Mathematical Model of the Inflationary Process (Part II)

Alexei Krouglov

796 Caboto Trail, Markham, Ontario L3R 4X1, Canada

Email: alexkrouglov@concordidea.com

ABSTRACT

Here I describe the dynamical model of a "pure" inflationary process and give a model-based definition of the inflation.

Journal of Economic Literature Classification Numbers: E 31.

Keywords: Inflation dynamics

1. Introduction

There are lots of talks about inflation in our life. Here I describe by means of mathematical model why the prices of commodities (in wide sense) grow and what purpose economical forces are trying to achieve by these processes.

I am using the dynamical differential equations originally developed in [1]. As well I am not somewhat satisfied how I tried to explain *the inflationary process* in [2]. Finally other descriptions of the inflation didn't come closer to the satisfactory explanation.

2. Model Description

The following situation constitutes the "pure" model of inflation.

At the beginning the commodity's supply (production) and demand are equal. There is no deficit on the market.

Then the commodity's demand is increased while the commodity's supply remains the same.

There is a situation of the commodity's deficit on the market. The deficit drives the commodity's prices to grow, which causes in return the commodity's demand down.

It is desirable to obtain that the commodity's supply and demand eventually become equal again and the commodity's price stabilizes.

The differential equations describing dynamics of commodity's supply, demand, and price provided below are taken from [3, 4].

Commodity's demand V_D and supply V_S are equal initially.

Therefore it holds $\frac{dV_S(t)}{dt} = \frac{dV_D(t)}{dt} = v_0$ until $t < t_0$ where $v_0 > 0$ is a constant.

Thus at time $t = t_0$ we have the following conditions $\frac{dV_D(t_0)}{dt} = v_0 + \boldsymbol{d}v$,

$\frac{dV_S(t_0)}{dt} = v_0$, $\frac{dP(t_0)}{dt} = 0$ and $V_D(t_0) = V_0 + \Delta V$, $V_S(t_0) = V_0$, $P(t_0) = P_0$.

We assume that for $t > t_0$ commodity's supply remains the same. Hence for $t > t_0$ only commodity's demand and price are changing according to equations [3].

$$\frac{d^2 V_S(t)}{dt^2} = 0 \qquad (1)$$

$$\frac{dP(t)}{dt} = l_2 \cdot (V_D(t) - V_S(t)) \qquad (2)$$

$$\frac{d^2 V_D(t)}{dt^2} = -\boldsymbol{l}_3 \cdot \frac{d^2 P(t)}{dt^2} \qquad (3)$$

In equations above the values $\boldsymbol{l}_2, \boldsymbol{l}_3 \geq 0$ are constants.

In the following paragraph I will derive the equations describing dynamics of commodity's demand and price.

3. Dynamics of Commodity's Supply, Demand and Price

Thus for $t > t_0$ we have the following differential equation from (1), (2) and (3).

$$\frac{dP(t)}{dt} = l_2 \cdot (V_D(t) - (V_0 + v_0 \cdot (t - t_0))) \qquad (4)$$

$$\frac{dV_D(t)}{dt} = -l_3 \cdot \frac{dP(t)}{dt} + (v_0 + dv) \qquad (5)$$

Therefore the values $V_D(t)$ and $V_S(t)$ are described for $t > t_0$ by the following formulas.

$$V_D(t) = \left(V_0 + \frac{dv}{l_2 \cdot l_3} \right) + v_0 \cdot (t - t_0) + \left(\Delta V - \frac{dv}{l_2 \cdot l_3} \right) \cdot e^{-l_2 \cdot l_3 \cdot (t - t_0)} \qquad (6)$$

$$V_S(t) = V_0 + v_0 \cdot (t - t_0) \qquad (7)$$

We further assume that $d\,v = 0$ and $\Delta V > 0$. It means that at time $t = t_0$ there is the commodity's deficit on the market.

Hence from (6) and (7) it takes place the following expression $V_D(t) \xrightarrow[t \to \infty]{} V_S(t)$.

It shows that the commodity's demand is converging to the commodity's supply.

Similarly it is fulfilled for the commodity's price.

$$\frac{dP(t)}{dt} = l_2 \cdot \Delta V \cdot e^{-l_2 \cdot l_3 \cdot (t - t_0)} \qquad (8)$$

Thus it takes place $\frac{dP(t)}{dt} \xrightarrow[t \to \infty]{} 0$, i.e. the commodity's price eventually stabilizes.

We can also derive the formula for the commodity's price.

$$P(t) = P_0 + \frac{\Delta V}{l_3} \cdot \left(1 - e^{-l_2 \cdot l_3 \cdot (t - t_0)} \right) \qquad (9)$$

4. Conclusion

Thus we can put together the following definition of the inflationary process.

Inflation is the economical process of eliminating the deficit of a commodity (or service) on the market by the means of increasing the commodity's price, which causes the decline in the commodity's demand.

Similarly, we can define the process of deflation.

These processes are illustrated in dynamics by the appropriate model.

References

1. A. Krouglov, "Mathematical Model of Simple Business Fluctuations," ewp-mac/9706009, available at http://econwpa.wustl.edu.
2. A. Krouglov, "Mathematical Model of the Inflationary Process," ewp-mac/9804001, available at http://econwpa.wustl.edu.
3. A. Krouglov, "Mathematical Description of Business Fluctuations," ewp-mac/9710002, available at http://econwpa.wustl.edu.
4. A. Krouglov, "Mathematical Model of Interdependency between Production and Price Fluctuations," ewp-mac/9709002, available at http://econwpa.wustl.edu.