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Equity premium and consumption sensitivity when the consumer–investor allows for unfavorable circumstances

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Abstract

Introducing one additional element due to possible misfortune to the return of each of two assets in the basic model of Samuelson (Rev. Econom. Statist. 51 (1969) 239) on optimum portfolio and consumption decisions, this paper resolves both the excess equity premium and the excess consumption sensitivity puzzles. This unified treatment provides a framework to study how important state variables will affect the change in aggregate consumption which is considered unpredictable in one formulation of the permanent income hypothesis. The implications of the theory agree with empirical results reported here and elsewhere. The theoretical framework appears to be simple and powerful as compared with alternative theories to explain the two puzzles. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Excessive equity premium and excessive consumption sensitivity are two interesting puzzles in economics. This paper attempts to solve them in a unified treatment. In the literature, some of which will be referred to in Section 6 below, the two puzzles are often treated separately. Surely, economists are well aware that portfolio and consumption decisions are interrelated since consumption from current income reduces savings which go to investment in assets to yield future income. In fact a unified treatment of portfolio and consumption decisions was provided by the classic papers of Samuelson

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(1969) and Merton (1969). However, these papers emphasize the portfolio selection aspects and have initiated the field of asset pricing. On the other hand, it was the work of Friedman (1957), followed by Hall (1978), which generated much subsequent research on consumption based on the permanent income hypothesis. This paper shows that it is fruitful to study the two subjects together using the basic model of Samuelson (1969).

The approach taken is to introduce an attitude of pessimism along the lines of Hansen et al. (1999). Without necessarily taking a stand on whether this approach is better than its alternatives, this paper shows how a pessimistic attitude can be incorporated into the basic model of Samuelson (1969) on optimum consumption and portfolio selection, and how the incorporation of such an attitude can explain both puzzles.

Under the consumption-based capital asset pricing theory the observed equity premium is considered excessive as pointed out by Mehra and Prescott (1985) and widely discussed in the literature (see Campbell et al., 1997, pp. 304–314). As in Hansen et al. (1999), we resolve this problem by assuming that there are additional elements introduced into the random components of stock and bond returns. Hansen et al. (1999) justify the additional elements by appealing to uncertainty on the part of the economic agent concerning the true model. Another interpretation of these additional elements is that even if the economic agent believes the model to be correct, she may be concerned with bad luck in the sense that her draw from the distribution of returns may be less favorable as captured by these additional elements. Whatever the interpretation of these elements, they are assumed to be controlled by an unfriendly nature. Unlike the model of Hansen et al. (1999), our model is nonlinear in the state and control variables and is much simpler. It modifies slightly the basic model of optimum consumption and portfolio selection of Samuelson (1969). We solve the equity premium puzzle by following the treatment in Campbell et al. (1997, p. 308) using our model. The Lagrange method as expounded in Chow (1997) is applied to solve the dynamic optimization problem of the consumer–investor who imagines the existence of an unfriendly nature in the framework of a stochastic dynamic game. Such a solution is termed a robust control solution in the literature on optimal control. Our model can explain the equity premium puzzle very easily.

The same model is also capable of solving the puzzle of excessive sensitivity of consumption to income. The set of first-order conditions derived from solving the above dynamic game contains information on both portfolio selection and consumption. The information includes the nature of response of the change in consumption to current income and expected change in income. Under the permanent income hypothesis changes in consumption are supposed to be uncorrelated with these income variables, but correlations will be induced by the allowance for unfavorable returns. In Section 2, the model is formulated in discrete time and the optimum robust control policy is derived. In Section 3, we use the model to solve the equity premium puzzle. In Section 4 the excessive consumption sensitivity puzzle is resolved. The effects of current income and expected change in income on the change in consumption are deduced. Some econometric evidence is reported which strongly support the implications of the model. Extension to the case of many assets is briefly treated in Section 5. Section 6 concludes.

2. Model formulation and the optimum robust policy

Assume that there are two assets in the financial market: a stock and a bond. Let $R_t = [R_{1,t} \;\; R_{2,t}]^{\mathrm{T}}$ be a vector of returns to these two assets, meaning that one dollar invested in asset i at the beginning of period t will result in $R_{i,t}$ dollars at the end of period t. The covariance matrix of R_t will be denoted by $\Sigma = (\sigma_{ij})_{2\times 2}$. The consumer–investor is assumed to construct a self-financing portfolio with the two assets and consume C_t during period t. Let w_t be the proportion of wealth invested in the stock. Following Samuelson (1969), the beginning-of-period value Z_t of such a portfolio is governed by

$$Z_{t+1} = (Z_t - C_t)[w_t \quad 1 - w_t]R_t. \tag{1}$$

Because of model uncertainty or pessimism, the consumer–investor is assumed to modify model (1) in her mind by adding a vector V_t to the vector R_t of returns and to imagine that nature will set V_t to minimize the following time-separable utility function. This modified formulation of Samuelson's model has two components. First, the random vector R_t is contaminated by a vector V_t. In the standard formulation the distribution of the random vector R_t is assumed to be known to the customer–investor who makes his decision based on this distribution. If he does not know the distribution for the random error in (1) he might think of the error as consisting of two components, R_t having a distribution which he can specify to the best of his knowledge and V_t which represents the deviation of his specified distribution from the truth unknown to him. Secondly, the modelling of V_t as being selected by nature in a way unfavorable to the decision maker captures the idea that he is pessimistic. He is concerned that something unfavorable may happen in a way that the specification of the distribution of R_t does not allow for. This idea is found in the literature on robust control theory. Rather than solving a standard optimal control problem, which corresponds to the solution in Samuelson (1969) in our case, robust control considers solutions that deviate from given assumptions of the model in such a way that if the model is incorrect the optimal policy is still robust.

Given this imagined behavior of nature who selects V_t that is unfavorable, the customer–investor is assumed to maximize the same utility function with respect to C_t and w_t. The imagined problem of nature is

$$\min_{\{V_t\}} \sum_{t=0}^{\infty} \beta^t \left[\mathrm{E}_t u(C_t) + \frac{1}{2}\theta V_t^{\mathrm{T}} \Sigma^{-1} V_t \right] \tag{2}$$

subject to the constraint (1) with V_t added to R_t. β is the discount factor and $u(C_t)$ is the period utility function. The term $(\theta/2)V_t^{\mathrm{T}}\Sigma^{-1}V_t$ is introduced as a cost to nature in order to restrict its setting of V_t. A quadratic form is customarily used to penalize a deviation of V_t from zero. Without this quadratic cost nature will set a value of V_t that is too unreasonable to give a useful model of consumption–investment behavior. The parameter θ is used to calibrate the degree of pessimism or concerned for error in specifying the error term in (1). It characterizes the behavior of the consumer. This formulation follows Hansen et al. (1999) except that our model (1) is nonlinear in the variables Z_t, C_i and w_t. Also we introduce the inverse of the covariance matrix of the returns in order to weight the quadratic loss for the nature's policy variables V_t instead of using just an identity matrix in its place. It is reasonable to penalize the variations of $V_{i,t}$ proportionally less when the variance of the associated return $R_{i,t}$ is large.

In the sequel, we apply the Lagrange method expounded in Chow (1997) to solve this dynamic game. After the introduction of the vector V_t representing pessimism or model uncertainly in the dynamic model, the Lagrange function for the imagined minimization problem of nature is

$$L = \sum_{t=0}^{\infty} \mathrm{E}_t \beta^t \Big\{ u(C_t) - \beta\lambda_{t+1}[Z_{t+1} - (Z_t - C_t)[w_t \quad 1-w_t](R_t + V_t)] + \frac{\theta}{2} V_t^{\mathrm{T}} \Sigma^{-1} V_t \Big\}, \tag{3}$$

where λ_t $(t = 0, 1, \ldots)$ are Lagrange multipliers. The optimal strategy for nature is derived from the first-order condition

$$\beta^{-t} \frac{\partial L}{\partial V_t} = \beta \mathrm{E}_t \lambda_{t+1} (Z_t - C_t) \begin{bmatrix} w_t \\ 1-w_t \end{bmatrix} + \theta \Sigma^{-1} V_t = 0.$$

Solving the equation yields nature's policy

$$V_t = -\frac{\beta}{\theta} (Z_t - C_t) \Sigma \begin{bmatrix} w_t \\ 1-w_t \end{bmatrix} \mathrm{E}_t \lambda_{t+1}. \tag{4}$$

Anticipating the unfriendly action of nature, the customer–investor revises her objective function by substituting (4) for V_t in L, with the quadratic penalty for nature omitted from her own utility function since this penalty is not her concern when she sets C_t and w_t, yielding

$$L^* = \sum_{t=0}^{\infty} \mathrm{E}_t \beta^t \Big\{ u(C_t) - \beta\lambda_{t+1} \left(Z_{t+1} - (Z_t - C_t) \left[w_t \quad 1-w_t \right] R_t \right) - \frac{\beta^2}{\theta} (Z_t - C_t)^2 (\mathrm{E}_t \lambda_{t+1})^2 [\sigma_{11} w_t^2 + \sigma_{22}(1-w_t)^2 + 2\sigma_{12} w_t (1-w_t)] \Big\}. \tag{5}$$

The extra term in the modified Lagrange function (5), to be denoted by

$$B(Z_t, C_t, w_t) = \frac{1}{\theta} [\beta (Z_t - C_t) \mathrm{E}_t \lambda_{t+1}]^2 [\sigma_{11} w_t^2 + \sigma_{22}(1-w_t)^2 + 2\sigma_{12} w_t (1-w_t)] \tag{6}$$

results from substituting V_t from (4) into the dynamic model in (3). Even after the penalty function for nature is omitted from the customer–investor's utility function in L^*, this term remains and can be interpreted as a burden to the customer–investor due to her attitude of pessimism, or as a cost of carrying out a robust policy. Her optimized utility in period t will be less by the amount of this burden. We note that this burden of pessimism $B(Z_t, C_t, w_t)$ is positive. Its partial derivative with respect to Z_t, to be denoted by B_Z, is positive. Its partial derivative B_C with respect to C_t is negative, and equals $-B_Z$. Its partial derivative B_w with respect to w_t at the optimum is positive as will be discussed below.

The customer–investor's optimization problem is solved by differentiating L^* partially with respect to C_t, w_t and Z_t to obtain the first-order conditions for these variables

$$\beta^{-t}\frac{\partial L^*}{\partial C_t} = u'(C_t) - \beta \mathrm{E}_t \lambda_{t+1}[w_t R_{1,t} + (1-w_t)R_{2,t}] - B_C(Z_t, C_t, w_t) = 0, \tag{7}$$

$$\beta^{-t}\frac{\partial L^*}{\partial w_t} = \beta(Z_t - C_t)\mathrm{E}_t \lambda_{t+1}(R_{1,t} - R_{2,t}) - B_w(Z_t, C_t, w_t) = 0, \tag{8}$$

$$\beta^{-t}\frac{\partial L^*}{\partial Z_t} = -\lambda_t + \beta \mathrm{E}_t \lambda_{t+1}[w_t R_{1,t} + (1-w_t)R_{2,t}] - B_Z(Z_t, C_t, w_t) = 0. \tag{9}$$

From (7) and (9) and using $B_C = -B_Z$, we derive the following well-known optimal consumption strategy:

$$u'(C_t) = \lambda_t,$$

implying $C_t = u'^{-1}(\lambda_t)$. By condition (7) the discounted marginal utility from consuming the returns to an optimally invested dollar has to exceed the marginal utility of consuming a dollar today by the marginal burden $B_Z = -B_C$. Compared with the standard solution without allowing for unfavorable returns, condition (7) implies that $u'(C_t)$ is smaller because B_C is negative. With diminishing marginal utility, current consumption is larger because the incentive to invest is reduced by the marginal burden of pessimism introduced into the model.

Given C_t, first-order conditions (8) and (9) determine w_t and λ_t. Condition (8) implies

$$\beta \mathrm{E}_t \lambda_{t+1}(R_{1,t} - R_{2,t}) = \frac{B_w(Z_t, C_t, w_t)}{(Z_t - C_t)} > 0. \tag{10}$$

By condition (10) the discounted marginal utility of consuming the earnings of one dollar invested in the more risky asset 1 has to exceed that of one dollar invested in asset 2 by the marginal burden of shifting one dollar to investing in asset 1 from asset 2 (the marginal burden of raising the fraction w of $(Z - C)$ dollars invested). That this marginal burden on the right-hand side of (10) is positive can be demonstrated by the following continuity argument. The partial derivative B_w is proportional to the derivative of the expression in square brackets in (6) with respect to w which equals

$$2[(\sigma_{11} + \sigma_{22} - 2\sigma_{12})w - \sigma_{22} + \sigma_{12}].$$

Consider the case when the second asset is almost risk-free, so that σ_{22} and σ_{12} are very small. Then the above expression is approximately $2\sigma_{11}w_t$ which is positive. In the standard solution without the burden of pessimism, the right-hand side of (10) is zero. One dollar invested in the risky asset 1 should provide a return that can be used to purchase consumption goods in the next period yielding the same expected utility as one dollar invested in asset 2. The allowance of unfavorable outcomes in risky investment requires a dollar invested in asset 1 to yield a higher discounted expected marginal unity than a dollar invested in asset 2. It leads to investing less in the risky asset as a larger expected return is required for investing in it.

From (9) and (10) we obtain

$$\begin{aligned}\lambda_t &= \beta \mathrm{E}_t \lambda_{t+1}[w_t R_{1,t} + (1-w_t)R_{2,t}] - B_Z \\ &= \beta \mathrm{E}_t \lambda_{t+1} R_{1,t} - [B_Z + (1-w)B_w/(Z_t - C_t)] \\ &= \beta \mathrm{E}_t \lambda_{t+1} R_{2,t} - [B_Z - w_t B_w/(Z_t - C_t)]. \end{aligned} \tag{11}$$

Without the burden of pessimism the second and third equality in Eq. (11) state that one dollar spent on consumption in period t will yield marginal utility λ_t which should be the same as the discounted expected utility of consuming in period $t+1$ the earning of one dollar invested in asset 1 or in asset 2. By the second equality of (11) the burden of pessimism requires the discounted expected utility from a dollar invested in the risky asset 1 to exceed the marginal utility of consuming one dollar today by the sum of two marginal burdens. The first B_Z is for investing the dollar optimally in the two assets. The second is for shifting one dollar from the $(1-w)/(Z-C)$ dollars invested in asset 2 to investing in the risky asset 1. By the third equality of (11), the requirement of spending one dollar in investing in asset 2 rather than current consumption is B_Z plus the reduction in marginal burden by shifting one of the $w/(Z-C)$ dollars investing in asset 1 to investing in asset 2. All first-order conditions from (7) to (11) are reduced to those of the traditional model when the burden of pessimism is zero or when the parameter θ tends to infinity.

3. Solution of the equity premium puzzle

In the standard case when the unfriendly nature is absent, the second terms on the right-hand side of the second and third equality of (11) are zero. Campbell et al. (1997, p. 130) presents the equity premium puzzle as follows. Let the utility function be $u(C_t) = (C_t^{1-\gamma} - 1)/(1-\gamma)$. The marginal unity is $u'(C_t) = C_t^{-\gamma}$, which is equal to λ_t. Divide both sides of (11) by λ_t and take logarithms to get, as an approximation,

$$\begin{aligned}\log\left[\beta \mathrm{E}_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{1,t}\right] &= \log\{1 + [B_Z + (1-w_t)B_w/(Z_t - C_t)]/\lambda_t\} \\ &= [B_Z + (1-w_t)B_w/(Z_t - C_t)]/\lambda_t. \end{aligned} \tag{12}$$

Denote the logarithms of $R_{1,t}$ and $R_{2,t}$ by the corresponding low-case letters $r_{1,t}$ and $r_{2,t}$. Similarly, denote the log of C_{t+1}/C_t by Δc_{t+1}. Applying the equation $\log \mathrm{E}_t X = \mathrm{E}_t \log X + (1/2)\,\mathrm{Var}_t(\log X)$ for a conditionally log-normal random variable X, we can rewrite Eq. (12) as

$$\begin{aligned}&\mathrm{E}_t(r_{1,t+1}) + \log\beta - \gamma \mathrm{E}_t(\Delta c_{t+1}) + \tfrac{1}{2}[\sigma_{11} + \gamma^2\sigma_c^2 - 2\gamma\sigma_{1c}] \\ &\quad = [B_Z + (1-w_t)B_w/(Z_t - C_t)]/\lambda_t. \end{aligned} \tag{13}$$

Performing the same operations on the third equality of Eq. (11), we obtain

$$\begin{aligned}&\mathrm{E}_t(r_{2,t+1}) + \log\beta - \gamma\mathrm{E}_t(\Delta c_{t+1}) + \tfrac{1}{2}[\sigma_{22} + \gamma^2\sigma_c^2 - 2\gamma\sigma_{2c}]\\&\quad = [B_Z - w_t B_w/(Z_t - C_t)]/\lambda_t.\end{aligned} \tag{14}$$

Subtracting (14) from (13) yields

$$\mathrm{E}_t(r_{1,t+1} - r_{2,t+1}) + \frac{\sigma_{11} - \sigma_{22}}{2} = \gamma(\sigma_{1c} - \sigma_{2c}) + B_w/[\lambda_t(Z_t - C_t)]. \tag{15}$$

Solving (15) for γ, we have

$$\gamma = \frac{1}{\sigma_{1c} - \sigma_{2c}}\left[\mathrm{E}_t(r_{1,t+1} - r_{2,t+1}) + \frac{\sigma_{11} - \sigma_{22}}{2} - B_w/[\lambda_t(Z_t - C_t)]\right]. \tag{16}$$

Inserting American data $\mathrm{E}_t(r_1 - r_2) = 0.0418$, $\sigma_{11} = 0.1674^2$, $\sigma_{22} = 0.0544^2$, $\sigma_{1c} = 0.0027$, $\sigma_{2c} = -0.0002$ in Table 8.1 of Campbell et al. (1997, p. 308) into the above equation with $B_w/[\lambda_t(Z_t - C_t)] = 0$, one finds a value of 18.7 for γ. This value is much larger than the maximum reasonable value of 10 and creates an equity premium puzzle.

The model proposed in this paper introduces the additional term $B_w/[\lambda(Z - C)]$ on the right-hand side of (16) under the robust control policy. This additional term is subtracted from the equity premium $\mathrm{E}_t(r_{1,t+1} - r_{2,t+1})$. Hence, if the equity premium is considered too high by a certain value, assigning the same value to this additional term will solve the puzzle. By (10) $B_w/(Z_t - C_t) = \beta(\mathrm{E}_t\lambda_{t+1}R_{1,t} - \mathrm{E}_t\lambda_{t+1}R_{2,t})$ is the discounted extra expected marginal utility of a dollar spent to invest in asset 1 as compared with asset 2 due to the attitude of pessimism. The additional term equals this discounted extra expected utility as a fraction of λ_t. By our calculations using Eq. (16), the sum of the first two terms in squared brackets equals 0.05433. If the fraction due to the burden of pessimism is 0.03, the term in squared brackets will be reduced to $0.05433 - 0.03 = 0.02433$. γ will be reduced from 18.7 to 0.02433/0.0029 or 8.39, which is an acceptable value. If the fraction is 0.0485, the term in square brackets will be $0.05433 - 0.0485 = 0.00583$ and γ will be reduced from 18.7 to 0.00583/0.0029 or 2.0, a value suggested by much of the empirical literature on real business cycles to be most reasonable. Without estimating the parameters of the model including θ it is not possible to find out what the fraction is. The above illustrative calculations only show that a modest modification to the incentive to invest in the risky asset as given by the first-order condition (10) which has a positive value equal to 3–4.8 percent of the marginal utility of consumption λ suffices to resolve the equity premium puzzle.

4. Solution of the consumption sensitivity puzzle

According to the permanent income hypothesis originated by Friedman (1957) and further developed by Hall (1978), changes in consumption are uncorrelated with permanent income. If current consumption is determined by permanent income, permanent income in period t cannot help predict the changes in consumption from period t to period $t+1$ because all of its effect on consumption is already contained in the consumption statistic of period t and, given that, the consumption of period $t+1$ must be determined by information not known at t. However, this implication of the permanent income hypothesis has been found to be empirically invalid by Flavin (1981) and others.

To study the implications of our model for the variables known at time t which will affect the change in consumption from t to $t+1$, we rewrite Eq. (13) as

$$\mathrm{E}_t(\Delta c_{t+1}) = \frac{1}{\gamma}\left\{\mathrm{E}_t(r_{1,t+1}) + \log\beta + \frac{1}{2}(\sigma_{11} + \gamma^2\sigma_c^2 - 2\gamma\sigma_{1c}) \right. \\ \left. - [B_Z + (1-w_t)B_w/(Z_t - C_t)]/\lambda_t\right\}. \tag{17}$$

The first implication is that the conditional expectation $\mathrm{E}_t r_{t+1}$ of r_{t+1} based on information at time t has a positive effect on the change of consumption from t to $t+1$. This important implication is missing in the studies of consumption by Hall (1978), Sargent (1978), Flavin (1981) and others because they conveniently assume that the rate of return to assets is a constant in a model of aggregate consumption. Under such an assumption it is obvious from Eq. (17) that without the additional term due to the burden of pessimism the change in consumption is not affected by any state variable including wealth in the model. In other words without introducing a robust policy one cannot apply the Samuelson (1969) model to solve the puzzle that changes in consumption are empirically affected by current state variables. However, to study aggregate consumption behavior it is fruitful to use the model of Samuelson (1969) or its continuous-time counterpart Merton (1969) as a starting point because the interaction of consumption and portfolio selection decisions is taken into account.

We now turn to the extra term in Eq. (17) due to the burden of pessimism. This term provides important information on how the state variables known at time t will affect Δc_{t+1} because it is a function of the state variables $Z_t, r_{1,t-1}$ and others that may be introduced. We will evaluate the derivatives of the expression in square brackets in (17) with respect to the important state variables using the definition of B given in (6). The quadratic function of w in (6) will be denoted by $K(w)$. By differentiating (6) the expression in square brackets in (17) equals

$$\frac{1}{\theta}\beta^2[2K(w) + (1-w)\,\mathrm{d}K/\mathrm{d}w](\mathrm{E}_t\lambda_{t+1})^2(Z_t - C_t).$$

We will not be concerned with the variation of w_t through time and treat it as a constant. The two most important state variables to study are Z_t and $\mathrm{E}_t Z_{t+1}$ (the latter a composite state variable) as they enter importantly in the last two terms in parentheses in the above expression. When the dynamic optimization problem associated with (5) is solved, one obtains three time invariant functions C, w and λ (see Chow, 1997, pp. 22–23). These are functions of the state variables Z_t and $r_{1,t-1}$ and $r_{2,t-1}$. (Recall that in Eq. (1) $R_{1,t}$ and $R_{2,t}$ are not known at the beginning of period t when Z_t is observed. Some authors prefer to date these returns at $t+1$.) Our discussion will concentrate on the state variable Z_t because the wealth variable has been singled out in the study of consumption sensitivity and the analysis of this variable carries over to wage income which will soon be introduced.

The sign of the derivative of the last displayed expression with respect to the state variable Z_t can be easily ascertained to be positive by observing that the derivative of C_t with respect to Z_t is smaller than one. The second important component of the last displayed expression is the square of $E_t\lambda_{t+1} = E_t\lambda(Z_{t+1},\ldots)$. Differentiating both sides of the first-order condition $\lambda = u'(C)$ with respect to Z and using the facts that marginal utility is a decreasing function of consumption and consumption has a positive derivative respect to Z, one deduces that λ is a decreasing function of Z. The conditional expectation $E_t\lambda_{t+1} = E_t\lambda(Z_{t+1},\ldots)$ can be differentiated with respect to E_tZ_{t+1} to yield a negative derivative by using a Taylor expansion of the function λ, taking the expectation of the expansion and using the fact that λ is a decreasing function of this argument. Therefore, the partial derivative of the expression in square brackets in (17) with respect to E_tZ_{t+1} is negative. Our model thus implies that in the regression (17) of Δc_{t+1} the coefficient of Z_t is negative, the coefficient of E_tZ_{t+1} is positive and the coefficient of $E_tr_{1,t}$ is also positive but may not equal $1/\gamma$ because $E_t\lambda(Z_{t+1}, r_{1,t})$ is a function of $E_tr_{1,t}$ also. To state these results in economic terms, note first that the marginal burden of pessimism as formulated in (17) consists of two important terms given at the end of the last displayed expression. The last term gives a negative effect of Z_t because a larger Z_t increases the marginal burden to invest in risky assets and will accordingly increase current consumption at the expense of future earnings from assets and thus future consumption. This makes the change in consumption smaller. On the other hand a larger expected Z_{t+1} through the second to the last term in the last displayed expression decreases the marginal burden to invest in risky assets and will thus decrease current consumption while creating more wealth for future consumption. This makes the change in consumption larger.

It is straightforward to extend our model to include wage income. Define income Y_t to include both wage income I_t and income from returns to assets. The extension is accomplished simply by changing the term $(Z_t - C_t)$ in all equations to $(Z_t + I_t - C_t)$, where I_t denotes a stochastic process whose realization in period t takes place before the consumption decision is made. In the extended model wage income I_t is a new state variable. However, in all expressions it is added to Z_t. We can go through the same arguments as before to evaluate the partial derivatives of the expression in square brackets in the extended Eq. (17) with respect to I_t and E_tI_{t+1}. The result is that in the regression of Δc_{t+1} the coefficients of these income variables have the same signs as the corresponding Z variables. Forming income Y as a linear combination of its two components will not affect the signs of the resulting coefficients.

To provide an empirical test of the above implications we have performed a regression of Δc_{t+1} on $E_tr_{1,t+1}$, y_t and E_ty_{t+1} based on Eq. (17) where c and r_1 are the same variables used in Campbell et al. (1997, p. 308) which were kindly supplied to us by John Campbell. We refer to these variables after Eq. (16) with c denoting the logarithm of real per capita consumption expenditures on nondurables and services. The variable y is the logarithm of real per capita disposable personal income from the DRI data bank. The two expectation variables are the forecasts each from an autoregression of four lags. Since, the data for y begin in 1929, our sample period is from

1933 to 1993 when the data used in the analysis of Eq. (16) end. The autoregression for forecasting r_1 was estimated using a longer sample period 1875–1993 as these data were used for the analysis of (16). The estimated regression is (standard errors in parentheses)

$$\Delta c_{t+1} = \underset{(0.01607)}{-0.00184} + \underset{(0.0471)}{0.1006}\mathrm{E}_t r_{1,t+1} - \underset{(0.1353)}{0.3641} y_t + \underset{(0.1382)}{0.3666}\mathrm{E}_t y_{t+1},$$

$$R^2 = 0.1656; \quad s = 0.0162$$

$$\mathrm{DW} = 1.509.$$

The low value of R^2 in this regression is unimportant for testing our theory. The important result is that all three coefficients are statistically significant and have the correct signs as expected from the theory. The above numerical estimates also agree with the findings of Campbell and Mankiw (1989) and Baxter and Jermann (1999) which contain regressions of Δc_{t+1} on $\mathrm{E}_t \Delta y_{t+1} = \mathrm{E}_t y_{t+1} - y_t$ and the rate of interest in period t. Campbell and Mankiw (1989) report regression coefficients of $\mathrm{E}_t \Delta y_{t+1}$ in the range 0.29–0.66. The coefficient is interpreted as the fraction of "rule-of-thumb" consumers who consume out of current income rather than permanent income. The regression reported in Baxter and Jermann (1999, p. 912) based on their benchmark difference-stationary model derived from a theory of household production (which corresponds to our model as we assume the first difference of log consumption to be stationary) has a coefficient 0.36 for $\mathrm{E}_t \Delta y_{t+1}$. The last result agrees exactly with the point estimate 0.36 in our regression for the coefficient of the same variable.

Flavin (1981, p. 1003) reports a coefficient of 0.112 for y_t in a regression of Δc_{t+1} on this and other lagged y's. This coefficient would result from our model if the coefficient of y_t in the autoregression to forecast y_{t+1} is 1.299 since $1.299 \times 0.3666 - 0.3641 = 0.112$. When we estimated an autoregression for forecasting y_{t+1} the coefficient of y_t turned out to be 1.296, extremely close to the value 1.299. The remarkable agreements between our regression results and the results of the other studies just cited should not be too surprising as we have used the same data. The point to stress is that a theory based simply on the allowance of unfavorable events can explain these coefficients at least as well as the competing and perhaps more complicated theories of aggregate consumption behavior.

5. Extension to a multi-asset model

The extension to a model with n assets is straightforward. Let the n assets be ordered according to their risk or the variances of their returns, the first having the largest variance. Let w be a column vector of $n-1$ fractions of total wealth and wage income net of consumption invested in the first $n-1$ assets, and $1 - \mathbf{1}'w$ be the remaining fraction invested in the least risky asset, where $\mathbf{1}$ denotes a column vector of ones. If $R_{1,t}$ denotes the vector of returns to the first $n-1$ assets and $R_{2,t}$ denotes the return to the last asset, first-order condition (7) remains the same except that the expression in squared brackets becomes $w'R_{1,t} + (1 - \mathbf{1}'w)R_{2,t}$. Eq. (8) also remains the same if the derivative on the left-hand side denotes a vector of derivatives with respect to the vector w_t and $(R_{1,t} - R_{2,t})$ on the right-hand side is replaced by $(R_{1,t} - \mathbf{1}R_{2,t})$ to denote a vector of differences in returns between the first $n-1$ assets and the least risky asset.

All analysis goes through accordingly. For example the left-hand side of Eq. (10) has $(R_{1,t} - \mathbf{1}R_{2,t})$ and B_w on the right-hand side is a vector of partial derivatives of the burden of pessimism with respect to the vector w. The vector Eq. (10) holds component by component, each being a comparison of one of the first $n-1$ assets with the least risky asset. Eq. (13) holds for any asset i if the subscript 1 is replaced by i and $(1-w)B_w$ is replaced by the fraction of wealth not invested in asset i times the marginal burden of increasing the investment in asset i by a fraction. The essence of the economics of optimal portfolio and consumption decisions when the consumer–investor allows for unfavorable outcomes has been discussed by considering the case of two assets.

6. Conclusions

Besides providing an exposition of a framework applicable to studying economic behavior that allows for the consideration of unfavorable events this paper has solved two interesting economic puzzles using such a framework. In terms of the equity premium puzzle, the explanation provided in this paper is very simple. It builds upon the basic model of Samuelson (1969) by introducing one additional element in the return to each asset to capture the possible occurrence of unfavorable outcomes. Using the first-order conditions derived by the Lagrange method, we can see clearly the effects of introducing these elements and how a reduction in the value of the risk aversion coefficient γ comes about. The marginal burden of pessimism requires that the more risky asset yield a higher expected return than otherwise as compared with the less risky asset. This theory does not require an appeal to habit persistence as suggested in Epstein and Zin (1991) and in Campbell and Cochrane (1999), an introduction of information asymmetry as in Zhou (1999), the use of unobserved state-variables, as in Hansen et al. (1999), the modelling of an overlapping generation economy where the consumer–investor is subject to a borrowing constraint as in Constantinides et al. (1998), or another more complicated modification of Samuelson's basic theory.

For the explanation of the consumption sensitivity puzzle the theory implies that the marginal burden of pessimism in the regression function of the change in consumption is a decreasing function of current income and an increasing function of expected income of the next period. The higher the current income the more concern will be regarding future returns to investment; this leads to an increase in current consumption relative to future consumption and generates a negative effect on the change in consumption. The higher the expected future income the smaller the marginal burden: this leads to a reduction in current consumption and an increase in investment to pay for future consumption. The theory does not introduce two types of consumers with one type consuming according to current income as in Campbell and Mankiw (1989, 1990, 1991), or a model of home production as in Baxter and Jermann (1999), or both habit persistence and household production as in Hansen et al. (1999). The consideration of unfavorable events itself, as manifested in the burden of pessimism or the cost of being robust, suffices for providing a set of important state variables and their properties in the explanation of the change in consumption which is considered unpredictable according to one form of the permanent income hypothesis. This theory is supported by statistical evidence both reported in this paper and in other papers on aggregate consumption. The theory of consumption and portfolio selection presented here appears to have resolved both puzzles very simply.

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