Comments on job market models

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abstract

We point out that the condition of maximization of the benefit is not $Y'(n) = \omega$ in general, due to the eventual dependence of the salary ω on the employement n. We show that in keynessian models, where the relation between ω and n is given, we need to correct the relation between the salary and rent, but in classical models, where we use the benefit to find a demand function, the maximization of the benefit do not permit it. This is a case in wich, regardless the interplay between offer and demand is present, the price and level of interchange of a good (human work) cannot be determined by the intersection of offer and demand functions.

1 review of the simplest model

Let us start by reviewing the job market in the simplest classical model. Let n be the employement, $Y(n)$ the rent, and ω the salary. Define the benefit as $B(n) = Y(n) - n\omega$. We want to find out what salary would be payed to n contracted workers, $\omega(n)$. That should be the salary such that the benefit is maximized. That happens when $B'(n) = 0$, that is, $\omega(n) = Y'(n)$. This is the demand for jobs. The jobs market completes with the offer curve, wich is supposed to be a growing function of the salary representing the number of workers willing to work for that salary. At equilibrium, the offer and demand should be equal, determining the rent, the employement and the salary.

This setting follows the paradigm of free market ideology, in wich the interplay between offer and demand is supposed to self regulate markets. Under this scheme there is no room for unemployement at equilibrium, because if too many people seek for jobs, employers lower the salary and so the offer get lower. Recall that in the keynessian approach the relation between employement and salary is supposed to be determined outside market, for instance by means of minimum salary regulations. In the next two sections we will not enter in any controversy about werther a model describes or not accurately

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any actual present or past economy or even if some model feature is or is not desirable, and will concentrate in the mathematical structure of the model and its internal consistency. In section 4 we will discuss some semirealistic modelling in order to establish connections with keynessian models. In the last section we give the conclussions.

2 The mistake

Lets see again the expression for the benefit

$$B(n) = Y(n) - n\omega \tag{1}$$

We found its maximum by deriving it respect to n and setting the result to zero. Now, the hypotesis made is that ω is an unknown function of n, the 'demand curve', and we forgot completely about $\omega'(n)$. The actual equation for ω is thus a first order differential equation

$$0 = Y'(n) - \omega(n) - n\omega'(n) \tag{2}$$

If we forget about ω', we are considering ω as a given parameter, so we would be in the simplest keynesian hypotesis. If we were considering a keynessian model, where the relation between ω and n is given in advance, then we are done, having in mind that now the condition for maximum benefit is 2 and not $\omega = Y'(n)$. Let's try to correct the classical model and find a demand curve solving 2. As it is a first order equation, it needs a boundary condition. We choose $B(0) = 0$ (no work, no benefit). Let us replace in 2 ω by g/n; then it becomes

$$g'(n) = Y'(n) \tag{3}$$

The most general solution for 3 is g(n)=Y(n)+C. Replacing it back into 1 we get $B(n) = -C$, so by the boundary condition we get $C = 0$, and the solution is

$$\omega(n) = \frac{Y(n)}{n} \tag{4}$$

The salary would be then the rent divided among the workers, and the benefit is zero. We got not the maximum but the minimum of the benefit, wich is the other condition for the anihilation of the derivative. Why 2 didn't show the maximum as another solution? Well, the maximum can be found by inspection, and correspond to $\omega = 0$ and $n = \infty$, so there is not a function $\omega(n)$ that maximizes $B(n)$.

3 What happened?

The last section was a cumbersome way to show something that could be told by simple inspection: if the only criterion to choose the salary is to maximize the benefit as given by 1, then it would be set to zero. So, there is not such a thing like a 'demand curve' $n(w)$. Of course, it doesn't mean that there is no job demand: if we want a benefit we have to employ workers. We can talk about offer curves: for instance the number n of people willing to work for the salary ω, or the number n of people seeking for a job while employers offer a salary ω (notice the subtle difference in definition, wich will be analyzed in the next section), but in order to maximize the $B(n)$ we need the actual relation between employement and salary, wich have to be modelled appart. No doubt that offer will make the employement a growing function of the salary, but its relation to some offer has to be postuled (hopefully derived) appart. Then, replacing in $B(n)$ this function and deriving respect to n, we find that the maximum benefit is 2.

So here is a price not determined by the intersection of an offer and demand function. Of course, we cannot ignore the key role that offer and demand plays in any trade, but offer-demand curves is not the most general model of offer-demmand interplay. We find that if we want to keep as premise the maximization of the benefit we have to abandon such description and pass to a keynessian approach. What if we reject benefit maximization? Well, if there aren't employers and we suppose that 'the society' tries to maximize the rent $Y(n)$, then the demand curve would be $\omega = \frac{Y(n)}{n}$[2], wich would intersect with an offer curve, and we would recover almost the construction. But then it would be strange because no one 'buys' human work, or they does it in an indirect way, so this 'salary' would not be strictly a 'price'.

[2] $Y(n)/n$, under the usual asumptions $Y'(n) > 0$ and $Y''(n) < 0$, is qualitatively similar to $Y'(n)$

4 employement models and contact with keynessian models

Let us look at the employement and offer functions. In general, the second should be somewhat higher than the first. The simplest posibility is that the employement at equilibrium equals the offer. But there is no a priori reason for that, and we should analyze it carefully. Let us look first at the offer function, in a society in wich most of the population earns its money from a job. First we should be clear about what offer are we talking about. The relevant offer in order to analyze unemployement is the number of workers seeking for a job while employers pay ω. Depending on prices, cultural background, laws, etc, there would be a salary considered as acceptable, say ω_0. As at least one person per domestic unit needs a job, there is also a minimum demand, say n_0. Until the salary reaches ω_0, the demand should rise only slightly from n_0. Once the offered salary is above ω, more people will fill tempted to seek for a job, and the demand should increase fastly until it reaches a value close to the total adult population. The situation is different of that of a pre-industrial society, in wich domestic units could choose between subsistence economy in the countryside or employement, so there is no an n_0. Let us look now at the employement function: it is reasonable that above ω_0 the employement almost follows the demand. That is because those people who doesn't need the job will eventually be discouraged if they do not find it, and in stationary situation the demand would drop close to the actual employement. Below ω_0 the situation changes drastically: people will look for jobs with salary ω_0 at least and will be offered a lower salary, so the less the salary is, the less people accept the job, thus employement fall with ω, and unemployement rises. The employement in such a stage thus follow the other thinkable offer function: the number of workers willing to work for the offered salary. It might be argued that eventually people would accept its poverty and finally if the conditions remain long enough both offer functions would equal. But such a cultural change may take decades, and is clearly a structural change. Of course there is no need for believing that the unemployed are allways the same, we could have some kind of dynamical equilibrium of people taking and leaving part-time jobs. So unemployement can be an 'equilibrium' solution in the usual sense of the expression. Anyway, getting used to poverty is not a desirable solution for unemployement: we should get both wealthiness and full employement.

This is a keynessian-like scenario. Take for instance the case in wich the demand is n_0 until ω_0, and then becomes rigid, and the employement function is completely rigid at ω_0. This is the only case in wich we have $Y'(n) = \omega$. This is the simplest keynessian hypothesis, so simple that it does not take into account any offer.

Of course we should insert this in a more complete model to see what happen with the prices. Werther we stop here in modelling employement, or we try a most complete model, is a matter of choice. It looks like, at least in the job market, the simplest models are the keynessian ones, because the classic ones are just wrong.