increase in the contract length.

3.2.3 Partial disinflation

In the more realistic case of a partial disinflation endogeneity matters even in the case of no strategic complementarities. As depicted in Figure 5, the recession is more intense than with exogenous rules, reversing the result obtained with full disinflation and strategic complementarities. The reason is that individual prices set after the announcement are higher with endogenous rules because they will remain fixed for a longer period during which the money supply will continue to increase. As in the case of full disinflation with strategic complementarities, the recession lasts longer with endogenous rules.

Another difference is that in the case of exogenous rules there will be no output effects after a time interval equal to the contract length, since every firm will be adjusting its price taking into consideration the new money growth rate and the distribution of adjustments will continue to be uniform. In the endogenous rules case, there will be output cycles because of the irregularities of the new distribution of adjustments.²⁴

3.2.4 Different initial inflation levels

Even in the particular case of no strategic complementarities and full disinflation, the model with endogenous rules allows us to appropriately compare the cost of disinflation for different initial inflation levels. In models with exogenous rules, if we take the contract length as fixed and compare disinflations from different initial inflation rates, the length of the recession is invariant. The initial inflation level affects only the intensity of the recession, as in Ball (1994) (Figure 6a). When the endogeneity of rules is taken into consideration, a higher initial inflation makes the recession more severe, but shorter (Figure 6b). The intuition is straightforward. A higher initial inflation implies shorter contracts and prices that are set foreseeing a higher inflation. The hangover effect of fixed prices is higher initially, inducing a stronger recession. When all prices are reset after the announcement, i.e. after a period of time equal to the initial contract length has elapsed, the recession is over. Thus, the recession is shorter when the initial inflation is higher because the time between adjustments is smaller.

We conclude that there is a trade-off between intensity and duration of the recession. As a consequence, the commonly held belief that it is easier to disinflate when inflation is higher because the degree of nominal rigidity is lower (see, for example, Blanchard 1997) must be qualified. This is only true if it is easier for the economy to bear the cost of a shorter but more intense recession.

²⁴The periods in which no adjustments take place will correspond to periods of output growth, since money growth is positive. After this interval, because the contract length is now longer, individual prices will be subject to larger adjustments when compared to the exogenous case, while the density of firms adjusting will be the one corresponding to the old steady state. Therefore the aggregate price will increase at a faster rate than money growth, reducing output gradually. This cycle will repeat itself because this irregular distribution will be replicated indefinitely.

4 Reevaluation at the time of announcement

One common criticism to time-dependent rules is that firms keep their prices fixed until the scheduled time for price adjustment, even if there is some change in the economic environment. This assumption does not seem to be harmful for moderate regime changes, since the cost of gathering information and making decisions is precisely one of the reasons why price adjustments do not occur continuously.²⁵ However, a credible and substantial change in monetary policy will probably become a free information that will not be ignored by price setters, since it could affect firms' profits substantially. In this case, it is sensible to combine the strict time-dependent model with the hypothesis that firms reevaluate their pricing policies at the time of the announcement.

In this section, we modify our model by assuming that the information about the monetary regime change is freely available, which implies allowing firms to re-evaluate their pricing policies at the time of the announcement. We assume that in order to adjust and observe innovations in their own market firms are still subject to the same adjustment/information costs. We then compare the results with the ones obtained under strict time-dependent rules.

At the time of announcement ($t = 0$), a firm i , which had its last adjustment at time $-T$, has the option of revising its planned adjustment time. Formally, its value function at time zero is given by:

$$V^R(-T) = \min_{\tau^R \in [0, \infty)} \left\{ \left[\int_0^{\tau^R} \{ \sigma^2 (T + s) + [x_i(-T) - E_0 [p_i^*(s)]]^2 \} e^{-\rho s} ds \right] + e^{-\rho \tau^R} F + e^{-\rho \tau^R} V(\tau^R) \right\}, \quad (15)$$

where τ^R is the new (revised) time for the next adjustment, V^R is the revised value function at time zero, and V is the value function defined by 12. The optimal choice of τ^R depends on the last time of adjustment $-T$. If there is an interior solution to 15, it should satisfy

²⁵Another possible criticism to this assumption is that they could infer the information about the optimal price from freely observable variables, such as their own output. While valid at the microeconomic level, this criticism is of no consequence for our aggregate results, since for a microeconomic departure of the model to have aggregate implications it is necessary that this departure results from simultaneous actions of a non-negligible number of firms in the same direction. However, the probability that a non-negligible number of firms receive a large idiosyncratic shock in the same direction, at the same time, is also negligible.

the first order condition:

$$[x_i(-T) - E_0 [p_i^*(\tau^R)]]^2 + \sigma^2 (T + \tau^R) = \rho F + \rho V(\tau^R) - V'(\tau^R). \quad (16)$$

Notice that adjusting immediately is one of the available options, implying a corner solution. If this choice is optimal for some $-T$, the value function becomes:

$$V^R(-T) = F + V(0). \quad (17)$$

In the case of no strategic complementarities, for any τ^R , $V(\tau^R) = V_0$, where V_0 corresponds to the expected present value of costs in the zero inflation steady state. Then the marginal cost of postponing the adjustment, which corresponds to the left-hand side of (16), becomes:

$$MgC(T, \tau^R) = [z^* - \mu T]^2 + \sigma^2 (T + \tau^R).$$

The marginal cost corresponds to the additional expected profit loss from deviating from the frictionless optimal price. The first term is the square of the deviation of the price with respect to the expected optimal price,²⁶ while the second term is the accumulated uncertainty about the idiosyncratic component. The marginal benefit of postponement, which corresponds to the right-hand side of (16), simplifies to:

$$MgB = \rho(F + V_0). \quad (18)$$

It corresponds to the sum of the flow benefits of postponing both the payment of the adjustment cost (ρF) and the total intertemporal costs evaluated as of the time of adjustment (ρV_0).

The marginal cost is increasing in τ^R , but depends also on the time elapsed since the last adjustment T , while the marginal benefit does not depend on either τ^R or T . There is always a set of T 's for which the marginal cost of postponing an adjustment at zero is lower than the marginal benefit. Thus, for those T 's, the new adjustment will be at some

²⁶Recall that z^* is the price deviation set at the beginning of the contract, under an inflationary steady-state. Thus, $z^* - \mu T$ is the expected price deviation T periods after the beginning of the contract.

time $\tau^R > 0$ such that the marginal cost of postponement equals the marginal benefit. If for some T the marginal cost of postponing the adjustment starting from zero is higher than the marginal benefit, then it will be optimal to adjust immediately.

Notice that the fall of inflation causes a reduction in the intertemporal costs, represented by the value function. Thus, the marginal benefit of postponement is suddenly reduced when disinflation is announced, but the marginal cost of postponement remains the same at time zero. Then the marginal benefit of postponement will become instantaneously smaller than the marginal cost for a set of firms, triggering immediate price adjustments.

Figure 7 shows the newly chosen time of adjustment after disinflation announcement, τ^R , as a function of the time elapsed since the last adjustment T for several initial inflation rates. All curves are concave, contrasting with the planned linear curves τ^P .²⁷ For a 3% initial inflation, only a small set of firms who had planned to adjust a little bit after the announcement chooses to reset their prices immediately. By comparing this curve with the curve of planned adjustment times, we see that, except for a small group of firms that adjusted “a long time ago” or that had adjusted recently, most firms choose to lengthen their contracts (Figure 8a). For a 10% initial inflation rate, the concavity is accentuated and the τ^R curve is no longer increasing (Figure 8b). The reason is that the firms that had just adjusted find themselves with too high a price for a zero inflation environment. Those firms will want to readjust sooner than the ones that adjusted earlier but have their prices closer to the expected optimal level. For a 30% inflation the pattern becomes more accentuated, and both firms that were closer to their next adjustment time and firms that had adjusted recently choose to reset their prices immediately (Figure 8c). Now the set of firms that chooses to postpone their adjustment times is reduced. Thus, with higher inflation, more firms decide to reset their prices at the time of announcement and less firms choose to lengthen their contracts.

In Figures 9a,9b, and 9c we display the output paths for disinflation with and without re-evaluation, for 3%, 10% and 30% initial inflations, respectively. For a 3% initial inflation rate, re-evaluation has little impact, and the output paths are similar. With higher initial inflation rates, the difference increases. Three features are noteworthy: the recession starts

²⁷Formally $\tau^P(-T) = -T + \tau^*(\mu)$.

immediately, is less deep, and is longer. The reason for the jump down in the output level is the substantial price resetting at time zero, with more upward than downward price adjustments. The recession is attenuated with respect to the strict time-dependent case because most of the hangover effect is mitigated by the anticipated adjustments of firms with high relative prices. Finally, the recession is longer because a group of firms with prices higher but close to the expected optimal level will choose to lengthen their contracts.²⁸

5 Conclusion

One of the main methodological weaknesses in the literature which relates nominal rigidities and costs of disinflation is that pricing rules are invariant to policy regimes. This paper tries to fill this gap. We had to proceed in three steps. First we rationalized fixed price time-dependent rules as optimal rules. Second, we derived a methodology for simultaneously finding optimal rules during disinflation experiments and aggregating pricing rules under non-steady state conditions. And finally, we used the methodology of aggregation in the disinflation experiments to evaluate their results.

The methodology we developed is fairly general, being based on Bellman equations for the individual problem and on a recursive mapping of the measure of firms adjusting at each time for aggregation. Furthermore, our methodology allows us to account for strategic complementarities in prices, which are often neglected in the literature due to the technical difficulties they pose.

The results show that the effort was not vain, that is, the endogeneity of rules matters. We can summarize our main findings as follows: i) so long as money growth is not cut to zero, disinflation tends to have a stronger negative effect on output than when assessed with invariant rules; ii) the recession tends to last longer in the endogenous rules setting; iii) a higher initial inflation generates a deeper and shorter recession.

We also modified the strict time-dependent model by allowing re-evaluation of pricing

²⁸One could think that firms with prices close but lower than the expected optimal level would do the same, neutralizing the effect. However, those firms had their last adjustment earlier than the ones with prices higher than the expected optimal level. As a consequence, they have higher marginal cost of postponement due to accumulated uncertainty about the idiosyncratic component.

policies when there is an important piece of news. Re-evaluation has non-trivial impacts on disinflation results. For a sufficiently low initial inflation the results are similar to the pure time-dependent model. For more sizeable inflations, three features are noteworthy: the recession starts immediately and abruptly, is less deep, and is longer than in the strict time-dependent model with endogenous rules.

The endogeneity of rules also allows proper examination of the role of credibility on the output costs of disinflation. This is done in a sequel paper (Bonomo and Carvalho, 2003).²⁹

²⁹This issue is examined by Ball (1995) in a fixed price time-dependent model with exogenous rules, and by Almeida and Bonomo (2002) in a model with endogenous state-dependent rules.

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Appendix A

Proof of Proposition 1: We apply the Implicit Function Theorem to (11) obtain:

$$\frac{d\tau}{d\mu}\mu = \frac{Num\left(\frac{d\tau}{d\mu}\mu\right)}{Den} < 0, \quad (\text{a.})$$

where

$$Num\left(\frac{d\tau}{d\mu}\mu\right) = -2(e^{\rho\tau} - 1)\mu^2\tau(2 + \rho\tau + e^{\rho\tau}(\rho\tau - 2)) < 0,$$

and

$$Den = \left\{ \begin{array}{l} (e^{\rho\tau} - 1)\rho(F\rho^2 + \sigma^2(e^{\rho\tau} - (1 + \rho\tau))) + \\ \mu^2(2e^{2\rho\tau}(\rho\tau - 1) - 2 - 4\rho\tau - \rho^2\tau^2 + e^{\rho\tau}(4 + 2\rho\tau - 3\rho^2\tau^2)) \end{array} \right\} > 0.$$

$$\frac{d\tau}{d\sigma} = \frac{Num\left(\frac{d\tau}{d\sigma}\right)}{Den} < 0, \quad (\text{b.})$$

where

$$Num\left(\frac{d\tau}{d\sigma}\right) = -2e^{-\rho\tau}(e^{\rho\tau} - 1)^2\sigma(1 - e^{\rho\tau} + \rho\tau e^{\rho\tau}) < 0.$$

$$\frac{d\tau}{d\theta} = 0. \quad (\text{c.})$$

$$\frac{d\tau}{dF} = \frac{Num\left(\frac{d\tau}{dF}\right)}{Den} > 0, \quad (\text{d.})$$

where

$$Num\left(\frac{d\tau}{dF}\right) = ((e^{\rho\tau} - 1)\rho)^2 > 0.$$

$$\frac{d\tau^2}{d\mu d\sigma}\mu = \frac{Num\left(\frac{d\tau^2}{d\mu d\sigma}\mu\right)}{Den^2}, \quad (\text{e.})$$

where

$$Num\left(\frac{d\tau^2}{d\mu d\sigma}\mu\right) = 4(e^{\rho\tau} - 1)^2\mu^2\rho\sigma\tau(e^{\rho\tau} - (1 + \rho\tau))(2 + \rho\tau + e^{\rho\tau}(\rho\tau - 2)).$$

To prove the sign of the expressions (a.) through (e.) we used the following inequality (for $x > 0$):

$$e^x > 1 + x, \quad (19)$$

which holds because of the expansion:

$$e^x = 1 + x + \sum_{i=2}^{\infty} \frac{x^i}{i!}.$$

Since the denominator is common to all expressions, we first prove its sign.

$$Den = \left\{ \begin{array}{l} [(e^{\rho\tau} - 1) \rho(F\rho^2 + \sigma^2 (e^{\rho\tau} - (1 + \rho\tau)))] + \\ [\mu^2 (2e^{2\rho\tau} (\rho\tau - 1) - 2 - 4\rho\tau - \rho^2\tau^2 + e^{\rho\tau} (4 + 2\rho\tau - 3\rho^2\tau^2))] \end{array} \right\}.$$

The first expression between square brackets is obviously positive because of (19). We label the second expression between square brackets as

$$Den_2(u) = \mu^2 (2e^{2u} (u - 1) - 2 - 4u - u^2 + e^u (4 + 2u - 3u^2)),$$

where $u = \rho\tau$. To prove its sign, first we notice that it is equal to 0 for $u = 0$. We take the derivative with respect to u to find:

$$Den'_2(u) = \mu^2 (-2(2 + u) + e^{2u}(4u - 2) + e^u(6 - 4u - 3u^2)).$$

Notice that the right hand side of the above expression is 0 for $u = 0$. Taking once again the derivative, we arrive at:

$$Den''_2(u) = \mu^2 (-2 + 8ue^{2u} + e^u(2 - 10u - 3u^2)).$$

This expression is again 0 for $u = 0$. Differentiating for the last time:

$$Den'''_2(u) = \mu^2 e^u (-8 - 16u - 3u^2 + 8e^u(1 + 2u)).$$

This expression is equal to 0 for $u = 0$, and is greater than 0 for $u > 0$. This latter result

follows from (19). Thus $Den_2''(u) > 0$ for $u > 0$, which implies that $Den_2'(u) > 0$ for $u > 0$, and finally that $Den_2(u) > 0$ for $u > 0$.

By an analogous process we found the signs of the numerators of expressions (a.) through (e.).

Appendix B

Here we show that in steady state the aggregate price level does, in fact, grow at rate μ . Using the method of undetermined coefficients we assume that the price level evolves according to $p(t) = a + bt$. We plug this expression into (6) and aggregate according to:

$$p(t) = \frac{1}{\tau^*} \int_0^{\tau^*} x(t-r) dr,$$

where $x(s)$ is the average price set by firms which adjust at s and we assumed that price adjustments are uniformly staggered over time.³⁰ Since the idiosyncratic shock is the only component specific to firm i and vanishes with the averaging,

$$x(s) = x_i(s) - e_i(s) = p_i^*(s) + z(s) - e_i(s).$$

We then find the expressions for a and b that are consistent with the resulting equation for $p(t)$. This yields:

$$b = \mu;$$

and

$$a = -\mu \left[\frac{1}{\theta\rho} - \frac{\tau^* (1 + e^{\rho\tau^*})}{2\theta(-1 + e^{\rho\tau^*})} \right].$$

The resulting expression for $p(t)$ depends on the unknown contract length τ^* :

$$p(t) = \mu t - \mu \left[\frac{1}{\theta\rho} - \frac{\tau^* (1 + e^{\rho\tau^*})}{2\theta(-1 + e^{\rho\tau^*})} \right].$$

³⁰This is a natural assumption for the steady state since the uniform distribution is the only time-invariant distribution.

We can now substitute this expression in (2), arriving at the following expression for $p_i^*(t)$:

$$\begin{aligned} p_i^*(t) &= \theta m(t) + (1 - \theta) p(t) + e_i \\ &= \mu t - (1 - \theta) \mu \left[\frac{1}{\theta \rho} - \frac{\tau^* (1 + e^{\rho \tau^*})}{2\theta (-1 + e^{\rho \tau^*})} \right] + e_i. \end{aligned}$$

Thus, in steady state, both $p(t)$ and $p_i^*(t)$ grow at the same constant rate μ .

Table 1: *Optimal Pricing Rule with Strategic Complementarities*

$$\theta = 0.1 \quad \sigma = 3\% \quad \rho = 2.5\% \quad F = 0.000595.$$

time	$\mu = 10\%$	$\mu = 100\%$	time
$t < 0$	0.63	0.152	$t < 0$
$t = 0$	1.154	1.154	$t = 0$
$t_1 = 0.22$	1.148	1.138	$t_1 = 0.18$
$t > 1.13$	1.155	1.155	$t > 0.98$

Obs.: we report contract lengths before disinflation ($t < 0$), at the time of the announcement ($t = 0$), at the time when the shortest contract after the announcement is reached (t_1) and when the new steady state contract length is reached.

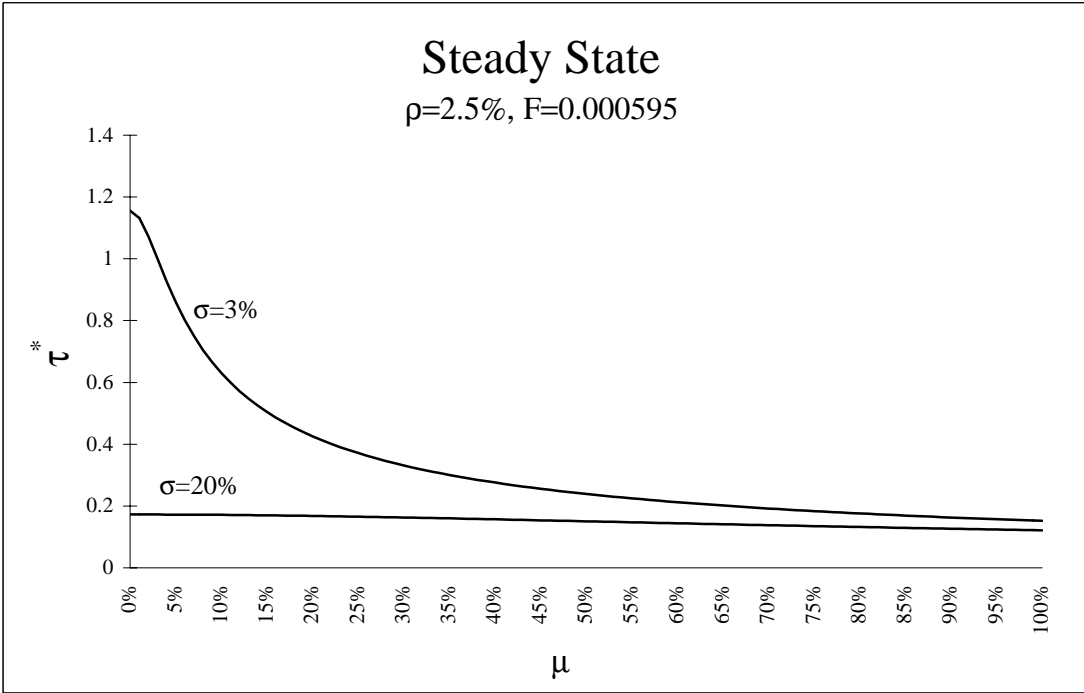


Figure 1a

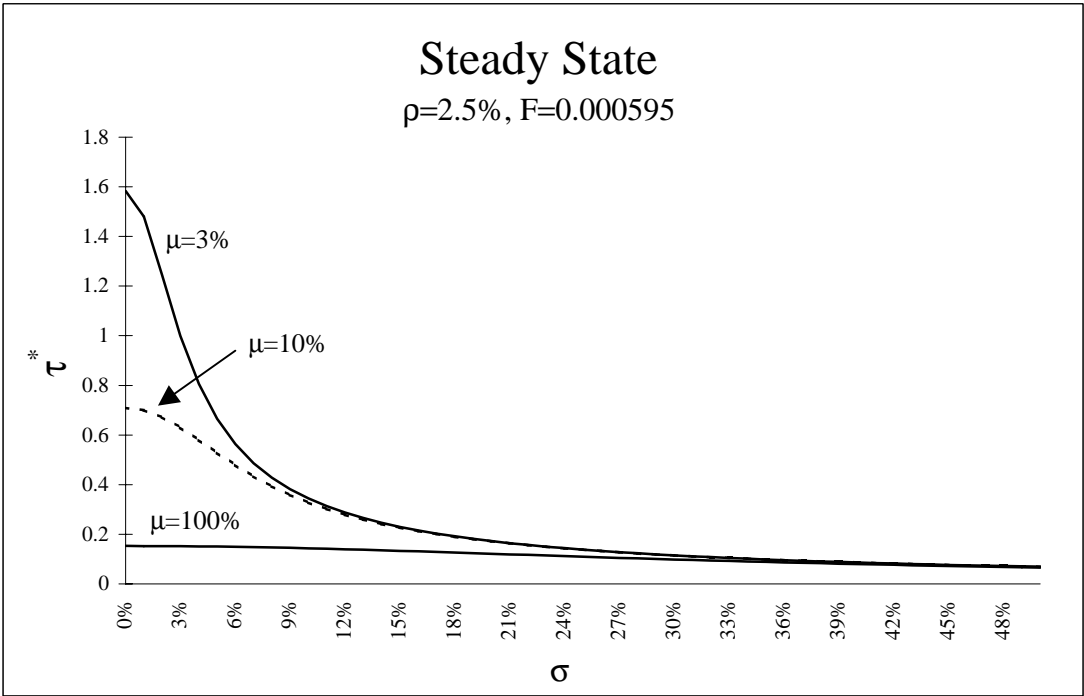


Figure 1b

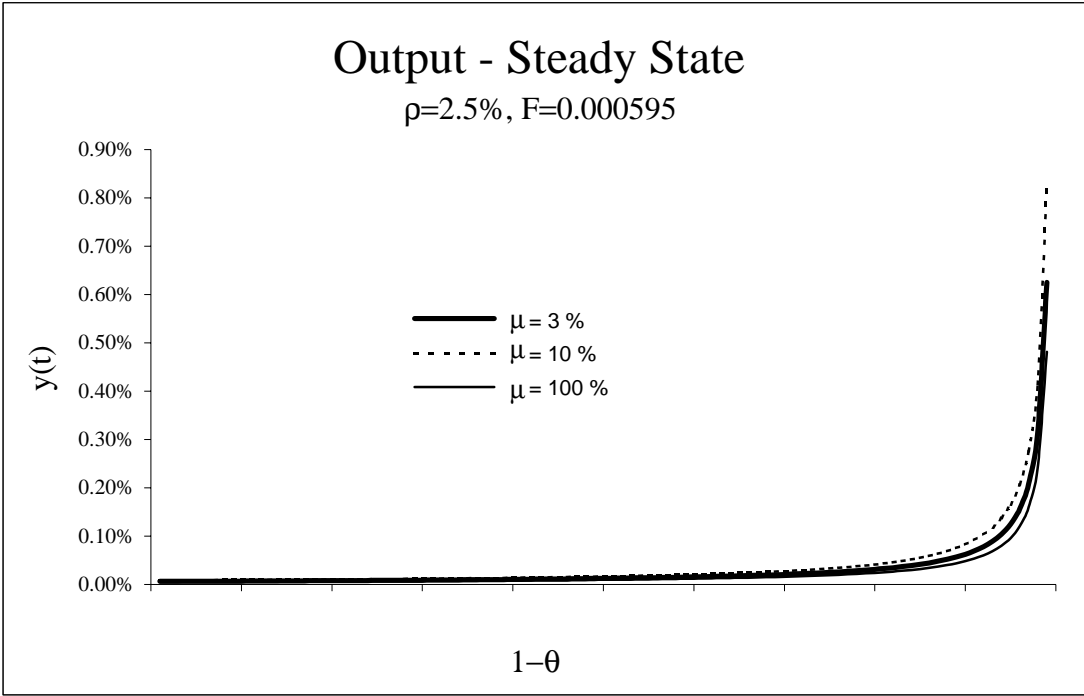


Figure 2

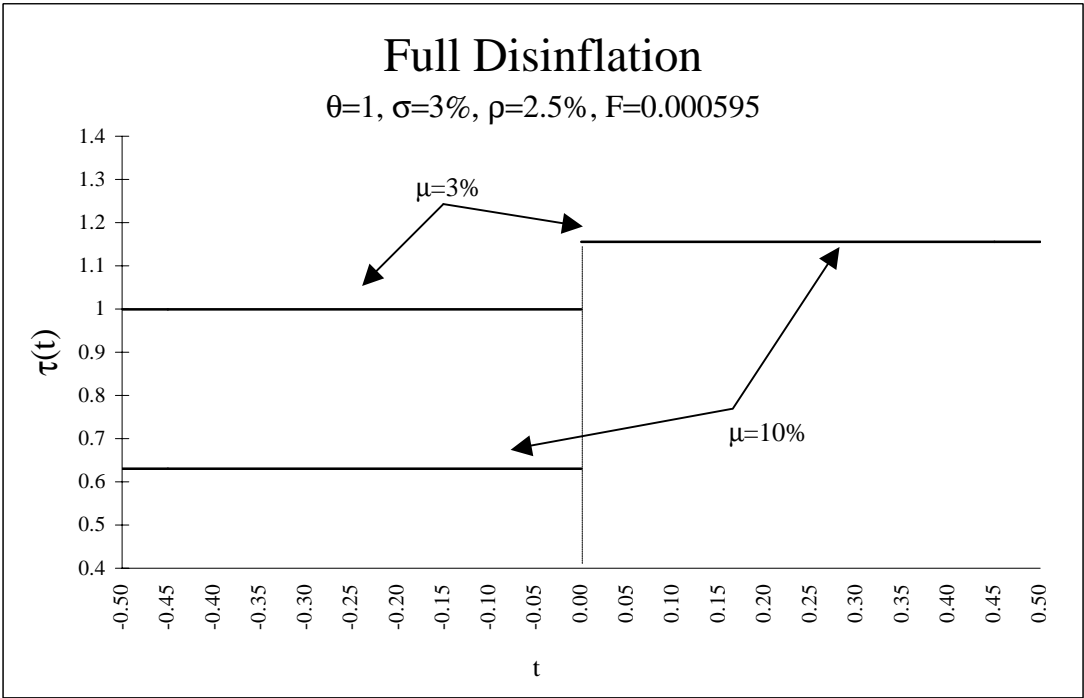


Figure 3

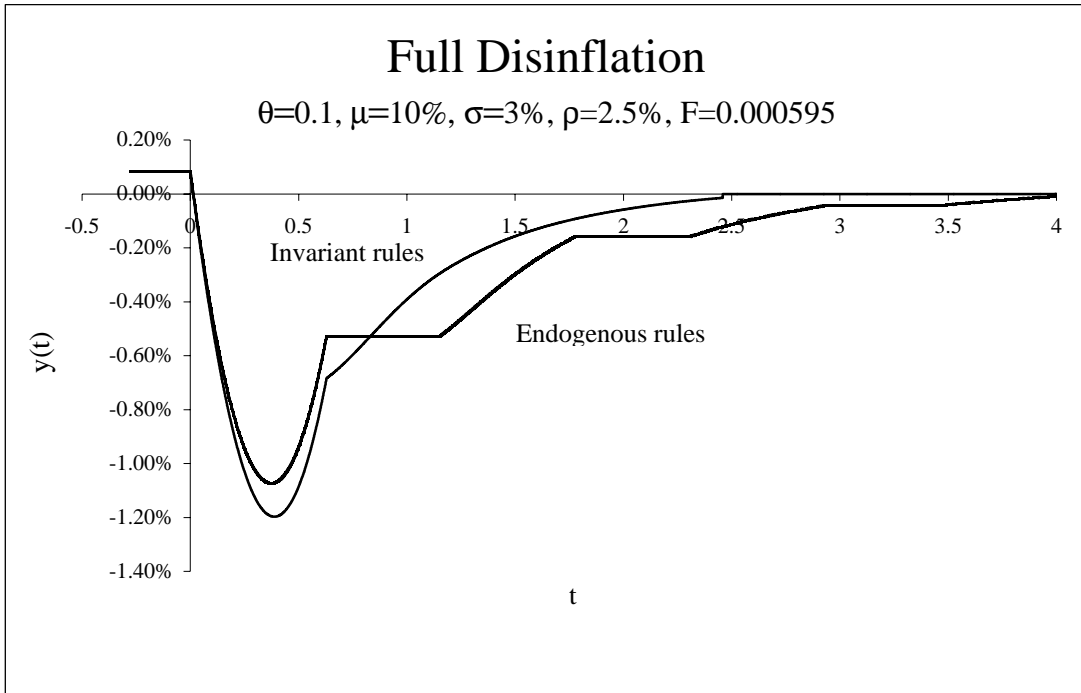


Figure 4a

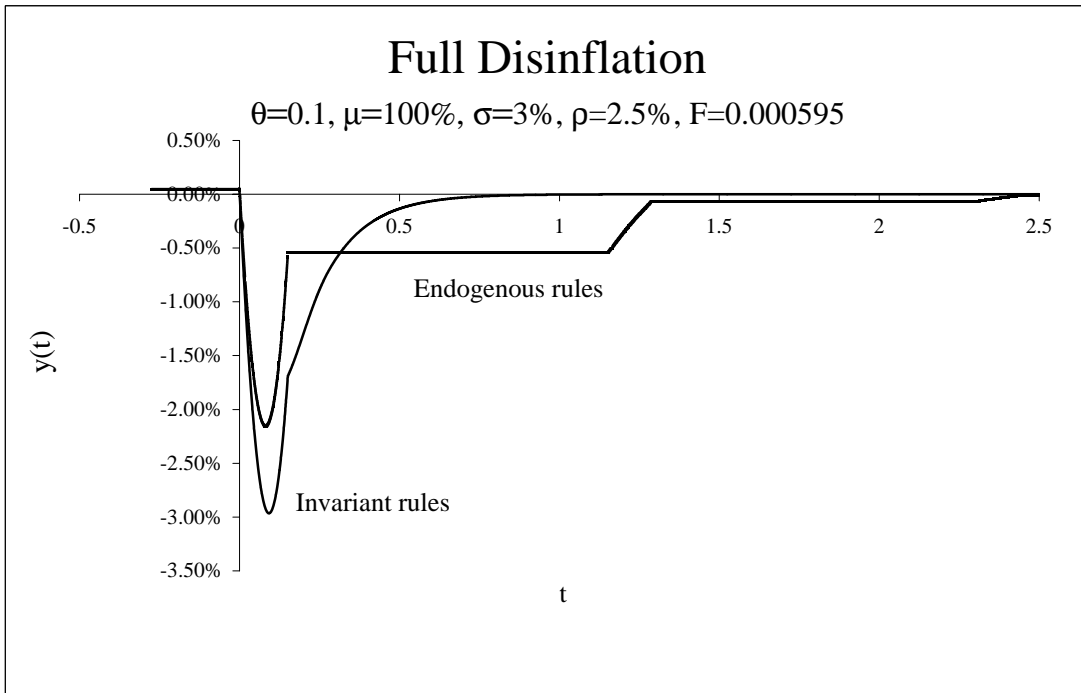


Figure 4b

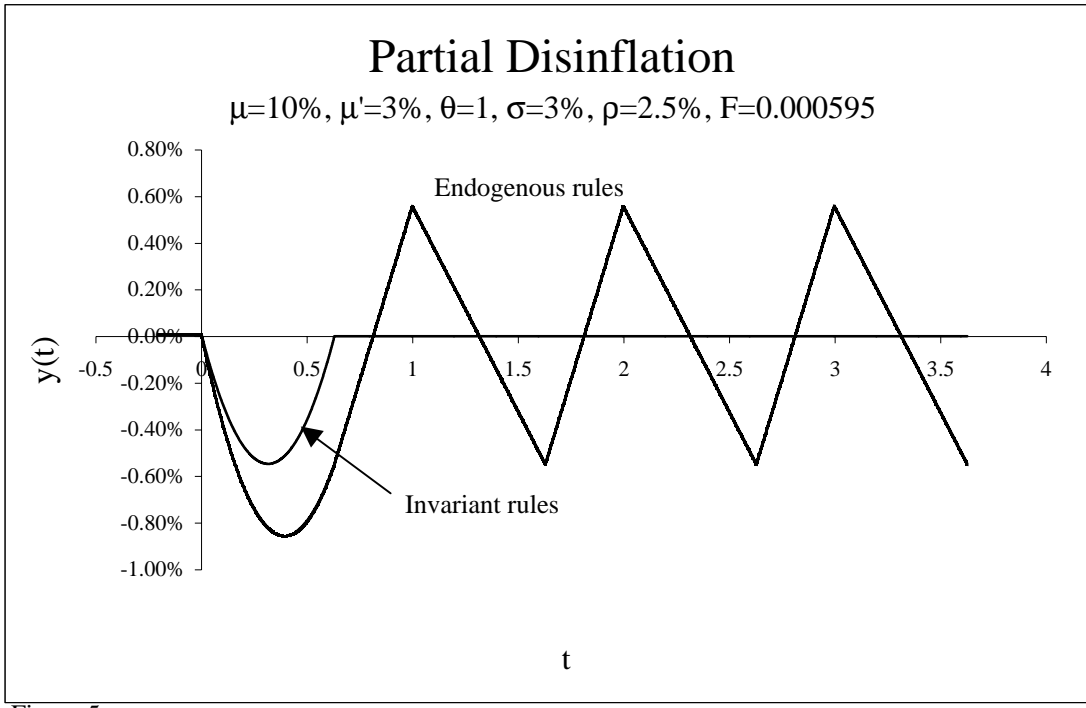


Figure 5

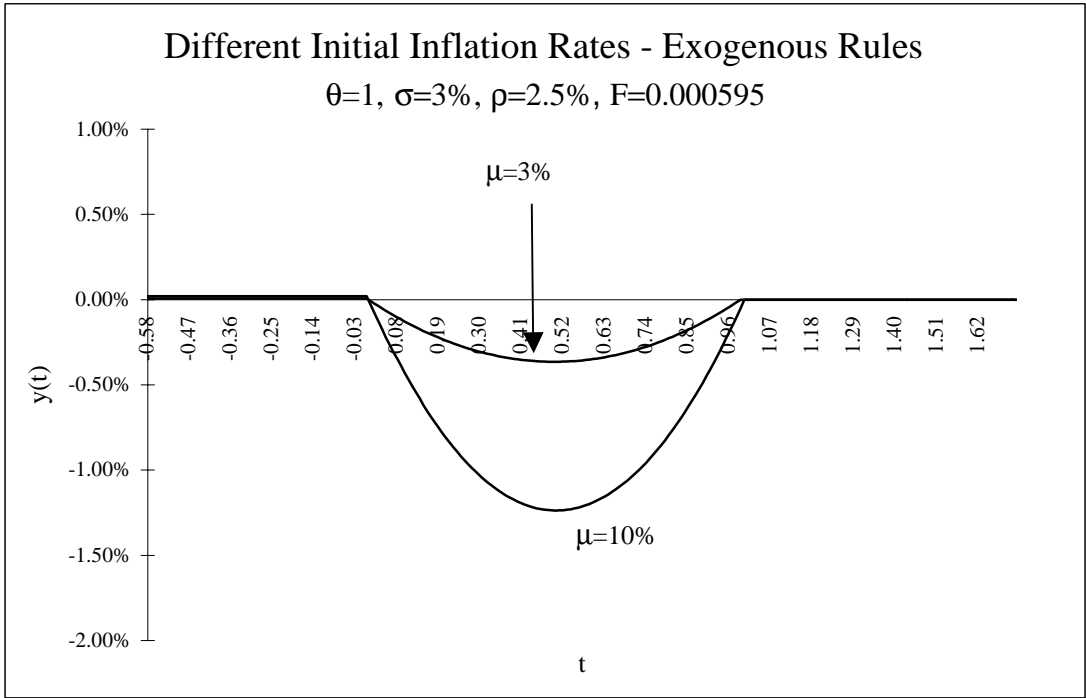


Figure 6a. Obs: In both paths contract length is optimal for a 3% inflation.

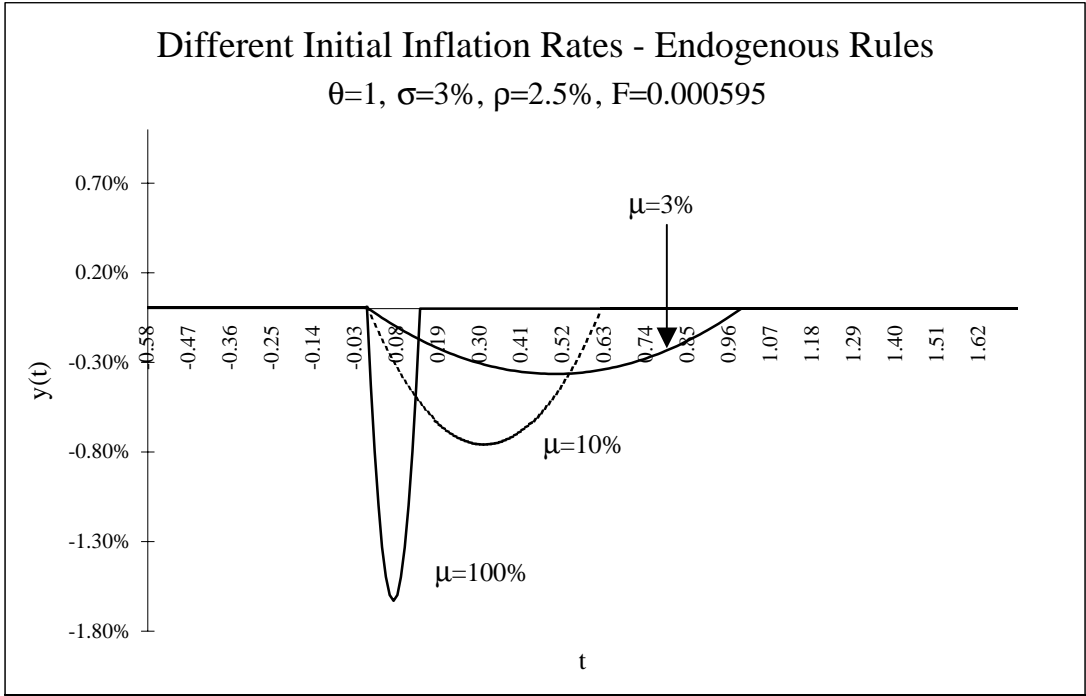


Figure 6b

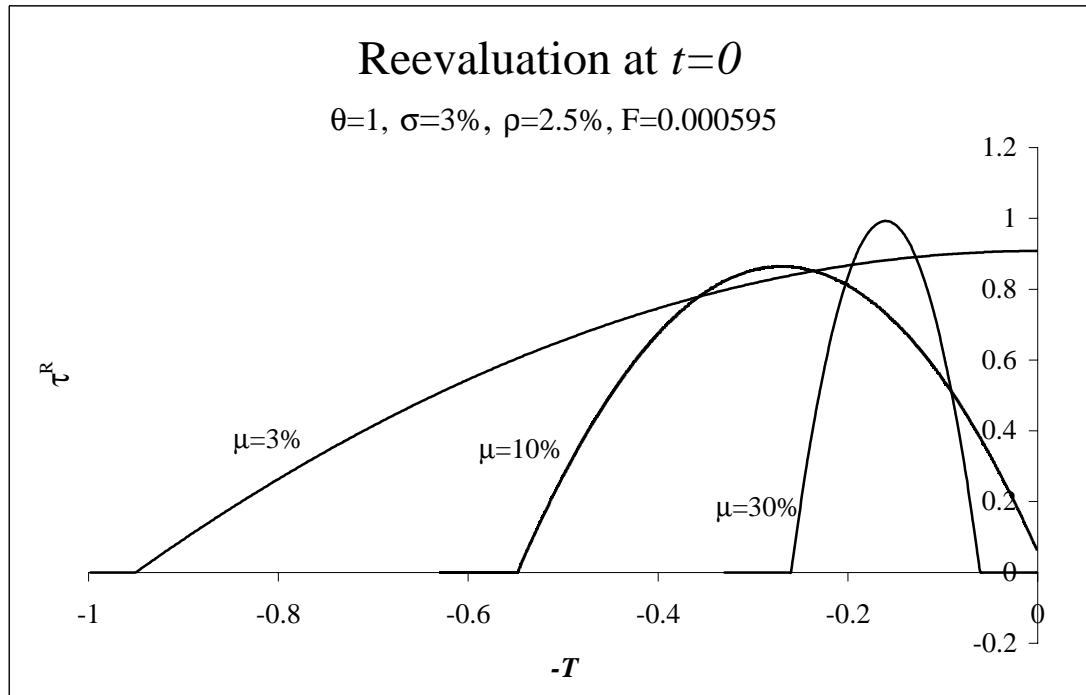


Figure 7

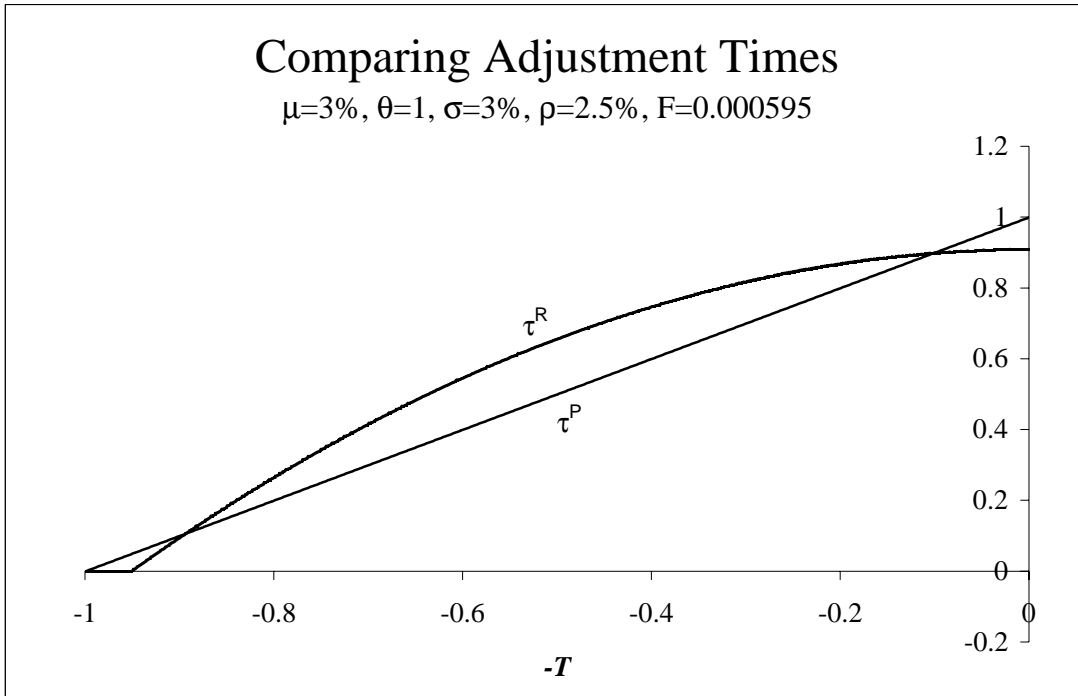


Figure 8a

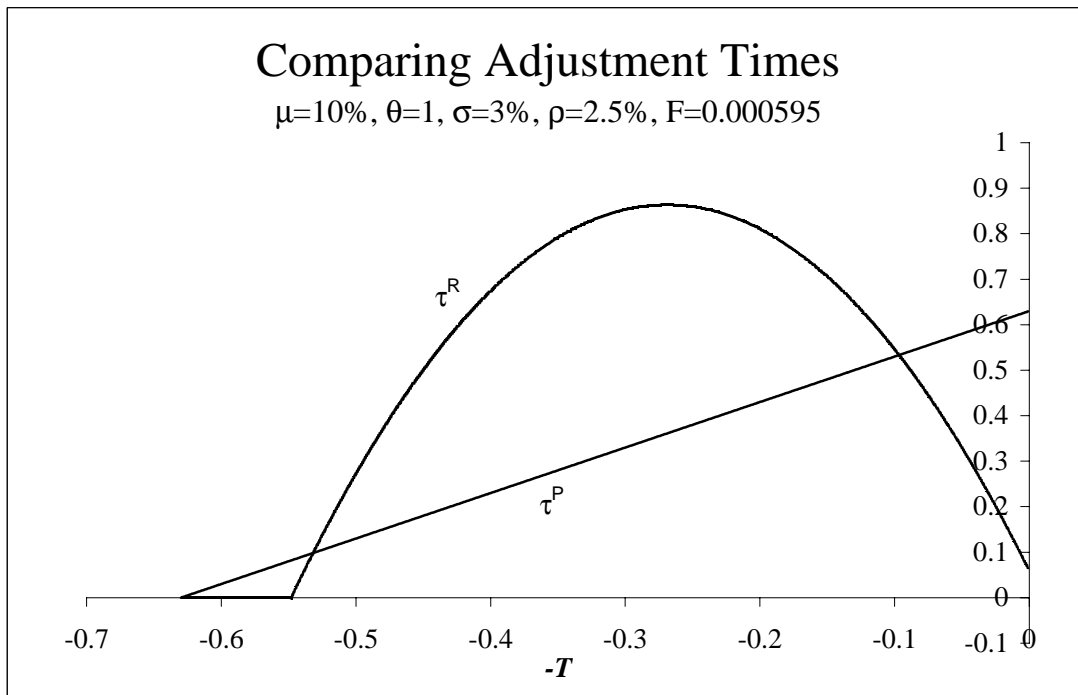


Figure 8b

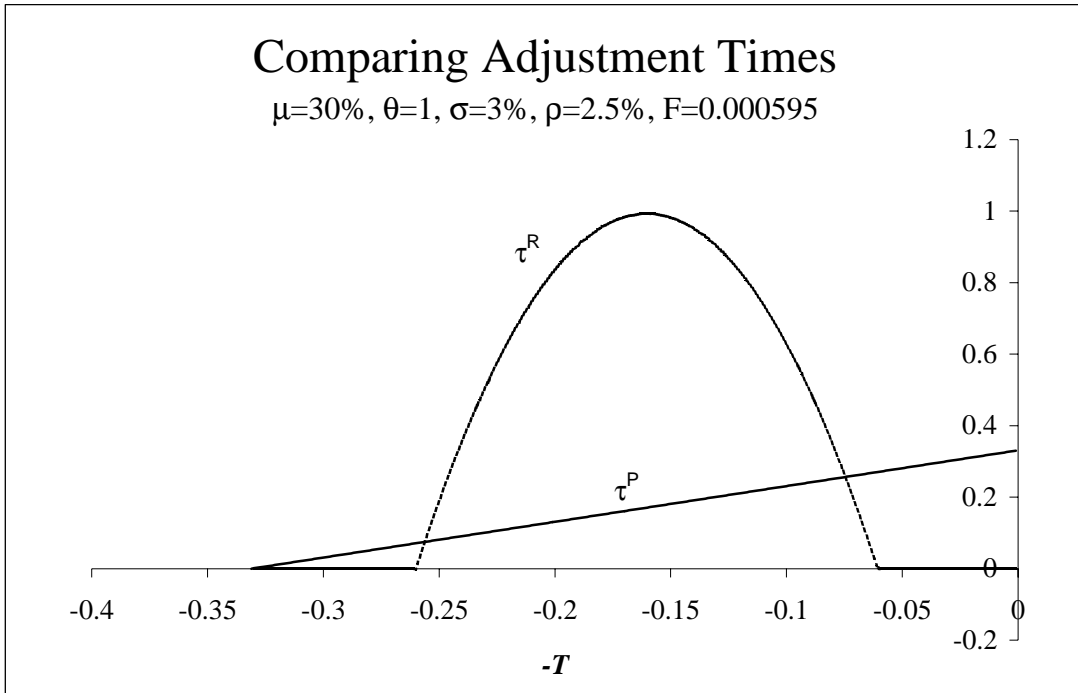


Figure 8c

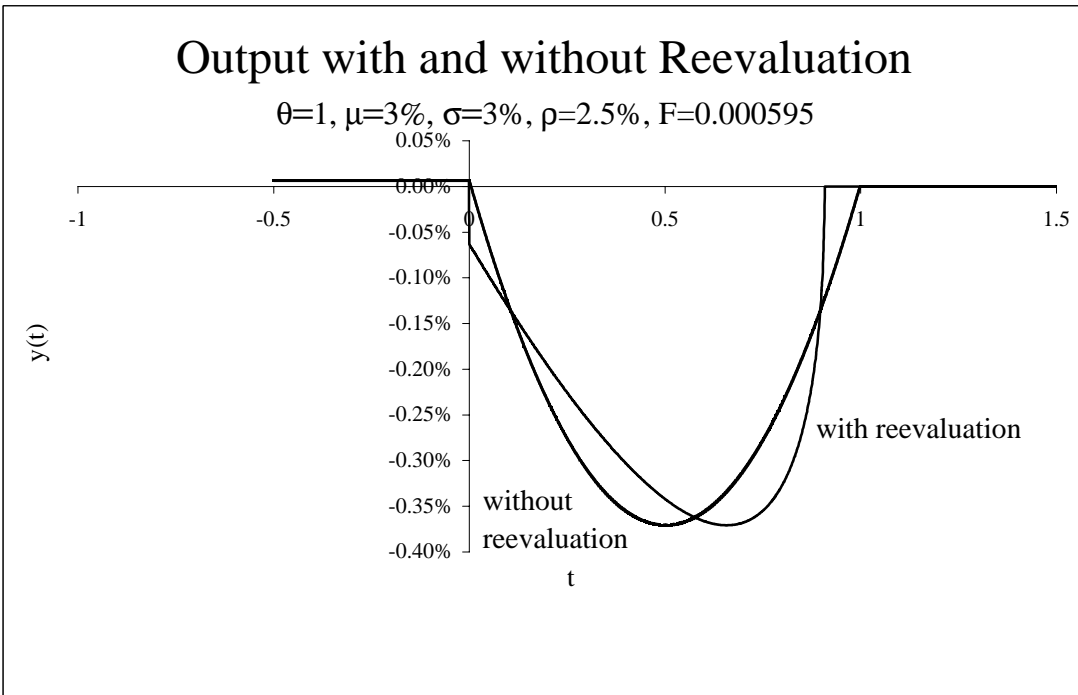


Figure 9a

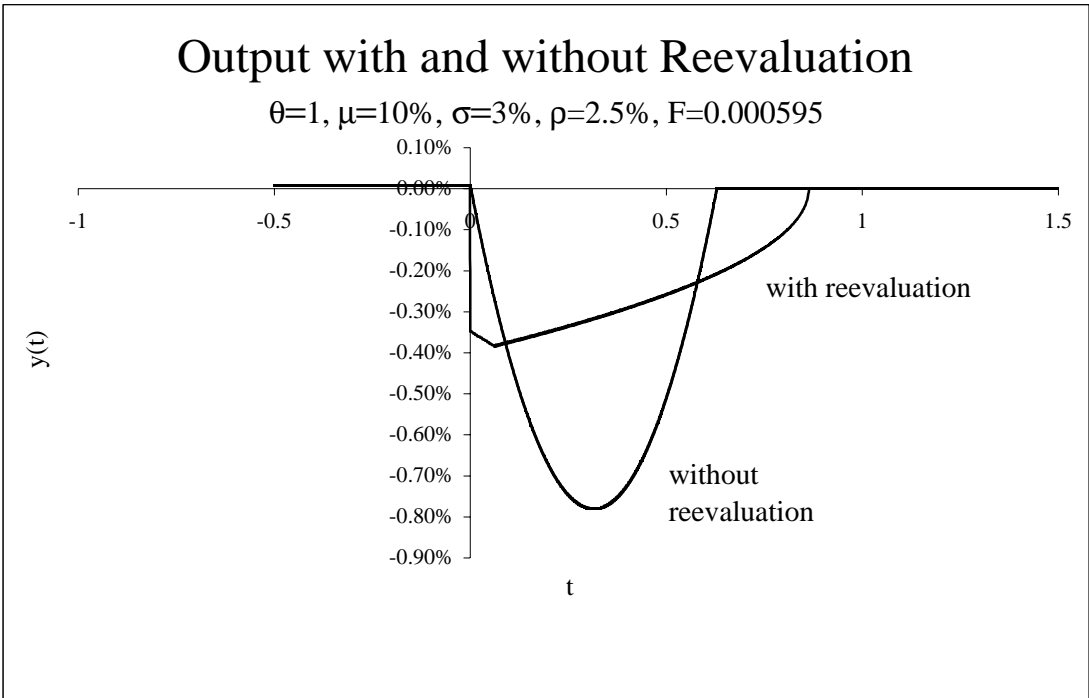


Figure 9b

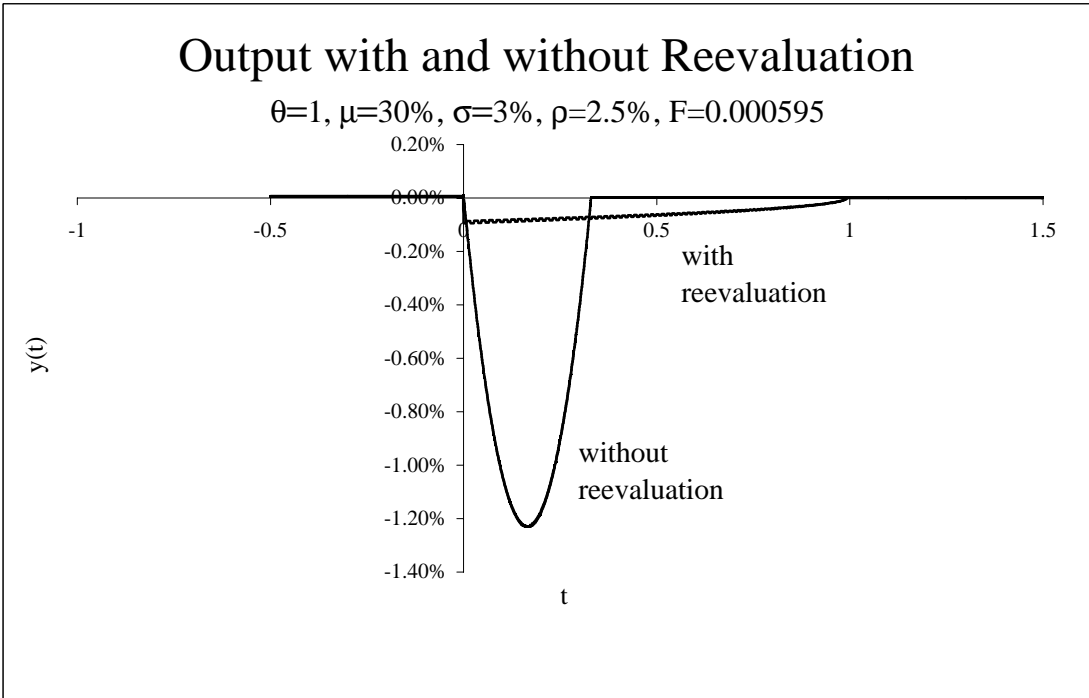


Figure 9c