Embodied technical change in a two-sector AK model

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Abstract

In this paper, we study a two-sector version of the AK model proposed by Rebelo (1991), where constant returns to capital are confined to the investment goods sector. We show that this setup, an endogenous growth extension to the model of Greenwood, Hercowitz, and Krusell (1997), reproduces important features of the U.S. NIPA data, namely the secular downward trend of the price of equipment investment relative to non-durable consumption and the increasing ratio of real equipment investment to real output. The main difference to the one-sector AK model lies in the existence of obsolescence costs, which decrease output growth if the elasticity of intertemporal substitution is larger than the saving rate.
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1 Introduction

Neoclassical growth theory builds on the famous *Kaldor* stylized facts, which postulate, among other things, the stationarity of aggregate ratios such as the investment to output ratio or the capital to output ratio. However, there is now a rich body of empirical research suggesting that some of these facts are in conflict with U.S. evidence. Based on data from the U.S. National Income Product Accounts (NIPA), Whelan [15] documents the following facts for the post world war II period:

(i) The price of equipment investment relative to the price of consumer nondurables and services has been declining permanently.

(ii) *Nominal* series of consumption and investment share a common stochastic trend with nominal output so that the consumption and investment shares in nominal output are stationary.

(iii) The ratio of *real* equipment investment to *real* output is non-stationary. Indeed, the growth rate of real equipment investment has been larger than that of real non-durable consumption, reconciling facts (i) and (ii).

Recent research addresses the inconsistency of facts (i) and (iii) with standard one-sector neoclassical growth models. Clearly, any growth model that takes the new evidence seriously needs more than just one sector, otherwise relative price trends cannot be addressed properly. Greenwood et al. [6], henceforth abbreviated GHK, based on the seminal contribution of Solow [14], were the first to make an attempt in this direction. Their model consists of a consumption goods sector, which benefits from exogenous disembodied technical progress, and of an investment goods sector, the efficiency of which also grows at an exogenous rate. Advances in the investment sector affect the consumption goods sector to the extent that firms acquire new capital goods: technical change is embodied. In this way, GHK are able to generate a permanent decline in the relative price of investment and a rising ratio of real equipment investment to real output, keeping nominal shares constant, as required by the stylized facts cited above.

This paper, by extending the framework developed by GHK, strives to develop the simplest possible endogenous growth model compatible with the cited evidence. In order to do so, we study the two-sector version of the AK model introduced by Rebelo [12] (chapter II). We assume that the technology in the investment goods sector is of the AK type, whereas the technology in the consumption goods sector exhibits diminishing returns, confining the origins of endogenous growth to the investment sector. In accordance with GHK's findings, in the proposed framework the relative price of investment trends downward and real investment growth outpaces consumption growth. However, the underlying mechanism is different. GHK generate the price trend by sectoral differences in the rate of technical progress, whereas our model relates the movement in relative prices to the asymmetric sectoral impact of capital accumulation.

Our key assumption endows the investment goods sector with constant returns to the reproducible factor (capital) while postulating decreasing returns in the consumption goods sector. This is in line with empirical evidence. Altuğ and Filiztekin [1] use data on U.S. manufacturing industries to estimate sectoral production functions. They find important sectoral heterogeneity with respect to returns to reproducible factors and conclude against increasing returns. Moreover, they generally fail to reject the null of constant returns to capital in the durable goods sectors although they do reject constant returns in the production of non-durable goods. In our stylized model, we capture this empirical regularity by letting the investment goods sector have constant, and the consumption goods sector decreasing returns to reproducible factors.

As proposed by Whelan [15], we measure the growth rate of real output by the so-called Divisia index, a continuous time approximation to the *chained Fisher index*, officially used in US growth accounting since 1996. This is necessary to account for the substitution effect that trends in relative prices usually bring about. We conclude that real output growth lies above the growth rate of non-durables consumption and below that of equipment investment, which is consistent with fact (iii).

In contrast to its one-sector version, in the two-sector AK model, the user cost of capital is augmented by an obsolescence cost term. Obsolescence costs show up in any model with a trending relative price of investment, as in the R&D driven models of embodied technical change proposed by Boucekkine et al. [2], Hsieh [7] or Krusell [9]. In the proposed AK model, the larger the decline rate of the relative price of investment, the larger the obsolescence cost term. This lowers the interest rate perceived by consumers and depresses consumption growth. However, whether the lower interest rate encourages or discourages capital accumulation depends on how the income effect associated with a change in the interest rate relates to the substitution effect, that is, whether the elasticity of intertemporal substitution is smaller or greater than unity. Obsolescence costs reduce (increase) the growth rate of real output if the elasticity of intertemporal substitution is larger (smaller) than the saving rate.

Moreover, the two-sector AK model is isomorphic to the model by Boucekkine et al. [3], which studies embodied technical change in a model of learning-by-doing. In the latter model, reallocating the efficiency of learning from the consumption goods sector to the investment goods sector generates a simultaneous increase in the decline rate of the relative price of investment and a reduction in the growth rate of aggregate output, matching the shift in US series experienced around the first oil shock. In the two-sector AK model, this exercise is equivalent to a reduction of the output elasticity in the consumption sector.

The remainder of the paper is organized as follows. Section 2 sets out the analytical framework and derives the main propositions; section 3 provides a discussion of the results and compares them with existing models; finally, section 4 concludes.

2 The Model

In this section we analyze a two-sector version of the AK model introduced by Rebelo [12]. The labor force is constant, and all quantities are in per capita terms. The capital stock per capita k_t is endogenously determined by explicit investment decisions. In the consumption sector, capital is combined with labor in a constant returns to scale production function. As in the models of Boucekkine et al. [2], Hsieh [7] and Krusell [9] the only source of endogenous growth lies in the investment goods sector. We model this in the simplest possible way, assuming that the technology in the investment goods sector features constant returns and capital is the only factor of production. For every point in time, the planner optimally divides the capital stock between investment goods production and consumption goods production.

The planner problem reads

$$\max \int_0^\infty \frac{c_t^{1-\sigma}}{1-\sigma} e^{-\rho t} \mathrm{d}t \tag{1}$$

$$\text{s.t. } c_t = (k_t^c)^\alpha \tag{2}$$

$$\dot{k}_t = (A - \delta)\, k_t - A k_t^c, \tag{3}$$

with $k_0 > 0$ given. c_t denotes per capita consumption and k_t^c is the per capita capital stock used in the production of consumption goods. As usual, σ (with $\sigma > 0$ and $\sigma \neq 1$) is the inverse of the intertemporal elasticity of substitution,

$\rho > 0$ denotes the subjective discount rate of the infinitely lived representative individual and $\alpha \in]0,1[$ is the output elasticity of capital in the consumption sector. At every point in time, $k_t - k_t^c$ units of capital are used in the investment sector according to the AK technology $i_t = A(k_t - k_t^c)$, giving rise to the law of motion of capital (3). $A > 0$ denotes the marginal productivity of capital in the investment sector and $\delta \in]0,1[$ its rate of depreciation. Note that capital is perfectly intersectorally mobile.

After substituting (2) in the objective (1), the control variable to the planner problem is k_t^c and the state variable is k_t. Denote by $\lambda_t e^{-\rho t}$ the costate. Then, the first order condition for the control can be written as

$$\alpha \left(k_t^c\right)^{\alpha(1-\sigma)-1} = A\lambda_t, \tag{4}$$

the Euler equation is

$$\frac{\dot{\lambda}_t}{\lambda_t} = -\left(A - \delta - \rho\right), \tag{5}$$

and the transversality condition associated to the problem reads

$$\lim_{t\to\infty} \lambda_t k_t e^{-\rho t} = 0. \tag{6}$$

From (5), the growth rate of λ_t is constant from $t = 0$ on. Denoting the growth rate of variable x by g_x, from equations (2) and (4) we derive the growth rates for consumption and the capital stock employed in the consumption sector:

$$g_c = \frac{\alpha}{\omega}\left(A - \rho - \delta\right), \tag{7}$$

$$g_k = \frac{1}{\omega}\left(A - \rho - \delta\right), \tag{8}$$

where $\omega \equiv 1 - \alpha(1-\sigma) > 0$. Both g_k and g_c are constant from $t = 0$ onwards.

Assumption 1. *Let the following parameter restriction hold:*

$$A - \delta > \rho > \alpha(1-\sigma)(A-\delta).$$

The first inequality in Assumption 1 is required for the growth rate of consumption, as given by equation (7), to be strictly positive. The second inequality ensures that the utility representation in equation (1) remains bounded at equilibrium.

Proposition 1. *Under Assumption 1, for all $t \geq 0$, the time path of capital is given by*

$$k_t = k_0 \, e^{g_k t}$$

and the initial value of the control is $k_0^c = \left(1 - \frac{\delta + g_k}{A}\right) k_0 > 0$.

Proof. See the appendix.■

As in the standard AK model, the economy is on its balanced growth path from $t = 0$; i.e. there are no transitional dynamics.[1] The balanced growth path of the model economy features $g_k = g_i$ and $g_c = \alpha g_k$. Since $\alpha \in]0, 1[$, consumption grows at a slower pace than investment and capital.

As stated in the introduction, in the U.S. two important secular trends are in sharp contradiction with Kaldor's stylized facts. First, the relative price of investment exhibits a secular downward trend. Second, the ratio of equipment investment to real output is steadily increasing. The standard one-sector AK growth model cannot account for these facts. The main proposition of this paper shows that, in contrast, a two-sector model with endogenous AK-type growth in the investment goods sector has predictions consistent with these empirical regularities.

Proposition 2. *In the proposed two-sector growth model, (i) the relative price of investment p_t is decreasing at rate $(1 - \alpha) g_k$, and (ii) the growth rate of output, g, defined by a Divisia quantity index, is constant and lies in the interval $]g_c, g_k[$, its exact position being determined by the saving rate.*

Proof. (i) In a competitive equilibrium, the marginal rate of transformation between the investment good and the consumption good has to be equal to the relative price of the investment good at every point in time. The feasibility constraint of our economy is described by the sectoral production functions and the aggregate endowment of capital. Thus, in per capita terms, at time t, the transformation curve of the economy is defined by the expression

$$c_t = (k_t - i_t/A)^{\alpha}.$$

Denoting the relative price of investment by p_t and equalizing it with the marginal rate of transformation (MRT) (for a given level of k_t^c) gives

$$p_t = \left| \frac{\mathrm{d}c_t}{\mathrm{d}i_t} \right| = \frac{\alpha}{A} \left(k_t^c\right)^{\alpha - 1}.$$

[1] Note that this is no longer true once investment decisions are assumed to be irreversible.

Clearly, as k_t^c grows over time, p_t has to decrease. More precisely,

$$\frac{\dot{p}_t}{p_t} = (\alpha - 1) g_k < 0. \tag{9}$$

(ii) In our context, using the Divisia index amounts to writing the growth rate of real output, g, as the weighted sum of the rates of growth of consumption and investment:

$$g = (1 - s) g_c + s g_k, \tag{10}$$

where the share of nominal investment in nominal output is given by the saving rate $s_t \equiv p_t i_t / (c_t + p_t i_t)$. Using the results derived above, the saving rate is constant and reads[2]

$$s = \frac{\alpha (\delta + g_k)}{A - (1 - \alpha)(\delta + g_k)}. \tag{11}$$

By Assumption 1, $s \in]0, 1[$ and hence $g \in]g_c, g_k[$.■

In Figure 1 we provide a graphical illustration of the equilibrium in the space (i, c). The expansion path Φ shows all pairs of c_t and i_t compatible with the equilibrium production structure implied by Proposition 1. Φ is found by substituting k_t out of the policy functions $c_t = \phi_c(k_t)$ and $i_t = \phi_i(k_t)$. In $(i, c)-$ space, Φ can easily be shown to be strictly concave and increasing in i_t. As the economy accumulates capital, it moves north-east along Φ. The relative price of investment goods at any point on Φ, say E, is found as the slope of the transformation curve at that point. Due to the assumed differences in the sectoral production functions, capital accumulation affects sectors asymmetrically and the transformation curve shifts out in an uneven way. Consequently, on the way from E to E', the relative price of investment falls from p to p' at a rate proportional to the rate at which the economy accumulates capital.

As Whelan [15] points out, to be consistent with NIPA data, the appropriate way to compute the growth rate of real output is to use the "*ideal chain index*" proposed by Irving Fisher. This index is the geometric average of a Paasche and a Laspeyres index and can be accurately approximated by the so-called Divisia index, which weights the growth rate of each component of output by its current share in the corresponding nominal aggregate (see Deaton and Muellbauer ([4]), pp. 174-5 for more details).

Since expression (10) plays a major role in our analysis, some additional remarks on the appropriate definition of real output growth may be useful.

[2] In the standard one-sector AK model (where $\alpha = 1$): $s_{AK} = (\delta + g_{AK})/A$.

Typically, a base year quantity index is computed as the average growth rate of real output in the different sectors, where base year prices are used as fixed weights. As Whelan [15] explains, in an economy with different sectoral growth rates, such a fixed-weight methodology results in *unsteady* aggregate growth. The growth rate will tend to increase, converging towards the growth rate of the faster growing sector. The reason for this pattern is the so-called substitution bias introduced by holding relative prices fixed over time. Those categories of output which exhibit faster growth in quantities typically also experience declining relative prices. Measured in prices of a base year, current output will become more and more expensive, as the fast-growing components are still weighted with high historical prices. Moreover, not only does this fixed-weight definition lead to unsteady growth, but the size of the substitution bias, and thus the computed growth rate, depend on the choice of the base-year: the farther in the past, the larger the error. In our model, since the weights used in the computation of the Divisia index are constant, the problem of unsteady growth is avoided.

This is why NIPA data is computed using a chained-type index equivalent to the Divisia index in continuous time. This method amounts to continually updating the prices used to calculate real output and to chain the index forward from an arbitrary base year on, in which nominal magnitudes have been set equal to real magnitudes. Moreover, Licandro et al. [10] provide theoretical support for the use of such a chained index in the framework of a two-sector exogenous growth model with embodied technical change (the GHK model). They compute a true quantity index, use official NIPA data to calibrate it for different parameter values, and show that their results come very close to what is obtained by applying NIPA's methodology.

3 Discussion

A model of technical change. GHK [6], p. 349, take the observed downward trend of the price of investment goods relative to the price of consumption goods as evidence for investment specific technical change. In their theoretical model, this price trend is generated by sectoral differences in the exogenous rates of technical progress. Boucekkine et al. [2], Hsieh [7] or Krusell [9] take up this idea and write up models where R&D driven endogenous technical progress is confined to the investment goods sector. Our model, instead, is not a model of technical *progress* but one of technical *change*. As Figure 1 makes clear, along the balanced growth path, the marginal rate of transformation falls at a steady rate. The technological description of the economy, given by the functional form of the sectoral production functions, does not evolve over time. Rather, it is the continuous increase in the relative endowment of capital that forces the marginal rate of transformation to decline over time. We refer to any change of the marginal rate of transformation, regardless of its origin, as technical change.

Note that along the balanced growth path the marginal productivity of capital in the consumption goods sector falls, whereas that of the investment goods sector remains constant at A. Therefore, in stark contrast to the above-mentioned R&D models, in the two-sector AK model the relative price trend is driven not by decreasing marginal costs in the investment goods sector, but by increasing marginal costs in the consumption goods sector.

Obsolescence costs. At this point of our argument, it is useful to analyze the decentralized version of our economy. Let firms own the capital stock and the representative consumer own the firms. The consumer's wealth is given by the value of her asset holdings, a_t. If we define r_t as the rate of return to this asset, the consumer's wealth evolves according to $\dot{a}_t = y_t + r_t a_t - c_t$, where y_t denotes labor income. y_t is given for the consumer since labor supply is perfectly inelastic and the labor market is competitive. The consumer maximizes lifetime utility (1) subject to the law of motion of wealth. The Euler equation associated with the optimal consumption path writes $\dot{c}_t/c_t = \sigma^{-1}\left[r_t - \rho\right]$. Let firms sell their output at the ongoing market price and use some of their revenue to purchase investment goods. They maximize the present value of their profits where the relevant discount rate is r_t. In this context, the user cost of capital is

$$u_t \equiv r_t + \delta - \frac{\dot{p}_t}{p_t}. \tag{12}$$

Then, the optimality conditions of firms in both sectors require that the marginal product of capital be equal to u_t. The sectoral allocation of capital is governed by the efficiency condition $(p_t)^{-1}\alpha\left(k_t^c\right)^{\alpha-1} = A = u_t$. Consequently, the user cost of capital is constant along the balanced growth path and identical to A. Moreover, the growth rates of k_t^c and p_t must have opposite signs and any increase in $\dot{p}_t/p$ must be entirely offset by a corresponding reduction in r_t. We can express the asset return rate by

$$r_t = A - \delta + \frac{\dot{p}_t}{p_t} \tag{13}$$

and the consumer's Euler equation by

$$g_c = \frac{1}{\sigma}\left(A - \delta - \rho + \frac{\dot{p}_t}{p_t}\right). \tag{14}$$

Substituting expression (9) for $\dot{p}_t/p_t$ in the Euler equation (14) yields exactly the centralized counterpart (7). Except for the term $\dot{p}_t/p_t$ equation (14) is identical to the Euler equation of the standard one-sector AK model. Thus, it is really the relative price change that makes the crucial difference. In this setting, the expected evolution of the relative price of investment is irrelevant for the firms' investment decision since the user cost of capital always remains fixed at A. However, the net interest rate r_t is affected by the growth rate of p_t. A larger decline rate of p_t results in a flatter consumption path (a lower growth rate g_c), giving rise to the typical intertemporal substitution effect. Thus, the term $\dot{p}_t/p_t$ acts as a cost and is referred to as capturing the so-called *obsolescence costs* typically associated with embodied technical change. Obsolescence costs lower the value of installed capital, therefore reducing the consumer's wealth. Higher obsolescence costs, do not, however, necessarily reduce aggregate output growth, as a simple inspection of equation (14) might suggest.

Differences to the one-sector version. Setting $\alpha = 1$, the two-sector AK model collapses to the standard one-sector AK model. Then $\dot{p}_t/p_t$ is clearly zero and there are no obsolescence costs. Does this imply that the one-sector AK-type economy grows faster than its two-sector version?

Denote the growth rate of the standard one-sector AK model by g_{AK}. Then the following proposition can be made:

Proposition 3.

$$\begin{aligned} (i) \quad g_{AK} &> g_c, \\ (ii) \quad g_{AK} &\gtrless g_k \Longleftrightarrow \frac{1}{\sigma} \gtrless 1, \\ (iii) \quad g_{AK} &\gtrless g \Longleftrightarrow \frac{1}{\sigma} \gtrless s. \end{aligned}$$

Proof. In the one-sector AK model the growth rates of consumption, output and capital are all equal to

$$g_{AK} \equiv \sigma^{-1} (A - \rho - \delta). \tag{15}$$

Parts (i) and (ii) of the proof involve comparing g_{AK} to the growth rates given by equations (7) and (8), which is obvious. To show part (iii), note that g can be expressed as

$$g = [(1 - s)\alpha + s]\,\omega^{-1} (A - \delta - \rho). \tag{16}$$

Then $g_{AK} \gtrless g \Longleftrightarrow \sigma^{-1} \gtrless [(1 - s)\alpha + s]\,\omega^{-1} \Longleftrightarrow \sigma^{-1} \gtrless s$. From (8), (10) and the definition of ω, s is a function of σ so that it is not a priori clear whether there are values for σ for which the above inequalities hold. The appendix shows that the equation $\sigma^{-1} = s(\sigma)$ has a unique interior solution $\sigma^* > 1$, so that the above inequalities can go either way.■

Note the role of the intertemporal elasticity of substitution in the comparision between g_{AK} and g_k. In the two-sector model, the interest rate being weighed down by obsolescence costs, the agent choses g_k larger than g_{AK} if the income effect outweighs the substitution effect, that is, if the intertemporal elasticity of substitution is smaller than unity.

In the comparison of (15) and (16) two differences stand out. The first relates to the terms σ^{-1} and ω^{-1}. To assign economic meaning to ω^{-1}, note that problem (1) to (3) can be rewritten as

$$\max \int_0^\infty \frac{x_t^{\alpha(1-\sigma)}}{\alpha(1-\sigma)} e^{-\rho t} \mathrm{d}t \quad \text{s.t.} \quad \dot{k}_t = (A - \delta) k_t - x_t$$

where $x_t \equiv A k_t^c$ denotes foregone investment. Then it is possible to compute the *elasticity of intertemporal substitution in foregone investment*, which is clearly equal to ω^{-1}. It measures the substitution between using capital to produce consumption goods today and producing investment goods today to produce consumption goods in the future. Clearly, with identical production functions for consumption and investment goods, ω^{-1} and σ^{-1} coincide, which is the case in the one-sector model. The second difference relates to the term $(1-s)\alpha + s$, which shows how the marginal effect of a change in g_k effects g, as implied by our definition of output growth (10). A fraction $1-s$ of capital is allocated to the consumption goods sector where it encounters decreasing returns given by α; the complementary fraction goes to the investment sector where returns are constant. In the one-sector model, this term is equal to unity.

Hence, for $g > g_{AK}$ two conditions must be satisfied: first, the elasticity of intertemporal substitution must lie below unity so that the income effect outweighs the substitution effect, generating a larger growth rate of capital; second, the saving rate has to be large enough so as to give sufficient weight to the investment sector when it comes to determining aggregate output growth.

1974. Greenwood and Yorukoglu [5] argue that the year of 1974 is a watershed in the history of US economic growth. First, the decline rate of the relative price of new equipment increased from about 3 to 4 percent. Second, labor productivity growth significantly decelerated. Third, the year of 1974 marked a period of decreased non-durable consumption growth, increased capital growth and decreased aggregate output growth. Greenwood

parameters kept constant	$A = 0.26, \delta = 0.1, \rho = 0.1, \sigma = 1.5$	
	'stylized' numbers	model predictions
pre 1974	$\dot{\mathbf{p}}_t/\mathbf{p}_t = \mathbf{-3}\%, g_k = 5\%$	$\boldsymbol{\alpha} = \mathbf{0.4}$ $g_c = 2\%, g = 3.1\%$
post 1974	$\dot{\mathbf{p}}_t/\mathbf{p}_t = \mathbf{-4}\%, g_k = 5.33\%$	$\boldsymbol{\alpha} = \mathbf{0.25}$ $g_c = 1.33\%, g = 2.4\%$

Table 1: 1974

and Yorukoglu argue that those phenomena are all related to the introduction of a new general purpose technology (GPT), namely information technology. Their argument relies on the assumption that introducing a new GPT makes existing human capital obsolete, requiring the slow build-up of new experience. In the framework of the two-sector AK model, the facts reported by Greenwood and Yorukoglu can be replicated by an unexpected negative shock on the output elasticity of capital in the production function of the consumption goods sector.

From equation (9) it can be seen that the decline rate of the relative price of investment increases if α is reduced, since $\partial\left|\dot{p}_t/p_t\right|/\partial\alpha = -g_k\sigma/\omega < 0$. The growth rate of consumption, g_c, is also reduced since $\partial g_c/\partial\alpha = g_k\omega^{-1} > 0$. However, the effect on g_k is ambiguous and depends on σ^{-1} being smaller or larger than unity: $\partial g_k/\partial\alpha = g_k\left(1-\sigma\right)/\omega$.

In the following exercise, documented in Table 1, we give a numerical illustration of the effects involved. Unfortunately, using equations (7), (8) and (9), it is impossible to assign values to g_c, g_k, and $\dot{p}_t/p_t$ independently. Thus, the two-sector AK model is underparametrized and cannot be used for calibration purposes. However, it may still be worthwhile to provide some quantitative intuition for the effects of a shock on α.

First we assign numbers to A, δ, ρ, and σ and keep them constant through the exercise. Next, we pick values for $\dot{p}_t/p_t$ and g_k roughly in line with pre-1974 data presented in Greenwood and Yorukoglu [5]. This choice fixes a value for α. Finally, we compute g_c and g_k from (7) and (8). For the period after 1974, we postulate changed values for $\dot{p}_t/p_t$ and g_k. These imply a decreased value of α, a lower g_c and a slightly decreased growth rate of aggregate output.

The numerical exercise is meant to illustrate that an adverse shock on the parameter α can generate a decline in the output growth jointly with an increase in the decline rate of the relative price of capital highlighted by Greenwood and Yorukoglu. However – in the light of our discussion comparing the two-sector to the one-sector version of the AK model – a shock on α can affect real output growth either way. Indeed, with the numbers above, choosing a sufficiently low value for σ^{-1} (smaller than .09) reverses the growth effect.

A model of learning-by-doing. Next, we compare our model to the one developed by Boucekkine et al. [3] (hereafter BdL), where endogenous growth is due to learning-by-doing in both the consumption and the investment goods sectors. The technological description of their model is given by

$$c_t + x_t = z_t k_t^{\eta}, \tag{17}$$
$$i_t = q_t x_t, \tag{18}$$

where the efficiency of production increases with cumulated net investment so that $z_t = k_t^{\gamma}$ and $q_t = Ak_t^{\lambda}$. The parameters $\lambda > 0$ and $\gamma > 0$ describe the efficiency of learning in the consumption and in the investment sector, respectively. In order to generate sustained growth, BdL assume $\lambda+\gamma+\eta = 1$. Thanks to constant returns to scale in the production of consumption goods, equations (17) and (18) can be written as

$$\begin{aligned} c_t &= (k_t^c)^{\gamma+\eta}, \\ i_t &= A(k_t - k_t^c)^{\lambda+\gamma+\eta} = A(k_t - k_t^c). \end{aligned}$$

After the change $\alpha = \gamma + \eta$, the two-sector AK model perfectly coincides with the optimal growth version of the learning-by-doing model. Thus, the two-sector AK model can be seen as the reduced form of a learning-by-doing model where firms internalize the learning externality.

Note that the efficiency of learning in the investment goods sector in BdL is inversely related to the elasticity of capital in the consumption sector in the two-sector AK model since $\lambda = 1 - \alpha$. From equations (8) and (9) and after substituting $\alpha = 1 - \lambda$, the decline rate of the relative price of investment can be written as

$$\frac{\dot{p}}{p} = -\frac{A - \delta - \rho}{1 + \sigma\left(\frac{1}{\lambda} - 1\right)},$$

which is an increasing function of the efficiency of learning in the investment goods sector relative to the consumption goods sector. Therefore, an adverse shock on the parameter α (as in the numerical example above) in the two-sector AK model is equivalent to a positive shock on λ in BdL's framework, with returns to capital in the investment sector kept constant. In that case, reducing the learning efficiency in the consumption sector comes with increasing it in the investment sector. BdL note that such a *reassignment* of learning efficiencies can account for the facts stressed by Greenwood and Yorukoglu [5]. Their exercise is equivalent to the one conducted in the subsection above in the framework of the two-sector AK model.

Remarks on the empirical evidence. Using data from 1950 to 1987 for 15 OECD countries, Jones [8] criticizes that the standard AK model cannot account for the observed coincidence of relatively stationary growth rates and upward-trending real investment rates. The two-sector AK model reconciles stationary output growth with trending investment rates and overcomes Jones' criticism. At the same time, it is a defense of the AK model.[3]

In a recent contribution, Restuccia and Urrutia [13] review the well-known negative and surprisingly robust correlation between the relative price of equipment investment and output per capita for a wide range of countries. In our model the relative price of investment goods is a decreasing function of the aggregate capital stock which is itself an increasing function of time. Thus, our model correctly predicts that countries which have entered the modern process of economic growth later than others, in autarky should exhibit a higher relative investment price. Moreover, again in line with evidence reviewed in Restuccia and Urrutia, our model shows a positive relationship between the relative price of equipment goods and the real investment ratio but an a priori ambiguous relation between aggregate growth and the decline rate of the relative investment price.

4 Conclusion

We analyze the two-sector AK model proposed by Rebelo [12] (section II) where constant returns to capital are confined to the investment goods sector. We show that this setup, an endogenous growth extension to the model of Greenwood et al. [6], reproduces important features of the U.S. NIPA data, namely the secular downward trend of the price of investment relative to consumption and the increasing ratio of real investment to real output. Using a Divisia quantity index, we demonstrate that real output grows faster than consumption but more slowly than investment. The main difference to the one-sector AK model lies in the existence of obsolescence costs, which tend to decrease output growth if the elasticity of intertemporal substitution is larger than the saving rate. We argue that the two-sector AK model is a model of technical change, not progress, and that it is perfectly isomorphic to the optimal growth version of the learning-by-doing model of Boucekkine et al. [3]. In particular, a decline in the output elasticity of capital in the consumption sector makes it possible to reproduce key features of the US

[3] See McGrattan [11].

growth experience after the year of 1974: i.e., a simultaneous increase in the decline rate of the relative price of investment, a larger growth rate of real equipment output and a somewhat decreased rate of growth of aggregate output. Finally, since the model is compatible at the same time with stationary output growth and upward-trending real investment, it overcomes Jones' [8] well-known critique of the AK model. At the same time, it is a defense of the AK model. Moreover, in line with overwhelming empirical evidence, the model predicts that less developed countries should feature higher relative prices of equipment investment relative to more advanced countries.

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A Proof of proposition 1

We start from equation (8):

$$\dot{k}_t = (A-\delta)\,k_t - Ak_t^c.$$

Using $e^{-(A-\delta)t}$ as the integrating factor, substituting $k_t^c = k_0^c e^{g_k t}$ and rearranging terms yields

$$e^{-(A-\delta)t}\left[\dot{k}_t - (A-\delta)\,k_t\right] = -e^{-(A-\delta-g_k)t}Ak_0^c.$$

The LHS can easily be recognized as $\frac{d}{dt}\left[e^{-(A-\delta)t}k_t\right]$ and the RHS as $\frac{d}{dt}\left[\frac{1}{(A-\delta-g_k)}e^{-(A-\delta-g_k)t}Ak_0^c\right]$. Integrating and dividing by $e^{-(A-\delta)t}$ gives

$$k_t = \frac{1}{(A-\delta-g_k)}e^{g_k t}Ak_0^c + Ce^{(A-\delta)t} \tag{A1}$$

where C and k_0^c are constants which can be determined using the initial condition $k_0 > 0$ and the transversality condition 6. Using expression (A1) and the law of motion for the state variable 5 in the transversality condition we get

$$\lim_{t\to\infty}\left\{\frac{A}{(A-\delta-g_k)}e^{-(A-\delta-g_k)t}+C\right\}=0. \tag{A2}$$

The first term in the braced brackets converges towards zero since $A-\delta-g_k > 0$. Therefore, the TVC requires the constant C to be zero. From (A1) we get $k_0 = Ak_0^c/(A-\delta-g_k)$.■

B Proof of proposition 3(iii)

It still needs to be shown that there exist intervals of values for which $\sigma^{-1} \lesseqgtr s$. In particular, it is not clear a priori whether there are values for σ such that $g > g_{AK}$. We need to prove that the equation $\sigma^{-1} = s$ has a solution σ^*. Clearly, s is a continuous function. Moreover it is decreasing in σ since

$$\frac{\partial s}{\partial \sigma} = -\left[\frac{\alpha}{A-(1-\alpha)(\delta+g_k)}\right]^2\frac{A}{\omega}g_k < 0. \tag{A3}$$

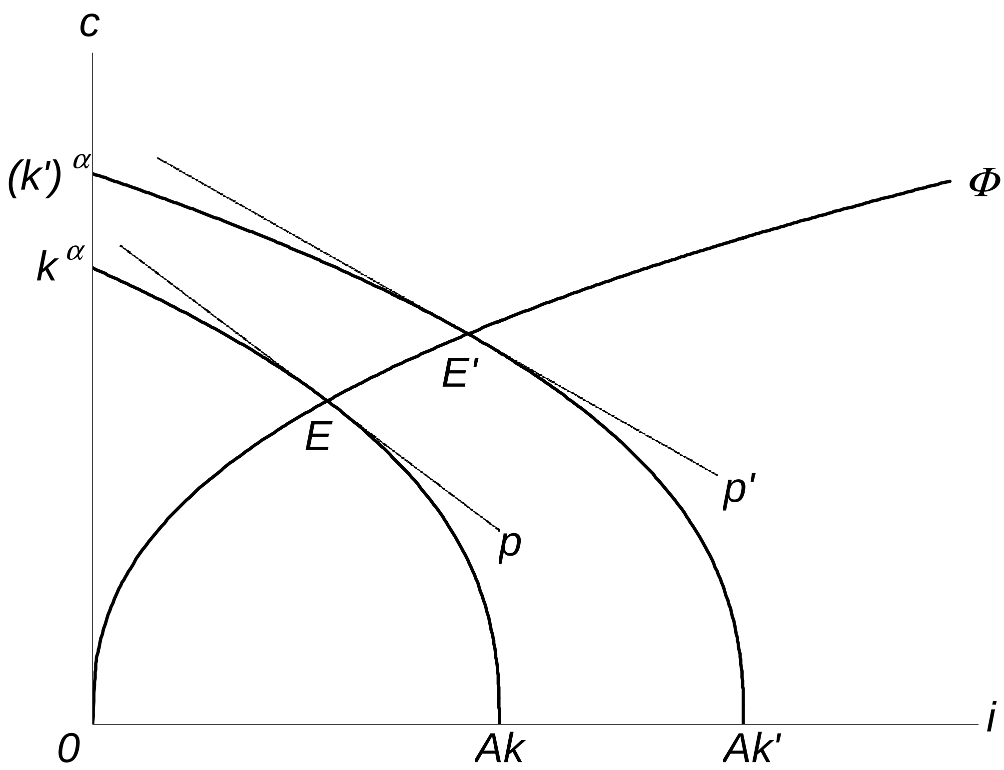
Moreover, as σ tends towards infinity, s converges towards the constant

$$s^- = \frac{\alpha\delta}{A-\delta+\alpha\delta} \tag{A4}$$

strictly larger than zero (by Assumption 1) and as σ tends towards zero, s converges towards the constant

$$s^+ = \frac{\alpha}{1-\alpha}\frac{A-(\rho+\alpha\delta)}{\rho+\alpha\delta} > s^-. \tag{A5}$$

Therefore it is clear that the there is a solution σ^* to the equation $\sigma^{-1} = s$ and that this solution is bounded below by $(s^+)^{-1}$. Since $\partial^2 s/\partial\sigma^2 > 0$, we can exclude oscillating behavior of $s(\sigma)$ around σ^{-1}, which is necessary and sufficient for unicity of σ^*. We conclude that if $\sigma < \sigma^* \iff \sigma^{-1} > s \iff g_{AK} > g$ and if $\sigma > \sigma^* \iff \sigma^{-1} < s \iff g_{AK} < g$.■



The economy on its expansion path Φ from E to E' $(k' > k)$.