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Business cycles asymmetry and monetary policy: a further investigation using MRSTAR models

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Abstract

This paper investigates the asymmetric effects of monetary shocks when the impact of monetary policy on real activity works through state-dependent variables. We use a nonlinear model, the multiple regime smooth transition autoregressive model, that allows the effects of shocks to vary across the business cycles when monetary innovations modify both the endogenous and state variables. Our impulse response functions show a history-dependence property. Indeed, hitting the economy at a given time induces persistence and asymmetric responses across histories and shocks. The empirical application concerns the US over the period 1975:1–1998:2.

JEL classifications: C32; E37; E32; E52

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1. Introduction

The past years have witnessed an increasing number of papers dealing with the asymmetry of business cycles. Although the idea is ancient, empirical studies have

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grown rapidly since the beginning of the 1980s. The recognition that structural changes affect the economies at any period has encouraged the use of multiple regimes models, instead of previous empirical approaches that mostly distinguished between two phases in business cycles: expansion and contraction. The varying slopes of expansion and contraction phases that induce time variations from the mean to the trough or peak of cycles, is an old stylized fact—at least it dates back to Mitchell (1927). However, for a long time, the problem has been the following: how can the theoretical concept be made operational? In an attempt to answer this question, econometricians have suggested the use of nonlinear time series models that enable the study of different dynamics over the business cycles. A plethora of papers on this topic started emerging in the 1980s and in the 1990s (see, among others, Neftçi, 1984; Falk, 1986; Lüükkonen and Teräsvirta, 1991; Teräsvirta and Anderson, 1992; Emery and Koening, 1992; Sichel, 1994; Ramsey and Rothman, 1996; Verbrugge, 1997; Pesaran and Potter, 1997; van Dijk and Franses, 1999).

Among the arguments that motivate the use of nonlinear structures, a simple idea is that the output fluctuations are influenced by variables that distort the business cycle shape. Such variables cause changes in regime in the sense that output variations follow a different time series process over different periods. This may be a cause of asymmetric dynamics. With regard to linear or VAR models, the 'asymmetry' of business cycles suggests that contractions last a longer period than expansions, or that shocks have stronger effects on certain variables during one of the two phases. With regard to nonlinear models, the meaning of 'asymmetry' is more general in the sense that we simply say that shocks have time-varying effects on the real activity. This variability occurs because the parameters of the equations describing the dynamics of the output change as a result of a regime-shift variable. Such a view modifies our comprehension of how demand and supply shocks contribute to movements in the real GDP over the business cycle. Indeed, when one perturbs the present to produce information on the dynamics of a nonlinear model, the response does not only depend on the sign of the shocks, but it is also a function of the history and of the magnitude of the shocks. This is a new challenge to econometricians.

In this paper, we study the effects of monetary policy on the real sector of the US economy, assuming that output fluctuations are governed by regime-shift models, here the multiple regime smooth transition autoregressive (henceforth MRSTAR) models. These models were introduced by van Dijk and Franses (1999) who analyzed how regime-shift variables cause asymmetries in the US business cycle. They generalized the smooth transition autoregressive (STAR) models that were extensively used in the literature.[1] Why is it interesting to use an MRSTAR model to evaluate the asymmetric effects of monetary policy on real GDP? If we were using a linear model (for instance a VAR process), we would proceed as follows. We would, firstly estimate a money–output equation, secondly create two series of respectively positive and negative monetary shocks, and thirdly study the properties

[1] STAR models were originally introduced by Lüükkonen et al. (1988) and Teräsvirta and Anderson (1992).

of impulse response functions (IRFs). In such a framework, the usual results obtained in the literature may be summarized as follows: (1) money does affect output strongly when monetary policy is restrictive and raises inflation when it is expansive; (2) the effects of money on output is greater during the contraction phases of business cycles and their impact on inflation are greater during expansion phases; (3) if prices adjust slowly, then only negative shocks affect the output. In a MRSTAR model, contractionary and expansionary monetary shocks lead asymmetric effects that differ significantly from those just mentioned. Indeed, the IRFs exhibit a time dependence property. The coefficients of the money–output equation are indeed state-dependent and vary according to transition variables that generate changes in the business cycle regimes. The regime-shift variables are economic indicators characterizing both the aggregate supply and the aggregate demand. For instance, the reaction of output to negative monetary shocks may be undetermined because the level of stocks and the production capacity act as state variables that condition the reaction of the GDP to money variations (see for similar arguments Wong (2000)). There are other state variables that may induce time variation of the elasticity of output to money. Firstly, due to the imperfect structure of the credit market, initial shocks by the central bank can be either smoothed or amplified by commercial banks. A variable representing the credit channel may thus be hypothesized as being regime-shifting (Galbraith, 1996). Secondly, the impact of monetary shocks on activity is also conditioned by the credibility of monetary policy. Financial variables such as interest rate differentials reflect the agents' expectations about future conditions of the business cycle. People may want to increase saving if they foresee a slowdown. In this case an expansive monetary policy might be ineffective. There is evidence in the literature that such behaviors induce asymmetric dynamics in the business cycle (Aftalion, 1997). Other examples of regime-shift variables could be evoked: the indexing rules that characterize the wage-price loop, the pricing rules on the good markets, the growth rate of federal expenditures, the output-gap. Whatever the case, it seems difficult to assume that a money–output equation has parameters that are invariant across alternative values of the regime-shift variables. In this paper, we use an MRSTAR model to see whether the state-dependent approach helps capturing the money nonneutrality on the business cycle. Our study concerns US quarterly data over the period 1975:1–1998:2.

The paper is organized as follows. Section 2 presents the MRSTAR model that is used to estimate the money–output equation. The endogenous variable is the variation of the GDP. The exogenous variables are, respectively, the growth rate of M1, a total productivity index variable and the federal budget deficit. The regime-shift—or transition—variables include the output-gap and financial variables that are indicators of the credit channel and interest rate term structure variables. Section 3 presents the econometric methodology and the results obtained for the US economy in Section 4, we give simulation results from generalized IRFs and compare the results obtained for STAR and MRSTAR models. This allows us to show evidence of asymmetry. Section 5 concludes the paper.

2. The money–output MRSTAR model

Throughout the paper, the influence of monetary impulses on real activity will be based on the propagation and impulse approach as initiated in earlier studies of the business cycle. As to the first point (the propagation mechanism), we describe the intrinsic structure of the economy using MRSTAR formulation. This allows us to discuss the nature of nonlinearity induced by time-dependent structural parameters.

2.1. General formulation of a MRSTAR model

A MRSTAR model for a univariate time series y_t can be formulated as follows (van Dijk and Franses, 1999):

$$y_t = \{\phi'_1 w_t[1 - F_1(s_{1t},\gamma_1,c_1)] + \phi'_2 w_t F_1(s_{1t},\gamma_1,c_1)\}[1 - F_2(s_{2t},\gamma_2,c_2)] + \{\phi'_3 w_t[1 - F_1(s_{1t},\gamma_1,c_1)] + \phi'_4 w_t F_1(s_{1t},\gamma_1,c_1)\} F_2(s_{2t},\gamma_2,c_2) + \varepsilon_t \quad (1)$$

where $\varepsilon_t \sim \text{iid}(0,\ \sigma^2)$, $w_t = (1,y_{t-1},\ldots,y_{t-p},x_{t1},\ldots,x_{tk})'$, $\phi_i = (\phi_{i0},\phi_{i1},\ldots,\phi_{im})'$ for $i = 1,\ldots,\ 4$ and $m = p + k$. F_1 and F_2 are logistic functions given by

$$F_i(s_{it},\gamma_i,c_i) = [1 + \exp(-\gamma_i(s_{it} - c_i))]^{-1} \quad (2)$$

where, for $i = 1,\ 2$, γ_i and c_i are scalars with $\gamma_i > 0$. The transition variables s_{ti} can be lagged endogenous variables or exogenous variables. The restrictions $\gamma_i > 0$ are identifying conditions. The slope parameters γ_i are indicators of the speed of the transition between two extreme regimes and the c_i are the half-way points between these regimes.

The model given by Eqs. (1) and (2) generalizes the original logistic STAR (LSTAR) model given as follows:

$$y_t = \phi'_1 w_t[1 - F(s_t,\gamma,c)] + \phi'_2 w_t F(s_t,\gamma,c) + \varepsilon_t \quad (3)$$

where F is a logistic function given by

$$F(s_t,\gamma,c) = [1 + \exp(-\gamma(s_t - c))]^{-1}. \quad (4)$$

This last model is able to describe asymmetric behavior where two extreme regimes have different dynamics with a smooth transition from one to the other one. It may be noted that, when $\gamma \to \infty$, the LSTAR model approaches a threshold autoregressive model with two regimes (Tong, 1990), and, when $\gamma \to 0$, it approaches a linear model.

STAR models can only accommodate two regimes. MRSTAR models are thus introduced in order to take into account more than two regimes. More precisely, they allow detecting four distinct regimes, each corresponding to some extreme values of the logistic transition functions F_1 and F_2:

$$\begin{cases} F_1=F_2=0 & : \quad y_t=\phi'_1 w_t+\varepsilon_t \\ F_1=1 \text{ and } F_2=0 & : \quad y_t=\phi'_2 w_t+\varepsilon_t \\ F_1=0 \text{ and } F_2=1 & : \quad y_t=\phi'_3 w_t+\varepsilon_t \\ F_1=F_2=1 & : \quad y_t=\phi'_4 w_t+\varepsilon_t \end{cases}. \tag{5}$$

The model is therefore locally linear in w_t. For our purpose, it is worthwhile noting that MRSTAR models can be considered as time-varying coefficient models—just as the STAR models. This can be shown by rewriting the model Eq. (1) as follows:

$$y_t=\pi_t w_t+\varepsilon_t \tag{6}$$

with

$$\pi_t=\{\phi'_1[1-F_1(s_{1t},\gamma_1,c_1)]+\phi'_2 F_1(s_{1t},\gamma_1,c_1)\} \times[1-F_2(s_{2t},\gamma_2,c_2)]+\{\phi'_3[1-F_1(s_{1t},\gamma_1,c_1)]+\phi'_4 F_1(s_{1t},\gamma_1,c_1)\}F_2(s_{2t},\gamma_2,c_2).$$

Therefore, the variability of parameters appears to be depending on both F_1 and F_2, and consequently nonlinearly on the transition variables s_{1t} and s_{2t}. In other words, the coefficients of w_t change smoothly with s_{1t} and s_{2t}.

2.2. Application to the money–output relationship

This section presents the MRSTAR formulation of the money–output equation and motivates the choice of explanatory and transition variables. The nonlinear specification that is introduced can be seen as a reduced form of a nonlinear structural model linking money and other financial variables to GDP. We do not seek the 'true' structural model, but simply use an equation that mimics the asymmetry of monetary policy over the business cycle when the switching of the economy between the different phases is governed by regime-shift—or transition—variables.

We use the following notations:

GDP: gross domestic product
M1: monetary index M1
DEF: federal deficit
PTY: Productivity
P: Price
SL3: 3-year interest rate term structure
SL10: 10-year interest rate term structure

WEDGE: Wedge[2]
CC: proxy of the credit channel
GAP: output-gap[3]

2.2.1. Economic motivations for the choice of the variables

2.2.1.1. The endogenous variable. The endogenous variable is ΔGDP.[4] Since we aim to model the asymmetry of the business cycle, it seems natural to consider the first-order difference of the logarithm of GDP instead of its level. Further, as will become clearer in Section 3, this choice is also motivated by the results of the stationarity tests. Although, we limit ourselves to the study of GDP, other indicators of the US business cycle could be considered: investment, consumption, employment. Also, a complete study requires the estimation of an equation relating money to inflation. To make the paper concise, these issues are not considered here but the reader is referred to Dufrénot et al. (2001).

2.2.1.2. The explanatory variables. Since the linear money–output equation is a particular case of a MRSTAR equation, it seems natural to choose, at the very least, the exogenous variables that are usually found in linear specifications. Most often, one finds the contemporaneous and lagged values of:

- the money growth rate ($\Delta M1_t$);
- the federal budget deficit (DEF_t);
- the treasury bill rate (as explained later, this variable is chosen here as a transition variable);
- the unexpected changes in the real price of energy (here, we use another proxy of the supply shocks: the variation of total productivity ΔPTY_t);
- the unemployment rate (this variable is omitted here).

2.2.1.3. The transition variables. First natural candidates for the choice of the transition variables are the lagged values of the endogenous and exogenous variables. Although this approach is common in many papers dealing with LSTAR and MRSTAR models, we add other variables that are channels for the transmission of monetary impulses to real activity: SL3, SL10, ΔCC, ΔWEDGE, GAP. The choice of these variables is motivated by the following arguments.

We first consider financial channels through the credit channel and the slope of the term structure of interest rates. The literature has indeed emphasized that the

[2] The wedge is measured as the real labor costs minus the hourly earnings plus the consumer price index. This variable measures the difference between the wages paid by employers and the wages earned by employees. It helps capturing the effects of the price-wage loop on some rigidities that render sluggish the response of the activity to monetary shocks.

[3] The output-gap is measured as the difference between the logarithm of GDP and the long-run trend (as given for instance by the Hodrick–Prescott filter) of the logarithm of GDP. Our data consist of the US output-gap series given by the O.E.C.D.

[4] Δ denotes the first-difference operator.

estimation of a money–output equation is sensitive to the choice of the variables that measure the monetary policy. The credit channel mechanism is consistent with, either the standard IS-LM model of output fluctuations—see Blinder (1987)—or with the microeconomic imperfections in the credit market—as suggested by Stiglitz and Weiss (1981). The proxy CC that we choose as a measure of the credit channel is the difference between the federal reserve base rate and commercial banks loan rate. We further consider financial series that contain information about the monetary policy: two interest rate term structures—SL3 and SL10—that are based on the difference between the 3- and 10-year treasury bonds and the rate of 3 month treasury bills. The interest rate term structure can be upward or downward sloping, depending upon whether the monetary policy is expansive or restrictive. There are some empirical evidence in the literature that the difference between long-term and short-term interest rates is a good predictor of real activity (Stock and Watson, 1989; Estrella and Hardouvelis, 1991). It has been shown in previous studies that financial variables can cause asymmetric responses in the reaction of output and inflation to money shocks. This issue is discussed by Galbraith (1996) and Aftalion (1997) who use threshold and STAR models to capture the asymmetric effects of monetary policy. In this paper, we show that such effects also emerge in the more general setting of the MRSTAR models.

More than the financial transmission mechanisms, there are other reasons why the coefficients of an equation relating money to output may be time-varying. Recent papers show that short-term Phillips curves are nonlinear. Even though the exact form of nonlinearity still remains ambiguous, it is argued that the nonlinear form comes from the following property: the slope of Phillips curves is a function of present and past macroeconomic conditions (for a synthesis of theoretical arguments, the reader is referred to Dupasquier and Richets (1998) and Yates (1998)). The factors that condition the slope of the Phillips curve include:

- the economy capacity constraint,
- the inflation volatility,
- the individual firms' adjustment costs,
- firms' market power in the good market,
- nominal rigidities in wages and prices.

With these factors, Phillips curves are concave or convex, thereby inducing asymmetric effects of output to money supply variations. The most important consequence, perhaps, is that asymmetric responses to money shocks are associated with the timing of monetary policy. Nothing indeed guarantees that a large variation in the monetary instrument is equivalent to successive small changes. Precisely, if the central bank allows deviations from the target for some time, a larger movement in the monetary instrument will be needed to achieve the desired level. This is due to the fact that a very small initial change in monetary policy may be amplified in the course of time because the effects on output and inflation are time-dependent. Conversely, a large movement in monetary aggregates does not necessarily imply a large response of output and inflation, if the dynamic effects go through channels that smooth the initial impulse and induce sluggishness in response to shocks. The fact that outside money may have an impact through 'real channels'—and not only through financial channels—has motivated recent empirical works. For instance, Elliason (1999) uses a STAR model as a reduced form of the Phillips curve and shows that money does have asymmetric effects on output in Australia and Sweden, while such effects are not very conclusive for the US. In this paper, we show that the same conclusion however holds for the US economy if one uses MRSTAR equations instead of STAR models. We stress that such 'real effects' of monetary policy exist, even outside the framework of real business cycles and even when the money supply is assumed not to be endogenous.

In view of the preceding remarks, the following transition variables are added to our equations. We choose a variable that captures the effects of capacity constraints. If the short-run Phillips curves are indeed nonlinear, then the consequences of money shocks may be larger when the economy operates close to capacity. Thereby, excess demand and excess supply situations may induce asymmetric responses of the output. The proxy that we choose to capture such effects is the output-gap, GAP. Besides, the effects of monetary impulses also depend on the microeconomic factors that render the response of output more or less sluggish. Shifts in the coefficients of the money–output equation are conditioned by the wage–price adjustment mechanism and by the pricing rules in the good markets (for theoretical arguments relating the pricing behaviors to state-dependent relationships between money and output, the reader may refer to Dotsey et al. (1999). Accordingly, we also consider the WEDGE as a possible transition variable.

2.2.2. The MRSTAR money–output equation

To capture the gradual and smooth changes in the impact of monetary policy, a model with time-varying parameters is more suitable than the standard linear formulation. The effect of monetary policy can be more or less strong and induces sluggish adjustment of the output, depending on the transmission mechanisms of money shocks. In this view, we consider the following specification:

$$\begin{aligned}\Delta GDP_t = & \left[A_{10} + \sum_{i=1}^{p} A_{1i}\Delta GDP_{t-i} + \sum_{j=0}^{q} A_{2j}\Delta M1_{t-j} \right.\\ & \left. + \sum_{k=0}^{r} A_{3k}\mathrm{DEF}_{t-k} + \sum_{l=0}^{s} A_{4l}\Delta PTY_{t-l}\right]\\ & + \left[B_{10} + \sum_{i=1}^{p} B_{1i}\Delta GDP_{t-i} + \sum_{j=0}^{q} B_{2j}\Delta M1_{t-j} \right.\\ & \left. + \sum_{k=0}^{r} B_{3k}\mathrm{DEF}_{t-k} + \sum_{l=0}^{s} B_{4l}\Delta PTY_{t-l}\right] F_1(s_{1t},\gamma_1,c_1)\\ & + \left[D_{10} + \sum_{i=1}^{p} D_{1i}\Delta GDP_{t-i} + \sum_{j=0}^{q} D_{2j}\Delta M1_{t-j} + \right.\\ & \left. + \sum_{k=0}^{r} D_{3k}\mathrm{DEF}_{t-k} + \sum_{l=0}^{s} D_{4l}\Delta PTY_{t-l}\right] F_2(s_{2t},\gamma_2,c_2)\end{aligned}$$

$$+\left[H_{10}+\sum_{i=1}^{p}H_{1i}\Delta GDP_{t-i}+\sum_{j=0}^{q}H_{2j}\Delta M1_{t-j}\right.$$
$$\left.+\sum_{k=0}^{r}H_{3k}\mathrm{DEF}_{t-k}+\sum_{l=0}^{s}H_{4l}\Delta PTY_{t-l}\right]F_1(s_{1t},\gamma_1,c_1)F_2(s_{2t},\gamma_2,c_2)+u_t$$

with $u_t \sim \mathrm{iid}(0,\ \sigma^2)$ s_1 and s_2 are the transition variables. With such a formulation, we define four regimes that are delimitated by the threshold parameters c_1 and c_2.

3. Econometric procedures and results for the US economy

The estimation of the MRSTAR money–output equation involves several steps. We test the stationarity of our series and we apply techniques that generalize the estimation procedures of one regime STAR models. In this section the econometric methodology is exposed and the results for the US economy are commented on.

3.1. The econometric methodology

Step 1. Stationarity tests
All variables are firstly tested for stationarity. This step is necessary to discriminate between the series that must be considered in level and those for which the first-difference must be calculated. This is helpful, notably for transition variables. For instance, one may want to know whether it is the interest term structures that influence the activity, or their slope (as a measure of the volatility of monetary policy). In this view, we use several tests. We first consider classical procedures such as Dickey and Fuller's augmented ADF test, and also tests of Phillips and Perron (1988), Kwiatkowski et al. (1992), Schmidt and Phillips (1992) and Elliot et al. (1996). We also use a mixing test. Since all these procedures are well-known in the literature, we do not present them here and refer the reader to the above papers.

Step 2. Estimate of a linear model
We specify a linear model, including lags on the endogenous and exogenous variables, by using the Akaike information criterion to select the appropriate lag lengths. Tests for residual autocorrelations are also required since omitted autocorrelations may cause rejection of the linearity hypothesis. At the end of this step, we denote w_t, the $m\times 1$ vector of explanatory variables.

Step 3. Testing linearity against STAR specifications
Because MRSTAR models are locally linear and also encompass STAR models, the next step is to test for linearity against STAR specifications for different transition variables s_t. If linearity is rejected for more than one transition variable, we choose the variable giving the lowest P-value. The procedure used is a LM-type test developed by Lüükkonen et al. (1988) and is based on the following auxiliary regression:

$$\hat{\nu}t=\beta'1wt+\beta'2wtst+\beta'3wtst2+\beta'4wtst3+\mu t,$$

where $\hat{\nu}_t$ is an element of $\{\hat{\nu}_s\}_{s=1}^{T}$ the vector of residuals obtained from the linear model estimated at step 2. The reason for using this auxiliary regression is the following. LSTAR models include nuisance parameters that are not identified under the null hypothesis of linearity. Several procedures have been suggested in the literature to cope with this problem. We adopt here an approach—exposed in Teräsvirta (1994) and adapted to MRSTAR by van Dijk and Franses (1999)—where Taylor expansion of the nonlinear function is used to form Lagrange Multiplier tests.[5]

The null hypothesis is H_0: $\beta_2=\beta_3=\beta_4=0$ against the alternative hypothesis H_1: $\beta_j\neq 0$ for at least one j, where the β_j's are $m\times 1$ vectors. The reader's attention is drawn on the following fact. In the general framework of linearity tests against LSTAR alternatives, all the variables should be tested as possible transition variables. However, in the framework of a money–output equation the choice of such variables are chosen in regard to the economic theory. Indeed, when the economy is affected by monetary policy, the asymmetric dynamics works through some specific mechanisms that are the 'transmission channels of monetary policy' (Section 2). So, instead of including directly M1 and productivity as transition variables, we choose some variables that are linked to the latter and that influence the dynamics of GDP: SL3, SL10, ΔCC, ΔWEDGE, GAP. Further, as shown in the results, when the lagged values of the GDP are included in the set of the transition variables, they are not selected by the LM tests.

Step 4. Distinguishing between LSTAR and ESTAR specifications

If the hypothesis of linearity is rejected, then it is necessary to choose the appropriate form of the transition functions defining the STAR model. In the literature, the functions that are the most often considered are the logistic and the exponential functions. In this view, one uses a sequence of nested hypotheses given by:

$$H_{01}:\beta_4=0,$$
$$H_{02}:\beta_3=0/\beta_4=0,$$
$$H_{03}:\beta_2=0/\beta_3=\beta_4=0.$$

The rejection of H_{01} implies the selection of a LSTAR model. If H_{01} is accepted while H_{02} is rejected, then the appropriate model is an ESTAR model. Accepting both H_{01} and H_{02} and rejecting H_{03} leads to a LSTAR model. Granger and Teräsvirta (1993) and Teräsvirta (1994) recommend to make the choice of the STAR model on the basis of the lowest P-value. If the test corresponding to H_{02} has the lowest P-value, then an ESTAR model is selected. If not, one chooses a LSTAR model. As we explain later on, applying this procedure to US data yields to retain a LSTAR model. So, at this stage, a LSTAR money–output equation is estimated.

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[5] For a detailed exposition of the testing procedures used for STAR models, the reader is referred to Granger and Teräsvirta (1993) and Teräsvirta (1994).

Step 5. STAR specifications against MRSTAR models

In the framework of STAR models, it seems interesting to test for the presence of a second transition function. This is done by testing the hypothesis of STAR specification against the alternative of a MRSTAR models. To do this, we rewrite Eq. (1) as follows:

$$y_t = \phi_1^{*\prime} w_t + \phi_2^{*\prime} w_t F_1(s_{1t},\gamma_1,c_1) + \phi_3^{*\prime} w_t F_2(s_{2t},\gamma_2,c_2) + \phi_4^{*\prime} w_t F_1(s_{1t},\gamma_1,c_1) F_2(s_{2t},\gamma_2,c_2) + \varepsilon_t, \tag{8}$$

with $\phi_1^* = \phi_1$, $\phi_2^* = \phi_2 - \phi_1$, $\phi_3^* = \phi_3 - \phi_1$ and $\phi_4^* = \phi_1 - \phi_2 - \phi_3 + \phi_4$. Suppose that the model selected, at step 4, was a LSTAR model—so that the problem is now to test the null hypothesis of a LSTAR model against the alternative of a MRSTAR model. $F_2(s_{2t}, \gamma_2, c_2)$ is replaced with a third-order Taylor expansion in the neighborhood of $\gamma_2(s_{2t} - c_2) = 0$ in order to obtain, after rearranging the different terms, the following model:

$$y_t = \theta'_1 w_t + \theta'_2 w_t F_1(s_{1t},\gamma_1,c_1) + \beta'_1 w_t s_{2t} + \beta'_2 w_t s_{2t}^2 + \beta'_3 w_t s_{2t}^3 + (\beta'_4 w_t s_{2t} + \beta'_5 w_t s_{2t}^2 + \beta'_6 w_t s_{2t}^3) F_1(s_{1t},\gamma_1,c_1) + \eta_t, \tag{9}$$

where the parameters vectors β_i are functions of the ϕ^*_j. The null hypothesis can be written as H^*_0: $\beta_i = 0$, $i = 1,\ldots,$ 6 and is tested by using again a LM-type test statistic as described by van Dijk and Franses (1999). The same rules as above apply (in case of several transition variables s_{2t}, we choose the variable that gives the statistic with the lowest P-value).

Step 6. Estimate of MRSTAR models

If H_0 is rejected at step 6, then MRSTAR models are estimated using algorithms such as BHHH, or BFGS. The great difficulty here is to find appropriate initial points that yield to the maximum of the objective function. Usually, this requires search procedures over different sets of initial values.

3.2. Application to the money–output equation

The sources and construction of data are presented in Appendix A. Our estimation covers the period from 1975:1 to 1998:2 (we use quarterly data). All series are in logarithm, except GAP, DEF, SL3, SL10, WEDGE and CC. The results of the different tests persuade us to allow for multiple regimes and two transition functions for the money–output equation.

3.2.1. Stationarity tests

We applied several stationarity tests (ADF, PP, KPSS, ERS, SP).[6] The results are available upon request to the authors. We briefly sketch here our main findings. For

[6] ADF, Augmented Dickey–Fuller. PP, Phillips–Perron. KPSS, Kwiatkowski–Phillips–Schmidt–Shin. ERS, Elliot–Rothenberg–Stock. SP, Schmidt–Phillips.

almost stationarity tests, we accepted the null hypothesis that the following variables are $I(1)$ at 5% level of confidence: GDP, M1, PTY, WEDGE, SL3, SL10, CC. This conclusion is quite surprising for SL3 and SL10 since term structure variables are usually found to be stationary in level (however, SL10 was stationary at 10% level of confidence). When the null hypothesis is stationarity instead of nonstationarity, as is the case in KPSS approach, the tests lead to the conclusion that these variables are $I(0)$. As expected, the variable DEF is $I(0)$. The conclusions for GAP are more ambiguous. According to ADF, PP and KPSS, this variable is $I(0)$, but if we look at ERS and SP, this hypothesis is rejected. The autocorrelation function—not reported here, however, suggests that GAP is a stationary variable.

We further studied the statistical properties of our variables by considering mixing tests. Mixing and stationarity properties share a common property, but the former is more general than the latter. Heuristically, the mixing property implies that the influence of the past observations of a series decreases gradually over time, so that in the very far future the past information is of no relevance to explain the stochastic patterns of a random variable. For a technical approach of mixing process, the reader can refer to Dufrénot and Mignon (2002). Here, we applied the R/S approach—suggested by Lo (1991). The mixing hypothesis was rejected for our variables, except for GAP, SL3, SL10 and DEF.

In summary, the results of the stationarity and mixing tests lead us to work with the following variables:

- variables measured in terms of first-differences: GDP, M1, PTY, WEDGE, P, CC;
- variables measured in level: SL3, SL10, GAP, DEF.

3.2.2. Linearity tests, STAR tests and estimates

The results obtained by applying steps 2–6 are reported in Appendices B and C. Appendix B contains the results of the different tests (linearity and STAR tests), while Appendix C reports the estimations (linear, STAR and MRSTAR equations).

The linear model is obtained using the Akaike information criterion and different tests for residual autocorrelation. The *P*-values reject serial correlation and the presence of conditional heteroskedasticity (see linear model in Appendix C).

The second column in Table 1 (Appendix B) shows the *P*-values of the LM test statistics when the hypothesis of linearity is tested against the alternative of STAR models for different transition variables, e.g. the contemporaneous and lagged observations of one of the following variables: GAP, SL3, SL10, ΔCC, ΔGDP, ΔWEDGE (see the first column). Remind that ΔM1 and ΔPTY are not included since the transition variables must capture the transmission channels of monetary policy. We, however, include ΔGDP since this variable has been found to be an important transition variable in the modelling of MRSTAR models (van Dijk and Franses, 1999). The delay parameter is over the range 0–8. Indeed, it has been found that long lags of the term structure are important for predicting the economic activity and a similar argument may hold for the other variables (Estrella and Mishkin, 1998). When linearity is rejected for more than one variable, we choose the variable with the lowest *P*-value (this variable is indicated in italic in the table).

It is seen that the LM tests succeeds to reject linearity in many cases and that GAP_{t-3} is selected as the first transition variable.

The application of the three nested tests for choosing between LSTAR and ESTAR specifications yields the following P-values

H_{01}: 0.0003
H_{02}: 0.1229
H_{03}: 0.4535

We thus conclude in favor of a LSTAR model. The second equation in Appendix C reports the results of the estimated LSTAR model together with some diagnostic tests in order to test its robustness. It must be pointed out that estimating LSTAR— and MRSTAR—models by nonlinear least squares is not always easy. For instance, the sequence of estimates for the transition parameter may converge rather slowly. Following Teräsvirta (1994), we scale the exponent of the transition function by dividing it by the empirical standard deviation σ_{thres} of the corresponding transition variable as follows

$$F(s_t,\gamma,c)=\left[1+\exp(-\gamma(s_t-c))/\sigma_{\text{thres}}\right]^{-1}. \tag{10}$$

This allows making γ scale-free and yields easier interpretations of its estimate. The variance ratio $\hat{\sigma}^2_{\text{LSTR}}/\hat{\sigma}^2_{\text{LIN}}$ gives an idea of the relative gain in the fit from using a LSTAR model instead of a linear model. It suggests here that the gain is good: 0.8668. The results further show that the null hypothesis of no autocorrelation is not rejected (statistic GB). Moreover, the model passes the test of no heteroskedasticity (statistic White). A similar conclusion holds for the test of no autoregressive conditional heteroskedasticity (statistics ARCH).

The third and fourth columns in Table 1 (Appendix B) report results of the F-version of the tests of LSTAR models against MRSTAR alternatives, on the basis of the estimated LSTAR model with the first transition variable. We indicate the P-values of the tests for various choices of the second transition variable. In view of the results reported in the third column, it seems that GDP_{t-3} is an appropriate transition variable. However, the residuals estimated in the LSTAR model may not be exactly orthogonal to the gradient matrix used in the standard test LSTAR/MRSTAR and the partial derivatives of the first transition function with respect to γ and c may approach zero functions. van Dijk and Franses (1999) thus introduce a modified version of the test, the P-value of which is given in the fourth column in Table 1. As shown by the authors, these tests generalize the remaining nonlinearity test of Eitrheim and Teräsvirta (1996): the transition variable s_t in their test is replaced here by the two transition variables s_{1t} and s_{2t}. We see in the fourth column of Table 1 that the variable with the lowest P-value is $SL10_{t-7}$.

In summary, there is some evidence for considering the multiple regimes version of STAR model. The last regression in Appendix C shows the estimation of the MRSTAR equation using the method of nonlinear least squares. To examine the robustness checks of the estimation, we test the usual hypotheses on the residuals (no serial autocorrelation, no autoregressive conditional heteroskedasticity,...). It is worth mentioning that for the same reasons expounded before, we scale the transition functions F_1 and F_2 by dividing their expression by respectively σ_{thres1} and σ_{thres2} the empirical standard deviations of the transition variables s_{1t} and s_{2t}. Several conclusions can be drawn from then results. The null hypotheses of no serial correlation is not rejected (statistic GB), as well as the hypotheses of no heteroskedasticity both unconditional and conditional (statistics WHITE and ARCH). Moreover, the relative gain in the fit from using a MRSTAR model instead of a linear model, characterized by $\hat{\sigma}^2_{\text{MRSTR}}/\hat{\sigma}^2_{\text{LIN}}$ is important (0.6197) and more interesting than the fit from using a simple LSTAR model (0.7150).

It is usually difficult to interpret the individual coefficients of the MRSTAR models. It is rather instructive to study their implications in terms of asymmetry by examining their generalized IRFs. This is done in the next section. Before doing this, some comments are in order.

In Appendix D, we report different figures. Fig. 1 shows the predicted values of ΔGDP from both linear and MRSTAR models. Fig. 2 gives the shape of the first transition function ordered over time and over the first transition variable. Fig. 3 shows the same curves for the second transition function. Finally, Fig. 4 depicts the distribution of observations of ΔGDP across the different regimes.

When analyzing these figures, it can be noted that the MRSTAR models describe the most 'turbulent' periods in the data, better than the linear model. The periods of 'explosive' growth rates are related to values of $\hat{F}_1$ and/or $\hat{F}_2$ close to 1. For instance, we distinctly see the aftermath of the second oil crisis in 1979 (Fig. 1). It must be noted that regime changes seem to occur less frequently after 1990:1 in comparison to the period before. This corroborates the observations on the US business cycle.

The shapes of the transition functions depend upon the values of the estimated parameters $\hat{\gamma}_1$ and $\hat{\gamma}_2$ that indicate how rapid the transition from 0 to unity is. Comparing Figs. 2 and 3, it is seen that the transition phases are quite smooth. We obtain $\hat{c}_2 = 0.2458$, which would mean that the GDP decreases when the long-term interest rate increases above the short-term interest rate by more than 0.24%. To explain this, note that the long-term interest rate corresponds to a decrease in the price of assets and an increase induces capital losses. This implies wealth effects that reduces aggregate consumption and thus the GDP. The reaction of the GDP occurs after a delay of seven quarters (the transition variable is SL10_{t-7}). This corroborates a commonly shared view that long lags of the term structures help predicting the activity, but it may also be attributable to inflation expectations that are central to the term structure transmission mechanism. If we consider the first transition function, we see that $\hat{c}_1 = -0.0723$, which means that a decrease of more than 7.23% of current production under the potential production induces a recession. Given the smoothness of the shape of the transition function, the decrease occurs gradually. Contraction phases due to the effect of the output-gap are illustrated by points located in regimes 1 and 2 in Fig. 4.

4. Measuring the effects of monetary shocks: generalized impulse response functions in presence of changing transition variables

We now complete the MRSTAR model by introducing the impulse problem. Should we proceed in the usual way, we would compute the generalized impulse response function (GIRF) by adding shocks to the MRSTAR equations and then study the dynamic properties of the response. In our case, things are more complicated in the sense that the transition variables changes every period in regard to the way monetary policy is introduced. Indeed, as explained below, since the output-gap and the inflation rate both enter the reaction function of the central bank, there are feedback effects between the monetary impulses and the transition variables. This interaction is a source of asymmetric dynamic.

4.1. The impulse problem: why do the transition variables change over time?

It is usually assumed in the literature that the central banks set the short-term interest rate according to a reaction function (the Taylor rule):

$$r_t = a\Delta P_t + b\mathrm{GAP}_t \tag{11}$$

where r_t is the short-term interest rate and a, b are weights on the inflation rate and on the output level. Further, the aggregate supply of the economy is described by the following equation:

$$\Delta P_t = \sum_{i=0}^{P} \phi_i \mathrm{GAP}_{t-i}. \tag{12}$$

By combining both these equations, we obtain an equation relating the contemporaneous value of the short-term interest rate and the lagged values of the output-gap:

$$r_t = \sum_{i=0}^{P} \theta_i \mathrm{GAP}_{t-i}. \tag{13}$$

The latter equation can be used as a basis for forecasts of the future short-term interest rate. This leads to the following expression:

$$E[r_{t+j}] = \sum_{i=0}^{j} \theta_i E[\mathrm{GAP}_{t+j-i}] + \sum_{i=j+i}^{P} \theta_i \mathrm{GAP}_{t+j-i}\, \cdot \tag{14}$$

Also, the short-term interest rate forecasts are usually described as a weighted sum of the contemporaneous short-term interest rate and long-term interest rates (l_t):

$$E[r_{t+j}] = \lambda_j r_t + (1-\lambda_j) l_t\, \cdot \tag{15}$$

The combination of the last two equations yields the following expression:

$$l_t - r_t = A_0 + \sum_{i=0}^{j} \tilde{\phi}\left[E(y_{t+j-i}) - y_{t-i}\right] + \sum_{i=j+1}^{P} \tilde{\theta}_i (y_{t+j-i} - y_{t-i}) \cdot \tag{16}$$

The term structure is thus a predictor of future output growth. Assuming that expectations of future growth rate are obtained by extrapolating past growth rates, we would obtain an equation relating the term structure to past growth rates.

Another remark is in order here. It is often argued that the term structure contains information about the credibility of monetary policy, notably through the expectations of future inflation rates. Assuming again adaptive expectations, we would obtain an equation relating the term structure to past inflation rates.

The preceding remarks have some implications on the way we choose to study the effects of the monetary policy. We introduce a reduced-form equation that relates $SL10_t$ to its past values and the lagged values of ΔGDP and ΔP:

$$SL10_t = \mu + \sum_{i=1}^{\tau} \phi_i SL10_{t-i} + \sum_{j=1}^{x} \theta_j \Delta GDP_{t-j} + \sum_{k=1}^{K} \phi_k \Delta P_{t-k} + \xi_t, \tag{17}$$

where $\xi_t \approx \text{iid}(0,\sigma_\xi^2)$. Given the state-dependence property of the MRSTAR models, it is worth noting that our formulation of the impulse mechanism implies feedback effects between the endogenous variables and the transition variable SL10. As we shall see, these feedbacks cause persistent asymmetric dynamics in the responses to monetary policy.

4.2. Generalized impulse response functions

Let us consider the time series model:

$$Y_t = F(Y_{t-1},\ldots,Y_{t-p}) + H_t V_t \tag{18}$$

where F is a known function, Y_t is $K\times 1$ random vector, V_t is a $K\times 1$ vector of iid random perturbations with zero means and finite variances, H_t is a $K\times K$ random matrix which is a function of $(Y_{t-1},\ldots, Y_{t-p})$. Upper-case letters design random variables and lower-case letters denote realizations of these random variables. In addition, we use the following notations: Ω_{t-1} is the information set used to forecast Y_t, ω_{t-1} is a particular realization of Ω_{t-1}.

The usual IRF is defined as the difference between two realizations of Y_{t+n} which are similar up to $t-1$. The first realization is such that a unique shock of size δ affects the system between t and $t+n$. The second realization, which is taken as the benchmark, assumes that the system is not hit by any shock between t and $t+n$. We define the IRF as follows:

$$\mathrm{IRF}(n,\delta,\omega_{t-1})=E[Y_{t+n}|V_t=\delta,V_{t+1}=\ldots=V_{t+n}=0,\omega_{t-1}] \\ -E[Y_{t+n}|V_t=V_{t+1}=\ldots=V_{t+n}=0,\omega_{t-1}] \quad (19)$$

for $n=1, 2, 3,\ldots$

For nonlinear models, the IRF generally depends upon ω_{t-1} the particular history chosen as the basis for comparison of the two realizations. It also depends on the sign and the size of the shock δ. Potter (1995) and Koop et al. (1996) report that, in this context, asymmetric responses occur in two main forms. Firstly, for any particular history, the effect of shocks of varying sizes and signs is not a simple scaling of a unit shock. Secondly, for the same shock but different histories, the response can differ markedly. In order to highlight these properties, it is more suitable to use a GIRF. The GIRF uses the concept of expectation operator conditioned on the history and/or shock. The response is thus an average of what might happen given the past and the present. More formally, the GIRF in the case of an arbitrary current shock υ_t and history ω_{t-1} is given by:[7]

$$\mathrm{GIRF}(n,\upsilon_t,\omega_{t-1})=E[Y_{t+n}|\upsilon_t,\omega_{t-1}]-E[Y_{t+n}|\omega_{t-1}] \quad (20)$$

for $n=0, 1,\ldots$ We see that the GIRF is the difference between two conditional expectations which are themselves random variables. We now describe how we compute the GIRF for the estimated MRSTAR models. Our simulations include the following steps.

Step 1. Creating vectors of monetary impulses

To study the impact of monetary policy on the real activity, we introduce monetary innovations in the estimated MRSTAR models. Our monetary innovations are computed as the residuals of Eq. (17).

Since the GIRF is a random variable, we need vectors of different monetary shocks. For each t, define

$$\Sigma'=(\xi_{1t},\xi_{2t},\ldots,\xi_{Rt}) \quad (21)$$

We construct Σ' as follows. We generate $(\chi+p+1)R$ parameters

$$(\mu_1,\ldots,\mu_R,\theta_{1j},\ldots,\theta_{Rj},\ldots,\phi_{1k},\ldots,\phi_{Rk}),$$

$j=1,\ldots,\ \chi$ and $k=1,\ldots,\ K$. These parameters are randomly sampled from Normal

[7] For another approach, see Gallant et al. (1993). These authors used a measure based on something that is similar to the mean of the GIRF, but they consider a different baseline forecast. As in the case of the usual impulse response function, they define the baseline forecast conditional on information up to time t.

laws $N(\hat{\mu},\hat{\sigma}^2_{\mu})$, $N(\hat{\theta}_j,\hat{\sigma}^2_{\theta j})$, $N(\hat{\phi}_k,\hat{\sigma}^2_{\phi k})$.[8] From (Eq. (17)), we construct ξ_{rt} ($r=1,\ldots,R$) as follows:

$$\xi_{rt}=\mathrm{SL10}_t-\mu_r-\sum_{i=1}^{\tau}\phi_{ri}\mathrm{SL10}_{t-i}-\sum_{j=1}^{\chi}\phi_{rj}\Delta GDP_{t-j}-\sum_{k=1}^{K}\phi_{rk}\Delta P_{t-k} \tag{22}$$

Step 2. Definition of forecasts of the endogenous variables without shocks

We iterate the system of the following equations to obtain forecasts of the endogenous variables ΔGDP and ΔP (which are represented here by the variable Δy). We denote the forecasts $\hat{\Delta y}_{r,t+1},\hat{\Delta y}_{r,t+2},\ldots,\hat{\Delta y}_{r,N}$ where $r=1,\ldots,R$, $t=\tau+1,\ldots,N$ and $N<T$. τ is the maximum lag in the regressors of the estimated MRSTAR model. We iterate the MRSTAR model and compute:

$$\begin{aligned}\Delta \mathrm{G\hat{D}P}_{r,t}=&\left[\hat{A}_{10}+\sum_{i=1}^{p}\hat{A}_{1i}\Delta\mathrm{G\hat{D}P}_{r,t-i}+\sum_{j=0}^{q}\hat{A}_{2j}\Delta M1_{t-j}+\sum_{k=0}^{r}\hat{A}_{3k}\Delta DEF_{t-k}\right.\\&\left.+\sum_{l=0}^{s}\hat{A}_{4l}\Delta PTY_{t-l}\right]+\left[\hat{B}_{10}+\sum_{i=1}^{p}\hat{B}_{1i}\Delta\mathrm{G\hat{D}P}_{r,t-i}+\sum_{j=0}^{q}\hat{B}_{2j}\Delta M1_{t-j}\right.\\&\left.+\sum_{k=0}^{r}\hat{B}_{3k}\Delta DEF_{t-k}+\sum_{l=0}^{s}\hat{B}_{4l}\Delta PTY_{t-l}\right]F_1(\hat{s}_{1t},\hat{\gamma}_1,\hat{c}_1)+\left[\hat{D}_{10}\right.\\&+\sum_{i=1}^{p}\hat{D}_{1i}\Delta\mathrm{G\hat{D}P}_{r,t-i}+\sum_{j=0}^{q}\hat{D}_{2j}\Delta M1_{t-j}+\sum_{k=0}^{r}\hat{D}_{3k}\Delta DEF_{t-k}\\&\left.+\sum_{l=0}^{s}\hat{D}_{4l}\Delta PTY_{t-l}\right]F_2(\hat{s}_{2t},\hat{\gamma}_2,\hat{c}_2)+\left[\hat{H}_{10}+\sum_{i=1}^{p}\hat{H}_{1i}\Delta\mathrm{G\hat{D}P}_{r,t-i}+\sum_{j=0}^{q}\hat{H}_{2j}\Delta M1_{t-j}\right.\\&\left.+\sum_{k=0}^{r}\hat{H}_{3k}\Delta DEF_{t-k}+\sum_{l=0}^{s}\hat{H}_{4l}\Delta PTY_{t-l}\right]\times F_1(\hat{s}_{1t},\hat{\gamma}_1,\hat{c}_1)F_2(\hat{s}_{2t},\hat{\gamma}_2,\hat{c}_2)+\hat{u}_t\end{aligned}$$

with $\hat{u}_t\sim \mathrm{NID}(0,\hat{\sigma}^2_{\mathrm{MRSTR}})$ and

$$\hat{s}_{2t}=\mathrm{S\hat{L}10}_t=\hat{\mu}+\sum_{i=1}^{\tau}\hat{\phi}_i\mathrm{S\hat{L}10}_{t-i}+\sum_{j=1}^{\chi}\hat{\theta}_j\Delta\mathrm{G\hat{D}P}_{t-i}+\sum_{k=1}^{K}\hat{\phi}_k\Delta\hat{P}_{t-k}+\xi_{rt}, \tag{23}$$

where

$$\xi_{rt}\approx \mathrm{iid}(0,\hat{\sigma}^2_{\xi}).$$

Step 3. Definition of forecasts of the endogenous variables with monetary impulses

[8] The '^' is used to indicate the estimated parameters from the regression of $\Delta\mathrm{SL10}_t$ on a constant, $\Delta\mathrm{GDP}_{t-j}$ and $\Delta\mathrm{P}_{t-k}$, $j=1,\ldots,\chi$ and $k=1,\ldots,K$.

We repeat step 2, but instead of the vector $\{\xi_{rt}\}$ we compute $\Delta\hat{GDP}_{r,t+n}$ using a vector $\{\xi^*_{rt}\}$ with $\xi^*_{rt}=\sigma_{\xi}\xi_{rt}I_t$ and $I_t=1$ if the monetary shock occurs at time t and $I_t=0$ otherwise. ξ_{rt} is the t-th observation of the vector of residuals defined in step 1. σ_{ξ} is the corresponding standard error. The forecasts obtained are noted $\Delta\hat{GDP}^{shocks}_{r,t+n}$. We do that for $r=1,\ldots, R$.

Step 4. Computation of the GIRF

We form the averages

$$\Delta\overline{GDP}_{t+n}=\frac{1}{R}\sum_{r=1}^{R}\left(\Delta\widehat{GDP}^{shocks}_{r,t+n}-\Delta\widehat{GDP}_{r,t+n}\right). \quad (24)$$

This gives us the GIRF of our MRSTAR model, for a given t.

4.3. Application to US data: comparing LSTAR and MRSTAR models

To compute the GIRFs, we choose initial shocks ξ corresponding respectively to regimes 1, 2, 3 and 4 (more precisely, the initial periods of the shocks are respectively 1975:4, 1991:4, 1984:1 and 1987:2 for regimes 1–4). We retain different values for the variance of the shocks σ_{ξ} and distinguish between positive and negative shocks. Given the definition of ξ_t in Eq. (17) and the definition of SL10, a positive monetary shock does correspond to an unexpected decrease of the short term interest rate. Conversely, a negative shock reflects an increase of the short term interest rate. ± 1, ± 2 means that we consider ± 1 and ± 2 times the residual standard deviation of the shock. Appendix E shows the figures corresponding to the nonlinear responses of the MRSTAR models. Note that the GIRFs are computed for the log level of GDP, by taking the cumulative sums of the GIRFs for the growth rate.

Several conclusions can be drawn from Figs. 5–8. Globally, all figures show some evidence of asymmetric dynamics in several ways. Firstly, for two out of the four regimes, the average response appears to be more magnified when shocks are negative than when they are positive (see regimes 2 and 3), while the opposite conclusion holds for regime 4. So, we have two types of asymmetric dynamics. One concerns the difference in the reaction of the economy to positive and negative and the second shows a 'regime-dependent' property. Secondly, it is important to note that the GDP is either positively or negatively correlated to monetary impulses, thereby changing with the regimes. This clearly appears by comparing figure with the others. Regime 1 (which corresponds to the years 1979:4–1983:4 and 1990:1–1991:4) shows evidence of the so-called 'consumption puzzle': the response to negative shocks are positive whereas the response to positive shocks are negative. Thirdly, we note that the way in which the magnification of the shocks occurs also change across regimes. For instance, negative shocks are magnified in regime 2, while positive shocks are magnified in regime 4.

Clearly, the above findings challenge the interpretation of asymmetry usually found in linear models. It is usually argued that expansionary monetary policies do not affect output, whereas restrictive monetary policies have significant negative effects on the activity output. Here, the effects of the shocks vary across time (due to regime-shifting phenomena). We see that the profiles of the IRFs vary with the initial period of the shocks. The fact that the response of the GDP is more damped in regimes 2 and 3, in comparison to regime 4, seems to indicate that in some periods most of the effects of the monetary policy on the real activity are carried by the inflation rate rather than by the output. However, this does not necessarily occur during expansion phases.

The time-varying effects are due to the feedback effects of our model. An initial shock in Eq. (17) induces a variation of SL10 and this modifies the value of the transition function F_2. These variations yield some modifications of the GDP in the MRSTAR equation and again of SL10 in Eq. (17). Depending upon whether the term structure varies above or under the threshold value $\hat{c}_2$, the economy either remains in the same regime, or is pushed in another one. Finally, one also notes that in our nonlinear framework, initial shocks have very persistent effects, since the GDP is never back to its initial value. This is an illustration of historical dependence.

We now compare the nonlinear GIRFs obtained for MRSTAR models with those computed for LSTAR models. We shock the transition variable common to both specifications (GAP_{t-3}) and again distinguish the initial periods of the shocks by the regime in which they occur. The comparative graphs of the GIRFs are shown in Appendix F.

At first glance, it seems that the graphs resemble each other in regard to the asymmetric dynamics that are produced (in both cases, we find some evidence of dependence to the sign and the size of the initial shock and we also observe a regime dependency). Despite these similarities, the LSTAR and MRSTAR models differ in one aspect. For three regimes out of four, the range of variation of the GIRFs is smaller for the MRSTAR models, thereby illustrating that adding more regimes to the LSTAR model yield a smoother dynamics. This findings corroborates what is usually found in the literature on nonlinear IRF: additive nonlinear terms (here the second transition function) reduces the dispersion of the effects of the initial shocks. This implies that shocks are more persistent in the case of MRSTAR models (the argument is notably true for regime 4, where there are more fluctuations in the response of the GDP for the LSTAR model).

5. Concluding remarks

In this paper, we have explored a new approach for studying the quantitative effects of monetary policy. The framework of regime-switching models such as the MRSTAR models allows reproducing some stylized facts, notably the asymmetric responses of the GDP. Also, the MRSTAR models help reproducing phenomena such as history-dependence, time variability of the impulse functions and sensitivity to the regime observed when the initial shock is produced.

This paper offers several extensions. First, it may be interesting to compare the results obtained here for the US economy with those of other O.E.C.D. countries. Secondly, the financial transmission channels of monetary policy are often considered without evoking the impact of volatility. Volatility can be a source of instability in the response functions. In this view, it may be worth extending the MRSTAR model by including nonlinear components in the error term. Thirdly, it might be interesting to calibrate and simulate MRSTAR models (instead of estimating them from data) and find the transition function parameters for which the models best reproduce the usual stylized facts on monetary policy.

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Appendix A: The data

We use quarterly data over the sample period 1975:1–1998:2 for the US economy. The sources are the IMF's International Financial Statistics and the O.E.C.D. database. We use of the following variables (seasonally adjusted and transformed in logarithm): M1 for the aggregate quantity of money, the federal fund rate (FED), the treasury bill rate (TBILL), 3-year and 10-year government bond yields (BOND3, BOND10), banks prime loan rate (BANK), the GDP volume, the real labor costs (RCL), hourly earnings (HEARNINGS), the total productivity index (PTY), the output-gap (GAP), all items consumer prices (P). We also consider the US budget deficit (DEF).

The following variables are also defined:

$$\mathrm{SL3}=\mathrm{BOND3}-\mathrm{TBILL} \quad \text{and} \quad \mathrm{SL10}=\mathrm{BOND10}-\mathrm{TBILL}$$

as proxies of interest rate term structures,

$$\mathrm{CC}=\mathrm{FED}-\mathrm{BANK}$$

as a proxy of the credit channel, and an indicator of the wedge is:

$$\mathrm{WEDGE}=\mathrm{RCL}-\mathrm{HEARNINGS}+P.$$

Appendix B: The tests

Table 1.

Appendix C: Estimations of ΔGDP models: linear, LSTAR and MRSTAR

Linear model:

$$\Delta GDP_t = \underset{(0.00)}{0.0042} + \underset{(0.89)}{0.0160}\Delta GDP_{t-1} - \underset{(0.08)}{0.1935}\Delta GDP_{t-2} - \underset{(0.98)}{0.0013}\Delta M1_t + \underset{(0.96)}{0.0034}\Delta M1_{t-1} + \underset{(0.01)}{0.1980}\Delta M1_{t-2} - \underset{(0.05)}{0.1522}\Delta M1_{t-3} + \underset{(0.00)}{0.7235}\Delta PTY_t - \underset{(0.05)}{0.0093}\mathrm{DEF}_t$$

$\hat{\sigma}^2_{\mathrm{LIN}} = 0.00005$, GB(1) = 0.1760 (0.67), GB(4) = 1.4494 (0.83), Skew = 0.5196, Kurt = 4.5704, BJ = 80.5545 (0.00), White = 27.4952 (0.97), ARCH(1) = 0.0691 (0.79), ARCH(4) = 0.6822 (0.95), AIC = −9.74169.

LSTAR model:

$$\Delta GDP_t = +\underset{(0.47)}{0.0054} - \underset{(0.39)}{0.0280}\Delta GDP_{t-1} - \underset{(0.20)}{0.0912}\Delta GDP_{t-2}$$

$$-\underset{(0.00)}{0.3088}\Delta M1_t + \underset{(0.17)}{0.1011}\Delta M1_{t-1} + \underset{(0.12)}{0.1235}\Delta M1_{t-2}$$

$$-\underset{(0.11)}{0.1276}\Delta M1_{t-3} + \underset{(0.00)}{0.9724}\Delta PTY_t - \underset{(0.49)}{0.0016}\mathrm{DEF}_t$$

$$+\Big[-\underset{(0.48)}{0.0051} + \underset{(0.32)}{0.0479}\Delta GDP_{t-1} - \underset{(0.00)}{0.8211}\Delta GDP_{t-2}$$

$$-\underset{(0.31)}{0.0535}\Delta M1_{t-3} + \underset{(0.00)}{0.3504}\Delta PTY_t + \underset{(0.39)}{0.0286}\mathrm{DEF}_t\Big]F_1$$

$$+\underset{(0.00)}{0.9850}\Delta M1_t - \underset{(0.00)}{0.3528}\Delta M1_{t-1} + \underset{(0.00)}{0.4537}\Delta M1_{t-2}$$

with

$$F_1 = \left[1 + \exp\left(-\underset{(0.00)}{1.9342}\left(\mathrm{GAP}_{t-3} - \underset{(0.00)}{1.1557}\right)/\sigma_{\mathrm{thres1}}\right)\right]^{-1}$$

$\hat{\sigma}^2_{\mathrm{LSTR}} = 0.00004$, $\hat{\sigma}^2_{\mathrm{LSTR}}/\hat{\sigma}^2_{\mathrm{LIN}} = 0.8668$, GB(1 = 0.3947 (0.52), GB(4) = 2.4550 (0.65), Skew = −0.2029, Kurt = 0.9858, BJ = 4.0254 (0.13), White = 42.9592 (0.51), ARCH(1) = 0.0040 (0.94), ARCH(4) = 4.2861 (0.36), AIC = −9.8774.

Table 1
Linearity test against STAR alternatives and LSTAR tests against MRSTAR alternatives

	LIN/STAR	LSTAR/MSTAR	LSTAR/MSTAR (modified)
$SL3_t$	0.0740	0.0132	0.0132
$SL3_{t-1}$	0.3149	0.2599	0.2088
$SL3_{t-2}$	0.5210	0.1055	0.1529
$SL3_{t-3}$	0.0272	0.0264	0.0241
$SL3_{t-4}$	0.1555	0.0891	0.1276
$SL3_{t-5}$	0.4733	0.0100	0.0383
$SL3_{t-6}$	0.4063	0.1398	0.0933
$SL3_{t-7}$	0.1708	0.1686	0.1192
$SL3_{t-8}$	0.0444	0.4340	0.3412
$SL10_t$	0.0951	0.0274	0.1551
$SL10_{t-1}$	0.2317	0.0500	0.0779
$SL10_{t-2}$	0.5188	0.0615	0.0415
$SL10_{t-3}$	0.0955	0.0023	0.0036
$SL10_{t-4}$	0.1814	0.1343	0.0604
$SL10_{t-5}$	0.3236	0.0084	0.0180
$SL10_{t-6}$	0.3413	0.2538	0.1204
$SL10_{t-7}$	0.6871	0.0121	*0.0009*
$SL10_{t-8}$	0.3275	0.1439	0.2702
ΔCC_t	0.1143	0.0448	0.1953
ΔCC_{t-1}	0.1069	0.3440	0.7157
ΔCC_{t-2}	0.2688	0.0084	0.0249
ΔCC_{t-3}	0.0053	0.0084	0.0125
ΔCC_{t-4}	0.6251	0.0513	0.2379
ΔCC_{t-5}	0.5898	0.1643	0.1774
ΔCC_{t-6}	0.6741	0.1546	0.0502
ΔCC_{t-7}	0.0898	0.2869	0.2407
ΔCC_{t-8}	0.4919	0.2829	0.2850
GAP_t	0.0061	–	–
GAP_{t-1}	0.2153	–	–
GAP_{t-2}	0.0655	–	–
GAP_{t-3}	*0.0008*	–	–
GAP_{t-4}	0.4935	–	–
GAP_{t-5}	0.7985	–	–
GAP_{t-6}	0.5926	–	–
GAP_{t-7}	0.4107	–	–
GAP_{t-8}	0.7145	–	–
$\Delta WEDGE_t$	0.0046	0.1957	0.1709
$\Delta WEDGE_{t-1}$	0.5658	0.1629	0.1401
$\Delta WEDGE_{t-2}$	0.4414	0.0345	0.3132
$\Delta WEDGE_{t-3}$	0.6008	0.2628	0.1474
$\Delta WEDGE_{t-4}$	0.3241	0.5756	0.5133
$\Delta WEDGE_{t-5}$	0.2850	0.2284	0.2975
$\Delta WEDGE_{t-6}$	0.4115	0.0008	0.0015
$\Delta WEDGE_{t-7}$	0.3428	0.1042	0.2830
$\Delta WEDGE_{t-8}$	0.0039	0.0455	0.0986
ΔGDP_{t-1}	0.3409	0.0225	0.0327
ΔGDP_{t-2}	0.0532	0.0045	0.0022
ΔGDP_{t-3}	0.6808	*0.0001*	0.0028
ΔGDP_{t-4}	0.0277	0.0003	0.0066

Table 1 *(Continued)*

	LIN/STAR	LSTAR/MSTAR	LSTAR/MSTAR (modified)
ΔGDP_{t-5}	0.9023	0.0965	0.1624
ΔGDP_{t-6}	0.4274	0.0298	0.0118
ΔGDP_{t-7}	0.9210	0.0972	0.0506
ΔGDP_{t-8}	0.2058	0.0512	0.0771

Note: The first column indicates the possible transition variables. The second column corresponds to the LM-type test exposed for instance in Teräsvirta (1994) in order to test linearity against STAR modeling for different transition variables. The third and fourth columns are the LM-type tests that are used to test a STAR model against a MRSTAR alternative; they are based on a Taylor expansion of the second transition function F_2; it consists in testing the nullity of supplementary parameters implied by this expansion for different possible transition variables s_{2t} (van Dijk and Franses (1999)). The last column is a modified version of this test that takes into account the possible problem of no orthogonalization of the residuals of the LSTAR model estimation with the gradient matrix (van Dijk and Franses (1999)). The numbers reported are P-values. We indicate in italic the lowest P-values that help selecting the transition variables that will be chosen for s_{1t} in the STAR models (second column) and for s_{2t} in the MRSTAR models (third and fourth columns).

MRSTAR model:

$$\Delta GDP_t = \underset{(0.38)}{0.0255} + \underset{(0.49)}{0.0007}\Delta GDP_{t-1} + \underset{(0.35)}{0.0423}\Delta GDP_{t-2}$$

$$-\underset{(0.00)}{0.6622}\Delta M1_t + \underset{(0.46)}{0.0098}\Delta M1_{t-1} + \underset{(0.11)}{0.1367}\Delta M1_{t-2}$$

$$-\underset{(0.00)}{0.3578}\Delta M1_{t-3} + \underset{(0.00)}{0.8174}\Delta PTY_t - \underset{(0.40)}{0.0276}\mathrm{DEF}_t$$

$$+\Big[-\underset{(0.33)}{0.0445} - \underset{(0.30)}{0.0567}\Delta GDP_{t-1} - \underset{(0.00)}{0.6497}\Delta GDP_{t-2}$$

$$+\underset{(0.00)}{0.7099}\Delta M1_t - \underset{(0.00)}{0.2704}\Delta M1_{t-1} + \underset{(0.00)}{0.3676}\Delta M1_{t-2}$$

$$-\underset{(0.11)}{0.1349}\Delta M_{t-3} + \underset{(0.00)}{0.2678}\Delta PTY_t + \underset{(0.30)}{0.0584}\mathrm{DEF}_t\Big]F_1$$

$$+\Big[-\underset{(0.44)}{0.0145} - \underset{(0.00)}{0.2674}\Delta GDP_{t-1} - \underset{(0.30)}{0.0564}\Delta GDP_{t-2}$$

$$+\underset{(0.30)}{0.0572}\Delta M1_t - \underset{(0.39)}{0.0300}\Delta M1_{t-1} - \underset{(0.00)}{0.3682}\Delta M1_{t-2}$$

$$+\underset{(0.00)}{0.6362}\Delta M1_{t-3} + \underset{(0.06)}{0.1721}\Delta PTY_t + \underset{(0.34)}{0.0455}\mathrm{DEF}_t\Big]F_2$$

$$+\Big[\underset{(0.36)}{0.0382} - \underset{(0.28)}{0.0628}\Delta GDP_{t-1} - \underset{(0.29)}{0.0615}\Delta GDP_{t-2}$$

$$+\underset{(0.00)}{0.8809}\Delta M1_t+\underset{(0.00)}{0.5722}\Delta M1_{t-1}+\underset{(0.05)}{0.1814}\Delta M1_{t-2}$$

$$-\underset{(0.14)}{0.1191}\Delta M1_{t-3}-\underset{(0.05)}{0.1859}\Delta PTY_t-\underset{(0.23)}{0.0830}\mathrm{DEF}_t\Big]F_1F_2$$

with

$$\begin{cases} F_1=\left[1+\exp\left(-\underset{(0.00)}{0.6617}\left(\mathrm{GAP}_{t-3}+\underset{(0.26)}{0.0723}\right)/\sigma_{\mathrm{thres1}}\right)\right]^{-1} \\ F_2=\left[1+\exp\left(-\underset{(0.00)}{0.7272}\left(\mathrm{SL10}_{t-7}-\underset{(0.01)}{0.2458}\right)/\sigma_{\mathrm{thres2}}\right)\right]^{-1} \end{cases}$$

$\hat{\sigma}^2_{\mathrm{MSTR}}=0.00003$, $\hat{\sigma}^2_{\mathrm{MRSTR}}/\hat{\sigma}^2_{\mathrm{LIN}}=0.6197$, $\hat{\sigma}^2_{\mathrm{MRSTR}}/\hat{\sigma}^2_{\mathrm{LSTR}}=0.7150$, GB(1) = 1.4045 (0.23), GB(4) = 3.9708 (0.40), Skew = −0.1542, Kurt = 0.4209, BJ = 0.8849 (0.64), White = 56.7334 (0.09), ARCH(1) = 1.7489 (0.18), ARCH(4) = 7.5819 (0.11), AIC = −10.1938.

Note: The *P*-values corresponding to the parameter estimates or the different test statistics are given in parentheses. $\hat{\sigma}^2_{\mathrm{LIN}}$, $\hat{\sigma}^2_{\mathrm{LSTR}}$ and $\hat{\sigma}^2_{\mathrm{MRSTR}}$, are the estimated variances of the residuals corresponding respectively to the linear, LSTAR and MRSTAR models, GB(q) denotes the Godfrey–Breusch statistic of the LM-type test for qth-order serial correlation in the residuals, Skew is the skewness coefficient, Kurt is the Kurtosis, BJ is the Jarque–Bera normality test for the residuals, White is the White heteroskedasticity test, ARCH denotes the Engle conditional heteroskedasticity test and AIC is the Akaike information criteria. σ_{thres}, σ_{thres1} and σ_{thres2} are the empirical standard deviations of the corresponding transition variables.

Appendix D: Predicted values of the GDP and the transition functions

Figs. 1–4.

Appendix E: Nonlinear impulse response functions: MRSTAR

Figs. 5–8.

Appendix F: Nonlinear impulse response functions: comparing LSTAR and MRSTAR models

Figs. 9–12.

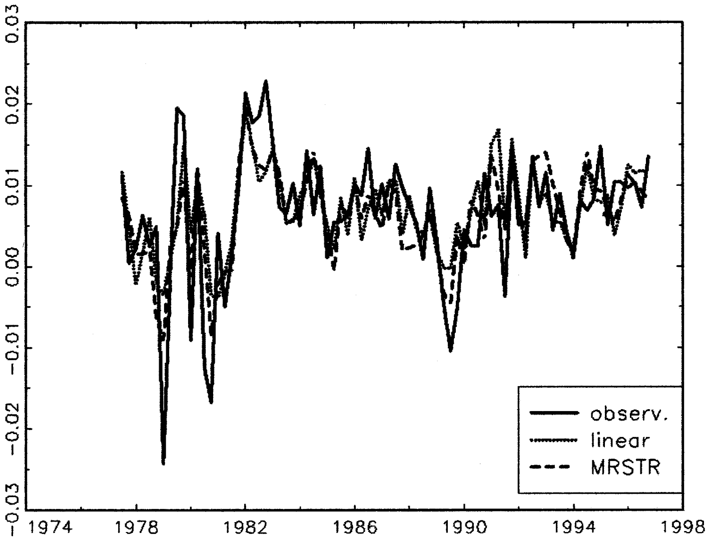


Fig. 1. Predicted values of DGP—linear and MRSTAR models.

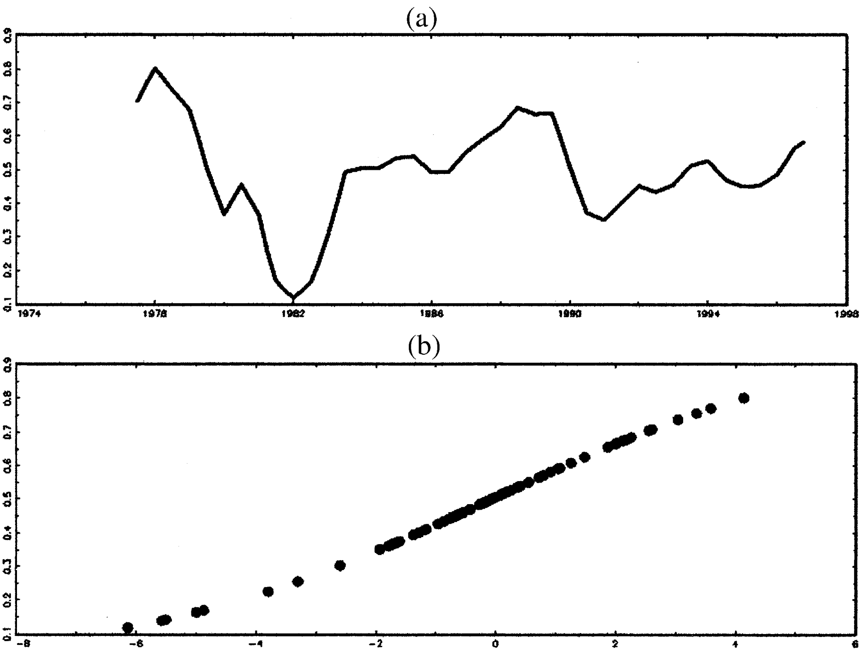


Fig. 2. Shape of the first transition function.

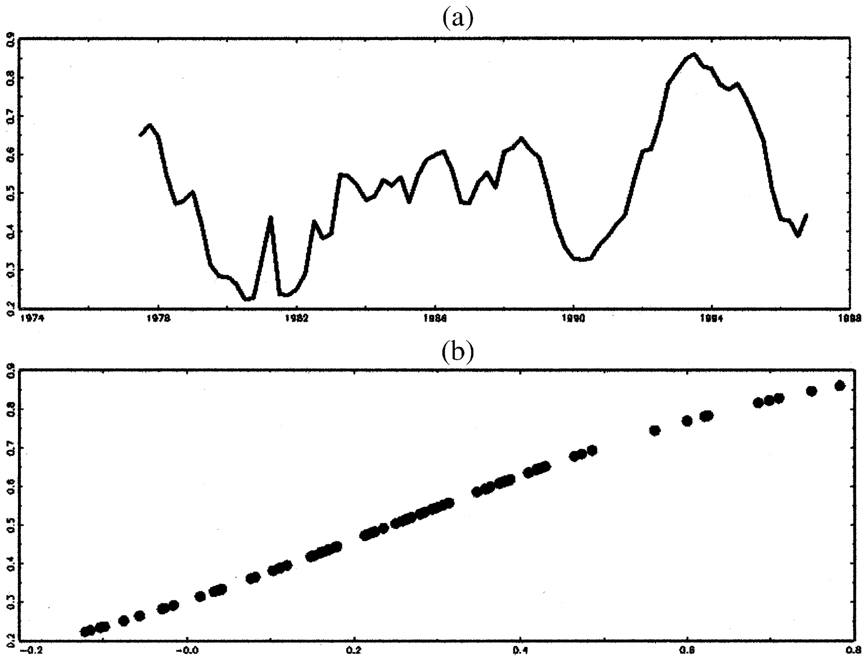


Fig. 3. Shape of the second transition function.

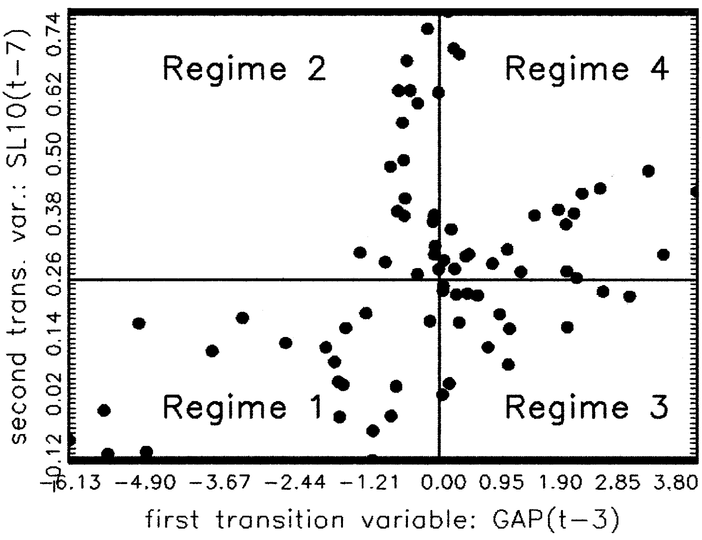


Fig. 4. Distributions of observations across the different regimes.

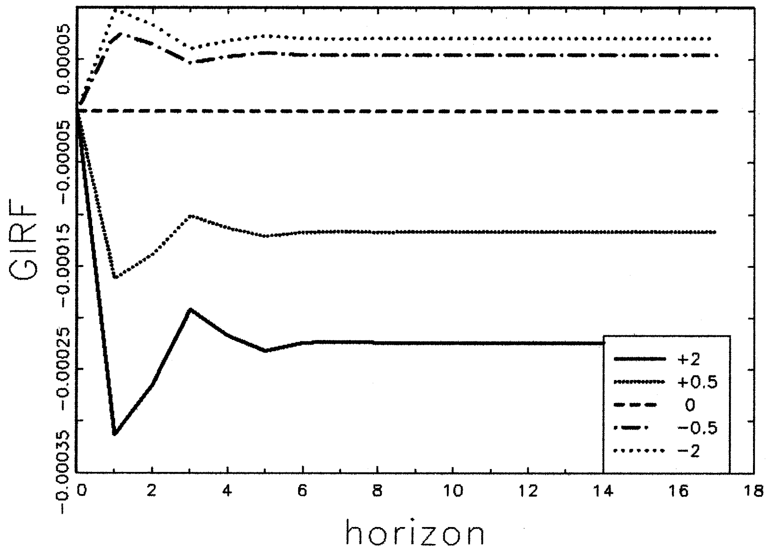


Fig. 5. GIRF—log level of GDP—Regime 1.

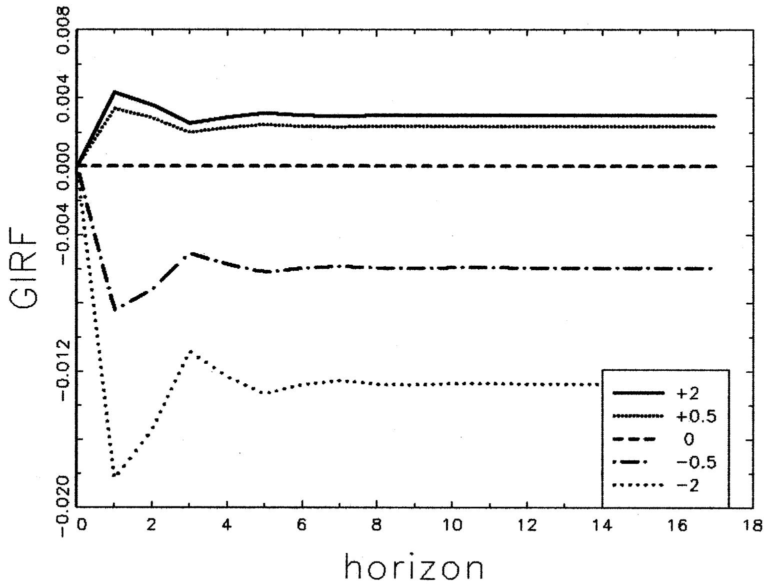
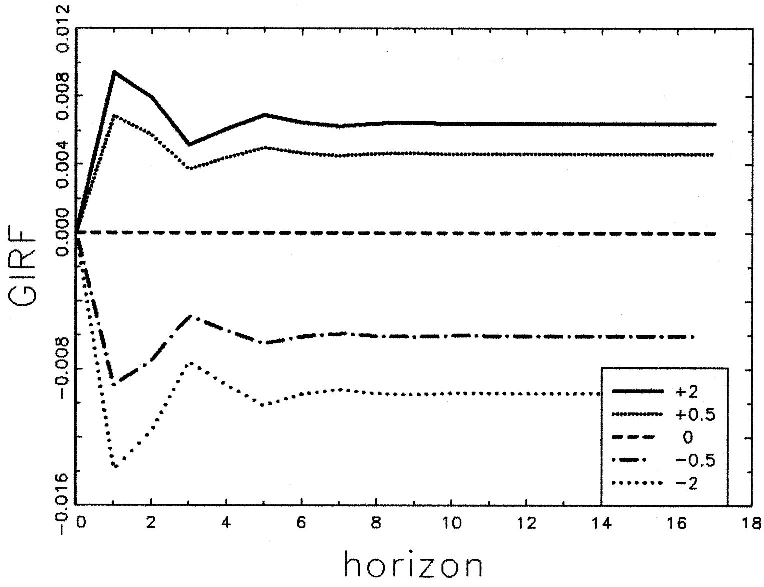


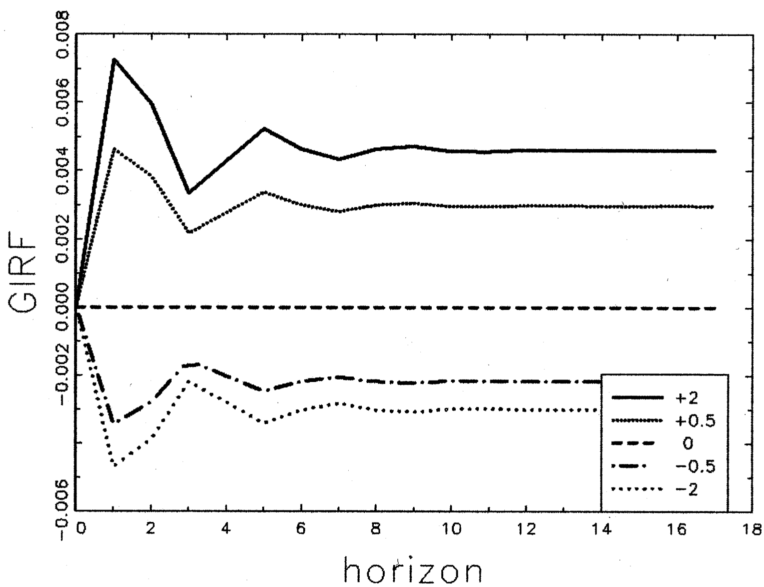
Fig. 6. GIRF—log level of GDP—Regime 2.

114



115 Fig. 7. GIRF—log level of GDP—Regime 3.

119



120 Fig. 8. GIRF—log level of GDP—Regime 4.

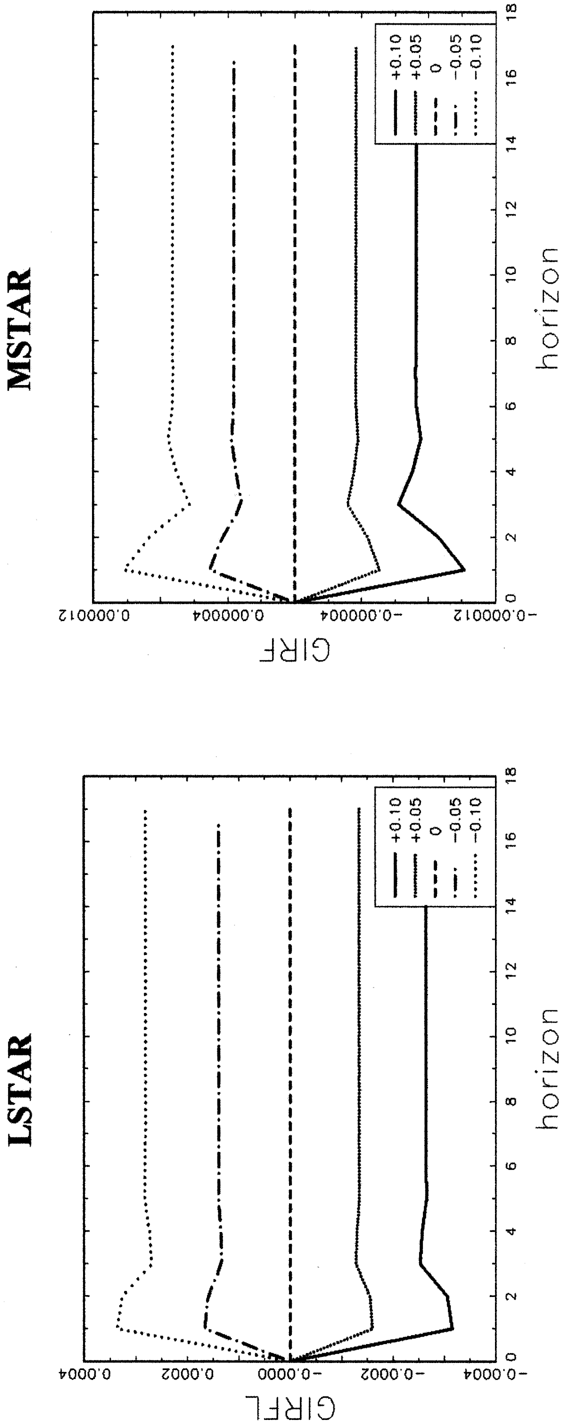


Fig. 9. GIRF—log level of GDP—Regime 1.

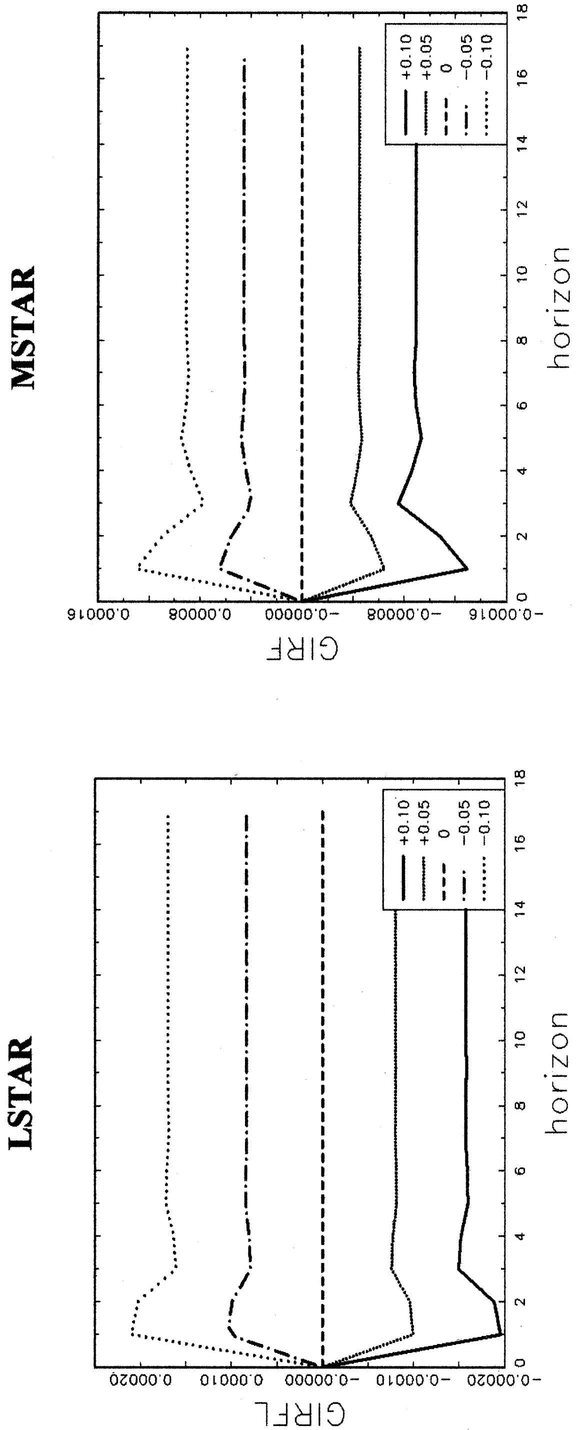


Fig. 10. GIRF—log level of GDP—Regime 2.

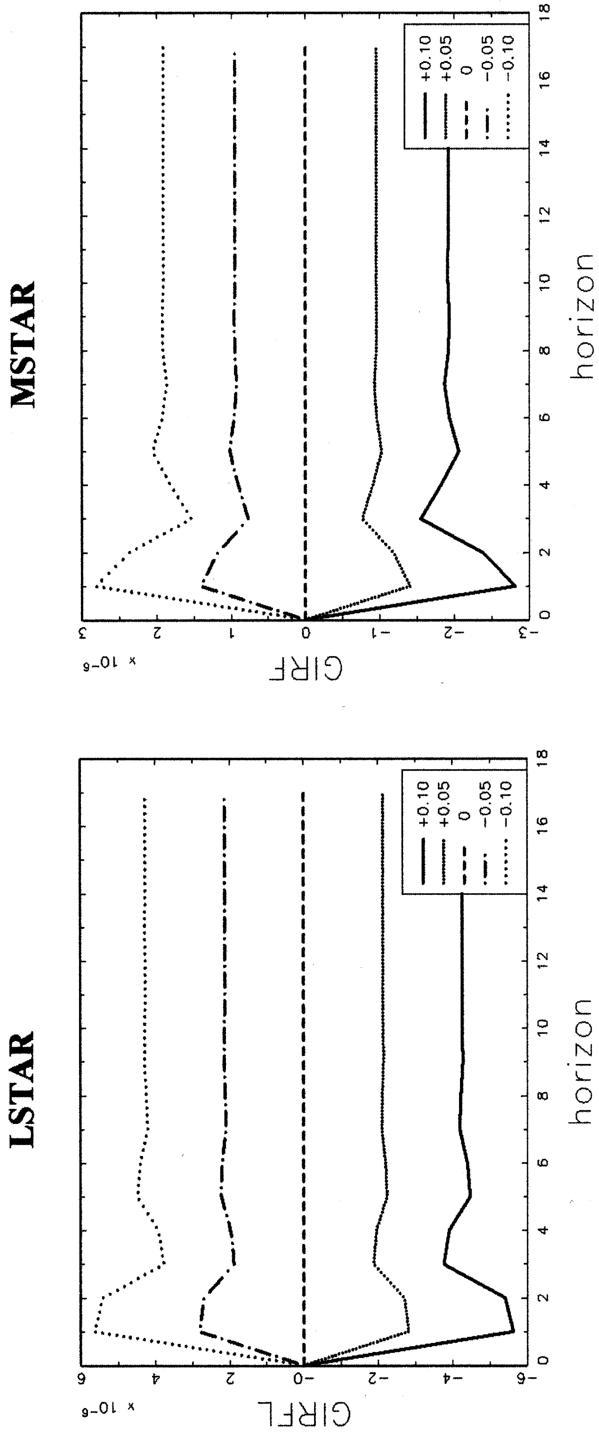


Fig. 11. GIRF—log level of GDP—Regime 3.

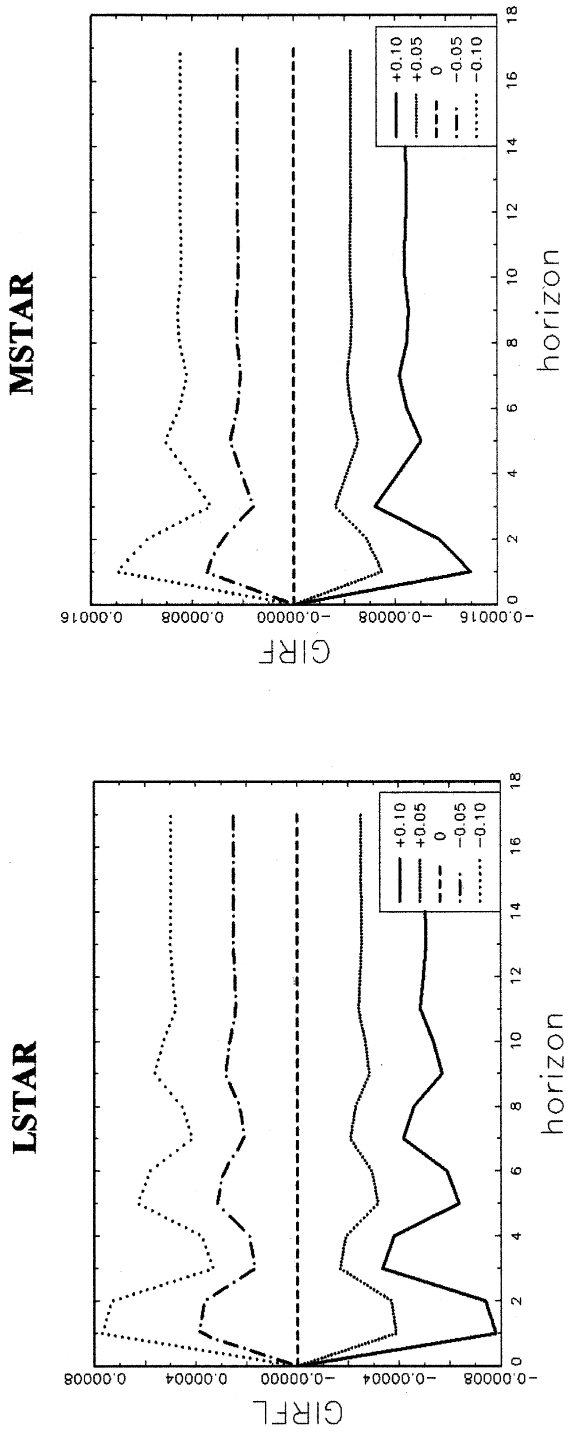


Fig. 12. GIRF—log level of GDP—Regime 4.

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