ELSEVIER Journal of Monetary Economics 41 (1998) 301–318

JOURNAL OF Monetary ECONOMICS

How the basic RBC model fails to explain US time series

Gregory C. Chow[a,*], Yum K. Kwan[b]

[a] Department of Economics, Princeton University, Princeton, NJ 08544, USA
[b] Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong

Abstract

By examining the reduced form equations implied by an RBC model this paper shows how it fails in explaining the dynamic characteristics of the US time series using time-domain analysis. In particular, by studying the serial correlation of the residuals of the reduced form and by introducing lagged dependent variables, important propagation mechanisms left out in the model can be clearly discerned and reformations to improve the model can be evaluated.

JEL classification: C3; C5; E2; E3

Keywords: Real business cycle model; Dynamic optimization

1. Introduction

In the study of macroeconomic fluctuations and growth using RBC models a standard practice is to compare the covariance properties of such models with those of historical time series. This paper suggests that the reduced form equations implied by the models should be carefully examined, in addition to the covariance properties of the structural form. A demonstration of this methodology will be given for a basic RBC model. That the simple RBC setup fails in explaining important dynamic elements in US time series is not controversial. The crucial question is how. By examining the serial correlation of the residuals of the reduced form and by introducing lagged dependent variables, important

*Corresponding author. Tel: (609)-258-4030; Fax: (609)-258-5561; e-mail:gchow@ariel.princeton.edu.

0304-3932/98/$19.00
PII S0304-3932(97)00074-3

propagation mechanisms left out by the model can be clearly discerned and reformations to improve the model can be evaluated. Our method is complementary to the frequency-domain approach of Watson (1993) and Chow and Levitan (1969). In particular, we demonstrate that lagged dependent variables dramatically affect the fit of the reduced form equations, suggesting that a simple and yet promising direction to enrich the basic RBC model is to build models with propagation mechanisms that feature lagged control variables as part of the state vector.

2. Formulation of the baseline real business cycle model

The problem facing the representative economic agent for an RBC model is

$$\max_{\{u_t\}_{t=0}^{\infty}} \left[\sum_{t=0}^{\infty} \mathrm{E}_t \beta^t r(s_t, u_t) \right] \tag{1}$$

subject to

$$s_{t+1} = f(s_t, u_t) + \varepsilon_{t+1} \tag{2}$$

where s_t is a vector of p state variables, u_t is a vector of q control variables, β is the discount factor, ε_{t+1} is a vector of random shocks to the economy, E_t is expectation conditioned on information at time t, and for convenience r and f are assumed to be differentiable and concave. The solution takes the form of a vector of optimal control equations $u_t = g_t(s_t)$. For the purpose of statistical analysis, a common practice is to approximate $g_t(s_t)$ by a time-invariant linear control equation system

$$u_t = Gs_t + g. \tag{3}$$

For the derivation of such a linear control equation system, see Chow (1975), (Chapter 12; 1983), Kydland and Prescott (1982), King et al. (1988a), and Chow (1993, 1997). Eq. (3) can explain only a part of the real world. To fit the observed data for u_{t+1}, a vector e_{t+1} of errors has to be added to Eq. (3), yielding for period $t+1$

$$u_{t+1} = Gs_{t+1} + g + e_{t+1}. \tag{4}$$

One is not justified to assume the statistical independence of e_{t+1} and s_{t+1}, but there is no need to. A reasonable assumption is that ε_{t+1} is uncorrelated with s_t and u_t in Eq. (2). If e_{t+1} is correlated with ε_{t+1} it is also correlated with s_{t+1} through Eq. (2), violating the independence assumption. A main point of this section is to show that there is no need to assume e_{t+1} to be independent of ε_{t+1} and s_{t+1} for the estimation and testing of the model consisting of Eqs. (2)

and (4) by standard statistical methods. This point is illustrated using the baseline RBC model presented by King et al. (1988a,b).

The model consists of two control variables u_{1t} and u_{2t} representing investment and labor input, respectively, and two state variables s_{1t} and s_{2t} denoting respectively $\ln z_t$ and capital stock at the beginning of period t, where z_t represents labor-augmenting productivity shocks in the Cobb–Douglas production function $q_t = s_{2t}^{1-\alpha}(z_t u_{2t})^{\alpha}$. The dynamic process is

$$\begin{aligned} s_{1t} &= \gamma + s_{1,t-1} + \varepsilon_t, \\ s_{2t} &= (1-\delta)s_{2,t-1} + u_{1,t-1}. \end{aligned} \tag{5}$$

The first equation assumes $s_{1t} = \ln z_t$ to be a random walk with a drift γ, ε_t being a random shock to technology. The second equation gives the evolution of capital stock s_{2t}, with δ denoting the rate of depreciation. The utility function r is assumed to be

$$r = \ln c_t + \theta \ln(1 - u_{2t}) \tag{6}$$

where $c_t = s_{2t}^{1-\alpha}\exp(\alpha s_{1t})u_{2t}^{\alpha} - u_{1t}$, the difference between output and investment; and $1 - u_{2t}$ denotes leisure.

Using the method of Chow (1993), one can solve the optimal control problem for the representative economic agent to yield optimal linear control equations of the form

$$\begin{aligned} u_{1t} &= g_{11}s_{1t} + g_{12}s_{2t} + g_1 + e_{1t}, \\ u_{2t} &= g_{21}s_{1t} + g_{22}s_{2t} + g_2 + e_{2t}, \end{aligned} \tag{7}$$

where e_{1t} and e_{2t} have been added to account for the deviations of the observed investment u_{1t} and labor input u_{2t} from their partially explained values derived from an incomplete theory. Given time-series data on u_{1t}, u_{2t}, s_{1t} and s_{2t}, the econometrician wishes to estimate and test this RBC model.

To do so we note that the state of technology, $s_{1t} = \ln z_t$, can be constructed by solving the production function, given output, capital, labor and the parameter α. Treating s_{1t} as observed, we have a model consisting of four simultaneous equations (Eqs. (5) and (7)) for four endogenous variables s_{1t}, s_{2t}, u_{1t} and u_{2t}. Note that the second equation for capital stock s_{2t} is nonstochastic and can be excluded from the system of simultaneous equations for the purpose of statistical analysis. It is a part of the deeper structure as the representative consumer-worker has utilized it, together with the utility function and the first of Eq. (5), in deriving the optimal control equations Eq. (7), thus imposing cross-equations restrictions on the parameters of the three remaining simultaneous stochastic equations. Since s_{2t} is the capital stock at the beginning of the period and is a function of only lagged variables, it is a predetermined variable in the system.

The three remaining simultaneous equations can be written in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 \\ -g_{11} & 1 & 0 \\ -g_{21} & 0 & 1 \end{bmatrix} \begin{bmatrix} s_{1t} \\ u_{1t} \\ u_2 t \end{bmatrix} = \begin{bmatrix} \gamma \\ g_1 \\ g_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_{1,t-1} \\ u_{1,t-1} \\ u_{2,t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ g_{12} \\ g_{22} \end{bmatrix} s_{2t} + \begin{bmatrix} \varepsilon_t \\ e_{1t} \\ e_{2t} \end{bmatrix}. \quad (8)$$

Allowing the covariance matrix of the three residuals, ε_t, e_{1t} and e_{2t}, to be of full rank, all parameters of the system Eq. (8) can be identified. The parameters include 7 coefficients and 6 elements of the residual covariance matrix. The 7 coefficients of Eq. (8) are derived through dynamic optimization from 5 deep parameters of the optimization process, including the discount factor β, θ of the utility function (6), α of the production function, and γ and δ of Eq. (5). The 6 elements of the residual covariance matrix are common to the derived model (8) and the optimization model appended by two error terms e_{1t} and e_{2t}. Through optimization the 5 deep structural parameters imply the 6 parameters in (g G) plus γ in Eq. (8). The latter seven parameters from Eq. (8) can be consistently estimated. They can be used to identify the five structural parameters. Given an optimal control algorithm to derive the seven parameters of Eq. (8) from the five deep parameters, one can estimate the latter by maximizing a likelihood function assuming normal residuals (as in (Chow (1983), Chapter 12)). The six variances and covariances can also be estimated as we do not assume the independence of e_{1t} and e_{2t} from ε_t. Using this example, we have shown that standard statistical inference in estimation and testing can be applied to RBC models, and there is no need to assume the errors of the control equations to be uncorrelated with the state variables in these equations.

3. Statistical estimation of the baseline RBC model

The discussion of the last section shows that the baseline RBC model has a reduced form which is a linear simultaneous equations model as given by Eq. (8). However Eq. (8) is not the form to be estimated because it contains a unit root in the first equation and the investment series is also nonstationary. To obtain a stationary model for estimation purpose we detrend the variables which incorporate a stochastic trend due to the random walk specification in log productivity. We then take logarithms of the detrended variables and linearize the model in the log detrended variables. Details for detrending and loglinearization can be found in Kwan and Chow (1996, 1997) and (Chow (1997), Chapter

9). It suffices here to point out that the optimal control equations for investment and labor supply are derived by solving three first-order conditions for optimum involving these two control variables and a Lagrange multiplier for the dynamic evolution of the capital stock. We detrend the original control variables and the capital stock variable and take logarithms of the detrended variables. The first-order conditions are then written as linear equations in the logarithmic detrended variables. Linearization is achieved by taking first-order Taylor expansion of the first-order conditions about the steady-state values of the detrended state and control variables for the deterministic version of the model under optimal control. Recalling the original state variables $s_1 = \ln z$ and $s_2 =$ capital stock k, we redefine the new state variables as $s_1 = \ln(z/z_{-1}) \equiv \ln\bar{z}$ and $s_2 = \ln(k/z_{-1}) \equiv \ln\bar{k}$. The control variables are $u_1 = \ln(i/z) \equiv \ln\bar{i}$, where investment i is by definition output minus consumption, and $u_2 \equiv \ln n$, where n is labor supply which need not be detrended. The optimal control equations for u_1 and u_2 are linear in the two state variables with coefficient matrix $(g\ \ G)$.

We use CITIBASE quarterly data for the United States from 1947.1 to 1993.4 in our estimation; all monetary figures are in constant 1987 dollars. Consumption is defined as expenditures on nondurables plus services (CITIBASE variables GCNQ + GCSQ). Investment is nonresidential private investment plus expenditures on consumer durables (CITIBASE variables GINQ + GCDQ). Output is the sum of consumption and investment by definition. Capital stock is the sum of nonresidential private capital and durable goods owned by consumers as reported in the *Survey of Current Business* (August 1994, p.62), loglinearly interpolated to quarterly frequency from the annual series. Labor input is defined as the ratio of total man-hours employed per week (CITIBASE variable LHOURS) to a weekly endowment of 112 h, converted to quarterly frequency by three-month averaging the monthly series. These figures are divided by population which is the number of persons over 16 yrs old (also loglinearly interpolated from annual to quarterly frequency). We report the results from estimating the three simultaneous equations in Eq. (8) by the method of maximum likelihood assuming trivariate normal residuals.

The structural parameters to be estimated are α, β, γ, δ and θ, representing respectively the labor exponent of the Cobb–Douglas production function, the discount factor of the utility function, the drift in the random walk model for log productivity, the rate of depreciation in the capital accumulation equation, and the weight of leisure relative to consumption in the utility function. Given the structural parameters, dynamic optimization is used to derive the reduced form parameters $(g\ \ G)$ in the two behavioral equations for investment (log detrended) and labor hours. These two equations are combined with the random walk equation for $\ln z$ to form a system of three simultaneous equations which is the log detrended version of Eq. (8). The log likelihood function for such a simultaneous equations system is maximized with respect to the structural parameters.

Using standard notation, we rewrite the log detrended version of Eq. (8) as

$$By_t + \Gamma x_t = e_t \tag{9}$$

where $y_t = (s_{1t}, u_{1t}, u_{2t})$ and x_t is a column vector consisting of s_{2t} and 1, and

$$B = \begin{bmatrix} 1 & 0 & 0 \\ -g_{11} & 1 & 0 \\ -g_{21} & 0 & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 & -\gamma \\ -g_{12} & -g_1 \\ -g_{22} & -g_2 \end{bmatrix}, \quad e_t = \begin{bmatrix} \varepsilon_t \\ e_{1t} \\ e_{2t} \end{bmatrix} \tag{10}$$

Eq. (9) with n observations can be written as

$$BY' + \Gamma X' = E'$$

where Y' is $3 \times n$, X' is $2 \times n$ and E' is $3 \times n$.

Notice that y_t involves $\ln z_t$ which is constructed from the data ($\ln q_t$, $\ln k_t$, $\ln n_t$) via the production function, conditional on parameter α. Since α is an unknown parameter to be estimated, we have to allow for the Jacobian in writing down the likelihood function. It is not hard to check that the Jacobian of the transformation from y_t to ($\ln q_t$, $\ln k_t$, $\ln n_t$) is $1/\alpha$. Assuming normal and serially uncorrelated e_t with covariance matrix Σ, the concentrated log-likelihood function (see Chow, 1983, pp. 170–171) is

$$\log L = \text{const} + n \log |B| - \frac{n}{2} \log |\hat{\Sigma}| - n \ln \alpha \tag{11}$$

where

$$\hat{\Sigma} = n^{-1}(BY' + \Gamma X')(YB' + X\Gamma'). \tag{12}$$

Maximum likelihood estimation of the five deep parameters requires iterations, and each iteration takes three steps: (a) given initial values of the deep parameters, compute G and g by a dynamic optimization algorithm (Chow, 1997, Chapter 9); (b) evaluate log L using Eq. (11) and the results of step (a); and (c) apply a maximization algorithm to revise the initial values of the deep parameters. Since the determinant of the B matrix is one, maximum likelihood estimation of the reduced form can be efficiently performed by Zellner's iterated seemingly unrelated regression (ISUR) together with a line-search algorithm, rather than searching all parameters simultaneously using a standard hill-climbing algorithm. Conditional on α, the log likelihood in Eq. (11) can be obtained by ISUR. Then the optimal α is found by a line-search algorithm in the interval between zero and one.

We will first present the estimates of the five structural parameters of the dynamic optimization model. In the next section we report (1) the restricted reduced form where the coefficients (g G) are derived from the structural parameters of the dynamic optimization model; (2) the unrestricted reduced form where the coefficients in (g G) are estimated freely without derivation from dynamic optimization; (3) unrestricted reduced form with one lagged dependent variable in each equation; and (4) unrestricted reduced form with other lagged variables. As it will become clear later, lagged dependent variables are pivotal in accounting for the missing dynamics in the baseline RBC model.

Maximum likelihood estimates of the five structural parameters of the model are given below with standard errors in parentheses:

$$\begin{array}{ccccc} \alpha = 0.5435 & \beta = 0.9770 & \gamma = 0.0033 & \delta = 0.0213 & \theta = 2.7956 \\ (0.0079) & (0.0004) & (0.0006) & (0.0003) & (0.0420) \end{array}$$

The covariance matrix of the estimated parameters is computed by inverting the Hessian of the log-likelihood function. The estimated values of the five parameters are all reasonable and in fact close to values normally chosen in calibration.

4. Studying dynamic properties of the model

Table 1 presents parameter estimates for several versions of Eq. (8). The first two columns are restricted reduced form, referring to the fact that the parameters (g G) in Eq. (8) are derived from the structural optimization model with the five structural parameters either (a) specified a priori as in model calibration, or (b) estimated by the method of maximum likelihood reported above. The five structural parameters imply through optimization (using $\bar{i}$ and n as control and $\bar{z}$ and $\bar{k}$ as state variables, and log-linearizing the decision functions) the reduced form parameters given in columns (a) and (b) in Table 1. The calibrated parameters are chosen to be $(\alpha, \beta, \gamma, \delta, \theta) = (0.58, 0.988, 0.004, 0.025, 3.2431)$, a set of values standard in the calibration literature as in Plosser (1989) and Watson (1993), among numerous others. Columns (c) – (e) are unrestricted reduced form, referring to the estimation of Eq. (8) as a system of three simultaneous equations without regard to the derivation from the structural optimization model.

Comparing the two restricted reduced forms, calibration gives a log likelihood of 1765.35 while estimation gives 2007.26. The likelihood ratio statistic registers a value of -2 times $(1765.35 - 2007.26)$, implying a critical value far exceeding any reasonable level of significance for a chi-squared distribution with 5 degrees of freedom. The calibrated parameters are therefore seriously discordant with the data, even assuming the structural model is correct. To assess the structural model, one may compare columns (b) and (c). It can be seen that the log likelihood of the structural model is 2007.26 while that of the unrestricted model is 2078.64. Twice the log likelihood ratio has a chi-squared distribution with 2 degrees of freedom under the null hypothesis that the specification of the optimization model is correct. Recall that the five structural parameters generate seven reduced form parameters in Eq. (8). Hence there are two restrictions

Table 1
Restricted versus unrestricted reduced form

	Restricted reduced form		Unrestricted reduced form		
	(a)	(b)	(c)	(d)	(e)
Eq. (1): $\ln \bar{z}_t$					
constant	0.004	0.0033	0.0037	0.0045	0.0039
		(0.0006)	(0.0009)	(0.0006)	(0.0008)
DW	2.082	2.04	2.15	2.288	2.211
Eq. (2): $\ln \bar{i}_t$					
constant	− 0.3684	1.1854	− 2.0131	− 0.0178	− 0.0210
			(0.0331)	(0.0412)	(0.0635)
$\ln \bar{z}_t$	0.6245	0.9274	− 1.0896	0.2536	0.4992
			(0.3813)	(0.2047)	(0.1394)
$\ln \bar{k}_t$	− 0.6245	− 0.9274	0.8646	0.0106	5.5688
			(0.0350)	(0.0244)	(1.5933)
$\ln \bar{i}_{t-1}$				0.9900	1.1061
				(0.0167)	(0.0564)
$\ln \bar{i}_{t-2}$					− 0.1255
					(0.0603)
$\ln \bar{z}_{t-1}$					5.7122
					(1.5791)
$\ln \bar{z}_{t-2}$					− 4.6467
					(1.5788)
$\ln \bar{k}_{t-1}$					− 10.5911
					(2.9691)
$\ln \bar{k}_{t-2}$					5.0627
					(1.5415)
DW	0.061	0.062	0.206	1.438	2.007
Eq. (3): $\ln n_t$					
constant	− 1.8923	− 0.8468	− 1.4842	− 0.0233	− 0.0235
			(0.0162)	(0.0163)	(0.0222)
$\ln \bar{z}_t$	0.2928	0.2962	− 0.4276	− 0.4927	− 0.2266
			(0.1869)	(0.0631)	(0.0530)
$\ln \bar{k}_t$	− 0.2928	− 0.2962	0.1203	− 0.0028	1.8632
			(0.0172)	(0.0065)	(0.5971)
$\ln n_{t-1}$				0.9859	1.0760
				(0.0108)	(0.0572)
$\ln n_{t-2}$					− 0.0953
					(0.0561)
$\ln \bar{z}_{t-1}$					2.0053
					(0.5874)
$\ln \bar{z}_{t-2}$					− 1.7581
					(0.6040)

Table 1 (continued)

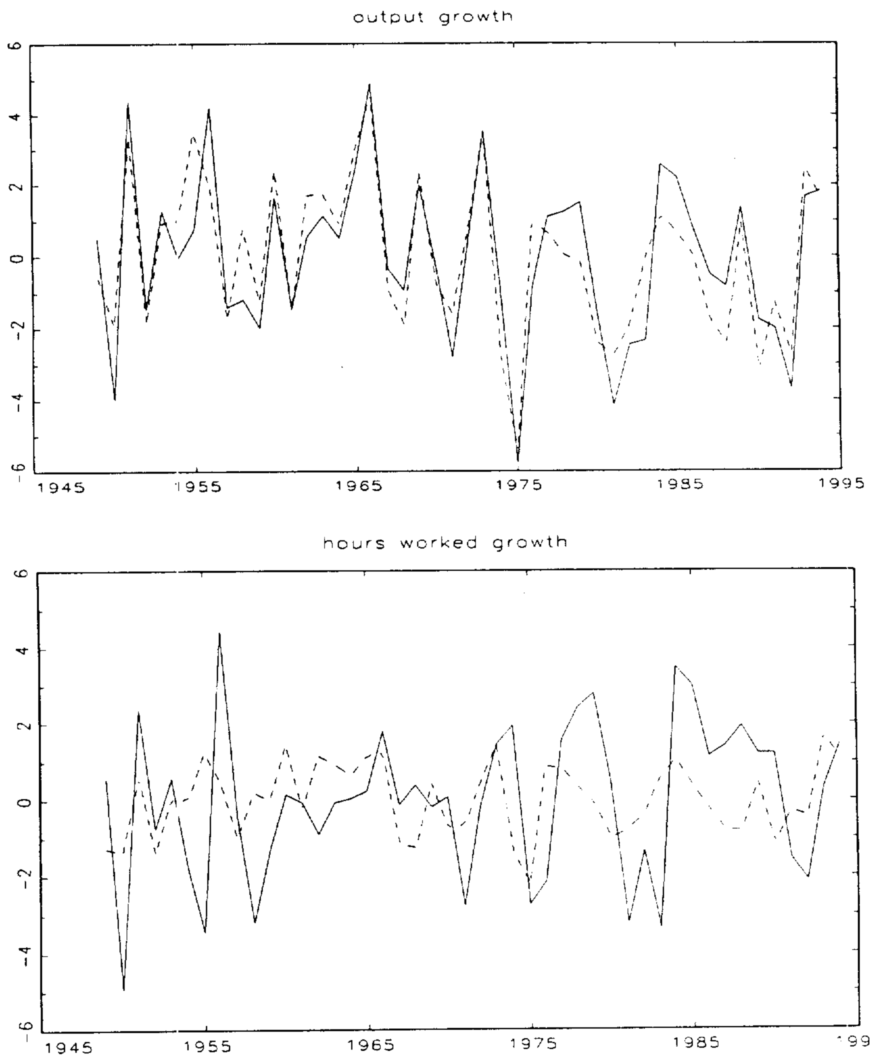
	Restricted reduced form		Unrestricted reduced form		
	(a)	(b)	(c)	(d)	(e)
$\ln \bar{k}_{t-1}$					− 3.7773
					(1.1504)
$\ln \bar{k}_{t-2}$					1.9211
					(0.5912)
DW	0.065	0.065	0.097	1.281	1.832
α	0.58	0.5435	0.6520	0.9698	0.7285
log likelihood	1765.35	2007.26	2078.64	2581.57	2602.58

Note: Standard errors in parentheses.

imposed on the seven parameters by the structural model. The chi-squared statistic is − 2 times (2007.26 − 2078.64), so much larger than the critical value under any reasonable level of significance. Again the model is strongly rejected.

Continuing to apply standard econometric methodology, we observe very small Durbin–Watson statistics of 0.062 for the investment equation and of 0.065 for the labor supply equation. The DW statistic is here used as a descriptive device to show the high first-order serial correlations of the residuals. Such strong serial correlations demonstrate clearly the two included explanatory variables have failed to explain the dynamics of the dependent variables in these two equations. This is something missed in the practice of calibration. To show how much dynamics is missing in the model, we present the unrestricted reduced form in column (c) of Table 1. The DW statistics for the two equations increase to 0.206 and 0.097, respectively, suggesting that one can explain the dynamics of the data better by abandoning the restrictions imposed by the structural model. We do not test the statistical significance of the improvement. Whatever the significance, the next column of Table 1 tells a more dramatic and revealing story. By adding lagged dependent variables in each of the investment and labor supply equations, we find that in the investment equation both current state variables become very insignificant, and in the labor supply equation only the log productivity variable remains significant. This model with lagged dependent variables shows very clearly how much dynamics is missed by the baseline RBC model. The last column of Table 1 reports unrestricted reduced form with other lagged variables. The DW statistic in each equation suggests that most of the positive serial correlations have been eliminated. The estimate 0.7285 for the labor exponent in the Cobb–Douglas production function is now more reasonable than the estimate 0.9698 given in the previous column.

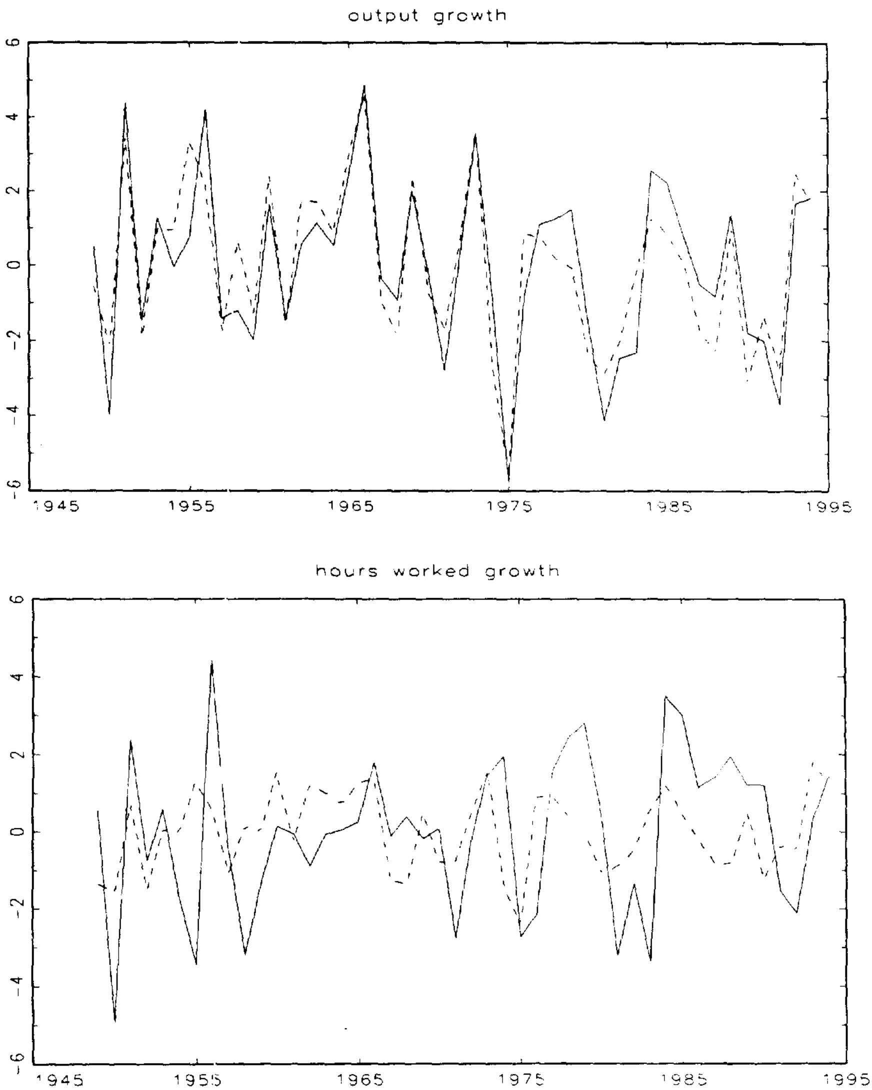
Comparing the actual growth rates of output and hours worked with the predicted values as in Plosser (1989) and shown here in Fig. 1a, Fig. 1b, Fig. 2a



Solid = actual, dashed = predicted.

Fig. 1. Calibrated structural model.

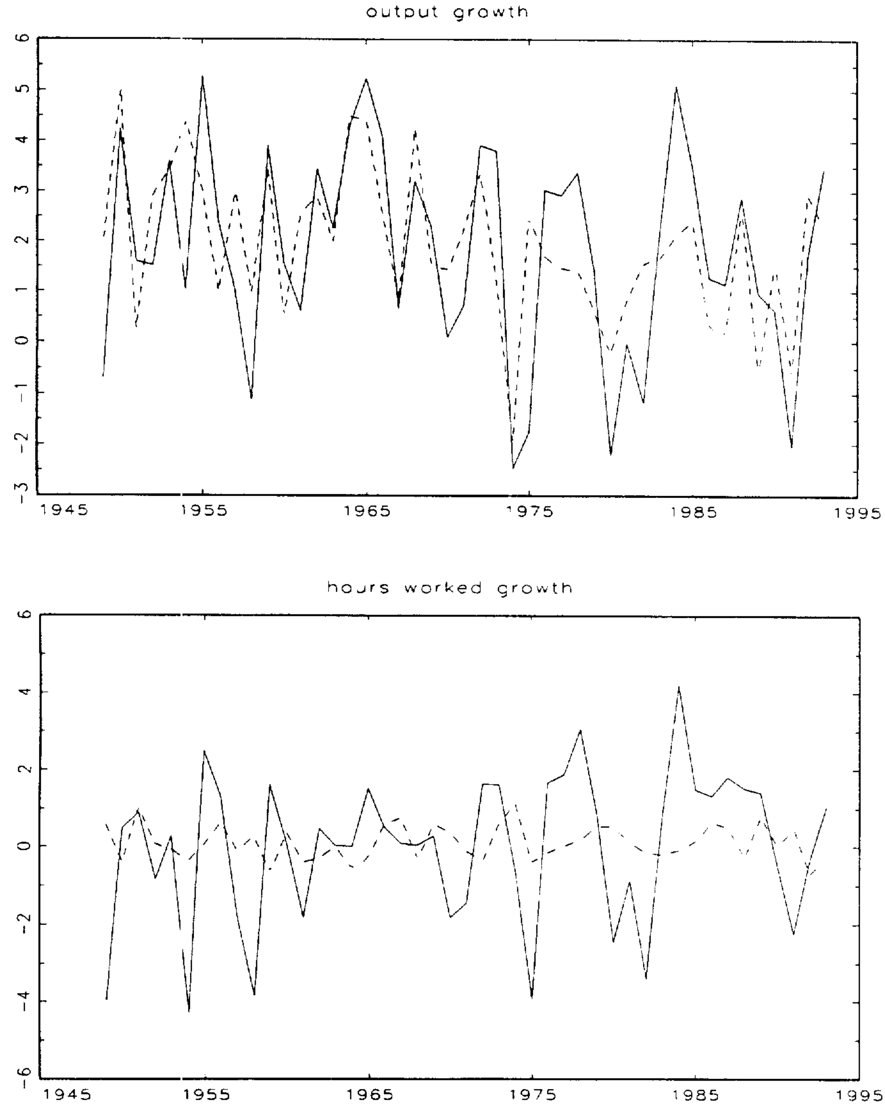
and Fig. 2b, we find that the calibrated model does quite well for output growth but misses some fluctuations in the growth rate of hours worked, as is well known in the literature. The estimated version does about as well for output growth but misses more fluctuations in the growth rate of hours worked. This



Solid = actual, dashed = predicted.

Fig. 2. Estimated structural model.

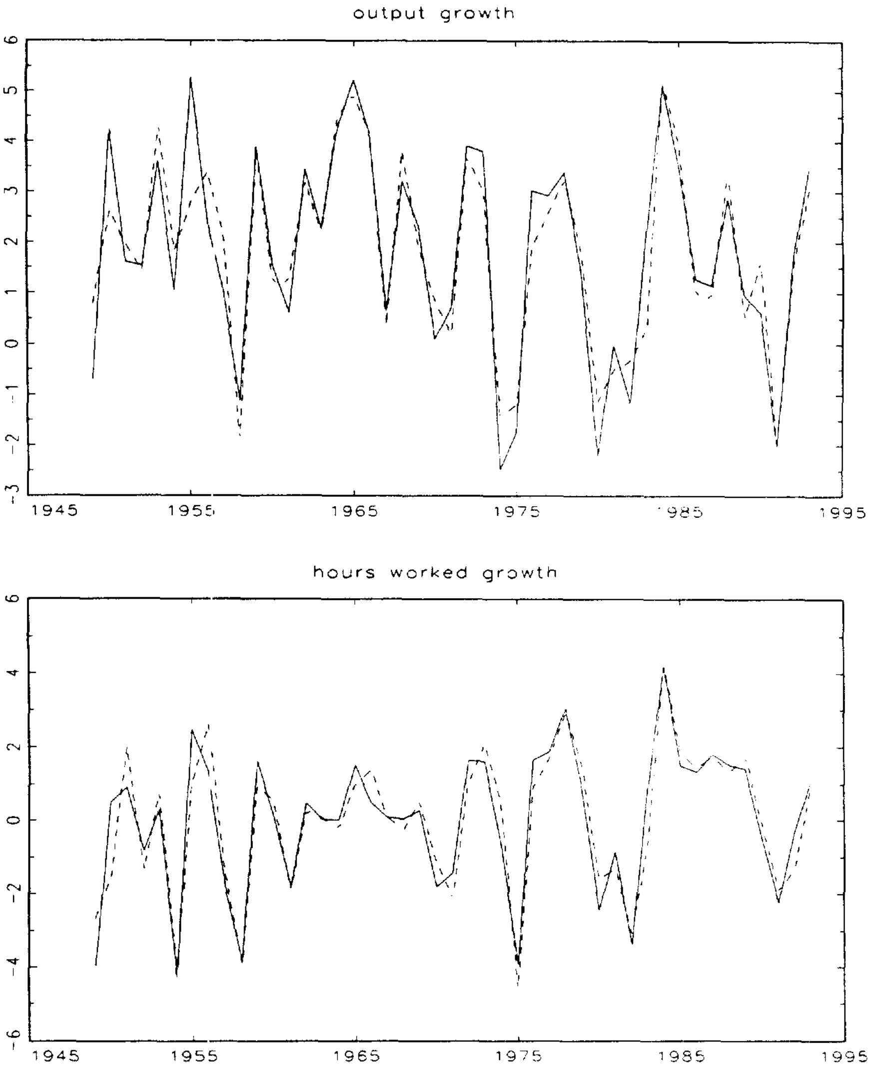
shows that by concentrating on fitting particular characteristics of the data, a calibrated model could do better in fitting those characteristics than estimation. However, for fitting all important characteristics, one would do better by relying on standard econometric methodology that can reveal the weaknesses of



Solid = actual, dashed = predicted.

Fig. 3. Unrestricted reduced form.

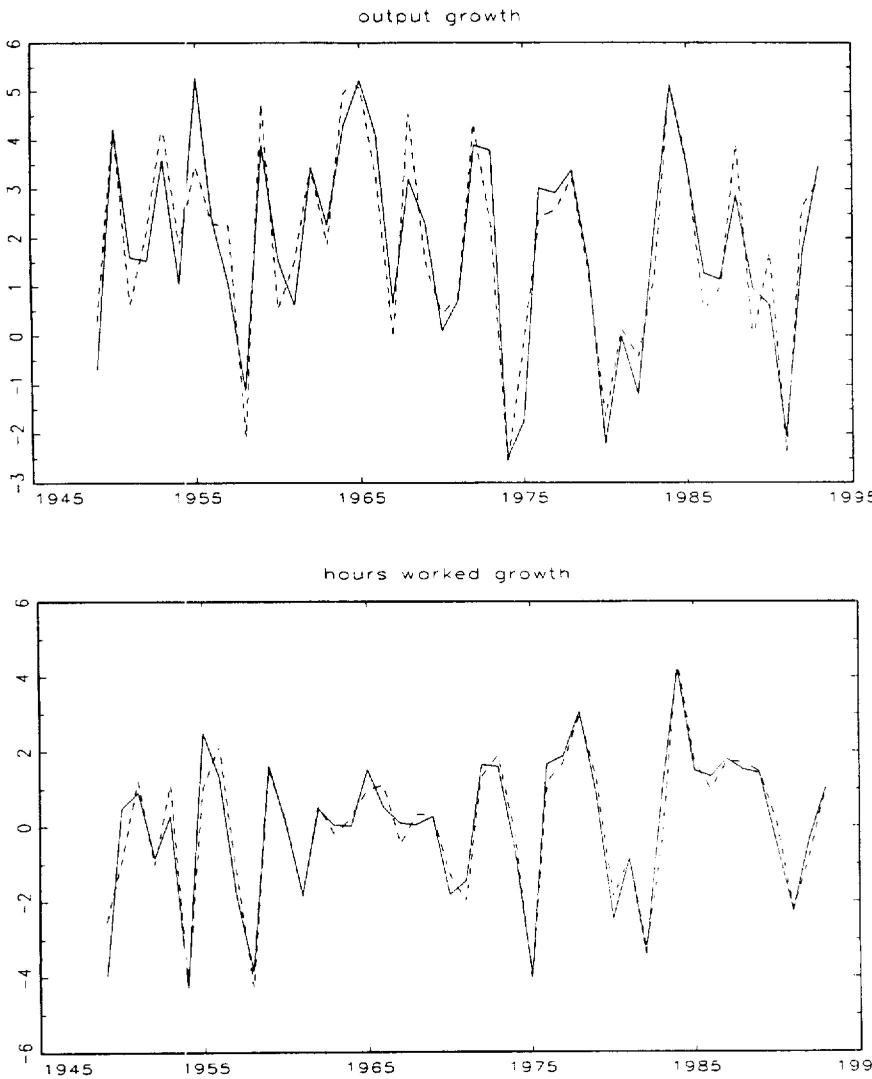
a calibrated model and indicate what the better fitting models are. Figs. 3–5 demonstrate this point. Fig. 3b shows that the unrestricted reduced form improves upon the restricted one by providing more fluctuations in the growth rate of hours worked, although it still does not fit as well as the calibrated model.



Solid = actual, dashed = predicted.

Fig. 4. Unrestricted reduced form with lagged dependent variables.

Introducing lagged dependent variables into the unrestricted reduced form, one improves dramatically the fit for the growth rates of both output and hours worked, as shown in Fig. 4a and Fig. 4b. Furthermore, introducing other lagged variables improves the fit significantly as has been demonstrated in column (e) in



Solid = actual, dashed = predicted.

Fig. 5. Unrestricted reduced form with other lagged variables.

Table 1, but the improvement can no longer be discerned by comparing the Plosser charts in Fig. 5a and Fig. 5b. We omit the charts for investment and consumption growth to save space, because the important point has already been made that by introducing lagged dependent variables in the reduced form, one can find out how the baseline RBC model fails in explaining the dynamic characteristics of time series data.

In Tables 2–6 we compare model predictions with actual data in terms of first and second moments for each of the five cases in Table 1, as is done in a calibration exercise. The moments are computed for detrended output, consumption, investment and hours worked (all in logarithm). Notice that the actual series are different for each cases because the detrending involves the log productivity series ln z which in turn depends on α.

Table 2
Calibrated structural model

Variables in log	Mean	Standard Deviation	Autocorrelation $\rho(1)$	$\rho(2)$	$\rho(3)$	Correlation with output	Correlation with actual
Panel A: actual							
Output	− 1.2900	0.0752	0.9775	0.9577	0.9370	1.0000	1.0000
Consumption	− 1.5557	0.0578	0.9689	0.9440	0.9171	0.9672	1.0000
Investment	− 2.7492	0.1463	0.9728	0.9428	0.9128	0.9403	1.0000
Hours	− 1.5976	0.0357	0.9559	0.9027	0.8419	0.5988	1.0000
Panel B: predicted							
Output	− 1.3242	0.0387	0.9760	0.9521	0.9276	1.0000	0.9669
Consumption	− 1.4423	0.0955	0.9760	0.9521	0.9276	1.0000	0.9418
Investment	− 3.1572	0.0966	0.9760	0.9521	0.9276	− 1.0000	− 0.9016
Hours	− 1.6567	0.0453	0.9760	0.9521	0.9276	− 1.0000	− 0.3747

Table 3
Estimated structural model

Variables in log	Mean	Standard Deviation	Autocorrelation $\rho(1)$	$\rho(2)$	$\rho(3)$	Correlation with output	Correlation with actual
Panel A: actual							
Output	− 1.2408	0.0839	0.9776	0.9575	0.9366	1.0000	1.0000
Consumption	− 1.5065	0.0665	0.9699	0.9444	0.9168	0.9748	1.0000
Investment	− 2.7000	0.1544	0.9741	0.9459	0.9179	0.9456	1.0000
Hours	− 1.5976	0.0357	0.9559	0.9027	0.8419	0.5543	1.0000
Panel B: predicted							
Output	− 1.2776	0.0485	0.9758	0.9515	0.9266	1.0000	0.9765
Consumption	− 1.3849	0.1095	0.9758	0.9515	0.9266	1.0000	0.9601
Investment	− 3.2585	0.1523	0.9758	0.9515	0.9266	− 1.0000	− 0.9126
Hours	− 1.6653	0.0486	0.9758	0.9515	0.9266	− 1.0000	− 0.3621

Table 4
Unrestricted reduced form

Variables in log	Mean	Standard Deviation	Autocorrelation $\rho(1)$	$\rho(2)$	$\rho(3)$	Correlation with output	Correlation with actual
Panel A: actual							
Output	− 1.3694	0.0614	0.9792	0.9619	0.9425	1.0000	1.0000
Consumption	− 1.6358	0.0448	0.9682	0.9468	0.9216	0.9440	1.0000
Investment	− 2.8268	0.1331	0.9722	0.9408	0.9084	0.9263	1.0000
Hours	− 1.5986	0.0355	0.9523	0.8947	0.8297	0.7402	1.0000
Panel B: predicted							
Output	− 1.3657	0.0566	0.9652	0.9446	0.9211	1.0000	0.9229
Consumption	− 1.6304	0.0392	0.8860	0.8733	0.8436	0.9843	0.8102
Investment	− 2.8269	0.1171	0.9771	0.9595	0.9393	0.9811	0.8800
Hours	− 1.5986	0.0173	0.9099	0.9093	0.8859	0.9257	0.4869

Table 5
Unrestricted reduced form with lagged dependent variables

Variables in log	Mean	Standard Deviation	Autocorrelation $\rho(1)$	$\rho(2)$	$\rho(3)$	Correlation with output	Correlation with actual
Panel A: actual							
Output	− 1.5852	0.0362	0.9552	0.9022	0.8424	1.0000	1.0000
Consumption	− 1.8515	0.0289	0.9462	0.8983	0.8531	0.7453	1.0000
Investment	− 3.0426	0.1051	0.9570	0.9037	0.8458	0.7914	1.0000
Hours	− 1.5986	0.0355	0.9523	0.8947	0.8297	0.9978	1.0000
Panel B: predicted							
Output	− 1.5807	0.0352	0.9238	0.8725	0.8151	1.0000	0.9441
Consumption	− 1.8456	0.0299	0.8705	0.8445	0.8015	0.7387	0.9034
Investment	− 3.0426	0.1029	0.9613	0.9064	0.8506	0.7557	0.9708
Hours	− 1.5986	0.0352	0.9568	0.8977	0.8287	0.9667	0.9755

Comparing the mean, standard deviation, and autocorrelation properties of the actual with the predicted in Tables 2 and 3, one cannot obviously see whether the estimated structural model is better than the calibrated structural model. The predicted autocorrelations from the unrestricted reduced form, the unrestricted reduced form with lagged dependent variables and the latter with other lagged variables continue to improve. The improvement in autocorrelation properties when lagged dependent variables are added reinforces the

Table 6
Unrestricted reduced form with other lagged variables

Variables in log	Mean	Standard Deviation	Autocorrelation $\rho(1)$	$\rho(2)$	$\rho(3)$	Correlation with output	Correlation with actual
Panel A: actual							
Output	− 1.4385	0.0513	0.9761	0.9547	0.9306	1.0000	1.0000
Consumption	− 1.7049	0.0358	0.9621	0.9373	0.9099	0.9064	1.0000
Investment	− 2.8960	0.1234	0.9686	0.9321	0.8938	0.9096	1.0000
Hours	− 1.5986	0.0355	0.9523	0.8947	0.8297	0.8273	1.0000
Panel B: predicted							
Output	− 1.4346	0.0499	0.9486	0.9306	0.9007	1.0000	0.9698
Consumption	− 1.6997	0.0354	0.8813	0.8662	0.8304	0.8917	0.9091
Investment	− 2.8960	0.1220	0.9653	0.9256	0.8879	0.8953	0.9867
Hours	− 1.5986	0.0350	0.9509	0.8887	0.8241	0.8058	0.9760

evidence provided in Table 1 on the improvement in the serial correlation of the residual in the equation explaining hours worked, as the Durbin–Watson statistic increased from 0.065 for the restricted reduced form to 1.281 for the unrestricted reduced form with lagged dependent variables, and to 1.832 for the unrestricted reduced form with other lagged dependent variables. Note that the negative correlation between hours and output in both the calibrated and the estimated structural models are very different from the actual positive correlation, a result that echoes the lack of fit in the labor supply equation vividly demonstrated by the Plosser charts (Fig. 1b and Fig. 2b) above. On the other hand, the correlation of hours with output is positive for the unrestricted reduced form, with and without lagged dependent variables, as in Tables 4–6. Together with Fig. 3b, Fig. 4b, Fig. 5b which show the dramatic improvement in goodness-of-fit, this demonstrates once again that by examining the unrestricted reduced form as is done in standard econometric practice, one can learn a lot about not only the empirical properties and potential weaknesses of a model, but also the direction for improvement. In the context of the baseline RBC model, our analysis suggests that the right direction to go is to enrich the model with propagation mechanisms that feature lagged control variables as part of the state vector.

Acknowledgements

We thank Francis Lui, Mark Watson, Chi-wa Yuen, participants in various seminars and especially Bob King for constructive comments that led to substantial revisions and improvements of the paper.

References

Chow, G.C., 1975. Analysis and Control of Dynamic Economic Systems. Wiley, New York.

Chow, G.C., 1983. Econometrics. McCraw Hill, New York.

Chow, G.C., 1993. Optimal control without solving the Bellman equation. Journal of Economic Dynamics and Control 17, 621–630.

Chow, G.C., 1997. Dynamic Economics: Optimization by the Lagrange Method. Oxford University Press, New York.

Chow, G.C., Levitan, R.E., 1969. Nature of business cycles implicit in a linear economic model. Quarterly Journal of Economics 83, 504–517.

King, R.G., Plosser, C.I., Rebelo, S.T., 1988a. Production, growth, and business cycles: I. The basic neoclassical model. Journal of Monetary Economics 21, 195–232.

King, R.G., Plosser, C.I., Rebelo, S.T., 1988b. Production, growth, and business cycles: II. New directions. Journal of Monetary Economics 21, 309–341.

Kwan, Y.K., Chow, G.C., 1996. Estimating economic effects of political movements in China. Journal of Comparative Economics 23, 192–208.

Kwan, Y.K., Chow, G.C., 1997. Chow's method of optimal control: A numerical solution. Journal of Economic Dynamics and Control 21, 739–752.

Kydland, F.E., Prescott, E.C., 1982. Time to build and aggregate fluctuations. Econometrica 50, 1345–1370.

Plosser, C.I., 1989. Understanding real business cycles. Journal of Economic Perspectives 3, 51–77.

Watson, M.W., 1993. Measures the fit for calibrated models. Journal of Political Economy 101, 1011–1041.