

Housing Collateral, Consumption Insurance and Risk Premia

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Abstract

In a model with housing collateral, the ratio of housing wealth to human wealth shifts the conditional distribution of asset prices and consumption growth. A decrease in house prices reduces the collateral value of housing, increases household exposure to idiosyncratic risk, and increases the conditional market price of risk. Using aggregate data for the US, we find that a decrease in the ratio of housing wealth to human wealth predicts higher returns on stocks. *Conditional* on this ratio, the covariance of returns with aggregate risk factors explains up to eighty percent of the cross-sectional variation in annual size and book-to-market portfolio returns. Regional risk-sharing patterns for US metropolitan areas lend direct support to the housing collateral channel. In times with a high housing collateral ratio, consumption growth is more strongly correlated across regions. Time-variation in the degree of risk-sharing induced by house price changes sheds new light on the consumption correlation puzzle.

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1 Introduction

We introduce housing into Lucas' (1978) endowment economy. The households in our economy trade contingent claims to insure against labor income risk. These claims have to be fully backed by the value of their housing stock. An increase in the value of housing wealth relative to human wealth, *the housing collateral ratio*, allows for more risk sharing and decreases the premia on risky assets.

When the collateral constraints do not bind, our model collapses to the standard consumption-based capital-asset-pricing model of Lucas (1978) and Breeden (1979). That model prices only aggregate consumption growth risk. It has been rejected by the data (e.g. Hansen and Singleton (1983)). Our paper addresses two shortcomings of the consumption-based capital-asset-pricing model (CCAPM).

First, because US aggregate consumption growth is approximately i.i.d., the CCAPM implies a market price of risk that is approximately constant. However, in the data, stock market returns are predictable. This suggests that the market price of aggregate risk varies over time (e.g. Fama and French (1988), Campbell and Shiller (1988), Ferson, Kandel and Stambaugh (1987), Whitelaw (1997), Lamont (1998) and Campbell (2000) for an overview).

Second, the covariance of asset returns with consumption growth explains only a small fraction of the variation in the cross-section of stock returns of firms sorted in portfolios according to size and value (book-value to market-value) characteristics (Fama and French (1992)). In response to this failure, Fama and French (1993) drop the connection between the stochastic discount factor and consumption growth and directly specify the stochastic discount factor as a linear function of the market return, the return on a small minus big firm portfolio, and a high minus low book-to-market firm portfolio. The empirical success of this three-factor model has motivated recent research on the underlying macroeconomic sources of risk for which their factors proxy (e.g. Bansal, Dittmar and Lundblad (2002), Lettau and Ludvigson (2001b), Santos and Veronesi (2001) and Cochrane (2001) for an overview).

The failure of the CCAPM reflects its imposing of perfect consumption insurance. In the data, there is strong empirical evidence against full consumption insurance at different levels of aggregation: at the household level (e.g. Attanasio and Davis (1996), Cochrane (1991b) and Nelson (1994)'s comment on Mace (1991)), the regional level (e.g. Hess and Shin (1998) and Ostergaard, Sorensen and Yosha (2002)) and the international level (e.g. Backus, Kehoe and Kydland (1992)). Blundell, Pistaferri and Preston (2002) find evidence for a degree of consumption insurance that varies over time.

Our paper addresses these issues in the context of an endowment economy, but follows Alvarez and Jermann (2000) in relaxing the assumption that contracts are perfectly enforceable. Depending on whether they have enough collateral, households sometimes cannot trade away all of their labor income risk. As in Lustig (2000), we allow households to file for bankruptcy. The new feature of our model is that each household owns part of the housing stock. Housing provides both utility services and collateral services. When a household chooses not to honor its debt repayments, it loses all

housing collateral but its labor income is protected from creditors. Defaulting households regain immediate access to credit markets. In equilibrium, all state-contingent promises are fully backed by the value of the housing stock. The lack of commitment gives rise to participation constraints whose tightness depends on the abundance of housing collateral. We measure this by the *housing collateral ratio*: the ratio of collateralizable housing wealth to non-collateralizable human wealth.

These constraints are motivated by the empirical importance of housing collateral. In the US, two-thirds of households own their house. For the median-wealth homeowner, home equity represents seventy percent of household net worth (Survey of Consumer Finance, 1998). Residential real estate wealth accounts for thirty-one percent of total household net worth and eighty-two percent of non-financial assets, while home mortgages make up sixty-eight percent of household liabilities (Flow of Funds, Federal Reserve, average for 1952-2001). Currently, the value of residential wealth is equivalent to the total household stock market wealth (\$13 trillion) and the mortgage market is the largest credit market in the US (\$5.6 trillion).

Relative to the benchmark model with fully-enforceable contracts, our theory modifies the stochastic discount factor. It adds a new component which is a function of the cumulative Lagrange multipliers on the households' participation constraint (*the Pareto-Negishi weights*). The household's Pareto-Negishi weight is increased whenever its constraint binds. Hence, the weights summarize the individual history of binding collateral constraints. We consider two variations of the model. In a first economy with frictionless rental markets, perfect aggregation obtains and the new component of the stochastic discount factor is the growth rate of a cross-sectional moment of the Pareto-Negishi weight distribution. In a second economy, households live in different regions and housing services can only be traded among households within a given region. The new component is the change in the Pareto-Negishi weight of the unconstrained household(s) relative to the other households in the economy. In either case, when a large fraction of households is constrained the new component is high. This mechanism increases the volatility of the stochastic discount factor relative to the benchmark model.

The key feature of the model is that the housing collateral ratio moves endogenously. It shifts the conditional distribution of household consumption growth between two benchmark economies. When the housing collateral ratio is low, households more frequently run into binding collateral constraints. To prevent a household from defaulting today, its current *and* future consumption must increase as a share of aggregate consumption. The economy is constrained in how much risk-sharing it can implement. In the limit, when the housing collateral disappears altogether, no risk sharing is possible and the economy is in autarky. In contrast, when the housing collateral ratio is high, the collateral constraints never bind; the economy achieves full insurance, like an economy without commitment problems.

The equilibrium Pareto-Negishi weight processes are functions of the primitives of the model: the preferences, the household endowment process, and the aggregate endowment process. In a companion paper, we obtain a recursive formulation and numerically solve for the equilibrium Pareto-Negishi weight processes for an economy with two agents (Lustig and VanNieuwerburgh

(2002)). However, for a large number of agents we run into the curse of dimensionality.

This paper takes a different route. Our empirical strategy is directly to specify a stochastic process for the Pareto-Negishi weights in a way consistent with the theory. The aim is to link the unobservable weight processes to the data on housing collateral. Theory disciplines this approach in two ways. The weight process is known in the polar cases of autarky and perfect commitment. We adopt a specification that allows the housing collateral ratio to shift the conditional distribution of household consumption growth between the polar cases.

On the basis of this specification, we impose a linear factor structure on the weight process that connects our model to the linear factor models in the empirical finance literature. Our model is a conditional version of the CCAPM with the housing collateral ratio as the conditioning variable. The housing collateral ratio summarizes the investor's time-varying information set. In the first economy, the risk of binding collateral constraints is captured by the housing collateral ratio and the interaction terms of the housing collateral ratio and the aggregate sources of risk. With non-separable preferences over housing services and consumption, the aggregate risk sources are consumption growth and rental price growth. In the second economy, the constraint risk is captured by the income and rental price growth of the unconstrained region relative to the other regions, and both terms interacted with the aggregate housing collateral ratio.

Our theory has three testable predictions. First, households demand a larger compensation for a given amount of aggregate consumption risk in times when the housing collateral ratio is low. This implies that the housing collateral ratio predicts aggregate stock returns over time. Second, a particular asset earns a larger risk premium if its returns are more correlated with consumption growth when the housing collateral ratio is low. Third, the model predicts less consumption insurance when the housing collateral ratio is low: consumption growth is more sensitive to income growth and rental price growth.

We test these three predictions using the following data. First, for the time-series predictability of returns, we use annual return data for the aggregate US stock market index. We measure the aggregate stock of housing collateral in three different ways: by the value of outstanding mortgages, by the value of residential real estate (structures and land) and by the value of residential fixed assets (structures). The housing collateral ratio is measured as the deviation from the cointegration relationship between the value of the aggregate housing stock and aggregate labor income. Second, for the cross-sectional exercise, we use twenty-five size and book-to-market portfolios, and the value-weighted market return. Third, for the risk-sharing tests, we construct a new panel data set for US metropolitan statistical areas on consumption, income and house prices.

Table 1 summarizes the predictions of our model and contrasts them with the predictions of the Lucas-Breeden model. The last column shows the data we use to test them. The ratio of collateralizable housing wealth to non-collateralizable human wealth is labelled my .

We find strong empirical support for each of the predictions. First, in the time series, the housing collateral ratio *does* predict stock returns, mainly at lower frequencies. Second, in the cross-section, our model explains between seventy and eighty percent of the variability in annual returns of the

	Perfect risk-sharing Consumption-CAPM	Limited risk-sharing Collateral-CAPM	Data Period
Time-series Predictability	no return predictability constant price of risk	my predicts returns my -varying price of risk	Excess market return and my (1889-2001)
Pricing Portfolios	Covariance of returns with risk factors	Covariance conditional on my	Aggregate factors (1926-01) Regional factors (1951-01)
Risk-sharing Across regions	$\text{Corr}(\Delta c, \Delta y) = 0$	$\text{Corr}(\Delta c, \Delta y)$ lower when my high	Regional data (1951-01)

Table 1: Predictions and Data for Empirical Exercises.

Fama-French portfolios. This fit is obtained for both variations of the model: the frictionless economy with aggregate pricing factors tested on aggregate data, and the economy with housing frictions and regional asset pricing factors tested on metropolitan data. For annual returns, this matches the empirical success of the Fama and French (1993) three-factor model and outperforms other consumption-based asset pricing models (e.g. Lettau and Ludvigson (2001b)). Third, we provide direct evidence for the underlying time-variation in risk-sharing. Using our metropolitan data set, we reject full consumption insurance. The degree of partial insurance decreases when the housing collateral ratio is low and varies substantially over time. This time variation in risk-sharing is direct evidence for the mechanism that drives our model and leads to the asset pricing predictions implied by it.

We organize the paper as follows. In section 2, we briefly discuss other related literature. Section 3 describes the environment and characterizes efficient and equilibrium allocations. The fourth section contains a discussion of our empirical strategy which bridges the gap between theory and data. Section 5 describes our data and section 6 shows how we measure the housing collateral ratio. Our empirical findings are summarized in section 7. Section 8 concludes. Appendix A contains details of the model, the computational method and the data. The most important figures and tables appear in the main text, all others in Appendix B.

2 Related Literature

Our paper is closest in spirit to the work of Lettau and Ludvigson (2001b). As in their paper, we develop a scaled version of the CCAPM. Our state variable my summarizes information about future returns on housing *relative* to human capital. It does not contain any direct information on the future returns on stocks. In contrast, the scaling variable in Lettau and Ludvigson (2001b) is the consumption-wealth ratio, which summarizes household expectations about future returns on the entire market portfolio, including financial wealth.

Our model contains three further important features. First, we allow preferences over non-durable and housing consumption to be separable or non-separable, and we find strong empirical evidence for the collateral effect in either case. In recent work, Piazzesi, Schneider and Tuzel (2002) argue that non-separability is important for pricing assets. They consider a representative agent who consumes nondurables and housing services. Suppose housing services and consumption are

complements; the agent commands a larger risk premium if returns and rental prices are positively correlated. They show that this *hedging effect* increases the explanatory power of the standard consumption capital asset pricing model for stock and bond returns.

Second, we model the outside option as bankruptcy with loss of all collateral assets. In Kehoe and Levine (1993), Krueger (2000), Krueger and Perri (2002), and Kehoe and Perri (2002) limited commitment is also the source of incomplete risk-sharing across US households and across countries respectively. In contrast, the outside option upon default is exclusion from future participation in financial markets.

Third, our paper features a general equilibrium economy with aggregate uncertainty and endogenous house price fluctuations. In contrast, life-cycle and portfolio choice models such as Fernandez-Villaverde and Krueger (2001), Cocco (2000), Yao and Zhang (2002), Flavin and Yamashita (2002) posit an exogenous price process for housing.

3 Setup

This section starts with a complete description of the environment in section 3.1. Section 3.2 sets up a planner problem. In this environment, households cannot commit ex ante to a consumption plan. This constrains the feasible allocations. We provide a complete characterization of these allocations using stochastic Pareto-Negishi weight processes. Section 3.3 introduces markets and offers a decentralization of these planner allocations. We show that the growth rate of an aggregated Pareto-Negishi weight process drives the consumption growth of the off-corner households and these households price the random payoffs. Rental markets are frictionless in section 3.3. In 3.4 we modify the problem and introduce housing market frictions.

3.1 Environment

We consider an endowment economy with N regions. Household i lives in region i and cannot move. There is a continuum of identical households in each region. These households are infinitely-lived.

Uncertainty s is an event that follows a Markov process. These events take on values on a discrete grid S . We use s^t to denote the history of events. S^t denotes the set of possible histories up until time t . $\pi(s^t|s_0)$ denotes the probability of a history s^t conditional on s^t .

Preferences We use $\{x\}$ to denote an infinite stream $\{x_t(s^t)\}_{t=0}^{\infty}$. There are two types of commodities in this economy: a consumption good and housing services. The consumption good cannot be stored. We let $\{c^i\}$ denote the stream of consumption and we let $\{h^i\}$ denote the stream of housing services of household i .

The households are infinitely lived and they rank streams $\{c^i\}$ and $\{h^i\}$ according to this

criterion:

$$U(\{c^i\}, \{h^i\}, \{b^i\}) = \sum_{s^t|s^0} \sum_{t=0}^{\infty} \delta^t \pi(s^t|s^0) u(c_t^i(s^t), h_t^i(s^t), b_t^i(s^t)).$$

where δ is the time discount factor and $\{b^i\} = \{b^{i,c}, b^{i,h}\}$ are sequences of taste shocks. The households have power utility over a CES-composite consumption good:

$$u(c_t^i, h_t^i, b_t^i) = \frac{\left[(e^{b_t^c} c_t^i)^\sigma + \psi \left(e^{b_t^h} h_t^i \right)^\sigma \right]^{\frac{1-\gamma}{\sigma}}}{1-\gamma},$$

$\psi > 0$ converts the housing stock into a service flow. The elasticity of substitution between c and h is $(1-\sigma)^{-1}$. Housing and non-durable consumption are complements if $1-\gamma-\sigma > 0$. Otherwise the two goods are substitutes.¹ We define $\phi = \left(\frac{1-\gamma-\sigma}{1-\sigma} \right)$.

Endowments Each of the households is endowed with a claim to a labor income stream $\{\eta^i\}$. The aggregate endowment of the consumption good is denoted $\{e\}$.

Each of the households is also endowed with a claim to a stream of housing services $\{\chi^i\}$. The aggregate endowment of housing services is denoted $\{h_t\}$. The aggregate endowments are the sum of the individual endowments:

$$\sum_{i=1}^N \eta_t^i(s^t) = e_t(s^t) \text{ and } \sum_{i=1}^N \chi_t^i(s^t) = h_t(s^t), \forall s^t, t \geq 0.$$

Commitment Technology A plan $\sigma^i = \{c^i, h^i\}$ is a complete description of household i 's consumption. We define $U^i(\sigma^i)(s^t)$ to be the household's continuation utility from a plan at node s^t :

$$U^i(\sigma^i)(s^t) = U(\{c^i\}, \{h^i\}, \{b^i\})(s^t).$$

The household cannot commit ex-ante to a plan. At each node s^t , it faces a participation constraint:

$$U^i(\sigma^i)(s^t) \geq \kappa_t^i(s^t) \forall s^t, t \geq 0, \tag{1}$$

where $\kappa_t^i(s^t)$ is the continuation value of the household upon default. For now, we take $\kappa_t^i(s^t)$ as given.

A household can choose to exercise its option to default on the plan in any state of the world. When it does so, its individual history is erased. Therefore we refer to this option as the *anonymity option*. From that node onwards, the household's future consumption plan only depends on the history of the economy.

The household cannot be excluded from the contract, because the planner cannot keep track of the household after it exercises the anonymity option. However, the planner can observe and seize

¹The preferences belong to the class of homothetic power utility functions of Eichenbaum and Hansen (1990). Special cases are separability ($1-\gamma-\sigma = 0$) and Cobb-Douglas preferences ($\gamma, \sigma = 0$).

its housing endowment $\{\chi^i\}$. After exercising the anonymity option, the household loses its claim to its endowment of housing services. The claim to its individual endowment of the consumption good $\{\eta^i\}$ is inalienable (Lustig (2000)).

Feasibility An allocation $(\sigma^i)_{i=1}^N$ describes the entire consumption plan for all of the households. An allocation $(\sigma^i)_{i=1}^N$ is said to be feasible if at all nodes of the event tree:

$$\sum_{i=1}^N c_t^i(s^t) \leq e_t(s^t) \text{ and } \sum_{i=1}^N h_t^i(s^t) \leq h_t(s^t) \text{ for all } s^t, t \geq 0. \quad (2)$$

An allocation $(\sigma^i)_{i=1}^N$ is said to be immune to the threat of bankruptcy if the allocation satisfies the participation constraint 1 for each household and at all nodes of the event tree.

3.2 Planner Problem

The planner computes the constrained efficient allocations. In this environment, the participation constraints depend on the shadow price of consumption and housing services in different states of the world. To compute these allocations, the planner solves a fixed point problem.

Let ℓ denote the $N \times 1$ -vector of initial Pareto-Negishi weights. The planner maximizes a weighted sum of the household's utilities subject to a participation constraint (1) in each node s^t and for each household:

$$Max_{\{c^i, h^i\}} \sum_{i=1}^N \ell^i \sum_{s^t | s^0} \sum_{t=0}^{\infty} \delta^t \pi(s^t | s^0) u(c_t^i(s^t), h_t^i(s^t), b_t^i(s^t)),$$

subject to the resource constraint in (2) for each s^t and subject to the participation constraint for each household, again for each node s^t .

Let $\{\gamma^i\}$ denote the sequence of multipliers on the participation constraints imposed on household i . To solve this convex programming problem with an infinite number of constraints, we introduce the Lagrangean:

$$Max_{\{c^i, h^i\}} Min_{\{\gamma^i\}} \sum_{i=1}^N \ell^i \sum_{s^t | s^0} \sum_{t=0}^{\infty} \left\{ \delta^t \pi(s^t | s^0) u(c_t^i(s^t), h_t^i(s^t), b_t^i(s^t)) \right. \\ \left. + \sum_{i=1}^N \gamma_t^i(s^t) \left[\sum_{s^\tau | s^t} \sum_{\tau=0}^{\infty} \delta^{t+\tau} \pi(s^{t+\tau} | s^t) u(c_{t+\tau}^i(s^{t+\tau}), h_{t+\tau}^i(s^{t+\tau}), b_{t+\tau}^i(s^{t+\tau})) - \kappa_t^i(s^t) \right] \right\},$$

subject to the resource constraint in (2) for each s^t . We define $\xi_t^i(s^t)$ to be household i 's cumulative Lagrange multiplier:

$$\xi_t^i(s^t) = \ell^i + \sum_{\tau=0}^t \sum_{s^\tau \preceq s^t} \gamma_\tau^i(s^\tau).$$

We refer to $\xi_t^i(s^t)$ as household i 's *Pareto-Negishi weight* in state s^t . $\{\xi^i\}$ is a non-decreasing stochastic process. If a household participation constraint binds, its weight increases to a cutoff level that depends only on s , the current event. If the constraint does not bind, its weight remains unchanged. This imputes limited memory to the allocations: a household's individual history is erased whenever it switches to a state with binding constraints.

Shadow Prices and Component Planner Problem We decompose the planner problem into a separate component planner problem for each household. Using this component planner problem, we can recover the household's Pareto-Negishi weight process for each household, starting at any arbitrary node s^t (Atkeson and Lucas (1992)).

Let $\{\tilde{\mu}\}$ denote the Lagrange multiplier process for the resource constraint for the consumption good in s^t . $\tilde{\mu}_t(s^t)$ is the planner's shadow price of consumption at node s^t . We will refer to this object as the shadow state price for consumption. Similarly, let $\{\tilde{\rho}\tilde{\mu}\}$ denote the Lagrange multiplier process for the resource constraint for the housing services in s^t .

For a given sequence of shadow state prices $\{\tilde{\mu}, \tilde{\rho}\}$, the planner allocates a stream $\{c^i, h^i\}$ to a household i (of mass zero). In a state s^t , the component planner problem for this household is:

$$Max_{\{c^i, h^i\}} U(\{c^i\}, \{h^i\}, \{b^i\})(s^t)$$

subject to the participation constraints (1) at all future nodes and subject to the shadow cost constraint:

$$\tilde{\Pi}_{s^t} [\{c^i\}] + \tilde{\Pi}_{s^t} [\{\tilde{\rho}h^i\}] \leq \tilde{\Pi}_{s^t} [\{\eta^i\}] + \tilde{W}_t^i(s^t). \quad (3)$$

$\tilde{\Pi}_{s^t} [\{d\}]$ denotes the shadow value of a dividend stream $\{d\}$, computed with $\{\tilde{\mu}\}$ as the state prices: $\tilde{\Pi}_{s^t} [\{d\}] = \sum_{s^\tau|s^t} \sum_{\tau=0}^{\infty} [\tilde{\mu}_{t+\tau}(s^\tau|s^t) d_{t+\tau}(s^\tau|s^t)]$. The variable $\tilde{W}_t^i(s^t)$ is net shadow wealth, net of labor income. At time 0, a household's shadow wealth is the value of its housing endowment: $\tilde{W}_0^i(s^0) = \tilde{\Pi}_{s^0} [\{\tilde{\rho}^i \chi^i\}]$. The vector of initial Pareto-Negishi weights ℓ is determined such that the shadow value of each household's consumption claim equals the shadow value of its initial endowment of housing services and labor income. $\tilde{W}_t^i(s^t)$ captures the effect of an individual household's history on its consumption plan. The shadow cost constraints (3) ensure that the planner satisfies its resource constraints in all histories.

The planner solves the following fixed point problem. For an initial guess of shadow state prices, he computes the Pareto-Negishi weight processes for each household and the implied allocation. The allocation gives rise to a new guess for the shadow state prices. The planner iterates on these two steps until convergence.

Participation Constraint By exercising the anonymity option, the household erases its individual history and sets $\tilde{W}_t^i(s^t) = 0$. The value of the anonymity option is the optimum of the component planner problem with $\tilde{W}_t^i(s^t) = 0$:

$$\kappa_t^i(s^t) = Max_{\{c^i, h^i\}} U(\{c^i\}, \{h^i\}, \{b^i\})(s^t)$$

subject to the participation constraints at all future nodes and such that the cost constraint in (3) is satisfied for $\tilde{W}_t^i(s^t) = 0$ (Lustig (2000)).

Because $\kappa_t^i(s^t)$ is monotonically increasing in $\tilde{W}_t^i(s^t)$, the participation constraints (1) can be stated as a non-negativity restriction on net wealth:

$$\tilde{W}_t^i(s^t) \geq 0 \text{ or } \tilde{\Pi}_{s^t} [\{c^i\}] + \tilde{\Pi}_{s^t} [\{\tilde{\rho}h^i\}] \geq \tilde{\Pi}_{s^t} [\{\eta^i\}], \forall s^t, t \geq 0 \quad (4)$$

The anonymity option depends on the shadow state prices $\{\tilde{\mu}, \tilde{\rho}\}$.

Solving the planner problem requires conjecturing shadow state prices, computing the outside options and the new, implied shadow prices until a fixed point is reached.

Constrained Optimality At the constrained Pareto-optimum, there is a mapping from the multipliers at s^t to consumption of both commodities. We refer to this mapping as the risk-sharing rule.

First, the planner equalizes the weighted marginal utility of consumption across households:

$$\xi_t^i(s^t) \delta^t \pi(s^t | s_0) u_c(c_t^i(s^t), h_t^i(s^t), b_t^i(s^t)) = \tilde{\mu}_t(s^t)$$

Marginal utility growth is determined by the growth of a household's individual weight, relative to the resource constraint multiplier:

$$\delta \pi(s^{t+1} | s_t) \frac{u_c(c_{t+1}^i(s^{t+1}), h_{t+1}^i(s^{t+1}), b_{t+1}^i(s^{t+1}))}{u_c(c_t^i(s^t), h_t^i(s^t), b_t^i(s^t))} = \frac{\tilde{\mu}_{t+1}(s^{t+1}) \xi_t^i(s^t)}{\tilde{\mu}_t(s^t) \xi_{t+1}^i(s^{t+1})}$$

Second, the planner equalizes marginal rates of substitution between consumption and housing services across households:

$$\tilde{\rho}_t(s^t) = \frac{u_h(c_t^i(s^t), h_t^i(s^t), b_t^i(s^t))}{u_c(c_t^i(s^t), h_t^i(s^t), b_t^i(s^t))} = \psi \left(\frac{h_t^i(s^t)}{c_t^i(s^t)} \right)^{\sigma-1}.$$

The equalization implies that the shadow state price of rental services is pinned down by the ratio of the aggregate housing endowment to the aggregate consumption endowment:

$$\tilde{\rho}_t(s^t) = \psi \left(\frac{h_t(s^t)}{e_t(s^t)} \right)^{\sigma-1}.$$

Let us abstract from taste shocks for now. A household's optimal consumption share and housing services share is a function of its own weight and an aggregate sum of these Pareto-Negishi weights:

$$c_t^i(s^t) = \frac{\xi_t^i(s^t)^{\frac{1}{\gamma}}}{\xi_t^a(s^t)^{\frac{1}{\gamma}}} e_t(s^t) \text{ and } h_t^i(s^t) = \frac{\xi_t^i(s^t)^{\frac{1}{\gamma}}}{\xi_t^a(s^t)^{\frac{1}{\gamma}}} h_t(s^t), \quad (5)$$

where $\xi_t^a(s^t)$ denotes the aggregate weight process $\sum_j \xi_t^j(s^t)^{\frac{1}{\gamma}}$. When a household switches to a state with a binding constraint, its consumption share experiences a sudden jump up. Everywhere else, its consumption share is drifting downwards.

Proposition 1. *A constrained optimal allocation is completely characterized by the sequence of household Pareto-Negishi weights $(\{\xi^i\})_{i=1}^N$ and the aggregate resource constraint multipliers $\{\tilde{\mu}, \tilde{\rho}\}$.*

We use m_{t+1}^a to denote the IMRS of an agent who consumes both of the aggregate endowments. Using the risk-sharing rules (5) for the unconstrained agents ($\xi_{t+1}^i = \xi_t^i$), the growth rate of the shadow state price is:

$$\frac{\tilde{\mu}_{t+1}(s^{t+1})}{\tilde{\mu}_t(s^t)} = \pi(s_{t+1}|s_t) m_{t+1}^a(s^t, s') (g_{t+1})^\gamma \quad (6)$$

The growth rate of the shadow state price for consumption consists of two parts: (1) the intertemporal marginal rate of substitution m_{t+1}^a of the representative agent² and (2) the growth rate g_{t+1} of the aggregate Pareto-Negishi weight process ξ_t^a . When many households are severely constrained in a state tomorrow, that state's shadow price increases, because the unconstrained households experience high marginal utility growth.

Collateral Supply The shadow price of housing services $\{\tilde{\rho}\}$ determines how much risk-sharing the planner can achieve. When the shadow price of housing services increases, the left hand side of the participation constraints (4) increases and the anonymity option becomes less appealing. The shadow value of the aggregate housing stock, $\tilde{\Pi}_{s^t}[\{\tilde{\rho}h\}]$, measures the collateral the planner has access to. We define the housing collateral ratio $my(s^t)$ as

$$my(s^t) = \frac{\tilde{\Pi}_{s^t}[\{\tilde{\rho}h\}]}{\tilde{\Pi}_{s^t}[\{e\}]} \quad (7)$$

It measures the tightness of the participation constraints. If this ratio is zero, no risk sharing is feasible. If the shadow relative price of housing services increases in a persistent way, this ratio increases and the planner can sustain more risk sharing.

These constrained efficient allocations can be decentralized using results by Alvarez and Jermann (2000). To do so, we introduce markets for all the assets and commodities.

3.3 Markets

We describe the trading arrangements in sequential markets, the household's problem and define a competitive equilibrium. We then argue that the equilibrium with sequential trading can be mapped into an Arrow-Debreu equilibrium in which all trading takes place at time zero. In a first step we assume that markets for housing services are frictionless. In a second step (section 3.4) we relax this assumption.

²The intertemporal marginal rate of substitution of the representative agent is a function of the aggregate consumption growth and the growth rate of the shadow state price of rental services:

$$m_{t+1}^a(s^{t+1}) = \delta \left(\frac{1 + \psi \tilde{\rho}_{t+1}(s^{t+1})^{\frac{1}{\sigma-1}}}{1 + \psi \tilde{\rho}_t(s^t)^{\frac{1}{\sigma-1}}} \right)^{\frac{1-\gamma-\sigma}{\sigma}} \left(\frac{e_{t+1}(s^{t+1})}{e_t(s^t)} \right)^{-\gamma}.$$

The financial markets are complete. Spot markets for housing services are frictionless. All prices are quoted in units of the consumption good. Households trade a complete set of contingent claims $a_t(s^t, s')$ in forward markets. $a_t(s^t, s')$ is a promise to deliver one of unit the consumption good if event s' is realized in the next period. These claims trade at a price $q_t(s^t, s')$. Households also trade shares $\omega_t(s^t)$ in the aggregate housing tree at a price $p_t^h(s^t)$. In the spot market, households trade housing services $h_t(s^t)$. The price of a unit of housing services is $\rho_t(s^t)$ in units of the consumption good.

Competitive Equilibrium In each node s^t , households face a separate *collateral constraint* for each event s' :

$$-a_t^i(s^t, s') \leq \omega_t^i(s^t) \left[p_{t+1}^h(s^{t+1}) + \rho_{t+1}(s^{t+1})h_{t+1}(s^{t+1}) \right], \text{ for all } s^t, s'. \quad (8)$$

All of a household's state-contingent promises are backed by the cum-dividend value of its holdings in the aggregate housing tree. At the start of the period, the household purchases goods in the spot market, contingent claims and shares in the financial market subject to a wealth constraint:

$$W_t^i(s^t) \geq c_t^i(s^t) + \rho_t^i(s^t)h_t^i(s^t) + \sum_{s'} q_t(s^t, s')a_t(s^t, s') + p_t^h(s^t)\omega_t^i(s^t).$$

Next period wealth is:

$$W_{t+1}^i(s^t, s') = y_{t+1}^i(s^t, s') + a_t^i(s^t, s') + \omega_t^i(s^t) \left[p_{t+1}^h(s^t, s') + \rho_{t+1}(s^t, s')h_{t+1}(s^t, s') \right].$$

Definition. Given an initial wealth distribution $\{W_0^i\}_{i=1}^N$ for each agent i , a competitive equilibrium is an allocation $\{c^i, h^i, a_{t-1}^i, \omega^i\}_{i=1}^N$ and a price vector $\{q_{t-1}, p^h, \rho\}$ such that (1) for given prices and initial wealth, $\{c^i, h^i, a_{t-1}^i, \omega^i\}$ solve household i 's maximization problem, (2) the markets for the consumption good and the housing services clear and (3) the market for contingent claims clears, (4) the shares in the aggregate housing tree sum to one.

The equilibria in the economy with sequential trading are equivalent to Kehoe and Levine (1993) equilibria, if the equilibrium interest rates are high enough (Alvarez and Jermann (2000)). These Kehoe-Levine equilibria are essentially Arrow-Debreu equilibria and hence the underlying allocations are (constrained) efficient. Appendix A.1 provides the details.

To show the equivalence, we define the market state price $\mu_t(s^t)$ as the product of the Arrow prices for the events along a path s^t :

$$\mu_t(s^t) = q_{t-1}(s^{t-1}, s')q_{t-2}(s^{t-1}) \dots q_0(s^1),$$

where $\mu_t(s^t)$ is the price at time 0 of a unit of consumption to be delivered at node s^t .

By iterating forward on the collateral constraints in (8), substituting for the time 0 budget constraint, and imposing a no-arbitrage condition on $\{p^h\}$, the sequence of collateral constraints

can be restated as a non-negativity constraint on net wealth in every history:

$$\Pi_{s^t} [\{c^i\}] + \Pi_{s^t} [\{h^i \rho\}] \geq \Pi_{s^t} [\{y^i\}], \quad \forall s^t, t \geq 0. \quad (9)$$

We use $\Pi_{s^t} [\{d\}]$ to denote the market value of a dividend stream computed with market state prices $\{\mu\}$.

Proposition 2. *If the interest rates are high enough, the equilibrium allocations are constrained efficient (Alvarez and Jermann (2000)).*

We can decentralize the constrained efficient allocations obtained from the planner's problem by equating the market state prices and the shadow state prices:

$$\mu_t(s^t) = \tilde{\mu}_t(s^t) \text{ and } \rho_t(s^t) = \tilde{\rho}_t(s^t)$$

The constrained efficiency follows from two facts. First, substituting the shadow state prices for $\mu_t(s^t)$ in (9) reproduces the planner's participation constraints, at each node s^t . Second, the first order conditions in the planner problem are identical to the first order conditions of the household in a competitive equilibrium.

Stochastic Discount Factor The solution to the planner problem delivers a list of Pareto-Negishi weight processes, one for each household. The aggregate weight process $\{\xi_t^a\}$ fixes consumption growth for the unconstrained households and these households price payoffs in that state of the world.

In this complete markets setting, there exists a unique and strictly positive stochastic process $\{m\}$ that satisfies the standard orthogonality condition (Harrison and Kreps (1979)):

$$E_t \left[m_{t+1} (s^{t+1}|s^t) R_{t+1}^j (s^{t+1}|s^t) \right] = 1, \quad (10)$$

for any return process $\{R^j\}$. $\{m\}$ is the *stochastic discount factor* process. It is the product of the standard representative agent stochastic discount factor m_{t+1}^a and the growth rate g_{t+1} of the aggregate Pareto-Negishi weights ξ_t^a :

$$m_{t+1} = m_{t+1}^a (g_{t+1})^\gamma \quad (11)$$

m_{t+1}^a is the IMRS of an agent who consumes the aggregate endowments. We refer to the growth rate g_{t+1} as the *aggregate weight shock*. The aggregate weight shock is large if many households are severely constrained. If none of the households is constrained, the shock is unity.

If the number of households N is arbitrarily large, we can define an aggregate event z_t that describes the current aggregate endowment growth rate and the current my_t . In this environment, Lustig (2001) shows that the aggregate shock $\{g\}$ depends only on the history of aggregate shocks z^t . This history tracks changes in the distribution of weights across households.

3.4 Housing Market Frictions

We introduce a second economy. The environment is unchanged, except for the tradeability of housing services. In the second economy, the planner cannot reallocate housing services across households of different types. In the decentralization, households can only trade housing services in the spot market with other households in region i .

Planner Problem The planner now faces a separate housing services resource constraint for each region:

$$\chi_t^i(s^t) \geq h_t^i(s^t), \forall i, \forall s^t, t \geq 0. \quad (12)$$

This gives rise to a different shadow price for housing services in each region i : $\tilde{\rho}_t^i(s^t)$.

Markets and Equilibrium Household i buys shares $\omega_t^{i,j}(s^t)$ in a real estate fund for household j . There is a consumption flow $\omega_t^{i,j}(s^t)\rho_t^j(s^t)h_t^j(s^t)$ in each period from household j to household i . In each node s^t , household i faces a separate collateral constraint for each event s' :

$$-a_t^i(s^t, s') \leq \sum_j \omega_t^{i,j}(s^t) \left[p_{t+1}^{h,j}(s^{t+1}) + \rho_{t+1}^j(s^{t+1})h_{t+1}^j(s^{t+1}) \right], \forall s^t, s'. \quad (13)$$

In equilibrium, each household has its own rental price $\rho_t^i(s^t)$. Housing market clearing requires that (12) holds with equality for all states.

Stochastic Discount Factor In this environment, different households face different rental prices and the aggregation result breaks down, except when preferences for housing and consumption are separable.

We provide a decomposition of the household's IMRS in a common component and a household-specific component. We let hatted variables denote the household-specific component of a variable, e.g. :

$$\log(\hat{\xi}_t^i) = \log(\xi_t^i) - \frac{1}{N} \sum_{i=1}^N \log(\xi_t^i) = \log(\xi_t^i) - \log^a(\xi_t).$$

We use i_{t+1}^* to denote the unconstrained household between s^t and s^{t+1} . The log stochastic discount factor can be approximated by a common component and a household-specific weight shock:

$$\log m_{t+1} \approx \log^a m_{t+1} - \Delta \log \hat{\xi}_{t+1}^{i_{t+1}^*}, \quad (14)$$

where $\log^a m_{t+1}$ is the IMRS evaluated at the average consumption growth $\Delta^a \log c_{t+1}$ and average rental price growth $\Delta^a \log \rho_{t+1}$. The weight shock is the growth rate of the unconstrained region's relative Pareto-Negishi weight.

This weight shock is large if many households are severely constrained. If none of the households is constrained, it is zero. The weight shocks are driven by my , the ratio of aggregate housing collateral wealth to aggregate human wealth.

4 Back-Solving

For given shadow state prices, the stochastic Pareto-Negishi weights determine the constrained efficient allocations. The allocations determine new shadow state prices, so that the planner solves a fixed-point problem. To compute the Pareto-Negishi weights, the planner takes a particular calibration of the household endowment process, the aggregate endowment process and the preferences as *inputs*. The Pareto-Negishi weights are the *outputs* of the planner problem. This is called “forward solving”. In a companion paper we characterize the solution to the planner problem recursively and solve numerically for the equilibrium Pareto-Negishi weight processes for an economy with two agents (Lustig and VanNieuwerburgh (2002)). Kehoe and Perri (2002) also follow a forward solving strategy in a two-agent world.

To estimate the model in this approach, we would have to fix a particular calibration of the model, compute the weights and compute the relevant asset pricing moments. We could then iterate on these steps to find the underlying parameter vector that minimizes the distance between the asset pricing moments for the model and for the data. This vector includes the parameters governing the individual endowment processes. For a large number of households, we run into the curse of dimensionality and the problem becomes computationally intractable.

In this paper we take a different route. We directly parameterize the outputs of the planner problem: the stochastic Pareto-Negishi weight process. We move the inputs, in particular, the underlying endowment processes, to the background and focus our efforts on linking the unobservable weight processes to the data. Theory strictly guides the specification. This approach is a version of “back-solving” (e.g. Sims (1990)).

We impose a linear factor structure on the weight process. As a result, the back-solving strategy delivers a linear pricing model:

$$m_{t+1} = -\theta F_{t+1},$$

where θ is a vector of constants and F_{t+1} is a vector of factors. This connects our model to the linear factor model tradition in the empirical finance literature.

In the first economy, with frictionless rental markets, we specify a stochastic process for the aggregate weight shock. This delivers a factor model that only depends on aggregate factors (section 4.1). In the second economy, with housing market frictions, we specify the regional weight shocks. This delivers a factor model that depends on regional factors and the housing collateral ratio (4.2).

In either case, the linear factor model gives rise to the β -representation described in 4.3, which is estimated in section 7.

4.1 Aggregate Factors

In the first economy, housing markets are frictionless and the aggregate weight shocks are driven only by aggregate variables.

We use $F_{t+1}^a = (\Delta \log c_{t+1}, A_t \Delta \log \rho_{t+1})$ to denote the vector of aggregate factors: consumption growth and rental price growth, scaled by the housing expenditure share $A_t = \frac{\rho_t h_t}{c_t + \rho_t h_t}$. We use

$\bar{F}^a = (\Upsilon_c, \Upsilon_\rho)$ to denote the unconditional mean of the aggregate factors.

To derive a linear pricing model $m_{t+1}(my_t, F_{t+1}^a)$, we *propose* a factor model $G_{t+1}(my_{t+1}, F_{t+1}^a)$ for the aggregate weight shocks $\{g\}$. The housing collateral ratio my_t is the conditioning variable.

Definition. *In the first economy, a complete description of the linear pricing model $m_{t+1}(my_t, F_{t+1}^a)$ consists of (1) a specification for the aggregate weight shocks $G_{t+1}(my_{t+1}, F_{t+1}^a)$ and (2) a process for the housing collateral ratio $my_{t+1}(my_t, F_{t+1}^a)$.*

The Housing Collateral Ratio $\{my\}$ is specified as an autoregressive process whose innovations are a linear combination C of the innovations to F_{t+1}^a :

$$my_{t+1} = \rho my_t + C_1 (\Delta \log c_{t+1} - \Upsilon_c) + C_2 (A_t \Delta \log \rho_{t+1} - \Upsilon_\rho). \quad (15)$$

The innovations to the aggregate factors are the structural innovations in our model.

Aggregate Weight Shocks The ratio my governs how much consumption can be transferred from good states to bad states by the planner. If this ratio is high enough, the planner can sustain perfect risk sharing. On the other hand, if this ratio is low enough, the planner cannot improve upon the autarchic outcome. The collateral ratio shifts the conditional distribution of tomorrow's Pareto-Negishi weights.

If the aggregate factors F^a are i.i.d., the weight shock g_{t+1} only depends on my_{t+1} (Lustig (2001)). In general, the aggregate weight shock can depend on the entire aggregate history of the economy $(F^a)^{t+1}$. If the constraints bind enough, the history dependence dies out rather quickly. We chose to impose a Markov structure on the aggregate weight shocks $\{g\}$.

We propose the following expression for $G_{t+1}(my_{t+1}, F_{t+1}^a)$:

$$\log G_{t+1} = my_{t+1} (B_1 + B_2 \Delta \log c_{t+1} + B_3 A_t \Delta \log \rho_{t+1}) + \varepsilon_{t+1}, \quad (16)$$

where B is a vector of constants and, by the Markov assumption, $E_t[my_{t+1}\varepsilon_{t+1}] = E_t[my_{t+1}\Delta \log c_{t+1}\varepsilon_{t+1}] = E_t[my_{t+1}A_t\Delta \log \rho_{t+1}\varepsilon_{t+1}] = 0$.

We can test for history dependence by including F_{t-k}^a for $k \geq 0$ on the right hand side.

Linear Factor Model The factor model for the weight shocks and the autoregressive process for my provide a complete description of the pricing model. By combining $G_{t+1}(my_{t+1}, F_{t+1}^a)$ and $my_{t+1}(my_t, F_{t+1}^a)$, the stochastic discount factor in (11) can be stated in terms of aggregate factors F_{t+1}^a and the state variable my_t : $m_{t+1}(my_t, F_{t+1}^a)$. A first-order Taylor approximation of this expression delivers our linear factor model:

$$m_{t+1} \approx \delta(1 - \theta^a F_{t+1}^a - \theta^c F_{t+1}^c - \gamma B_1 C \bar{F}^a - \gamma (B_2, B_3) F_{t+1}^a F_{t+1}^{a'} C' - \gamma \varepsilon_{t+1}), \quad (17)$$

where the representative agent factors F_{t+1}^a and constraint factors F_{t+1}^c are:

$$\begin{aligned} F_{t+1}^a &= (\Delta \log(c_{t+1}), A_t \Delta \log(\rho_{t+1}))' \\ F_{t+1}^c &= (my_t, my_t \Delta \log c_{t+1}, my_t A_t \Delta \log \rho_{t+1})', \end{aligned}$$

with associated factor loadings

$$\begin{aligned} \theta^a &= (\gamma + \gamma B_2 C \bar{F}^a, \phi + \gamma B_3 C \bar{F}^a) \\ \theta^c &= (\gamma \rho B_1, \gamma \rho B_2, \gamma \rho B_3) \end{aligned}$$

The constraint factors interact the aggregate factors F_{t+1}^a with the state variable my_t . We impose two additional restrictions. First, we assume $E_t \left[(F_{t+1}^a F_{t+1}^{a'})_{kl} \tilde{R}_{t+1}^{e,j} \right] = 0$ for $k, l = 1, 2$, where $\tilde{R}_{t+1}^{e,j}$ denotes the (mean-zero) unexpected returns.³ This assumption and the law of iterated expectations imply that the quadratic terms in F^a will drop out in the unconditional beta-representation (25). Second, we assume $E_t \left[\varepsilon_{t+1} R_{t+1}^{j,e} \right] = 0$. By the law of iterated expectations this implies $E \left[\varepsilon_{t+1} R_{t+1}^{j,e} \right] = 0$.

Case 1: Separable Preferences When utility is separable, the equity risk premium is determined by the conditional covariance of its returns with consumption growth and a state-varying market price of risk:

$$E_t \left[R_{t+1}^{e,j} \right] \approx \gamma \left[1 + B_2 C \bar{F}^a + \rho B_2 my_t \right] Cov_t \left(\Delta \log c_{t+1}, R_{t+1}^{e,j} \right) \quad (18)$$

If B_2 is zero, the expression collapses to the standard CCAPM of Lucas (1978) and Breeden (1979). The market price of consumption risk is constant and equal to the coefficient of relative risk aversion γ . In contrast, our theory predicts an increase in the size of the aggregate weight shock when aggregate consumption growth is low, driven by an increase in idiosyncratic risk. Consumption growth has an effect on the liquidity shock: $B_2 < 0$. When my is low, the market price of consumption risk is high.

Case 2: Non-Separable Preferences Non-separability introduces a second covariance in the risk premium equation: the covariance with rental price changes. Under complementarity of nondurable consumption and housing ($\phi > 0$), households want to hedge by investing in assets that deliver high returns when consumption is scarce, that is when the rental price of housing services increases. This hedging risk is the focus of recent work by Piazzesi et al. (2002).

If B_3 is zero, the market price of rental price risk is constant and equal to the degree of complementarity between consumption and housing services in the utility function ϕ . In contrast, if $B_3 < 0$, the market price of rental price risk is high when my is low.

³Conditional joint normality of returns and aggregate factors is sufficient for this assumption.

4.2 Regional Factors

The second economy has housing market frictions. We use regions as the unit of analysis and directly work with the region-specific weights instead of the aggregate weight shock. Let $F_{t+1}^i = \left(\Delta \log(\hat{y}_{t+1}^i), \hat{A}_t^i \Delta \log(\hat{\rho}_{t+1}^i) \right)'$ denote the vector of regional factors: income growth and scaled rental price growth for region i in deviation from the cross-region mean.

To derive a linear pricing model $m_{t+1}(my_t, F_{t+1}^a, F_{t+1}^i)$, we *propose* a factor model for the regional weight shocks $G_{t+1}^i(my_{t+1}, F_{t+1}^i)$. The housing collateral ratio my_t is the conditioning variable.

Definition. *In the second economy, a complete description of the linear pricing model $m_{t+1}(my_t, F_{t+1}^a, F_{t+1}^i)$ consists of (1) a specification for the regional weight shocks $G_{t+1}^i(my_{t+1}, F_{t+1}^i)$ and (2) a process for the aggregate housing collateral ratio $my_{t+1}(my_t, F_{t+1}^a)$.*

Consumption Euler Equation Regional consumption is a non-linear function of the Pareto-Negishi weights. This function satisfies the first order condition of the planner problem and the resource constraint. We define the log consumption share $\log(\hat{c}_t^i)$ as the deviation of log consumption for agent i from the cross-region mean. Using the planner's first order condition we approximate the region-specific component of consumption growth as:

$$\Delta \log(\hat{c}_t^i) \approx \frac{1}{\gamma} \Delta \log(\hat{\xi}_t^i) - \frac{\phi}{\gamma} \hat{A}_{t-1}^i \Delta \log(\hat{\rho}_t^i) + \frac{1}{\gamma} \nu_t^i, \quad (19)$$

where ν_t^i is a taste shock residual and A_t^i is the expenditure share of housing services.⁴ This is an approximation because the nonlinear term in the utility function is expanded around $\rho_t^i = \rho_{t-1}^i$. The details of this derivation are in appendix A.2.

Region-specific consumption growth is driven by region-specific weight shocks, region-specific rental price changes weighted by the housing expenditure share A^i and taste shocks for housing and non-durable consumption summarized in ν .

Regional Weight Shocks There are two polar cases for which the Pareto-Negishi weight shock is known explicitly : (1) the autarkic outcome and (2) the perfect risk-sharing outcome. The housing collateral ratio shifts the allocations between these two outcomes.

We use my^{\min} to denote the housing collateral ratio below which only the autarkic allocations can be sustained, and we use my^{\max} to denote the collateral ratio above which perfect risk-sharing can be sustained.

First, when $my_t < my^{\min}$, the planner chooses the autarkic allocations and region-specific consumption growth equals region-specific income growth: $\Delta \log(\hat{c}_{t+1}^i) = \Delta \log(\hat{y}_{t+1}^i)$. Using equation

⁴In particular: $\nu_t^i = (1 - \gamma) \Delta \hat{b}_t^{i,c} - \phi \hat{A}_{t-1}^i \Delta \hat{b}_t^{i,c} + \phi \hat{A}_{t-1}^i \Delta \hat{b}_t^{i,h}$ and $A_t^i = \frac{\rho_t^i h_t^i}{c_t^i + \rho_t^i h_t^i}$.

(19), the region-specific weight is given by:

$$\Delta \log \left(\hat{\xi}_{t+1}^{i,ub} \right) \approx \gamma \Delta \log \left(\hat{y}_{t+1}^i \right) + \phi \hat{A}_t^i \Delta \log \left(\hat{\rho}_{t+1}^i \right) - \nu_{t+1}^i.$$

This is the upper bound (*ub*) on size of the weight shock $\Delta \log \left(\hat{\xi}_{t+1}^i \right)$.

Second, when $my_t > my^{\max}$, the planner can implement the perfect risk-sharing allocations and the region-specific weight shock is zero:

$$\Delta \log \left(\hat{\xi}_{t+1}^{i,lb} \right) = 0.$$

This is the lower bound (*lb*) on the size of the weight shock $\Delta \log \left(\hat{\xi}_{t+1}^i \right)$.

The housing collateral ratio shifts these weight shocks between the upper bound and the lower bound. We restrict this shifting effect of my to be *linear*. We propose this simple factor model for the regional weight shocks $\Delta \log \left(\hat{\xi}_{t+1}^i \right)$:

$$G_{t+1}^i(F_{t+1}^i, my_{t+1}) \approx \gamma (\beta_1 - \beta_2 my_{t+1}) \Delta \log \left(\hat{y}_{t+1}^i \right) + \phi (\beta_1 - \beta_2 my_{t+1}) \hat{A}_t^i \Delta \log \left(\hat{\rho}_{t+1}^i \right) - \nu_{t+1}^i, \quad (20)$$

where the coefficients β_1 and β_2 are functions of the housing collateral ratios my^{\max} and my^{\min} :

$$\beta_1 = \frac{my^{\max}}{my^{\max} - my^{\min}}, \quad \beta_2 = \frac{1}{my^{\max} - my^{\min}}.$$

Equation (20) expresses the region-specific weight shocks as a linear function of region-specific income growth, rental price growth and the interaction terms with the housing collateral ratio.

Without the linearity assumption, a constrained region's weight shock $\Delta \log \left(\hat{\xi}_{t+1}^i \right)$ depends on its individual history $(F^i)^t$. For an unconstrained region, the weight shock cannot depend on its own history because all unconstrained regions equate their marginal utility growth. Because we impose the same weight shock process (20) for constrained and unconstrained regions, the Markov structure of the unconstrained weight shocks implies a Markov structure for the constrained weight shocks as well: $\Sigma_{i_{constr}} \Delta \log \left(\hat{\xi}_{t+1}^i \right) = -\Sigma_{i_{unconstr}} \Delta \log \left(\hat{\xi}_{t+1}^i \right)$. By virtue of this symmetry, the relative regional weight processes, $G_{t+1}^i(my_{t+1}, F_{t+1}^i)$ are Markov.

Linear Factor Model By combining $G_{t+1}^i(my_{t+1}, F_{t+1}^i)$ and $my_{t+1}(my_t, F_{t+1}^a)$ from equation (15), the stochastic discount factor in (14) can be stated in terms of aggregate factors F_{t+1}^a , the regional factors F_{t+1}^i and the state variable my_t : $m_{t+1}(my_t, F_{t+1}^a, F_{t+1}^i)$. We use i_{t+1}^* to denote the region with the smallest weight shocks: $\min_i \{G_{t+1}^i\}$. A first-order Taylor approximation delivers the linear pricing model:

$$m_{t+1} \approx \delta(1 - \theta^a F_{t+1}^a - \theta^{lc} F_{t+1}^{lc} + \gamma \beta_2 C(F_{t+1}^a - \bar{F}^a)) \Delta \log \left(\hat{y}_{t+1}^i \right) + \phi \beta_2 C(F_{t+1}^a - \bar{F}^a) \hat{A}_t^i \Delta \log \left(\hat{\rho}_{t+1}^i \right) + \nu_{t+1}^i. \quad (21)$$

where the representative agent and the constraint factors are:

$$\begin{aligned} F_{t+1}^a &= (\Delta \log^a(c_{t+1}), A_t^a \Delta \log^a(\rho_{t+1}))' \\ F_{t+1}^c &= \left(\Delta \log \left(\hat{y}_{t+1}^{i*} \right), my_t \Delta \log \left(\hat{y}_{t+1}^{i*} \right), \hat{A}_t^{i*} \Delta \log \left(\hat{\rho}_{t+1}^{i*} \right), my_t \hat{A}_t^{i*} \Delta \log \left(\hat{\rho}_{t+1}^{i*} \right) \right)', \end{aligned}$$

with associated factor loadings

$$\begin{aligned} \theta^a &= (\gamma, \phi) \\ \theta^c &= (\gamma \beta_1, -\gamma \rho \beta_2, \phi \beta_1, -\phi \rho \beta_2) \end{aligned}$$

We impose two restrictions. First, we assume $E_t \left[(F_{t+1}^a - \bar{F}^a) \Delta \log \left(\hat{y}_{t+1}^{i*} \right) \tilde{R}_{t+1}^{e,j} \right] = 0$ and $E_t \left[(F_{t+1}^a - \bar{F}^a) \hat{A}_t^{i*} \Delta \log \left(\hat{\rho}_{t+1}^{i*} \right) \tilde{R}_{t+1}^{e,j} \right] = 0$.⁵ This assumption and the law of iterated expectations imply that the quadratic terms drop out when testing (25). Second, we assume $E_t \left[\nu_{t+1}^{i*} R_{t+1}^{j,e} \right] = 0$. By the law of iterated expectation, $E \left[\nu_{t+1}^{i*} R_{t+1}^{j,e} \right] = 0$.

Constraint Risk The household is compensated for shocks to the Pareto-Negishi weights that are correlated with returns. This collateral constraint risk is captured by the four constraint factors.

Case 1: Separable Preferences When utility is separable, the equity risk premium is determined by the conditional covariance of returns with average consumption growth and with the growth rate in the income share of the unconstrained region:

$$E_t \left[R_{t+1}^{e,j} \right] \approx \gamma Cov_t \left(\Delta \log^a c_{t+1}, R_{t+1}^{e,j} \right) + \gamma [\beta_1 - \rho \beta_2 my_t] Cov_t \left(\Delta \log \left(\hat{y}_{t+1}^{i*} \right), R_{t+1}^{e,j} \right) \quad (22)$$

The latter risk factor has a state-varying market price of risk. If β_1 and β_2 are zero, only risk associated with average consumption growth is priced. When $\beta_1 > 0$ a decrease in the unconstrained region's income share decreases her relative weight shock. This increases the equity risk premium. If $\beta_2 > 0$, the market price of constraint risk associated with $\Delta \log \left(\hat{y}_{t+1}^{i*} \right)$ is high when my is low.

Case 2: Non-Separable Preferences When utility is non-separable, the equity risk premium is also determined by the conditional covariance of returns with average rental price growth and with the rental price growth of the unconstrained region. If β_1 and β_2 are zero, only rental price risk associated with average rental price changes is compensated. When $\phi > 0$ (complementarity) and $\beta_1 > 0$, our theory predicts a decrease in the unconstrained region's weight shock when her relative rental price growth is low. This increases the equity risk premium. If $\phi > 0$ and $\beta_2 > 0$, the market price of constraint risk associated with $\hat{A}_t^{i*} \Delta \log \left(\hat{\rho}_{t+1}^{i*} \right)$ is high when my is low.

⁵Conditional joint normality of asset returns and the factors F_{t+1}^a , $\Delta \log \left(\hat{y}_{t+1}^{i*} \right)$ and $\hat{A}_t^{i*} \Delta \log \left(\hat{\rho}_{t+1}^{i*} \right)$ is sufficient.

4.3 Unconditional β -Representation

To summarize, the discount factor is decomposed into a representative agent and a constraint component:

$$m_{t+1} = -\theta F_{t+1}, \quad (23)$$

where θ is a vector of constants, $\theta = (1, \tilde{\theta})$ and $\tilde{\theta} = (\theta^a, \theta^c)$ and $F_{t+1} = (1, \tilde{F}_{t+1})$. $\tilde{F}_{t+1} = (F_{t+1}^a, F_{t+1}^c)'$ is a vector of representative agent and constraint pricing factors. The aggregate and regional models each have a distinct set of factors. Each factor has a macroeconomic interpretation.

If θ were time-varying, the conditional orthogonality conditions in (10) would not imply unconditional orthogonality conditions. Here, the vector of constraint factors contains the original factors scaled by the housing collateral ratio my_t . my_t is the conditioning variable that summarizes the investor's information set. The stochastic discount factor contains the conditioning information through the scaled constraint factors. The model (23) can be tested using the unconditional orthogonality conditions of the discount factor and excess asset returns j :

$$E \left[m_{t,t+1} R_{t+1}^{e,j} \right] = 0. \quad (24)$$

Using the definition of the risk-free rate and the covariance, the unconditional factor model in (23) implies an unconditional β -representation:

$$E \left[R_{t+1}^{e,j} \right] = \delta \bar{R}^f \tilde{\theta} Cov \left(\tilde{F}_{t+1}, R_{t+1}^{e,j} \right) = \tilde{\lambda} \tilde{\beta}^j, \quad (25)$$

where \bar{R}^f is the average risk-free rate, $\tilde{\beta}^j$ is asset j 's risk exposure $\tilde{\beta}^j = Cov \left(\tilde{F}, \tilde{F}' \right)^{-1} Cov \left(\tilde{F}, R^{e,j} \right)$ and $\tilde{\lambda}$ is a transformation of the parameter vector $\tilde{\theta}$: $\tilde{\lambda} = \delta \bar{R}^f \tilde{\theta} Cov \left(\tilde{F}, \tilde{F}' \right)$.⁶ The unconditional β -representation in (25) is the restriction we test in section 7.2.

5 Data

In the empirical section (section 7) we use three sets of variables: financial variables, aggregate macroeconomic variables and regional macroeconomic variables. All variables are annual and for the United States.

5.1 Financial Data

In a first time-series exercise we just use the return on the aggregate stock market. In a second exercise we use a cross-section of stock portfolios, sorted by size and value characteristics.

⁶Lettau and Ludvigson (2001b) point out that $\tilde{\lambda}$ does not have a straightforward interpretation as the vector of market prices of risk. The market prices of risk λ depend on the conditional covariance matrix of factors which is unobserved.

Market Return The market return is the cum-dividend return on the Standard and Poor's composite stock price index. The market return is expressed in excess of a risk-free rate, the annual return on six-month prime commercial paper. The returns are available for the period 1889-2001 from Robert Shiller's web site.

Size and Book-to-Market Portfolios We use twenty-five portfolios of NYSE, NASDAQ and AMEX stocks, grouped each year into five size bins and five value (book-to-market ratio) bins. Size is market capitalization at the end of June. Book-to-market is book equity at the end of the prior fiscal year divided by the market value of equity in December of the prior year. Portfolio returns are value-weighted. We also include the market return R^{vw} , the value-weighted return on all NYSE, AMEX and NASDAQ stocks. We refer to this set of 26 test assets as $T1$. All returns are expressed in excess of an annual return on a one-month Treasury bill rate (from Ibbotson Associates). The returns are available for the period 1926-2001 from Kenneth French's web site and are described in more detail in Fama and French (1992). The first column of table 15 shows mean and standard deviation for the 26 excess returns in $T1$.

5.2 Aggregate Macroeconomic Data

Price Indices Aggregate rental prices ρ_t are constructed as the ratio of the CPI rent component p_t^h and the CPI food component p_t^c . Data are for urban consumers from the Bureau of Labor Statistics for 1926-2001. The price of rent is a proxy for the price of shelter and the price of food is a proxy for the price of non-durables. We use the rent and food components because the shelter and non-durables components are only available from 1967 onwards. Two-thirds of consumer expenditures on shelter consists of owner-occupied housing. The BLS uses a rental equivalence approach to impute the price of owner-occupied housing. Because ρ_t is a relative *rental* price, our theory is conceptually consistent with the BLS approach. We also use the all items CPI, p_t^a , which goes back to 1889. All indices are normalized to 100 for the period 1982-84.

Housing Collateral We use three distinct measures of the housing collateral stock HV : the value of outstanding home mortgages (mo), the market value of residential real estate wealth (rw) and the value of the owner-occupied and tenant occupied residential fixed assets (fa). The first two time series are from the Historical Statistics for the US (Bureau of the Census) for the period 1889-1945 and from the Flow of Funds data (Federal Board of Governors) for 1945-2001. The last series is from the Fixed Asset Tables (Bureau of Economic Analysis) for 1925-2001. The rw and fa -series differ by the value of land. Appendix A.5 provides detailed sources. Real per household variables are denoted by lower case letters. The real, per household housing collateral series, hv , is constructed using the all items CPI from the BLS, p^a , and the total number of households, N , from the Bureau of the Census.

Consumption and Income Consumption is non-durable consumption C , measured by expenditures on food and apparel. Food and apparel are the only two items for which we have data prior to 1930. This is unproblematic because the correlation between the growth rate in real per household consumption on food and apparel and the growth rate in real per household consumption of non-durables and services excluding housing services is 0.97 for the period 1930-2001.

The housing expenditure share, A , is the ratio of rent expenditures to the sum of expenditures on rent, food and apparel.

The income endowment in the model corresponds to an after-government income concept; it includes net transfer income. Aggregate income Y is labor income plus net transfer income. Nominal data are from the Historical Statistics of the US for 1926-1930 and from the National Income and Product Accounts for 1930-2001. Consumption and income are deflated by p^c and p^a and divided by the number of households N .

5.3 Regional Macroeconomic Data

To estimate the model with housing frictions (section 7.2.3) and to estimate the consumption Euler equation (section 7.3), we construct a panel data set for US regions. In particular, we collect data for the thirty largest metropolitan areas. Thirteen of the regions are metropolitan statistical areas (MSA). The other seventeen are consolidated metropolitan statistical areas (CMSA), comprised of adjacent and integrated MSA's. The regions combine for 47 percent of the US population. The metropolitan data are annual for 1951-2001. Most CMSA's did not exist at the beginning of the sample. For consistency we keep track of all constituent MSA's and construct a population weighted average for the years prior to formation of the CMSA (see table 10 in appendix A.5).

Price Indices The CPI all items index $p_t^{i,a}$, the rent component $p_t^{i,h}$ and the food component $p_t^{i,c}$ are available at the metropolitan level (BLS).

Figure 9 (at the end of the main text) shows the evolution of the relative rental price $\rho_t^i = p_t^{i,h}/p_t^{i,c}$ for the Bay Area, St.-Louis and the US average. The Bay Area and St.-Louis have the most divergent rental prices among all regions in our sample. The plot reveals a large and slow-moving *common* component in relative rental prices.

Consumption and Income Inter-regional risk-sharing studies use retail sales data as a proxy for non-durable consumption (DelNegro (1998) and references therein). We collect retail sales data from the annual Survey of Buying Power published by Sales & Marketing Management (S&MM). Non-durable consumption for region i , C_t^i , is total retail sales minus hardware and furniture sales and vehicle sales. From the same source we obtain the number of households in each region, N_t^i . Real per household consumption c^i is nominal non-durable consumption deflated by $p_t^{i,c}$ and divided by the number of households N_t^i .

Because we have very limited data for the housing expenditure share at the regional level, we assume that the regional expenditure share equals the aggregate one: $A_t^i = A_t, \forall t$.

Disposable personal income Y_t^i is also from S&MM. Disposable personal income consists of labor income, financial market income and net transfers. The latter two contain a potentially important insurance component. Therefore we also use labor income plus net transfers from the Regional Economic Information System.

Appendix A.5 compares non-durable retail sales and disposable income with aggregate consumption and income data (Table 12), with metropolitan non-durable consumption data from the Consumption Expenditure Survey (BLS, 1986-2000, Table 13) and with metropolitan labor income data plus transfers from the REIS for 1969-2000 (Table 14). The correlation between the growth rates of aggregate real non-durable consumption per household and the metropolitan average of real non-durable retail sales per household is 0.77. Also, our metropolitan data are highly correlated with the metropolitan data from the BLS and the REIS.

There are no complete CPI data for Baltimore, Buffalo, Phoenix, Tampa and Washington. There are no complete consumption and income data for Anchorage. Elimination of these regions leaves us with annual data for 23 metropolitan regions from 1951 until 2001. This is the regional data set we use in sections 7.2.3 and 7.3.

6 Measuring the Housing Collateral Ratio

This section measures the new state variable, the housing collateral ratio my . my is defined as the ratio of collateralizable housing wealth to non-collateralizable human wealth. Human wealth is unobserved. Following Lettau and Ludvigson (2001a), we assume that the non-stationary component of human wealth H is well approximated by the non-stationary component of labor income Y . In particular, $\log(H_t) = \log(Y_t) + \epsilon_t$, where ϵ_t is a stationary random process. The assumption is valid in a model in which the expected return on human capital is stationary (see Jagannathan and Wang (1996) and Campbell (1996)).

Cointegration Log, real, per household real estate wealth ($\log hv$) and labor income plus transfers ($\log y$) are non-stationary. According to an augmented Dickey-Fuller test, the null hypothesis of a unit root cannot be rejected at the 1 percent level. This is true for all three measures of housing wealth ($hv = mo, rw, fa$).

If a linear combination of $\log hv$ and $\log y$, $\log(hv_t) + \varpi \log(y_t) + \chi$, is trend stationary, the components $\log hv$ and $\log y$ are said to be stochastically cointegrated with cointegrating vector $[1, \varpi, \chi]$. We additionally impose the restriction that the cointegrating vector eliminates the deterministic trends, so that $\log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi$ is stationary. A likelihood-ratio test (Johansen and Juselius (1990)) shows that there is one cointegration relationship between housing collateral and labor income plus transfers. Table 2 reports the results of this test and of the vector error correction estimation of the cointegration coefficients:

$$\begin{bmatrix} \Delta \log(hv_t) \\ \Delta \log(y_t) \end{bmatrix} = \alpha [\log(hv_t) + \varpi \log(y_t) + \vartheta t + \chi] + \sum_{k=1}^K D_k \begin{bmatrix} \Delta \log(hv_{t-k}) \\ \Delta \log(y_{t-k}) \end{bmatrix} + \varepsilon_t. \quad (26)$$

The K error correction terms are included to eliminate the effect of regressor endogeneity on the distribution of the least squares estimators of $[1, \varpi, \vartheta, \chi]$. The housing collateral ratio my is measured as the deviation from the cointegration relationship:

$$my_t = \log(hv_t) + \hat{\varpi} \log(y_t) + \hat{\vartheta}t + \hat{\chi}.$$

The OLS estimators of the cointegration parameters are superconsistent: They converge to their true value at rate $1/T$ (rather than $1/\sqrt{T}$). The superconsistency allows us to use the housing collateral ratio my as a regressor without need for an errors-in-variables standard error correction (see section 7).

The housing collateral ratios for the three housing collateral measures are labelled $mymo$, $myrw$ and $myfa$. The constructed housing collateral ratio is stationary. The null hypothesis of a unit root is rejected at the five percent level for $mymo$ and $myfa$ and at the ten percent level for $myrw$. For the common sample period 1925-2001, the correlation between $mymo$ and $myrw$ is 0.85, 0.75 between $mymo$ and $myfa$ and 0.83 between $myrw$ and $myfa$.

<i>mymo</i>	ϖ	ϑ	χ	<i>LHR</i>	<i>K</i>
1889-2001	-.958**	-.015**	1.88	27.6*	7
1946-2001	-.489	-.023**	2.13	44.8**	8
<i>myrw</i>	ϖ	ϑ	χ	<i>LHR</i>	<i>K</i>
1889-2001	-.576**	-.009**	-.124	42.13**	18
1946-2001	-.691**	-.010**	.075	28.28*	8
<i>myfa</i>	ϖ	ϑ	χ	<i>LHR</i>	<i>K</i>
1925-2001	-.907**	-.001	-.234	35.47**	6
1946-2001	-.854**	-.002*	-.122	30.35*	9

Table 2: Cointegration Relationship. The second through fourth columns show coefficient estimates for the cointegration relationship. Significance at the 5% level is denoted by a *, significance at the 1 % level by **. The fifth column shows the likelihood ratio statistic of the Johansen cointegration test. It assumes a linear trend in the data, a constant and a trend in the cointegration relationship. The 5% critical value is 25.32, the 1% critical value is 30.45. The last column shows the number of error correction terms in the regression. They are chosen on the basis of a likelihood ratio test. The first panel is for the log real value of outstanding mortgages per household *mo*. The second panel is for the log real market value of real estate per household *rw*. The third panel is for the log real value of owner- and tenant-occupied housing for non-farm persons *fa*. All cointegration relationships are estimated with log real per capita labor income plus net transfers income y . The cointegration relationship is estimated for the entire sample period (1889-2001 for *mo* and *rw* and 1925-2001 for *fa*) and for the post-war subsample. Coefficient estimates for D_k are not reported.

Figure 1 displays my between 1889 and 2001. All three series exhibit large persistent swings, especially in the two decades between 1925-45. They reach a maximum deviation in 1932-33. Mortgage debt is 83 percent above its joint trend with human wealth. Land and structures and residential fixed asset wealth are 40 percent and 47 percent above their joint trends with human wealth. The series reach a minimum in 1944, when $mymo$ is $-.80$, $myrw$ is $-.43$ and $myfa$ is $-.29$.

The second row of each panel of Table 2 reports error-correction estimations for the subsample post-1946. Figure 2 shows the cointegration residuals my for that post-war period. Residential wealth and fixed assets residuals $myrw$ and $myfa$ are plotted against the right axis, mortgage residuals $mymo$ against the right axis. Housing collateral wealth fluctuates within 30 percent

Figure 1: Housing Collateral Ratio 1889-2001. Measured by outstanding home mortgages (mo), non-farm residential wealth (rw) and residential fixed asset wealth (fa) relative to human wealth (y)

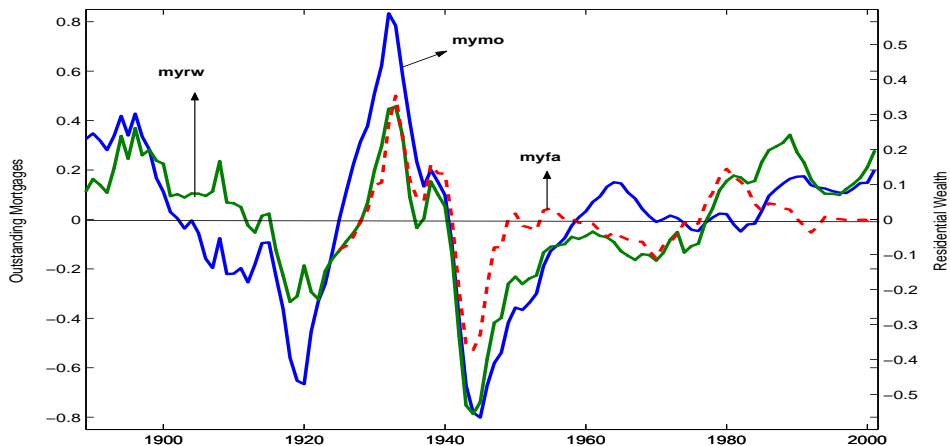
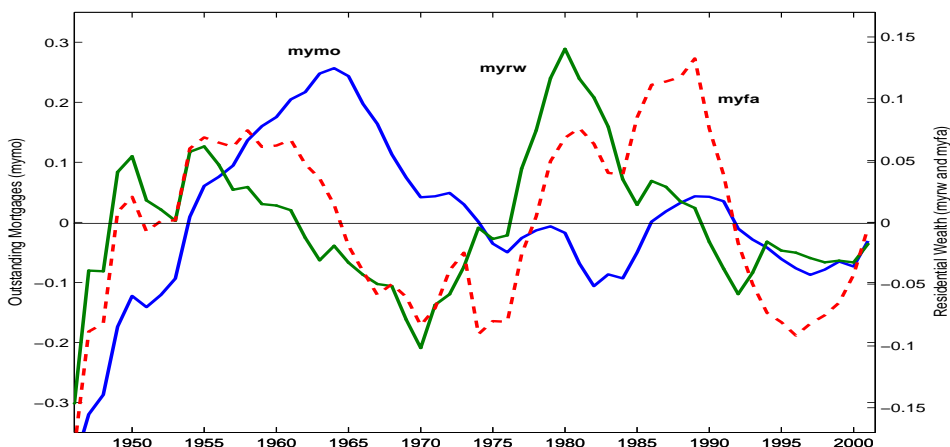


Figure 2: Housing Collateral Ratio 1946-2001.



below and above the long-run trend with human wealth.

7 Empirical Evidence of the Collateral Effect

We address two empirical failures of the CCAPM. In section 7.1 we provide evidence that the housing collateral ratio predicts stock returns. This suggests that the market price of risk is not a constant but a function of my . Second, in contrast to the CCAPM, our model can help account for a large fraction of the cross-sectional variation in size and book-to-market portfolio returns (section 7.2). The failure of the CCAPM model is not surprising because it relies on perfect

consumption insurance. In section 7.3 we provide evidence that consumption growth is imperfectly correlated across US metropolitan areas. Furthermore, we show that cross-sectional correlation of consumption growth is higher when the housing collateral ratio my is high. This time variation in risk-sharing is direct evidence for the mechanism that drives our model and underlies the asset pricing results.

7.1 Time-Series Predictability

The model generates predictable variation in risk premia on stocks. The reward for risk is higher when housing collateral is scarce. We find empirical support for this negative relationship.

Many financial and macroeconomic variables have forecasting power for the market return.⁷ A subset of those variables, such as the investment-capital ratio (Cochrane (1991a)), the consumption-wealth ratio (Lettau and Ludvigson (2001a)) and the labor income - consumption ratio (Santos and Veronesi (2001)), are macroeconomic variables. These variables are correlated with or forecast the business cycle. In contrast, the housing collateral ratio is a more low-frequency variable. A spectral decomposition reveals that at least two-thirds of the variation in the housing collateral ratio is situated at horizons longer than 20 years. The power spectrum in figure 10 (at the end of the text) reaches its peak at frequencies below $2\pi/20$. As for the cyclical properties of my , the spectrum displays a smaller hump at $\pi/4$, a frequency associated with a long recession (8 years). Figure 11 adds NBER recession dates to figure 1. In many episodes, the collateral ratio increases until after the start of the recession and only starts to decline near the end.

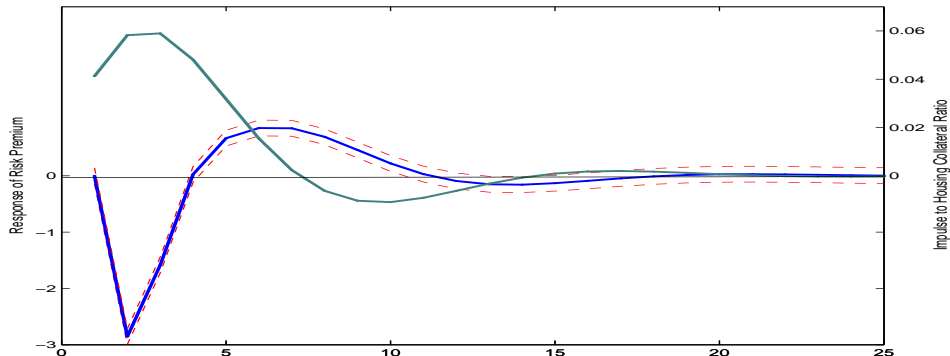
VAR A vector autoregression provides a first window on the data generating process for one-year excess returns and the housing collateral ratio. We study the response of excess returns to an innovation to my . Figure 3 shows the negative response of the excess return to an orthogonal innovation in $myfa$, the my measure for fixed assets. The initial drop in the equity risk premium is followed by a further decrease which persists for multiple years.⁸ The effect is large: A 4 percentage point innovation to $myfa$ causes a 2.9 percentage point decrease in the equity risk premium. In 1942, $myfa$ declined by 20 percentage points in one year. The impulse response estimates suggest a 14 percentage point increase in the risk premium. Figures 12 and 13 (at the end of the text) show a similar pattern for the other two measures of the housing collateral ratio.

Long Horizon Predictability To illustrate the economic effect of return forecastability over a longer period, we study long-horizon excess returns. We define the K -year continuously compounded excess return as $r_{t+K}^{vw,K} = (r_{t+1}^{vw,1} + \dots + r_{t+K}^{vw,1})$ where $r_t^{vw,1}$ equals $\log(1 + R_t^{vw,e})$. Figure

⁷The dividend-price and dividend-earnings ratio, the treasury bill rate, the term spread between long-term government bonds and treasury bills and the default spread between low- and high-grade corporate bonds are financial variables with forecasting power for excess returns. See Cochrane (2001), Ch. 20 for an overview.

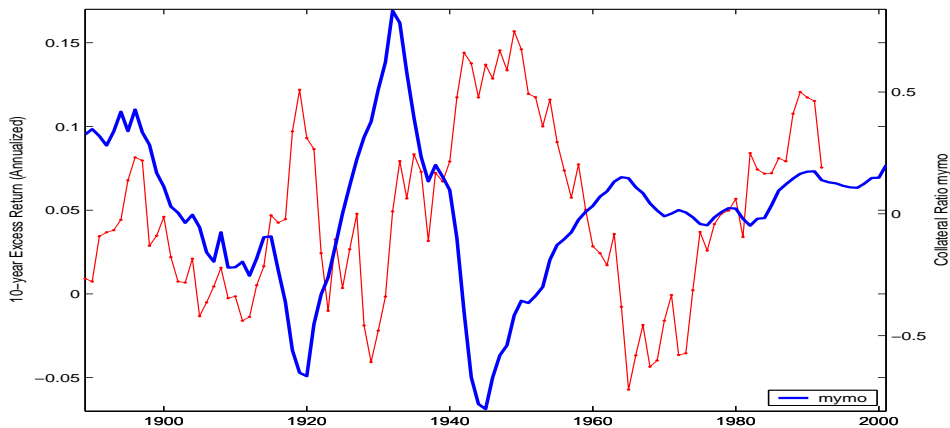
⁸The optimal lag length for the VAR is two years according to the Aikake Information criterion. The covariance matrix of innovations has small off-diagonal elements, i.e. their innovations have a small common component. Therefore, changing the ordering of the variables $R^{vw,e}$ and my in the VAR does not affect the impulse-responses.

Figure 3: Response of the One-Year Excess Return to Impulse in Collateral Ratio $myfa$. The dashed lines represent one standard error above and below the response graphs, generated by Monte Carlo simulation (5,000 repetitions)



4 shows the housing collateral ratio ($mymo$) and the annualized ten-year excess return. The series exhibit a negative correlation of -0.51 . Regressions of the one- to ten-year cumulative stock returns on the housing collateral ratio ($mymo$) provide further evidence of predictability.

Figure 4: 10-year Excess Market Return on the Collateral ratio $mymo$.



There are two econometric problems with the ordinary least squares regression:

$$r_{t+K}^{vw,K} = b_0 + b_{my}my_t + e_{t+1}. \quad (27)$$

First, because the forecasting variable my is a slow-moving process, the least squares estimator of the coefficient on my , b_{my}^{LS} , suffers from persistent regressor bias in small samples (Stambaugh (1999)). Second, because $r_{t+K}^{vw,K}$ contains overlapping observations, the standard errors on b_{my}^{LS} need to be corrected for serial correlation in the residuals e . Asymptotic corrections as advocated by Hansen and Hodrick (1980) have poor small sample properties. Ang and Bekaert (2001) find that use of those standard errors leads to over-rejection of the no-predictability null. To address the

persistent regressor bias and the serial correlation issues we conduct a bootstrap exercise, detailed in appendix A.3.

The first row of table 3 shows the least squares coefficient estimate on my for the period 1889-2001. The fourth row contains the estimates for the postwar period. All coefficients on the housing collateral ratio are negative: A negative housing collateral ratio predicts high future risk premia. The R^2 of the least-squares regression increases with the horizon, to 54 percent in the postwar period (second row). The third row in each panel reports the small-sample coefficient estimates, generated by bootstrap. With few exceptions, the bias is small. At every horizon, the coefficient estimates remain negative. The fourth row reports the p -value of a two-sided test of no predictability, generated by bootstrap. It measures the likelihood of observing the least squares coefficient estimate when returns are in fact unpredictable. For $K \geq 5$, there is evidence against the null-hypothesis at the 10 percent level.

1889-2001	1	2	3	4	5	6	7	8	9	10
b_{my}^{LS}	-.06	-.12	-.17	-.23	-.31	-.39	-.46	-.52	-.56	-.57
R^2	.01	.02	.03	.04	.06	.08	.11	.13	.13	.12
b_1 small sample	-.06	-.13	-.09	-.17	-.48	-.40	-.34	-.53	-.56	-.56
p -value	[.17]	[.18]	[.28]	[.22]	[.09]	[.05]	[.11]	[.04]	[.05]	[.08]
1946-2001	1	2	3	4	5	6	7	8	9	10
b_{my}^{LS}	-.18	-.47	-.80	-1.14	-1.41	-1.69	-2.01	-2.39	-2.68	-2.94
R^2	.03	.08	.19	.27	.30	.33	.39	.46	.50	.54
b_1 small sample	-.22	-.50	-.79	-.81	-.87	-1.09	-1.80	-2.33	-2.65	-2.81
p -value	[.33]	[.20]	[.17]	[.13]	[.24]	[.26]	[.07]	[.09]	[.06]	[.06]

Table 3: Long-Horizon Predictability Regressions. The results are for the regression $R_{t+K}^{vw,e} = b_0 + b_{my}my_t + \epsilon_{t+K}$, where $R_{t+K}^{vw,e}$ are cumulative excess returns on the value weighted market portfolio over a K -year horizon. The housing collateral ratio my is $mymo$. The first row reports least squares estimates for b_{my} . The second row reports the R^2 for this regression. The third row reports small sample coefficient estimates generated by bootstrap (see A.3). The fourth row gives the p -value of the null hypothesis of no predictability, also generated by bootstrap. The second panel shows results for the postwar period using the postwar housing collateral ratio $mymo$.

7.2 Cross-Sectional Results

Size and book-to-market value are asset characteristics that challenge the standard CCAPM. Historically, small firm stocks and high book-to-market firm stocks have higher returns. In the post-war period, the size premium has largely disappeared, but the value premium is still prominent. The CCAPM yields large pricing errors on book-to-market stocks: This is the value premium puzzle. The new asset pricing factors in our model substantially improve the fit of the cross-section of returns. The average pricing errors are cut in half. This is the case for both the aggregate (section 7.2.2) and the regional asset pricing model (section 7.2.3). In section 7.2.4, we compare the fit of our model to other asset pricing models. First we briefly discuss the computational procedure.

7.2.1 Computational Procedure

The coefficient vector θ in equation (24) can be estimated using the Fama and MacBeth (1973) two-stage regression procedure or the Hansen and Singleton (1982) generalized method of moments

procedure. We opt for the two-stage Fama-MacBeth procedure and estimate the unconditional β -representation $E \left[R_{t+1}^{e,j} \right] = \tilde{\lambda} \tilde{\beta}^j$. In a first time-series stage, for each asset separately, excess returns are regressed on factors to uncover the $\tilde{\beta}$'s. In a second cross-sectional stage, average excess returns are regressed on the $\tilde{\beta}$'s from the first stage to obtain the market prices of risk $\tilde{\lambda}$. Appendix A.4 describes the procedure in more detail.

The regional factor model introduces a new computational issue, not present in the representative agent framework. The sequence of unconstrained agents $\{i^*\}$ and the vector θ that minimizes pricing errors on the cross-section of asset portfolios have to be determined jointly. Appendix A.4 describes a fixed-point algorithm that addresses this issue. The algorithm can be used in any model in which a shifting subset of agents prices the assets.

7.2.2 Results: Aggregate Asset Pricing Factors

We use aggregate macroeconomic data and the Fama-MacBeth procedure to investigate the explanatory power of the aggregate asset pricing factors in (17) for the cross-section of excess returns on size and book-to-market portfolios $T1$.

Table 4 reports the estimates for the market price of risk $\tilde{\lambda}$ obtained from the second-stage of the Fama-MacBeth procedure. Below the estimates for $\tilde{\lambda}$, we report conventional standard errors and Shanken (1992) standard errors, which correct for the fact that the $\tilde{\beta}$'s are generated regressors from the first time-series step. Since all returns are in excess of a risk free rate, according to the theory, the intercept in the cross-sectional regressions should be zero.

Row 1 shows the standard CCAPM. It explains 15 percent of the cross-sectional variation in excess returns on the size and book-to-market portfolios. With non-separable preferences, the change in relative rental prices scaled by the housing expenditure share is an additional asset pricing factor. The hedging effect increases the R^2 to 44 percent (row 2). Rows 3 through 8 investigate the collateral effect. With separable preferences, the new asset pricing factors are the housing collateral ratio my and consumption growth scaled by my . The fit improves to 70 - 77 percent for the respective measures of the housing collateral ratio (rows 3-5). With non-separable preferences, the interaction term of my with rental price growth is an additional asset pricing factor and the fit improves slightly (rows 6-8). With conditioning variable $myfa$, our *collateral-CAPM* explains 81 percent of the cross-sectional variation in portfolio returns.

The time-invariant market price of consumption risk, predicted by the standard CCAPM, is overly restrictive. A decrease in the housing collateral ratio increases the market price of consumption risk (row 3-5): We estimate $\lambda_{my.c} < 0$. This time-varying reward for consumption risk is predicted by the model.

The estimation reveals two weaknesses, which are common to all consumption-based models (see section 7.2.4 for a comparison with other models). First, the intercept in the cross-sectional regression, $\tilde{\lambda}_0$ should be zero. Its estimate is positive and significant. This suggests the models in

	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\rho$	$\tilde{\lambda}_{my}$	$\tilde{\lambda}_{my.c}$	$\tilde{\lambda}_{my.\rho}$	R^2
1 CCAPM	8.28 (2.39) [2.68]	3.86 (2.57) [3.01]					15.0 11.5
2 Hedging Effect	8.15 (2.39) [3.31]	3.89 (2.57) [3.67]	-1.44 (.51) [.73]				43.8 38.9
3 Separable Prefs. mymo	10.21 (2.60) [3.77]	1.46 (2.41) [3.77]		.08 (.06) [.10]	-1.53 (1.07) [1.61]		69.8 65.6
4 Separable Prefs. myrw	8.30 (2.38) [3.37]	1.96 (2.50) [3.66]		.05 (.04) [.06]	-.66 (.50) [.74]		72.1 68.3
5 Separable Prefs. myfa	6.68 (2.25) [3.01]	1.17 (2.48) [3.42]		-.01 (.02) [.03]	-.71 (.37) [.52]		77.0 73.8
6 Non-Separable Prefs. mymo	10.26 (2.95) [4.32]	2.02 (1.85) [2.85]	-.81 (.44) [.67]	.07 (.06) [.10]	-1.40 (1.00) [1.53]	.15 (.20) [.31]	70.3 62.9
7 Non-Separable Prefs. myrw	9.60 (2.66) [4.28]	4.25 (1.92) [3.21]	-1.02 (.48) [.80]	.04 (.04) [.07]	-.63 (.48) [.81]	.15 (.09) [.16]	75.0 68.7
8 Non-Separable Prefs. myfa	7.68 (2.44) [3.94]	2.91 (1.85) [3.12]	-.06 (.40) [.67]	.00 (.02) [.04]	-.71 (.36) [.60]	.18 (.08) [.14]	81.4 76.7

Table 4: Cross-Sectional Results with Aggregate Pricing Factors. The asset pricing factors are $\Delta \log(c_{t+1})$ in row 1, $\Delta \log(c_{t+1})$ and $A_t \Delta \log(\rho_{t+1})$ in row 2, $\Delta \log(c_{t+1})$, my_t , $my_t \Delta \log(c_{t+1})$ in rows 3-5 and $\Delta \log(c_{t+1})$, $A_t \Delta \log(\rho_{t+1})$, my_t , $my_t \Delta \log(c_{t+1})$ and $my_t A_t \Delta \log(\rho_{t+1})$ in rows 6-8. The housing collateral variable is *mymo* in rows 3 and 6, *myrw* in row 4 and 7 and *myfa* in row 5 and 8. The estimation is done using the Fama-MacBeth procedure. The set of test assets is *T1*. The sample period is 1926-2001. OLS standard errors are in parenthesis, Shanken (1992) corrected standard errors are in brackets. The last column reports the R^2 and the adjusted R^2 just below it.

table 4 do a poor job pricing the risk-free rate. Second, the Shanken standard-error correction is large. This is because the macro-economic factors have a low sample variance and the size of the standard-error correction is inversely related to this variability.

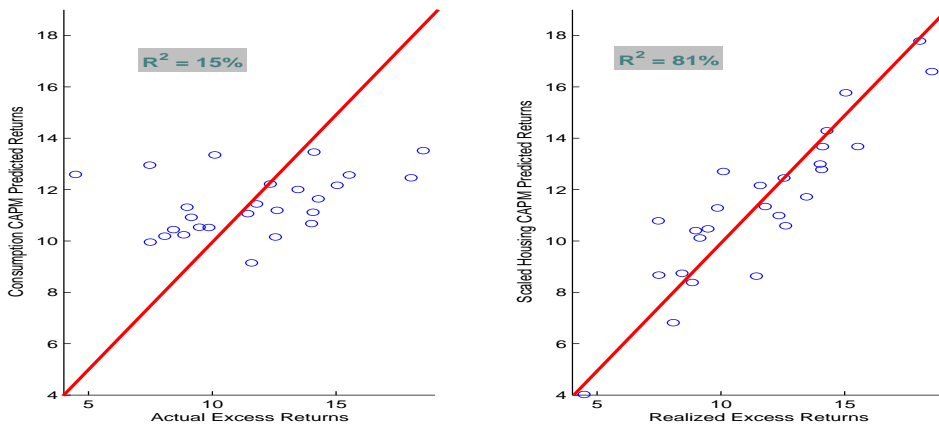
As a robustness check, we relax the Markov assumption on the aggregate weight shock $\{g\}$. We allow for history dependence by including an additional lag of the aggregate factors F_t^a in the empirical specification of the weight process: $G_{t+1}(my_{t+1}, F_{t+1}^a, F_t^a)$. The fit of the cross-sectional estimation does not improve. We conclude that the Markov assumption on $\{g\}$ is not too restrictive.

Figure 5 compares the CCAPM and the collateral-CAPM. The left panel plots the average realized excess return on each of the 26 portfolios in *T1* against the return predicted by the standard CCAPM. It also shows the 45 degree line. This panel illustrates that the CCAPM fails to account for the variation in excess return across portfolios: The predicted returns spread along a horizontal line. The right panel, which corresponds to the estimates in row 8 of table 4, shows the returns predicted by the collateral-CAPM. The size and value portfolios line up along the 45 degree line.

Table 16 (at the end of the text) reports the average pricing errors on each of the 26 portfolios in *T1*. Relative to the CCAPM, the collateral-CAPM eliminates the overpricing of growth stocks

and the underpricing of value stocks. The average pricing error across portfolios is 3 percentage points for the CCAPM (first column, second to last row) but only half as large for the collateral-CAPM (last column). The errors are comparable in size and sign to the Fama and French (1993) three-factor model (second column of table 16 and section 7.2.4). The last row of the table shows a χ^2 -distributed test statistic for the null hypothesis that all pricing errors are zero. The collateral-CAPM is the only model for which the hypothesis of zero pricing errors cannot be rejected.⁹

Figure 5: CCAPM and Collateral-CAPM - Aggregate Pricing Factors. Left Panel: Realized average excess returns on 25 Fama-French portfolios and the value weighted market return against predicted excess returns by standard Consumption CAPM. Right Panel: against predicted returns by Collateral-CAPM.



Time-Varying Betas Why does the collateral-CAPM help explain the value premium? In the model, a stock’s riskiness is determined by the covariance of its returns with aggregate risk factors *conditional* on the state variable my . The conditional covariance reflects time-variation in risk premia. If time variation in risk premia is important for explaining the value premium, then stocks with high book-to-market ratios should have a larger covariance with aggregate risk factors in risky times, when my is low, than in less risky times, when my is high. This is the pattern we find in the data.

We estimate the risk exposure (the β ’s) for each of the twenty-five size and book-to-market portfolios and the value weighted market return. This is the first step of the Fama-MacBeth two-step procedure. To make the point more forcefully we impose separable preferences over housing and non-durable consumption:

$$R_{t+1}^{e,j} = \tilde{\beta}_0^j + \tilde{\beta}_c^j \Delta \log c_{t+1} + \tilde{\beta}_{my}^j my_t + \tilde{\beta}_{my,c}^j my_t \Delta \log c_{t+1}. \quad (28)$$

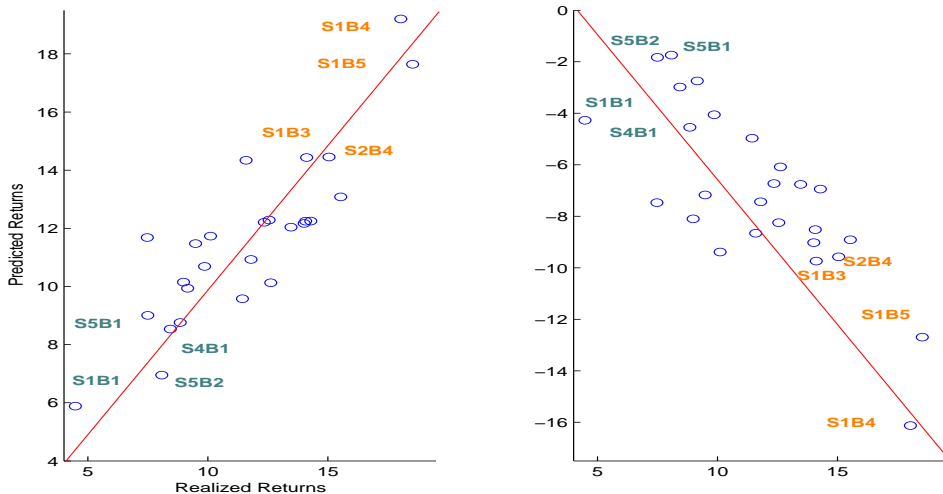
Equation (28) allows the covariance of returns with consumption growth to vary with my . For

⁹Because of the sampling error in the regressors the Shanken correction for the χ^2 test statistics is large. This correction substantially reduces the test statistic (see A.4). The result that the collateral-CAPM fails to reject the null hypothesis of zero pricing errors should be interpreted in this light.

each asset j , we define the conditional consumption beta as $\beta_t^j = \tilde{\beta}_c^j + \tilde{\beta}_{my,c}^j my_t$. We estimate equation (28) and compute the average consumption beta in good states, defined as times in which my is one standard deviation above zero, and in bad states (risky times) when my is one standard deviation below zero. Table 17 shows that the high book-to-market portfolios (B4 and B5) have a consumption β that is large and positive when housing collateral is scarce and negative in times of collateral abundance. The opposite is true for growth portfolios (B1 and B2).

The left panel of figure 6 shows that the value portfolios (B4, B5) have a high return and the growth portfolios (B1, B2) have a low return. The right panel plots realized excess returns against $\tilde{\beta}_{my,c}^j$, the exposure to the interaction term of the housing collateral ratio with aggregate consumption growth. Growth stocks in the upper left corner have a low exposure to collateral constraint risk whereas value stocks have a large exposure. So, value stocks, are riskier than growth stocks because their returns are more highly correlated with the aggregate factors when risk is high (my is low) than when risk is low (my is high). Because the estimate of $\tilde{\lambda}_{my,c}$ and $\tilde{\beta}_{my,c}^j$ are negative, value stocks are predicted to have a higher risk premium. The value premium is the compensation for the risk of binding collateral constraints.

Figure 6: Collateral CAPM: The Value Premium. Left Panel: Realized average excess returns on 25 Fama-French portfolios and the value weighted market return against excess returns predicted by the collateral-CAPM with $myrw$. Right Panel: Realized average excess returns against $\tilde{\beta}_{my,c}$ exposure to interaction term of my_t and $\Delta \log c_{t+1}$.



When preferences are non-separable, the change in rental prices and its interaction term with the housing collateral ratio enter as additional regressors in equation (28). Table 18 at the end of the paper shows the $\tilde{\beta}^{vw}$ -estimates for the value weighted market return. Not only does the covariance of the market return with consumption growth decrease with my , the covariance with rental price growth does as well ($\tilde{\beta}_{my,\rho}^{vw} < 0$). The fit of the time-series regression improves from five to twenty percent once the scaled factors (the interaction terms) are included. This result shows that the covariance of the aggregate US stock market return with the aggregate risk factors is not

constant as predicted by the static CCAPM, but varies with my .

7.2.3 Results: Regional Asset Pricing Factors

We use metropolitan macroeconomic data (1951-2001) and the iterative Fama-MacBeth procedure to investigate the explanatory power of the regional asset pricing factors in (21) for the cross-section of excess returns on size and book-to-market portfolios $T1$.

Table 5 displays the estimates for the market prices of risk $\tilde{\lambda}$. In addition, the table reports the estimates for the underlying parameter vector $\tilde{\theta}$ implied by the estimates for $\tilde{\lambda}$.

The R^2 of the regressions are 67-72 percent, similar to the fit of the scaled aggregate factor model in section 7.2.2. We find support for the collateral effect in the regional factor model. The compensation for bearing collateral constraint risk is higher in times of collateral scarcity. The price of risk associated with the income growth of the unconstrained region decreases with my : $\tilde{\lambda}_{my,\dot{y}} < 0$ and significant according to conventional standard errors. Estimates $\theta_4 < 0$ and $\theta_1 > 0$ imply a positive estimate for β_2 . This is consistent with the collateral effect predicted by the theory. The interaction term between the housing collateral ratio and the rental price growth of the unconstrained region has a small and insignificant price of risk attached to it. Finally and in contrast with the aggregate factor model, the intercept $\tilde{\lambda}_0$ is no longer statistically different from zero.

The left panel of figure 7 plots realized excess returns for the twenty-six portfolios in $T1$ against the returns predicted by the collateral-CAPM with regional factors (row 3 in table 5). Value firms (B4, B5) have a higher exposure to constraint risk in risky times, when my is low, than in good times, when my is high. The coefficient $\tilde{\beta}_{my,\dot{y}}$ is large and negative for value stocks (right panel). The opposite is true for growth stocks (B1, B2). This pattern is analogous to the one in figure 6. Stocks that pay low returns when the income growth of the unconstrained region decreases demand a risk premium, because constraint risk is high. In those times housing collateral is extra valuable. The value premium is a compensation for the fact that value stocks pay low returns in times that households face a high risk of binding collateral constraint and housing collateral values are low.

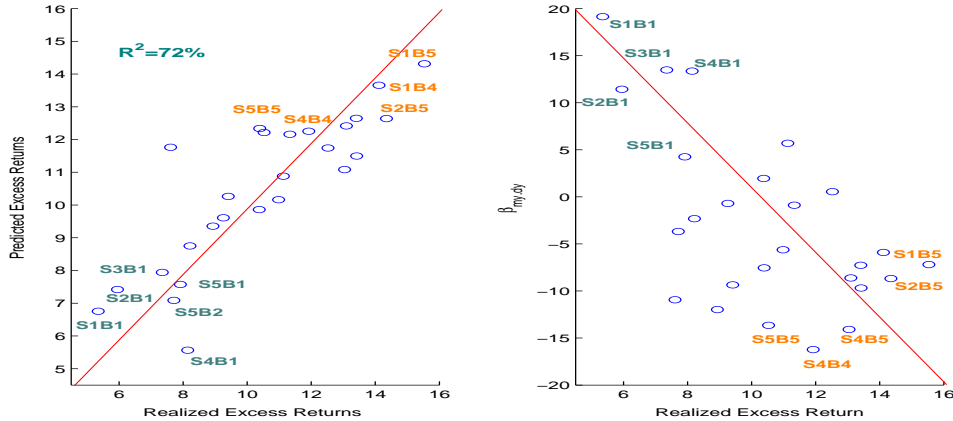
The cross-sectional fit does not hinge on a high degree of risk aversion. The implied estimate for the coefficient of risk aversion is lower than unity: $\theta_1 = 0.48, 0.09$ and 0.02 in rows 1-3. The evidence on the degree of complementarity between housing services and consumption, ϕ , is mixed. The estimate for $\theta_2 = \phi$ is -0.11 in row 1 but $.96$ and 1.24 in rows 2 and 3. The former implies substitutability, the latter two complementarity. The coefficients $\tilde{\lambda}_{\dot{\rho}}$ and $\tilde{\lambda}_{my,\dot{\rho}}$ are estimated imprecisely, so that θ_5 and θ_6 are uninformative about ϕ . This confirms previous work which has found mixed evidence on the sign and magnitude of the intratemporal elasticity of substitution between durable (housing) and non-durable consumption.¹⁰

¹⁰An exception is Ogaki and Reinhart (1998) who reject a Cobb-Douglas specification using aggregate (quality-adjusted) consumption data on durable goods for the period 1951:1-1983:4. They find a point estimate for ϕ of -1.66 . Many quantitative papers such as Fernandez-Villaverde and Krueger (2001), Cocco (2000) and Yao and Zhang (2002) assume a unit elasticity, corresponding to Cobb-Douglas preferences.

	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\rho$	$\tilde{\lambda}_{\hat{y}}$	$\tilde{\lambda}_{my.\hat{y}}$	$\tilde{\lambda}_{\hat{\rho}}$	$\tilde{\lambda}_{my.\hat{\rho}}$	R^2
1 <i>mymo</i>	4.43 (3.81) [7.02]	2.32 (1.04) [1.95]	.58 (.33) [.63]	-2.26 (2.57) [4.78]	-4.81 (2.57) [4.83]	-1.10 (.21) [.41]	.34 (.37) [.70]	67.4 57.1
θ		.48	-.11	.05	-.27	.79	.71	
2 <i>myrw</i>	5.07 (3.87) [7.26]	.95 (.81) [1.55]	.73 (.28) [.55]	-1.72 (1.43) [2.74]	-4.81 (2.13) [4.10]	.10 (.25) [.48]	.04 (.82) [1.56]	71.1 62.1
θ		.09	.96	-.07	-.29	2.61	-.76	
3 <i>myfa</i>	1.61 (3.95) [7.52]	.82 (.81) [1.59]	.81 (.30) [.60]	-2.44 (2.27) [4.35]	-5.72 (3.90) [7.50]	-.02 (.45) [.86]	.01 (.97) [1.87]	71.8 62.9
θ		.02	1.24	-.13	-.24	2.26	-.59	

Table 5: Cross-Sectional Results with Regional Asset Pricing Factors. Results are for the iterative Fama-MacBeth procedure described in appendix with cutoff level 0.9. The asset pricing factors consists of: $\Delta \log^a(c_{t+1})$, $A_t \Delta \log^a(\rho_{t+1})$, $\Delta \log(\hat{y}_{t+1}^{i*})$, $my_t \Delta \log(\hat{y}_{t+1}^{i*})$, $A_t \Delta \log(\hat{\rho}_{t+1}^{i*})$, and $my_t A_t \Delta \log(\hat{\rho}_{t+1}^{i*})$. The sequence $\{i_t^*\}$ is the sequence of unconstrained metropolitan regions. The coefficient vector θ consists of γ , ϕ , $\gamma\beta_1$, $-\gamma\rho\beta_2$, $\phi\beta_1$ and $-\phi\rho\beta_2$. The second column gives the zero- β return $\tilde{\lambda}_0$. OLS standard errors are in parentheses, Shanken (1992) corrected standard errors are in brackets. Row 1 is for *mymo*, row 2 for *myrw* and row 3 for *myfa*. The set of test assets is *T1*. The period is 1952-2001, the longest period with metropolitan data.

Figure 7: Cross-Sectional returns - Regional Asset Pricing Factors. Left Panel: Realized average excess returns on 25 Fama-French portfolios and the value weighted market return against predicted excess returns with *myfa*. Right Panel: realized excess returns against exposure to interaction term of my_t and $\Delta \log \hat{y}_{t+1}^{i*}$.



Sensitivity Analysis We investigate the robustness of the regional asset pricing results to three sources of misspecification.

First, in the iterative Fama-MacBeth procedure, the region with the largest decrease in Pareto-Negishi weight is selected to price all assets. Because the weight processes themselves are unobservable, the selection is based on our specification in equation 20. The specification relates the weight changes to observable changes in the income share and relative rental price. However, the relationship between the weights and the observables also contains an error term which measures unobservable preference changes. The error term is not taken into account when selecting the

unconstrained agent. As a result, when preference shocks are large, the algorithm may select the wrong region to price the assets. To investigate the effects of this omission on the parameter estimates, we conduct a sensitivity analysis described in appendix A.4. It amounts to giving a weight less than one, in a weighted average of intertemporal marginal rates of substitution, to the region that is predicted to be the unconstrained region. From table 9 (in appendix A.4) we conclude that our estimation procedure is robust to the omission of region-specific preference shocks in the selection of the region that prices the assets.

Second, region-specific income and rental price growth variables are deviations from a cross-region average. Up until now, this average weighted regions *equally*. As a robustness check, we estimate the model with variables that are deviations from a *population-weighted* average. The population weight for region i at time t is defined as its number of households divided by the total number of households in all 23 regions at time t . The coefficient estimates remain largely unchanged, as reported in table 19. The R^2 of the model is between 60 and 66 percent depending on the collateral measure.

Third, the model assumes that consumption has the same price in each region: trade of non-durables is costless. The real exchange rate for non-durables, defined as a region's non-durable price level to the population-weighted, cross-region average of non-durable price levels, is one in the model. In the metropolitan data, annual changes in real exchange rates for non-durables are between -4.6 and +3.8 percent. To bring the data in line with the model, we express asset returns in regional consumption units. Regional real exchange rates do not vary enough to affect the asset pricing results in any substantial way. Table 20 shows that the fit of the model and the coefficient estimates are largely unchanged.

As a final robustness check, we use regional consumption data *directly* instead of proxying the regional Pareto-Negishi weight process (equation 20). The stochastic discount factor now contains the unconstrained agents' consumption growth and relative rental price growth as the only asset pricing factors. We find similar estimates for γ and ϕ : 0.26 and 0.03. In recent work, Brav, Constantinides and Geczy (2002) find that higher order moments of the distribution of consumption growth across US households affect asset prices, in particular the variance and skewness. Here, the higher order cross-sectional moment relevant for asset pricing is the minimal consumption growth across households (regions).

7.2.4 Comparison Across Models

The cross-sectional explanatory power of the collateral-CAPM proposed in this paper compares favorably to other asset pricing models. Table 6 compares return-based asset pricing models in rows 1-3 with consumption-based models in rows 4-6.

The capital asset pricing model relates the returns on stocks to their correlation with the return on the wealth portfolio. In the standard CAPM of Lintner (1965), the return on the wealth portfolio is proxied by the market return R^{vw} (row 1). It explains 36 percent of annual returns. Because stock market wealth is a very incomplete total wealth measure, Jagannathan and Wang

(1996) include the return on human wealth in the return on the wealth portfolio. The R^2 in row 2 increases slightly to 39 percent. In contrast to Jagannathan and Wang (1996), we assume that human wealth cannot be traded. In our model, human wealth affects financial returns only through the housing collateral ratio. In addition, our model points towards another often ignored source of wealth: housing. In the representative agent economy of Santos and Veronesi (2001) the ratio of labor income to consumption lc predicts stock returns. Times in which investors finance a large fraction of consumption out of labor income rather than out of stock dividend income (lc is high), are less risky. The risk premium is lower: $\tilde{\lambda}_3 < 0$ in row 3. Their conditional CAPM explains 53 percent of the annual returns.¹¹

The Fama and French (1993) three-factor model adds a size and a book-to-market factor to the standard CAPM. The size factor is the return on a hedge portfolio that goes long in small firms and short in big firms (smb). The value factor is the return on a hedge portfolio that goes long in high book-to-market firms and short in low book-to-market firms (hml). This model accounts for 79 percent of the cross-sectional variation in annual returns (row 7). There is a 3.4 percent point size premium and a 6.3 percent point value premium in our sample. Given its good fit, this model serves as the empirical benchmark.

In contrast to the previous models, the consumption-based asset pricing models measure the riskiness of an asset by its covariance with marginal utility growth. One of the objectives of this literature has been to identify macroeconomic sources of risk that can explain the empirical success of the Fama and French (1993) size and book-to-market factors. The fourth row reports the standard CCAPM of Breeden (1979). The only factor is consumption growth. It explains 15 percent of the cross-sectional variation in returns. Lettau and Ludvigson (2001b) explore a conditional version of the CCAPM with the consumption-wealth ratio as scaling variable. The ratio is measured as the deviation from the common trend in consumption, labor income and financial wealth (cay). Periods with high cay indicate high expected future returns, thereby rationalizing a high propensity to consume out of wealth. The market price of consumption risk increases in times with low cay (recessions). The Lettau-Ludvigson model explains 33 percent of the annual cross-sectional variation.¹² In contrast to the consumption-wealth ratio (cay), our conditioning variable does not contain direct information on future returns.

Model 6 is our collateral-CAPM with aggregate pricing factors and scaling variable $myfa$.¹³ For parsimony we omit the interaction term with rental price growth. The model goes a long way in accounting for the cross-sectional differences in returns on the 25 Fama-French portfolios. The

¹¹The authors also investigate a scaled version of the CCAPM, as we do, but their results for the scaled CCAPM are not as strong as for the scaled CAPM. We construct their scaling variable as the ratio of annual labor income to total consumption expenditures, for the period 1926-2001. We rescale the scaling variable lc : $\tilde{lc}_t = 1 + \frac{lc_t - E(lc)}{std(lc)}$.

¹²We construct the cay variable for the period 1926-2001 using log real per household total consumption expenditures (c), log real per household labor income plus transfers (y/l) and log real per household financial wealth (fw). We find evidence for one cointegration relationship between the three variables. The estimated relationship we find with annual data is $cay = c - 0.294fw - 0.702y/l + 0.452$. We follow Lettau and Ludvigson and rescale the scaling variable cay : $\widetilde{cay}_t = 1 + \frac{cay_t - E(cay)}{std(cay)}$.

¹³We rescale the my variable: $\widetilde{my}_t = 1 + \frac{my_t}{std(my)}$.

R^2 of 78 percent comes close to the fit of the Fama-French model.

Model	$\tilde{\lambda}_0$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$	R^2
1	-1.42	10.38				36.3
Static CAPM	(3.95)	(4.59)				33.7
Lintner	[4.41]	[5.66]				
2	-0.12	9.17	3.75			39.1
Human Capital-CAPM	(3.80)	(4.47)	(1.81)			33.8
Jagannathan-Wang	[4.41]	[5.71]	[2.30]			
3	-.68	9.52	.09	-8.60		52.9
lc -conditional CAPM	(4.17)	(4.75)	(0.24)	(7.87)		46.5
Santos-Veronesi	[5.49]	[6.68]	[0.34]	[11.00]		
4	8.17	3.86				15.0
Static CCAPM	(2.39)	(2.57)				11.5
Breeden-Lucas	[2.68]	[3.01]				
5	11.64	1.36	-.62	-9.32		33.3
cay -conditional CCAPM	(3.10)	(2.47)	(.28)	(6.97)		24.3
Lettau-Ludvigson	[4.23]	[3.48]	[.40]	[9.69]		
6	6.49	.64	.32	-.04	-6.39	78.1
Collateral-CAPM	(2.26)	(2.46)	(.44)	(.20)	(4.75)	74.0
this paper	[3.31]	[3.71]	[.66]	[0.32]	[7.18]	
7	11.03	-2.97	3.37	6.33		79.3
Three-factor model	(4.77)	(5.32)	(1.84)	(1.80)		76.4
Fama-French	[5.46]	[6.55]	[2.74]	[2.65]		

Table 6: Model Comparison: 7 models, 1927-2001. Row 1: factor is $R_{t+1}^{vw,e}$. Row 2: factors are $R_{t+1}^{vw,e}$ and $R_{t+1}^{hc,e}$. Row 3 factors: $R_{t+1}^{vw,e}$, lc_t and $lc_t R_{t+1}^{vw,e}$. Row 4: $\Delta \log(c_{t+1})$. Row 5: $\Delta \log(c_{t+1})$, cay_t , $cay_t \Delta \log(c_{t+1})$. Row 6 is the housing model: $\Delta \log(c_{t+1})$, $A_t \Delta \log(\rho_{t+1})$, $myfa_t$, and $myfa_t \Delta \log(c_{t+1})$. Row 7: $R_{t+1}^{vw,e}$, $R_{t+1}^{smb,e}$, and $R_{t+1}^{hml,e}$. The second column gives the zero- β return $\tilde{\lambda}_0$. OLS standard errors are in parenthesis, Shanken corrected standard errors are in brackets.

7.3 Regional Risk-Sharing

The third and last empirical exercise uses regional data to test the collateral effect *directly*. Using metropolitan income, consumption and housing data we examine risk-sharing patterns directly. We find evidence that US metropolitan regions share a larger proportion of idiosyncratic risk when the housing collateral ratio is high. We interpret the findings as direct support for the mechanism that drives the asset pricing predictions.

In the complete markets model with perfect commitment, consumption growth is equal across agents and uncorrelated with any source of idiosyncratic risk. In our metropolitan data, income growth is more strongly correlated across regions than consumption growth. The time average of the cross-sectional correlation of consumption growth is 0.27, lower than the cross-correlation of labor income growth of 0.48. From the perspective of the complete markets model this is a puzzle, also known as the ‘‘quantity anomaly’’ Time-variation in the degree of risk-sharing due to changes in the value of housing collateral sheds new light on the consumption correlation puzzle.

Quantifying the Extent of Risk-Sharing The consumption Euler equation in (19) is the central object in our test of the collateral effect. After substitution of $\Delta \log(\hat{\xi}_t^i)$ from (20), it relates region-specific consumption growth to region-specific income growth, rental price growth and the aggregate housing collateral ratio:

$$\begin{aligned} \Delta \log(\hat{c}_t^i) \approx & (\beta_1 - \beta_2 my_t) \Delta \log(\hat{y}_t^i) + \phi(\beta_1 - 1 - \beta_2 my_t) \hat{A}_{t-1}^i \Delta \log(\hat{\rho}_t^i) \\ & + (1 - \beta_1 + \beta_2 my_t) \nu_t^i. \end{aligned} \quad (29)$$

Recall that ν contains the preference shifters and that β_1 and β_2 are a function of the lower and upper bounds on the housing collateral ratio:

$$\begin{aligned} \nu_t^i &= (1 - \gamma) \Delta \hat{b}_t^{i,c} - \phi \hat{A}_{t-1}^i \Delta \hat{b}_t^{i,c} + \phi \hat{A}_{t-1}^i \Delta \hat{b}_t^{i,h} \\ \beta_1 &= \frac{my^{\max}}{my^{\max} - my^{\min}}, \quad \beta_2 = \frac{1}{my^{\max} - my^{\min}}. \end{aligned}$$

With separable preferences and fully-enforceable contracts, regional non-durable consumption growth only varies with average non-durable consumption growth $\Delta \log^a c_t$. This captures aggregate risk. Limited enforceability leads to partial insurance. Consumption is allocated with regard to income shocks: The equilibrium consumption share increases for a household with above average income growth ($\beta_1 > 0$). A higher housing collateral ratio my allows for more risk sharing: the correlation between consumption share and income share growth decreases ($\beta_2 > 0$).

With non-separable preferences and fully-enforceable contracts, the region-specific component of rental price growth affects consumption growth. Under complementarity ($\phi > 0$), consumption growth is higher in regions with below-average rental price growth ($\Delta \log \hat{\rho}_t^i < 0$). Limited enforceability and non-separability interact. The equilibrium consumption share increases for regions with below-average rental price growth. A higher my allows for more risk sharing: the correlation between consumption share and region-specific rental price growth decreases ($\beta_2 > 0$).

The degree of risk sharing can be summarized by a simple statistic:

$$\Omega_t = 1 - (\beta_1 - \beta_2 my_t).$$

When $my_t = my^{\max} > 0$, $\Omega_t = 1$: 100 percent of idiosyncratic risk is shared. When $my_t = my^{\min} < 0$, $\Omega_t = 0$: 0 percent of idiosyncratic risk is shared. Ω_t measures the distance between the Pareto frontier of the limited commitment economy and the frontier for the autarkic economy. $1 - \Omega_t$ measures the distance between the Pareto frontier of the limited commitment economy and the frontier of the perfect enforcement economy.

Testing Full Consumption Insurance In the benchmark model with perfect commitment there is full consumption insurance. All agents equate their intertemporal marginal rates of substitution. Variables capturing idiosyncratic risk should not change the consumption distribution (Cochrane (1991b) and Mace (1991)). Only aggregate risk factors change the intertemporal marginal rate of

substitution over time. The null hypothesis of full insurance is $H_0 : \beta_1 = \beta_2 = 0$ in equation (29).

Identification and Econometric Issues Before proceeding to the estimation of equation (29), we define the error process $\varepsilon_t^{i,c}$. We allow for multiplicative measurement error ζ_t in log idiosyncratic consumption levels. This implies a MA(1) structure for measurement error in consumption growth. The error process contains preference shocks and measurement error:

$$\varepsilon_t^{i,c} = (1 - \beta_1 + \beta_2 my_t) \nu_t^i + \zeta_t - \zeta_{t-1}.$$

We make one assumption: $E[\nu_t^i my_{t-k}] = 0, \forall k \geq 0$. Since only aggregate variables affect the aggregate housing collateral ratio my and only region-specific preference shifts enter in ν^i , the assumption follows from the theory.

Correlation between residuals and regressors renders least squares estimators of the parameters in equation (29) inconsistent. Therefore, it is important to understand when such correlation arises. When $\Delta \log \hat{y}_t^i$ and $\Delta \log \hat{\rho}_t^i$ are cross-sectionally independent of $\varepsilon_t^{i,c}$, the regressors and residuals are orthogonal. This assumption is clearly violated for household-level data. For example, a family expansion changes preferences for housing $\Delta \hat{b}_t^{i,h}$ and affects labor supply and hence $\Delta \log(\hat{y}_t^i)$. However, at the metropolitan level such demographical shocks average out when aggregating over households. Indeed, the metropolitan data show that household-level characteristics such as average household size, age of head, and female labor supply are very similar across the 23 metropolitan areas. In contrast, an adverse shock to an industry predominantly located in one region, a large population influx in a region, or a change in the demographical composition may affect all households in one region alike. Such shocks certainly affect regional rental price growth and per household income growth, but it is less obvious that they affect preference changes or measurement error in a systematic way. Only when they do, income and rental price changes need to be instrumented to obtain consistent estimates. In that case, an instrumental variables estimator that uses 2-period and 3-period leads of dependent and independent variables as instruments is consistent (Arellano and Bond (1991)).

Time-Variation in Degree of Risk-Sharing Table 7 shows the estimation results for the consumption Euler equation:

$$\Delta \log(\hat{c}_t^i) = \beta_0^i + (\beta_1 - \beta_2 my_t) \Delta \log(\hat{y}_t^i) + (\beta_3 - \beta_4 my_t) A_{t-1} \Delta \log(\hat{\rho}_t^i) + \varepsilon_t^{i,c}.$$

The degree of risk sharing Ω_t moves over time. The last column reports the lowest, median and highest degree of risk-sharing observed in the sample period. A strong collateral effect implies a wide interval Ω :

$$\Omega = \left[1 - \left(\hat{\beta}_1 - \hat{\beta}_2 my^{lo} \right), 1 - \left(\hat{\beta}_1 - \hat{\beta}_2 my^{med} \right), 1 - \left(\hat{\beta}_1 - \hat{\beta}_2 my^{hi} \right) \right].$$

Two-thirds of disposable income shocks are shared on average (rows 1-3, last column). The

estimate is fifty percent when labor income plus transfers growth is used (rows 7-9). Similar magnitudes are found for the US states (e.g. Hess and Shin (1998)), but they are much lower than the 80-95 percent found for household data (e.g. Krueger (2000)). The null hypothesis of complete consumption insurance is strongly rejected. The p-value for a Wald test of $\beta_1 = \beta_2 = 0$ is 0.00 for all rows in table 7.

We find that the correlation of regional consumption and income is lower when housing collateral is abundant: β_2 is always positive in rows 1-9. This is the sign predicted by the theory. Furthermore, the last column shows that the time-variation in the degree of risk sharing is substantial. For low values of the collateral ratio, risk sharing is as low as 43 percent. In times of abundant housing collateral, up to 93 percent of income shocks can be shared (row 3). In 1945, real estate wealth was 33 percent below its trend with human wealth ($myfa = -.33$). The estimates for β_1 and β_2 from row 3 imply that the economy was in autarky.

The estimates for β_1 through β_4 are consistent with $\phi < 0$, i.e. substitutability between housing and non-durable consumption in the utility function. A Wald test for $H_0 : \phi = \frac{\beta_1 - 1}{\beta_3} = 0$ has a p-value of 0.00 in rows 1 through 3. The estimates for β_2 and β_4 are also consistent with $\phi < 0$, but the evidence for non-separability is not as strong. The p-value for $H_0 : \phi = \frac{\beta_4}{\beta_2} = 0$ is .24, .06 and .52 in rows 1-3 respectively.

	β_1	σ_{β_1}	β_2	σ_{β_2}	β_3	σ_{β_3}	β_4	σ_{β_4}	R^2	Ω
1 - <i>mymo</i>	.33	.03	.26	.19	.85	.13	-1.96	1.20	9.6	[57, 67, 72]
2 - <i>myrw</i>	.36	.03	1.53	.53	.88	.12	-3.83	1.71	9.9	[50, 64, 84]
3 - <i>myfa</i>	.36	.03	2.05	.62	.90	.12	-1.37	1.93	10.0	[43, 62, 93]
4 - <i>mymo</i>	.34	.02	.36	.26	.96	.06	-1.11	.71	9.1	[64, 67, 72]
5 - <i>myrw</i>	.34	.02	1.27	.32	.93	.05	-2.36	.79	9.4	[54, 63, 82]
6 - <i>myfa</i>	.36	.02	1.17	.32	.93	.06	-.61	1.00	9.4	[52, 63, 80]
7 - <i>mymo</i>	.49	.02	.52	.30	.91	.08	-1.23	.81	14.1	[48, 52, 60]
8 - <i>myrw</i>	.47	.02	.29	.30	.91	.06	-2.84	.87	14.0	[50, 52, 57]
9 - <i>myfa</i>	.49	.02	.56	.31	.95	.06	-1.64	1.10	14.0	[45, 50, 59]
10 - <i>myrw</i>	0.52	0.08	5.53	1.43	0.68	0.21	-9.61	2.42		[0, 48, 100]
11 - <i>myfa</i>	0.58	0.08	2.67	1.48	0.28	0.22	-6.17	3.32		[15, 39, 79]

Table 7: Testing the Collateral Effect with Aggregate Collateral Measures. Rows 1-3 are for the period 1952-2001 (1069 observations). They use *mymo*, *myrw* and *myfa* respectively and idiosyncratic income is disposable personal income. Rows 4-6 are identical to rows 1-3 but are for the period 1970-2000 (702 observations). Regressions 7-9 use labor income plus transfers, available only for 1970-2000. The coefficients on the fixed effect, β_0^i , are not reported. The interval Ω is calculated using the estimated intercept and slope coefficient on income and the minimum, median and maximum realized values for *my* during the estimation period. Estimation is by feasible GLS allowing for both cross-section heteroskedasticity and contemporaneous correlation. Rows 10-11 are by instrumental variable estimation (3SLS). Instruments are a constant, $\log(\hat{y}_{t+2}^i)$, $\log(\hat{y}_{t+3}^i)$, $\Delta\hat{\rho}_{t+2}^i$, $\Delta\hat{\rho}_{t+3}^i$, $\log(\hat{c}_{t+2}^i)$, $\log(\hat{c}_{t+3}^i)$, $myrw_{t+2}$, $myrw_{t+3}$ resp. $myfa_{t+2}$, $myfa_{t+3}$. The period is 1952-1998 (997 observations).

Row 10 (11) of table 7 report instrumental variable (3SLS) estimates where income and rental price changes are instrumented by 2 and 3-period leads of independent and dependent variables. The collateral ratio is *myrw* (*myfa*). On average forty to fifty percent of income shocks are shared. The instrumental variables estimates imply a substantial degree of time-variation in risk-sharing. For the residential wealth measure *myrw*, Ω covers the entire unit interval.

Regional Housing Collateral So far we have used aggregate collateral measures only. This is consistent with our theoretical setup in which households are allowed to own a fraction of the housing stock in different regions. When households are restricted to be the full owner of the housing stock in their region, regional collateral measures affect risk-sharing. In that case, the planner is prevented to reallocate housing endowments across regions. as a result, houses are priced off the region-specific intertemporal marginal rate of substitution, rather than off the IMRS of the unconstrained agents. This modifies the collateral constraints. We pursue this additional restriction on allocations in Lustig and VanNieuwerburgh (2002). Here we want to test this prediction empirically.

We find that the regional collateral variables lend additional support to the collateral effect. In particular, regions with a higher home ownership rate and regions with a higher value of housing wealth are better able to smooth regional income shocks. Table 8 summarizes the findings and appendix A.5 describes the regional collateral data in detail.

Regions with a higher home-ownership rate can sustain a higher degree of risk sharing (row 1). β_2 is positive and measured precisely. The minimal and maximal home ownership rate observed in the sample imply variation in risk-sharing between 52 and 66 percent.

Likewise, a higher regional housing collateral value increases the degree of risk-sharing. The log deviation of the collateral value in region i from the cross-sectional median, \widehat{hv}_{it} , is the measure of regional collateral abundance used in row 2. The estimate β_2 is statistically unambiguously positive. Time variation in the regional collateral variable implies variation in risk-sharing between 44 and 74 percent.

Rows 3 to 5 show that when both the interaction term with my and the interaction term with \widehat{hv}_{it} are included, the regional collateral effect remains, whereas the aggregate collateral interaction term have mostly insignificant coefficient estimates.

	β_1	σ_{β_1}	β_2	σ_{β_2}	β_3	σ_{β_3}	β_4	σ_{β_4}	R^2	Ω
1 - HO^i	.69	.07	.48	.12	.74	.29	-.57	.47	13.8	[52, 61, 66]
2 - \widehat{hv}_i	.45	.02	.09	.02	1.05	.05	.18	.07	13.6	[44, 54, 74]
3 - \widehat{hv}_i	.45	0.02	.09	.02	.94	.06	.17	.07	13.6	
$mymo$.01	.25			-1.49	.62		
4 - \widehat{hv}_i	.44	.02	.08	.02	1.03	.05	.19	.07	13.5	
$myrw$			-.72	.33			-.79	.75		
5 - \widehat{hv}_i	.43	.02	.08	.02	1.09	.05	.19	.07	13.7	
$myfa$.62	0.29			.78	.74		

Table 8: Testing the Collateral Effect with Regional Collateral Measures. Rows 1 and 2 estimate $\Delta \log(\hat{c}_t^i) = \beta_0^i + (\beta_1 - \beta_2 X_t^i) \Delta \log(\hat{y}_t^i) + (\beta_3 - \beta_4 X_t^i) \Delta \log(\hat{p}_t^i) + \varepsilon_t^{i,c}$. In regression 1 X^i is the region-specific home-ownership rate (594 observations). In regression 2, X^i is \widehat{hv}_{it} , the log ratio of $hv_t^i = HV_t^i/p_t^{i,a}$ to the cross-sectional median hv_t^{med} (574 observations). The interval Ω denotes the minimum, median and maximum degree of inter-regional risk sharing. It is calculated using the estimated intercept and slope coefficient on income and the minimum, median and maximum actual values for the home ownership rate and \widehat{hv}_{it} in the years 1975-2000 and across regions. In regression 3-5, X^i includes both the interaction term of regression 2 and the interaction term between income share growth and the aggregate collateral measures $mymo$ (regression 3), $myrw$ (regression 4) and $myfa$ (regression 5). The coefficient on the interaction term with my is reported in the columns β_2 and β_4 in the second row. In all regressions y is labor income plus transfers. The coefficients on the fixed effect, β_0^i , are not reported. Estimation is by feasible GLS allowing for both cross-section heteroskedasticity and contemporaneous correlation. All regressions are for the period 1975-2000, the longest period with metropolitan house price data.

8 Conclusion

This paper develops a general equilibrium asset pricing model with housing collateral. Agents have to back up their state-contingent promises with the value of their house. Time variation in the price of housing induces time variation in the economy's ability to share labor income risk. In recessions, the collateral value is low and there is an endogenous increase in idiosyncratic risk. Agents demand a higher risk premium to hold equity.

Empirical evidence supports the collateral effect. Conditional on the housing collateral ratio, the consumption-CAPM explains the cross-sectional variation in size and book-to-market portfolio returns as well as the Fama and French (1993) model. This is true for a first version of the model with frictionless rental markets, which we test with aggregate data as well as for a second version with housing market frictions, which we test with regional data. Using the same regional data set, we find direct evidence for the mechanism underlying the pricing results: time variation in the extent of risk-sharing. Our theory only predicts strong consumption growth correlations when my is high. The data seem to support this qualification; conditioning on my weakens the consumption growth puzzle for US regions. Our hope is that fluctuations in housing collateral can help understand risk-sharing patterns at the household and the international level.

A fully calibrated version of the model is capable of generating values for the mean and volatility of the excess return, dividend-price ratio and risk-free rate in line with historical stock market return and the T-bill rate moments (Lustig and VanNieuwerburgh (2002)). Persistence in the pricing kernel, coming from the persistence of the housing collateral ratio, helps reconcile a low term premium with a high equity premium (Alvarez and Jermann (2001)).

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A Appendix

A.1 Competitive Equilibrium with Time-Zero Markets

We show under which conditions the sequence of budget constraints and collateral constraints in the sequential market setup can be rewritten as one time-zero budget constraint and the collection of collateral constraints shown in equation (9). The proof strategy follows Sargent (1984) (Ch. 8). We then formally state the household problem and define an equilibrium in which all trading takes place at time zero. Finally, we outline a procedure to compute time-zero equilibria. We show that the Pareto-Negishi weights in the planner problem (in the main text) are the cumulative multipliers on the collateral constraints in the time-zero equilibrium.

Budget Constraint First, we show how the Arrow-Debreu budget constraint obtains from aggregating successive sequential budget constraints. The sequential budget constraint for a household is:

$$c_t^i + \rho_t h_t^i + p_t^h \omega_t^i + \sum_{s'} q_t(s') a_t^i(s') \leq \theta_t^i y_{t+1}^i(s') + a_{t+1}^i(s') + [p_{t+1}^h(s') + \rho_{t+1}(s') h_{t+1}(s')] \omega_t^i = W_{t+1}^i(s')$$

Multiply the second equation by $q_{t+1}(s')$ and sum over states. Then substitute the expression for $\sum q_{t+1}(s') a_{t+1}(s')$ into the first equation.

$$c_t^i + \rho_t h_t^i + \sum_{s'} q_{t+1}(s') W_{t+1}^i(s') \leq W_t^i + \sum_{s'} q_{t+1}(s') y_{t+1}^i(s') + \omega_t^i \left(\sum_{s'} q_{t+1}(s') [p_{t+1}^h(s') + \rho_{t+1}(s') h_{t+1}(s')] - p_t^h \right)$$

Similarly, for period $t + 1$

$$c_{t+1}^i + \rho_{t+1} h_{t+1}^i + \sum_{s''} q_{t+2}(s'') W_{t+2}^i(s'') \leq W_{t+1}^i + \sum_{s''} q_{t+2}(s'') y_{t+2}^i(s'') + \omega_{t+1}^i \left(\sum_{s''} q_{t+2}(s'') [p_{t+2}^h(s'') + \rho_{t+2}(s'') h_{t+2}(s'')] - p_{t+1}^h \right)$$

Substituting the expression for $t + 1$ into the expression for t by substituting out W_{t+1} we get

$$\begin{aligned} & c_t^i + \rho_t h_t^i + \sum_{s'} q_{t+1}(s') [c_{t+1}^i + \rho_{t+1} h_{t+1}^i] + \sum_{s'} \sum_{s''} q_{t+1}(s') q_{t+2}(s'') W_{t+2}^i(s'') \leq \\ & W_t^i + \sum_{s'} q_{t+1}(s') y_{t+1}^i(s') + \sum_{s'} \sum_{s''} q_{t+1}(s') q_{t+2}(s'') y_{t+2}^i(s'') + \omega_t^i \left(\sum_{s'} q_{t+1}(s') [p_{t+1}^h(s') + \rho_{t+1}(s') h_{t+1}(s')] - p_t^h \right) + \\ & \sum_{s'} q_{t+1}(s') \omega_{t+1}^i \left(\sum_{s''} q_{t+2}(s'') [p_{t+2}^h(s'') + \rho_{t+2}(s'') h_{t+2}(s'')] - p_{t+1}^h \right). \end{aligned}$$

Let Π_{s^t} be the value of a dividend stream $\{d\}$ starting in history s^t priced using the market state prices $\{\mu\}$:

$$\Pi_{s^t}[\{d\}] = \sum_{j \geq 0} \sum_{s^{t+j} | s^t} \mu_{t+j}(s^{t+j}) d_{t+j}(s^{t+j}),$$

where for a given path s^{t+j} following history s^t , μ is defined as

$$\mu_{t+j}(s^{t+j} | s^t) = q_{t+j}(s^{t+j} | s^{t+j-1}) q_{t+2}(s^{t+2} | s^{t+1}) \dots q_{t+1}(s^{t+1} | s^t).$$

Repeating the successive substitutions, the budget set is given by

$$\Pi_{s^t}[\{c^i\}] + \Pi_{s^t}[\{\rho h^i\}] \leq W_t^i - y_t^i + \Pi_{s^t}[\{y^i\}] \quad (30)$$

under 2 assumptions: (1) the transversality condition

$$\lim_{j \rightarrow \infty} \sum_{s^{t+j}} \mu_{t+j}(s^{t+j}) W_{t+j}^i(s^{t+j}) = 0, \quad (31)$$

is satisfied and (2) there are no arbitrage opportunities:

$$p_{t+j-1}^h(s^{t+j-1}) = \sum_{s^{t+j}|s^{t+j-1}} q_{t+j}(s^{t+j}) \left[p_{t+j}^h(s^{t+j}) + \rho_{t+j}(s^{t+j}) h_{t+j}(s^{t+j}) \right], \quad \forall j \geq 0, \forall s^{t+j} \quad (32)$$

If the latter condition were not satisfied, a household could achieve unbounded consumption by investing sufficiently high amounts in housing shares ω and financing this by borrowing. This is a feasible strategy because ownership shares in the housing tree are collateralizable.

Because $W_0^i = y_0^i + \Pi_{s^0} [\{\rho\chi_0^i\}]$ we obtain the Arrow-Debreu budget constraint

$$\Pi_{s^0} [\{c^i\}] + \Pi_{s^0} [\{\rho h^i\}] \leq \Pi_{s^0} [\{\rho\chi^i\}] + \Pi_{s^0} [\{y^i\}]$$

Collateral Constraints Second, we show the equivalence between the collateral constraints of the sequential markets setup and the solvency constraint in the static economy. The sequential collateral constraints are:

$$\left[p_t^h(s) + \rho_t(s) h_t(s) \right] \omega_{t-1}^i + a_t^i(s) \geq 0,$$

and the collateral constraints in a history s^t :

$$\Pi_{s^t} [\{c^i\}] + \Pi_{s^t} [\{\rho h^i\}] \geq \Pi_{s^t} [\{y^i\}]. \quad (33)$$

The equivalence follows if and only if

$$a_t^i(s) + \left[p_t^h(s) + \rho_t(s) h_t(s) \right] \omega_{t-1}^i = \Pi_{s^t} [\{c^i - \rho h^i - y^i\}].$$

But this follows immediately from the budget constraint (30) holding with equality and the definition of W :

$$W_t^i(s) - y_t^i(s) = a_t^i(s) + \left[p_t^h(s) + \rho_t(s) h_t(s) \right] \omega_{t-1}^i.$$

Allocations are feasible if, at all nodes of the event tree:

$$\sum_{i=1}^N c_t^i(s^t) \leq e_t(s^t) \text{ and } \sum_{i=1}^N h_t^i(s^t) \leq h_t(s^t) \text{ for all } s^t, t \geq 0. \quad (34)$$

An allocation is immune to the threat of default if the allocation satisfies the time-zero collateral constraint (33) for each agent. Under conditions (31) and (32) an allocation that is feasible and immune to the threat of default in sequential markets is feasible and immune to the threat of default in time-zero markets.

The equivalence implies that the portfolio shares ω^i in the sequential economy are indeterminate.

Household Problem Households purchase a complete contingent consumption plan $\{c^i\}$ at market state prices $\{\mu\}$. In addition, they can set up a housing services plan $\{h^i\}$ at market state prices $\{\mu\rho\}$. For expositional purposes we abstract from the preference shifts.

The household solves:

$$\sup_{\{c^i\}, \{h^i\}} U(c^i, h^i)$$

subject to the time-zero budget constraint (30) and an infinite sequence of collateral constraints (33), one for each s^t .

Competitive Equilibrium

Definition 2. For given $(W_0^i)_{i=1}^N$ a competitive equilibrium in time-zero markets is a feasible allocation $\{h_t^i(s^t), c_t^i(s^t)\}$ and a list of market state prices $\{\mu_t(s^t|s_0), \rho_t(s^t|s_0)\}$ such that (i) for given prices, the households solve their optimization problem and (ii) the markets for the consumption good and the housing services clear, for all s^t .

Computation It is more convenient to work with the dual problem for the household. Given Arrow-Debreu prices $\{\mu, \rho\}$ the household minimizes the cost of delivering utility w_0^i to itself:

$$C^i(w_0^i, s_0) = \min_{\{c^i, h^i\}} \left(c_0^i(s_0) + h_0^i(s_0)\rho_0(s_0) \right) \quad (35)$$

$$+ \sum_{s^t} \mu(s^t|s_0) \left(c_t^i(s^t|s_0) + h_t^i(s^t|s_0)\rho_t(s^t|s_0) \right) \quad (36)$$

subject to the promise-keeping constraint

$$U_0(w_0^i, s_0, c^i, h^i) \geq w_0^i$$

and the collateral constraints (33).

The initial promised value w_0^i is determined such that the household spends its entire initial wealth:

$$C^i(w_0^i, s_0) = \Pi_0 \left[\left\{ y + \rho\chi^i \right\} \right].$$

The above problem is a convex programming problem. We first set up the saddle point problem and then make it recursive by defining cumulative multipliers (Marcet and Marimon (1999)). Let $\{\gamma_t^i(s^t)\}$ denote the sequence of multipliers on the constraints of agent i . Define a cumulative multiplier at each node: $\zeta_t^i(s^t) = 1 - \sum_{s^t} \gamma_t^i(s^t)$. Finally, we rescale the market state price $\hat{\mu}_t(s^t|s_0) = \mu_t(s^t|s_0)/\delta^t \pi_t(s^t|s_0)$. By using Abel's partial summation formula and the law of iterated expectations to the Lagrangian, we obtain an objective function that is a function of the cumulative multiplier process ζ^i :

$$D(c^i, h^i, \zeta^i; w_0^i, s_0) = \sum_{t \geq 0} \sum_{s^t} \left\{ \delta^t \pi(s^t|s_0) \left[\begin{array}{c} \zeta_t^i(s^t|s_0) \hat{\mu}_t(s^t|s_0) (c_t^i(s^t) + \rho_t(s^t) h_t^i(s^t)) \\ + \gamma_t^i(s^t) \Pi_{s^t} [\{y^i\}] \end{array} \right] \right\}$$

such that

$$\zeta_t^i(s^t) = \zeta_{t-1}^i(s^{t-1}) - \gamma_t^i(s^t), \quad \zeta_0^i = 1$$

Then the **recursive dual** saddle point problem is given by:

$$\inf_{\{c^i, x^i\}} \sup_{\{\zeta^i\}} D(c^i, x^i, \zeta^i; w_0^i, s_0, h_0^i) \quad (37)$$

such that

$$\sum_{t \geq 0} \sum_{s^t} \delta^t \pi(s^t|s_0) u(c_t^i(s^t), h_t^i(s^t)) \geq w_0^i$$

To keep the mechanics of the model in line with standard practice, we re-scale the multipliers. Let $\xi_t^i(s^t) = \kappa^i / \zeta_t^i(s^t)$, where κ^i is the multiplier on the promise keeping constraint. The cumulative multiplier ξ^i is a non-decreasing stochastic sequence (sub-martingale). If the constraint binds, it goes up, else it stays put.

First Order Necessary Conditions The f.o.c. for c^i is :

$$\hat{\mu}(s^t) = \xi_t^i(s^t) u_c(c_t^i(s^t), h_t^i(s^t)).$$

Upon division of the first order condition of agent i and j , the following restriction on the joint evolution of marginal utilities over time and across states must hold:

$$\frac{u_c(c_t^i(s^t), h_t^i(s^t))}{u_c(c_t^j(s^t), h_t^j(s^t))} = \frac{\xi_t^j(s^t)}{\xi_t^i(s^t)}.$$

Growth rates of marginal utility of non-durable consumption, weighted by the multipliers, are equalized across agents:

$$\frac{\xi_{t+1}^i}{\xi_t^i} \frac{u_c(c_{t+1}^i, h_{t+1}^i)}{u_c(c_t^i, h_t^i)} = \frac{\hat{\mu}_{t+1}}{\hat{\mu}_t} = \frac{\xi_{t+1}^j}{\xi_t^j} \frac{u_c(c_{t+1}^j, h_{t+1}^j)}{u_c(c_t^j, h_t^j)}.$$

The time zero ratio of marginal utilities is pinned down by the ratio of multipliers on the promise-keeping constraints. For $t > 0$, it tracks the stochastic weights ξ^i . From the first order condition w.r.t. ξ^i we obtain a reservation weight policy:

$$\xi_t^i(s^{t+1}) = \begin{cases} \underline{\xi}_t^i(s^{t+1}) & \text{if } \xi_t^i(s^t) < \underline{\xi}_t^i(s^{t+1}) \\ \xi_t^i(s^t) & \text{elsewhere} \end{cases} \quad (39)$$

and the collateral constraints hold with equality at the bounds:

$$\Pi_{s^t} \left[\left\{ c_t(s^t; \underline{\xi}^i) + \rho h^i(s^t; \underline{\xi}^i) \right\} \right] = \Pi_{s^t} \left[\left\{ y^i \right\} \right].$$

Equivalence AD-Equilibrium and Efficient Allocation The Arrow-Debreu competitive equilibrium is equivalent to the constrained efficient allocation from the planner problem for $\mu = \tilde{\mu}$ and $\rho = \tilde{\rho}$. The equivalence arises from two facts. First, the first-order conditions of the Arrow-Debreu equilibrium correspond to the first-order conditions of the planner problem. Second, the collateral constraints in equation (33) correspond to the participation constraints in the planner economy. Note that the cumulative multiplier processes on the collateral constraints are the Pareto-Negishi weights in the planner problem.

A.2 Euler Equation and Stochastic Discounter

This appendix derives the consumption Euler equation and the expression for the stochastic discount factor in the economy with housing market frictions.

Euler equation The relative price of housing services ρ_t^i is the marginal rate of substitution between housing and consumption.

$$\rho_t^i = \frac{p_t^{ih}}{p_t^{ic}} = \psi \left(\frac{h_t^i}{c_t^i} \right)^{\sigma-1} e^{\sigma(b_t^{i,h} - b_t^{i,c})}$$

A_t^i denotes the fraction of housing consumption to non-durable consumption and housing consumption of agent i at time t :

$$A_t^i = \frac{p_t^{ih} h_t^i}{p_t^{ic} c_t^i + p_t^{ih} h_t^i} = \frac{\rho_t^i h_t^i}{c_t^i + \rho_t^i h_t^i}$$

Using the definition of ρ , the marginal utility of non-durable consumption can be manipulated to get:

$$\begin{aligned} u_c(c_t, h_t, b_t) &= \left(e^{b_t^{i,c}} \right)^\sigma c_t^{\sigma-1} \left[\left(e^{b_t^{i,c}} c_t \right)^\sigma + \psi \left(e^{b_t^{i,c}} h_t \right)^\sigma \right]^{\frac{1-\gamma-\sigma}{\sigma}} \\ &= \left(e^{b_t^{i,c}} \right)^\sigma c_t^{\sigma-1} \left(e^{b_t^{i,c}} c_t \right)^{1-\sigma-\gamma} \left[1 + \psi \left(\frac{e^{b_t^{i,h}}}{e^{b_t^{i,c}}} \right)^\sigma \left(\frac{h_t}{c_t} \right)^\sigma \right]^{\frac{1-\gamma-\sigma}{\sigma}} \\ &= \left(e^{b_t^{i,c}} \right)^{1-\gamma} c_t^{-\gamma} \left[1 + \psi \left(\frac{e^{b_t^{i,h}}}{e^{b_t^{i,c}}} \right)^\sigma \left(\frac{e^{b_t^{i,h}}}{e^{b_t^{i,c}}} \right)^{\frac{-\sigma}{\sigma-1}} \left(\frac{\rho_t^i}{\psi} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{1-\gamma-\sigma}{\sigma}} \\ &= \left(e^{b_t^{i,c}} \right)^{1-\gamma} c_t^{-\gamma} \left[1 + \psi^{\frac{-1}{\sigma-1}} \left(\frac{e^{b_t^{i,h}}}{e^{b_t^{i,c}}} \right)^{\frac{-\sigma}{\sigma-1}} \left(\rho_t^i \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{1-\gamma-\sigma}{\sigma}} \end{aligned}$$

Use this expression in the first order condition for non-durable consumption

$$\begin{aligned} \log(q_t) &= \log(\xi_t^i) + (1-\gamma)b_t^{i,c} - \gamma \log(c_t^i) + \\ &\quad \left(\frac{1-\gamma-\sigma}{\sigma} \right) \log \left[1 + \psi^{\frac{-1}{\sigma-1}} \left(\frac{e^{b_t^{i,h}}}{e^{b_t^{i,c}}} \right)^{\frac{-\sigma}{\sigma-1}} (\rho_t^i)^{\frac{\sigma}{\sigma-1}} \right] \end{aligned} \quad (40)$$

Denoting first differences by Δ , we have

$$\begin{aligned} \Delta \log(c_t^i) &= \frac{1}{\gamma} \left[\Delta \log(\xi_t^i) - \Delta \log(q_t) \right] + \left(\frac{1-\gamma}{\gamma} \right) \Delta b_t^{i,c} + \\ &\quad \left(\frac{1-\gamma-\sigma}{\sigma\gamma} \right) \Delta \log \left[1 + \psi^{\frac{-1}{\sigma-1}} \left(\frac{e^{b_t^{i,h}}}{e^{b_t^{i,c}}} \right)^{\frac{-\sigma}{\sigma-1}} (\rho_t^i)^{\frac{\sigma}{\sigma-1}} \right] \end{aligned} \quad (41)$$

We *approximate* the non-linear term in the marginal utility using the following two facts: (1) approximative equivalence of log changes and percentage changes: if $x_{t+1} > x_t$ then $\frac{x_{t+1}-x_t}{x_{t+1}} < \Delta \log(x_{t+1}) < \frac{x_{t+1}-x_t}{x_t}$, and (2) $\% \Delta(1+x_{t+1}) = \frac{x_{t+1}-x_t}{x_t} \frac{x_t}{1+x_t}$.

The log change in the bracketed term is approximately equal to

$$\begin{aligned} &\approx \% \Delta \left[\psi^{\frac{-1}{\sigma-1}} \left(\frac{e^{b_t^{i,h}}}{e^{b_t^{i,c}}} \right)^{\frac{-\sigma}{\sigma-1}} (\rho_t^i)^{\frac{\sigma}{\sigma-1}} \right] \left(\frac{\psi^{\frac{-1}{\sigma-1}} \left(\frac{e^{b_{t-1}^{i,h}}}{e^{b_{t-1}^{i,c}}} \right)^{\frac{-\sigma}{\sigma-1}} (\rho_{t-1}^i)^{\frac{\sigma}{\sigma-1}}}{1 + \psi^{\frac{-1}{\sigma-1}} \left(\frac{e^{b_{t-1}^{i,h}}}{e^{b_{t-1}^{i,c}}} \right)^{\frac{-\sigma}{\sigma-1}} (\rho_{t-1}^i)^{\frac{\sigma}{\sigma-1}}} \right) \\ &\approx \frac{\sigma}{\sigma-1} A_{t-1}^i \left[\Delta \log(\rho_t^i) - \Delta b_t^{i,h} + \Delta b_t^{i,c} \right] \end{aligned}$$

When taking averages across agents and subtracting the expression for the average from the expression for individual consumption, changes in the state price deflator drop out as a time fixed effect. Idiosyncratic consumption growth is a function of $\hat{A}_{t-1}^i \Delta \log(\hat{\rho}_t^i)$, $\Delta \hat{b}_t^{i,c}$ and $\Delta \hat{b}_t^{i,h}$:

$$\Delta \log(\hat{c}_t^i) \approx \frac{1}{\gamma} \Delta \log(\hat{\xi}_t^i) - \frac{\phi}{\gamma} \hat{A}_{t-1}^i \Delta \log(\hat{\rho}_t^i) + \frac{1}{\gamma} \nu_t^{i,c} \quad (42)$$

where $\nu_t^{i,c}$ contains the preference shifters.

$$\begin{aligned} \nu_t^{i,c} &= (1-\gamma) \Delta \hat{b}_t^{i,c} - \phi \hat{A}_{t-1}^i \Delta \hat{b}_t^{i,c} + \phi \hat{A}_{t-1}^i \Delta \hat{b}_t^{i,h} \\ \phi &= \left(\frac{1-\gamma-\sigma}{1-\sigma} \right) \end{aligned}$$

It is understood that

$$\begin{aligned} \hat{A}_{t-1}^i \Delta \log(\hat{\rho}_t^i) &= A_{t-1}^i \Delta \log(\rho_t^i) - \frac{1}{I} \sum A_{t-1}^i \Delta \log(\rho_t^i) \\ &= A_{t-1}^i \Delta \log(\rho_t^i) - A_{t-1}^a \Delta \log^a(\rho_t) \end{aligned}$$

and likewise for $\hat{A}_{t-1}^i \Delta \hat{b}_t^i$.

Intertemporal Marginal Rate of Substitution The marginal utility of non-durables can be written as a function of c_t^i and A_t^i :

$$u_c(c_t, h_t, b_t) = \left(e^{b_t^{i,c}} \right)^{1-\gamma} \left(c_t^i \right)^{-\gamma} \left(1 - A_t^i \right)^{-\frac{1-\gamma-\sigma}{\sigma}}$$

The log of the individual IMRS is

$$\log m_{t+1}^i = \log \delta + (1 - \gamma)\Delta b_{t+1}^{i,c} - \gamma\Delta \log(c_{t+1}^i) - \left(\frac{1 - \gamma - \sigma}{\sigma}\right)\Delta \log(1 - A_{t+1}^i)$$

Using the approximation described above, we write the last term as a function of rental price changes.

$$\begin{aligned} \log m_{t+1}^i &\approx \log \delta - \gamma\Delta \log(c_{t+1}^i) - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)A_t^i\Delta \log(\rho_{t+1}^i) \\ &+ (1 - \gamma)\Delta b_{t+1}^{i,c} - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)A_t^i\Delta b_{t+1}^{i,c} + \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)A_t^i\Delta b_{t+1}^{i,h} \end{aligned}$$

Taking averages across agents

$$\begin{aligned} \log m_{t+1}^a &\approx \log \delta - \gamma\Delta \log^a(c_{t+1}) - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)A_t^a\Delta \log^a(\rho_{t+1}) \\ &+ (1 - \gamma)\Delta b_{t+1}^{a,c} - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)A_t^a\Delta b_{t+1}^{a,c} + \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)A_t^a\Delta b_{t+1}^{a,h} \end{aligned}$$

Therefore,

$$\begin{aligned} \log m_{t+1}^i &\approx \log m_{t+1}^a - \gamma\Delta \log(\hat{c}_{t+1}^i) - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \log(\hat{\rho}_{t+1}^i) \\ &+ (1 - \gamma)\Delta \hat{b}_{t+1}^{i,c} - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \hat{b}_{t+1}^{i,c} + \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \hat{b}_{t+1}^{i,h} \end{aligned}$$

Substituting for the idiosyncratic consumption growth expression (42) from the previous section:

$$\begin{aligned} \log m_{t+1}^i &\approx \log m_{t+1}^a - \Delta \log(\hat{\xi}_t^i) + \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \log(\hat{\rho}_{t+1}^i) - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \log(\hat{\rho}_{t+1}^i) \\ &- (1 - \gamma)\Delta \hat{b}_{t+1}^{i,c} - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \hat{b}_{t+1}^{i,c} - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \hat{b}_{t+1}^{i,h} + \\ &(1 - \gamma)\Delta \hat{b}_{t+1}^{i,c} - \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \hat{b}_{t+1}^{i,c} + \left(\frac{1 - \gamma - \sigma}{1 - \sigma}\right)\hat{A}_t^i\Delta \hat{b}_{t+1}^{i,h} \end{aligned}$$

which simplifies to

$$\log m_{t+1}^i \approx \log m_{t+1}^a - \Delta \log(\hat{\xi}_t^i)$$

A.3 Bootstrap Procedure

The bootstrap procedure addresses the persistent-regressor bias and serial correlation in the OLS residuals. We compute small-sample coefficient estimates and small-sample p-values for the null hypothesis of no predictability.

A univariate specification test shows that my is best described by an AR(2) process. If annual returns are unforecastable, the data generating process for $(r_{t+1}^{vw,e}, my_{t+1})$ is

$$r_{t+1}^{vw,1} = b_0 + e_{t+1}^1 \quad (43)$$

$$my_{t+1} = c_0 + c_1 my_t + c_2 my_{t-1} + e_{t+1}^2 \quad (44)$$

where e is a bivariate i.i.d. mean zero process. Under the no-predictability null, K -period returns have a MA(K) error structure because of overlapping observations. We estimate the long-horizon excess return regressions for $K = 1, 2, \dots, 10$:

$$r_{t+K}^{vw,K} = b_0 + b_1 my_t + e_{t+K}^1 + \phi_1 e_{t+K-1}^1 + \phi_2 e_{t+K-2}^1 + \dots + \phi_{K-1} e_{t+1}^1. \quad (45)$$

The bootstrap method consists of the following steps.

step 1 Jointly estimate the coefficient on my and the $K - 1$ moving average coefficients in the $r^{vw,K}$ and the coefficients in the AR(2) specification for my .

step 2 Draw a sample of length T with replacement from (e_t^1, e_t^2) .

step 3 For given $my_0 = my_1 = 0$ and parameter estimates from step 1, build up time series for $r^{e,K}$ and my_{t+1} recursively from equations (45) and (44).

step 4 Estimate the coefficients in the return equation. Let the coefficient on my be b_1^* .

step 5 Repeat steps 1 through 4 $N = 5,000$ times.

The small sample bias equals $\frac{1}{N} \sum b_{1n}^* - b_1^{LS}$.

The second bootstrap exercise proceeds as the first, except it imposes the null hypothesis of no predictability in step 1. In step 4, let the coefficient on my be b_1^{**} . The p-value is the frequency of observing estimates b_1^{**} smaller than the least-squares estimate b_1^{LS} .

A.4 Iterative Fama-MacBeth Procedure

Standard Fama-MacBeth The estimation problem is $E[R_{t+1}^{e,j}] = \tilde{\lambda} \tilde{\beta}^{j'}$ with $\tilde{\lambda} = \theta Cov(F_{t+1}, F'_{t+1})$ and $\tilde{\beta}^j = Cov(F_{t+1}, F'_{t+1})^{-1} Cov(F_{t+1}, R_{t+1}^{e,j})$.

First, for each j , the vector $\tilde{\beta}^j$ is obtained from the time-series regression of returns on the factors. Given the limited length of the time series, the β are estimated using 1 regression over the entire sample instead of a rolling regression.

$$R_t^{e,j} = \beta_0^j + \tilde{\beta}_{cm}^{j'} \bar{f}_t^{cm} + \tilde{\beta}_{lc}^{j'} \bar{f}_t^{lc} + \epsilon_t^j \quad t = 1, 2, \dots, T \quad (46)$$

Let $\Sigma = E[\epsilon_t \epsilon_t']$ be the $N \times N$ covariance matrix of the errors $\epsilon_t = [\epsilon_t^1, \dots, \epsilon_t^N]$. Second, for each t , a cross-sectional regression of returns on the estimated $\tilde{\beta}^j$ uncovers estimates for $(\tilde{\lambda}_{cm}, \tilde{\lambda}_{lc})$ and the zero- β return $\tilde{\lambda}_0$:

$$R_t^{e,j} = \tilde{\lambda}_0 + \tilde{\beta}_{cm,t}^{j'} \tilde{\lambda}_{cm,t} + \tilde{\beta}_{lc,t}^{j'} \tilde{\lambda}_{lc,t} + \alpha_t^j \quad j = 1, 2, \dots, J \quad (47)$$

The estimator for the price of risk is the time series average of the estimated second stage coefficients: $\tilde{\lambda} = \frac{1}{T} \sum_{t=1}^T \tilde{\lambda}_t$. The α_t^j are the pricing errors, $\hat{\alpha}^j = \frac{1}{T} \sum_{t=1}^T \alpha_t^j$. The sampling error for $\hat{\lambda}$ and the covariance matrix for $\hat{\alpha}$ are given by:

$$\begin{aligned} var(\hat{\lambda}_{FMB}) &= \frac{1}{T^2} \left[\sum_{t=1}^T (\hat{\lambda}_t - \hat{\lambda})^2 \right] \\ cov(\hat{\alpha}_{FMB}) &= \frac{1}{T^2} \left[\sum_{t=1}^T (\hat{\alpha}_t - \hat{\alpha})(\hat{\alpha}_t - \hat{\alpha})' \right] \end{aligned}$$

The pricing errors are the basis for a goodness of fit statistic $\hat{\alpha} cov(\hat{\alpha})^{-1} \hat{\alpha}$. In a regression with K factors the statistic has a χ^2 distribution with $J - K$ degrees of freedom. A second measure of fit is the R^2 constructed from the cross-sectional variance of the time-averages (denoted by a bar) of errors and returns for each of the portfolios:

$$R_{FMB}^2 = 1 - \frac{var(\bar{\hat{\alpha}}_t^i)}{var(\bar{R}_{t+1}^i - \bar{R}_t^i)}$$

The β 's are generated regressors from a first stage time-series analysis. Generally, this error in variables problem gives rise to an underestimation of the asymptotic variance-covariance matrix. The standard errors for λ and α are corrected following Shanken (1992). Let the matrix Σ_f be the covariance matrix of the factors F . The Shanken correction to the variance of the estimator $\hat{\lambda}$ and the covariance matrix of pricing errors α is:

$$\begin{aligned} Var(\hat{\lambda}_{corr}) &= Var(\hat{\lambda}_{uncorr}) \left(1 + \hat{\lambda}' \Sigma_f^{-1} \hat{\lambda} \right) + \frac{1}{T} \Sigma_f \\ cov(\hat{\alpha}_{corr}) &= \left(1 + \hat{\lambda}' \Sigma_f^{-1} \hat{\lambda} \right) cov(\hat{\alpha}_{uncorr}). \end{aligned}$$

Cochrane (2001) (pp.241-242) describes a GMM procedure that carries out the time series and cross-sectional

estimation jointly. It corrects for serial correlation and conditional heteroskedasticity in the residuals ϵ_t^i and for correlation of α_t^i across assets.

Iterative Algorithm With limited commitment, the unconstrained agent is not known and her identity changes over time. We propose the following fixed-point algorithm. Make an initial guess for the parameter vector, θ^0 . Given θ^0 , the log IMRS for each agent and each period is $\log m_{t+1}^i = -\theta^0 F^i(\theta^0)$. Second, for each t , find the unconstrained agent $i_t^* = \arg \max \{m_t^i\}$. The SDF in period t is $m_t = m_t^{i_t^*}$. Third, knowledge of $\{i_t^*\}_{t=1}^T$ enables construction of the asset pricing factors $\{F_t^{i_t^*}\}$. Fourth, apply the two-step Fama-MacBeth procedure described above. This gives estimates for the market price of risk $\tilde{\lambda}$. Fifth, map $\tilde{\lambda}$ back into a parameter vector θ

$$\theta = \text{Cov}(F_{t+1}, F'_{t+1})^{-1} \tilde{\lambda}$$

This is a new guess for the parameter vector, θ^1 . It generally implies different values for the factors $\{F_t^{i_t^*}\}_{t=1}^T$ because $\{i_t^*\}_{t=1}^T$ is different. We repeat the steps 2-5 until a fixed point of the map $\theta^{k+1} = \mathcal{T}(\theta^k)$ is found. Convergence on θ is a consistency requirement that the estimated market prices of risk imply the same sequence of unconstrained agents that were used to compute them.

Continuation Method In general, fixed-point iterations are not globally convergent. The fixed point problem above is potentially poorly behaved because in every iteration the sequence of unconstrained agents $\{i_t^*\}_{t=1}^T$ and hence the relevant asset pricing factors changes. The *max*-operator, which identifies the unconstrained agent, violates standard regularity conditions. We approximate the max-operator by a differentiable function:

$$m_t = \max \{m_t^i\} \approx \frac{\exp(\Lambda \log m_t^i)}{\sum_{i=1}^I \exp(\Lambda \log m_t^i)} m_t^i. \quad (48)$$

For $\Lambda = \infty$, we select the agent with the maximum IMRS, for $\Lambda = 0$ the SDF is an equal weighting of all agents' IMRS.

A continuation method uses local convergence properties to improve chances of global convergence (see Judd (1998)). The idea is to construct a sequence of fixed point problems that ultimately leads to the problem of interest. A natural starting point is $\theta = \mathcal{T}(\theta; \Lambda)$, with $\Lambda = 0$. Equal weighting cancels the asset pricing factors under limited commitment: $\bar{f}^{lc} = 0$. Only aggregate factors \bar{f}^{cm} matter and we are in the case of perfect enforcement. We use the solution θ^{*1} to the fixed point problem $\theta = \mathcal{T}(\theta; \Lambda^0)$ as an initial guess to the problem $\theta = \mathcal{T}(\theta; \Lambda^1)$. We solve a sequence of intermediary problems for $0 = \Lambda^0 < \Lambda^1 < \dots < \Lambda^k < \Lambda^{k+1} < \dots < \Lambda^K$, using the solution of the Λ^k problem as an initial guess to the Λ^{k+1} problem. Each intermediary problem satisfies the consistency requirement and can be solved quickly. The algorithm terminates when

$$\frac{1}{T} \sum_{t=1}^T \max_i \frac{\exp(\Lambda \log m_t^i)}{\sum_{i=1}^I \exp(\Lambda \log m_t^i)} > c$$

In every period, the unconstrained agent receives a weight of $c \in [0, 1]$ on average.

Sensitivity Analysis for Regional Asset Pricing Results The stochastic discount factor is a weighted average of the intertemporal marginal rates of substitution (IMRS) of all regions. When all regions receive equal weight in the average we are in the perfect commitment world. With 23 regions, the region with the highest predicted IMRS receives an average weight of $\frac{1}{23}$. Equal weighting obtains for $\Lambda = 0$. When, in every period, the region with the largest intertemporal marginal rate of substitution receives a weight of one we have implemented equation (14). Giving the unconstrained region weight one corresponds to $\Lambda = +\infty$. In between these two polar cases, we give a strictly positive weight to other regions according to the function (48).

In table 5 of section 7.2.3, we choose $\Lambda = 20$, which implies that the unconstrained region receives a weight of .9 on average. In table 9 we solve the fixed point problem for a range of Λ 's from 0.1 to 20. Each row of table 9

Λ	max-index (%)	R^2	$\tilde{\theta}_1$	$\tilde{\theta}_2$	$\tilde{\theta}_3$	$\tilde{\theta}_4$	$\tilde{\theta}_5$	$\tilde{\theta}_6$
.25	14	56.2	.32	.64	.10	-.52	-3.21	-1.91
.50	19	58.2	.28	.67	.03	-.36	2.17	-1.32
.75	25	60.7	.23	.69	-.00	-.31	1.78	-1.09
1	34	68.8	.26	.98	-.22	-.17	2.85	-1.03
2	56	70.6	.14	.99	-.16	-.22	2.69	-.87
3	70	71.2	.08	.98	-.12	-.25	2.68	-.80
5	81	71.4	.08	.96	-.09	-.27	2.66	-.76
10	90	71.1	.05	.96	-.08	-.28	2.66	-.76
15	92	71.1	.09	.96	-.07	-.29	2.64	-.77
20	94	71.2	.09	.96	-.07	-.29	2.61	-.76

Table 9: Cross-Sectional Results with Regional Asset Pricing Factors: Sensitivity Analysis. The first column is the parameter Λ in the temperature function (see iterative Fama-MacBeth procedure in A.4). The second column reports the average (over time) weight on the unconstrained region’s (predicted) intertemporal marginal rate of substitution in the stochastic discount factor. The third column reports the R^2 for the second stage of the Fama-MacBeth procedure, evaluated at the fixed point of the iteration. The other rows report the coefficient estimates (the fixed point $\tilde{\theta}$ itself). The period is 1951-2001. The set of test assets is $T1$.

shows the estimates for the underlying parameters $\tilde{\theta}$, for a different parameter Λ . The two main findings are: (i) the fit of regional asset pricing factors improves by increasing Λ and (ii) both the parameter estimates and the R^2 remain virtually identical beyond $\Lambda \geq 2$. That is, for a large range of weights on the region that is *predicted* to be unconstrained, we obtain similar results from the estimation procedure (parameter estimates and R^2).

A.5 Data Appendix

Aggregate Housing Collateral For, 1889-2001, residential non-farm mortgage debt outstanding (series N262) is based on Grebler-Blank-Winnick “Capital Formation in Residential Real Estate: Trends and Prospects”, 1956. Non-farm residential real estate (series N196) includes land and structures. Estimates are for housekeeping dwellings and exclude hotels, dormitories, motels, clubs, etc. The series is based on David L. Wickens’ “Residential Real Estate”, NBER, 1941. For 1945-2001, home mortgages are loans secured by one-to four-family properties, including owner-occupied condominium units. The total includes second mortgages on these properties and home-equity loans. We only consider the household sector. The market value of real estate wealth (table B.100, row 4) includes land and structures, inclusive vacant land, vacant homes for sale, second homes and mobile homes.

Metropolitan Area The concept of a metropolitan areas is that of a core area containing a large population nucleus, together with adjacent communities having a high degree of economic and social integration with that core. They include metropolitan statistical areas (MSA’s), consolidated metropolitan statistical areas (CMSA’s), and primary metropolitan statistical areas (PMSA’s). An area that qualifies as an MSA and has a population of one million or more may be recognized as a CMSA if separate component areas that demonstrate strong internal, social, and economic ties can be identified within the entire area and local opinion supports the component areas. Component areas, if recognized, are designated PMSA’s. If no PMSA’s are designated within the area, then the area remains an MSA.

The S&MM survey uses the definitions of MSA throughout the survey and of CMSA when CMSA’s are created. We use the 30 metropolitan areas described in table 10. Before the creation of the CMSA’s, we keep track of all separate MSA’s that later form the CMSA in order to obtain a consistent time series. For example, the Dallas-Forth Worth CMSA consists of the Dallas MSA and Forth Worth MSA until 1973 and of the combined area thereafter.

The total number of households in the 30 metropolitan areas is 47 percent of the US total in 2000 compared to 40 percent in 1951. The total number of households are from the Bureau of the Census. Most of the increase occurs before 1965. Likewise, the 30 metropolitan areas we consider contain exactly 47 percent of the population in 1999 (see tables 10 and 12, first column).

Table 10: Metropolitan Areas. Total population numbers (in thousands) are displayed next to the metropolitan areas. For the Consolidated Metropolitan areas (CMSA), the constituent MSA's are listed and the fraction of their population in the total of the CMSA is shown next to their name. All numbers are from the Regional Economic Information System of the Bureau of Economic Analysis for the year 2000.

Anchorage (AK), MSA	261	Miami CMSA	3,897
Atlanta (GA), MSA	4,145	Miami, FL	58.1%
Baltimore (MD), MSA	2,557	Fort Lauderdale, FL	41.9%
Boston CMSA	6,068	Milwaukee CMSA	1,691
Boston, MA-NH	58.6%	Milwaukee-Waukesha, WI	88.8%
Worcester, MA-CT	8.7%	Racine, WI	11.2%
Lawrence, MA-NH	6.7%	Minneapolis (MN-WI) MSA	2,797
Lowell, MA-NH	5.1%	New York CMSA	21,134
Brockton, MA	4.3%	New York, NY	45.5%
Portsmouth-Rochester, NH-ME	4.2%	Bergen-Passaic, NJ	6.6%
Manchester, NH	3.4%	Bridgeport, CT	0.5%
Nashua, NH	3.3%	Dutchess County, NY	1.2%
New Bedford, MA	3.2%	Danbury, CT	0.4%
Fitchburg-Leominster, MA	2.5%	Jersey City, NJ	3.0%
Buffalo (NY), MSA	1,169	Middlesex-Somerset-Hunterdon, NJ	5.6%
Chicago CMSA	9,176	Monmouth-Ocean, NJ	5.4%
Chicago, IL	90.3%	Nassau-Suffolk, NY	13.5%
Gary, IN	6.9%	Newburgh, NY-PA	1.8%
Kenosha, WI	1.6%	Newark, NJ	9.9%
Kankakee, IL	1.1%	New Haven-Meriden, CT	6.2%
Cincinnati CMSA	1,983	Stamford-Norwalk, CT	0.6%
Cincinnati, OH-KY-IN	92.6%	Trenton, NJ	1.7%
Hamilton-Middletown, OH	7.4%	Waterbury, CT	0.5%
Cleveland CMSA	2,946	Philadelphia CMSA	6,194
Cleveland-Lorain-Elyria, OH	76.4%	Philadelphia, PA-NJ	82.4%
Akron, OH	23.6%	Wilmington, NC	9.5%
Dallas CMSA	5,254	Atlantic-Cape May, NJ	5.7%
Dallas, TX	67.4%	Vineland-Millville-Bridgeton, NJ	2.3%
Fort Worth-Arlington, TX	32.6%	Phoenix - Mesa MSA	3,276
Denver CMSA	2,597	Pittsburgh (PA), MSA	2,356
Denver, CO	81.7%	Portland CMSA	2,273
Boulder-Longmont, CO	11.3%	Portland-Vancouver, OR-WA	84.7%
Greeley, CO	7.0%	Salem, OR	15.3%
Detroit CMSA	5,463	Saint Louis (MO-IL), MSA	2,606
Detroit, MI	81.4%	San Diego (CA), MSA	2,825
Ann Arbor, MI	10.6%	San Francisco CMSA	7,056
Flint, MI	8.0%	San Francisco, CA	24.6%
Honolulu (HI), MSA	876	San Jose, CA	23.9%
Houston CMSA	4,694	Oakland, CA	34.1%
Houston, TX	89.5%	Vallejo-Fairfield-Napa, CA	7.4%
Galveston-Texas City, TX	5.3%	Santa Cruz-Watsonville, CA	3.6%
Brazoria, TX	5.2%	Santa Rosa, CA	6.5%
Kansas City (MO-KS), MSA	1,782	Seattle CMSA	3,562
Los Angeles CMSA	16,440	Seattle-Bellevue-Everett, WA	67.9%
Los Angeles-Long Beach, CA	58.1%	Tacoma, WA	19.8%
Orange County, CA	17.4%	Bremerton, WA	6.5%
Riverside-San Bernardino, CA	20.0%	Olympia, WA	5.8%
Ventura, CA	4.6%	Tampa (FL), MSA	2,404
		Washington,DC-MD-VA-WV, PMSA	4,948

Regional Housing Collateral Following Case, Quigley and Shiller (2001), we construct the market value of the housing stock in region i as the product of four components:

$$HV_t^i = N_t^i HO_t^i HP_t^i V_0^i$$

V_0^i is the median house price for detached single family housing from the US Bureau of the Census for 2000. For the CMSA's, it is constructed as a population weighted average of the median home value for the constituent MSA's. Population data are from the REIS.

HP_t^i is the housing price index from the Office of Federal Housing Enterprise Oversight, based on the weighted repeat sales method of Case and Shiller (1987). It measures house price increases in detached single family homes between successive sales or mortgage refinancing of the identical housing unit. The index is available from 1975 onwards for all MSA's. We construct an index for the CMSA's as a population weighted average of the MSA's.

There is a literature on quality-controlled house price indices. They broadly fall into two categories. Hedonic methods capture the contribution of narrowly defined dwelling unit and location characteristics to the price of a house in a certain region (number of bedrooms, garage, neighborhood safety, school district, etc.). Out of sample, houses are priced as a bundle of such characteristics. Repeat sales indices are based on houses that have been sold or appraised twice. Because they pertain to the same property, they control for a number of hedonic characteristics (bedrooms, neighborhood safety, etc.). See Pollakowski (1995) for a literature review and a description of data availability. The OFHEO database contains 17 million transactions over the last 27 years.

Home ownership rates HO_t^i are from the US. Bureau of the Census. We combine home ownership rates for 1980, 1990 and 2000 from the Decennial Census with annual home ownership data for the largest 75 cities for 1986-2001, also from the Bureau of the Census. We project a home ownership rate for 1986 using the 1980 and 1990 number and the annual changes in the national home ownership rate. We use the changes in the major cities to infer MSA-level changes between 1986 and 1990. Between 1981 and 1986 and 1975 and 1979 we apply national changes to the MSA's. This procedure captures most of the regional and time series behavior of home-ownership rates. Figure 8 shows a gradual increase in the US home-ownership starting in 1965, only interrupted by a decline in the period 1980-95. Table 11 illustrates the large regional differences in the median home value and home ownership rate in 1980 and 2000.

Regional collateral values contain a large common component. A principal component analysis on the real variable $hv_t^i = HV_t^i/p_t^{i,a}$ shows that the first principal component explains 74% of the total variation in regional housing values. The largest three principal components explain 95% of the variation. The first principal component of the region-specific \widehat{hv}_{it} is 55 percent and the first 3 principal components account for 92 percent of the variation in the idiosyncratic collateral value.

Deflation All variables are expressed in real terms. We deflate by the region-specific price index. Let $p_t^{i,a}$ be the price level and $\pi_t^{i,a}$ be the inflation rate at time t for all items in the consumption basket of region i . Likewise, let $p_t^{i,c}$ ($p_t^{i,h}$) be the price level and $\pi_t^{i,c}$ ($\pi_t^{i,h}$) be the inflation rate at time t for non-durable consumption (housing consumption) of agent i . The nominal value of the housing stock is deflated by $p_t^{i,a}$. The shadow rental price ρ_t^i equals $p_t^{i,h}/p_t^{i,c}$. Denote real per household quantities by lower case letters. The change in the idiosyncratic component of income and consumption is defined as

$$\begin{cases} \Delta \log(\hat{y}_t^i) = (\Delta \log(p_t^{i,a} y_t^i) - \pi_t^{i,a}) - \frac{1}{I} \sum_{i=1}^I (\Delta \log(p_t^{i,a} y_t^i) - \pi_t^{i,a}) \\ \Delta \log(\hat{c}_t^i) = (\Delta \log(p_t^{i,c} c_t^i) - \pi_t^{i,c}) - \frac{1}{I} \sum_{i=1}^I (\Delta \log(p_t^{i,c} c_t^i) - \pi_t^{i,c}) \end{cases}$$

Consumption and Income in Detail We collect data from the Survey of Buying Power (and Media Markets), a special September issue of the magazine Sales and Marketing Management. The data are proprietary and we thank S&MM for permission to use them. We use five series and reproduce the S&MM definitions below.

Total retail sales measures sales from five major store groups considered to be the primary channels of distribution for consumer goods in local markets. Store group sales represent the cumulative sales of all products and or services handled by a particular store type, not just the product lines associated with the name of the store group. The five store groups are: food stores, automotive dealers, eating and drinking places, furniture, home furnishings and appliance stores, and general merchandize stores. Total retail sales reflect net sales. Receipts from repairs and other services by retailers are also included, but retail sales by wholesalers and service establishments are not.

Table 11: Median Home Value and Home-Ownership Rate. The table shows median home values for 1980 and 2000 (in thousands of nominal dollars) and the home ownership rate for 1980 and 2000. All data are from the US Bureau of the Census, Decennial Survey 1980 and 2000

MSA	V ₈₀	V ₀₀	HO ₈₀	HO ₀₀
Washington, DC (PMSA)	79.9	178.9	54.3	64.0
Baltimore, MD (PMSA)	51.4	134.9	60.0	66.9
Atlanta, GA (MSA)	47.7	135.3	61.4	66.4
Miami, FL (CMSA)	57.0	126.1	61.5	63.2
Dallas-Fort Worth, TX (CMSA)	45.6	100.0	64.7	60.4
Houston, TX (CMSA)	52.8	89.7	59.1	60.7
Tampa, FL (MSA)	39.9	93.8	71.7	70.8
San Francisco, CA (CMSA)	98.4	353.5	55.8	57.8
Los Angeles, CA (CMSA)	87.6	203.3	53.8	54.8
San Diego, CA (MSA)	90.0	227.2	55.1	55.4
Portland, OR (CMSA)	60.8	165.4	63.2	63.0
Seattle, WA (CMSA)	66.0	195.4	63.8	62.9
Honolulu, HI (MSA)	129.5	309.0	49.9	54.6
Anchorage, AK (MSA)	89.2	160.7	56.6	60.1
Denver, CO (CMSA)	69.1	179.5	63.0	66.4
Phoenix, AZ (MSA)	59.2	127.9	68.7	68.0
New York, NY (CMSA)	62.5	203.1	44.2	53.0
Philadelphia, PA (CMSA)	42.2	122.3	67.7	69.9
Boston, MA (CMSA)	52.0	203.0	54.8	60.6
Pittsburgh, PA (MSA)	42.7	68.1	69.0	71.3
Buffalo, NY (MSA)	39.7	89.1	63.7	66.2
Chicago, IL (CMSA)	62.8	159.0	58.5	65.2
Detroit, MI (CMSA)	43.5	132.6	70.2	72.2
Milwaukee, WI (CMSA)	59.2	131.9	61.1	62.1
Minneapolis-St. Paul, MN (MSA)	62.3	141.2	67.2	72.4
Cleveland, OH (CMSA)	52.1	117.9	66.6	68.8
Cincinnati, OH (CMSA)	47.9	116.5	63.8	67.1
St. Louis, MO (MSA)	41.8	99.4	68.2	71.4
Kansas City, MO-KS (MSA)	43.5	104.7	66.4	67.9
Tampa, FL (MSA)	59,925	85,248	73.0	71.0

Automotive dealer sales are sales by retail establishments primarily engaged in selling new and used vehicles for personal use and in parts and accessories for these vehicles. This includes boat and aircraft dealers and excludes gasoline service stations.

Furniture, home furnishings and appliance store sales measures sales by retail stores selling goods used for the home, other than antiques. It includes dealers in electronics (radios, TV's, computers and software), musical instruments and sheet music, and recordings.

Households measures the number of households, defined by the Census which includes all persons occupying a housing unit. A single person living alone in a housing unit is also considered to be a household. The members of a household need not be related.

Effective Buying Income is an income measure of income developed by S&MM. It is equivalent to disposable personal income, as produced by the BEA in the NIPA tables. It is defined as the sum of labor market income, financial income and net transfers minus taxes. *Labor income* is wages and salaries, other labor income (such as employer contributions to private pension funds), and proprietor's income (net farm and non-farm self-employment income). *Financial income* is interests (from all sources), dividends (paid by corporations), rental income (including imputed rental income of owner-occupants of non-farm dwellings) and royalty income. *Net transfers* is Social Security and railroad retirement, other retirement and disability income, public assistance income, unemployment compensation, Veterans Administration payments, alimony payments, alimony and child support, military family allotments, net

winnings from gambling, and other periodic income minus social security contributions. *Taxes* is personal tax (federal, state and local), non-tax payments (fines, fees, penalties, ...) and taxes on owner-occupied nonbusiness real estate. Not included is money received from the sale of property, the value of income in kind (food stamps, public housing subsidy, medical care, employer contributions for persons), withdrawal of bank deposits, money borrowed, tax refunds, exchange of money between family members living in the same household, gifts and inheritances, insurance payments and other types of lump-sum receipts. Income is benchmarked to the decennial Census data.

We create a *durable retail sales* series by adding automotive dealer sales and furniture, home furnishings and appliance store sales. *Non-durable retail* sales is total retail sales minus durable retail sales.

Comparison with Aggregate Data We construct aggregate non-durable retail sales per households and compare it to aggregate non-durable consumption per household. The aggregate consumption data are from the National Income and Product Accounts. The two nominal time series are very similar. Non-durable metropolitan retail sales per household are on average 17 percent higher than national non-durable consumption per household. Their correlation between their growth rates is 0.77. The one exception is 1999 when retail sales grow at a rate of 19.6 percent compared to 5.6 percent for non-durable consumption. We believe this is an anomaly in the data and deflate the 1999 retail sales so that the metropolitan average growth rate equals the national one. This correction is identical across areas.

We compare the sum of motor vehicles and parts and furniture and household equipment for the US. to the metropolitan data on automotive dealer sales and furniture, home furnishings and appliance store sales. Nationwide, these two categories of consumption make up 84 percent of all durable purchases. Sales are higher by an average of 30 percent. The pattern of the two series mimic each other closely. The correlation between national durable consumption growth and the average metropolitan durable retail sale growth is 0.80. For 1999 the sales data show a much bigger increase than the durable consumption data (27 percent versus 8.6 percent). As for non-durables, we correct the 1999 metropolitan retail sales for this discrepancy. We refer to the two series as *metropolitan* non-durable and durable *consumption* per household.

Effective buying income (EBI) per household corresponds to the BEA's disposable income (personal income minus personal tax and non-tax payments). The S&MM income data are tracking disposable income closely. There are a two discrete jumps in the EBI time-series (1988 and 1995), but the concept remains disposable, personal income. The S&MM is not precise as to which income categories were excluded between 1987 and 1988 and between 1994 and 1995. From comparing the definition of EBI before and after the changes, it seems to us that the most important changes are the exclusion of other labor income (such as employer contributions to pension plans, ...) and income in kind (such as food stamps, housing subsidies, medial care,...). To obtain a consistent time-series, we correct the S&MM income data by the ratio of average EBI to disposable income from the NIPA. This correction is identical across areas. We refer to this series as *metropolitan disposable income* per household. Table 12 summarizes.

Table 12: Aggregate Metropolitan and US data

Year	HH (000)	metr. HH to US HH (%)	nond sales per HH (\$)	nond. sales to nondur. cons.	dur. sales per HH (\$)	dur. sales dur. cons	EBI per HH (\$)	EBI to disp. inc.
1951	17,623	39.4	3,008	1.23	799	1.36	5,959	1.15
1960	23,080	43.7	3,519	1.22	899	1.26	7,711	1.11
1970	28,332	44.7	4,688	1.09	1,180	1.05	11,936	1.03
1980	36,144	44.7	9,683	1.12	2,660	1.24	24,975	1.00
1990	41,784	44.8	15,418	1.15	5,531	1.37	43,698	0.95
2000	49,379	47.2	24,741	1.30	11,888	1.90	56,566	0.83

Comparison with CEX Data We compare the SM&M data to the non-durable and durable consumption data from the Consumer Expenditure Survey (CEX). Based on household data, the Bureau of Labor Statistics (BLS)

provides metropolitan averages for 13 overlapping two-year periods (1986-87 until 1994-95 and 1996-97 until 1999-2000). The two data sources have 25 regions with full data in common. Buffalo is in the CEX sample until 1994-95 and is replaced by Tampa, Denver and Phoenix from 1996-97 onwards.

Consumption expenditures on non-durables are defined as in Attanasio and Weber (1995): It includes food at home, food away from home, alcohol, tobacco, utilities, fuels and public services (natural gas, heating fuel electricity, water, telephone and other personal services), transportation (gasoline and motor oil, public transportation), apparel and services (clothes, shoes, other apparel products and services), entertainment, personal care products and services, reading, and miscellaneous items. Durable consumption includes vehicle purchases and household furnishings and equipment. Consumption expenditures on housing services measure the cost of shelter. $p_t^{ih} h_t^i$ is comprised of owned dwellings, rented dwellings and other lodging. The CEX imputes the cost for owner-occupied dwellings by adding up mortgage interest rates, property taxes and maintenance, improvements, repairs, property insurance and other expenditures. The average expenditure share on housing was 31.5 percent in 2000.

Non-durable and housing services consumption add up to 55-60 percent of total annual consumption expenditures. Excluded consumption items are consumer durables (furniture, household supplies), vehicle purchases, insurance (vehicle, life, social security), health care and education.

For each area, we construct bi-annual averages from the S&MM consumption data. The correlation between all data cells is 0.77 for non-durables and 0.66 for durables. The average correlation across regions is 0.88 for non-durables and 0.73 for durables. We conclude that the metropolitan sales data give an accurate measure of consumption on non-durables and durables at the metropolitan level.

We also compare the bi-annual averages of before-tax income from the CEX with the metropolitan disposable income. The correlation is high for each region. The average correlation across regions is 0.94 and is 0.91 for all data cells jointly. Table 13 summarizes the correlations by region for the 25 areas with all 13 periods.

Table 13: Correlations CEX and S&MM by Metropolitan Area.

MSA	Nond.Cons	Dur.Cons	Income
Washington, DC (PMSA)	0.926	0.660	0.973
Baltimore, MD (PMSA)	0.973	0.791	0.956
Atlanta, GA (MSA)	0.740	0.522	0.944
Miami, FL (CMSA)	0.533	0.399	0.922
Dallas, TX (CMSA)	0.939	0.839	0.917
Houston, TX (CMSA)	0.936	0.955	0.932
Los Angeles, CA (CMSA)	0.836	0.845	0.944
San Francisco, CA (CMSA)	0.921	0.797	0.981
San Diego, CA (MSA)	0.838	0.511	0.961
Portland, OR (CMSA)	0.989	0.932	0.973
Seattle, WA (CMSA)	0.928	0.841	0.935
Honolulu, HI (MSA)	0.858	0.409	0.956
Anchorage, AK (MSA)	0.931	0.601	0.847
New York, NY (CMSA)	0.952	0.727	0.957
Philadelphia, PA (CMSA)	0.812	0.698	0.932
Boston, MA (CMSA)	0.876	0.515	0.799
Pittsburgh, PA (MSA)	0.921	0.759	0.846
Chicago, IL (CMSA)	0.803	0.601	0.953
Detroit, MI (CMSA)	0.960	0.534	0.956
Milwaukee, WI (CMSA)	0.792	0.636	0.949
Minneapolis-St, Paul, MN (MSA)	0.940	0.863	0.972
Cleveland, OH (CMSA)	0.881	0.878	0.956
Cincinnati, OH (CMSA)	0.898	0.864	0.974
St. Louis, MO (MSA)	0.881	0.815	0.945
Kansas City, MO-KS (MSA)	0.958	0.708	0.961
Average	0.881	0.708	0.938

Comparison with REIS Data Disposable income contains two important channels of insurance. It includes income from financial markets and the net income from government transfers and taxes. For consumption to fully capture income smoothing, the income concept should exclude smoothing that takes place through financial markets, credit markets and through the federal tax and transfer system. The Regional Economic Information System (REIS) of the BEA allows us to construct separate series for labor market income, financial market income and net transfers for each metropolitan area.

For the overlapping period 1969-2000, we compute the correlation between the idiosyncratic component of log disposable income, $\log(\hat{y}_t^{i,d})$, from the S&MM and labor income plus transfers $\log(\hat{y}_t^{i,lt})$ from the REIS. Table 14 shows that the correlation is generally high, but with a few exceptions (Miami, Cincinnati, Milwaukee). The average correlation is 0.64. This imperfect correlation is due to a combination of measurement error in income and insurance through financial markets. The discrepancy warrants use of both income measures in the empirical analysis.

Table 14: Correlation Regional Disposable (S&MM) and Labor Income plus Transfers (REIS)

South and West	Coeff.	Northeast and Midwest	Corr.
Washington, DC (PMSA)	0.79	New York, NY (CMSA)	0.84
Baltimore, MD (PMSA)	0.42	Philadelphia, PA (CMSA)	0.82
Atlanta, GA (MSA)	0.73	Boston, MA (CMSA)	0.73
Miami, FL (CMSA)	-0.18	Pittsburgh, PA (MSA)	0.57
Dallas, TX (CMSA)	0.63	Buffalo, NY (MSA)	0.77
Houston, TX (CMSA)	0.86	Chicago, IL (CMSA)	0.76
Los Angeles, CA (CMSA)	0.85	Detroit, MI (CMSA)	0.74
San Francisco, CA (CMSA)	0.65	Milwaukee, WI (CMSA)	0.12
San Diego, CA (MSA)	0.75	Minneapolis-St. Paul, MN (MSA)	0.70
Portland, OR (CMSA)	0.57	Cleveland, OH (CMSA)	0.90
Seattle, WA (CMSA)	0.60	Cincinnati, OH (CMSA)	-0.23
Honolulu, HI (MSA)	0.84	St. Louis, MO (MSA)	0.54
Anchorage, AK (MSA)	0.80	Kansas City, MO-KS (MSA)	0.57
Phoenix, AZ (MSA)	0.83		
Denver, CO (CMSA)	0.67	Average	0.64

B Tables and Figures

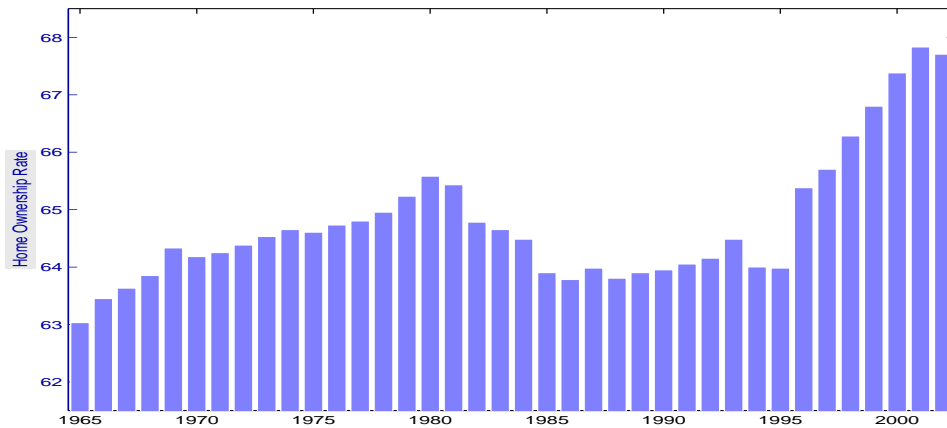
<i>T1</i>	Mean	St. Dev	<i>T2</i>	Mean	St. Dev.	<i>T3</i>	Mean	St.Dev.
RVW	12.3	20.3	RVW	12.3	20.3	RVW	12.3	20.3
S1B1	8.3	37.6	S1	20.4	41.8	Food	13.1	19.5
S1B2	14.0	37.2	S2	17.7	36.9	Beer	16.5	36.0
S1B3	18.0	35.6	S3	16.6	31.7	Smoke	15.2	23.7
S1B4	21.9	44.4	S4	16.2	30.3	Games	14.5	33.0
S1B5	22.4	37.3	S5	15.6	28.2	Books	14.2	29.8
S2B1	11.3	31.6	S6	15.2	26.9	Hshld	12.9	22.9
S2B2	16.2	29.7	S7	14.7	25.5	Clths	13.1	31.2
S2B3	17.9	30.2	S8	13.8	23.1	Hlth	14.7	22.1
S2B4	18.9	32.5	S9	13.2	19.2	Chemis	13.3	25.0
S2B5	19.4	32.6	S10	11.7	22.8	Txtls	12.8	33.6
S3B1	12.8	29.9	B1	11.5	19.4	Cnstr	12.4	26.8
S3B2	15.7	27.6	B2	12.4	19.7	Steel	11.9	30.0
S3B3	16.8	26.7	B3	12.1	22.6	FabPr	12.8	26.3
S3B4	17.3	27.4	B4	11.9	22.3	ElcEq	14.7	31.8
S3B5	18.2	32.4	B5	13.9	23.2	Autos	15.7	35.0
S4B1	12.7	23.5	B6	13.7	24.5	Carry	14.7	31.7
S1B2	13.4	25.2	B7	14.6	26.8	Mines	11.0	27.2
S4B3	15.3	25.4	B8	16.4	28.8	Coal	12.6	28.7
S4B4	16.4	26.6	B9	17.5	32.6	Oil	13.4	22.7
S4B5	17.9	34.3	B10	17.2	33.4	Util	11.4	22.7
S5B1	11.9	21.1	M1	2.6	28.2	Telcm	11.7	20.2
S5B2	11.4	19.2	M2	7.8	25.4	Servs	17.4	37.5
S5B3	13.0	21.6	M3	8.0	22.4	BusEq	15.1	26.3
S5B4	13.7	25.1	M4	10.7	22.1	Paper	13.2	21.9
S5B5	15.5	33.2	M5	11.2	20.3	Trans	11.1	25.6
			M6	12.0	20.0	Whlsl	12.2	30.8
			M7	13.6	20.9	Rtail	14.1	25.7
			M8	16.1	22.5	Meals	15.3	35.3
			M9	18.2	24.5	Fin	13.7	24.4
			M10	21.6	27.5	Other	11.8	29.4

Table 15: Annual Portfolio Returns 1926-2001. Time-series mean and standard deviation of portfolio returns in excess of the one-year return on a one month T-bill. The first three-columns are for the twenty-five size and book-to-market portfolios. For comparison, the next six columns show returns on two other sets of test assets that are often used in cross-sectional exercises. The first one consists of ten size, ten book-to-market and ten momentum portfolios (*T2*). The momentum portfolios measure winners and losers: Stocks that pay high returns in a given year and pay high returns in the subsequent year are high momentum stocks. The last three columns are for thirty industry portfolios (*T3*). All data are from Kenneth French, except the momentum portfolio returns which are from Chris Lundblad.

Portfolio	CCAPM	Fama-French	Collateral-CCAPM
RVW	2.00	-.40	.31
S1B1	8.11	3.75	-.46
S1B2	3.24	2.28	2.60
S1B3	-.65	-.85	-.44
S1B4	-5.58	-1.88	-.25
S1B5	-5.01	-2.59	-1.93
S2B1	5.48	2.19	3.31
S2B2	-.13	-.97	-1.36
S2B3	-2.96	-1.13	-1.29
S2B4	-2.88	-.95	.72
S2B5	-2.96	-.22	-1.85
S3B1	2.33	-1.24	1.42
S3B2	-.35	-1.10	-.46
S3B3	-1.42	-.50	-2.02
S3B4	-1.45	-.02	-1.74
S3B5	-2.65	1.21	.00
S4B1	1.39	-2.59	-.47
S1B2	1.05	.89	.99
S4B3	-.37	.19	-2.80
S4B4	-2.40	.42	-.10
S4B5	-3.33	-.67	-1.02
S5B1	2.11	-2.71	-1.26
S5B2	2.46	.46	1.18
S5B3	1.77	.27	.96
S5B4	.66	1.56	1.42
S5B5	-2.44	.60	.57
Average	3.098	1.536	1.457
χ^2	62.54**	55.54**	28.48

Table 16: Average Pricing Errors. Pricing errors from the Fama-MacBeth Regressions with aggregate pricing factors. The set of returns is the value weighted market return and the 25 size and book-to-market portfolio returns. The second column reports errors from the consumption CAPM, the third from the three-factor Fama-French model and the last column reports average errors from the collateral CAPM with scaling variable *myfa*. The last two rows report the square root of the average squared pricing errors and the χ^2 statistic for the null hypothesis that all pricing errors are zero. The degrees of freedom are 25, 23 and 21 respectively. Two stars denote rejection of the hypothesis at the 1 percent level.

Figure 8: Home-Ownership rate in the U.S.



Portfolio	All States	Good States	Bad States
RVW	.673	.432	1.056
S1B1	.796	1.509	-.334
S1B2	1.742	.543	3.642
S1B3	1.790	.729	3.472
S1B4	2.226	-.598	6.702
S1B5	2.037	.157	5.018
S2B1	1.464	.831	2.469
S2B2	1.383	.643	2.555
S2B3	1.244	-.018	3.244
S2B4	1.694	-.038	4.438
S2B5	1.547	.310	3.509
S3B1	1.220	.290	2.694
S3B2	1.225	.244	2.779
S3B3	1.065	.347	2.202
S3B4	1.288	.434	2.641
S3B5	1.343	.050	3.393
S4B1	.666	.340	1.181
S1B2	.996	.012	2.556
S4B3	.920	.493	1.596
S4B4	.922	-.283	2.831
S4B5	1.156	-.413	3.642
S5B1	.471	.613	.246
S5B2	.539	.330	.869
S5B3	.884	.389	1.670
S5B4	.894	.069	2.201
S5B5	.610	-.518	2.398

Table 17: Consumption Betas. Consumption betas are computed as $\beta_t = \beta_c + \beta_{c.my}my_t$. Good states are states where $myfa$ is one standard deviation below zero and bad states are times where $myfa$ is one standard deviation above zero. The third and fourth column report the average consumption betas in good states and bad states respectively. Lettau and Ludvigson (2001b) do the same exercise for their scaling variable, the consumption-wealth ratio.

	$\tilde{\beta}_0$	$\tilde{\beta}_c$	$\tilde{\beta}_\rho$	$\tilde{\beta}_{my}$	$\tilde{\beta}_{my.c}$	$\tilde{\beta}_{my.\rho}$	R^2
1	5.50 (3.05)	.59 (.38)					4.72
2	5.03 (3.35)	.72 (.45)	1.16 (3.45)				5.17
3	2.73 (3.62)	1.08 (.51)	4.36 (2.49)	-.07 (.08)	-.02 (.01)	-.16 (.05)	19.85
4	3.69 (2.70)	.90 (.29)	2.83 (2.10)	-.00 (.15)	-.04 (.02)	-.30 (.09)	18.55
5	2.90 (2.87)	1.07 (.28)	4.14 (2.15)	.11 (.36)	-.07 (.03)	-.43 (.10)	19.75

Table 18: Time-Series Analysis for Aggregate Stock Market Return. The regression is: $R_{t+1}^{e,j} = \tilde{\beta}_0^j + \tilde{\beta}_c^j \Delta \log c_{t+1} + \tilde{\beta}_\rho^j A_t \Delta \log \rho_{t+1} + \tilde{\beta}_{my}^j my_t + \tilde{\beta}_{my.c}^j my_t \Delta \log c_{t+1} + \tilde{\beta}_{my.\rho}^j A_t \Delta \log \rho_{t+1}$. The first row just includes $\Delta \log(c_{t+1})$. The second row adds $A_t \Delta \log(\rho_{t+1})$. The last three rows add the interaction terms with my_t . The scaling variable is my_{mo} in regression 3, my_{rw} in regression 4, and $myfa$ in regression 5. my is multiplied by 100. Data are for 1926-2001. Newey-West heteroskedasticity and autocorrelation corrected standard errors are in parentheses.

	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\rho$	$\tilde{\lambda}_{\hat{y}}$	$\tilde{\lambda}_{my.\hat{y}}$	$\tilde{\lambda}_{\hat{\rho}}$	$\tilde{\lambda}_{my.\hat{\rho}}$	R^2
1 <i>mymo</i>	4.70 (4.10) [7.33]	2.49 (1.09) [1.98]	.71 (.34) [.63]	-2.36 (2.37) [4.29]	-5.20 (2.63) [4.81]	-.11 (.22) [.41]	.33 (.39) [.71]	65.6 54.7
θ		.42	-.02	.07	-.24	.71	.52	
2 <i>myrw</i>	5.53 (3.41) [6.29]	1.94 (1.06) [1.99]	.67 (.29) [.55]	-2.17 (1.10) [2.09]	-5.60 (2.27) [4.28]	.22 (.42) [.78]	.48 (.63) [1.19]	64.0 52.7
θ		.27	.49	-.07	-.22	-1.16	.87	
3 <i>myfa</i>	4.38 (3.79) [6.71]	2.08 (1.10) [1.97]	.72 (.29) [.54]	-3.00 (2.39) [4.25]	-3.72 (3.88) [6.92]	-.03 (.45) [.80]	-.05 (.87) [1.56]	60.0 47.4
θ		.25	.51	-.33	-.07	1.76	-.44	

Table 19: Cross-Sectional Results with Regional Asset Pricing Factors: Population-weighted data. Results are for the iterative Fama-MacBeth procedure described in appendix with cutoff level 0.9. The asset pricing factors consists of: $\Delta \log^a(c_{t+1})$, $A_t \Delta \log^a(\rho_{t+1})$, $\Delta \log(\hat{y}_{t+1}^{i*})$, $my_t \Delta \log(\hat{y}_{t+1}^{i*})$, $A_t \Delta \log(\hat{\rho}_{t+1}^{i*})$, and $my_t A_t \Delta \log(\hat{\rho}_{t+1}^{i*})$. The sequence $\{i_t^*\}$ is the sequence of unconstrained metropolitan regions. The average factors are population weighted, using metropolitan data on the number of households. The variable with a hat denote deviations from the population-weighted average. The coefficient vector θ consists of γ , ϕ , $\gamma\beta_1$, $-\gamma\beta_2$, $\phi\beta_1$ and $-\phi\beta_2$. The second column gives the zero- β return $\tilde{\lambda}_0$. OLS standard errors are in parenthesis, Shanken (1992) corrected standard errors are in brackets. Row 1 is for *mymo*, row 2 for *myrw* and row 3 for *myfa*. The set of test assets is T1. The period is 1952-2001, the longest period with metropolitan data.

	$\tilde{\lambda}_0$	$\tilde{\lambda}_c$	$\tilde{\lambda}_\rho$	$\tilde{\lambda}_{\hat{y}}$	$\tilde{\lambda}_{my.\hat{y}}$	$\tilde{\lambda}_{\hat{\rho}}$	$\tilde{\lambda}_{my.\hat{\rho}}$	R^2
1 <i>mymo</i>	4.63 (4.19) [8.10]	3.18 (1.28) [2.50]	.82 (.37) [.73]	-1.48 (2.65) [5.18]	-4.40 (2.91) [5.73]	-.15 (.24) [.48]	.41 (.43) [.83]	63.7 52.2
θ		.53	-.07	.14	-.22	.11	.87	
2 <i>myrw</i>	5.34 (3.46) [6.43]	2.12 (.99) [1.89]	.70 (.28) [.54]	-1.84 (1.39) [2.63]	-4.91 (1.94) [3.69]	.40 (.44) [.84]	.79 (.63) [1.19]	63.0 51.3
θ		.31	.54	-.07	-.22	-1.04	.83	
3 <i>myfa</i>	4.87 (3.78) [6.88]	2.82 (1.30) [2.39]	.77 (.30) [.57]	-2.43 (2.20) [4.02]	-2.52 (3.79) [6.96]	.03 (.44) [.81]	.02 (.85) [1.55]	59.1 46.1
θ		.37	.37	-.32	-.06	1.74	-.44	

Table 20: Cross-Sectional Results with Regional Asset Pricing Factors: Population-Weighted Data and Real Exchange Rate Correction. The exercise is identical to the one in table 19 except that the asset pricing factors (F^a, F^{lc}) are converted into aggregate (national) consumption units. They are multiplied by the ratio of the regional non-durable price index to the population-weighted cross-region average non-durable price index (the regional real exchange rate). This conversion is equivalent to converting the asset returns (expressed in national units) into regional consumption units. The non-durable price index used is the food component of the CPI (1951-2001). Results are similar for the non-durables component of the CPI, available only for 1967-2001 and are not reported.

Figure 9: Regional variation in ratio of rent to food component of CPI.

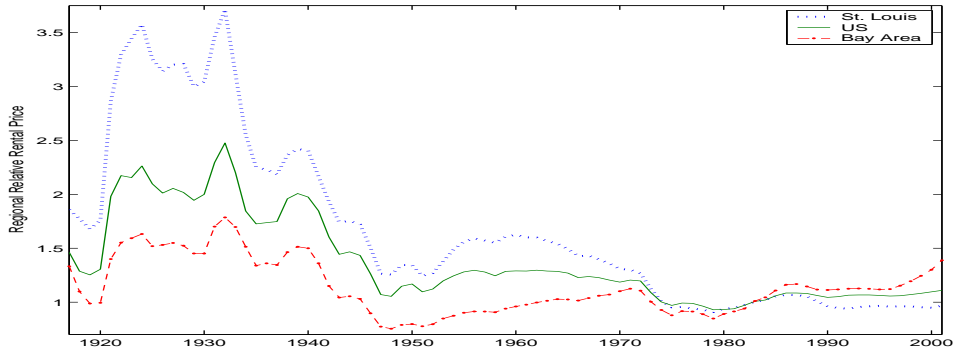


Figure 10: Power Spectral Density of Housing Collateral Ratio. The three lines are the cointegration deviations between labor income and one of our three different measures of housing collateral. For mymo, 92 percent of the variance occurs at frequencies below $\pi/10$. For myrw that is 83 percent and for myfa 65 percent.

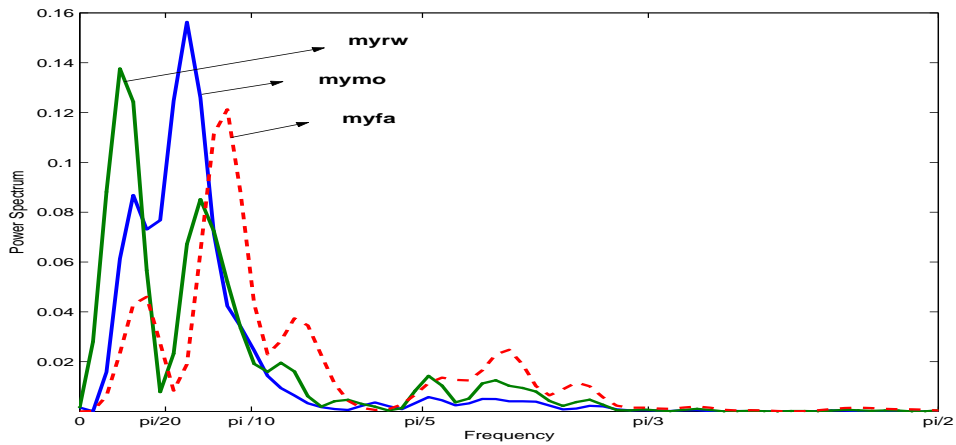


Figure 11: Collateral Ratio and NBER Recession Dates.

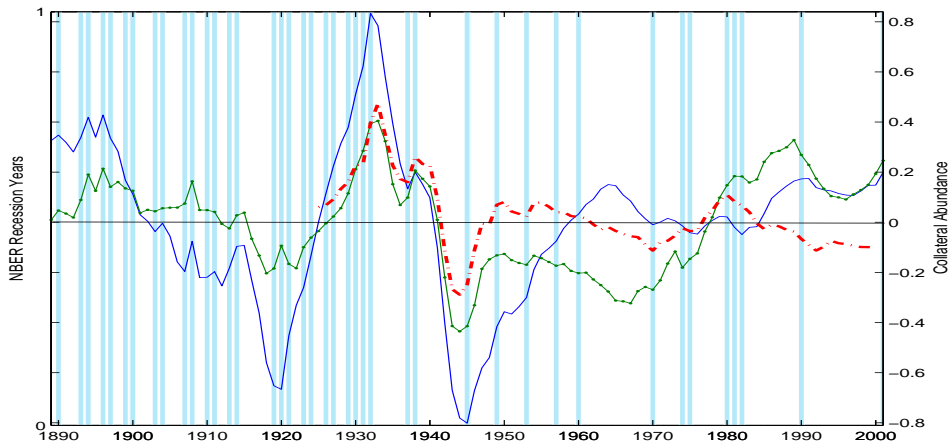


Figure 12: Response of the One-Year Excess Return to Impulse in Collateral Ratio *mymo*.

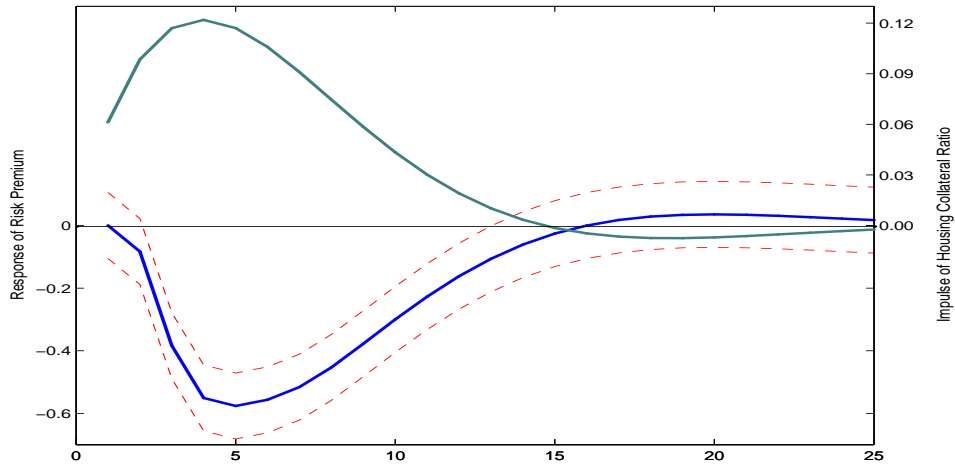


Figure 13: Response of the One-Year Excess Return to Impulse in Collateral Ratio *myrw*.

