Monetary Policy and Business Cycle Analysis in an Optimising Model with Expectations Lags

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ABSTRACT

Monetary models of the business cycle often neglect the importance of investment and the capital stock in the monetary transmission mechanism. Most of the recent literature assumes either investment adjustment costs or ignores capital altogether. This paper re-takes the arguments put forward by Kydland and Prescott (1982) and Christiano and Todd (1996), namely, that firms face a planning period before undertaking investment expenditures. The resulting model is able to replicate some of the most salient characteristics of the business cycle, including the lags from monetary policy actions to output.

JEL Classification: E20, E22, E32, E52

Key Words: Business Cycles, New Keynesian Phillips Curve, Monetary Policy, Real Rigidities, Investment Lags.

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Dale Henderson, Richard Holt, Saqib Jafarey, Patrick Minford and Carl Walsh provided many invaluable comments. Thanks are also due to seminar participants at Liverpool University, Cardiff Business School and ZEI Summer School for helpful discussions. Any remaining errors are the sole responsibility of the author.

1 Introduction

Recent research in macroeconomics focuses on nominal rigidities to account for output fluctuations and the role of monetary policy in stabilisaton policy, by choosing the relative variability of inflation and output. In this context most models can be summarised into an IS curve, a New Keynesian Phillips Curve arising from sticky prices and a description of monetary policy, usually an interest rate rule, such as the Taylor rule[2]. The reputed strengths of this approach are that the equations are obtained from optimising behaviour on the part of households and firms and the methodology is intertemporal in nature, allowing for a detailed study of the monetary transmission mechanism (as defined by McCallum 1999[3] and Taylor, 1995) and optimal monetary design.

As highlighted by Ellison and Scott (2000), a characteristic of sticky price models is the high volatility of investment that they generate at business cycle frequencies. This is most clearly seen from the linearised Euler equation for capital obtained from a simple RBC model:

$$\hat{k}_{t+1} = E_t \hat{y}_{t+1} - \lambda \hat{r}_t \qquad (1)$$

With sticky prices, changes in nominal interest rates translate one for one into changes in real rates, with a standard value of λ this implies a 15 per cent[4] fall in the capital stock, which translates into a fall in investment of 600 per cent.

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[2] See for example McCallum (1999)

[3] i.e. attention is devoted to both the effect of shocks and of the systematic component of monetary policy.

[4] The coefficient λ depends on the capital-output ratio and the rate of time preference.

The general response to this problem has taken one of two forms: to assume investment adjustment costs[5] or to do away with capital altogether[6]. The exclusion of the capital stock in monetary policy analysis is often implemented on the grounds that it does not exhibit substantial volatility at business cycle frequencies[7]. However, this approach eliminates one of the benefits of the SDGE approach: that it is inherently intertemporal. As King and Rebelo (2000) argue "the process of investment and capital accumulation can be very important for how the economy responds to shocks". The alternative approach of including investment adjustment costs raises an additional problem in that these are difficult to quantify; moreover, it could equally be argued that one should include labour adjustment shocks.

This paper presents a model that overcomes the shortcomings mentioned above, showing that it is important to include the capital stock when studying short run behaviour and providing an alternative formulation to the optimal investment decision on the part of firms. The model also includes variable capacity utilisation for two reasons. First, it leads to a flatter marginal cost curve[8] by making the marginal product of labour less responsive to changes in the labour input. Second, variable capacity utilisation amplifies technological shocks, allowing these to be of much lower than is required in standard RBCs[9].

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[5] Casares (2001)
[6]Such as Jeanne (1997), Rotemberg and Woodford (1999) and McCallum and Nelson (1997) among others. Alternatively, McCallum and Nelson(1999) simply treat investment as exogenous.
[7] This is the argument put forward by McCallum and Nelson (1997).
[8] See King and Rebelo (2000) p. 51.
[9] On this, the Prescott (1986a,b) Summers (1986) debate provides some useful insights.

The approach adopted in this paper is consistent with the argument put forward by Ball and Romer (1990), namely, that real rigidities are necessary in order to generate substantial effects of monetary shocks

The model presented in this paper contains four key features::sticky prices á la Calvo[10], predetermined investment decisions, variable capacity utilisation and some degree of real wage rigidity arising from nominal contracts.

The Calvo (1983) model will be used for ease of comparison with the results of other authors[11], as it has become the standard pricing formulation in recent research due to its tractability, despite some serious limitations. In particular the standard Calvo equation:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \phi \tilde{y}_t \qquad (2)$$

with $\tilde{y}$ representing the output gap, does not satisfy the natural rate hypothesis[12] since the latter implies that monetary policy cannot maintain a positive output gap by any sustained policy and that credible disinflations are expansionary. Moreover, as noted by Erceg (1999), a monetary policy that ensures complete inflation stabilisation implies output stabilisation, eliminating the tradeoff in the relative variabilities of inflation and output[13], as exemplified in the papers contained in Taylor (1999).

[10] The Calvo equation belongs to the time dependent class of pricing equations.
[11] This approach was pioneered by Yun (1996) among others.
[12] That is if $E(\hat{\pi}_t - \hat{\pi}_{t-1}) \neq 0$ then $E\tilde{y} \neq 0$.
[13] The Taylor menu as defined by Uhlig (2001).

2 The Model.

2.1 Firms.

The model is characterised by monopolistic competition in the intermediate-goods market. Final goods producing firms operate in perfectly competitive markets by combining a continuum of intermediate goods indexed $i \in [0,1]$.

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \qquad \theta \succ 1 \tag{3}$$

Letting P_t and P_{jt} denote the price of the final good and the intermediate good j at time t, respectively, profit maximisation implies:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{4}$$

with P_t described as the aggregate price level.

2.2 Households.

Households are identical their number is normalised to 1, so that aggregate and per capita quantities are the same in equilibrium (Jeanne 1997). The representative household maximises its utility function *U*, the expectation of the discounted sum of instantaneous (felicity) utility flows conditional on information available at time *t*.

The utility function has two arguments, consumption and real money balances. The utility function is of the CRRA and the following first and second order conditions are assumed to hold:

$$U = E_t \sum_{t=0}^{\infty} \beta^t u(c_t, m_t) \qquad (5)$$

$$U_c \succ 0$$
$$U_{cc} \prec 0$$
$$U_m \succ 0$$
$$U_{mm} \prec 0$$

$$\lim_{x \to 0} u_x(c,m) = \infty, \qquad \lim_{x \to \infty} u_x(c,m) = 0 \qquad for\ x = c, m$$

There are only two arguments in the utility function: the composite consumption good c_t[14]and real money balances ($\frac{M}{P}$). Households maximise their utility function at time *t* in the face of uncertainty, hence the expectations operator E_t is used.

2.3 The Production Function.

Households produce the output, $y_t(i)$ with a Cobb-Douglas production function containing four arguments: labour, n_t^d, capital, k_t, capacity utilisation, h_t and the state of technology, z_t. The production function $y_t = f(k_t h_t, n_t, z_t)$ is homogeneous of degree one in its inputs:

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[14] $c_t = \left(\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$

$$y_t(i) = (k_t h_t)^{\alpha} (n_t^d)^{1-\alpha} e^{z_t} \tag{6}$$

It is assumed that sales of the good produced by the household are determined by the demand function

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \tag{7}$$

where P_t ,Y_t denote aggregate levels of prices and output, respectively.

Households save a proportion of their output in order to increase the capital stock and replace depreciated capital**.** Moreover, the rate of depreciation will be dependent on the rate of capital utilisation. In particular, it will be assumed that the depreciation rate will be an increasing function of the rate of utilisation.

Therefore, the transition equation for the capital stock is of the form:

$$k_{t+1} = [1 - \delta(h_t)] k_t + x_t \tag{8}$$

And for the depreciation rate, the following form is specified:

$$\delta(h_t) = d + \frac{h_t^{1-\varpi}}{1-\varpi} \tag{9}$$

This functional form resembles the one presented by Greenwood et al (1988) except for the inclusion of the constant d. The reason for this is that otherwise the model's steady state solution, as will be seen below, yields an average rate of utilisation of implausibly low value[15].

In the case of the labour market, households supply one unit of their labour inelastically every period and they purchase labour inputs at a real wage rate w_t.[16]

The household's real budget constraint is:

$$\left(\frac{P_t(i)}{P_t}\right)^{1-\theta} Y_t + T_t = c_t + k_{t+1} - [1-\delta(h_t)]k_t + w_t(n_t - 1) + m_t - \frac{m_{t-1}}{1+\pi_t} + \frac{b_{t+1}}{1+r_t} - b_t \qquad (10)$$

The household maximises (5) subject to (6)-(10).

The Euler equations for the representative household are then:

$$u_{c,t} - \lambda_t = 0 \qquad (11)$$

$$u_{m,t} - \lambda_t + \beta E_t \frac{\lambda_{t+1}}{1+\pi_{t+1}} = 0 \qquad (12)$$

$$-\lambda_t w_t + \mu_t f_{n,t} = 0 \qquad (13)$$

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15 Greenwood et al's steady state capital utilisation rate yields a value of less than 30 per cent.

16 Hence in equilibrium n =1.

$$\delta'(h_t)k_t\lambda_t - f_{h,t}\mu_t = 0 \tag{14}$$

$$-\lambda_t + \beta E_t\lambda_{t+1}\left[f_{k,t+1} + 1 - \delta\right] = 0 \tag{15}$$

$$\frac{-\lambda_t}{1+r_t} + \beta E_t\lambda_{t+1} = 0 \tag{16}$$

$$(1-\theta)\lambda_t Y_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta}\frac{1}{P_t} + \theta\mu_t Y_t\left(\frac{P_t(i)}{P_t}\right)^{-(1+\theta)}\frac{1}{P_t} = 0 \tag{17}$$

In the case of equation (12) and taking into account the functional form already specified, this can be rewritten as:

$$\delta'(h_t)k_t = \left(\frac{\theta-1}{\theta}\right)f_{h,t} \tag{18}$$

which equates the marginal cost of higher capital utilisation resulting in a higher depreciation rate, to the marginal benefit, the increase in output. (18) can be simplified to:

$$h_t^{\gamma} = f_{k,t} \tag{19}$$

2.4 The Pricing Equation.

Equation (17) describes the optimal price, $P_t(i)$ for the monopolistic firm when it is able to change its price freely. In many similar models (e.g. McCallum and Nelson, 1997) the Calvo (1983) equation was assumed and inflation was dependent on the output gap as a proxy for marginal cost. Here, the dependence of inflation on marginal cost will be made explicit, yielding a New Keynesian Phillips curve with strong micro foundations.

It is assumed that each firm can re-optimise its nominal price with a constant probability equal to $1-\eta$, otherwise it will have to maintain the previous period's price with an exogenous probability equal to η[17]. The ability to re-optimise the price does not vary across firms and time.

Then, the Euler equation under these assumptions becomes:

$$\sum_{j=0}^{\infty}\left[(1-\theta)\beta^j\eta^j E_t\left(\lambda_{t+j}P_t(i)^{-\theta}P_{t+j}^{\theta-1}Y_{t+j}\right)+\theta\beta^j\eta^j E_t\left(\mu_{t+j}P_t(i)^{-(1+\theta)}P_{t+j}^{\theta}Y_{t+j}\right)\right]=0 \qquad (20)$$

Simplifying for $P_t(i)$, noting that the real marginal cost, s_t, is equal to $\frac{w_t}{f_{n,t}}$ and using equation (11) one can solve for the firm's optimal price:

[17] Christiano, Eichenbaum and Evans (2001) explore the implications of varying the criteria for firms unable to re-optimise.

$$P_t(i) = \left(\frac{\theta - 1}{\theta}\right) E_t \left[\frac{\sum_{j=0}^{\infty} \beta^j \eta^j \lambda_{t+j} s_{t+j} P_{t+j}^{\theta} Y_{t+j}}{\sum_{j=0}^{\infty} \beta^j \eta^j \lambda_{t+j} P_{t+j}^{\theta-1} Y_{t+j}} \right] \tag{21}$$

Hence in steady state:

$$\overline{P} = \frac{\theta}{\theta - 1} \bar{s} \overline{P} \tag{22}$$

since in equilibrium $P(i) = P$.

The linear approximation to (21) then yields the Calvo pricing equation[18]

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta} \hat{s}_t \tag{23}$$

This inflation equation has the characteristic of being forward-looking in nature, unlike older generation Keynesian pricing equations. Moreover, the relationship between inflation and marginal cost is strongly based on theory, with the coefficient on marginal cost having a precise meaning. Thus the frequency of price adjustment is the crucial factor affecting the sensitivity of the inflation rate to changes in marginal costs. Moreover, given the controversy surrounding the cyclical behaviour of marginal cost, this formulation avoids using the output gap as a proxy.

[18] The tildes above the variables denote percentage deviations from steady state.

2.5 Wages.

Having discussed the model in terms of marginal cost, which can be represented (in deviations from steady state) as the difference between the real wage and the marginal product of labour, it is now necessary to posit a formulation for the determination of the real wage. Here, an approach due to Casares (2001) will be used because, as will be seen, a small degree of real wage rigidity[19] will impart substantial persistence to the main variables of the model. It is assumed that there are two kinds of contracts. In the first, the nominal wage is equal to the previous period's wage adjusted by last period's expectation of the rate of inflation in the current period:

$$W_t = W_{t-1}\left(1 + E_{t-1}\pi_t\right) \tag{24}$$

which implies:

$$\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + E_{t-1}\hat{\pi}_t \tag{25}$$

The second contract adjusts nominal wages each period by the steady state rate of inflation:

$$W_t = W_{t-1}(1 + \overline{\pi}) \tag{26}$$

[19] Ball and Romer (1990) and Romer (1996) argue that real rigidities are necessary for sticky price models to perform well. This is also the approach adopted by Jeanne (1998).

Hence in linear form:

$$\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t \tag{27}$$

Assuming that a fraction ϕ of all contracts are applied using the first scheme, then the aggregate real wage equals:

$$\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + \phi E_{t-1}\hat{\pi}_t \tag{28}$$

For $\phi = 0$, all contracts simply adjust the real wage to the steady state rate of inflation and are thus not very sensitive to the business cycle. For $\phi = 1$, all contracts are adjusted to expectations of inflation and given that inflation varies over the business cycle so will the wage.

2.6 Monetary Policy.

Modelling actual UK monetary policy with a simple rule is notably difficult[20]. Nevertheless, Nelson (2001) has argued that the UK monetary authorities have followed a Taylor rule since 1992[21]. Here, it will be assumed that the monetary authorities follow a simple Taylor rule with smoothing:

$$R_t = (1-\mu_3)\left[\mu_1 E_{t-1}\pi_t + \mu_2 E_{t-1}\tilde{y}_t\right] + \mu_3 R_{t-1} + \varepsilon_{R,t} \tag{29}$$

[20] See Nelson (2001).
[21] Although issues relating to measurement and observability of the output gap are not tackled.

where μ_3 reflects the degree of interest rate smoothing thought to apply in practice and ε_{Rt} is the monetary policy shock. In an optimising context, the coefficients in the Taylor rule will normally be obtained from the minimisation of a loss function on the part of the monetary authorities[22]. In this context, it should be noted that the Taylor rule as described above is operational, as described by McCallum and Nelson. Further justification for the Taylor rule applying to the UK is the fact that it has been defined by the future governor of the Bank of England[23] as a 'restatement of the obvious'.

2.7 The Linear Model.

The above system of Euler equations and resource constraints can be linearised around the steady state yielding the following system:

Collecting all Euler equations and resource constraints:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{b(1-\Phi)}{\Phi}(\hat{m}_{t+1} - \hat{m}_t) - \frac{1}{\Phi}\hat{r}_t \tag{30}$$

$$\hat{m}_t = \hat{c}_t - \frac{1}{R}\hat{R}_t \tag{31}$$

$$\hat{R} = \hat{r}_t + E_t \hat{\pi}_{t+1} \tag{32}$$

[22] The loss function is normally a quadratic function, so that both positive and negative deviations of inflation and output from their respective targets are regarded as incurring the same loss.
[23] Mervyn King (1999), quoted in Nelson (2001).

$$\hat{r}_t = \alpha\beta \frac{Y}{K} E_t(\hat{f}_{k,t+1}) \tag{33}$$

$$\hat{f}_{k,t} = \hat{y}_t - \hat{k}_t \tag{34}$$

$$\hat{y}_t = \alpha\hat{k}_t + \alpha\hat{h}_t + (1-\alpha)\hat{n}_t + z_t \tag{35}$$

$$\frac{Y}{K}\hat{y}_t = \frac{C}{K}\hat{c}_t + \delta\hat{x}_t \tag{36}$$

$$\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \delta\hat{x}_t - h^{\gamma}\hat{h}_t \tag{37}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(1-\eta)(1-\beta\eta)}{\eta}\hat{s}_t \tag{38}$$

$$\tilde{y}_t = \hat{y}_t - \alpha\hat{k}_t - \alpha\hat{h}_t - z_t = (1-\alpha)\hat{n}_t \tag{39}$$

$$\hat{s}_t = \hat{w}_t - \hat{f}_{n,t} \tag{40}$$

$$\hat{f}_{n,t} = \hat{y}_t - \hat{n}_t \tag{41}$$

$$\gamma\hat{h}_t = \hat{f}_{k,t} \tag{42}$$

$$R_t = (1-\mu_3)\left[\mu_1 E_{t-1}\pi_t + \mu_2 E_{t-1}\tilde{y}_t\right] + \mu_3 R_{t-1} + \varepsilon_{R,t} \tag{43}$$

$$\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + \phi E_{t-1}\hat{\pi}_t \tag{44}$$

Compared to the simple sticky price model, the inclusion of variable capacity utilisation leads to higher flexibility of output in response to shocks. Hence technology shocks will have a greater impact under this set up, or alternatively, smaller shocks are required to produce empirically plausible output fluctuations. Nevertheless, as mentioned in the introduction, the model will display excessive elasticity of investment with respect to the real interest rate.

The approach adopted in this paper to overcome this high elasticity is to assume that some variables are chosen several quarters in advance. In particular, it will be assumed that households choose the levels of consumption and capital before they observe the shock. The assumption is crucial in the case of capital if the model is to yield positive results and is also the most empirically plausible. It seems unlikely that most investment expenditure would be chosen in the current period[24]. In this context the real rigidities arising from this assumption are similar to those from the time-to-build approach whereby new investments do not reach fruition until several quarters later. As in Christiano and Todd (1996), the it is assumed that firms decide on their investment expenditure that will come to fruition in the future, before the state of the economy is known.

[24] Christiano and Todd (1996) suggest that firms engage in 'Time-to-Plan', that is, part of the process of capital accumulation involves a period of planning expenditures.

Because the capital stock is chosen in advance, the Euler equation (33) is re-written as:

$$\hat{k}_{t+1} = E_{t-2}\left[\hat{y}_{t+1} - \frac{(1+\bar{r})}{\alpha\frac{Y}{K}}\hat{r}_t\right] \tag{45}$$

and for consumption, it should be noted that household optimisation sets current consumption as a function of expected future consumption and the real interest rate, hence it is forward looking in nature. If consumption is chosen one period in advance, the Euler equation (43) becomes[25]:

$$\hat{c}_t = E_{t-1}\left[\hat{c}_{t+1} - \frac{b(1-\Phi)}{\Phi}(\hat{m}_{t+1} - \hat{m}_t) - \frac{1}{\Phi}\hat{r}\right]_t \tag{46}$$

Having consumption chosen one period in advance is necessary because otherwise it becomes excessively volatile. The reason for this could be interpreted as consumption absorbing shocks that affect demand, given that investment expenditure has been predetermined.

There is an added effect when the capital stock and consumption are predetermined for several periods: the lagged response of the model economy to the different shocks affecting the system. The reason for postulating the above process for real wages and the

[25] This is the approach adopted by Rotemberg and Woodford (1999) to fit US data.

inclusion of expectational lags is to allow for some degree of real rigidity that maintains a relatively flat marginal cost curve.

3 Calibration.

To assess the quantitative performance of the model it is necessary to calibrate it. The model is calibrated to UK data. To ensure consistency with other related studies the values taken will be the same as those of Ellison and Scott (2000). The β, δ *and* α are set to 0.99, 0.025 and 0.4436 respectively. For technology, ρ_z = 0.95 and σ_z = 0.00925. Coefficients are chosen to ensure that steady state labour supply is equal to 0.31, and *b* is set to 0.005 as in Walsh (1998).

For the coefficients relating to the monopolistic firm: the elasticity of demand, θ, will be set to 2.64.

Table 1 presents a list of all calibrated values for the sticky-price variable utilisation model.

Table 1: Calibration	
Parameter	Value
α	0.4436
β	0.9939
δ	0.025
η	0.1
θ	2.64

ρ_z	0.95
σ_z	0.0925
σ_{eR}	0.0135
μ_1	1.27
μ_2	0.47
μ_3	0.17
$\bar{h}$	0.75

The value for the rate of time preference, β, is chosen so as to be consistent with a 0.6 per cent quarterly value for the interest rate[26]; The parameter η, from the pricing equation, reflects the probability that a firm will be unable to change its price in a given quarter. In choosing 0.1, we are adopting a much lower value than that found in the literature. The reason for this is that the inclusion of variable capacity utilisation and some degree of real wage rigidity generate a 'high-substitution economy', making the marginal cost curve more horizontal in the short run. As a result, the amount of price rigidity necessary to allow for shifts in demand to be satisfied is greatly reduced. For the real wage equation, the value ϕ=0.5 was used as a benchmark.

The coefficients on the monetary policy rule simply serve as a benchmark and are taken from Nelson (2001).

With respect to the steady state value of capacity utilisation, the long-run average from CBI surveys derived by Holland and Scott (1998) 0.75 will be used.

[26] Calculated as the difference between average nominal interest rate and inflation for the period 1955Q1 1997Q4.

4 Impulse Responses.

Figure 1 shows the percentage effect of technology shock equivalent to 1 standard deviation. Because both consumption and investment are predetermined at *t*, the technology shock has a lagged effect, and through the production function, there is a fall in employment, gradually returning to equilibrium. According to Galí (2000) there is some evidence in favour of this negative employment effect. Although this is characteristic of sticky price models this runs counter to traditional RBC models. Other responses resemble those of similar models. The inflation rate shows a sharp fall, falling 1.6 per cent before returning to long-run equilibrium, whereas the rise in investment expenditure and output are long lasting as a consequence of the persistence of the technology shock. Consumption peaks in the period after the shock as a consequence of being predetermined for one period. Nevertheless, as with investment and output, it remains above the steady state for a substantial period.

Compared to a simple sticky price model without expectations lags[27], the present model does not suffer from unrealistically high volatility of the endogenous variables, although employment is substantially volatile. This is because it acts as a buffer variable, absorbing the shocks since consumption and investment have been pre-determined.

How does this model economy react to a monetary policy shock? This is shown in Figure 2, where most of the variables display a high degree of persistence. The effect of the technology shock on output, capacity utilisation, investment and employment are remarkably similar, showing a gradual decline and reaching a trough after approximately three quarters. In the case of the real interest rate, the monetary policy shock and the falls

[27] Such as, for example the one presented by Ellison and Scott (2000).

in inflation combine to compound the effect on the real interest rate, rising by more than the percentage increase in the nominal rate.

The effect of the monetary policy shock on inflation is not persistent. However, it is not immediate either, rather, it takes two quarters for the effect of the shock to achieve its full impact. Furthermore, the falling inflation rate has had a positive effect on real wages. The lack of substantial persistence in the inflation process is not necessarily a drawback, however. Although models displaying sticky inflation rigidity are commonly used[28] it is not clear whether inflation persistence arises from the internal propagation mechanism or the economy or is simply a reflection that the money supply is a persistent process.

The remarkable characteristic of these impulse responses is the lagged nature of the response coefficients. Most structural, sticky-price models display an instantaneous response to shocks and then persistence is generated by positing adjustment costs. In this respect and in the quantitative response of the variables when compared to Bank of England estimates[29], this model shows a marked improvement over previous ones. The smaller effect on output from interest rate shocks estimated at the Bank of England could be achieved in this model if the expectations lags were longer, although this would come at the cost of becoming a less plausible assumption. Interestingly, the Bank of England estimates an initially negligible, or even positive, response of output to the interest rate shock, which is what the model below shows.

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[28] Nelson (1998) evaluates different dynamic optimisation models of inflation in their ability to explain its characteristics.
[29] See Bank of England (1997).

6 Conclusions.

Current sticky price models of the business cycle ignore capital or introduce investment adjustment costs in order to avoid some well known unrealistic responses to monetary policy shocks. Casares and McCallum (2000) have argued that sticky price models with endogenous capital/investment choices and no adjustment costs “appear to be less appropriate than ones with exogenous investment” (p. 28). This paper has presented a simple calibrated model where it is shown that this is not the case. Rather, the inclusion of an endogenous capital/investment decision can be crucial in understanding the propagation mechanism in response to shocks. By allowing the investment decision to be predetermined for more than one period, the model is able to overcome two key difficulties surrounding basic sticky price models, namely, the high volatility of the variables and the immediate impact of monetary policy shocks. In this context the one of the key results of the present model is particularly noteworthy: the lagged response of the endogenous variables to monetary shocks[30], a feature that most SDGE models have difficulty replicating.

If the above representation captures some important features of the propagation mechanism of business cycles, models that neglect investment and the capital accumulation process will ignore one of the primary channels in the monetary transmission mechanism and have the potential to yield misleading results when conducting monetary policy and business cycle analysis.

[30] This result has been found, for example, in Christiano et al (2001).

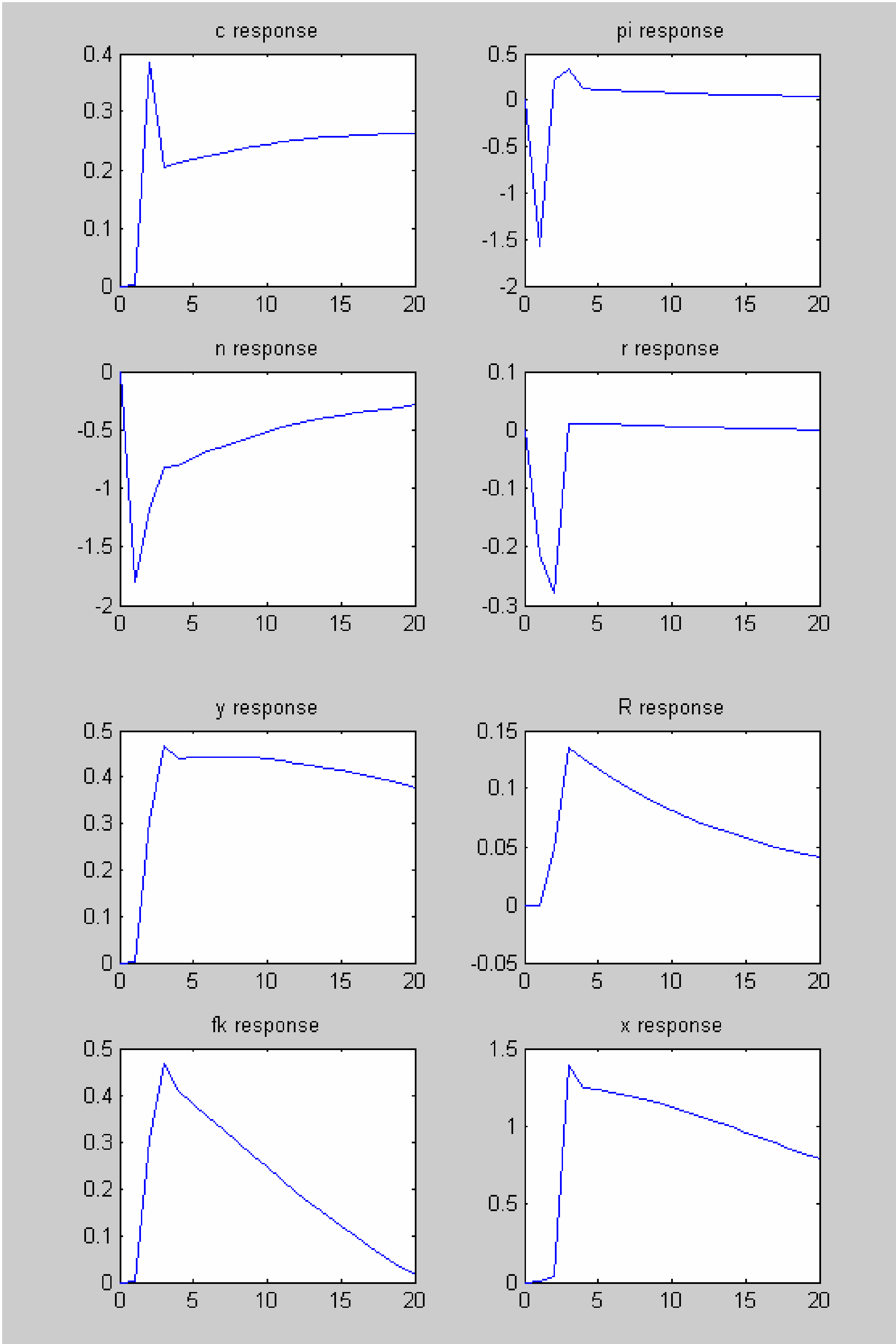

Figure 1:Impact of a Technology Shock.

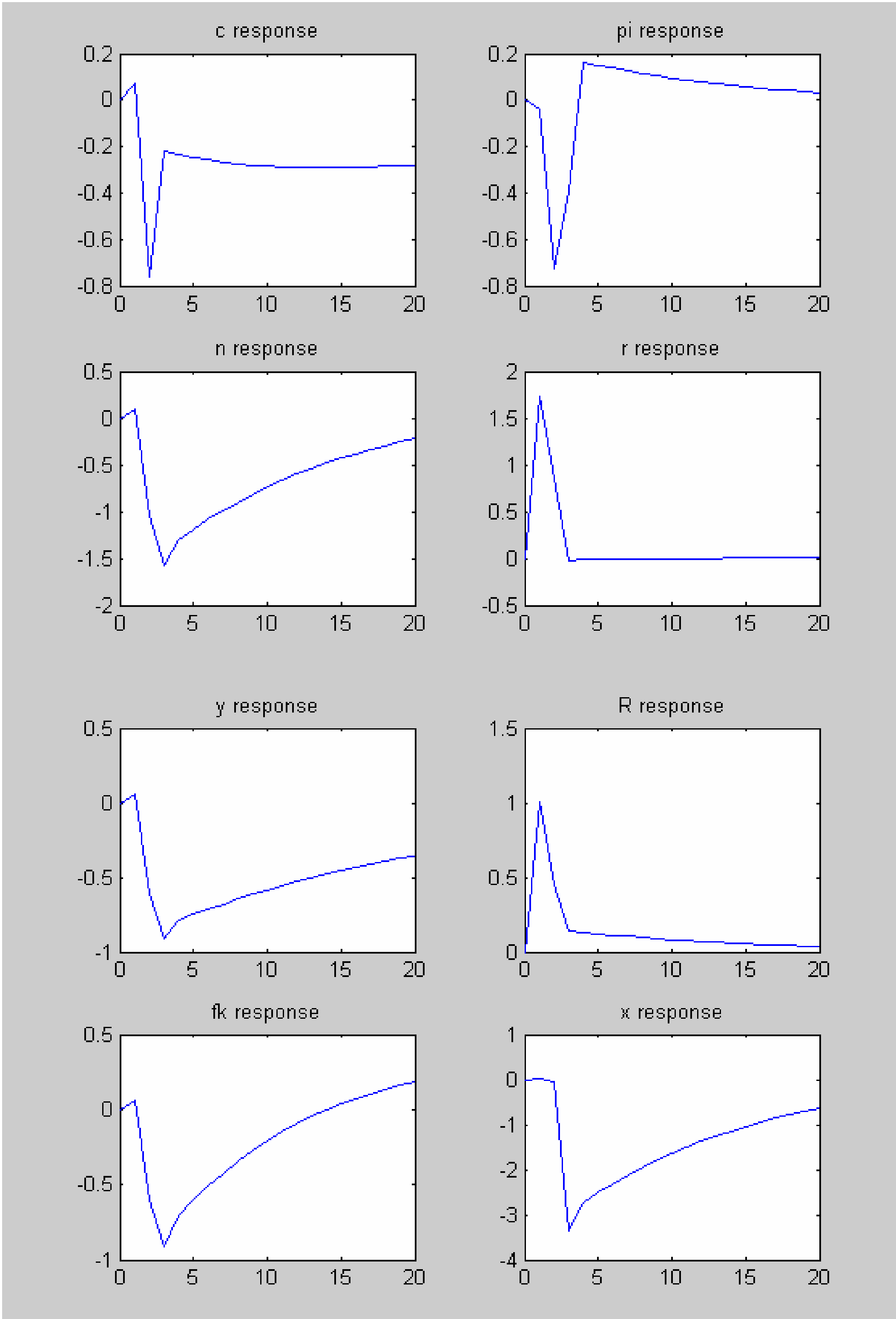

Figure 2: Impact of a Monetary Policy Shock.

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APPENDIX C

A SIMPLE SOLUTION TO LINEAR RATIONAL EXPECTATIONS MODELS BY UNDETERMINED COEFFICIENTS.

The method below extends the procedure suggested by McCallum (1999).

Let the model be of the form:

$$A_1 E_t y_{t+1} + A_2 x_{t+1} = A_3 y_t + A_4 x_t + A_5 u_t \tag{C.1}$$

$$x_{t+1} = B_1 y_t + B_2 E_{t-1} y_t + B_3 x_t + B_4 u_t \tag{C.2}$$

$$u_{t+1} = R u_t + \varepsilon_{t+1} \tag{C.3}$$

Where y is a *yx1* vector of endogenous choice variables, x is a *kx1* vector of endogenous state variables, u is a *ux1* vector of exogenous state variables and ε is a vector of white noise variables.

The minimum state variable (MSV) solution (McCallum 1983, 1998, 1999) is[31]:

$$y_t = \Omega x_t + \Gamma_1 u_{t-1} + \Gamma_2 \varepsilon_t \tag{C.4}$$

$$x_{t+1} = \Pi_1 x_t + \Pi_2 u_{t-1} + \Pi_2 \varepsilon_t \tag{C.5}$$

which implies:

[31] Alternative solution methods are provided by Christiano (1998), Urrutia (1998), Klein (1997).

$$E_t y_{t+1} = \Omega\Pi_1 x_t + (\Omega\Pi_2 + \Gamma_1 R)u_{t-1} + (\Omega\Pi_3 + \Gamma_1)\varepsilon_t \tag{C.6}$$

$$E_{t-1} y_t = \Omega x_t + \Gamma_1 u_{t-1} \tag{C.7}$$

Substituting these into (C.1) and (C.2):

$$A_1[\Omega\Pi_1 x_t + (\Omega\Pi_2 + \Gamma_1 R)u_{t-1} + (\Omega\Pi_3 + \Gamma_1)\varepsilon_t] + A_2[\Pi_1 x_t + \Pi_2 u_{t-1} + \Pi_3 \varepsilon_t] =$$
$$A_3[\Omega x_t + \Gamma_1 u_{t-1} + \Gamma_2 \varepsilon_t] + A_4 x_t + A_t(Ru_{t-1} + \varepsilon_t) \tag{C.8}$$

$$\Pi_1 x_t + \Pi_2 u_{t-1} + \Pi_3 \varepsilon_t = B_1(\Omega x_t + \Gamma_1 u_{t-1} + \Gamma_2 \varepsilon_t) + B_2(\Omega x_t + \Gamma_1 u_{t-1}) + B_3 x_t + B_4(Ru_{t-1} + \varepsilon_t) \tag{C.9}$$

Collecting terms in x_t:

$$\begin{bmatrix} A_1 & A_2 \\ 0 & I \end{bmatrix} \begin{bmatrix} \Omega\Pi_1 \\ \Pi \end{bmatrix} = \begin{bmatrix} A_3 & A_4 \\ (B_1 + B_2) & B_3 \end{bmatrix} \begin{bmatrix} \Omega \\ I \end{bmatrix} \tag{C.10}$$

For u_{t-1}:

$$A_1(\Omega\Pi_2 + \Gamma_1 R) + A_2\Pi_2 = A_3\Gamma_1 + A_5 R \tag{C.11}$$

$$\Pi_2 = (B_1 + B_2)\Gamma_1 + B_4 R \tag{C.12}$$

For ε_t:

$$A_1(\Omega\Pi_3 + \Gamma_1) + A_2\Pi_3 = A_3\Gamma_2 + A_5 \qquad \text{(C.13)}$$

$$\Pi_3 = B_1\Gamma_2 + B_4 \qquad \text{(C.14)}$$

Letting A and B denote the two square matrices in (C.10), the QZ decomposition can be applied to both A and B. This will provide two unitary (invertible) matrices, Q and Z, such that $QAZ = S$ and $QBZ = T$, where both S and T are triangular.

Premultiplying (C.10) by Q yields:

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix}\begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix}\begin{bmatrix} \Omega\Pi_1 \\ \Pi_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix}\begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix}\begin{bmatrix} \Omega \\ I \end{bmatrix} \qquad \text{(C.15)}$$

The second row can be written as:

$$S_{22}(H_{21}\Omega + H_{22})\Pi_1 = T_{22}(H_{21}\Omega + H_{22}) \qquad \text{(C.16)}$$

which will be satisfied for:

$$\Omega = -H_{21}^{-1}H_{22} \qquad \text{(C.17)}$$

Writing out the first row and using the solution in (C.17):

$$\Pi_1 = (H_{11}\Omega + H_{12})^{-1}S_{11}^{-1}T_{11}(H_{11}\Omega + H_{12}) \qquad \text{(C.18)}$$

For the coefficients on u_{t-1}, combining (C.11) and (C.12) yields:

$$vec(\Gamma_1) = (I + R' \otimes (D_2^{-1} A_1)) vec(D_2^{-1} D_3) \qquad \text{(C.19)}$$

for

$$D_2 = (A_1 \Omega + A_2)(B_1 + B_2) - A_3;$$

and

$$D_3 = (A_5 - D_1 B_4) R$$

Π_2 can be found directly from (C.12).

Finally, for ε_t, combining (C.13) and (C.14):

$$\Gamma_2 = (D_1 B_1 - A_3)^{-1} (A_5 - D_1 B_4 - A_1 \Gamma_1) \qquad \text{(C.20)}$$

Again, Π_3 can be found from (C.14) directly.