Squeezing the Interest Rate Smoothing Weight with a Hybrid Expectations Model*

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November 12, 2002

Abstract

Successful descriptions of the short-term nominal interest rate *inertial* behavior have frequently been obtained with small scale macro models in which a Central Banker minimizes a loss function containing an argument labelled as *interest rate smoothing.* The *rationale* for this argument is not straightforward; indeed, there has been a lively debate about it among academics. In this paper we perform a positive exercise to evaluate the relationship existing between private *rational expectations* and the interest rate smoothing argument. Our findings strongly support rational expectations as an element capable to remarkably reduce the importance of the interest rate smoothing weight in replicating the observed path of the federal funds rate. However, we find a predominance of adaptive expectations in shaping the future paths of inflation ad output gap. Our results also suggest that the Fed has followed a 'Strict Inflation Targeting' strategy under Greenspan's regime.

Keywords: Central Banker, interest rate smoothing, rational expectations, hybrid Phillips curve, hybrid IS curve.

JEL Classification System: C51, E52.

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[*] First draft: July 2002. I thank Richard Dennis, Carlo Favero, Francesca Monti, Davide Pettenuzzo, Saverio Simonelli, Ulf Söderstrom, and Paolo Surico for valuable comments and suggestions. The usual disclaimers apply. Author's e-mail account: efrem.castelnuovo@uni-bocconi.it

1 Introduction

The use of small-scale macro models for the analysis of monetary policy-making has become a standard approach in the last decade or so. Typically, the Central Banker (CB hereafter)'s problem is characterized by the presence of a penalty function whose minimization is subject to some constraints. In the penalty function we usually find relevant aggregates such as the rate of inflation and a measure of output gap, or unemployment rate, while the constraints are nothing but the representation of the economic environment. Then, the CB moves a short-term interest rate to minimize his expected loss value.

Indeed, there seems to be little discussion about the fact that in general a monetary authority aims at maintaining stable prices and a steady real growth.[1] As far as the formalization of the economy is concerned, many recent empirical studies have been based on simple AD-AS representations of the economy.[2] Interestingly enough, the mathematical solution of the CB's problem turns out being an interest rate path featured by frequent reversals, reversals due to the willingness of the CB to tackle the various shocks affecting the economy. In fact, in reality we observe smooth paths of the policy rates; this tendency has been labeled as *interest rate smoothing*.[3] In order to capture this feature of the policy rate, many authors have added to the CB's loss function an *interest rate smoothing argument*, i.e. an argument

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[1] Cecchetti (2001) reviews some of the reasons why it is good for the Society to have stable inflation and growth. We briefly recall here that inflation is a tax on the monetary base, it causes distortions in investments decisions leading to lower-than-optimal stock of capital, it creates noise in the price system and, when it is very high, it diverts resources to control it and it injects uncertainty in the economy. About real aggregates, Ramey and Ramey (1995) and McConnell and Perez-Quiros (2000) present evidence of a strong *negative* correlation existing between output volatility and growth.

[2] A very incomplete list includes Ball (1999), Rudebusch and Svensson (1999,2002), Smets (1999), Nessén and Vestin (2000), Ozlale (2001), Dennis (2002), Söderlind, Söderstrom, and Vredin (2002), Dennis and Söderstrom (2002), Favero and Rovelli (2002), Castelnuovo and Surico (2002), and Rudebusch (2001,2002a,b,c).

[3] Rudebusch (1995), Goodhart (1997), Lowe and Ellis (1998), Sack and Wieland (2000), and Srour (2001) are examples of studies focused on the interest rate smoothing evidence. Interestingly, in a recent contribution Rudebusch (2002a) sustains that the monetary policy inertia observed at a quarterly frequency is just an illusion. However, English, Nelson, and Sack (2002) run a direct test on the existence of the CB's sluggish adjustment strategy, and they find it statistically significant, so casting some doubts on Rudebusch (2002a)'s 'illusionist' conjecture.

function of the interest rate *change*.[4]

On the contrary, authors such as Goodhart (1999), Sack (2000), Sack and Wieland (2000), and Cecchetti (2001) claim that a smooth interest rate may very well be the solution of a problem in which there is *no* role for the smoothing goal in the CB's penalty function. In fact, private sector's rational expectations, uncertainties regarding the dynamics in the economy, and measurement-errors problems could induce monetary authorities to implement a cautious policy. From this standpoint, the interest rate smoothing element embedded into the loss function is basically a measure of the *residual* capturing all what is not formalized in the model.

In this paper we focus our attention on the relationship between interest rate smoothing and rational expectations (RE hereafter). Our work aims at understanding *how much* explanatory power a small macro model may gain when passing from a backward looking formalization of the economic dynamics to a representation in which there is room for RE.[5] To do so, we use an encompassing AD-AS model á la Rudebusch (2002b) which, under some identifying conditions, may collapse to a backward looking, hybrid, or fully forward looking illustration of the linkages existing among inflation, output gap, and the policy rate. For each different tuple of structural parameters identifying the economic framework, we calibrate the value to be attributed to the interest rate smoothing argument in order to fit the actual federal funds rate at best.

A comparison of the results obtained with a fully backward looking model with those stemming from our hybrid version of the economy enables us to state that RE is a very important ingredient capable to largely explain the observed interest rate persistence. Indeed, the gains in terms of descriptive power turn out being quite large. To our knowledge, in this literature this is the first effort oriented at *quantitatively* assessing the role of RE in designing

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[4] Reasons for a structural concern of the volatility of the interest rate change by monetary authorities may be monetary policy credibility (Mishkin, 1999), fears of financial markets disruption (Goodfriend, 1991), inflation bias under discretion (Woodford, 1999, and Amato and Laubach, 1999). In this paper, we will consider instead the interest rate smoothing as a 'residual' capturing all what is not specified in our model.

[5] All along the paper we will use the terms 'forward looking agents' or 'rational agents' in association with a model embedding the 'rational expectations' component. Indeed, as pointed out by Sims (2001), a backward looking model is not inconsistent with agents having forward looking behavior if past variables represent sufficient statistics for the description of expectations formation. However, it should be noticed that a backward representation does not explicitely separate expectations from other sources of dynamics, e.g. economic shocks.

these small macro models.

Our calibration exercises also suggest that in a framework like the one we employed in our investigation a predominant weight should be attributed to the adaptive part; nevertheless, the importance of the RE component remains remarkable. Finally, there is empirical evidence of little concern of the CB for the volatility of the output gap with respect to the variance of inflation, i.e. Alan Greenspan seems to have followed a 'Strict Inflation Targeting' scheme, so deserving the label 'inflation nutter' as defined in King (1997).

The map of the paper is the following. Section 2 describes the model formalizing the economy. In Section 3 we discuss our strategy for evaluating the importance of the RE ingredient. In Section 4 we highlight and comment our findings. Section 5 collects some insights on the importance of RE. In Section 6 we deepen our analysis with a robustness check. Section 7 reviews some other possible ingredients potentially capable to ulteriorly reduce the interest rate smoothing' s weight in the loss function. Section 8 concludes and, after the References, a Technical Appendix explaining the algorithm we use in order to tackle the optimal stochastic regulator problem is provided.

2 Modeling the Central Banker's problem

Our hypothesis is that a CB solves an optimal control problem to determine the optimal path of its control variable, i.e. the short term nominal interest rate. The period loss function reads as follows:

$$L_t = (\bar{\pi}_t - \pi^*)^2 + \lambda(y_t)^2 + \mu(i_t - i_{t-1})^2 \tag{1}$$

where π_t represents the inflation rate (the log difference of prices between time t and time t-1), π^* is the inflation target, $\bar{\pi}_t = \frac{1}{4}\sum_{s=0}^{3}\pi_{t-s}$ stands for the four-quarter average inflation, y_t is the output gap defined as log difference between actual and potential output, and i_t is the short-term nominal interest rate (e.g. the federal fund rate). A couple of comments on our definition of the loss function are needed. Regarding the inflation targeted by the CB, we think of it as being an average-inflation, because we believe that this average may better represent the will of the monetary authority to monitor the inflation rate in different periods, rather than just in the 'current one'. Our definition of the output gap implies that the target for the level of output set by the CB is the potential output, as probably done by the Fed (Blinder, 1997). [6] Finally, in (1) the weight λ represents the relative preference of the CB over the output gap with respect to inflation. Instead, given our interpretation of the interest rate smoothing argument, the weight μ should be seen as a residual component, needed in order to replicate at best the observed path of the federal funds rate.

It is common belief that the CB solves an *intertemporal* optimization problem. That is why we shape his problem as follows:

$$\underset{\{i_t\}}{Min} E_t \sum_{j=0}^{\infty} \delta^j L_{t+j} \tag{2}$$

As explained in Rudebusch and Svensson (1999), when the discount rate $\delta \rightarrow 1$, equations (1) and (2) can be rewritten as follows:

$$\underset{\{i_t\}}{Min} E(L_t) = Var(\bar{\pi}_t - \pi^*) + \lambda Var(y_t) + \mu Var(i_t - i_{t-1}) \tag{3}$$

So, the conditional mean (2) collapses to its unconditional counterpart, which is equal to the weighted sum of the unconditional variances of the loss function's arguments. From now on, we will consider equation (3) as our objective function.

We now turn to the representation of the economic environment. We adopt a model á la Rudebusch (2002b), which reads as follows:

$$\pi_{t+1} = \gamma_\pi E_t \bar{\pi}_{t+4} + (1-\gamma_\pi) \sum_{j=1}^{4} \alpha_{\pi j} \pi_{t-j+1} + \alpha_y y_t + \varepsilon_{t+1} \tag{4}$$

$$\begin{aligned} y_{t+1} \; = \; & \gamma_y E_t y_{t+2} + (1-\gamma_y) \sum_{j=1}^{2} \beta_{yj} y_{t-j+1} \\ & -\beta_r (i_t - E_t \bar{\pi}_{t+4}) - (1-\beta_r)(\bar{i}_t - \bar{\pi}_t) + \eta_{t+1} \end{aligned} \tag{5}$$

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[6] Indeed, the monopoly-power held by firms in the underlying structure of the economy might lead to think about a CB willing to set a higher target level, given that the equilibrium production in case of monopolistic competition is lower than the socially desirable one. However, by introducing a target greater than the potential output, the CB would face an inflation bias problem (Barro and Gordon, 1983). That is why in our study the output gap level target is equal to zero.

where $\bar{i}_t = \sum_{s=1}^{3} i_{t-s}$, γ_π represent the 'degree of forwardness' of the dynamic Phillips curve (4), while γ_y and γ_r are the weights of the RE respectively of the expected demand and of the expected real interest rate in the IS equation (5). A few comments are due here. First of all, following some researchers' example (e.g. Fuhrer and Moore, 1995; Clarida, Galí and Gertler, 1999; Rudebusch and Svensson, 1999,2002), we admit a stochastic element in the Phillips curve, the *cost-push* shock ε_t^π, which is responsible of the short-run trade-off existing between inflation and output gap. We also admit a demand shock in the IS curve, namely ε_t^y. In this latter curve, we consider the possibility of having a 'hybrid' representation of the short-term real interest rate; we do so to be consistent with the overall 'hybrid' economic set up we want to consider for performing our exercises. Finally, it is easy to notice that, when $\gamma_\pi = \gamma_y = \gamma_r = 1$, this model collapses to the well-known 'New Neoclassical Synthesis' model by Goodfriend and King (1997).

In this framework the transmission of the monetary policy action happens with some lags.[7] This is in line with what the observation of the real economy seems to suggest, i.e. a change in the interest rate level affects the output gap with a certain delay, and even with a larger delay the inflation rate, as underlined in Christiano, Eichembaum and Evans (1998,2001). Moreover, Söderlind, Söderstrom, and Vredin (2002) verifies how this model is capable (under some parametrization) to broadly match the features of the historical series.[8]

The model (4)-(5) may be re-written in its state space form as follows:

$$A_0 \begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A_1 \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + B_1 i_t + v_{t+1} \qquad (6)$$

where A_0 and A_1 are squared matrices of size $(n1 + n2)$, B_1 is a $((n1 + n2) \times 1)$ columns vector, x_{1t} is a $(n1 \times 1)$ column vector of predetermined state variables (with $n1 = 9$), which is defined as $x_{1t} = [\pi_t \ \pi_{t-1} \ \pi_{t-2} \ \pi_{t-3}$

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[7] The timing of the game is the following: at the beginning of each period private agents form their expectations; then, the interest rate level is optimally fixed by the Central Bank, and last the demand and supply shocks strike the economy.

[8] In particular, they find that a CB with a small concern for output stability, but a large preference for inflation and interest rate stability, delivers a path for these three variables very much in line with the data. Furhermore, they find a small degree of forward-looking behavior in inflation, and a very large one for the output gap.

$\mathrm{y}_t\ \mathrm{y}_{t-1}\ \mathrm{i}_{t-1}\ \mathrm{i}_{t-2}\ \mathrm{i}_{t-3}]'$, and x_{2t} is a $(n2 \times 1)$ column vector of forward-looking jump variables (with $n2 = 4$), which is $x_{2t} = [\mathrm{E}_t\pi_{t+3}\ \mathrm{E}_t\pi_{t+2}\ \mathrm{E}_t\pi_{t+1}\ \mathrm{E}_t\mathrm{y}_{t+1}]'$.[9]

The CB's aim is that of optimally setting the path of the interest rate i_t in order to minimize the expected loss (3) subject to the law of motion (6). Söderlind (1999) proves the optimality of the following linear feedback rule

$$i_t = -Fx_{1t} \tag{7}$$

where F is $(1 \times n1)$ row vector whose elements are convolutions of the structural parameters of the equations (4)-(5) and the coefficients attached to the arguments in the objective function.[10]

The model (6)-(7) is capable of replicating the dynamics present in the economy. Figure 1 shows some impulse response functions. It is immediate to notice that shocks to output and inflation are followed by *gradual* movements of the policy rate, the graduality being justified by the presence of a strictly positive interest rate smoothing weight. After a demand shock, the central bank must drive downwards the output gap rendering it negative, in order to tackle the inflationary pressure. The volatile pattern shown by the inflation rate may be due to the will of the CB to target annual inflation. Instead, in response to a cost-push shock, the CB raises the short-term nominal interest rate, so depressing the real economy. This induces the return of the average inflation rate to its target, at the cost of periods of under-production.

With this model at hand, we can calibrate the value of the weight μ in order to find the optimal simulated interest rate that most closely replicate Greenspan's federal funds rate. In the next section we describe our calibration strategy.

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[9] A description on how to conveniently set up and solve the optimal control problem proposed in this paper is provided in the Technical Appendix.

[10] Notice that, in performing our simulations, we use the discretionary solution, because we believe that a credible CB implements time-consistent strategies (as also pointed out by Jensen, 2002). However, in our study the discretionary solution with average inflation targeting and the one under commitment and 'standard' inflation targeting are indeed quantitatively very close. In fact, the model we use in this analysis - Rudebusch (2002b)'s - does not imply the existence of a large difference between the solution under commitment and the one under discretion, as found out by Dennis and Söderstrom (2002).

3 Our Econometric Strategy

The aim of our exercise is to fit the policy rate set by Alan Greenspan in the sample 1987:Q3-2001:Q1.[11] In doing so, we consider two different set of identification restrictions of equations (4)-(5). The first one - our Benchmark specification - is featured by $\gamma_\pi = \gamma_y = \gamma_r = 0$.[12] We want to have this model to understand how large is the weight that we have to assign to the parameter μ in order to replicate the historical path of the federal fund rate while employing a backward looking model. The second set of restrictions identify our Hybrid version of the model, featured by the presence of RE. Referring once more to equations (4)-(5), we are in this case allowing for the presence of strictly positive values for the parameters γ_π, γ_y, and γ_r. Notice that we are not exogenously fixing those weights; instead, we want to calibrate them to get the best possible fit of the federal fund rate. So, when the Hybrid version of the model is considered, we will *jointly* calibrate the weight μ and the parameters γ_π, γ_y, and γ_r.

To attribute sensible values to the coefficients of the model (3), (4), and (5), i.e. γ_π, γ_y, γ_r, α_s, β_s, λ, and μ, we then implement the following strategy:

1) we OLS estimate the parameters α_s and β_s of our Benchmark specification, i.e. we estimate equations (4)-(5) subject to the constraint $\gamma_\pi = \gamma_y = \gamma_r = 0$. Our estimates, together with some details of our econometric exercise, are reported in Table 1. A key parameter for the transmission of the monetary policy is the interest rate elasticity β_r. Notably, our point estimate - 0.073 - is statistically in line with that of Rudebusch (2002b).[13]

2) We exogenously fix a value for the relative preference λ. We do so because, in performing our calibration exercise, we want to control for all the possible sources of influence of the trade off existing between goodness of fit (i.e. the distance between the actual federal fund rate and the simulated ones)

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[11] The choice of Greenspan's period is suggested both by sample-length consideration (he has been in charge since the third quarter of 1987, a sample longer than those featured by other chairmen's conducts) and by our willingness of comparing our findings with the available literature, which mostly concentrates on the post-Volcker era. The choice of a single-chairman sample is due to our belief on the *idiosyncratic* preferences that each CB is featured by.

[12] A backward looking framework like this has been used for monetary policy analyses by Ball (1999), Rudebusch and Svensson (1999, 2002), Favero and Milani (2001), Ozlale (2001), Favero and Rovelli (2002), and Castelnuovo and Surico (2002), among the others.

[13] Instead, probably due to the different samples considered, it is much lower than those provided by Clark, Laxton and Rose (1996) - 0.16 - and Smets (1999) - 0.9.

and (inverse of) magnitude of the 'unexplained part' (i.e. the magnitude of the weight μ). That is why we have to select a sensible value for λ, and set it before performing the calibration. Notice that we are discussing about a structural preference of our set-up, i.e. the relative weight that Alan Greenspan has attributed to the volatility of the output gap with respect to the volatility of the average inflation rate.

Indeed, it is possible to find many different sensible values for the relative preference parameter λ in the literature. Focusing on backward representations of the economy á la Rudebusch and Svensson (1999, 2002), Favero and Rovelli (2002) estimate with GMM the Euler conditions of the CB's problem, finding a (statistically insignificant) value of 0.00125. Ozlale (2001) exploits Kalman-filtering and estimates a value of 0.525. Dennis (2002) gets 0.815 with a FIML approach. Finally, Castelnuovo and Surico (2002) calibrate a value equal to 1. With a slightly different underlying representation of the economy, Cecchetti, Flores-Lagunes, and Krause (2001) find negligible values for sub-samples regarding the '80s and '90s, while Cecchetti and Ehrmann (2001)'s results support a value of about 1/4. For the same period, but with a VAR representation of the economy, Salemi (1995) finds very low relative weights for the output gap with respect to inflation. We somehow arbitrarily fix a benchmark value of $\lambda = 0.5$; however, we check for the robustness of our results by considering also values such as 0.0, 0.2, and 1.0.

3) Given steps 1) and 2), we can perform the calibration of the remaining parameters μ, γ_π, γ_y, and γ_r.[14] We do so by implementing a grid-search based on a minimum-distance criterium. In particular, we compute, *per each battery* $j : [\mu^j, \gamma_\pi^j, \gamma_y^j, \gamma_r^j]$, an optimal simulated interest rate $i^{sim,j}$ to be compared with the actual one i^{actual}.[15] For our calibration we exploit the following measure of Distance:

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[14] To have a more easily manageable problem optimal control problem, we demean all the variables involved in our study. As argued by Dennis (2000), this operation does not affect the derivation of the CB's weights in the loss function, but it constraints the average inflation target π^* to be equal to zero, which is equivalent to the sample mean (2.54% in Greenspan's sample) in an undemeaned world. Actually, our analysis is meant to identify the weights of the CB's loss rather than the targets *per se*. A number of papers cover this latter issue, including Judd and Rudebusch (1998), Sack (2000), Dennis (2002), and Favero and Rovelli (2002).

[15] In performing our calibration exercise, we consider values belonging to the interval [0.1;0.9] for the forwardness coefficients γ_π and γ_y (step-length: 0.1), while [0.0;1.0] for γ_r. Finally, for the weight μ we took into account values belonging to the interval [0.0;10.0] (step-length: 0.1).

$$Distance(i^{simulated}, i^{actual}) = \sqrt{\frac{\sum_{t=1}^{T}(i_t^{simulated} - i_t^{actual})^2}{T}} \qquad (8)$$

With this measure of distance we can pick up the simulated interest rate $i^{sim,j*}$ (i.e. the one delivering the minimum distance) implied by the calibrated vector $[\mu^*, \gamma_\pi^*, \gamma_y^*, \gamma_r^*]$. We recall here that, when our Benchmark model is employed, the calibration exercise just regards the weight μ, given the identifying restriction $\gamma_\pi = \gamma_y = \gamma_r = 0$.[16]

Notice that our calibration strategy relies on the assumption of optimal behavior undertaken by Greenspan in the period analyzed. As pointed out by Cecchetti, McConnell, and Perez-Quiros (1999), this is equivalent to assume that Greenspan has operated along the efficiency-frontier that defines the trade-off between inflation and output gap, otherwise labelled as 'Taylor Curve' (Taylor, 1979). Moreover, our search for the optimal weight μ assumes that the parameters of our economy remains unvaried after a variation in the policy rule. Given the presence of RE, the calibration of our hybrid model is not affected by the Lucas (1976) critique. Instead, our exercise with the backward looking specification of the economy is hit by that. Of course, if a variation in the policy rule led to remarkable changes of the structural parameters of the economy, our empirical analysis would risk to be flawed. However, the empirical relevance of the Lucas critique in this context seems to be discussable. In fact, Rudebusch (2002c) shows that with an AD-AS backward looking model like the one used in this study the empirical relevance of the critique is not overwhelming; further evidence concerning this latter point is provided by Estrella and Fuhrer (1999).

4 Our Findings

In this section we present our findings. In Table 2/Panel a) we collect the results of our calibration exercise which are unconditional with respect to the parameter value μ. Indeed, in absence of RE, the value of this parameter is quite large, and it has to be judged as economically unacceptable. If we believed that the CB could indeed have an interest rate smoothing goal, this

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[16] In performing these calibrations we do not take into account the 'natural' Zero Lower Bound constraint (see Amirault and O'Reilly, 2001, for a survey on this problem).

goal would surely not be 2.1 times more important than the volatility of inflation. Since we think of the smoothing argument as being a sort of 'catch all' approximating omitted (potentially important) components, then such a large value might signal the existence of an omitted variable problem. In fact, when adding RE to the model, our results change quite dramatically. The weight attached to the smoothing argument collapses to 0.5, so providing us with an important support in favor of RE as being a central element for correctly representing the economic dynamics. Some descriptive statistics seem to support this intuition. In fact, both for the mean and for the standard deviation of the interest rate *level* the simulated interest rate is much closer than the one deriving from the Benchmark model. As far as the standard deviation of the interest rate *change* is concerned, it is actually difficult to distinguish between the two models; however, we recall that the Benchmark formulation needs an incredible value of 2.1 to replicate the historical data.

What if we control for the weight μ? Table 2/Panel b) collects the results coming from simulations in which the value 0.5 (the calibrated weight μ in the Hybrid model case) has been imposed also to our Benchmark. This is done in order to *quantitatively* assess the role of RE. Notably, all the descriptive statistics are clearly in favor of the Hybrid Model. In particular, with such a low μ, the Benchmark model's simulated policy exhibits an excessive volatility both in levels and in first differences. Moreover, the distance reduction gained when passing from the Benchmark model to the Hybrid one is about 60%. This can loosely be seen as a measure of the bit of the observed smoothness that the Benchmark model is not capable to justify - that is where much of the need for a high value of μ comes from - and that RE help explaining. Although not exhaustive, it seems to us definitely important.

In Table 2/Panel c) we collect the values of the feedback rule (7)'s coefficients. Those referring to the actual rule have been OLS estimated on the basis of the historical time-series, while the simulated ones are the solution of the CB's control problem. In fact, the coefficients concerning all the contemporaneous and lagged expressions of inflation and output gap are numerically closer to the actual one, while intriguing differences arise when the lagged interest rates are considered. In fact, the first lag shows a positive sign in all the feedback rules; however, while the estimated one has a very large value (1.259), the simulated rates attributes to that explanatory part a value of 0.576 (Benchmark) and 0.562 (Hybrid). Notice that there are three lags of the federal fund rate in this feedback rules. When considering the sum of the coefficients of all the three, we end up having an overall impact in the actual feedback rule which is equal to 0.853, while the Benchmark reads

0.511 vs. a value for the Hybrid equal to 0.562. Hence, the overall impact of the lagged rates is about the same in the simulated rates, and lower than that of the actual one. The true difference between the simulated policy variables seems to be due to the diverse response they have with respect to inflation and output gap. Once more, the smoother response shown by the Hybrid rate finds its justification in the RE effect.

Table 2/Panel c) suggests that there is a numerical difference in the coefficients of the Benchmark vs. Hybrid specifications' feedback rules. However, it should be understood that this numerical difference *per se* is not enough to plot so diverse simulated rates as those in Figure 2. Indeed, it is the presence of the RE element that induce the degree of smoothness that features the optimal rate in the Hybrid context. To prove this statement, we take the optimal feedback rule *coefficients* coming from the Hybrid specification, and we use it together with the Benchmark model to simulate the economic development. We then compute some descriptive statistics of the outcoming simulated policy rate. Table 3 presents our findings. As it is possible to see, the more gradual response to the economic evolution does not induce such a smoother track of the federal fund rate *per se*; indeed, descriptive statistics improve only timidly, and the distance reduction (2.42%) is negligible. When adding RE to the model without varying the feedback coefficients (Table 3, move from the third to the fourth row), we may observe a quite striking net effect. Indeed, expectations seem to be of great help in explaining the policy rate inertial behavior.

5 The Importance of RE

What is the economic *rationale* for this result? Why are RE so important in describing the observed smooth path of the policy rate, which is to say in *squeezing the interest rate smoothing weight*? To understand this, it is useful to refer to recent contributions by Woodford (1999,2001). He shows how in a model with forward looking agents it may be efficient to implement an inertial policy rate. In particular, Woodford (2001, pag. 15) writes:

" *When the effects of policy depends crucially upon private sector expectations about future policy as well, it is generally optimal for policy to be history-dependent, so that the anticipation of later policy responses can help to achieve the desired effect upon private sector behavior.*"

Then, the possibility of influencing private agents' expectations throughout the implementation of an inertial policy rate is the reason why, *ceteris* *paribus*, an optimally determined policy rate will show a higher degree of inertia in presence of rational agents than when a fully adaptive private sector is taken into account. Of, course, this has strong implications for our residual weight μ. In fact, given that our aim is to replicate the observed smooth actual rate, it should be clear that with an economy featured by RE, just a moderate weight attached to the interest rate argument will be sufficient to trigger beneficial expectations on the future inflation rate which will imply a lower level of the current inflation rate, so rendering monetary actions more effective. Instead, in an economy characterized by fully adaptive agents, Woodford's suggestion regarding the optimal monetary policy inertia does not find any room; in this case, the optimal policy rate shows frequent reversals, given that the CB has to off-set white noise economic shocks. Then, with such a backward looking model at hand, a researcher will need to impose an high weight on the interest rate smoothing argument in order to fit the facts.

6 Robustness Check: Some Considerations

We perform a robustness check of our results focussing on the value of the relative preference parameter λ. Our findings are contained in Tables 4-6, and refers to values such as 0.0, 0.2, and 1.0. We list here some intuitions that may be gained when looking at our sensitivity analysis.

1) RE dramatically reduce the importance of the interest rate smoothing argument in the loss function from a descriptive viewpoint. In fact, the conditional distance reduction got when embedding into the model forward looking agents goes from a minimum of 46.67% (case with $\lambda = 1.0$) up to 81.12% ($\lambda = 0.0$).

2) RE seem to be particularly important as far as the *real* interest rate in the AD equation is concerned; indeed, all along our sensitivity exercises, γ_r turns out being equal to 1. Instead, the percentage of firms and households fully forward looking seems to be low: γ_π assumes values such as 0.1 or 0.2, while γ_y figures like 0.2 or 0.3. These figures are in line with many studies regarding the importance of 'rules of thumb' in agents' decisions (for the Phillips curve, see e.g. Roberts, 1998, 2001; Lindè, 2002; Rudd and Whelan, 2001; Rudebusch, 2001; Söderlind, Söderstrom, and Vredin 2002; relatively to the IS equation, Fuhrer and Rudebusch, 2002). Nevertheless, RE a key role in our positive exercise.

3) The smallest value of our distance measure is the one related to the

framework in which the output gap weight is zero. This means that if we calibrated the preference λ over the grid [0.0; 0.2; 0.5; 1.0] we would find that the first figure is the one that most closely represents Greenspan's preferences. A possible interpretation of this result is provided by Dennis (2002) and Favero and Rovelli (2002), who underline the role of the output gap as leading indicator for future inflation. In other words, the output gap would not find any role in Greenspan's penalty function, but still it could be very much important for the determination of the optimal policy rate; in fact, the output gap turns out being a statistically relevant explanatory variable in the estimated feedback rule (Tables 2, 4-6/Panel c). An alternative possible explanation, in the spirit of the evidence on output gap uncertainty in Smets (1999), Estrella and Mishkin (1999), and Wieland (1998), is that monetary authorities may have placed a low weight on the most poorly measured goal, or yet, that the market productivity growth of the 90s may have drastically reduced any concern of output stabilization. A similar result is also present in Cecchetti, Flores-Lagunes, and Krause (2001).[17]

7 Still a Positive μ: Possible Reasons

The RE ingredient considerably reduces the weight to be attributed to the interest rate smoothing argument in order to replicate the data. Nevertheless, our calibration exercises confirm that a positive μ is needed in order to obtain a smooth equilibrium path of the federal fund rate. This calls for other, complementary ingredients to be added to our framework in order to replicate the historical policy rate and reduce the 'quick fix' component μ. Sack and Wieland (2000) suggest other two possible reasons for having policy cautiousness: parameter uncertainty and measurement error affecting the data. We discuss them here, also adding a few considerations on learning, model uncertainty, and financial markets.

Parameter Uncertainty

In the real world, monetary policy-making is an exercise undertaken in an uncertain environment (Goodhart, 1999). It is well known that, in a linear quadratic scheme like the one we exploit in the present analysis, additive

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[17] Interestingly, our set of calibrated parameters, i.e. λ=0, μ=0.4, γ_π=0.1, γ_y=0.3, and γ_r=1, turns out being very close to the one in SSV (2002)'s work, i.e. λ=0.1, μ=0.5, γ_π=0.1, γ_y=0.5. Notice that SSV perform their calibration by taking into account not only some moments of policy rate, inflation rate, and output gap, but also the standard deviation of inflation and output shocks.

uncertainty does not matter for the solution of the regulator problem because of the certainty equivalence principle (see Ljungqvist and Sargent, 2000, ch. 4). Instead, a much more interesting case is that of multiplicative uncertainty á la Brainard (1967), i.e. the uncertainty directly affecting the parameters regulating the dynamics of the economy. Brainard (1967), Blinder (1997), and Sack and Wieland (2000) explain that when the CB is partially ignorant about the key-parameters of the economy, he finds it optimal to implement prudent monetary policies in response to shocks affecting the economy, since in this way it is possible to reduce the 'uncertainty cost', i.e. the possibility of leading the economy to an undesirable status.[18] Söderstrom (1999) and Sack (2000) demonstrate that with VARs representations of the economic environment it is possible to replicate fairly well the policy rate path by taking into account parameter uncertainty even in absence of an interest rate smoothing target. However, there is not a complete agreement yet regarding the link between this type of uncertainty and the optimal CB's behavior. In fact, Söderstrom (2002) suggests that uncertainty related to the persistence of the inflation mechanism may induce CBs to implement an aggressive strategy, in order to avoid bad outcomes in the future.[19] Robust-control oriented works, such as those by Sargent (1999), Stock (1999), Tetlow and von sur Muchlen (2001), and Onatski and Stock (2002) show that the best possible reaction of the CB to the worst-scenario drawn by the Nature is an aggressive monetary action.[20] Finally, with the use of small scale models and focusing just on a few key-parameters, Estrella and Mishkin (1999), Peersman and Smets (1999), and Rudebusch (2001) claim that parameter uncertainty is not so important from a quantitative viewpoint.

Measurement Error

Orphanides (1998) offers an important contribution regarding the noise affecting the data. His point is intuitive: CB should respond to shocks cautiously, because it is difficult to understand if the one under consideration is a pure economic shock, or instead just a measurement error (or a mix between the two). In fact, this presumed optimal gradualism is challenged if we

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[18] In his paper Brainard (1967) adds that this result is driven by the low covariance level between the instrument and the state variables; indeed, a high covariance could overturn the result.

[19] Anyhow, Söderstrom (2002) underlines also that parameter uncertainty related to other dynamics in the economy lead to the standard CB's optimal response to shocks.

[20] However, this results hinges on the worst scenario that the Nature can draw (or a scenario in its neighbourhood), apparently discarding all the other ex-ante possible scenarios.

shape the CB's problem with a linear-quadratic scheme. In this case, the certainty equivalence principle suggests that additive shocks (like measurement errors) do not influence the optimally determined feedback coefficients if we refer to fully state contingent optimal rules. Instead, when simple rules (e.g. á la Taylor 1993) are taken into account, the increase in volatility caused by measurement errors does matter, and can then be seen as a reason for implementing cautious policies.

Learning

Up to now, we have discussed about a 'fixed' degree of uncertainty. In practice, monetary policy makers' beliefs about the relevant structural parameters as well as the precision of the estimates are likely to be time-varying, above all if the unknown parameters are themselves changing (Sack, 1998; Wieland, 2000). As Sack and Wieland (2000) suggest, since the degree of uncertainty about the relevant parameters varies over time, the incentive for cautious policy making is not constant. However, the lagged interest rate could be an indicator capable to capture the effect of parameter uncertainty in a simple policy rule, because it represents the optimal decision implemented under a similar degree of uncertainty in the previous period.

Notice that, under the learning hypothesis, there may be some justifications for large movements of the interest rate, i.e. the will to more effectively learn about parameters, an effect referred to as *experimentation* (see Bertocchi and Spagat, 1993, and Caplin and Leahy, 1996). Nevertheless, Sack and Wieland (2000) comment that even in case of experimentation the optimal policy is still found to be less aggressive in most situations than a policy that disregards parameter uncertainty.[21]

Model Uncertainty

McCallum (1999) sustains that a good policy rule is the one that is capable to perform well across many different models. In fact, not only a CB is uncertain about the key-parameters of the equations formalizing the economy; indeed, he is uncertain regarding the structure of the equations themselves. From the descriptive side, recent empirical contributions by Favero and Milani (2001) and Castelnuovo and Surico (2002), conducted in a class of linear backward looking models, show that model uncertainty indeed helps explaining the observed policy rate behavior.

Financial Constraints

A positive value of the μ parameter might also be the expression of the

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[21] Regarding this point, it is worth to signal a comment by a former Vice-Chairman of the Fed, Alan Blinder (1998, p.11): "You don't conduct experiments on a real economy solely to sharpen your econometric estimates".

concern that the CB has for the financial markets, markets that are thought as being very reactive to large swings of the nominal interest rate (Goodfriend, 1991; Blinder, 1997; Mishkin, 1999). An interesting point tackling this view is provided by Cecchetti (2001). In fact, he claims that large jumps in the policy instruments could be disruptive only if financial markets are relatively certain that it will never happen. Instead, if market participants expect that new information can precipitate large and sudden interest rate changes, then they will defend themselves by building up institutions that can withstand the potential disruptions this would otherwise cause. In synthesis, the only reason that people believe smooth interest rates enhance financial stability is because interest rate has been smooth up to now.

8 Conclusions

The interest rate smoothing argument has been debated quite intensively in the past few years. From a positive perspective this argument is often needed in order to generate the observed persistence of the policy rate. In fact, in small scale, fully backward looking models the interest rate smoothing has usually got a puzzling high relative value.

In this paper we show that rational expectations may play a big role in partially solving this puzzle. Indeed, by comparing the outcomes stemming from a backward looking model with those deriving from a calibrated hybrid one, we found that the rational expectations component seems to be necessary for obtaining a sensible description of the federal funds rate path. Implicitely, this suggests that Greenspan has seriously taken into account private sector's expectations when setting US monetary policy.

Our calibration exercise also reveals that Greenspan does not seem to have had a high concern on the output gap *per se*; indeed, he has probably considered it as a good leading indicator for future inflation. Other robust findings are those regarding the degrees of forwardness. While households seem to be very much forward looking relatively to the real interest rate in the IS schedule, the same does not hold true when looking at the dynamic AD-AS curves. Our results support the idea that rules of thumb are important from descriptive purposes when small macro models are employed.

Although very much important, rational expectations are not sufficient to render the interest rate smoothing argument negligible when performing positive exercises. We feel that learning, model uncertainty, and real-time data are the most credible candidates for a further reduction (if not the complete elimination) of the interest rate smoothing argument in the CB's penalty function. We plan to undertake endeavors along these lines early in the future.

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10 Technical Appendix

The algorithm to solve the optimal control problem faced by the CB is more easily understandable if the model representing the economy is written in its state-space form.[22] Consider equations (4) and (5), which we rewrite here below:

$$\begin{aligned}\pi_{t+1} &= \gamma_\pi E_t(\frac{\pi_{t+1}+\pi_{t+2}+\pi_{t+3}+\pi_{t+4}}{4}) \\ &\quad +(1-\gamma_\pi)(\alpha_{\pi 1}\pi_t+\alpha_{\pi 2}\pi_{t-1}+\alpha_{\pi 3}\pi_{t-2}+\alpha_{\pi 4}\pi_{t-3})+\alpha_y y_t+\varepsilon_{t+1}\end{aligned} \tag{9}$$

$$\begin{aligned}y_{t+1} &= \gamma_y E_t y_{t+2}+(1-\gamma_y)(\beta_{y1}y_t+\beta_{y2}y_{t-1}) \\ &\quad -\beta_r\gamma_r[i_t-E_t(\frac{\pi_{t+1}+\pi_{t+2}+\pi_{t+3}+\pi_{t+4}}{4})] \\ &\quad -\frac{\beta_r(1-\gamma_r)}{4}(i_t+i_{t-1}+i_{t-2}+i_{t-3}-\pi_t-\pi_{t-1}-\pi_{t-2}-\pi_{t-3})+\eta_{t+1}\end{aligned} \tag{10}$$

To solve the optimal control problem, we basically have to compute the expectations terms $E_t(\pi_{t+4})$ and $E_t y_{t+2}$. Noticing that $\pi_{t+1}=E_t\pi_{t+1}+\varepsilon_{t+1}$ and $y_{t+1}=E_t y_{t+1}+\eta_{t+1}$ (where ε_{t+1} and $\eta_{t+1}$are white noise), it is then possible to write (9) and (10) as follows:

$$\begin{aligned}\frac{\gamma_\pi E_t\pi_{t+4}}{4} &= (1-\frac{\gamma_\pi}{4})E_t\pi_{t+1}-\frac{\gamma_\pi}{4}E_t\pi_{t+2}-\frac{\gamma_\pi}{4}E_t\pi_{t+3} \\ &\quad -(1-\gamma_\pi)(\alpha_{\pi 1}\pi_t+\alpha_{\pi 2}\pi_{t-1}+\alpha_{\pi 3}\pi_{t-2}+\alpha_{\pi 4}\pi_{t-3})-\alpha_y y_t\end{aligned} \tag{11}$$

$$\begin{aligned}\gamma_y E_t y_{t+2}+\beta_r\gamma_r E_{t+4} &= E_t y_{t+1}-(1-\gamma_y)(\beta_{y1}y_t+\beta_{y2}y_{t-1}) \\ &\quad +\beta_r\gamma_r[i_t-E_t(\frac{\pi_{t+1}+\pi_{t+2}+\pi_{t+3}}{4})] \\ &\quad +\frac{\beta_r(1-\gamma_r)}{4}\sum_{j=0}^{3}(i_{t-j}-\pi_{t-j})\end{aligned} \tag{12}$$

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[22] This technical appendix is very much in line with the one in Söderlind, Söderstrom, and Vredin (2002).

Notice that, as already specified in the text, we compute the discretionary solution of the problem, given that it is time-consistent. To find it, we use Söderlind (1999)'s strategy.[23] This strategy requires a precise distinction of the elements involved in the problem between state (predetermined) and jump (forward-looking) variables. So, we define the $(n1x1)$ vector of predetermined state variables as follows $(n1 = 9)$:

$$x_{1t} = \left[\begin{array}{ccccccccc} \pi_t & \pi_{t-1} & \pi_{t-2} & \pi_{t-3} & y_t & y_{t-1} & i_{t-1} & i_{t-2} & i_{t-3} \end{array} \right]' \tag{13}$$

and the $(n2x1)$ vector of forward-looking jump ones as here below $(n2 = 4)$:

$$x_{2t} = \left[\begin{array}{cccc} E_t\pi_{t+3} & E_t\pi_{t+2} & E_t\pi_{t+1} & E_t y_{t+1} \end{array} \right]' \tag{14}$$

Since we are solving a stochastic problem, we also define the $n1x1$ vector of shocks to the predetermined variables as:

$$v_{1t} = [\varepsilon_t, 0_{1x3}, \eta_t, 0_{1x4}]' \tag{15}$$

Then, the state-space representation of the problem is the following:

$$A_0 \left[\begin{array}{c} x_{1t+1} \\ E_t x_{2t+1} \end{array} \right] = A_1 \left[\begin{array}{c} x_{1t} \\ x_{2t} \end{array} \right] + B_1 i_t + v_{t+1} \tag{16}$$

where

$$v_{t+1} = \left[\begin{array}{c} v_{1t+1} \\ 0_{n2x1} \end{array} \right] \tag{17}$$

and where the matrices A_0, A_1, and B_1 read as follows:

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[23] The Gauss and Matlab routines for solving the optimal stochastic regulator problem presented in this Technical Appendix can be found in Söderlind's webpage, i.e. http://www.hhs.se/personal/PSoderlind/.

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\gamma_\pi}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\beta_r \gamma_r}{4} & 0 & 0 & \gamma_y \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \widetilde{\alpha}_{\pi 1} & \widetilde{\alpha}_{\pi 2} & \widetilde{\alpha}_{\pi 3} & \widetilde{\alpha}_{\pi 4} & -\alpha_y & 0 & 0 & 0 & 0 & -\frac{\gamma_\pi}{4} & -\frac{\gamma_\pi}{4} & (1-\frac{\gamma_\pi}{4}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \widehat{\beta}_r & \widehat{\beta}_r & \widehat{\beta}_r & \widehat{\beta}_r & \widetilde{\beta}_{y1} & \widetilde{\beta}_{y2} & -\widehat{\beta}_r & -\widehat{\beta}_r & -\widehat{\beta}_r & -\frac{\beta_r \gamma_r}{4} & -\frac{\beta_r \gamma_r}{4} & -\frac{\beta_r \gamma_r}{4} & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0_{1x6} & 1 & 0_{1x5} & \beta_r \gamma_r - \widehat{\beta}_r \end{bmatrix}'$$

where $\widetilde{\alpha}_{\pi j} = -(1-\gamma_\pi)\alpha_{\pi j}, \widehat{\beta}_r = -\frac{\beta_r(1-\gamma_r)}{4}, \widetilde{\beta}_{yj} = -(1-\gamma_y)\beta_{yj}$.

To obtain the standard state-space representation, we just have to premultiply (16) by A_0^{-1}, so obtaining

$$\left[\begin{array}{c} x_{1t+1} \\ E_t x_{2t+1} \end{array} \right] = A \left[\begin{array}{c} x_{1t} \\ x_{2t} \end{array} \right] + Bi_t + v_{t+1} \tag{18}$$

with $A = A_0^{-1} A_1$ and $B = A_0^{-1} B_{1.}$[24]

It is useful to express also the CB's objective function in a compact form. To do so, it is necessary to write down the vector of the arguments targeted by the CB. This vector is defined as:

$$z_t = \left[\begin{array}{ccc} \bar{\pi}_t & y_t & \Delta i_t \end{array} \right]' \tag{19}$$

Notice that, given our choice of working with demeaned variables which renders easier the management of the optimal stochastic regulator problem, π^* is normalized to be equal to zero.

The goal variables included in vector (19) can be expressed via the following formula:

$$z_t = C_x x_t + C_i i_t \tag{20}$$

where

$$C_x = \left[\begin{array}{ccccccccccccc} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

and

$$C_i = \left[\begin{array}{ccc} 0 & 0 & 1 \end{array} \right]'$$

---

[24] Notice that the requirement for having $det(A_0) \neq 0$ is that $\gamma_\pi, \gamma_y \neq 0$. That is why, when identifying the Benchmark model (i.e. fully backward looking model) in our exercise, we do not set those weights to a zero value. Instead, we set them equal to 0.0001. This is a shortcoming deriving from our choice of using the procedure elaborated by Paul Söderlind (1999) for solving RE models. Richard Dennis made us notice that, if we were to solve for the optimal discretionary rule using the *structural* form of the model rather than the *state-space* form, this problem would vanish. For further information abour this point, see Dennis (2000). Notice also that $A_0^{-1} v_{t+1} = v_{t+1}$, since A_0 is block diagonal with an identity matrix as its upper left block and the lower block of $v_{t+1}$is equal to zero.

The CB attributes to the quadratic transformation of the arguments in (19) different weights. We normalize the weight on the average inflation rate to one, and we attribute relative weights to the other targets, as follows:

$$L_t = \bar{\pi}_t^2 + \lambda y_t^2 + \mu \Delta i_t^2 \tag{21}$$

which can be re-expressed as:

$$L_t = z_t^{'} K z_t \tag{22}$$

where K is a $3x3$ diagonal matrix containing the relative concerns of the CB. K is shaped in this way:

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \mu \end{bmatrix} \tag{23}$$

Using (20), the period loss function (22) can be re-expressed as follows:

$$\begin{aligned} L_t &= \begin{bmatrix} x_t^{'} & i_t^{'} \end{bmatrix} \begin{bmatrix} C_x^{'} \\ C_i^{'} \end{bmatrix} K \begin{bmatrix} C_x & C_i \end{bmatrix} \begin{bmatrix} x_t \\ i_t \end{bmatrix} \\ &= x_t^{'} C_x^{'} K C_x x_t + x_t^{'} C_x^{'} K C_i i_t + i_t^{'} C_i^{'} K C_x x_t + i_t^{'} C_i^{'} K C_i i_t \\ &= x_t^{'} Q x_t + x_t^{'} U i_t + i_t^{'} U' x_t + i_t^{'} R i_t \end{aligned}$$

where $x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}$,

and where $Q = C_x^{'} K C_x$, $U = C_x^{'} K C_i$, $R = C_i^{'} K C_i$.

Hence the CB's optimal control problem is given by the conventional Bellman equation

$$J_t = E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} (x_\tau^{'} Q x_\tau + x_\tau^{'} U i_\tau + i_\tau^{'} U' x_\tau + i_\tau^{'} R i_\tau) \tag{24}$$

subject to the law of motion of the economy (18). As already written in the text, it turns out that the optimal discretionary policy is a rule for the interest rate as a linear function of the predetermined variables in the vector x_{1t}, i.e.

$$i_t = -Fx_{1t} \tag{25}$$

Notice an important equivalence. Rudebusch and Svensson (1999) underline how, when the discount factor $\delta \to 1$, the intertemporal loss function (24) approaches the unconditional mean of the period loss function. Hence, we can write is as

$$E(L_t) = Var(\bar{\pi}_t^2) + \lambda Var(y_t^2) + \mu Var(\Delta i_t^2) \tag{26}$$

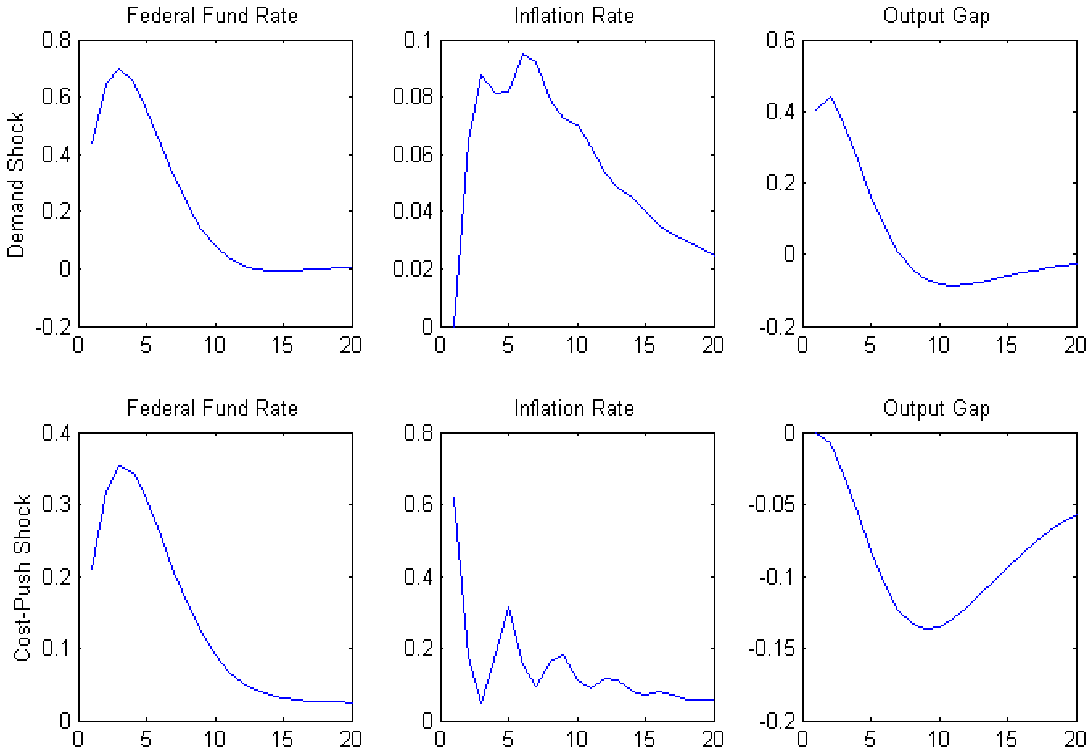
which makes it clear how the role of the CB is indeed that of stabilizing the economy.

After having

1) initialized the vector x_{10} with historical observations,
2) attributed to the vector x_{20} nil values,
3) determined the relative weights λ and μ in the loss function (26),
4) set the values of the key-coefficients $\alpha_s, \beta_s, \gamma_\pi, \gamma_y$, and γ_r in the matrices A_0, A_1, and B,
5) stored the structural residuals into the vector v_t, and
6) computed the optimal feedback coefficients in F,

we can exploit expressions (18) and (25) in order to simulate how the economy would have evolved if the CB had implemented the policy rule solution of the optimal control problem. Finally, given the simulated time-series for π_t, y_t, and i_t, it is easy to compute the value of the expected loss (26).

Figure 1: Impulse Response Functions



Note : one standard deviation shocks. The Impulse Response Functions were computed by considering the following calibrated parameters' values: $\lambda = 0.5$, $\mu = 0.5$, $\gamma_\pi = 0.1$, $\gamma_y = 0.2$, $\gamma_r = 1$.

Table 1. Estimates of the AD-AS backward looking structure

Phillips Curve

$$\pi_{t+1} = \alpha_{\pi 1}\pi_t + \alpha_{\pi 2}\pi_{t-1} + \alpha_{\pi 3}\pi_{t-2} + \alpha_{\pi 4}\pi_{t-3} + \alpha_y y_t + \varepsilon_{t+1}$$

Parameter	$\alpha_{\pi 1}$	$\alpha_{\pi 2}$	$\alpha_{\pi 3}$	$\alpha_{\pi 4}$	α_y
Point Estimate	0.282	-0.025	0.292	0.385	0.141
St. Deviation	0.133	0.134	0.134	0.136	0.054

Adjusted R^2*: 0.58;* $\sigma_\varepsilon=0.66$.

Aggregate Demand Curve

$$y_{t+1} = \beta_{y1}y_t + \beta_{y2}y_{t-1} + \beta_r(\bar{i}_t - \bar{\pi}_t) + \eta_{t+1}$$

Parameter	βy1	βy2	βr
Point Estimate	1.229	-0.244	-0.073
St. Deviation	0.136	0.149	0.078

Adjusted R^2*: 0.93;* $\sigma_\eta=0.51$.

Variables definition and specification

π_t is the quarterly inflation rate in the GDP chain-weighted price index (P_t) in percent at an annual rate (i.e. $\pi_t \equiv 4(p_t - p_{t-1})$, where $p_t = 100 lnPt$);

y_t is the output gap (approximately $q_t - q_t^*$, where $q_t \equiv 100 lnQ_t$ and $q_t^* \equiv 100 lnQ_t^*$ with Q_t defined as chain-weighted real GDP and Q_t^* defined as potential GDP as estimated by the Congressional Budget Office);

i_t is the Federal Fund rate;

the upper-barred variables indicates simples averages taken over the contemporaneous observation and the previous three lags of the variable in consideration.

All the variables have been demeaned, so no constants appear in the model here above written.

The estimates refer to the sample 1987:3-2001:1, US quarterly data. The data used for performing these estimates are downloadable at the following URL: http://research.stlouisfed.org/fred2/. The estimates have been performed by OLS equation by equation; the correlation between the two equations turns out being very low (0.137). Last, the Andrew (1993)'s test cannot reject the null of structural stability for both equations.

Table 2. Calibrated Interest Rate Smoothing Weights, Policy Rates Descriptive Statistics, and Feedback Coefficients

Panel a): Unconditional Analysis

Interest Rate	**Calibrated μ**	**Mean**	**Std(i_t)**	**Std(Δi_t)**
Actual	-	0	1.8175	0.4723
Optimal: Benchmark Model	2.1	0.8074	2.9924	0.6364
Optimal: Hybrid Model	0.5	0.5692	1.7942	0.5074

Note: the calibration has been performed by fixing the weight of the output gap **λ = 0.5**, and by searching for the weight μ capable to minimize the distance between the actual and the optimal rate, distance computed as average of the sum of squared deviations with respect to the historical rate. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$). The Hybrid Model is featured by $\gamma_\pi = 0.1$, $\gamma_y = 0.2$, $\gamma_r = 1$.

Panel b) : Conditional Analysis (μ = 0.5)

Interest Rate	**Mean**	**Std(i_t)**	**Std(Δi_t)**	**Correlation w.r. to Actual**	**Distance w.r. to Actual**	**Distance reduction**
Actual	0	1.8175	0.4723	-	-	-
Optimal: Benchmark Model	0.8431	3.4839	0.9272	0.9087	2.1445	-
Optimal: Hybrid Model	0.5692	1.7942	0.5074	0.9411	0.8519	60.24%

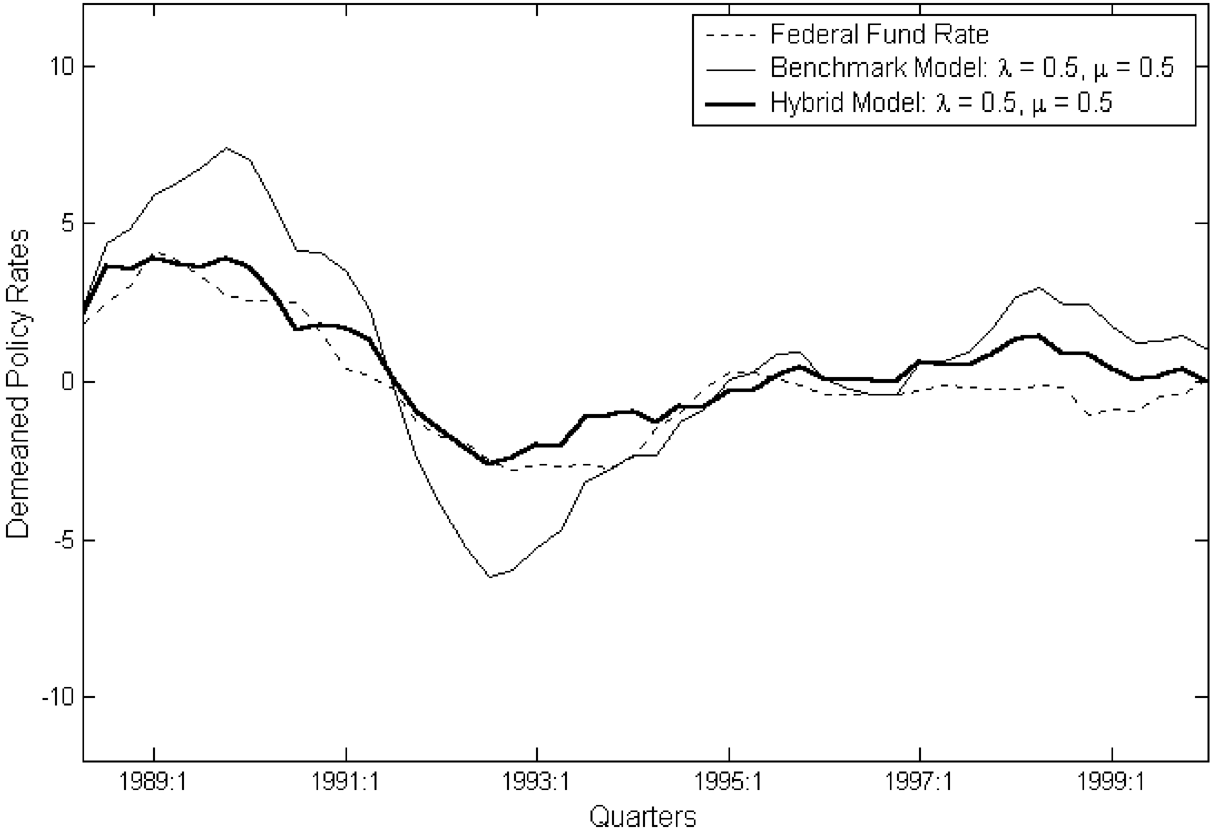
Note: figures obtained by fixing the weight of the output gap **λ = 0.5**, and the weight on the interest rate smoothing **μ = 0.5**. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$). The Hybrid Model is featured by $\gamma_\pi = 0.1$, $\gamma_y = 0.2$, $\gamma_r = 1$.

Panel c) : Policy Rule Coefficients

Feedback Rules	π_t	π_{t-1}	π_{t-2}	π_{t-3}	y_t	y_{t-1}	i_{t-1}	i_{t-2}	i_{t-3}
Actual Rule	.212 (.07)	.043 (.08)	.151 (.08)	-.177 (.09)	.346 (.10)	-.265 (.11)	1.259 (.14)	-.398 (.20)	-.008 (.12)
Optimal: Benchmark Model	.380	.274	.277	.160	1.256	-.295	.576	-.043	-.022
Optimal: Hybrid Model	.347	.242	.229	.113	1.128	-.273	.562	0	0

Note: these coefficients refer to the state-contingent feedback rule
$i_t = f_1 \pi_t + f_2 \pi_{t-1} + f_3 \pi_{t-3} + f_4 \pi_{t-4} + f_5 y_t + f_6 y_{t-1} + f_7 i_{t-1} + f_8 i_{t-2} + f_9 i_{t-3}$
The coefficients of the Actual Rule have been OLS estimated by considering the sample 1987:3 – 2001:1, quarterly data (standard deviations in brackets). Those of the Optimal Rules have been computed by minimizing the penalty function (equation 3 in the paper) taking into account the law of motion of the economy (equation 6). The structural parameters' values are those indicated in this Figure/Panel b).

Figure 2: Policy Rates Behavior - Benchmark vs. Hybrid Model



Note: the preference parameter on the volatility of average inflation is normalized to one. λ denotes the preference on output gap stabilization, while μ that on the interest rate change. The optimal paths show the values that the variables would have taken if the Fed had historically implemented the optimal policy rule. The Benchmark Model is featured by the following parameters' values: $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$. Hybrid Model's values: $\gamma_\pi = 0.1$, $\gamma_y = 0.2$, $\gamma_r = 1$.

Table 3. Statistics on RE Descriptive Power

Interest Rate	Mean	Std(i_t)	Std(Δi_t)	Correlation w.r.t. to Actual	Distance w.r.t. to Actual	Distance Reduction
Actual	0	1.8175	0.4723	-	-	-
Simulated: Benchmark Model with Benchmark Rule (CFC)	0.8431	3.4839	0.9272	0.9087	2.1445	-
Simulated: Benchmark Model with Hybrid Rule (IFC)	0.8192	3.4267	0.8849	0.9068	2.0925	2.42%
Simulated: Hybrid Model ($\gamma_\pi = 0.1$, $\gamma_y = 0.2$, $\gamma_r = 1$) *with Hybrid Rule (CFC)*	0.5692	1.7942	0.5074	0.9411	0.8519	60.24%

Note: figures obtained by fixing the weight of the output gap $\boldsymbol{\lambda} = \mathbf{0.5}$, and the weight on the interest rate smoothing $\boldsymbol{\mu} = \mathbf{0.5}$. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$). CFC stands for 'Consistent Feedback Coefficients', i.e. coefficients stemming from the relative optimal control problem. IFC stands for 'Inconsistent Feedback Coefficients', i.e. coefficients stemming from another optimal control problem.

Table 4. Calibrated Interest Rate Smoothing Weights, Policy Rates Descriptive Statistics, and Feedback Coefficients

Panel a): Unconditional Analysis

Interest Rate	**Calibrated μ**	**Mean**	**Std(i_t)**	**Std(Δi_t)**
Actual	-	0	1.8175	0.4723
Optimal: Benchmark Model	10	0.5369	2.5432	0.3876
Optimal: Hybrid Model	0.4	0.1890	1.5930	0.4026

Note: the calibration has been performed by fixing the weight of the output gap $\boldsymbol{\lambda} = \mathbf{0.0}$, and by searching for the weight μ capable to minimize the distance between the actual and the optimal rate, distance computed as average of the sum of squared deviations with respect to the historical rate. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$). The Hybrid Model is featured by $\gamma_\pi = 0.1$, $\gamma_y = 0.3$, $\gamma_r = 1$.

Panel b) : Conditional Analysis ($\mu = 0.4$)

Interest Rate	**Mean**	**Std(i_t)**	**Std(Δi_t)**	**Correlation w.r. to Actual**	**Distance w.r. to Actual**	**Distance reduction**
Actual	0	1.8175	0.4723	-	-	-
Optimal: Benchmark Model	0.6079	4.7276	1.0106	0.8672	3.3049	-
Optimal: Hybrid Model	0.1890	1.5930	0.4026	0.9478	0.6241	81.12%

Note: figures obtained by fixing the weight of the output gap $\boldsymbol{\lambda} = \mathbf{0.0}$, and the weight on the interest rate smoothing $\boldsymbol{\mu} = \mathbf{0.4}$. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$). The Hybrid Model is featured by $\gamma_\pi = 0.1$, $\gamma_y = 0.3$, $\mu_r = 1$.

Panel c) : Policy Rule Coefficients

Feedback Rules	π_t	π_{t-1}	π_{t-2}	π_{t-3}	y_t	y_{t-1}	i_{t-1}	i_{t-2}	i_{t-3}
Actual Rule	.212	.043	.151	-.177	.346	-.265	1.259	-.398	-.008
	(.07)	(.08)	(.08)	(.09)	(.10)	(.11)	(.14)	(.20)	(.12)
Optimal: Benchmark Model	.450	.319	.323	.184	.856	-.193	.666	-.028	-.014
Optimal: Hybrid Model	.422	.292	.275	.148	.765	-.179	.628	0	0

Note: these coefficients refer to the state-contingent feedback rule

$i_t = f_1 \pi_t + f_2 \pi_{t-1} + f_3 \pi_{t-3} + f_4 \pi_{t-4} + f_5 y_t + f_6 y_{t-1} + f_7 i_{t-1} + f_8 i_{t-2} + f_9 i_{t-3}$

The coefficients of the Actual Rule have been OLS estimated by considering the sample 1987:3 – 2001:1, quarterly data (standard deviations in brackets). Those of the Optimal Rules have been computed by minimizing the penalty function (equation 3 in the paper) taking into account the law of motion of the economy (equation 6). The structural parameters' values are those indicated in this Figure/Panel b).

Table 5. Calibrated Interest Rate Smoothing Weights, Policy Rates Descriptive Statistics, and Feedback Coefficients

Panel a): Unconditional Analysis

Interest Rate	**Calibrated μ**	**Mean**	**Std(i_t)**	**Std(Δi_t)**
Actual	-	0	1.8175	0.4723
Optimal: Benchmark Model	7.4	0.6303	2.5787	0.4309
Optimal: Hybrid Model ($\gamma_\pi = 0.2$, $\gamma_y = 0.2$, $\gamma_r = 1$)	0.5	0.4443	1.9436	0.4968

Note: the calibration has been performed by fixing the weight of the output gap $\boldsymbol{\lambda} = \mathbf{0.2}$, and by searching for the weight μ capable to minimize the distance between the actual and the optimal rate, distance computed as average of the sum of squared deviations with respect to the historical rate. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$).

Panel b) : Conditional Analysis ($\mu = 0.5$)

Interest Rate	**Mean**	**Std(i_t)**	**Std(Δi_t)**	**Correlation w.r. to Actual**	**Distance w.r. to Actual**	**Distance Reduction**
Actual	0	1.8175	0.4723	-	-	-
Optimal: Benchmark Model	0.7198	3.8366	0.9031	0.9060	2.4134	-
Optimal: Hybrid Model ($\gamma_\pi = 0.2$, $\gamma_y = 0.2$, $\gamma_r = 1$)	0.4443	1.9436	0.4968	0.9474	0.7721	68.01%

Note: figures obtained by fixing the weight of the output gap $\boldsymbol{\lambda} = \mathbf{0.2}$, and the weight on the interest rate smoothing $\boldsymbol{\mu} = \mathbf{0.5}$. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$).

Panel c) : Policy Rule Coefficients

Feedback Rules	π_t	π_{t-1}	π_{t-2}	π_{t-3}	y_t	y_{t-1}	i_{t-1}	i_{t-2}	i_{t-3}
Actual Rule	.212 (.07)	.043 (.08)	.151 (.08)	-.177 (.09)	.346 (.10)	-.265 (.11)	1.259 (.14)	-.398 (.20)	-.008 (.12)
Optimal: Benchmark Model	.393	.280	.284	.163	1.018	-.235	.625	-.034	-.018
Optimal: Hybrid Model	.366	.254	.240	.129	.923	-.221	.597	0	0

Note: these coefficients refer to the state-contingent feedback rule

$$i_t = f_1 \pi_t + f_2 \pi_{t-1} + f_3 \pi_{t-3} + f_4 \pi_{t-4} + f_5 y_t + f_6 y_{t-1} + f_7 i_{t-1} + f_8 i_{t-2} + f_9 i_{t-3}$$

The coefficients of the Actual Rule have been OLS estimated by considering the sample 1987:3 – 2001:1, quarterly data (standard deviations in brackets). Those of the Optimal Rules have been computed by minimizing the penalty function (equation 3 in the paper) taking into account the law of motion of the economy (equation 6). The structural parameters' values are those indicated in this Figure/Panel b).

Table 6. Calibrated Interest Rate Smoothing' s Weights, Policy Rates' Descriptive Statistics, and Feedback Coefficients

Panel a): Unconditional Analysis

Interest Rate	**Calibrated μ**	**Mean**	**Std(i_t)**	**Std(Δi_t)**
Actual	-	0	1.8175	0.4723
Optimal: Benchmark Model	2.6	0.9166	2.7908	0.6353
Optimal: Hybrid Model	1.3	0.6977	1.6167	0.4314

Note: the calibration has been performed by fixing the weight of the output gap $\boldsymbol{\lambda} = \mathbf{1.0}$, and by searching for the weight μ capable to minimize the distance between the actual and the optimal rate, distance computed as average of the sum of squared deviations with respect to the historical rate. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$). The Hybrid Model is featured by $\gamma_\pi = 0.1$, $\gamma_y = 0.2$, $\gamma_r = 1$.

Panel b) : Conditional Analysis (μ = 1.3)

Interest Rate	**Mean**	**Std(i_t)**	**Std(Δi_t)**	**Correlation w.r. to Actual**	**Distance w.r. to Actual**	**Distance reduction**
Actual	0	1.8175	0.4723	-	-	-
Optimal: Benchmark Model	0.9395	3.0101	0.7677	0.8916	1.8648	-
Optimal: Hybrid Model	0.6977	1.6167	0.4314	0.9248	0.9945	46.67%

Note: figures obtained by fixing the weight of the output gap $\boldsymbol{\lambda} = \mathbf{1.0}$, and the weight on the interest rate smoothing $\boldsymbol{\mu} = \mathbf{1.3}$. The Benchmark Model is the backward looking model (i.e. $\gamma_\pi = 0.0001$, $\gamma_y = 0.0001$, $\gamma_r = 0$). The Forward Looking Agents Model is featured by $\gamma_\pi = 0.1$, $\gamma_y = 0.2$, $\gamma_r = 1$.

Panel c) : Policy Rule Coefficients

Feedback Rules	π_t	π_{t-1}	π_{t-2}	π_{t-3}	y_t	y_{t-1}	i_{t-1}	i_{t-2}	i_{t-3}
Actual Rule	.212 (.07)	.043 (.08)	.151 (.08)	-.177 (.09)	.346 (.10)	-.265 (.11)	1.259 (.14)	-.398 (.20)	-.008 (.12)
Optimal: Benchmark Model	.262	.190	.192	.111	1.032	-.245	.613	-.036	-.018
Optimal: Hybrid Model	.248	.179	.164	.088	.968	-.238	.587	0	0

Note: these coefficients refer to the state-contingent feedback rule
$i_t = f_1 \pi_t + f_2 \pi_{t-1} + f_3 \pi_{t-3} + f_4 \pi_{t-4} + f_5 y_t + f_6 y_{t-1} + f_7 i_{t-1} + f_8 i_{t-2} + f_9 i_{t-3}$
The coefficients of the Actual Rule have been OLS estimated by considering the sample 1987:3 – 2001:1, quarterly data (standard deviations in brackets). Those of the Optimal Rules have been computed by minimizing the penalty function (equation 3 in the paper) taking into account the law of motion of the economy (equation 6). The structural parameters' values are those indicated in this Figure/Panel b).