Can Sectoral Shifts Generate Persistent Unemployment in Real Business Cycle Models?

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Abstract

This paper extends the standard Real Business Cycle (RBC) model to incorporate sectoral shifts in unemployment. Using relative sectoral technology and sectoral tastes shocks, combined with labor adjustment costs across sectors, we assess the possibility of generating persistent aggregate unemployment. We calibrated the models to Canadian data and found that the introduction of sectoral labor mobility with adjustment costs improves the ability of the standard real business cycle model to match the observed persistence in unemployment. Empirically, we estimated a Vector Auto-Regressive model (VAR) and successfully matched the models' overshooting of labor due to the adjustment costs. The results suggest that government policies aimed to alleviate the unemployment burden should pay closer attention to sectoral phenomena, specifically to sectoral labor mobility.

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1 Introduction

Well documented stylized facts regarding observed employment variability and output persistence pose a hurdle for Real Business Cycle (RBC) models' success.[1] Relative to the stylized facts, standard RBC models consistently generate lower variability in employment and lower persistence in unemployment. Prescott (1986) reported that observed employment is twice as volatile as the one simulated from the standard RBC economy. Standard RBC models generate a substantially smaller volatility in employment than that in the data.[2] Campbell (1994) argued that explaining a decline of three percent employment in recession, requires one to assume a seven percent decrease in technology, a number which is obviously unrealistic. Further, regarding persistence and variability, Cogley and Nason (1995) concluded that actual output dynamics are more persistent than those generated from standard RBC models.[3] The failure of the standard RBC models to generate an adequate match for employment variability and the absence of a strong propagation mechanism sparked the search for alternatives that could generate the observed employment variability and unemployment persistence.[4]

Persistence in unemployment has long been investigated at the theoretical and applied levels (Hall 1998, p. 34). Whenever evidence of unemployment persistence is found, how fast policies can decrease the unemployment rate depends on the persistence mechanism. Also, in the pres-

[1] For a complete review of RBC models, refer to the series of discussion papers in the Economic Journal (1995) and Stadler (1994).
[2] In U.S. data, the variance of hours worked relative to the variance of output equals 0.95 percent. A standard RBC model generates a ratio of 0.52 percent.
[3] They pointed out that this heavy dependence and similarity of characteristics between the shock and the simulated series is only a symptom of a weak propagation mechanism that project the shock characteristics onto the simulated series.
[4] For example, Boldrin, Christiano and Fisher (2000) outlined that habit persistence and limited labor mobility were necessary to generate [output] persistence.

ence of unemployment persistence, any disinflation policies based on the unemployment rate can prove very costly in terms of lost output.

While many have studied the impact of aggregate variability on regional fluctuations, few studies have investigated the impact [5] of sectoral/regional factors on aggregate variability. Several recent studies suggested different mechanisms by which the law of large numbers [6] can be weakened. Mechanisms such as asymmetries, threshold effects, non-linear settings and monopolistic competition have proved useful in modelling the effects of inter-sectoral shocks on aggregate employment (see Boldrin and Woodford (1990) and Scheinkman (1990)). For example, by avoiding the law of large numbers and assuming that *some sectors are more important* input-suppliers than others, Horvath (1997) simulated greater aggregate volatility from sector-specific shocks.

Lilien's (1982) observation [7] of sectoral labor mobility led Davis (1987) to argue that allocative

[5] Few investigated the implications of sectoral shocks on the aggregate economy. Phelan and Trejos (2000) showed that isolated sectoral shifts can have important aggregate implications, even if the size of the 'impulse' is small [a one-time permanent military cut-back shock in the 1990s]. They concluded that a one-time change in the fundamentals (technologies) that determine the sectoral composition of the economy could prompt a significant downturn, which persisted and propagated across sectors into a recession. Dupor (1996) considered the aggregate effects of sector-specific shocks to production. The study concluded that the law of large numbers holds and that such a modelling strategy is unnecessary to explain the business cycle character. If all sectors are *equally important*, and labor were mobile between sectors, then the sectoral law of large numbers implies that their effect on the aggregate economy would average out to zero. Therefore, one can neglect the effects of sectoral shocks on the aggregate economy. Murphy, Shleifer and Vishny (1989) argued that labor immobility across sectors is of central importance in explaining cross-sectoral movement of outputs and labor inputs. For models that emphasized the importance of sectoral phenomena, see Cooper and Haltiwanger (1990), Basu and Fernald (1997), and Horvath (1997).

[6] The sectoral law of large number states that 'given that the economy is made out of a large number of sectors, a sectoral shock to the economy will move labor between sectors and will have no effect on the aggregate level of activity'.

[7] Using time series analysis, Lilien (1982) argued that half of the variance in unemployment is due to sectoral labor mobility.

disturbances [8] can have a large influence on aggregate unemployment fluctuations. [9] In general, sectoral shock models focus on the costly adjustment of labor between sectors. These models assume that the unemployed workers spend time searching for a match when moving between sectors (search unemployment) or incur training costs to join a different sector (structural unemployment). In this setup, the sectoral law of large numbers does not hold (because of the adjustment costs), and recessions are periods of costly inter-sectoral labor adjustment. There is a now growing consensus on the importance of the sectoral shifts hypothesis. In our view, multi-sector analysis is crucial in explaining unemployment persistence.

The empirical observation that motivates the models in this paper is that of unemployment persistence. Our objective is to construct a model that generates a similar to the data unemployment persistence. To achieve this, we enrich the standard RBC model with inter-sectoral labor mobility combined with labor adjustment costs. [10] We use employment dynamics at the

[8] Davis (1987) argued that allocative disturbances have a large influence on aggregate unemployment fluctuations. By 'allocative disturbances', Davis (1987, p. 326) meant "... the events that impinge on the economy by inducing a costly, time-consuming reallocation of specialized resources ...". Economists are aware of the important potential of allocative disturbances on aggregate fluctuations since Ricardo's *Principles* in 1817. However, the idea of using 'allocative disturbances' as a channel for a propagation mechanism in business cycle models was only presented after Lilien's (1982) observation.

[9] Samson (1985) found evidence that the 'dynamic reallocation' model best fit the Canadian data. Mills, Pelloni and Zervoyianni (1996) tested for the presence of the sectoral shifts hypothesis in U.K. data. Lu (1996) used both quarter and annual data on both one-digit and two-digit U.S. code industries, and reported no evidence of the sectoral shifts hypothesis. The study concluded that the significance of Lilien' results diminished at quarterly level data suggesting that Lilien results might be a special case. Corak and Jones (1995) investigated the influence of sectoral unemployment benefits on the persistence of aggregate unemployment. They concluded that no evidence of a direct mechanism - through which the unemployment benefits overhaul in 1977 influenced the level and the persistence of aggregate unemployment - was found. Others explored the sectoral model implications to: assess the substitution between labor supply decisions across sectors (see Cooper and Haltiwanger (1990)), or to explain aggregate increasing returns to scale and the procyclicality of aggregate productivity (see Baily, Hulten and Campbell (1992), Burnside (1996), and Basu and Fernald (1997)). Long and Plosser (1983) and Horvath (1997) emphasized multi-sector models wherein 'intermediate input linkage' generated aggregate persistence [but not for the employment level].

[10] Cogley and Nason (1995, p. 492) concluded that "Models that incorporate labor adjustment costs

sectoral level to generate fluctuations in output and to generate unemployment persistence that will match empirical regularities. We argue that persistent aggregate unemployment is a result of sectoral phenomena - such as relative technology shocks or relative product demand shocks - and emerges due to adjustment costs to labor mobility across sectors. Specifically, we integrate a two-sector framework into a stochastic general dynamic equilibrium model to assess the validity of Lilien's hypothesis.

The paper is organized as follows. Section 1.1 describes the models. Section 2 explains the intuition of the models. Sections 3 and 4 discuss the size of the shock, the calibration and sensitivity to the calibrated parameters. Section 5 reports and discusses the results of the models. Section 6 addresses one important aspect of the results and presents the empirical Vector-Autoregressive (VAR) estimations. Finally, Section 7 concludes.

This paper presents two multi-sector RBC models.[11] For simplicity, both models use a log-linear utility function[12] that allows for a convex cost function $c(\Delta N_{1t}, \Delta N_{2t})$ to capture the costly movement of labor between sectors. The $c(.,.)$ function can be viewed as capturing search unemployment (the time invested by workers in finding a job) or structural unemployment (due

are partially successful."

[11] Another weakness of the standard RBC is its inability to predict [match] the positive serial autocorrelation in business cycle output growth rates. Real output growth rates are positively serially correlated and the serial autocorrelation is significantly higher than zero for lags of one and two quarters (Cogley and Nason 1995). This discrepancy between model generated and business cycle data is present in a wide class of standard RBC models. To match this serial autocorrelation, Schmitt-Grohé (2001) emphasized a model with sector-specific external increasing returns to scale, Burnside, Eichenbaum and Rebelo (1993) proposed employment lags in the labor hoarding process, and Bills and Cho (1994) emphasized the use of *adjustment costs*. Oi (1962) suggested that these costs explain the stylized fact of productivity leading output. By treating labor as a quasi-fixed factor, wherein booms, firms increase their output but the labor input is a quasi-fixed factor, training costs are modelled as labor adjustment costs. Empirically, Weinberg (1999, p. 23) reported evidence of slow employment adjustment process. Others suggested solving the problem at the impulse level - instead at the propagation mechanism level - by using an AR(2) technology shock or government shock.

[12] The log-linear utility is selected to induce an intertemporal elasticity of substitution for leisure that equals to one.

to retraining costs when switching between sectors).

The single consumer is assumed to be representative of the society as a whole.[13] Given a single agent in this economy and convexity, there is a unique optimum to this maximization problem. This optimum is the unique competitive equilibrium allocation and supports the Pareto optimum.[14] Therefore, one can solve for the social planner's problem using concave programming techniques. Representative agents' preferences are represented by a utility function that is time separable and state independent. The two sectors are characterized by a strong complementarity in the production process and a highly specialized labor input.

[13] For a comprehensive development of the representative agent in macroeconomics modeling, refer to Hartley (1997). A change in the level of her utility reflects and is equivalent to a change in the overall level of social welfare. An increase (decrease) in her utility implies an improvement (loss) in social welfare. The actual numerical value of utility is irrelevant. A change in the utility level provides a measure of the direction of welfare change. Dinwiddy and Teal (1988, p. 104) noted that "This convention is commonly used by economists wishing to abstract from questions of distribution in order to concentrate upon problems dealing with the allocation of resources." With only a representative consumer, questions regarding the distribution of wealth do not arise. If there are two or more consumers, with differing factor endowments and/or utility functions, then economic change will clearly have different consequences for each. For simplicity, assume the case of two consumers. If both gain or both lose, the calculation of welfare change is unambiguous. However, should one gain and the other lose, computing the value of welfare change is difficult without some explicit value judgments (e.g., the Nash equilibrium, the Bergson-Samuelson welfare function).

[14] Given local nonsaturation and no externalities, competitive equilibria - which exists for this l_∞ commodity space economy (Bewley (1972) theorems) - are Pareto Optima (using the competitive welfare theorems of Debreu (1954)). The space l_∞consists of all sequences $x = (x_1, x_2, ...)$, $x_n \in R$, that are bounded in the norm $\| x \|_\infty = \sup_i | x_i |$. This space is important for the two welfare theorems. The space l_∞ ensures that assumptions 15.3 and 15.5 (Stokey and Lucas [with Prescott] (1989, p. 455)) hold for the preferences and technologies of interest. For infinite horizon stochastic optimal growth models, any space of the l_p spaces other than l_∞ causes serious difficulties. Stokey and Lucas (1989) defined this space (pp. 447-449), emphasized its role in the two welfare theorems (pp. 458-460), and explained its extension to stochastic growth models (p. 462).

1.1 MODEL I (Sectoral Technology Shocks)

To study the dynamics of a two-sector model in industries which are characterized by strong complementarities in the production process and a highly specialized labor input, Model I makes the following assumptions: a) Since many manufacturing processes can be characterized by fixed or almost fixed proportions, the representative firm's production function is assumed to exhibit perfect complementarity in the labor input across sectors and constant returns to scale between labor and capital, b) the representative agent incurs a cost in terms of leisure to move labor across sectors, c) the cost function is quadratic and d) the sector-specific shock to the labor input in sector i is inversely symmetric to the one in sector j. The first three assumptions reflect labor specialization in each sector, which imposes a cost to move between sectors. Assumption d) is necessary for the shocks to be 'pure' allocation shocks.

Specifically, assumption d) is made to ensure that a sector-specific technology shock does not shift the aggregate production function. Since there are only two sectors, a relative shock to sector 1 implies a shock in reverse direction - and *equal* in magnitude - in sector 2. Therefore, labor demand increases in sector 1, and decreases in sector 2. This setup ensures that the aggregate production function is stable and any employment variation in the model is to be considered as structural, not aggregative. This symmetry is useful for investigating 'pure' sectoral' shock effects. These technology shocks shift the sectoral labor demands and leave the aggregate production function intact. Relative to sector 2, a shock to sector 1 increases the labor demand in sector 1 and decreases it in sector 2. Without symmetry, it is difficult to isolate the effects of a sectoral shock from those of a general productivity shock, since all shocks would entail a mixture of both. The social planner is assumed to be faced with the following problem.

$$\max_{(C_t, K_{t+1}, N_{1t}, N_{2t})_{t=0}^{\infty}} \left\{ E \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \gamma \ln(T - N_{1t} - N_{2t} - c(\Delta N_{1t}, \Delta N_{2t})) \right] \right\} \tag{1}$$

subject to

$$Y_t = A K_t^{\alpha} \left[\min(\theta_1 N_{1t}, \theta_2 N_{2t}) \right]^{1-\alpha} \tag{2}$$

$$I_t = K_{t+1} - (1-\delta) K_t \tag{3}$$

$$c(\Delta N_{1t}, \Delta N_{2t}) = d \cdot \left[f(N_{1t} - N_{1t-1}) \right]^2 + d \cdot \left[f(N_{2t} - N_{2t-1}) \right]^2 \tag{4}$$

$$C_t + I_t \leq Y_t \tag{5}$$

$$T \geq N_{1t} + N_{2t} + c(\Delta N_{1t}, \Delta N_{2t}) \tag{6}$$

$$N_{1t} \geq 0 \qquad N_{2t} \geq 0 \tag{7}$$

where $f(z) \equiv \max(z, 0)$, $c(\Delta N_{1t}, \Delta N_{2t})$ denotes the cost function to move labor between sectors 1 and 2, with d denotes a cost parameter and T is the total time endowment of the agent. So there is a cost only if there is an increase in employment. The representative firm chooses the minimum level of employment. If employment increases in sector 1, it decreases in sector 2. Moving employment to sector 1 from sector 2 will impose a cost on the representative agent in terms of lost leisure. A is the aggregate shock (here constant). θ_i denotes the sector-specific shock. The shock θ_i follows a Markov process[15] which is governed by the following transition probability matrix

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{21} \\ \lambda_{12} & \lambda_{22} \end{pmatrix} \tag{8}$$

where $\lambda_{ij} \equiv \Pr(z_t = j | z_{t-1} = i)$. In this setup, the Bellman equation to be solved by the social planner, subject to the above constraints, is

[15] For the theoretical derivations and implications of Markov processes, see Norris (1997).

$$
\begin{aligned}
v(N_{1t-1}, N_{2t-1}, K_t, z_t) = \max_{(N_{1t}, N_{2t}, K_{t+1})} & \{\ln C_t + \gamma \ln[T - N_{1t} - N_{2t} \\
& -d \cdot (f(N_{1t} - N_{1t-1}))^2 - d \cdot (f(N_{2t} - N_{2t-1}))^2] \\
& +\beta E_t v(N_{1t}, N_{2t}, K_{t+1}, z_{t+1})\} \qquad (9)
\end{aligned}
$$

where z denotes the state of the economy (θ_1, θ_2). Given the imposed symmetry of the problem by the 'min' function between sector 1 and sector 2 technologies, we define the sector specific shock as $\theta_1 \equiv 1/\theta_2$ and let $\theta \equiv \theta_1$. Under the symmetry condition, the Bellman equation for being in state 1 can be rewritten as,

$$
\begin{aligned}
v(N_{1t-1}, N_{2t-1}, K_t, 1) = \max_{(N_{1t}, N_{2t}, K_{t+1})} & \{\ln C_t + \gamma \ln[T - N_{1t} - N_{2t} \\
& -d \cdot (f(N_{1t} - N_{1t-1}))^2 - d \cdot (f(N_{2t} - N_{2t-1}))^2] \\
& +\beta[\lambda_{11} v(N_{1t}, N_{2t}, K_{t+1}, 1) \\
& +\lambda_{12} v(N_{1t}, N_{2t}, K_{t+1}, 2)\} \qquad (10)
\end{aligned}
$$

We also impose a symmetry condition on the transition matrix Λ. The transition probability to move from state 1 to state 2 (λ_{12}) equals the transition probability to move from state 2 to state 1 (λ_{21}). In this setup, the disequilibrium wage differentials that will exist across sectors are eliminated when the labor input is perfectly mobile and the cost function $c(\Delta N_{1t}, \Delta N_{2t})$ equals zero.

Following a shock, employment falls in sector 1 and rises in sector 2. However, in sector 1, firms may choose not to reduce employment to the minimum level implied by its production function, thereby firing all unproductive workers, because keeping some of these workers is expected to reduce the adjustment costs of increasing employment as its desired output increases in the following period. Note that, the labor thus hoarded does not produce any output in the current period. The derivative of the utility function with respect to sector 1 employment is,

$$\frac{\partial U_t}{\partial N_{1t}} = \frac{-\gamma}{L_t} + \beta \left[\lambda_{11} \frac{\gamma}{L_{t+1_{|z=1}}} d\Delta N_{1_{|z=1}} + \lambda_{12} \frac{\gamma}{L_{t+1_{|z=2}}} d\Delta N_{1_{|z=2}} \right] \geq 0 \tag{11}$$

$$\lambda_{11} \frac{\Delta N_{1_{|z=1}}}{L_{t+1_{|z=1}}} + \lambda_{12} \frac{\Delta N_{1_{|z=2}}}{L_{t+1_{|z=2}}} \geq \frac{1}{dL_t \beta} \tag{12}$$

$$\lambda_{11} \frac{N_{1t} - N_{1t-1}}{L_{t+1_{|z=1}}} + \lambda_{12} \frac{\theta^2 \left(N_{1t} - N_{1t-1} \right)}{L_{t+1_{|z=2}}} \geq \frac{1}{dL_t \beta} \tag{13}$$

where L_t denotes leisure in period t. The last inequality is derived using Table 1, which summarizes the change in sectoral employment as a function of the state of the economy.[16]

[Insert Table 1 here]

At the simulated optimal solution, we verified that the effect of an increase in N_{1t} is negative with respect to sector's 2 employment, so that workers are always employed in fixed proportions between the two sectors, i.e., there is no labor hoarding. The reason for maximizing over sector 1 labor and capital is as follows. Given the perfect complementarity between sector 1 and sector 2 labor, there will always exist a fixed proportion between them. Therefore, maximizing over the grid of sector 1 labor and then computing sector 2 labor from this value is similar to maximizing over both values of sector 1 and sector 2 labor. See section 4 for details.

[16] The first term on the right hand side of equation (11) is the present cost of increasing labor in sector 1 in terms of lost leisure. This cost is a function of the weight of leisure in the utility function. The second term is the discounted value of the expected future marginal utility benefit arising as a consequence of increasing labor in sector 1 in the current period. Note that this depends on the state of the shock in the next period. In the case of an interior solution, a positive marginal benefit implies that firms in sector 1 are inclined to hoard labor in the current period, since that it would reduce the adjustment costs in the following period. Equation (13) gives the marginal benefit from increasing employment in sector 1 above the fixed proportions level.

1.2 MODEL II (Sectoral Taste Shocks)

We assume the following: a) the representative firm's production technology is identical for both sectors, b) the representative agent incurs a cost in terms of leisure to move labor across sectors, c) the cost function is quadratic, and d) the sector-specific tastes shocks to consumption are inversely symmetric. In brief, we adopt the same assumptions as for Model I except that there is no capital in this economy. We explicitly model the two goods' markets and study the dynamics of the economy subjected to tastes shocks. The social planner is faced with the following problem.

$$\max_{(C_{1t},C_{2t},N_{1t},N_{2t})_{t=0}^{\infty}} \left\{ E\sum_{t=0}^{\infty}\beta^t\left[\theta_1\ln C_{1t}+\theta_2\ln C_{2t}+\gamma\ln(T-N_{1t}-N_{2t}-c(\Delta N_{1t},\Delta N_{2t}))\right]\right\} \quad (14)$$

subject to

$$\begin{aligned} C_{1t} &= A_1 N_{1t}^{1-\alpha} && (15)\\ C_{2t} &= A_2 N_{2t}^{1-\alpha} && (16)\\ c(\Delta N_{1t},\Delta N_{2t}) &= d\cdot(f(N_{1t}-N_{1t-1}))^2 + d\cdot(f(N_{2t}-N_{2t-1}))^2 && (17)\\ T &\geq N_{1t}+N_{2t}+c(\Delta N_{1t},\Delta N_{2t}) && (18)\\ N_{1t} &\geq 0 \qquad N_{2t}\geq 0 && (19) \end{aligned}$$

where the same notation as Model I applies. We assume that $A = A_1 = A_2$. Let θ_i denotes the sector-specific tastes shock. The Bellman equation solved by the social planner is,

$$\begin{aligned} v(N_{1t-1},N_{2t-1},z_t) = \max_{(N_{1t},N_{2t})} & \{\theta_1\ln C_{1t}+\theta_2\ln C_{2t}+\gamma\ln[T-N_{1t}-N_{2t} \\ & -d\cdot(f(N_{1t}-N_{1t-1}))^2-d\cdot(f(N_{2t}-N_{2t-1}))^2] \\ & +\beta E_t v(N_{1t},N_{2t},z_{t+1})\} \qquad (20) \end{aligned}$$

where z denotes the state of the economy (θ_1, θ_2). Again, given the symmetry of the problem imposed by assumption (c), one can define the sector specific shock as $\theta_1 \equiv 1/\theta_2$ and let $\theta \equiv \theta_1$. The Bellman equation for being in state 1 and in state 2 can be rewritten as

$$\begin{aligned} v(N_{1t-1}, N_{2t-1}, 1) = \max_{(N_{1t}, N_{2t})} & \{\theta \ln C_{1t} + \theta^{-1} \ln C_{2t} \\ & +\gamma \ln[T - N_{1t} - N_{2t} \\ & -d \cdot (f(N_{1t} - N_{1t-1}))^2 - d \cdot (f(N_{2t} - N_{2t-1}))^2] \\ & +\beta[\lambda_{11} v(N_{1t}, N_{2t}, 1) + \lambda_{12} v(N_t, N_{2t}, 2)]\} \end{aligned} \tag{21}$$

$$\begin{aligned} v(N_{1t-1}, N_{2t-1}, 2) = \max_{(N_{1t}, N_{2t})} & \{\theta^{-1} \ln C_{1t} + \theta \ln C_{2t} \\ & +\gamma \ln[T - N_{1t} - N_{2t} \\ & -d \cdot (f(N_{1t} - N_{1t-1}))^2 - d \cdot (f(N_{2t} - N_{2t-1}))^2] \\ & +\beta[\lambda_{21} v(N_{1t}, N_{2t}, 1) + \lambda_{22} v(N_t, N_{2t}, 2)]\} \end{aligned} \tag{22}$$

This model (Model II) is similar to Model I in terms of wage differentials whenever the cost function is zero and labor is perfectly mobile. For the computation of real output in Model II see Endnotes, no. 1.

2 The Models' Intuition

Allowing a two-sector framework is one way of introducing the missing dynamics and strengthening the weak propagation mechanism in the standard RBC model. The mechanism by which workers lose jobs in response to an adverse technology shock and the slow process of re-employment, is the propagation mechanism of the persistent periods of slack. Assuming that optimizing agents encounter no market failure and that productivity shocks are serially independent across sectors, a sector-specific shock will have its primary effect on the originating sector.

This effect will depend on how large or small this sector is relative to the economy. Knowledge of these effects will allow policy makers to address unemployment in a more appropriate sectoral manner instead of just focusing on the aggregate level.

The aggregate production function exhibits constant returns to scale in Model I. This assumption reflects the empirical assessment of the Canadian production structure reached in Paquet [17] and Robidoux (1997). For Model II, each sectoral production function is constant. If one adds a fixed and sector specific amount of capital, say $\overline{K}^{\alpha}$, to each production function, then the modified production functions would exhibit constant returns to scale.

Model I emphasizes *relative* sectoral technology shocks. The argument is based on the following. While technological change leads to job losses in certain industries - e.g., in the manufacturing sector - it does not imply that employment must fall at the aggregate level. Therefore, we adopted a relative technology shock to keep the aggregate level insulated from the shock, while allowing for differential sectoral responses. The only reason for unemployment in both models is the labor reallocation process, which is costly and not instantaneous.

Some economists have argued that, with similar technological trends in the U.S. and Canada, it is unlikely that technological change can lead to a relatively high and persistent unemployment in Canada when it does not have that effect in the U.S. (Sharpe (1999, p. 31)). We view this argument as flawed for the following reason. While it is widely accepted that both countries tend to face similar technological trends, the Canadian economy suffers from *gaps* across the

[17] Once the Solow residuals were corrected for capacity utilization in the U.S. and Canada, Paquet and Robidoux (1997) concluded that - over the period from 1962Q1 to 1993Q4 and from 1970Q1 to 1993Q4 for the U.S. and Canada, respectively - the U.S. and Canadian market structures were well described by constant returns to scale.

spectrum of industries. Relative to the U.S. economy, some industries are non-existent in the Canadian economy and others are poorly represented. These gaps impinge on workers mobility, making labor movement across industries more difficult and time consuming - e.g., the aerospace and manufacturing industries. We argue that, faced with a technology shock similar to that in the U.S., the Canadian economy will experience higher persistence in terms of output and unemployment deviations, and that these can be captured by labor adjustment costs. This persistence is also affected by the particular nature of the institutional structures and public programs in Canada.

Due to the presence of the 'min' function in the production function in Model I, at the steady state

$$\theta N_1 = \theta^{-1} N_2 \tag{23}$$

$$N_2 = \theta^2 N_1 \tag{24}$$

and total labor supply equals $N_1 + N_2 = (1 + \theta^2)N_1$. In Model II, total employment equals to the sum of employment in the two sectors. During recessions, matching workers to jobs is time-consuming and costly in terms of lost time. Following an adverse relative sectoral shock, jobs are destroyed in one sector while new ones are created in the other one. Workers are willing to move to the sector with the high demand for labor but have to engage in a search process. This search process increases non-cyclical unemployment. Some workers find themselves with the wrong skills to move to the other sector, and as time goes on, other unsuccessful job finders suffer a deterioration of skills. These raise non-cyclical unemployment. Therefore, an adverse shock increases the natural rate of unemployment and decreases output. In this paper, the aim is not to explain the job search or the loss of skills processes. Our focus is on explaining the increase in the natural rate of unemployment through a sectoral shock. The impulse in Model I, is a relative technology shock, while in Model II, the impulse is a relative tastes shock that increases the product demand in one sector and reduces it in the other.

To explain the sharp rise in unemployment during recessions, one is inclined to make use of adjustment costs to labor mobility. These costs impinge on labor mobility following an adverse productivity shock. As mentioned earlier, these costs can be interpreted as 'search costs' or 'retraining costs'. The former provides an explanation of the increase in frictional unemployment, while the latter provides an explanation of the increase in structural unemployment. The end result is that an adverse sectoral supply shock (sectoral productivity shock) will increase unemployment and reduce output.

If one is able to quantify the magnitude of the increase in the natural rate of unemployment relative to the general level of unemployment from the model, then a clear policy response is in sight. At the aggregate level, the problem is the following. The unemployment rate increases sharply during recessions. Part of this increase is due to an increase in the natural rate[18] and part is due to cyclical unemployment. The proposed models suggest that a good explanation of the increases in the former is the reallocation of labor and the appropriate policy should deal independently with each of the two causes of changes in unemployment. If most of the increase in unemployment is due to the cyclical component, then an aggregate demand policy could alleviate the burden. If the increase in unemployment is due to a fluctuation in the natural rate, then a supply policy such as eliminating (or reducing) barriers to labor market adjustment

[18] Changes in the natural rate includes any transitional changes in unemployment resulting from the reallocation of labour between sectors.

and costly regulations would be appropriate. We next address the issues of calibration and the size of the shock.

3 Size of the Shock and Calibration

Our interest in simulating our RBC models with different shocks size[19] is sparked by the question posed by[20] Bianchi and Zoega (1996). They emphasized the size of the shock issue and asked "Does the size of the shock matter in explaining unemployment persistence?" They concluded that most of the persistence was accounted for by a few large shocks rather than by numerous small shocks.

To investigate whether the size of the shock matters, we calibrated[21] the models' such that the steady state workweek hours match the one in the business cycle data. For the magnitude of the shocks, we use $\theta = \{1.1, 1.15, 1.2, 1.25, 1.3\}$, i.e., we investigate shocks with relative sizes of 10 percent to 30 percent. Values of θ around 1.20 are chosen so that the model yields a steady

[19] Technically, the size of each industry can be measured as the proportion of the industry output relative to the total economy-wide output. The size of each industry shock can be proxied by the mean of the industry Solow residuals (corrected for capital utilization) à la Burnside, Eichenbaum and Rebelo (1995). Once computed, the respective mean can be used to calibrate the size of the industry shock.

[20] Using statistical analysis based on switching regression models (Markovian regime shifts in the mean) and non-parametric density estimation techniques (as an exploratory tool to investigate the data) they identified and quantified the size of the shift in the unemployment series mean of 17 OECD countries. The annual data covered the period 1960-1993. They criticized the use of linear time series models in which the mean is constant (time invariant), as is the case with ARMA models. Therefore, they proposed a time series Markov switching regime type model, in which the unemployment mean is a function of the state of the economy. They concluded that large annual changes in the unemployment mean (large shocks) are consistent with the hysteresis models of unemployment. For Canada, a shift was found in 1975. Note that in Canada, the unemployment insurance reform took place in 1972.

[21] For an excellent exposition of the merits of calibration versus estimation, see Quah (1995), and for the statistical aspects of calibration in macroeconomics, see Gregory and Smith (1993).

state value of N equal to 0.20 which matches the average workweek as a fraction of total hours over the time period. Since the week contains 168 hours, 20 percent for hours worked equals 33.6 hours per week on the job. Note also that a workweek of 40 hours implies that N^* equals 0.238, a value that is not far from the chosen 0.20. The shocks are generated using a Markov transition probability matrix. The probability to stay in the same state λ_{11} is usually set to equal the serial correlation coefficient of the sectoral Solow residual. To do so, we computed the serial correlation coefficients for different sectors' multifactor productivity.

[**Insert Table 2 here**]

We choose an upper bound value of 0.92 for λ_{11} in the transition matrix Λ. For symmetry purpose, we set $\lambda_{22} = \lambda_{11}$. The values of λ_{12} and λ_{21} are computed directly from λ_{11} and λ_{22}.

Independent evidence on an appropriate value for D (the adjustment cost parameter) is not available.[22] For our calibration of D, we follow Cardia (1991) and Greenwood, Hercowitz and Krusell (1992) in setting the adjustment cost parameter so that the generated series match the variance of employment in the business cycle data. We investigate the robustness of the results at the following grid for the adjustment cost parameter $D = \{5,\ 10,\ 15\}$.

The value of A (constant) is set for each model to ensure that the model possesses a steady state on the grid mesh, so that A is a function of the steady state values of the decision variables. Table 3 reports the value of A for each frequency for Model I. Note that A is not a function of the adjustment cost parameter D.

[22] Note that $d = 0.5\ D$. In the literature, d is used as the adjustment cost parameter. Here, we calibrate and report our results in terms of D.

[Insert Table 3 here]

To calibrate the proposed models over the period from 1980 to 1996, we relied on the empirical results of Goldstein (1998) who examined the projections of Canadian long-term economic growth prepared by various forecasters. In our production technology, α is capital's share in income, δ is the capital depreciation parameter and γ denotes the momentary leisure shape parameter. The leisure shape parameter $\gamma = 2/3$ implies that two-thirds of the household time is allocated to non-market activities (see Drolet and Morissette (1997)) and the elasticity of the labor supply equals 2. The same value was used by Prescott (1986). ρ denotes the rate of time preference and β denotes the discount factor. T is the units of time endowment in each period. The calibrated parameters for the models are reported in Table 4,

[Insert Table 4 here]

Each model is simulated with all possible combinations of $D = \{5,\ 10,\ 15\}$ and $\theta = \{1.1,\ 1.15, 1.2,\ 1.25,\ 1.3\}$. Therefore, in total, 30 models were simulated. The rationale for these simulations is to investigate the sensitivity of the results to the calibrated parameters. [23] We use a random number generator to determine the incidence of a shock. [24]

[23] For robustness, we also simulated each model at two time frequencies (quarterly and annual). In total, 60 models were simulated. For space, we do not report the results of the annual models. Upon request, these are available from the authors.

[24] To simulate the time series, the procedure is as follows. First, assume that the economy is resting in state 1 with probability λ_{11} to stay in the same state for the next period. Second, generate a uniformly distributed random number. If the random number is higher than λ_{11}, then the economy will move to state 2. If state 1 is the state wherein sector 2 enjoys the high value of θ (high productivity in model I and high product demand in model II) and sector 1 collects $1/\theta$, then when the economy moves to state 2, the role of θ is switched for both sectors. The random number generator is used to simulate the models. For example, assume as described that the economy is in state 1 and sector 2 is the high θ sector ($\theta = 1.2$). If the value of the random number is higher than λ_{11}, then sector 1 enjoys a shock of $\theta = 1.2$, which implies that sector 2 is experiencing a shock of $\theta = 1/1.20 = 0.83$. For model I, this shock translates into a 20 percent increase in the labor demand in sector 1 and a 17 percent decrease

4 Algorithm, Robustness and Validity

Different numerical methods, as outlined in Taylor and Uhlig (1990), can be used to solve the models. They compared seven different numerical methods: the value-function grid, the quadrature value-function grid, the linear-quadratic, backsolving, the extended-path, the parametrizing expectations and the least-squares projections. One of their conclusions was that if the measuring stick is the 'closeness' of the numerical solution to the true decision rule, then grid methods are "... likely to do very well" Taylor and Uhlig (1990, p. 16). They pointed that when 'computing time' is the measuring stick, linear-quadratic approximation methods exhibit financially significant savings in terms of computing time. In our case, we chose 'closeness' as the measuring stick and used the value-function grid method.

The value-function grid method relies on approximating the continuous valued problem by a discrete-valued one. It evaluates and iterates on the Bellman equation over a grid of points with respect to the choice variables. For Model I, the choice variables are capital and labor. For Model II, the choice variables are sector 1 and sector 2 labor.

Model I was maximized over 20,000 grid points of capital and sector 1 labor. The value for sector 2 labor was computed from sector 1 labor. Sector 2 employment is computed as $N_2 = \theta^2 N_1$. Total employment was set to $N = (1 + \theta^2)N_1$. The mesh size for Model I differed across the frequencies, annually and quarterly. For capital, it is set to 0.2 for the annual frequency and

in the demand for labor in sector 2. For model II, this shock translates into a 20 percent increase in the demand for sector 1 goods and a 17 percent decrease in the demand for sector 2 goods. The range of analysis is chosen to cover the range from a small shock (10 percent) to a relatively large shock (30 percent).

0.02 for the quarterly one. For sector 1 labor, the mesh is set to 0.009 for the annual frequency and 0.0003 for the quarterly one. Model II was maximized over 22,500 grid points of sector 1 and sector 2 labor. The mesh size was set to 0.006 for all sub-models. All grids were centered around the steady state.

Once an approximate solution is computed, the computation of the error bound on the Bellman equation is carried out (see Judd 1998, pp. 413-414). The contraction property used to iterate the value function implies that each iteration satisfy the inequality, $\| V^{sol} - V^k \|_\infty \leq 1/(1-\beta) \| V^{k+1} - V^k \|_\infty$. The iterations were stopped at the first iterate where $\| V^{k+1} - V^k \|_\infty \leq \epsilon^V (1-\beta)$. The last inequality becomes the convergence rule given one's goal ϵ^V. This implies that the initial convergence-stopping rule is $\epsilon = \epsilon^V (1-\beta)$. Numerically, we set the stopping rule to $1.E-10$. This rule implies that the following value for ϵ^V was used $1.E-12$. When $| V^k - V^{k-1} | < 1.E-10$, the iterations stop and the policy rules are computed from the steady state. Once they are computed, the variables are simulated and their properties are investigated.

We studied the models' results in the neighborhood of local parameter perturbation. To do so, we simulated the models by fixing all calibrated parameters but one and decided to investigate the effect of adjustment cost sizes, relative shock sizes and the frequency used on the results. As outlined by Kim and Pagan (1995, p. 381), we computed the 'sensitivity elasticities' for the models' calibrated parameters Θ. These elasticities are based on the Taylor series expansion of a function of the calibrated parameters $g(\Theta)$ around Θ^* featured in the model, where $g(\Theta)$ is defined as the ratio of the standard deviations of the model output to the sample GDP. Formally, $g(\Theta) \simeq g(\Theta^*) + [\partial g/\partial\Theta]_{\Theta=\Theta^*} (\Theta - \Theta^*)$. In terms of proportionate changes, $(g(\Theta) - g(\Theta^*))/g(\Theta^*) \simeq \sum_{j=1}^{p} \eta_j \left[(\Theta_j - \Theta_j^*)/\Theta_j^*\right]$, where, $\eta_j \equiv \{[\partial g(\Theta)/\partial\Theta_j] [\Theta_j/g(\Theta)]\}_{\theta=\theta^*}$. η_j is the sensitivity elasticity for the j^{th} coefficient. These elasticities are computed numerically by perturbing the coefficients of interest. Table 5 reports the models' elasticities with respect to the adjustment cost parameter D and the size of the shock θ.

[**Insert Table 5 here**]

At low levels of the adjustment cost [from 5 to 10], if one changes D by 1 percent, Model I (shock size $\theta = 1.2$) implies a change of 0.907 percent in the ratio of the model output's standard deviation relative to the standard deviation of the business cycle GDP data. For similar conditions (i.e., fixed shock size), Model II implies a change of -0.04 percent in the ratio of the model's output standard deviation relative to the standard deviation of the business cycle GDP data. Overall, relative to Model II, the results of Model I are more sensitive to changes in the adjustment cost parameter and to the size of the shock. The sensitivity elasticities for Model II imply that regardless of the shape of the adjustment cost parameter, the effects of a sectoral tastes shock are robust in terms of output variability.

In the absence of formal educational institutions that facilitate labor mobility across sectors (i.e., high adjustment cost parameter $D = 15$), a 1 percent change in D influences considerably the model output variability. If the parameter D can be thought of as an index that measures the absence, the rigidity or the presence of institutions that facilitate labor mobility in the economy, then a small policy change can influence the severity of output lost during a recession that is generated by a sectoral technological change.

With the exception of the case of low size of the shock (i.e., $\theta = 1.15$), Model II results are insensitive to the change in the size of the shock. The size of the sectoral technology shock in

Model I is very important to the model's results on output variability. Overall and at almost all levels of adjustment costs, output variability is very sensitive to the size of the sectoral technology shock.

A pattern that emerges from Table 5 is that, as the size of the sectoral technology shock increases, the sensitivity elasticity decreases. This implies that output variability is very sensitive to large sectoral technology shocks.[25] This result concurs with the findings of Cogley and Nason (1995, p. 492) and Bianchi and Zoega (1996).

5 Stochastic General Equilibrium Results

This section reports and analyses the results of the simulated models, their characteristics and their ability to match business cycle data. In what follows, 'output' is used to describe the real GDP simulated series, and 'GDP' is used to refer to the real business cycle data.

[**Insert Table 6 here**]

[**Insert Table 7 here**]

[**Insert Table 8 here**]

[25] To investigate further the sensitivity of model I, we present parameter sensitivity analysis in the Appendix. The Appendix Figures illustrate the sensitivity of consumption and output to the capital depreciation parameter δ and show that the labor supply is an increasing function of the size of the sectoral technology shock. As θ increases, unemployment and the labor supply increase.

Table 6 reports the empirical regularities of the Canadian business cycle data.[26] Tables 7 and 8 report the simulation results for models I and II. In general, the highest variation for output is produced by the models which includes the highest shock size ($\theta = 1.3$) and the highest adjustment cost parameter ($D = 15$). For Model I - Table 7 - smaller technology shocks generate a GDP match to output variability. However, the propagation mechanism highly amplifies the effects of these shocks on employment. All submodels generate a higher than data employment variability. Relative to output, investment variability is matched at low level of adjustment cost (D=5) combined with $\theta = 1.2$. For Model II - Table 8 - it takes a small tastes shock to generate a match for output variability. Contrary to the results of Model I, employment variability is smaller that the data, while output variability is higher.

At higher level of adjustment costs, the propagation mechanism directs the bulk of the effect of the shock to output and employment in Model I. Relative to Model I, the dynamics of Model II seem to absorb a greater part of the shock. It is also plausible that by design, technology shocks generate higher output and employment variability. Interestingly in both models, and irrespective of the size of the adjustment costs, a small size of the shock generates output variability that can match the data. The adjustment costs are crucial to explain employment

[26] The data covers the period from 1976 to 1999. It is for gross domestic product (GDP), employment (EMP), consumption (CONS) and investment (INVST). The Cansim labels are: GDP (D14872), EMP (D980662), CONS (D14842) and INVST (D14851). GDP is measured from the expenditure side at 1992 market prices. Consumption is measured as expenditure on consumer goods and services. Investment is measured as business gross fixed capital formation. Employment is measured as actual hours. Note that measuring employment as the total number of people above 15 years of age who are employed, reduces employment variability with respect to GDP. Relative to GDP, actual hours are more variable than total employment. All series are: in 1992 dollars, in log form and de-trended using the Hodrick-Prescott filter. The reform of the Canadian system of unemployment insurance was introduced in the early 1970s. Mixing two different policy regimes induces parameter instability, therefore we concentrated on a single regime. To circumvent the effect of the change in policy on the data, we used post-1976 data.

variability. The models imply that, in a recession caused by a relative sectoral technology shock, the adjustment costs of labor mobility across sectors amplify the effect of the shock and lead to higher unemployment.[27]

Figures 1 and 3 graph the impulse responses of employment for Models I and II, respectively. The essence of adjustment costs is captured in the way employment adjusts. Following a relative sectoral shock, total employment decreases. The time it takes to revert to its original state is due to the adjustment costs. The higher the adjustment cost (parameter D) is, the longer it takes for employment to revert back to its steady state level and the greater the deviation in it.

[**Insert Table 9 here**]

From Table 9, it is apparent that Model I generates relatively stronger decline and higher unemployment persistence. After one period, employment declines by 9.52 and 4.5 percent for models I and II, respectively. The severity of the fall of employment is positively correlated with the adjustment costs. The decrease in employment in Model I is double that of Model II. Adjustment costs play a stronger role in Model II when combined with a high value of D. Two important results are reported in Table 9. First, the absence of institutions that facilitate sectoral labor mobility (D=15) imply higher unemployment persistence. Second, sectoral technology shocks generate deeper recessions relative to sectoral taste shocks. Next, we investigate the merits of each model relative to its performance in replicating observed labor productivity characteristics.

[27] Also, we computed and graphed the output autocorrelations from the simulated series. We found that models I and II explain (at best) two-thirds of the first lag autocorrelation.

5.1 The Average Productivity of Labor (APL)

In most simulated RBC models, the correlation between the average productivity of labor (APL) and GDP is positive. A positive technology shock increases the demand for labor and output. Such shocks are responsible for the generated positive labor productivity, a result that matches the observed positive correlation in business cycle data. In periods of booms, workers produce more output during each hour worked than they do during a recession. One of the strong points of the basic RBC model is that, to generate a procyclical APL, one needs an aggregate productivity shock. Without an aggregate productivity shock, an increase in labor during booms will reduce the APL because of the diminishing marginal product of labor. Therefore, a stable aggregate production function generates a countercyclical average productivity of labor.

Figures 2 and 4 illustrate the impulse responses of labor productivity for both models. For Model I, following a relative technology shock, the reallocative process of employment across sectors reduces total employment and increases the APL. A slow reallocation process, due to the presence of adjustment costs, results in decreasing APL. Therefore, the APL is countercyclical and sectoral technology driven shocks can generate countercyclical movements in the APL.

We also investigate whether procyclical movements in the APL can be generated without an aggregate productivity shock. While Hall (2003) had proposed a preference shock in a multi-sector [asymmetric] model, we choose to use the impulse mechanism in Model II. Model II focuses on changing labor demand without a shift in the production function. In Model II, households' relative tastes change such that they demand higher quantities of a specific good (sector 2 good) relative to the other (sector 1 good). Firms answer by supplying more of the desired good and by increasing their derived demand for labor in this sector. This generates the observed procyclical labor productivity, so that Model II can provide a non-technology driven explanation for procyclical productivity. However, our results show that labor productivity is coincident with output, as opposed to leading in observed data.

6 Overshooting of the adjustment process

Figures 1 and 3 show an apparent overshooting of employment due to the adjustment process. For Model I, a relative technology shock reduces total employment and output. Since output equals consumption plus investment, this reduction in output must be matched by a reduction in consumption and/or investment. Given the preference for smoothing consumption by the representative household, a large reduction in consumption to match the loss in output is undesirable. Therefore, investment falls by more than the reduction in consumption. This reduction in investment produces a reduction of capital over subsequent periods, linked by the law of motion for capital (i.e., the time-to-build characteristic). The reduction in capital acts as a negative wealth effect that impacts on the households' decisions. The representative agent responds by increasing labor supply and reducing consumption and leisure. This effect, when combined with the cost of adjustment in terms of leisure lost to move across sectors, produces overshooting (Figure 1). This theoretical outcome led us to investigate its empirical counterpart.

We estimated a bivariate Vector Auto-Regressive[28] (VAR) model between the growth rate of

[28] VAR models are linear dynamic models which postulate that all the variables in the system of equations are endogenous. The rationale for focusing on linear systems is that, since monthly and quarterly macroeconomic time series are usually well approximated by linear processes (see Brock and Sayers (1988)), non-linearity in the conditional mean is of marginal interest unless one examines higher-

total employment and a measure of manufacturing sectoral reallocation. The VAR model is identified using a *slightly* different identification from the one proposed in Blanchard and Quah (1989) [See Endnotes, no. 2]. They assumed that aggregate demand shocks have no long-run effect on the *level* of GNP. A similar restriction was used by Schmitt-Grohé (2001, p. 1147) and by Davis and Haltiwanger (1999, p. 1244) wherein it was labeled as the 'Neutrality Restriction'.

In our bivariate VAR, y_{1t} refers to the growth rate of total employment computed as the difference in logs. To investigate the Blanchard-Quah identification scheme, we were inclined to use total employment rather than the rest of employment for the long-run restriction to be meaningful. y_{2t} refers to the square of the growth rate of the fraction of manufacturing employment relative to total employment. Formally, $y_{2t} = \ln[(s_t - s_{t-1})/s_{t-1}]^2 = 2\ln[|s_t - s_{t-1}|/s_{t-1}]$, where s_t denotes the share of sectoral employment. We consider y_{2t} as a proxy for the sectoral reallocation of employment. Using the square of the growth rate of s_t implicitly assumes (at least as an approximation) that increases and decreases in s_t have symmetric effects on employment. This implicitly assumes that the adjustment cost of moving employment into manufacturing is roughly the same as the adjustment cost of moving labor out of manufacturing and into another sector. y_{2t} treats the percentage decreases in the sector's employment share symmetrically with increases. Some support of this assumption is provided by Campbell and Fisher (2000, p. 1329) who found that using symmetric per-job adjustment costs, produced reasonable results in their simulations. As in our proposed theoretical models, we symmetrically treat increases and decreases in the sector's share of employment. As labor is reallocated across sectors, a decrease in the share of employment in one sector implies an increase in the share

frequency data. For details on the mathematical derivations of VAR, see Rozanov(1967), Brockwell and Davis(1989) and Quah(1993).

of employment of other sectors. y_{2t} is computed as $2\ln[|s_t - s_{t-1}|/s_{t-1}]$. We bound $|s_t - s_{t-1}|$ from below by 10^{-8} to avoid instances of constant employment share. This functional form is arbitrary. We also tried several other transformations of y_{2t},including $y_{2t} = \ln[(s_t - s_{t-1})^2/s_{t-1}]$, $y_{2t} = (s_t - s_{t-1})/s_{t-1}$ and $y_{2t} = s_t$ but found similar results. Let the structural (primitive) VAR be,

$$By_t = \Gamma_0 + \Gamma_1 y_{t-1} + \Gamma_2 y_{t-2} + \cdots + \Gamma_p y_{t-p} + \varepsilon_t \tag{25}$$

and the reduced form VAR be,

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + e_t \tag{26}$$

with

$$\varepsilon_t \sim (0, \not\Sigma) \qquad \text{and} \qquad e_t \sim (0, D) \tag{27}$$

where

$$\not\Sigma \equiv \begin{bmatrix} \sigma^2_{\varepsilon_1} & 0 \\ 0 & \sigma^2_{\varepsilon_2} \end{bmatrix} \qquad \text{and} \qquad D \equiv \begin{bmatrix} \sigma^2_{e_1} & \sigma_{e_1 e_2} \\ \sigma_{e_2 e_1} & \sigma^2_{e_2} \end{bmatrix} \tag{28}$$

The assumption that the covariance of structural shocks is zero, implies that we are treating these shocks as 'pure' structural shocks (Enders 1995, p. 325) and that they are uncorrelated at all leads and lags. The same assumption was made by Blanchard and Quah (1989, p. 659). The assumption that the two disturbances are uncorrelated does not restrict the channels through which 'pure' structural shocks affect y_t. The 'pure sectoral' shock refers to the component of the shock that is orthogonal to the 'pure aggregate' shock. This is similar to Schmitt-Grohé (2001, p. 1147). This interpretation is reasonable and useful in understanding the dynamics of sectoral reallocation shocks. Starting with the reduced form VAR , the innovations of the reduced form can be written in terms of uncorrelated structural error terms,

$$e_t = Ge_t + \varepsilon_t \tag{29}$$

where G is a matrix with zeros on the diagonal. Let $B = I - G$ and $A = B^{-1}$. Therefore, the relationship between D and $\not\Sigma$ can be presented as follows. $\not\Sigma = BDB^T$ and $D = A\not\Sigma A^T$. D is decomposed into PP^T, where $P = C(1)^{-1}G$. $C(1)$ is the long-run multiplier sum of the ∞-MA coefficients. G is the lower Cholesky decomposition of $C(1)D(C(1))^T$. $B = P^{-1}$ and $\not\Sigma$ is the identity matrix. Here, we assume that the 'pure' sectoral shock has no long-run effect on the level of employment while it does have short-run effects on the level of employment because of the adjustment costs of moving labor across sectors.

We view shocks affecting y_{1t} as 'aggregate' shocks that impinge directly on employment growth. However, these shocks can also indirectly influence y_{2t}. For example, an inflow into the labor force will in the first instance feed into the sectoral labor markets. A favorable aggregate technology shock will also shift labor demand between the sectors. Therefore, we assume the existence of an indirect channel that transmits the effect of an aggregate shock into sectoral employment growth.

We also propose that 'pure' sectoral shocks that influence y_{2t} have an indirect influence on y_{1t}. For example, sector-specific tastes shocks can display such an impulse. For instance, the demand for more nutritious food products at the beginning of the 1980s increased relative to the demand for other food products. This relative increase for the product of one sector relative to others shifted the firms' derived demand for factor inputs, such as labor. The demand for labor in declining industries decreased. Also, relative technological shocks across industries would have produced a similar pattern in the labor market.

Whenever labor is immobile and costly to move across sectors, aggregate employment is likely to fall during the adjustment period following a shock. Therefore, we assume the existence of an indirect influence on aggregate employment. This influence is transitory and reflects the time it takes labor to fully adjust across sectors. These effects are typical of models with adjustment costs (Sargent 1986, p. 399). Consequently, in the long run, we assume that a 'pure' sectoral shock to y_{2t} does not have a long-run effects on the level of total employment. As in Blanchard and Quah (1989), these two assumptions - that the structural shocks are uncorrelated and that the structural shocks to y_{2t} have no long-run effect on the level of employment - exactly identify the model. Similar to Blanchard and Quah (1989, p. 671), we interpret the 'pure' sectoral shock as a shock (or the portion of a shock) that is unaffected by a total employment shock. The existence of a propagation mechanism that delays the adjustment of the variables to a shock can be captured by the lags in a structural VAR model.

Given these assumptions, we proceeded to estimate a structural VAR using the Blanchard-Quah identification.[29] For our identifying restrictions imposed on the VAR, see Endnotes, no. 3. We report Figure 5 for the impulse responses of the VAR. The standard deviations for the estimated impulse responses are usually carried out through the bootstrap resampling technique [Endnotes, no. 4] or by normal density approximation.[30]

[29] To serve as sensitivity analysis for the Blanchard-Quah identification, we estimated the VAR using the Bernanke-Sims identification scheme under the assumption of independent shocks. Similar results were concluded and not reported for space.
[30] We bootstrap the residuals 1000 times for each impulse response. We choose the residuals at the same time period for each equation to preserve the contemporaneous relationship. Then we simulate

Prior to estimating the VAR models, we used the multivariate AIC and Schwarz criteria to select the lag length for both models. On this basis, we estimated the VAR model at lag 8. Two dummy variables were added to the list of the exogenous variables in the VAR to account for the structural breaks identified by the graphs [Endnotes, no. 5]. Using the Likelihood Ratio test, we tested for 1 lag and 4 lags exclusions. The exclusion of 1 lag tests the null hypothesis that the last lag equals zero. The latter exclusion tests the joint null; the that the last four lags equal zero. Each hypothesis was rejected at the 5% level. The VAR results are reported in Endnotes, no. 6.

Figure 5 illustrates the accumulated impulse response to a reallocative manufacturing shock. The initial effect of the shock on employment is negative and equals 16.4 percent. Moving labor across sectors - combined with adjustment costs - implies a decrease in employment. Given the transitory nature of the shock, employment returns to its initial pre-shock level after 4 quarters. In terms of persistence, the effect of the shock is felt for a minimum of 10 quarters. The initial negative effects last only for 2 quarters. After 6 quarters, employment *overshoots* its long-run steady state level and then returns to it after 9 quarters. The labor adjustment process from manufacturing to total employment lasts for 8 quarters. Coming out of a recession and following a decline in wealth (due to the loss of labor income), workers supply more labor during the adjustment and capital build up processes.

Note that this empirical *overshooting characteristic* is similar to the ones generated from the

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the system using the new residuals, the coefficients and the actual series as initial values. We estimate the VAR and compute the impulse responses. We repeat this exercise 1000 times, then we calculate the 95 percent coverage (i.e., the 2.5 and the 97.5 percentiles) of the impulse responses. This method uses the percentile approach described in Mooney and Duval (1993, pp. 36-37) and Stine (1990, pp. 249-250).

proposed RBC models (Model I and Model II, see Figures 1 and 3) wherein the size and the timing of overshooting were positively correlated with the size of the shock and with the cost of adjustment parameter.

7 Conclusions

The success of RBC models is usually measured in the literature by their ability to mimic general business cycle correlations/moments. This paper added the criterion of explaining the observed unemployment persistence. Our simulations examined the dynamics between sectoral shocks and unemployment. Specifically, they tried to answer the questions: How much of the increase in structural unemployment in recessions is due to sectoral reallocation? Which impulse and propagation mechanisms, if any, can generate persistence in unemployment similar to that in the data?

At the absolute level, sectoral reallocation and adjustment costs combined with a relative sectoral tastes (relative sectoral technology) shock produced a range of variations in unemployment. Depending on the size of the shock and the degree of difficulty in moving across sectors, the volatility in unemployment was found to be between 10 percent and 37 percent. While this range is narrower than the one suggested by Lilien (1982), our results do encompass the observation by Campbell and Kuttner (1996, p.113) that sectoral reallocation is responsible for *at least* 27 percent of aggregate unemployment variation.

Our relative sectoral-technology-shock model [Model I] dominates our relative sectoral-tastes-shock model [Model II] with respect to higher unemployment variance. However, Model's I results are more sensitive to the calibrated parameters, while Model's II performs poorly in terms of output volatility. Our results show that relative sectoral-tastes shocks can successfully produce procyclical labor productivity without recourse to a technology shock. However, the generated labor productivity is cyclically coincident, and not leading as in the data. Both models show partial success in matching empirical regularities.

Both our models successfully generate unemployment persistence. A smaller adjustment cost tends to generate higher persistence for a technology shock than for a taste shock. In the absence of institutions that ease labor mobility across sectors (i.e., higher adjustment costs) unemployment displays persistence regardless of the source of the shock. The variance of employment is positively related to the adjustment costs parameter, such that a policy that is aimed at reducing these costs could significantly reduce it. Comparing relative sectoral shocks of the same magnitude to technology and tastes, the former produces higher employment volatility, longer unemployment persistence and a deeper recession. It takes a smaller sectoral technology shock and a relatively larger sectoral tastes shock to generate a similar decrease in employment. Among the contributions of this paper is the ability of its models to re-produce the fluctuations in employment during the adjustment process, following a sectoral shock. This theoretical investigation (Figures 1 and 3) is successful in capturing the empirical wealth effect displayed in Figure 5.

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8 Endnotes (not for publication)

(1) The computation of real output in Model II is done by solving the inter-temporal representative maximization problem,

$$\max_{C_1,C_2} L = \theta \ln C_1 + \frac{1}{\theta} \ln C_2 + \lambda(M - C_1 - PC_2) \quad (30)$$

$$\frac{\partial L}{\partial C_1} = \frac{\theta}{C_1} - \lambda = 0 \quad (31)$$

$$\frac{\partial L}{\partial C_2} = \frac{1}{\theta C_2} - \lambda P = 0 \quad (32)$$

$$\frac{\theta}{C_1} = \lambda = \frac{1}{\theta P C_2} \quad (33)$$

$$P = \frac{C_1}{\theta^2 C_2} \quad (34)$$

where P is the price of good 2 relative to good 1 and M refers to income. Similarly, $1/P$ is the price of good 1 relative to good 2. Nominal output equals $C_1 + PC_2$, and real output is computed at a base year price, $C_1 + P_{(0)}C_2$.

(2) They proposed an identifying assumption based on a long-run economic description of the VAR system. In this setup, both variables must be in stationary form. Re-write the VAR system in its infinite Moving Average (∞-MA) notation as,

$$y_{1t} = \sum_{k=0}^{\infty} c_{11}(k)\ \varepsilon_{y_{1t-k}} + \sum_{k=0}^{\infty} c_{12}(k)\ \varepsilon_{y_{2t-k}} \quad (35)$$

$$y_{2t} = \sum_{k=0}^{\infty} c_{21}(k)\ \varepsilon_{y_{1t-k}} + \sum_{k=0}^{\infty} c_{22}(k)\ \varepsilon_{y_{2t-k}} \quad (36)$$

or equivalently in its compact matrix form,

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{12}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{y_{1t}} \\ \varepsilon_{y_{2t}} \end{bmatrix} \quad (37)$$

where $\begin{bmatrix} \varepsilon_{y_{1t}} \\ \varepsilon_{y_{2t}} \end{bmatrix}$ ~independent White Noise with $\Sigma_\varepsilon \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. $C_{ij}(L)$ are polynomials in the lag operator L such that the individual coefficients of $C_{ij}(L)$ are denoted by $c_{ij}(k)$. For example,[31] the second coefficient of $C_{21}(L)$ is $c_{21}(2)$. The coefficients of $C_{11}(L)$ represent the impulse responses of a $\varepsilon_{y_{1t}}$ shock on y_{1t}. For convenience, the shocks' variances are normalized to 1. $E(\varepsilon_{y_{1t}}, \varepsilon_{y_{2t}}) = 0$ implies that both structural shocks are uncorrelated. The key underlying argument is that one assumes that $\varepsilon_{y_{1t}}$ is the portion of the (economic) shock that does not change (orthogonal to) in response to a change in $\varepsilon_{y_{2t}}$, and vice versa. Since $E(\varepsilon_{y_{1t}}, \varepsilon_{y_{2t}}) = 0$, one interprets $\varepsilon_{y_{2t}}$ as a shock (or the portion of a shock) that is unaffected by a total employment shock, i.e., 'pure' sectoral shock. For a similar

[31] In general, $C_{11}(L) = c_{11}(0) + c_{11}(1)L + c_{11}(2)L^2 + ...$

discussion, see Blanchard and Quah (1989, p. 671). Since y_t is stationary, neither shock has a long-run effect on y_t. Also, assuming that $\varepsilon_{y_{2t}}$ has no effect on the long-run *level* of y_{1t} amounts to setting $\sum_{k=0}^{\infty} c_{12}(k) = 0$. In Blanchard and Quah (1989, p. 657), y_{1t} and y_{2t} referred to the growth rate of GNP and the unemployment rate, respectively. $\varepsilon_{y_{1t}}$ and $\varepsilon_{y_{2t}}$ denoted aggregate demand and aggregate supply shocks, respectively. They assumed that aggregate demand shocks have no long-run effect on the *level* of GNP. Formally, they set $\sum_{k=0}^{\infty} c_{11}(k) = 0$.

(3) Consider the bivariate first-order VAR,

$$y_{1t} = b_{10} - b_{12}y_{2t} + \gamma_{11}y_{1t-1} + \gamma_{12}y_{2t-1} + \varepsilon_{y_{1t}} \quad (38)$$
$$y_{2t} = b_{20} - b_{21}y_{2t} + \gamma_{21}y_{1t-1} + \gamma_{22}y_{2t-1} + \varepsilon_{y_{2t}} \quad (39)$$

Reallocation of labor in response to a 'pure' sectoral shock occurs whenever manufacturing's share in total employment either increases or decreases. $\varepsilon_{y_{1t}}$ and $\varepsilon_{y_{2t}}$ denote 'aggregate' and 'pure' sectoral shocks, respectively. $\sum_{k=0}^{\infty} c_{12}(k) = 0$ is equivalent to assuming that 'pure' sectoral shocks have no long-run effect on the level of total employment. A 'pure' sectoral shock - when combined with labor adjustment costs in terms of moving workers across sectors - redistributes employment across sectors and does not affect the total employment level in the long-run. Since the total employment and the 'pure' sectoral shocks are not observed, the issue is to recover them from the VAR estimation. The reduced form of the VAR is

$$y_t = A(L)y_{t-1} + e_t \quad (40)$$

where $A(L)$ is a 2x2 matrix with elements equal to the polynomials $A_{ij}(L)$ with coefficients denoted by $a_{ij}(k)$. e_{1t} is the one-step ahead forecast error for y_{1t}, i.e., $e_{1t} = y_{1t} - E_{t-1}y_{1t}$. From the ∞-MA representation, the one-step ahead forecast error for y_{1t} is $c_{11}(0)\varepsilon_{y_{1t}} + c_{12}(0)\varepsilon_{y_{2t}}$. Therefore,

$$e_{1t} = c_{11}(0)\varepsilon_{y_{1t}} + c_{12}(0)\varepsilon_{y_{2t}} \quad (41)$$

and similarly for y_{2t}. In compact form,

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} c_{11}(0) & c_{12}(0) \\ c_{21}(0) & c_{22}(0) \end{bmatrix} \begin{bmatrix} \varepsilon_{y_{1t}} \\ \varepsilon_{y_{2t}} \end{bmatrix} \quad (42)$$

If the coefficients $c_{ij}(0)$ were known, it would be possible to recover $\varepsilon_{y_{1t}}$ and $\varepsilon_{y_{2t}}$ from the regression residuals e_{1t} and e_{2t}. Blanchard and Quah (1989) showed that using (42) and the long-run restriction ($\sum_{k=0}^{\infty} c_{11}(k)\varepsilon_{y_{1t-k}} = 0$), there are four restrictions to be used to exactly identify the four $c_{ij}(0)$ coefficients. The four restrictions are,

$$Var(e_1) = c_{11}(0)^2 + c_{12}(0)^2 \quad (43a)$$
$$Var(e_2) = c_{21}(0)^2 + c_{22}(0)^2 \quad (43b)$$
$$E(e_1e_2) = c_{11}(0)c_{21}(0) + c_{12}(0)c_{22}(0) \quad (43c)$$
$$\sum_{k=0}^{\infty} c_{11}(k)\varepsilon_{y_{1t-k}} = 0 \quad (43d)$$

The system (43) is four equations in four $c_{ij}\,(0)$ unknowns. Therefore, one can recover the coefficients and exactly identify the VAR. For our analysis here, the fourth restriction is replaced by $\sum_{k=0}^{\infty} c_{12}(k)\varepsilon_{y_{2t-k}} = 0$. To transform this restriction into its VAR representation, the following algebraic derivation must be carried. First, rewrite the VAR as,

$$y_t = A(L)Ly_t + e_t \tag{44}$$

Next, some transformations are necessary,

$$[I - A(L)L]\, y_t = e_t \tag{45}$$

$$y_t = [I - A(L)L]^{-1}\, e_t \tag{46}$$

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \frac{\begin{bmatrix} 1 - A_{22}(L)L & A_{12}(L)L \\ A_{21}(L)L & 1 - A_{11}(L)L \end{bmatrix}}{\mid I - A(L)L \mid} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{47}$$

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \frac{\begin{bmatrix} 1 - \sum_{k=0}^{\infty} a_{22}(k)L^{k+1} & \sum_{k=0}^{\infty} a_{12}(k)L^{k+1} \\ \sum_{k=0}^{\infty} a_{21}(k)L^{k+1} & 1 - \sum_{k=0}^{\infty} a_{11}(k)L^{k+1} \end{bmatrix}}{\mid I - A(L)L \mid} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{48}$$

$$y_{1t} = \frac{\left(1 - \sum_{k=0}^{\infty} a_{22}(k)L^{k+1}\right) e_{1t} + \left(\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}\right) e_{2t}}{\mid I - A(L)L \mid} \tag{49}$$

$$y_{1t} = \frac{\begin{bmatrix} \left(1 - \sum_{k=0}^{\infty} a_{22}(k)L^{k+1}\right)(c_{11}(0)\varepsilon_{y_{1t}} + c_{12}(0)\varepsilon_{y_{2t}}) \\ + \left(\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}\right)(c_{21}(0)\varepsilon_{y_{1t}} + c_{22}(0)\varepsilon_{y_{2t}}) \end{bmatrix}}{\mid I - A(L)L \mid} \tag{50}$$

Making the assumption that $\varepsilon_{y_{2t}}$ has no long-run effect on the log level of employment implies,

$$0 = \left(1 - \sum_{k=0}^{\infty} a_{22}(k)L^{k+1}\right) c_{12}(0)\varepsilon_{y_{2t}} + \left(\sum_{k=0}^{\infty} a_{12}(k)L^{k+1}\right) c_{22}(0)\varepsilon_{y_{2t}} \tag{51}$$

Setting the long-run restriction $\sum_{k=0}^{\infty} c_{12}(k)\varepsilon_{y_{2t-k}}$ equals 0, yields

$$0 = \left(1 - \sum_{k=0}^{\infty} a_{22}(k)\right) c_{12}(0) + \left(\sum_{k=0}^{\infty} a_{12}(k)\right) c_{22}(0) \tag{52}$$

The last equation presents the fourth restriction needed for our identification. Equations (43a), (43b), (43c) and (52) are four equations in four unknowns used to identify the coefficients $c_{11}(0)$, $c_{12}(0)$, $c_{21}(0)$ and $c_{22}(0)$. The method proceeds by estimating the reduced VAR, then computing the variance-covariance matrix of the residuals. Once computed, one calculates the sums $\sum_{k=0}^{p} a_{22}(k)$ and $\sum_{k=0}^{p} a_{12}(k)$ then proceed to compute the $c_{ij}(0)$ coefficients. Using these coefficients and the VAR residuals (e_{1t}, e_{2t}), one can identify the entire sequences of $\varepsilon_{y_{1t-k}}$ and $\varepsilon_{y_{2t-k}}$,

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} c_{11}(0) & c_{12}(0) \\ c_{21}(0) & c_{22}(0) \end{bmatrix} \begin{bmatrix} \varepsilon_{y_{1t}} \\ \varepsilon_{y_{2t}} \end{bmatrix} \tag{53}$$

Based on information criteria for lag selection, a first order VAR is rarely chosen for estimation. Finally, proceed with impulse response function analysis.

(4) There are few methods for computing confidence intervals for impulse responses. These are the delta-method, the bootstrap, the bias-adjusted bootstrap, the asymptotic parametric inference methods and the Bayesian Monte-Carlo integration method. See Runkle (1987) for a detailed analysis and see Ripley (1987, p. 175) for the properties of the standard errors of the impulse responses estimates using bootstrap methods. Fachin and Bravetti (1996) examined the performance of bootstrap and asymptotic parametric inference methods. They concluded that the bootstrap delivered superior results in terms of both length of the confidence interval and coverage when the variance of the forecast error is considered. See Fachin and Bravetti (1996, p. 339) for details. Wright's new proposal is a size-adjusted delta based method. In an attempt to overcome the low coverage of traditional methods that compute confidence intervals for the impulse responses in a vector autoregression, Wright (2000) proposed a new approach. This approach relies on the Normality assumption of the innovations and the lag order. His proposed confidence interval controls for coverage and addresses the coverage versus width trade-off. For our study here, we adopt the bootstrap method (Runkle 1987) in the Classical VAR. As mentioned, Sims, Stock and Watson (1990) argued that a VAR in levels in the presence of cointegration implies that the estimates of the parameters of interest are inefficient, but consistent.

(5) Outliers are defined as observations generating observed residuals of a magnitude exceeding, in absolute value, three times the standard deviation of fitted residuals. See Favero (2001, p. 142) for the definition.

(6) Endnotes Table E.1 reports the Jarque-Bera normality test, the Lagrange-multiplier serial autocorrelation test and the Lagrange-multiplier ARCH test for the reduced form VAR residuals. Evidence of deviations from normality appears only for EMP M/T, where normality is rejected. The residuals of the reduced form are serially uncorrelated and no evidence of conditional heteroskedasticity is found. Endnotes Table E.2 presents some statistics of interest regarding the estimated reduced form VAR. The value of $\rho(EMP\ T, EMP\ M/T) = -0.024$, which indicates that ordering is unimportant if one assumes the Cholesky decomposition.

Endnotes Table E.3 reports the VAR results under two identification schemes; Bernanke-Sims and Blanchard-Quah. The matrices notation B, $D^{0.5}$ and $B^{-1}D^{0.5}$ are used to be consistent with the notation on page 29. Note the difference in matrix D across the identifications. From the matrices, the initial effect of a one standard deviation structural shock on the variables is computed from $B^{-1}D^{0.5}$. The initial effect of a one percent standard deviation 'pure' sectoral shock on the growth rate of total employment is negative. The growth of total employment decreases whenever a 'pure' sectoral shock occurs. This result suggests the presence of adjustment costs that impinge on labor mobility. The usefulness of normalized variables in the VAR lies in the easier interpretation of the impulse response figures. For example, the first point of the impulse response curve is given by $B^{-1}D^{0.5}$.From the matrix $B^{-1}D^{0.5}$, (Endnotes Table E.3), in the initial period of the shock, a one standard deviation structural 'aggregate' shock leads to 60.8 percent increase in employment growth and to 20.4 percent increase in the square of the manufacturing employment growth rate. In the initial period of the shock, a one standard deviation structural 'pure' manufacturing shock leads to 84 percent increase in the square of the manufacturing employment growth rate. A 'pure' sectoral shock decreases the employment growth rate by 16.4 percent in the initial period. The accumulated responses to a structural shock (one standard deviation) for MA(16) and MA(∞)are,

$$y_{1t} = \sum_{k=0}^{16} c_{11}(k)\ \varepsilon_{y_{1t-k}} + \sum_{k=0}^{16} c_{12}(k)\ \varepsilon_{y_{2t-k}} \tag{54}$$

$$y_{2t} = \sum_{k=0}^{16} c_{21}(k)\ \varepsilon_{y_{1t-k}} + \sum_{k=0}^{16} c_{22}(k)\ \varepsilon_{y_{2t-k}} \tag{55}$$

$$y_{1t} = \sum_{k=0}^{\infty} c_{11}(k)\ \varepsilon_{y_{1t-k}} + \sum_{k=0}^{\infty} c_{12}(k)\ \varepsilon_{y_{2t-k}} \tag{56}$$

$$y_{2t} = \sum_{k=0}^{\infty} c_{21}(k)\ \varepsilon_{y_{1t-k}} + \sum_{k=0}^{\infty} c_{22}(k)\ \varepsilon_{y_{2t-k}} \tag{57}$$

MA($k = 16$)	MA($k = \infty$)
$y_{1t} = 3.229\ \varepsilon_{y_{1t-16}} - 0.027\ \varepsilon_{y_{2t-16}}$ $y_{2t} = -0.594\ \varepsilon_{y_{1t-16}} + 1.156\ \varepsilon_{y_{2t-16}}$	$y_{1t} = 3.232\ \varepsilon_{y_{1t-k}}$ $y_{2t} = -0.599\ \varepsilon_{y_{1t-k}} + 1.118\ \varepsilon_{y_{2t-k}}$

The initial effect of a one standard deviation shock is given by the matrix $P \equiv B^{-1}D^{0.5}$. Note that PP^T is the variance covariance matrix of the residuals. To transform the initial impact to a structural shock of 1 - rather than a one standard deviation - one normalizes the $B^{-1}D^{0.5}$ matrix such that the sum of each row equals one. For the normalized variables, the effect of a 'pure' sectoral shock on employment growth is negligible after 4 years (16 steps in Endnotes Table E.3). By construction, in the long-run (∞-steps in Endnotes Table E.3), the accumulated influence of the Blanchard-Quah sectoral 'pure' shock is zero on the level of total employment.

Endnotes Table E.4 reports the forecast error variance decomposition of a structural shock that equals one. It determines the proportion of the k-step ahead forecast error variance of the ith variable attributable to a shock to the jth variable. Each period in this table should be read as follows. The first (second) row of each cell refers to the variance of the first (second) variable. The first (second) element is the k-period variance proportion in the first variable attributable to a shock to the first (second) variable. Note that each row sums to 100 percent.

When one assumes that there is no long-run effect on the level of employment following a 'pure' manufacturing shock, after 4 years, a manufacturing reallocation shock is responsible for 13.87 percent variance in the growth rate of employment. Endnotes Table E.5 reports the reduced form coefficients estimates. To compute the structural form coefficients, one has to multiply the reduced form coefficients by the respective rows of matrix $B^{-1}D^{0.5}$ (from Endnotes Table E.3).

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Endnotes Table E.1: Residual Analysis

	Jarque-Bera Normality Test JB(2)	Ljung-Box Residual Autocorrelation LB(24)	Lagrange Multiplier Residual ARCH ARCH (24)
EMP T	0.9526 [0.6211]	14.1949 [0.5842]	8.5763 [0.9984]
EMP M/T	6.1818 [0.0455]	22.2956 [0.1339]	26.0556 [0.3504]

[Significance]

Endnotes Table E.2

Reduced-Form Residuals		
σ (EMP T)	0.6305	
σ (EMP M/T)	0.8646	
ρ (EMP T, EMP M/T)	-0.0248	
Multivariate Normality		
Skewness	22.8451 [0.0000]	
Kurtosis	744.7952 [0.0000]	
Joint	767.6402 [0.0000]	
Log-Likelihood	-185.1554	
Log-Determinant of the Residual variance-covariance Matrix	-1.2142	
AIC	-0.2985	
BIC	0.8089	
Estimated Sum of the VMA(∞) coefficients And Standard Errors	2.0388 (0.6362) -0.5494 (0.2497)	0.4394 (0.5212) 0.4827 (0.2046)
LR Test for exclusion of the		
Last Lag χ (4)	4.5230 [0.3398]	
Last 4 Lags χ (16)	14.6581 [0.5498]	

[Significance]
(Standard Error)

Endnotes Table E.3

	Blanchard-Quah
Matrix B	B, where D = inv(B)*SIGMA*inv(B)' EMP T EMP M/T 0.9385 0.2023 -0.2851 0.9385
Matrix $D^{1/2}$	1.4879 0.0000 0.0000 1.8600
LR Test for Overidentification	
LR ~ χ (1)	
Significance Level	
Matrix $B^{-1} D^{1/2}$	EMP T EMP M/T 1.4879 -0.4009 0.4520 1.8600
Accumulated Effect of a Normalized Structural Shock (One Standard Deviation)	
Out to 16 Steps	EMP T EMP M/T 3.22949 -0.02754 -0.59407 1.15616
Out to ∞ Steps	EMP T EMP M/T 3.23218 -0.00000 -0.59925 1.11819
Normalized Variables	
Matrix $B^{-1} D^{1/2}$	EMP T EMP M/T 0.6088 -0.1640 0.2042 0.8401
Out to 16 Steps	EMP T EMP M/T 1.32140 -0.01127 -0.26832 0.52221
Out to ∞ Steps	EMP T EMP M/T 1.32250 0.00000 -0.27066 0.50505

Endnotes Table E.4

	Blanchard-Quah
Forecast Error Variance Decomposition	
Period 0	0.93231 0.06769 0.05576 0.94424
Period 1	0.94313 0.05687 0.11404 0.88596
Period 2	0.94359 0.05641 0.11218 0.88782
Period 3	0.93947 0.06053 0.11449 0.88551
Period 4	0.93041 0.06959 0.11563 0.88437
Period 5	0.93119 0.06881 0.12724 0.87276
Period 6	0.90323 0.09677 0.15808 0.84192
Period 7	0.88975 0.11025 0.15832 0.84168
Period 8	0.87364 0.12636 0.15946 0.84054
Period 9	0.86566 0.13434 0.16279 0.83721
Period 10	0.86542 0.13458 0.16294 0.83706
Period 11	0.86415 0.13585 0.16404 0.83596
Period 12	0.86411 0.13589 0.16458 0.83542
Period 13	0.86225 0.13775 0.16572 0.83428
Period 14	0.86181 0.13819 0.16541 0.83459
Period 15	0.86198 0.13802 0.16588 0.83412
Period 16	0.86122 0.13878 0.16615 0.83385

Endnotes Table E.5

Model C-I
Reduced Form Coefficients Values

	EMP T	EMP M/T
1. EMP T{1}	0.49455	-0.31253
2. EMP T{2}	-0.12157	0.12191
3. EMP T{3}	0.01768	-0.03662
4. EMP T{4}	0.05490	-0.04470
5. EMP T{5}	0.19273	-0.10146
6. EMP T{6}	-0.05469	-0.13565
7. EMP T{7}	0.06917	-0.02125
8. EMP T{8}	-0.04663	0.08206
9. EMP M/T{1}	0.05899	-0.05755
10. EMP M/T{2}	-0.00936	-0.23199
11. EMP M/T{3}	0.07738	-0.01185
12. EMP M/T{4}	0.07122	-0.03441
13. EMP M/T{5}	0.06735	-0.04075
14. EMP M/T{6}	0.18015	-0.15871
15. EMP M/T{7}	0.07017	0.00516
16. EMP M/T{8}	-0.15738	-0.13333
17. DUM1	-4.92409	1.23483
18. DUM2	-3.64221	-2.25146
19. Constant	0.38133	3.83456

7. Endnotes (Not for publication)

Representative Agent's Problem

Maximise $$E\sum_{t=0}^{\infty}\left(\ln(c_t)+\gamma\cdot\ln\left(1-n1_t-n2_t-d(n1_{t+1}\cdot -n1_t)^2-d(n2_{t+1}\cdot -n2_t)^2\right)\right)$$

Subject to $$c_t+k_{t+1}-(1+\delta)\cdot k_t = A\cdot(k_t)^{\alpha}\cdot\left(\min\left(\theta n1_t,\theta^{-1}n2_t\right)\right)^{1-\alpha}$$

$$n1_t+n2_t\le 1$$

Euler Equations - given the symmetry just solve for state 1

For n $$\frac{(1-\alpha)\cdot A\cdot(k_t)^{\alpha}\cdot\theta^{1-\alpha}\cdot(n_t)^{-\alpha}}{c_t}-\frac{\gamma\cdot(1+\theta^2)}{\left[1-(1+\theta^2)\cdot n_t\right]} = 0$$

For k $$\frac{-1}{c_t}+\beta\cdot\frac{\alpha\cdot A\cdot(k_{t+1})^{\alpha-1}\cdot(\theta\cdot n_{t+1})^{1-\alpha}-\delta}{c_{t+1}} = 0$$

Steady State

$$\left[(1-\alpha)\cdot A\cdot(k)^{\alpha}\cdot\theta^{1-\alpha}\cdot n^{-\alpha}\cdot\left[1-(1+\theta^2)\cdot n\right]\right] = \left[\gamma\cdot(1+\theta^2)\right]\cdot\left[A\cdot k^{\alpha}\cdot(\theta\cdot n)^{(1-\alpha)}-\delta\cdot k\right]$$

$$\alpha\cdot\beta\cdot A\cdot k^{\alpha-1}\cdot\theta^{1-\alpha}\cdot n^{1-\alpha} = 1+\delta$$

Set Parameters:

$\alpha := 0.35$	Capital's Share of GDP	$A := 10$	Technology Aggregate
$\beta := 0.96$	Discount Factor	$\theta := 1.2$	Labour Shock
$\delta := 0.06$	Depreciation Rate	$\gamma := 2$	Weight of Leisure in U function

$$\rho := \left(\frac{1}{\beta}\right)-1 \qquad \rho = 0.042$$

Initialise Variables

$n := 0.5$ Labour

$k := 9$ Capital Stock

$$M := \left(\frac{\rho + \delta}{\alpha \cdot A \cdot \theta^{1-\alpha}}\right)^{\frac{1}{\alpha - 1}}$$

$$M = 277.732$$

$$co := \frac{\gamma \cdot \left(1 + \theta^2\right) \cdot \left(A \cdot \theta^{1-\alpha} - \delta \cdot M^{1-\alpha}\right)}{(1 - \alpha) \cdot A \cdot \theta^{1-\alpha}}$$

$$co = 5.957$$

Given

$$n = \frac{1}{co + \left(1 + \theta^2\right)}$$

$$k = M \cdot n$$

$$\text{Find}(n, k) = \begin{pmatrix} 0.119 \\ 33.075 \end{pmatrix}$$

Steady State Analysis

Given

$$n = \frac{1}{co + \left(1 + \theta^2\right)}$$

$$k = M \cdot n$$

$$g1(\alpha, \beta, \theta, \delta, A, \gamma) := \text{Find}(n, k) \qquad g1(\alpha, \beta, \theta, \delta, A, \gamma) = \begin{pmatrix} 0.119 \\ 33.075 \end{pmatrix}$$

$$\text{Labour}(\alpha, \beta, \theta, \delta, A, \gamma) := g1(\alpha, \beta, \theta, \delta, A, \gamma)_0 \qquad \text{Labour}(\alpha, \beta, \theta, \delta, A, \gamma) = 0.119$$

$$\text{Capital}(\alpha, \beta, \theta, \delta, A, \gamma) := g1(\alpha, \beta, \theta, \delta, A, \gamma)_1 \qquad \text{Capital}(\alpha, \beta, \theta, \delta, A, \gamma) = 33.075$$

$$\text{Output}(\alpha,\beta,\theta,\delta,A,\gamma) := A\cdot\left[\left(\text{Capital}(\alpha,\beta,\theta,\delta,A,\gamma)^{\alpha}\right)\cdot(\theta\cdot\text{Labour}(\alpha,\beta,\theta,\delta,A,\gamma))^{1-\alpha}\right]$$

$$\text{Output}(\alpha,\beta,\theta,\delta,A,\gamma) = 9.608$$

$$\text{Investment}(\alpha,\beta,\theta,\delta,A,\gamma) := \delta\cdot\text{Capital}(\alpha,\beta,\theta,\delta,A,\gamma)$$

$$\text{Investment}(\alpha,\beta,\theta,\delta,A,\gamma) = 1.985$$

$$\text{Consumption}(\alpha,\beta,\theta,\delta,A,\gamma) := \text{Output}(\alpha,\beta,\theta,\delta,A,\gamma) - \text{Investment}(\alpha,\beta,\theta,\delta,A,\gamma)$$

$$\text{Consumption}(\alpha,\beta,\theta,\delta,A,\gamma) = 7.623$$

$$\text{LabourSupply}(\alpha,\beta,\theta,\delta,A,\gamma) := \left(1+\theta^{2}\right)\cdot\text{Labour}(\alpha,\beta,\theta,\delta,A,\gamma)$$

$$\text{LabourSupply}(\alpha,\beta,\theta,\delta,A,\gamma) = 0.291$$

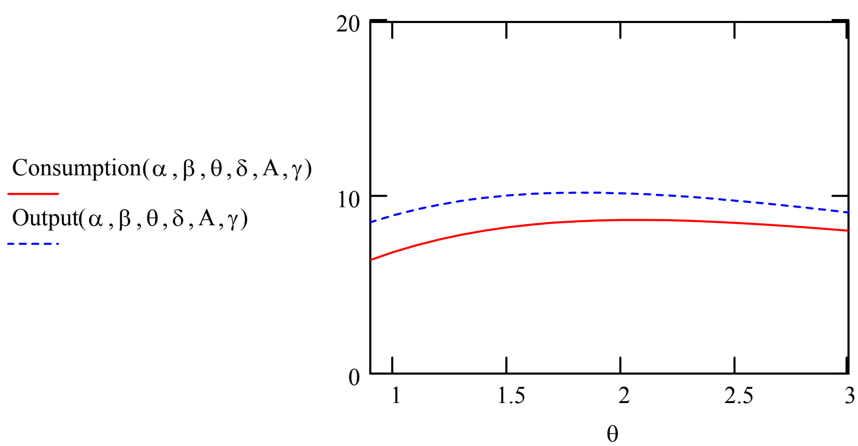
Create a range variable. This will be the variable whose value you are interested in changing.

As an example, let's look at the effect of varying the utility weight on leisure, *A*

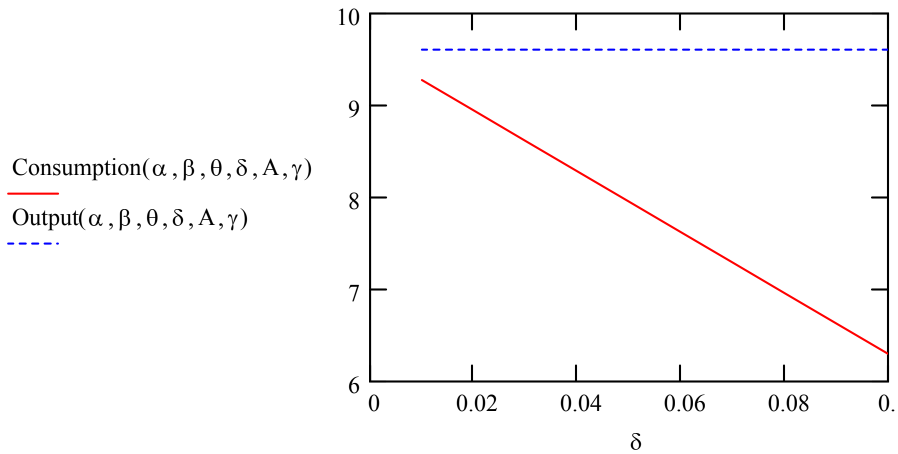
$\theta := 0.9, 1 .. 3$

In the graphs below, the user should alter the x-axis label. The graphs will then be redrawn.

Graph of Effect of depreciation on Consumption, Investment, and Output



$\delta := 0.01, 0.02 .. 0.1$
$\theta := 1.2$



$\theta := 0.5, 0.6 .. 2 \quad \delta := 0.06$

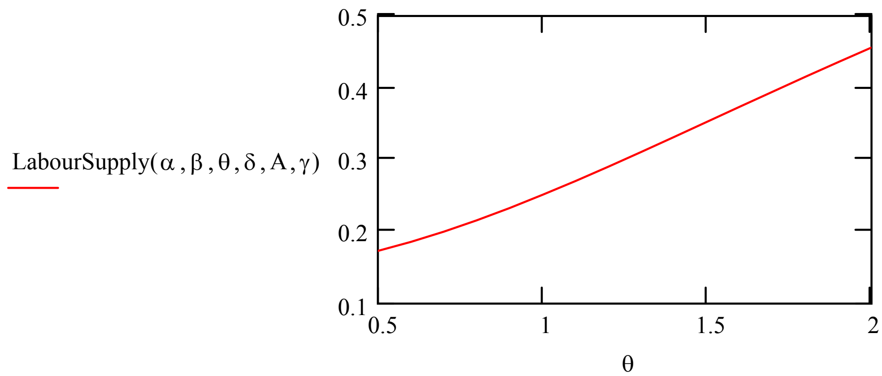


Table 1
Model I - Sectoral Employment

MODEL I	Change of employment in sector 1	Change of employment in sector 2
Previous state was low in sector 1 Present state is low in sector 1	$N_{1,t} - N_{1,t-1}$	$\theta^2(N_{1,t} - N_{1,t-1})$
Previous state was high in sector 1 Present state is high in sector 1	$N_{1,t} - \theta^2 N_{1,t-1}$	$\theta^2 N_{1,t} - N_{1,t-1}$

Table 2
Canadian Multifactor Productivity by Sector

Canadian Data - HP Filtered	**Serial Correlation**		
	K = 1	**K = 2**	**K = 3**
Real Gross Domestic Product	0.889	0.706	0.514
Multifactor Productivity: Sector GOODS	0.861	0.743	0.651
Multifactor Productivity: Sector MANUFACTURING	0.875	0.757	0.657
Multifactor Productivity: Sector SERVICES	0.892	0.762	0.635

Source: CANSIM data D14872, I700601, I700606 and I700602, respectively.

Table 3
Values for A

Model I	$\theta = 1.10$	$\theta = 1.15$	$\theta = 1.20$	$\theta = 1.25$	$\theta = 1.30$
A	3.155	3.259	3.365	3.474	3.586

Table 4
Calibrated Parameters

$\alpha = 0.35$	$\delta = 0.06$	$\gamma = 2/3$	$\rho = 0.01$	$\beta = 0.99$	$\lambda_{11} = 0.92$	$T = 1$

Table 5
Sensitivity Elasticities

MODEL I		**MODEL II**	
$\boldsymbol{\theta = 1.2}$	$\boldsymbol{\eta}$	$\boldsymbol{\theta = 1.2}$	$\boldsymbol{\eta}$
D=10	0.90	D=10	0.04
D=15	0.51	D=15	-0.10
$\mathbf{D = 5}$	$\boldsymbol{\eta}$	$\mathbf{D = 5}$	$\boldsymbol{\eta}$
$\theta = 1.15$	16.28	$\theta = 1.15$	2.71
$\theta = 1.2$	20.87	$\theta = 1.2$	1.91
$\theta = 1.25$	8.50	$\theta = 1.25$	-1.78
$\theta = 1.3$	3.72	$\theta = 1.3$	2.84
$\mathbf{D = 10}$	$\boldsymbol{\eta}$	$\mathbf{D = 10}$	$\boldsymbol{\eta}$
$\theta = 1.15$	6.76	$\theta = 1.15$	-0.60
$\theta = 1.2$	6.13	$\theta = 1.2$	0.21
$\theta = 1.25$	4.26	$\theta = 1.25$	0.02
$\theta = 1.3$	4.07	$\theta = 1.3$	0.02
$\mathbf{D = 15}$	$\boldsymbol{\eta}$	$\mathbf{D = 15}$	$\boldsymbol{\eta}$
$\theta = 1.15$	7.10	$\theta = 1.15$	2.64
$\theta = 1.2$	5.35	$\theta = 1.2$	1.60
$\theta = 1.25$	4.66	$\theta = 1.25$	0.03
$\theta = 1.3$	4.19	$\theta = 1.3$	0.72

Table 6
Sample Moments: Quarterly Canadian Data, 1976.1 to 1999.4

Variable	**Std. Dev.**	**Relative St. Dev.**	**Cross-Corr with GDP**	**Autocorrelations 1**	**2**	**3**
GDP	1.60	1.00	1.00	0.88	0.70	0.51
EMP	1.75	1.09	0.88	0.84	0.68	0.50
INVST	5.29	3.30	0.64	0.87	0.67	0.44
CONS	1.31	0.81	0.86	0.82	0.70	0.55

Table 7
Model I Simulation Results

Model I		St. Dev. GDP	St. Dev. Relative to Output EMP	INVST	CONS
DATA		0.016	1.09	3.30	0.81
D = 5,	$\theta = 1.1$	0.0005	0.0001	4.713	3.204
D = 5,	$\theta = 1.15$	0.001	1.457	2.549	1.625
D = 5,	$\theta = 1.2$	0.013	1.466	3.388	0.581
D = 5,	$\theta = 1.25$	0.020	1.460	2.874	0.443
D = 5,	$\theta = 1.3$	0.024	1.455	2.847	0.420
D = 10,	$\theta = 1.1$	0.013	1.425	2.635	0.632
D = 10,	$\theta = 1.15$	0.018	1.439	2.789	0.512
D = 10,	$\theta = 1.2$	0.025	1.419	2.773	0.487
D = 10,	$\theta = 1.25$	0.030	1.422	2.830	0.435
D = 10,	$\theta = 1.3$	0.035	1.412	2.762	0.467
D = 15,	$\theta = 1.1$	0.016	1.409	2.701	0.572
D = 15,	$\theta = 1.15$	0.023	1.404	2.692	0.522
D = 15,	$\theta = 1.2$	0.030	1.405	2.712	0.487
D = 15,	$\theta = 1.25$	0.037	1.408	2.737	0.501
D = 15,	$\theta = 1.3$	0.044	1.392	2.746	0.500

Table 8
Model II Simulation Results.

Model II		**St. Dev. GDP**	**St. Dev. Relative to Output**	
			EMP	**CONS**
DATA		0.016	1.09	0.81
D = 5,	$\theta = 1.1$	0.019	0.448	0.950
D = 5,	$\theta = 1.15$	0.021	0.395	0.838
D = 5,	$\theta = 1.2$	0.023	0.459	0.792
D = 5,	$\theta = 1.25$	0.022	0.722	0.902
D = 5,	$\theta = 1.3$	0.024	0.668	0.803
D = 10,	$\theta = 1.1$	0.023	0.564	0.815
D = 10,	$\theta = 1.15$	0.023	0.564	0.815
D = 10,	$\theta = 1.2$	0.023	0.564	0.815
D = 10,	$\theta = 1.25$	0.023	0.836	0.910
D = 10,	$\theta = 1.3$	0.023	1.022	0.969
D = 15,	$\theta = 1.1$	0.019	0.582	0.955
D = 15,	$\theta = 1.15$	0.022	0.583	0.862
D = 15,	$\theta = 1.2$	0.024	0.678	0.844
D = 15,	$\theta = 1.25$	0.024	0.911	0.916
D = 15,	$\theta = 1.3$	0.024	1.090	0.969

Table 9
Summary of the Impulse Responses for Employment

Relative to the Steady State, Percentage Change in Employment following a 20 percent shock ($\theta = 1.2$)	Quarters after the Shock				
	T=1	T=2	T=3	T=4	T=5
Model I					
D = 5	9.52	3.63	2.04	0.00	0.00
D = 10	12.24	5.89	3.17	1.36	0.46
D = 15	14.05	7.70	4.08	1.81	0.91
Model II					
D = 5	4.50	3.00	1.50	0.00	0.00
D = 10	6.00	3.00	3.00	1.50	0.00
D = 15	7.50	4.50	3.00	1.50	1.50

The employment mesh size is 0.0003 and 0.006 for models I and II, respectively.

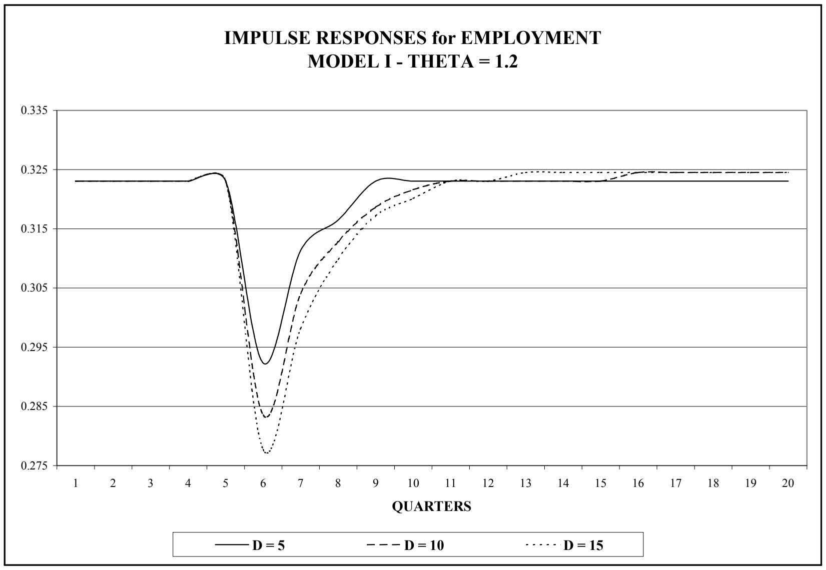


Figure 1. Impulse Responses for Employment (Model I)

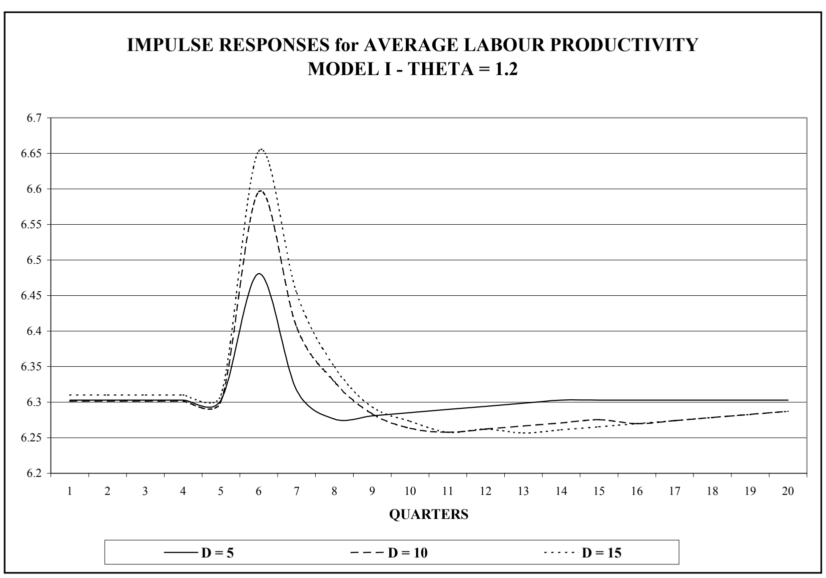


Figure 2. Impulse Responses for Average Labor Productivity (Model I)

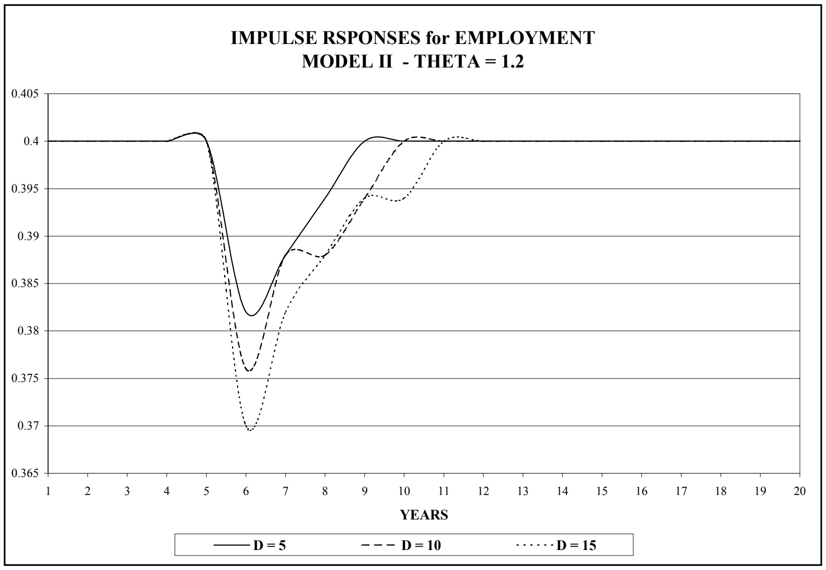


Figure 3. Impulse Responses for Employment (Model II)

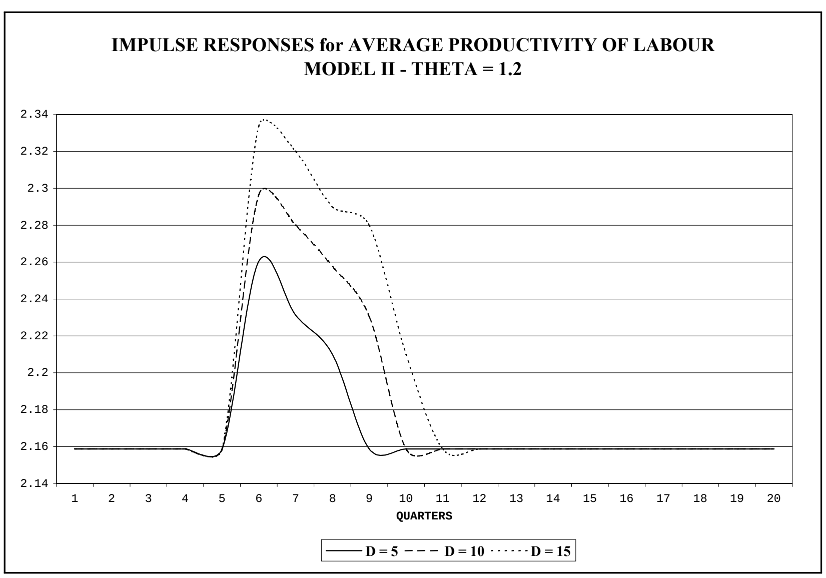


Figure 4. Impulse Responses for Average Labor Productivity (Model II)

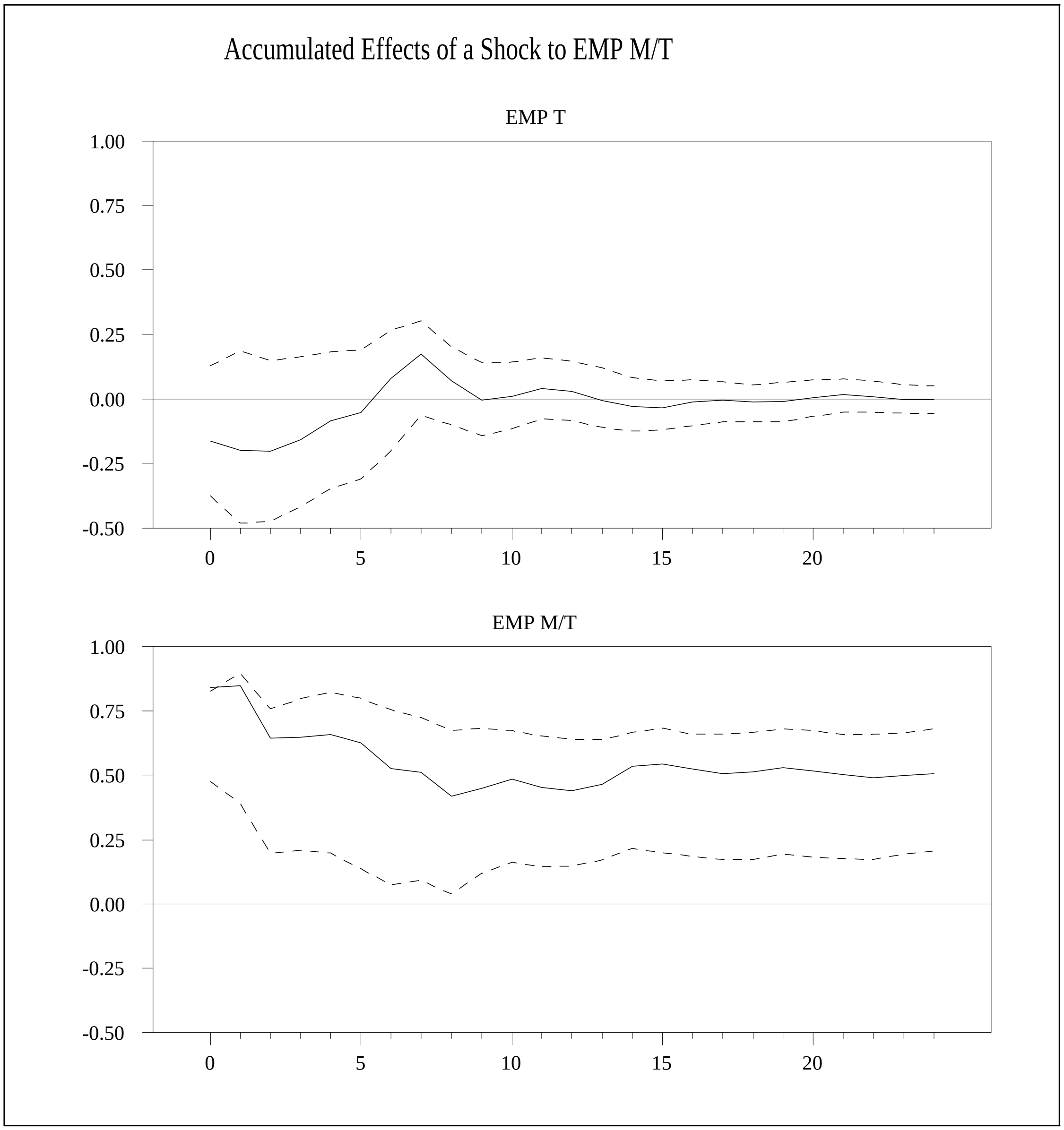


Figure 5. VAR - Impulse Responses