On modelling markets

abstract

We analyze critically the usual analysis of free (that is, unreguled by a central authority) market behavior of prices and traded amounts. We find that quite usually either the offer function or the demand function cannot be defined meaningfully, and other models must be set-up. We show also that profit maximization is most times uncorrectly used and wrongly calculated. We show that the absence of regulation (perfect price and production flexibility) and perfect competence does not necesarily lead to equilibrium between offer and demmand.

1 Introduction: The problem and the method

Let us isolate, from a given economy, some good whose traded amount per capita and unit time we call q, and whose unitary price we denote by P. We suppose that the good is traded in a market with perfect competence in wich every other relevant variable is kept fixed. The accepted description for this phenomenon is to postule the existence of an offer function $q_{offered}(P)$, supposed to be the amount that good owners are willing to sell at every price, and a demand funcion $q_{demanded}(P)$, the amount of good that people would buy at every price. At equilibrium, offer and demand should equal, and outside equilibrium there is a tendency to equilibrium as traders adjust prices and amounts. This is a quite simple and elegant description that gives sensible outputs and permit the introduction of dynamics because it includes the description 'off shell'. However, as Economy is a factical science, we should feel unsatisfied until we can say that observations confirm our description. We should try to measure offer and demand functions, or to derive them from something more fundamental. Even more, we would like to be predictive, and from a few data extract lots of other data (measurable a posteriori) between phenomena that would seem unrelated outside a specific theory.

Let us analyze more deeply what is the meaning of offer and demand functions. If we want contact with observations (econometric data) we must work with magnitudes defined with the same precision. This doesn't mean that we have to enter into the technical details of data analysis and statistical inference, such details are operative in character but doesn't get the spirit of the concept. Let's introduce the following criterion for well definition of variables:

definition: We will say that any cuantitative concept is *meaningfull* if we can in principle design an experiment in order to measure it.

Note that we say 'in principle', having in mind that the experiment could be insane, cruel, or simply prohibitive. We do not intend to actually perform the experiment, nor to suggest practical methods to get econometric input; we just want to be sure that we have sharply defined concepts. Let me stress that it is a purely theoretic instrument, in wich the 'gendankenexperiment' is used not to give a way to measure the quantity but to define it.

But that a concept is meaningfull doesn't mean it is usefull: once such definition is given, we should start searching for the right variables to build a theory. This takes a lot of trial and error. The criterion is to get variables with wich to construct a theory as accurate and predictive as possible. Maybe some of the magnitudes described here (which are mere refinements of the magnitudes usually defined in theoretical economy) can be set in a powerfull theory, as predictive as thermodynamics, or maybe they are first trials in a long search. However, the only work of refining the concepts in order for them to be meaningfull shows that some known models are too loose, or wrong, and that there are many subtelties that makes the apparently so sensible offer-demand scenario not acurate or even not consistent.

With this definition in hand, let's come back to demand and offer. Can we think in how to measure demand? For definiteness, let us suppose that the good is a consumption one, wich has to be produced, say, in a factory. If we, the observers, are rich enough and have a police strong enough to avoid the formation of black markets, we could think in just taking one town and sell the good at some uniform price, then take another town and sell it at other price, and so on. The demand, intended as the amount of a consumption good that people would buy for some price, is thus a meaningfull concept.

An important remark is in order here: once a sharp meaning (adecuate for a quantitative magnitude) is assigned to a concept, we must be carefull with the way in wich we use the name: we must not guide by its semantics. For instance, demand of a product could be thought as the need for it, but once we give the operational definition above we must abandon thinking that way. If we want to define such a thing as a need for something we should think in another measuring procedure. This remark is all-important at the moment of interpreting results, when for instance politicians, trusting the 'mathematical' character of economic theories, give potentially harmfull interpretations of it. That the price of food is fixed in a free market means that anyone will get as much food as he can afford, but doesn't mean that he get enough not to starve.

What about the offer function? How much would firms produce for a given price? How could we measure it? Following the logic above, we could forbid people in some town to buy the good, and buy to the producers at a fixed price as much as they want to sell. But it would be ill-defined: if the price makes it a profitable business, they will sell their whole production capacity, and if it is not, they will not produce anything. If we think in the inverse function, that is, for a given production level, how much they would sell the good if they do not know anything about the demand, the answer would be 'infinity'. So, we cannot define meaningfully something like an offer function, independently of the demand. Indeed, in practice facturers fixes prices after a 'market study' (they look at the demand) while consumers just buys if they can afford it.

2 The 'principle' of maximum profit

It is customary to think that, as capital owners will want to earn 'as much as possible', we can find some offer or demands by maximizing globally their profit. But we should be aware that the existence of perfect competence may put aditional conditions, so such maximization should be performed very carefully. Let us start analyzing the global profit for facturers of a consumption good, with production q, price P and all the rest of the prices or costs fixed:

$$G(q,P) = qP - C(q) \tag{1}$$

where $C(q)$ is the cost of producing q [1]. Many times the following (wrong) reasoning is done: as the benefit is maximum the price and cost are related: we derive respect to q and as it is a maximum we equate to zero, getting

[1]$(C(q)$ is in fact a random variable if we suppose that there are many factories each with a cost function $c(q_f)$; as each factory would sell its product at random performance, the cost of producing globally $q = \Sigma_f q_f$ depends on its distribution among the q_f. We will avoid the complication and work with means.

$C'(q) = P$. Let us enphatically stress that it is wrong: as facturers can choose at will both q and P we are dealing with a two-variable function, whose extrema are found by fixing both partial derivatives to zero. That is $0 = \frac{\partial}{\partial q}G = P - C'(q)$, $\frac{\partial}{\partial P}G = q$, with solutions $q = 0$, $P = C(0)$, wich is the minimum. The set of points given by $P = C'(q)$ is that of the so called 'chair points'. This function has no absolute (finite) maximum, and we could maximize it only in the presence of some link between q and P (the demand, for instance).

So, should we try to maximize G? Well, in the case that too few factories produce the good, if they agree, their only limit would be the demand, and we should maximize G with that link. But if there are many producers, the consumers could choose the cheaper, so a factory should offer competitive prices. The overall maximization of G in that case would be meaningless, even while individuals try to get as much profite as possible.

In the case in wich there is only one unreguled producer, as explained above, he will have to decide the level of production and price, having in mind the biggest profit. We can find the biggest profit inserting the (inverse of the) demand function $P(q)$, and deriving respect to q:

$$0 = \frac{d}{dq}G(q, P(q)) = P + qP'(q) - C'(q) \tag{2}$$

Note that one couldn't use the expression 1 to find an offer function, because then $P(q)$ would be unknown, having eq 2 as its differential equation, whose solution is just the unitary cost (the profit is zero). No maximum can be extracted from 1 without a link between q and P.

Let us stress that the imposibility of having an offer function in this case in no way means that there is not an offer. The offer is the willing to maximize profit, it just cannot be represented by a function of the price.

3 Competitive offer

We now face the case in wich there are many producers of the consumption good. Here, contrary to the monopolic case, demand will have little incidence on price and cost production will become very important. As explained above, we cannot use global profit maximization in this context. However, the presence of competence allows to define a kind of offer function: we could fix a 'test demand' q_0 and divide it according to the capacity of every factory, buying the part of a given factory if it offer the good at the lowest price plus a little porcentual tollerance. We would measure (by changing the q_0) an offered price $P(q)$.

Let us try to understand how this offer arises. The minimum conceivable price is the unitary cost $C(q)/q$. What about the profit? Let us ignore the eventual existence of 'golondrine' actionists for simplicity and consider factory owners and managers. The profit is thinkable as their's 'salary': they decide to owe a factory of the good as they could do other things for a living, so its earnings are in some sense 'ruled by a market', we could consider it a part of the job market. Let us suppose that there are N factory owners, with 'salary' W (as we are considering all prices but that of the good as fixed, we take W as given); then the profit is NW, and can be considered as a fixed cost and included in $C(q)$. The good is thus offered at price

$$P_{offer}(q) = \frac{C(q) + NW}{q} \tag{3}$$

What does this offer look like? Well, first we have the fixed costs $C(0) + NW$ and then a growing function $C(q) - C(0)$ (wich grows very possibly slower than linearly) until the maximum capacity is reached. So, the price offered is a decreasing function of the production, contrary to the classical conception. So it could easily happen that both curves do not intersect. If the demand function remains below this offer, first other markets (as the job market or that of some ingredients) could evolve at a new equilibrium; if it is not enough, some factories will have to close (the fixed costs are reduced), wich is a structural (drastic) case. In the opposite case (demand above offer), if other market adjustements do not make reaching equilibrium, we are before an underinvestment.

4 An intermediate product market: coupled markets

Let us try an intermediate product, say yeast. The offer will present the same behavior as stated above for consumption goods. What about the demand? Well, we can do exactly the same experiment as we did for consumption goods and measure an 'undiscrimined' demand $q_y^{(u)}(P_y)$, and if our only interest is to analyze this market we would be done. But in such setting we loose control of prices like that of bread: we take the bread facturers, bread market and consumption as a blackbox, thus from the analysis the yeast would be a consumption. If we want to consider it as 'intermediate', we should consider as given the demand for consumptions factured using it, and see what the producer (of the intermediate and consumption goods) do. Do we have enough information to describe the intermediate product market?. Well, let us see: there is (if so) a whole rank of prices and productions such that all business involved are profitable. We can measure another demand, amenable to the case: we could regulate the prices of all consumtions in wich production the good is used, thus obtaining a discriminate demand

$$q_y^{(d)} = f(P_y; P_1, ..., P_N) \tag{4}$$

where P_i are the (fixed by us) prices of the consumptions. If we do everything OK, we should be able to put $q_y^{(u)}$ in terms of $q_y^{(d)}$ by 'solving out' the consumption markets. This is a refinement of the former analysis, when we let other variable to vary. Recall that we have the N $q_i(P_i)$ demand functions for the consumtions, and suppose we need α_i^{-1} intermediate product to facture unit consumption; then we have the relation

$$q_y^{(d)}(P_y; P_1, ..., P_N) = \sum_{i=1}^{N} \alpha_i q_i(P_i) \tag{5}$$

Thus it seems that we can put the price of the intermediate product in terms of the prices for the consumptions, what would be strange indeed. A closer analysis shows what is happening: if P_y makes their production profitable at the prices P_i, the demand will be fixed by the demands for the consumptions and $q_y^{(d)}$. will be independent of P_y. If we release the prices for the consumptions (keeping P_y fixed), their markets, if they reach their equilibria, will fix prices that will depend on P_y, so we have

$$q_y^{(u)}(P_y) = q_y^{(d)}(P_y; P_1(P_y), ..., P_N(P_y)) \tag{6}$$

We have kept an explicit dependence on P_y in $q^{(d)}$ to remind us that for some intermediate product prices certain consumption markets could become unprofitable. We now can proceed with the intermediate product market as above, thus determining its price.

Notice that higly nontrivial relations start to become apparent (and calculable) under this scheme: once the price of an ingredient is allowed to vary according to its market, if the demand for a final product change by any reason, it will affect the prices of all final products through that ingredient. Of course, in actual economies all prices are regulated by theirs markets, so when we consider the whole economy we should expect a very complicated description. In fact, it is usual in quantitative sciences that the whole system is untractable and we must conform ourselves with pasting the separate pictures, with the help of experimental data.

5 Classical-like cases

What made the classical reasoning fail that way? Well, the main reason for this behavior is that facturers has no interest in keeping its production: their job is to make and sell the whole production. The classical mind works for a more symmetric situation, where both traders may be interested in being owners. This is the case in a market of values: gold, actions, currencies, buildings, used cars, etc. Notice that in those cases a measuring procedure as that described in the introduction (banning the trade of the good and buying or selling as much as market want) is well defined for both offer and demand.

Even when most of modern econommies' markets do not behave in a 'classical' way, still the key ones that rules investment do (money market and the like) so we can't just throw those ideas away. But we must be aware that we have to be carefull every time we analyze a market in defining everything such it is meaningfull.

6 Job market

This is probably the most difficult market, because everything gets coupled through employement and salary. This is also the most delicate, as it gets into the kernel of political controversy. So its analysis deserves the greatest of the cares.

First, let us start by measuring job offer. Keeping the prices fixed, we offer a job at salary ω to as many people as want it. Thus this offer is the number n of people willing to work for the salary ω, in accordance with the classical concept. Here we should make an important remark: even if it is possible to reach an equilibrium and all the $n(\omega)$ get a job, this doesn't mean that there is not unemployement. The unemployement is the number of people that seek for a job and don't get it, and the fact that one wants to work doesn't mean that one accepts working for the salary ω. So, the idea that in a free job market unemployement is automatically absent is a mistake arising from a misunderstanding of the meaning of the job offer.

Now lets turn our attention to job demmand. Here we are in a situation simmilar to product offer: if we forced the employers to take n workers and ask the firsts to fix salaries, they would choose not paying at all. Let us analyze the employers' profit:

$$G(n, \omega) = Y(n) - n\omega \tag{7}$$

Where Y is the brute rent. As before, we should see if we have to maximize the profit, and if so, how to do it. Again, the global maximization only makes sense if the employer is monopolic, as workers may prefer to work for the best payer. As before, it is wrong to maximize by equating to zero the partial derivative respect to n, so the condition $\omega = Y'(n)$ is not right. We must perform the maximization with links, and the only bound that the monopolic employer finds not to pay zero is the offer. In the case of an unreguled monopolic employer the equilibrium salary is $Y'(n) = \omega_{offered}(n) + n\omega'_{offered}(n)$.

The case of competitive employement is parallel to that of competitive product offer: we may consider that owner earnings are fixed (say, by other markets) so the salary would be

$$\omega_{demmand}(n) = \frac{Y(n) - NW}{n} \tag{8}$$

wich tends to be a growing function, as can be seen in a similar way to that of competitively offered prices

7 Conclussions

We have been analyzing market behavior by artificially fixing some variables. Closed economies do not permit that, and all markets couples simultaneously.

The discussion above shows that in the simplest macroeconomic models (classical, keynessian, etc) profit maximization is used widely and wrongly. Also the usual assumption that in absence of regulations and under perfect competence the equilibrium is allways reached has proved wrong. At first look it seems hopeless to put a closed realistic model (even with crude simplifications) in wich all variables are set looking for its equilibrium. Indeed this is not that surprising: there is no natural system in nature wich can be studied at once, from first principles and with all relevant variables set free. One sets special cases, uses what he observes, isolate variables and study that aspect alone. Then the theory permits relating or predicting other behaviors if the theory turns to be correct, or if a predicted connection fails to happen according to observations, we have to change our theory.

An example from physics is particularly illustrative. take electromagnetism, where the movement of electric charges produce electromagnetic fields, and an electromagnetic field controls the movement of electric charges in it. It is quite straightforward to calculate the fields when the movement of the charges is given, and to calculate charge trayectories when the fields are given, but putting charges to self-consistently determine their motions in the field generated by themselves is an outstanding unsolved problem. In particular, a single electron in its own field is unbelievly difficult and not well understood yet. This situation in quantitative sciences is very usual. It is so quite conceivable that economy, the most quantitative of the social sciences, can be given the same status of 'exact' as physics or chemistry, if the variables, concepts and theories are well chosen.