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LABOR MARKET REGIMES AND MONETARY POLICY

N. Acocella, G. Di Bartolomeo, and D.A. Jr. Hibbs

Abstract

In this paper we propose straightforward extensions of multi-union, monopolistic competition models appearing in the recent literature on the macroeconomic effects of monetary policy. We extend these models from the Stackelberg equilibrium to the Nash equilibrium under variations in labor market regime in order to evaluate propositions about non-neutrality of monetary policy.

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1. Introduction

The aim of this paper is twofold. First, we use a simplified version of the monopolistic competition model laid out in Coricelli *et al*. (2000)[1] to evaluate macroeconomic outcomes associated with different labor market regimes, which are defined by the rigidity of wage contracts. The labor market is rigid when unions set contract wages in advance (commitment) and cannot alter their claims after the monetary authority acts. It is flexible where unions do not sign binding contracts and have the capacity to vary wages ex-post in reaction to realizations of monetary policy.

Second, we evaluate results concerning non-neutrality of monetary policy obtained in recent literature. In particular, we use a multi-union, monopolistic competition framework to evaluate Acocella and Di Bartolomeo's (2002) specification of necessary and sufficient conditions for monetary policy non-neutrality. Our objective is to help clarify the reasons for differences in macroeconomic outcomes associated with the different labor market regimes.

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* Nicola Acocella and Giovanni Di Bartolomeo work at the University of Roma *La Sapienza*; Douglas A. Jr. Hibbs works at the University of Göteborg. We thank two anonymous referees for useful comments.

[1] We use the Coricelli *et al*. (2000) framework because of its generality; with simple assumptions it nests a number of the other models.

The rest of the paper is organized as follows. The next section introduces the baseline model. Section 3 solves the players' problems. Section 4 considers different regimes. We first consider the more common case of the Stackelberg equilibrium, and we then extend the model to the Nash case. Section 5 discusses the results we obtain in the context of the debate on monetary policy non-neutrality. Section 6 concludes.

2. The economic setup

We consider a simple economy where a central bank interacts with several labor unions and firms. The basic setup is taken from Coricelli *et al*. (2000) as amended with some additional assumptions that simplify the exposition.[2] The central bank determines aggregate demand by setting the money supply. Product prices and wages are determined by the firms and unions, respectively, which act in an imperfectly competitive environment.[3] We assume n unions of equal size, with $n \in [1, +\infty]$, and a continuum of monopolistic firms of mass one, each producing one good. Unions are indexed by *i*. All the firms associated with union $i \in [1, n]$ in wage bargaining are indexed by *ij* and without loss of generality are assumed to be located in the interval $\left(\frac{i-1}{n}, \frac{i}{n}\right)$.

[2] The Appendix gives a full derivation of the economic setup, as well as a description of the assumptions made to simplify the exposition.

[3] Competitive markets can be viewed as a limiting case. They have been analyzed elsewhere: Cubitt (1995) and Cukierman and Lippi (1999) for example investigate unionized labor markets with competitive goods markets and Blanchard and Kiyotaki (1987) treat the case of imperfect competition without trade unions.

Firm(s) *ij* maximize a one period profit function under demand and production technology constraints. Firm level demand is given by:

(1) $$Y_{ij}^{d}=\left(\frac{P_{ij}}{P}\right)^{-\eta}\frac{M}{P}$$

where P_{ij} and P are the individual firm price and general price level, respectively. The latter is conveniently defined as the geometric average of the individual firms' prices. $\eta>1$ is the elasticity of demand facing the individual firm with respect to its relative price. Firm-level demand is also affected by aggregate demand, which is assumed equal to the real money supply, $\frac{M}{P}$.

Each firm owns a production technology using labor inputs only and exhibiting decreasing returns to scale:

(2) $$Y_{ij}=L_{ij}^{\alpha} \qquad \alpha\in(0,1)$$

where Y_{ij} and L_{ij} are the output supply and the labor input of firm *ij*.

Using equations (1) and (2), firm *ij*'s conditions for profit maximization under monopolistic competition can be written as log-linear equations for product price and labor demand:

(3) $$p_{ij}-p=\theta\left[(1-\alpha)(m-p)+\alpha(w_i-p)\right]$$

(4) $$l_{ij}^{d}=\theta\left[-\eta(w_i-p)+m-p\right]$$

where $\theta=\frac{1}{\eta-(\eta-1)\alpha}>0$ and lower case variables denote logs of the corresponding upper case variables.

Equation (3) can be rewritten in terms of unemployment among union *i*'s members as:

(5) $$u_i = \theta\left[\eta\left(w_i - p\right) - \left(m - p\right)\right]$$

By averaging equations (3) and (4), one obtains (as shown in the Appendix) aggregate reduced forms for the price level and unemployment as:

(6) $$p = \alpha w + \left(1-\alpha\right)m$$

(7) $$u = w - m$$

By using equation (6) we can rewrite equation (5) as:

(8) $$u_i = \theta\left[\eta w_i - \alpha\left(\eta - 1\right)w\right] - m$$

Equation (8) says that unemployment for union *i*'s members is positively related to union *i*'s wage claims but negatively related to the average wage. The effect of union *i*'s claim always dominates the average effect since $\eta > \alpha\left(\eta-1\right)$. Unemployment is also decreasing in the nominal money supply. Taking account of the fact that the average nominal wage is equal to $w = \sigma w_i + \left(1-\sigma\right)w_{-i}$, where $\sigma = \frac{1}{n}$ is union *i*'s membership,[4] and w_{-i} is the average nominal wage set by the other unions, equation (8) becomes:

(9) $$u_i = \theta\left\{\left[\eta - \alpha\sigma\left(\eta-1\right)\right]w_i - \alpha\left(1-\sigma\right)\left(\eta-1\right)w_{-i}\right\} - m$$

Equation (9) is union *i*'s reduced form unemployment rate. In setting the nominal wage union *i* faces two opposite effects. First, by raising its nominal wage, it decreases the demand for labor and, thus, employment because of the higher labor costs imposed on the firm. Second, by raising its

[4] Recall that we have *n* unions of equal size.

wage, the union increases the average wage which by itself makes firm *ij* more competitive. The former effect of course always dominates the latter.

3. Optimal policies

3.1 The central bank's problem

We assume that the central bank seeks to minimize the following quadratic loss function subject to (6) and (7):

(10) $$V = \frac{\beta}{2}\pi^2 + \frac{u^2}{2}.$$

$\pi = p - p_{-1}$ is the inflation rate. $\beta \in (0, +\infty)$ is the central bank's inflation aversion parameter. For a one period optimal policy the lagged price level, p_{-1}, is given parametrically. So without loss of generality it can be set to zero, which allows us to speak of current prices and inflation rates interchangeably, as in Cubitt (1995).

By solving the central bank's problem we obtain the optimal wage-contingent monetary policy rule:

(11) $$m = -\phi w,$$

where $\phi = \frac{\alpha(1-\alpha)\beta - 1}{(1-\alpha)^2\beta + 1}$, $\phi \in \left(-1, \frac{\alpha}{1-\alpha}\right)$ and $\phi'(\beta) > 0$.

When $\beta < \frac{1}{\alpha(1-\alpha)}$, $\phi < 0$ and wage increases are accommodated; the central bank is liberal or "populist".[5] The reverse is true for $\beta > \frac{1}{\alpha(1-\alpha)}$;

[5] See, e.g., Guzzo and Velasco (1999).

$\phi > 0$ and the central bank is conservative. The accommodation parameter ϕ therefore defines the central bank's effective degree of conservativeness.[7]

3.2 The labor unions' problem

Recall that the number of unions is equal to $n \in [1, +\infty)$. Each union seeks to minimize its preference function defined by the membership's real wage $(w_i - p)$ and unemployment rate (u_i):

$$(12) \qquad U_i = \gamma(w_i - p) - \frac{u_i^2}{2} \qquad\qquad i \in \{1, 2, ...n\}$$

Parameter γ is the union's weight associated with the real wage rate. It can be interpreted as an index of the labor market distortion induced by the unions' existence.

The first order condition for union i is:

$$(13) \qquad \gamma(1 - Z_\pi) - u_i Z_{u_i} = 0$$

where the operators $Z_x = \frac{\partial x}{\partial w_i}$ for $x \in \{\pi, u_i\}$ depend on the regime facing a union because a union's information set varies by regime as will be clear from the next section.[8]

[7] When ϕ equals the lower bound of its domain, we observe the ultra-liberal central bank, i.e. a central bank that takes account of unemployment only. With ϕ equal to the upper bound of its domain, we observe the ultra-conservative central bank, i.e. a central bank that takes account of inflation only.

[8] In other words, in the flexible wage regime (Nash equilibrium) the Zs are computed by differentiating equations (6) and (8) with respect to w_i. By contrast in the rigid labor market regime (Stackelberg equilibrium), they are equal to the derivatives of (6) and (8) subject to equation (11). In differentiation, recall that $w=\sigma w_i+(1-\sigma)\, w_{-i}$.

4. Labor market regimes

4.1 Regimes and information setting

Equilibrium outcomes of the game are obtained by solving equations (11) and (13) under the unions' information constraint. Different equilibrium concepts are associated with different information settings and associated specifications of expectations,[9] and therefore, they vary by labor market regime.

Following Ljungqvist and Sargent's discussion of equilibrium concepts and expectations, the Stackelberg equilibrium implies that unions must form rational expectations ex-ante about the central bank's policy, and cannot change nominal wages thereafter. Therefore, this regime is associated with sticky wages set one period in advance. By contrast, in a Nash equilibrium environment each player forms (rational) expectations of the other's best response policy. A Nash game therefore implies flexible wages — both players can always change the value of their control variables. Notice, however, that a Nash induced flexible wage setting does not mean that the labor market is perfectly competitive because of the unions' monopolistic power in setting the nominal wages.

Elasticities associated with the two equilibrium concepts are summarized in Table 1.

[9] An excellent discussion on the relationship among equilibrium concepts, information sets, and rational expectations is provided by Ljungqvist and Sargent (2000: Chapter 16).

Table 1. Elasticities

	Stackelberg information set	*Nash information set*
Z_{π}	$(\alpha-\phi+\alpha\phi)\sigma$	$\alpha\sigma$
Z_{u_i}	$\frac{\alpha\sigma+(1-\alpha\sigma)\eta}{\alpha+(1-\alpha)\eta}+\sigma\phi$	$\frac{\alpha\sigma+(1-\alpha\sigma)\eta}{\alpha+(1-\alpha)\eta}$

Elasticities under the rigid wage setting regime (Stackelberg equilibrium) are computed by differentiating equations (6) and (9) with respect to w_i after substituting *m* from (11) into the union's FOC in (13). Elasticities associated with the flexible wage setting (Nash equilibrium) are obtained by differentiating equations (6) and (9) with respect to w_i, taking *m* as given.

All the elasticities are positive and $(1-Z_{\pi})=Z_{w_i-p}$. By using the elasticities in Table 1 in equations (11) and (13) we obtain the outcomes of the two labor market regimes in the next subsections.

4.2 Rigid labor markets

The Stackelberg equilibrium (rigid labor market equilibrium) is found by considering a two-stage game solved backwards. In the first stage the central bank solves its problem (equation (11)). In the second stage unions simultaneously solve their problems (equation (13)) in light of the optimal wage-contingent monetary policy rule (equation (11)). Therefore, the *Zs* associated with the Stackelberg solution are computed by considering equations (6), (9), and (11) (see Table 1).

By substituting the Zs into equation (13), after some algebra we find the following first order condition for union i:[10]

(14) $$\gamma\left(1-\hat{Z}_{\pi}\right)-\hat{Z}_{u_i}\left(\kappa_1 w_i-\kappa_2 w_{-i}\right)=0 \Rightarrow w_i=\frac{\kappa_2}{\kappa_1}w_{-i}+\frac{\gamma}{\kappa_1}\frac{\hat{Z}_{w_i-p}}{\hat{Z}_{u_i}}$$

with: $\kappa_1=\left[\eta-\alpha\sigma(\eta-1)\right]\theta+\sigma\phi=\hat{Z}_{u_i}$, $\kappa_2=\left[\alpha(1-\sigma)(\eta-1)\right]\theta-(1-\sigma)\phi$.

Each union reacts to an increase in the average wage of other unions by raising its nominal wage if $\alpha(\eta-1)\theta>\phi$ (which implies $\frac{\kappa_2}{\kappa_1}>0$). Reactions are however less than proportional, since $\kappa_1>\kappa_2$. By contrast, if $\alpha(\eta-1)\theta<\phi$, union i reacts by decreasing its nominal wage. Thus the more conservative the central bank is (the larger is ϕ), the more likely it is that monetary policy disciplines the unions' wage policies.

Equation (14) represents a system of n equations. By imposing the symmetry condition $w=w_i=w_{-i}$ and solving, we obtain the equilibrium nominal wage, $w=\frac{\gamma}{\kappa_1-\kappa_2}\left(\frac{\hat{Z}_{w_i-p}}{\hat{Z}_{u_i}}\right)$, which can be written as:

(15) $$w=\frac{\left[1-(\alpha-\phi+\alpha\phi)\sigma\right](\alpha+\eta-\alpha\eta)}{\left\{\eta\left[1-\sigma(\alpha-\phi+\alpha\phi)\right]+\alpha\sigma(1+\phi)\right\}(1+\phi)}\gamma$$

Unemployment and inflation rates are directly derived from equation (15) and the aggregate reduced form equations of the model are (remark that $m=-\phi w$):

[10] We indicate the Zs associated with the rigid wage regime with an hat.

(16) $$u=\frac{\left[1-(\alpha-\phi+\alpha\phi)\sigma\right](\alpha+\eta-\alpha\eta)}{\eta\left[1-(\alpha-\phi+\alpha\phi)\sigma\right]+\alpha\sigma(1+\phi)}\gamma$$

(17) $$\pi=\frac{\left[1-(\alpha-\phi+\alpha\phi)\sigma\right](\alpha+\eta-\alpha\eta)(\alpha-\phi+\alpha\phi)}{\left\{\eta\left[1-\sigma(\alpha-\phi+\alpha\phi)\right]+\alpha\sigma(1+\phi)\right\}(1+\phi)}\gamma$$

Equation (16) is a function of the effective degree of central bank conservativeness. Hence monetary policy is non-neutral. After some tedious comparative statics algebra, both u and π can be shown to be decreasing in the effective degree of conservativeness.[11] Therefore, Coricelli's *et al.* (2000) result holds: the more conservative is the central banker, the lower are inflation and unemployment.

By rewriting equation (16) as $u=\gamma\hat{Z}_{w_i-p}\big/\hat{Z}_{u_i}$, notice that $\kappa_1-\kappa_2=1+\phi$, the channel used by monetary policy to affect real variables in the presence of a wage wedge[12] becomes clear. Monetary policy influences union wage choices by affecting the utility trade off between the real wage and unemployment, since both $\hat{Z}_{w_i-p}$ and $\hat{Z}_{u_i}$ are function of the effective degree of central bank's conservativeness, ϕ. This channel is different from the standard mechanism based on unions' inflation aversion—first introduced by Gylfason and Lindbeck (1994). Now, unions moderate their wage claims not because inflation has direct utility costs, but rather out of the fear that the central bank's reaction to inflation will make the real wage-employment trade-off implied by their objective functions less favorable.

[11] Parameter ϕ is the only parameter that depends on central bank's preferences (β).

[12] In this paper a wage wedge between the wage relevant for the firms' labor demand and that relevant for the unions arises: each firm *ij*, in fact, deflates the nominal wage w_i by its own-product price and each union *i* deflates the nominal wage by the average product price.

4.3 Flexible labor markets

In the Nash case each union maximizes its preference under the aggregate price constraint (6) and the firm *ij*'s labor demand (9) by taking the money supply and other unions' nominal wages as given. Union *i*'s first order condition is still equation (13), but now the elasticities faced by the union do not depend on monetary policy, and therefore, the Zs must be recomputed using constraints (6) and (9) only. By doing so, and substituting the resulting Zs into equation (13), more tedious algebra yields union *i*'s reaction function:

(18) $$w_i = \frac{\eta - \alpha(\eta-1)}{\eta - \alpha\sigma(\eta-1)} m + \frac{\alpha(1-\sigma)(\eta-1)}{\eta-\alpha\sigma(\eta-1)} w_{-i} +$$

$$+\gamma(1-\alpha\sigma)\left[\frac{\eta-\alpha(\eta-1)}{\eta-\alpha\sigma(\eta-1)}\right]^2 \quad i \in \{1,2,...n\}$$

Union *i*'s always reacts to money expansion by raising its wage claims since the coefficient of m in equation (18) is always positive. Moreover, unlike the case in the rigid wage regime, union *i* always reacts to increases in the average wage of other unions by raising its nominal wage. By taking account of the parameter magnitudes it is easy to check that each union always reacts less than proportionally to the central bank, with the exception of the single union case. If there is only one union, wage policy completely neutralizes monetary policy since the reaction coefficient on m is equal to one.

The Nash equilibrium is found by solving the system of $n+1$ equations formed by equations (11) and (18). By imposing the symmetry condition

$w = w_i = w_{-i}$, we obtain the following equilibrium values for the players' control variables:

$$w = \left(\frac{\eta-\alpha(\eta-1)}{\eta-\alpha\sigma(\eta-1)}\right)\left(\frac{1+(1-\alpha)^2\beta}{\beta(1-\alpha)}\right)(1-\alpha\sigma)\gamma \tag{19}$$

$$m = \left(\frac{\eta-\alpha(\eta-1)}{\eta-\alpha\sigma(\eta-1)}\right)\left(\frac{1-\alpha\beta(1-\alpha)}{\beta(1-\alpha)}\right)(1-\alpha\sigma)\gamma \tag{20}$$

After substitution into (6) and (7), the optimal settings for *w* and *m* in (19) and (20), respectively, imply the following equilibrium macroeconomic outcomes:

$$u = \frac{(1-\alpha\sigma)\left[\eta-\alpha(\eta-1)\right]}{\eta-\alpha\sigma(\eta-1)}\gamma \tag{21}$$

$$\pi = \frac{(1-\alpha\sigma)\left[\eta-\alpha(\eta-1)\right](\alpha-\phi+\alpha\phi)}{\left[\eta-\alpha\sigma(\eta-1)\right](1+\phi)}\gamma. \tag{22}$$

Since the degree of conservativeness is the only parameter dependent on the central bank's preferences, it is clear that monetary authority cannot affect the real variable as, i.e., the unemployment rate (*u*). Thus monetary policy is neutral in the flexible wage regime. Moreover, inflation is decreasing in the effective degree of conservativeness ϕ, which, it should be recalled, is an increasing function of β. Hence Rogoff's standard results apply. First, unemployment is not affected by central bank policy. Second, the more conservative the central bank is, the lower inflation is.

Again equation (21) reflects the unions' utility trade off between the real wage and unemployment. However, now monetary policy is neutral since neither $Z_{w_i - p}$ or Z_{u_i} depend on the effective degree of central bank's conservativeness, ϕ.

5. Some propositions concerning necessary and sufficient conditions for non-neutrality in the light of model outcomes

The different outcomes associated with the different regimes depend on the monetary policy non-neutrality proposition. Therefore, it is important to discuss the reasons and conditions for monetary non neutrality.

In a recent paper Acocella and Di Bartolomeo (2002) have shown that a necessary but not sufficient condition for non-neutrality is that the unions directly or indirectly take account of inflation, in addition to the real wage and/or employment. Sufficient conditions for non-neutrality are either (i) that unions can pre-commit their wage policies or (ii) that the marginal rate of substitution between employment (unemployment) and inflation in the unions' preference is a function of inflation.

In order to evaluate Acocella and Di Bartolomeo's non-neutrality proposition in our context, two facts have to be noted.[13]

1. For $\sigma < 1$ each union indirectly takes account of inflation because a "wage wedge" between the firm and the union exists:[14] Unions deflate nominal wages by the general price level whereas the firm's wage deflator is own product price.

[13] See Acocella and Di Bartolomeo (2002) for a more detailed discussion on the two arguments.

[14] This is not the case for σ=1.

2. The marginal rate of substitution between unemployment and inflation (which recall is here the price level) does not depend on the latter, since union *i*'s preference function is linear in the real wage.

In our model the necessary condition for non-neutrality is satisfied, whereas one of the sufficient conditions is not. Therefore, non-neutrality requires satisfaction of the wage pre-commitment condition, which holds in the rigid labor market (Stackelberg) regime.

The comparisons between equations (21) and (16) are consistent with Acocella and Di Bartolomeo's claims: If unions take (indirectly) account of inflation, a wage commitment implies non-neutrality. By contrast, since the marginal rate of substitution between unemployment and inflation/prices in the union's preference function does not depend on the latter, the Nash equilibrium yields monetary neutrality.

Now assume that only one union exists (i.e. $\sigma = 1$). In this case there is no wage wedge, since the real wage relevant for the union is equal to that relevant for the firm *ij*. Therefore, unions do not weight inflation. In fact, the union's preference can be rewritten in terms of real variables only (i.e. in terms of unemployment). The unemployment rate associated with the rigid labor market regime (cf. equation (16)) is

(23) $u = (1-\alpha)\gamma$.

Equation (23) is the traditional result of the policy game literature (see, among the others, Acocella and Ciccarone, 1997). Hence monetary neutrality also characterizes the Stackelberg regime when unions give no weight to inflation, directly or indirectly (e.g. through the existence of a wage wedge). Again this result conforms to the necessary condition underlined by Acocella and Di Bartolomeo (2002).

Assuming that firms act in a perfectly competitive market also implies that the wage wedge does not exist, even if several unions interact in the labor market.[15] A competitive goods market is obtained by positing an infinite elasticity of product demand with respect to firm *ij'*s relative price. By taking this limit, equation (16) becomes equation (23). Therefore, non-neutrality again vanishes and Acocella and Di Bartolomeo's (2002) claims are confirmed.

Finally it should be noticed that if inflation appears directly (in a quadratic form) in the union's objective function, instead of indirectly as in our model, the marginal rate of substitution between (real wage) unemployment and prices depends on inflation levels. Therefore, non-neutrality always holds[16] as predicted by the Acocella and Di Bartolomeo's (2002) conditions for non-neutrality.

6. Conclusions

In this paper we considered a standard model of imperfectly competitive markets for goods and labor where a central bank and several unions strategically interact. Unions had the capacity to set wages and firms had the capacity to set prices, subject to downward sloping labor and product demand functions. We extended the standard setup by investigating the macroeconomic consequences of two different labor market regimes.

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[15] Because the price that all unions face is the same and it is given.

[16] See Cukierman and Lippi (1999) and Coricelli *et al*. (2000).

A rigid wage regime was defined by the presence of binding wage contracts such that nominal wages could not be adjusted ex-post, that is, after realizations of monetary policy. A flexible wage regime was defined by an institutional framework in which unions and the central bank simultaneously determine wages and the money supply, respectively, according to the expected behavior of the rival's policy.

We found that if only one union is present in the labor market, standard Rogoff results hold no matter which regime is in place. By contrast, if there is a multiplicity of unions, the rigid wage regime is associated with monetary non-neutrality, whereas the flexible wage regime is associated with monetary neutrality.

These results confirm conclusions of recent studies attempting to identify the channels of monetary policy non-neutrality. In particular, results are in line with two propositions specifically derived for the single-union policy games. First, if a union is not inflation-averse, money non-neutrality can arise only when the union indirectly weights inflation. In our case unions weight inflation because of the existence of a wage wedge, which arises between the real wage relevant for workers (normed to the general price level) and that relevant for the firm (normed to product price).

Second, concern about inflation is not a sufficient condition. It is predicted that non-neutrality is ensured if either the marginal rate of substitution between unemployment and prices in the union's preference function depends on prices, or unions can pre-commit their nominal wages. In our case, the marginal rate of substitution between unemployment and prices in the union's preference function does not depend on prices. Non-neutrality can arise only if unions pre-commit (rigid wage regime).

Appendix – The baseline monopolistic competition model

This appendix describes the economic setup where players act, it mainly correspond to the first stage of the Coricelli's *et al.* (2000) game. In the goods market monopolistic competition is assumed. A continuum of identical firms is evenly distributed over the unit interval and their total mass is one. Firms face the same demand and technological constraints.

The demand for firm *ij*'s product depends on its relative price and aggregate demand:

$$\text{(A.1)} \quad Y_{ij}^{d} = \left(\frac{P_{ij}}{P} \right)^{-\eta} \frac{M}{P}$$

where P_{ij} and P are individual firm price and the general price level. The general price level is defined as the geometric average of the prices of individual firms. $\eta > 1$ is the elasticity of demand facing the individual firm with respect to its relative price, and $\left(\frac{M}{P} \right)$ is the real quantity of money.

The aggregate price level can be conveniently written (in logarithms) as: $\ln(P) \equiv p = \sum_{i=1}^{n} \int_{\frac{1}{n}}^{\frac{i-1}{n}} p_{ij} dj = \int_{0}^{1} p_{ij} dj$ with $p_{ij} \equiv \ln(P_{ij})$.

Each firm owns a production technology using only labor input and exhibiting decreasing returns to scale:

(A.2) $Y_{ij} = L_{ij}^{\alpha}$ $\alpha \in (0,1)$

where Y_{ij} and L_{ij} are the output supply and the labor input of firm *ij*. The production function implies that the labor requirement for any level of output is $L_{ij} = Y_{ij}^{1/\alpha}$.

Each firm seeks to maximize its real profit, given by:

(A.3) $\Pi_{ij} = \frac{P_{ij}}{P} Y_{ij}^{d} - \frac{W_i}{P} L_{ij}$.

The usual profit maximization condition that marginal revenue equals marginal cost may thus be written:

(A.4) $\frac{P_{ij}}{P}\left[1-\frac{1}{\eta}\right] = \frac{W_i}{P}\frac{1}{\alpha} Y_{ij}^{\frac{1-\alpha}{\alpha}}$.

By substituting equation (A.1) into equation (A.4), we obtain the optimal relative price of firm *ij* as a function of the real wage and real money balances:

(A.5) $\frac{P_{ij}}{P} = \left(\frac{\mu}{\alpha}\frac{W_i}{P}\right)^{\frac{\alpha}{\alpha+\eta(1-\alpha)}} \left(\frac{M}{P}\right)^{\frac{1-\alpha}{\alpha+\eta(1-\alpha)}}$

where $\mu = \left(1-\frac{1}{\eta}\right)^{-1} > 1$ is the mark-up.

Equation (A.5) states that the optimal relative price of firm *ij*'s is higher the higher is the real wage it faces and the higher are real money balances.

Equating equation (A.1) and (A.2), $\left(\frac{P_{ij}}{P}\right)^{-\eta} \frac{M}{P} = L_{ij}^{\alpha}$, and using equation (A.5) we can derive an expression for employment in firm *ij*:

(A.6) $L_{ij} = \left(\frac{\mu}{\alpha}\frac{W_i}{P}\right)^{-\frac{\eta}{\alpha+\eta(1-\alpha)}}\left(\frac{M}{P}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}}$.

and taking the logs of expression (A.6), we derive firm *ij*'s demand for labor:

(A.7) $l_{ij}^d = \frac{1}{\alpha+\eta(1-\alpha)}\left[m-p-\eta(w_i-p)+\eta\ln(\alpha)-\eta\ln(\mu)\right]$

By taking the logs, equation (A.5) becomes:

(A.8)

$$p_{ij}-p = \frac{1}{\alpha+\eta(1-\alpha)}\left\{(1-\alpha)(m-p)+\alpha\left[(w_i-p)+\ln(\mu)-\ln(\alpha)\right]\right\}$$

Averaging equation (A.8) over the firms and rearranging we obtain the reduced form for the equilibrium general price level:

(A.9) $p = \alpha w + (1-\alpha)m + p_0$

where $p_0 = \alpha\left[\ln(\mu)-\ln(\alpha)\right]$

We find log aggregate employment by taking the average of (A.7) and substituting in for log aggregate price given by (A.9):

(A.10) $l = m - w + l_0$

where $l_0 = \frac{p_0}{\alpha}$

From equation (A.10) the reduced form for unemployment rate can also be derived.

(A.11) $u = \overline{l} - l = \overline{l} - (m-w) - l_0$

where $\overline{l}$ is the logarithm of aggregate labor supply, which also coincides with the per firm labor supply since the total mass of firms is equal to one.

In order to obtain a model simpler to handle, we abstract from the location parameters in equations (A.9), (A.10), and (A.11) by neglecting p_0, l_0, and $\overline{l}$. The unemployment rate is therefore proportional to $-l$.

Notice that the omission of $\overline{l}$, l_0, and p_0 is without loss of generality since these parameters do not affect solutions to the games because of the linear-quadratic functional forms of objective functions. A rigorous proof is available upon request.

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