Can Skill Biased Technological Progress Have a Role in the Decline of the Savings Rate?*

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Abstract

This paper explores the consequences of skill biased technological progress on the savings rates. The literature, both theoretical and empirical, on the causes and consequences of skill biased technological progress in the past few years has burgeoned considerably. So has the literature on declining household savings, motivated by the American experience over the past couple of decades. I present a general equilibrium model where declining savings rates emerges as an outcome of exogenously driven skill biased technological progress. The link between the two is attributed to optimizing behavior of altruistic households. In an overlapping generations model, parents are assumed to derive utility from both spending on their children's education and making monetary transfers (or bequests). I show that increases in the growth rate of skill biased technological change causes a shift in allocations away from bequests in favor of education-leading to a decline in domestic capital accumulation. The analysis is extended to incorporate life cycle savings both under certainty and uncertainty regarding the timing of death.

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1 Introduction

The personal savings rate of the US economy since the early eighties has been declining and has now touched levels below zero. Explanations for this decline are multifarious and unresolved. Candidates include the changing demographic structure, increasing ratio of wealth to income (reinforced by a belief of good times in the future), an increase in the discount rate, huge transfers of wealth to the aged (who have a higher marginal propensity to consume) through public institutions, increased access to credit allowing greater consumption smoothing, etc.[1] From the early eighties the economy has also experienced what observers believe to be a rapid pace of technological change with the benefits of this change accruing disproportionately to those with higher levels of skills. This phenomenon of "skill-biased technological change" is considered to be the cause of a monotonic rise in income inequality experienced in the past two decades.[2]

Though traditionally average earnings for the more educated have been greater than that for the less educated, in the eighties and nineties the differences have soared. Murphy and Welch (1992) noted that the college high school wage differential for all experience levels rose from 40% in 1963 to 48% in 1970 and then fell to 38% in the late 1970s only to soar to 58% in 1989. Restricting attention to those with only ten years of work experience, the ratio rose from 28% in the beginning of the 1980s to 69% by the end of that decade. The dispersion in the returns to education is also reflected in the average returns which have undergone dramatic changes in the post-war era. Mincer (1998) notes that returns grew in the sixties, fell in the seventies to reach a low level of 4%, then rebounded in the eighties and reached heights of 12% or more in the first half of the nineties. Bound and Johnson (1992) and Katz and Murphy (1992) drew attention to the important role of the "demand" for more educated and skilled workers in driving these patterns. Kreuger (1993) suggested that the advent of the computer has had a role to play in these changes. Autor, Katz and Kreuger (1998) reconfirm the above findings by showing that the wage premium continued to rise in the nineties and that the most rapid skill upgrading has taken place in the more computer intensive sectors of the economy.

The literature trying to explain the decline in savings is quite extensive. Gokhale, Kotlikoff and

[1] See Parker (1999) for a thorough review of these arguments.

[2] Freer international trade is also often considered to be a cause of the increase in income inequality. However there now seems to be a greater consensus that skill biased technological change is the culprit.

Sabelhaus (1996) decompose changes in savings due to cohort specific consumption propensities, those due to intergenerational redistribution of resources, changes in government spending patterns and changes in demographics. They argue that redistribution of resources from the youth to the elderly and the higher consumption propensities of the latter group can explain the decline in savings. Gale and Sabelhaus (1999), take Nordhaus' quote above seriously and try to re-measure savings. For example, after augmenting household savings with data on real capital gains, they suggest that savings is at its highest in the past forty years. They note that adding human capital, research and development, and other intangible capital would further raise the savings rate. Parker (1999) undertakes a thorough examination of existing explanations of the savings decline. He argues that neither wealth changes nor the changing age distribution can completely explain the savings decline since they post date the consumption boom . He also rules out the importance of explanations based on income by age distribution, financial innovations and intergenerational fiscal transfers. Parker notes that financial innovation can explain a third of the decline and wealth changes another fifth. This implies that these two explanations are enough to account for more than half of the decrease. He speculates that since consumption is a forward looking variable, the consumption boom maybe linked to households learning about high levels of output in the future. Other possibilities include a shift in household preferences and finally he does not rule out that a combination of some of the arguments presented so far might explain the decline but questions the ability of a monocausal argument.

This paper argues that increasing returns to education may be an important, and as yet overlooked explanation for the decline in savings. Standard economic theory suggests that investments in education should rise in response to an increase in its rate of return. This notion lies at the core of the model developed in this paper. Indeed, Mincer (1998) shows that enrollment rates for the period 1967-1990 responded positively to the wage premia and predicted that an upward trend in enrollment was forthcoming. The second element of the model is the assumption that households are altruistic, and parents care about the amount they spend on their children's education and on the amount that they allocate as financial transfers. Finally in order to maximize the effectiveness of their transfers, parents allocate funds to these two types of transfers until the returns are equalized- an assumption that is fairly standard since its introduction to the literature by Becker and Tomes (1986). In the presence of increases in skill biased technological progress, parents find it more profitable to spend on their children's education rather than leave bequests. Following a permanent increase in skill biased technological progress, it is then possible to show that the ratio of bequests to output declines while the share of educational expenditures rise.

After establishing the main result, I then consider some important extensions to the model. A decline in the bequest ratio is not the same as a decline in the savings rate. Individuals save for a number of reasons with bequests being only one of them. Other important constituents include life cycle savings and precautionary savings. To check for the robustness of the model I then extend it to include life cycle savings as well. It turns out that the conditions which generate a decline in bequests will also generate a decline in life cycle savings and therefore a decline in the aggregate savings rate too. Finally, a concern of those working on the nature of bequests, has been the relative importance of intentional bequests to unintentional ones. To allow for both types of transfers, I further introduce uncertainty regarding the timing of death. The results continue to be robust. Allowing uncertain longevity means that the theoretical framework lends itself also to analyzing the effects of increasing life expectancy in the presence of technological progress. Increasing life expectancy implies that individuals will save more for retirement, will allocate less to their children and accidental bequests will also fall. Therefore, increasing life expectancy will have both positive and non positive effects on savings. I show that it is possible that increases in life expectancy can not only reduce bequests but life cycle savings in the long run as well.

In addition to suggesting a novel explanation for the savings decline, the model draws attention to the possible trade-off between physical capital accumulation and human capital accumulation. The macroeconomic consequences of whether intergenerational transfers are in the form of financial or human wealth was recognized by Blinder (1976). He observed that human capital transfers enter into the consumption expenditure side of the national income account whereas financial bequests enter the savings side. The recent work in economic growth (e.g. Mankiw, Romer and Weil (1992)) has argued that the human capital stock in an economy is an important factor in explaining differences in growth rates across nations. Our model suggests that the decline in the savings rate is nothing but the failure on the part of the national accounts to reflect a change in the nature of accumulation by the household: an endogenous substitution of human capital wealth for financial wealth in a period where technological improvements favor skills.

Given that there are so many competing explanations for the savings decline, how much more of a story can human capital contribute? To consider the argument presented here, one needs to look at the magnitude of changes in investment in human capital. In practice, the true measure of investment in human capital is anybody's guess. The share of aggregate education expenditures, in GDP has hardly changed over the past few decades and hovers at around seven percent. However these are direct expenditures and do not include opportunity costs and activities which are complementary to human capital. Fraumeni and Jorgenson (1990) attempted to measure investment in education based on its contribution to output (i.e. an incomes based approach as opposed to a cost based approach). If one looks at education's contribution only to market based activities, their estimate of aggregate investment in education as a share of GDP turns out to be 45%. Mincer (1990) in his comment on their paper noted that the rate of discount used by Jorgenson and Fraumeni was half of what it should have really been. That reduces the percentage roughly to 22.5%- still a very high number. Mincer himself prefers cost based approaches and notes that investment in education would end up being 14% of GNP (However, in his 1998 paper, Mincer quotes a lower share of 9.1% of GNP for 1980 of which 5% is in post secondary education). Adding on the job training takes this number up by another 6% to 20%. Though not related to the model presented here, one can conjecture that investment in on the job training is likely to rise during periods of rapid technological change.

To get an idea of the changes in investment in education, as a starting point it is useful to look at what happened to enrollments in higher education during the period in which savings declined. If one concentrates on the period from 1984 (the last year in which the US experienced a double digit savings rate and declined almost monotonically from then on) to 1998, it turns out that enrollment rates during this period jumped up by 26%. Figure 1 depicts the movement of the enrollment rate in college for the 18-29 cohort.It suggests that the percentage attending an institution of higher education jumped by almost 45% during the entire period 1980-98. Further there is an a clear upward spike from 1986 onwards. Mincer's prediction that there should be a supply response seems to have borne out. These trends suggest that there must have been a sizeable increase in investments in education.

In order to get some sense of how much education expenditures might have risen, I undertook a crude extrapolation from the numbers provided in Mincer (1998). He notes that adding opportunity costs and direct expenditures on post secondary education implies a share of 5.0% for post secondary education in 1980. Between 1980-1998 enrollments have risen by 37%. Roughly, this means an increase in the share of education relative to GNP of 1.88% during that period. During the same period personal savings as a share of GNP fell from 7.4% to 2.6%, implying that increased expenditure on education can potentially explain 39% of this decline. Though this might seem like an overestimate, it is also conservative in the sense that it increases expenditures only based on increases in enrollments and rules out increases simply because individuals are willing to spend more per head in the face of increasing returns.

To derive an alternative estimate of the possible magnitude of the role of expenditures in education in explaining the savings decline, I looked at the weights given to education expenditures in the consumer price index over the years. These weights include only tuition, books and fees and no other expenditures and stand at a paltry 2.62% in 1998 (this translates to a 2.5% share in terms of personal disposable income). In 1980 this number was even lower at 0.87% (which translates to 0.76% of personal disposable income). Between 1980 and 1998 the savings rate as a share of personal disposable income fell by 6.5%. Based on these numbers increases in educational expenditures as a share of personal disposable income, can explain 28.2% of the savings decline. If we close in further to the period 1984-98, which is at the heart of the savings decline, personal savings as a share of personal disposable income declined by 4.5 percentage points from 8.2% to 3.65%, while the share of education rose by 0.9% points. This also suggests a prima facie case for explaining almost 23% of the decline in the savings rate.

Clearly both these exercises suggest that increasing returns to education is likely to have had an important role in explaining the savings rate. One could easily argue that direct measures such as these do not capture the entire picture and can even be underestimates. Measures of investment in human capital do not generally include activities complementary to formal education. This is particularly to true in the case of pre college education. As the returns to college education rise, and securing admission becomes more competitive, parents to spend a lot more on their children's school education. This includes but is not restricted to costs incurred in tutoring and extra-curricular activities (which one might argue increases the "cognitive skills" of the child) in order to enhance their children's prospects.[3] Adding on the job training would presumably increase the importance of education spending specially in a period during which rapid technological change forces firms to retrain their workers.

In addition to the late twentieth century, are there any other periods during which the negative

[3]In addition to underestimating investment in education in the accounting sense, there is anecdotal evidence suggesting increasing "time costs" have been ignored as well. See "The Parent Trap", Cover story in Newsweek (Jan 29, 2001).

relationship between savings and the return to education might be operative? Figure 2 plots the movements of returns to education, the personal savings rate and the private savings rate through the previous century for the United States. The savings rates are annual data from the national accounts while the return to education are decade estimates for returns to college for young men from Goldin and Katz (1999). The graph, which also includes smooth quadratic fits for all three variables, seems to suggest that a strong negative relationship might have persisted between the two upto 1950 as well and again in the seventies. Only during the fifties and the sixties does the negative relationship seem not to hold.[4]

The theoretical framework in this paper relies on the assumption that intergenerational transfers are an important source of capital accumulation in the economy. The contribution of such transfers relative to life cycle savings as the primary source of wealth accumulation was first brought to attention by Kotlikoff and Summers (1981). They suggested that almost eighty percent of the stock of wealth accumulated by households in 1974 was attributable to intergenerational transfers. In addition to stocks, they also looked at flows and estimated that intergenerational transfer flows may have been as high as sixty five percent of household wealth. Further, they noted that this number is likely to be biased downwards since it did not include gifts not recorded in any data source.

Within intergenerational transfers, one may distinguish between bequests and other types of transfers such as tuition support, loans and transfers of businesses to children. This question of inclusion is one of the main points of disagreement between the proponents of the intergenerational transfers and proponents of the life cycle savings story- best exemplified in the debate between Kotlikoff (1988) and Modigliani (1988). Modigliani is opposed to parental support in college education as being defined as an intergenerational transfer. He argues that this is a transfer of human wealth and should not be incorporated into any measure of non-human wealth. Kotlikoff responded by arguing that as long as the transfer is one of funds, the point is immaterial. Clearly this distinction is germane to the modeling strategy here. The model relies on a distinction between the two not only because the behavior of their respective returns differ but also because education expenditures, as noted by Modigliani, does not go towards asset accumulation but pays for current

[4]One can argue that for the first half of the twentieth century what should matter more are returns to schooling rather than college. In fact, the return to schooling also seems to have dropped drastically (from approximately 12% to 5%) between 1914 to 1950 (see figure 4, Goldin and Katz(1999)).

consumption.[5] Finally, and here there is less disagreement between the two, is the importance of intentional bequests versus unintentional bequests. They both agree that a large part of intergenerational transfers might be unintentional- precautionary savings and life cycle savings that were not consumed until death will become intergenerational transfers. I extend the model to include life cycle savings and uncertain life expectancy and the basic intuition still goes through. On the whole, elements of the debate are well captured in the framework adopted in this paper. Kotlikoff's argument that both types of intergenerational transfers matter implies that the entire optimization exercise carried out below is not over a variable of insignificant magnitude; Modigliani's recognition of the distinction between different types of transfers lies at the center of this paper and both of their observations on unintentional bequests is also taken care of. Finally, the basic insight of the model that parents might bequeath less when they perceive their children's income to be growing faster was noted also by Meade (1966). However he did not concern himself with issues of human capital and changes in the nature of transfers. In Meade's analysis, parents are interested in ensuring that children's consumption is above a certain minimum. Thus the nature of altruism is also different.

The next section of the paper develops a simple model with bequests being the sole constituent of savings. Section 3 of the paper incorporates life cycle savings. The first subsection deals with no uncertainty regarding life spans. The second subsection further develops the model by introducing uncertainty and therefore allowing unintentional bequests as well.

2 The Basic Model

Consider a small open overlapping generations economy in a perfectly competitive world with unrestricted capital flows. Economic activity extends over infinite discrete time. In every period the economy produces a single homogenous good that can be used for consumption, investment in physical capital and investment in human capital. The good is produced by physical capital and human capital augmented raw labor. The supply of all factors of production is determined endogenously. In particular the supply of human capital and physical capital is determined by the state of technology which determines the relative rates of return.

[5]Though as mentioned above, it is more appropriately viewed as an investment

2.1 Production in the Economy

In every period the final good is produced under a constant returns to scale neoclassical production function that is subject to technological progress. The output produced at time t is,

$$Y_t = Y(K_t, A_tH_t) \equiv A_tH_tf(k_t);\ \ k_t \equiv K_t/(A_tH_t)$$

where A_t represents a skill-augmenting productivity parameter. K_t and H_t represent physical and human capital stocks respectively in any given period, t. The production function $f(k_t)$ is strictly monotonic increasing, strictly concave satisfying the neoclassical boundary condition that ensure the existence of an interior solution to the producer's profit maximizing problem.

Producers operate in a perfectly competitive environment. Given the wage rate per efficiency unit of labor and the rate of return to capital at time t, w_t and r_t respectively, producers employ the optimal amount of capital K_T and human capital H_t, in order to maximize profits, i.e. $K_t, H_t = \arg\max[A_tH_tf(k_t) - w_tH_t - r_tK_t]$. Competitive factor markets ensure that factor prices equal their marginal products,

$$r_t = f'(k_t)$$

$$w_t = A_t[f(k_t) - f'(k_t)k_t] \equiv A_tw(k_t)$$

H_t, the human capital stock is the economy's stock of raw Labor augmented by an efficiency parameter h,

$$H_t = h_tL$$

where L represents the population in the economy and is fixed and h represents the human capital augmenting factor. Without loss of generality, I assume that L is normalized to 1. The production function for human capital, is simple. Human capital for a single individual, in any period, h_t is a simple concave function of the educational expenditures in the past period, e_{t-1} subject to a fixed cost, $\bar{e}_{t-1}$,

$$h_t = (e_{t-1} - \bar{e}_{t-1})^{\theta} \qquad \theta \in (0,1) \tag{1}$$

where $\bar{e}_{t-1} = A_{t-1}^{\frac{1}{1-\theta}}\bar{e}$. The fixed cost is allowed to change over time since it is unlikely that such costs remain fixed in the presence of technological progress.

The world rental rate is assumed to be fixed at r. Since the small economy permits unrestricted international lending and borrowing the domestic rental rate is fixed at the international value, i.e.

$$r_t = r$$

Consequently the ratio of capital to efficiency units of labor is also fixed in every period at $f'^{-1}(\bar{r}) \equiv \bar{k}$ and the wage rate per efficiency unit of labor is

$$w_t = A_t w(\bar{k}) \equiv \bar{w} A_t$$

From equation(1) it follows that an individual receives a final wage of $\bar{w} A_t h_t$.

Capital is assumed to depreciate completely in every period. This assumption ensures that there is a clear correspondence between inherited wealth in this model and household savings as defined in national accounts.[6] Technological progress is exogenous and takes place at a constant rate of growth, g,

$$A_{t+1} = (1+g) A_t$$

Finally, one can note that the nature of technological progress is indeed *skill-biased* in the sense that an increase in the growth rate will lead to an increase in the wage ratio of those who invest more in education relative to those who invest less, i.e.

$$\frac{\partial \left(\frac{\bar{w} A_t h_t(e_{2,t-1})}{\bar{w} A_t h_t(e_{1,t-1})} \right)}{\partial g} > 0 \qquad \forall e_{2,t-1} > e_{1,t-1} > \bar{e}_{t-1},$$

2.2 The household

Most overlapping generations models usually incorporate only one type of transfer -either human capital or bequests. Usually this depends on the focus of the model. Very few include both. Becker and Tomes (1986) takes cognizance of both types of transfers and it is their style which is adopted here. However they are concerned with issues of inheritability of endowments and intergenerational mobility. A similar preference structure in a dynamic setting is employed by Galor and Moav (1999) but abstracts from technological progress. Zilcha (1996) incorporated both bequests and human capital investments into the parents' utility function to study economic growth. However the functional form is of a constant returns Cobb-Douglas nature where human capital and bequests

[6] In fact in a closed economy version, the two will be exactly the same. The only difference arising from the small open economy assumption is that some of the inherited wealth may be invested abroad.

enter as separate arguments. In such a setup an individual's preferences structure might imply a relatively higher share of wealth being allocated to bequests relative to education spending or vice versa. One can then compare growth trajectories that emerge under the two different cases. Such a preference structure is obviously too restrictive to answer the question posed here. Ishikawa (1975) also addresses the issues of investment in children's education and bequests but is more concerned with the role of family values and income distribution. A lot of the work in overlapping generational models of human capital formation also usually assumes that individuals make their own educational investments. In practice however this is not completely correct. Rosenzweig and Wolpin (1993) show that intergenerational transfers begin with huge amounts earmarked by parents to support their children's human capital accumulation.

In each period a generation is born. The economy is characterized by the coexistence of two generations simultaneously. Individuals are assumed to care about the total transfer they make to their children. At any period t, members of one generation (generation, $t+1$), are economically inactive and spend time receiving an education. Members of another generation (generation, t), work and earn their livelihood. With their income and the bequest they receive, members of generation t in time period t consume the final good, support the younger generation's education and make bequest allocations. All transactions take place at the beginning of each period and shocks to the growth rate of technological progress also take place then. Therefore an individual is aware if her income changed because of a shock and adjusts expenditures accordingly.

Since an individual is not economically independent in the first period of her life, utility is defined over expenditures in the second period. In addition to her own consumption, she derives utility from the sum of the value of total transfers it makes to its offsprings. Transfers can take place in the form of investments in human capital and bequests. Altruism is directed only towards the young and not towards the old. In any period t, the only source of savings therefore are bequests of agents of period t.[7]

The utility function is assumed to be of the logarithmic form and is separable in consumption and transfers. For a member of generation t, the optimization problem is as follows,

$$\max_{c_t, e_t, b_t} U_t^t = \alpha \ln c_t^t + \gamma \ln \omega_t \tag{2}$$

[7]In this economy firms do not have profits. Hence the terms "domestic" and "households" will be used interchangeably when I talk about wealth accumulation from sources within the country.

subject to,

$$c_t^t + \omega_t = \bar{w}A_t h_t + rb_{t-1} \tag{3}$$

where

$$\omega_t = e_t + b_t \tag{4}$$

and

$$\alpha < 1, \gamma < 1 \text{ and } \alpha + \gamma = 1$$

ω_t is the total transfer made by a member of generation t. The budget constraint says that the sum of all expenditures is equal to the sum of an individuals labor earnings, $\bar{w}A_t h_t$ and her inheritances, rb_t. A superscript t implies that the value of the variable in question was chosen by a member of generation t. A subscript tell us in which period that variable belongs to. The superscript is avoided for variables for which there is no such confusion.

Utility maximization implies that the following solutions for c_t^t and ω_t will hold,

$$c_t^t = \alpha\left(\bar{w}A_t h_t + rb_{t-1}\right) \tag{5}$$

$$b_t + e_t = \gamma\left(\bar{w}A_t h_t + rb_{t-1}\right) \tag{6}$$

Given the current specification, the model by itself does not provide a method for calculating how the total transfers should be divided between bequests and human capital investment. The decision regarding this is assumed to come from an arbitrage equilibrium. That is, the net rate of return on education and financial assets are equalized (Becker and Tomes (1986)). This implies that, since $r_t = r$ is fixed, investment in education will be undertaken until a point such that,

$$r_e = r$$

where r_e is denotes the marginal rate of return on human capital investments. Given the human capital production function, the condition can be rewritten as,

$$\bar{w}A_{t+1}h'(e_t) \equiv r_e = r \tag{7}$$

$$\Rightarrow \bar{w}A_{t+1}\theta(e_t - \bar{e}_t)^{\theta-1} = r$$

$$\Rightarrow e_t^* = \left(\frac{\bar{w}A_{t+1}\theta}{r}\right)^{\frac{1}{1-\theta}} + \bar{e}_t$$

$$\Rightarrow e_t^* = \left(\frac{\bar{w}A_{t+1}\theta}{r}\right)^{\frac{1}{1-\theta}} + A_t^{\frac{1}{1-\theta}}\bar{e} \tag{8}$$

This is the optimal value of human capital investment. Clearly as the level of technology improves the optimal amount of education increases. Further as the international rate of return rises, investment in education declines. Substituting e_t^*, one can calculate the optimal value of an individuals earnings,

$$\bar{w}A_t h_t = \bar{w}A_t\left(\frac{\bar{w}A_{t+1}\theta}{r}\right)^{\frac{\theta}{1-\theta}} \tag{9}$$

Further substituting the optimal value of e_t^* in equation (6) gives the optimal value for b_t,

$$b_t = \gamma\left(\bar{w}A_t h_t + r b_{t-1}\right) - \left(\frac{\bar{w}A_{t+1}\theta}{r}\right)^{\frac{1}{1-\theta}} - A_t^{\frac{1}{1-\theta}}\bar{e} \tag{10}$$

2.3 Dynamics

In this section, the evolution of the savings rate, the share of education in total output and aggregate output is analyzed. In order to keep track of the savings rate in the economy, I introduce some notation: let $\tilde{y_t}$ and y_t represent the output per capita and output per efficiency unit in period t. Since the capital stock per efficiency unit in the economy is fixed, $y_t = \bar{y}\ \forall t$. Then,

$$\tilde{y_t} = \bar{y}A_t h_t. \tag{11}$$

Similarly,

$$\tilde{k_t} = \bar{k}A_t h_t \tag{12}$$

The savings ratio which is equivalent to the bequest ratio is the amount of bequests per individual divided by output per capita and is defined as Z_t. Therefore,

$$Z_t = \frac{b_t}{\tilde{y_t}} = \frac{b_t}{\bar{y}A_t h_t} \tag{13}$$

Substituting the value of b_t from equation (10),

$$Z_t = \frac{\gamma\left(\bar{w}A_t h_t + r b_{t-1}\right)}{\bar{y}A_t h_t} - \frac{e_t^*}{\bar{y}A_t h_t} \tag{14}$$

This simplifies to:

$$Z_t = \frac{\gamma\bar{w}}{\bar{y}} + \frac{\gamma r Z_{t-1}}{(1+g)^{\frac{1}{1-\theta}}} - \frac{\theta\bar{w}(1+g)^{\frac{1}{1-\theta}}}{\bar{y}r} - \frac{\phi}{\bar{y}} \tag{15}$$

where $\phi = \frac{\bar{e}}{(\bar{w}\theta/r)^{\frac{\theta}{1-\theta}}}$.

Equation (15) is the central dynamic equation for the model. The stability of the dynamics are clearly dependent on parameter values. In order to assure that the system does not exhibit monotonically explosive or contractionary behavior, I make the following restriction,[8] :

Assumption 2.1 $(1+g)^{\frac{1}{1-\theta}} \geq \gamma r \geq \theta(1+g)^{\frac{1}{1-\theta}} + \frac{\phi r}{\bar{w}}$

Proposition 2.3.1 *An increase in the rate of growth of technological progress, g leads to a decline in the savings rate:*

$$\frac{\partial Z_t}{\partial g} < 0$$

Proof. From equation (15),

$$\frac{\partial Z_t}{\partial g} = -\frac{1}{1-\theta} \frac{\gamma r Z_{t-1}}{(1+g)^{\frac{1}{1-\theta}-1}} < 0$$

Proposition 2.3.2 *Under assumption (2.1), An increase in the rate of growth of technological progress, g, leads to a decline in steady state the savings rate:*

$$\frac{\partial \bar{Z}}{\partial g} < 0$$

Proof. From equation (15), the steady state value of the savings ratio is,

$$\bar{Z} = \frac{\bar{w}}{\bar{y}} \frac{\left[\gamma - \frac{\theta(1+g)^{\frac{1}{1-\theta}}}{r} - \frac{\phi}{\bar{w}}\right]}{1 - \frac{\gamma r}{(1+g)^{\frac{1}{1-\theta}}}}$$

It is easy to see that $\frac{\partial \bar{Z}}{\partial g} < 0$.

Figure 3 shows the phase space for Z_t. A positive shock to g leads to both these propositions. There are two reasons why this happens. In any given period, the agent receives some amount as a bequest, decided by her parent. Following the shock to the growth rate, this amount remains unchanged since it was decided earlier but output per capita in the current period rises and thus lowers the bequest to output ratio. Additionally, the shock to the growth rate raises the investment made by the agent in her children's education. This further reduces the bequest to output ratio. If the economy was initially at the steady state then clearly the savings rate drops and continues to do so until it reaches the new lower steady state. On the other hand, it is possible that the economy was initially at a savings rate which was below the original steady state. In that situation,

[8]This assumption has no effect on the effect of technology shocks on the transitional path.

an increase in g will reduce the savings rate both in the next period and also in the steady state, but in transition it may rise.

Finally in terms of output per capita, the economy exhibits balanced growth irrespective of the dynamics of the savings ratio. Since the capital stock is fixed, growth is driven completely by technological progress and human capital accumulation. The latter is in turn a function of technological progress as well. The growth rate is therefore completely a function of g:

$$\frac{\Delta \tilde{y}_t}{y_t} = \frac{g}{1-\theta}$$

Note that this is exactly equal to the rate of growth of education expenditures, $\frac{\Delta \tilde{e}_t}{e_t}$ with the implication that the ratio of education per capita to output per capita is constant at $\frac{\theta \bar{w}(1+g)^{\frac{1}{1-\theta}}}{\bar{y}r}$.

While the intuition for the story is simple, one may argue that the model above does not incorporate some issues very well. In particular, the technological change that seems to drive the increasing returns to education without changing the returns to capital hinges on the assumption of the small open economy. With a closed economy such technological progress will be neutral to both physical and human capital if one were to assume a Cobb Douglas production function (which is not ruled out by the nature of the assumed production function). Also, it would translate to total factor productivity growth which has actually decreased and not increased during the past two decades. To address some of these issues, appendix A presents a static closed economy model with a constant elasticity of substitution production function. The production function allows one to clearly differentiate between technologies that make physical capital more productive and that which makes human capital more productive. It is easy to show that an increase in the technological parameter for human capital relative to that for physical capital causes a reallocation of resources in favor of the former and a decline in the savings rate.

3 Introducing Life Cycle Savings

As discussed in the introduction, the empirical literature on wealth accumulation is divided between proponents of the importance of life cycle savings and intergenerational transfers.[9] In this section we extend the model to include life cycle savings as well. The first subsection presents the model without any uncertainty involved. Therefore there are no unintentional bequests. The second

[9] It must be added that there is also considerable support for the precautionary savings hypothesis which I have not discussed in this paper.

subsection introduces uncertainty and allows such types of bequests to occur. This complicates the analysis somewhat but the story still goes through.[10]

3.1 No Uncertainty

The economy is now characterized by the coexistence of three generations simultaneously. At any period t, members of generation $t+1$, are economically inactive and spend time receiving an education as before. Members of generation t are working and earning their livelihood. With their income and also the bequest they receive they consume the final good, support the younger generation's education, make bequest allocations for the younger generation and save a fraction of their income for retirement. The third generation live off their own savings.[11]

An individual's utility is now defined over second and third period consumption and from expenditures on their children. As before, the utility function is assumed to be of the logarithmic form and is separable in consumption and transfers. For a member of generation t, the optimization problem is now,

$$\max_{c_t, c_{t+1}, \omega_t} U_t^t = \alpha \ln c_t^t + \beta \ln c_{t+1}^t + \gamma \ln \omega_t \tag{16}$$

subject to,

$$c_t^t + \omega_t + s_t = \bar{w} A_t h_t + r b_{t-1} \tag{17}$$

$$r s_t = c_{t+1}^t \tag{18}$$

where

$$\alpha + \beta + \gamma = 1 \tag{19}$$

Utility maximization implies that the following solution for c_t^t, c_{t+1}^t and ω_t will hold,

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[10] As far as I am aware, no model has so far incorporated both intentional and unintentional transfers. This is not surprising since the model does run into more than one difference equation. However the simplifying assumption of the small open economy keeps things tractable.

[11] Note that bequests here are not so in the strict sense of the term. Individuals receive them before the death of their parents and hence are literally *inter-vivos intergenerational transfers* which earn a return. It is easy to see that even if it were a bequest at death, the maximization problem and budget constraints would be unchanged. This is because the bequest allocation will earn a return for over two periods. That would be compensated by the fact that it needs to be discounted one period to make it comparable to the returns to investment in education -leaving the situation unaltered. The only way it would matter is that it would impose some additional liquidity constraints. Since the model will not incorporate issues of liquidity constraints on education expenditures, this assumption only plays a simplifying role. Further allowing perfect capital markets would remove this restriction as well. The evidence on borrowing constraints is ambiguous. Shea (1998), Cameron and Heckman (1998) and Cameron and Taber (2000) find no evidence that borrowing constraints play a role in college choices. However Acemoglu and Pischke (2000) show evidence that family income is an important determinant.

$$c_t^t = \alpha\left(\bar{w}A_th_t + rb_{t-1}\right) \tag{20}$$

$$c_{t+1}^t = \beta r\left(\bar{w}A_th_t + rb_{t-1}\right) \tag{21}$$

$$b_t + e_t = \gamma\left(\bar{w}A_th_t + rb_{t-1}\right) \tag{22}$$

Similar to Z_t in equation (15), one can define a new variable X_t which is the life cycle savings ratio (i.e. life cycle savings of the representative agent divided by output per capita),

$$X_t = \frac{s_t}{\tilde{y}_t} = \frac{s_t}{\bar{y}A_th_t} \tag{23}$$

The evolution of X_t can be characterized by,

$$X_t = \frac{\beta\left(\bar{w}A_th_t + rb_{t-1}\right)}{\bar{y}A_th_t} \tag{24}$$

$$\Rightarrow X_t = \frac{\beta\bar{w}}{\bar{y}} + \frac{\beta r Z_{t-1}h_{t-1}A_{t-1}}{h_tA_t} \tag{25}$$

Further simplification leads us to the following expression:

$$X_t = \frac{\beta\bar{w}}{\bar{y}} + \frac{\beta r Z_{t-1}}{(1+g)^{\frac{1}{1-\theta}}} \tag{26}$$

Equation (15) for Z_t ((henceforth referred to as the "bequest ratio"), in section 2 continues to be the main dynamical equation for the model and also drives X_t . As before, the bequest ratio falls as a consequence of technological progress. The steady state level of the life cycle savings ratio is given by,

$$\bar{X} = \frac{\beta\bar{w}}{\bar{y}} + \frac{\beta r \bar{Z}}{(1+g)^{\frac{1}{1-\theta}}} \tag{27}$$

Proposition 3.1.1 *An increase in the rate of growth of technological progress, g, leads to an immediate decline in the life cycle savings rate*

Proof.

$$\frac{\partial X_t}{\partial g} = -\frac{1}{1-\theta}\frac{\beta r Z_{t-1}}{(1+g)^{\frac{1}{1-\theta}+1}} < 0$$

A simple inspection of the life cycle savings ratio's difference equation (26) clearly indicates that the latter must also fall. The reason is that the second component of the life cycle savings ratio, the ratio of bequests from the earlier period to the current per capita output, has fallen.

This component is similar to the component in Z_t in equation (15) and the explanation of why this falls is the same -interest income is based on allocation made in the previous period. Increases in productivity in the current period raises output per capita but does not alter the bequest of the earlier period on which interest is earned. This reduces the ratio of interest income to current output per capita. Note that when one considers savings ratios as opposed to absolute savings, the pure income effect does not play a role. An increase in income of an individual from higher wages due to technological progress is exactly offset by a similar increase in output per capita (i.e. the first term of equation (26) is unchanged).

Proposition 3.1.2 *An increase in the rate of growth of technological progress, g, leads to a decline in the steady state level of life cycle savings, $\bar{X}$.*

Proof.

$$\frac{\partial \bar{X}}{\partial g} = -\frac{1}{1-\theta} \frac{\beta r \bar{Z}}{(1+g)^{\frac{1}{1-\theta}+1}} + \frac{\beta r}{(1+g)^{\frac{1}{1-\theta}}} \frac{\partial \bar{Z}}{\partial g} < 0$$

since $\frac{\partial \bar{Z}}{\partial g} < 0$.

Interestingly, in the steady state not only does an increase in g lead to a decline in the life cycle savings ratio but the effect is actually magnified compared to the effect on the bequest ratio. This is because in addition to the direct negative effect discussed already, the fact that steady state life cycle savings ratio is a positive function of the steady state bequest ratio, means that the former must further decline because of the decline in the latter.

Corollary 3.0.1 *An increase in the rate of technological progress, g, leads to an immediate decline in the aggregate savings rate, $X_t + Z_t$.*

Proof. Follows from Propositions 2.3.1 and 3.1.1.

Corollary 3.0.2 *An increase in the rate of technological progress, g, leads to a decline in the steady state value of the aggregate savings rate, $\bar{X} + \bar{Z}$.*

Proof. Follows from Propositions 2.3.2 and 3.1.2.

It is straightforward to see that aggregate savings ratio, $X_t + Z_t$ is a negative function of the rate of growth of technological progress in transition.[12] Finally in terms of output per capita, the

[12]There exists a class of models which argue that higher growth causes higher savings rates. To the extent that growth depends on technological progress, the results seem contradictory. However these models rely on other assumptions such as habit formation, for example Carroll and Weil (1994). Further it should be stressed again, that the framework adopted here of a small open economy is purely a modeling vehicle. In the case of the CES production function in the appendix, technological progress can be skill biased or physical capital biased with the savings rate rising if it is the latter.

economy continues to exhibit balanced growth irrespective of the dynamics of the savings ratio.

3.2 Introducing Uncertainty

The controversy regarding unintentional versus intentional bequests in wealth accumulation has been hotly debated over the past couple of decades since Kotlikoff and Summers (1981)published their findings. On the whole the literature suggests that unintentional bequests are more important that intentional ones. Clearly, if the theory presented here is to have any empirical significance, then it should be robust to the inclusion of unintentional bequests. Incorporating both unintentional bequests and intentional bequests in a unified model can be problematic. One has to allow for heterogeneity of agents since wealth accumulation along a dynasty depends on its mortality history. Abel (1985) first introduced explicitly unintentional bequests and derived the intra-cohort distribution of bequests, wealth and consumption. He then went on to analyze the effect of actuarially fair social security on national wealth. However intentional bequests were not included and neither were human capital investments. Zhang, Zhang and Lee (1999) develop the Abel model further to incorporate public education and re-examine fiscal policy issues. They also allow the interest rate to be endogenous. Both of these papers use overlapping generations frameworks and assume that individuals *can* live for two periods but a certain fraction of them die at the end of the first period. This creates unintentional bequests. It also means that even though agents maybe similar at the initial time period eventually there will be a distribution of wealth. The modeling strategy adopted here is similar in this respect but is quite different from then on. This is necessitated by the different preference structure assumed so far in my model: allowing intentional bequests and human capital. Compared to those papers I am less interested in the actual distribution of wealth at every time period and more interested in the savings ratio in the aggregate. This does not mean that distributional issues can be ignored - after all the aggregate does depend on the way the underlying distribution evolves every time period.

Individuals as in section 3.1, live for three periods but now are face the probability that they will not survive beyond their second period in life, i.e. they live upto the end of their working lives but may die at the beginning of retirement. The probability that this may happen is p where $p \in (0, 1)$. With the introduction of uncertain lifespans, an individual, therefore now seeks to maximize her modified utility function,

$$\max_{c_t, c_{t+1}, \omega_t} U_t^t = \alpha \ln c_t^t + \beta(1-p) \ln c_{t+1}^t + \gamma \ln \omega_t \tag{28}$$

subject to the same restrictions as before (equations (20) to (22) above).

As stated before the problem is that now there are groups of individuals who have different b_t given their dynasty's mortality record. Consider one of these subgroups i. Within this subgroup, all individuals receive the same bequest, or Z_t^i -the bequest ratio. They all face the same probability of death and have the same budget constraint. And importantly they all make the *same decision regarding how much to invest in human capital, how much to set aside for a financial transfer and how much for life cycle savings.* Lets assume that they all decide on intentional bequests of an amount IB_t^i and life cycle savings, LC_t^i.[13] What happens next is that a fraction p of them will die at the end of the second period of their lives. Therefore a fraction p of their descendants receive bequests equal to $IB_t^i + LC_t^i$ while another fraction receives bequests equal to IB_t^i. This means that in the next generation a fraction p make their decisions based on $IB_t^i + LC_t^i$ and another fraction $(1-p)$ based on IB_t^i. This seems to suggest that the two variables (X_t and Z_t) might follow a well defined method of evolution in the aggregate. However to be able to do that, as a starting point, I need to make the following assumption:

Assumption 3.1 *At time $t = 0$ all individuals are alike and have the same endowment W_0.*

The above assumption allows for the economy in time period zero to be characterized by a representative agent. Further, both b_0 (bequests allocated at $t = 0$) and s_0 (life cycle savings allocated at $t = 0$) will be the same for all individuals. For the rest of the paper, Z_t is the average bequest ratio. This means that it includes both incidental bequests and intentional ones. The word "average" means that it is the weighted average of the bequest ratios of all groups in society at any given time period, t (it is easy to figure out that the number of subgroups in any period is 2^t under assumption (3.1)). In Appendix B it is shown that, indeed one can figure out well defined paths for both Z_t and X_t and the results are derived more thoroughly. It turns out that the economy is now characterized by a system of two linear difference equations,

$$X_t = a_1 + b_1 X_{t-1} + c_1 Z_{t-1} \tag{29}$$

$$Z_t = a_2 + b_2 X_{t-1} + c_2 Z_{t-1} \tag{30}$$

[13] The level of educational investment is spent while they are still alive and therefore that is not altered.

where,

$$a_1 = \frac{\beta(1-p)\bar{w}}{(1-\beta p)\bar{y}} \quad a_2 = \left(\frac{\gamma}{(1-\beta p)} - \frac{\theta(1+g)^{\frac{1}{1-\theta}}}{r} - \frac{\phi}{\bar{w}}\right)\frac{\bar{w}}{\bar{y}} \tag{31}$$

$$b_1 = \frac{rp\beta(1-p)}{(1-\beta p)(1+g)^{\frac{1}{1-\theta}}} \qquad b_2 = \frac{rp\gamma}{(1-\beta p)(1+g)^{\frac{1}{1-\theta}}} \tag{32}$$

$$c_1 = \frac{r\beta(1-p)}{(1-\beta p)(1+g)^{\frac{1}{1-\theta}}} \qquad c_2 = \frac{r\gamma}{(1-\beta p)(1+g)^{\frac{1}{1-\theta}}} \tag{33}$$

In order to ensure that the system is stable, the following assumption is made[14],

Assumption 3.2 $1 \geq \frac{r(\gamma+\beta(1-p)p)}{(1+g)^{\frac{1}{1-\theta}}(1-\beta p)}$

The solution to this dynamical system is

$$X_t = \frac{((Z_0-\bar{Z})+p(X_0-\bar{X}))(\beta(1-p))}{(\gamma+\beta(1-p)p)}\lambda_1^t + \bar{X} \tag{34}$$

$$Z_t = \frac{((Z_0-\bar{Z})+p(X_0-\bar{X}))\gamma}{(\gamma+\beta(1-p)p)}\lambda_1^t + \bar{Z} \tag{35}$$

where

$$\lambda_1 = \frac{r(\gamma+\beta(1-p)p)}{(1+g)^{\frac{1}{1-\theta}}(1-\beta p)} \leq 1 \tag{36}$$

$$\bar{X} = \frac{\bar{w}}{\bar{y}}\frac{\beta(1-p)(1-\theta)\bar{w}}{(1-\beta p)}\Lambda(g) \tag{37}$$

$$\bar{Z} = \Omega(g)\Lambda(g) \tag{38}$$

$$\text{where} \qquad \Lambda(g) = \frac{1}{1-\lambda_1(g)}, \qquad \Lambda'(g) < 0$$

$$\text{and} \qquad \Omega(g) = \frac{\bar{w}}{\bar{y}}\left(\frac{\gamma}{1-\beta p} - \frac{\phi}{\bar{w}} - \frac{\theta(1+g)^{\frac{1}{1-\theta}}}{r} + \frac{p\beta(1-p)}{(1-\beta p)}\left(\theta + \frac{\phi r}{\bar{w}(1+g)^{\frac{1}{1-\theta}}}\right)\right),$$

$$\Omega'(g) < 0$$

The dynamical system here is governed by only one eigenvalue, λ_1. The second eigenvalue, $\lambda_2 = 0$, is zero since $b_1c_2 = b_2c_1$. The phase diagram is depicted in Figure 4. Due to the fact that

[14]This assumption is a slight modification of assumption 3.1. It reduces to the left inequality of assumption 3.1 once I set $p = 0$.

one eigenvalue is zero, it means that X_t and Z_t cannot be at any arbitrary point in the phase space depicted in the figure.[15] In fact the two variables exhibit a rather well defined path of motion -both of them must always be on the convergent path AA in time $t > 0$.[16] In other words, the movement of these variables are similar when there were no unintentional bequests. It turns out that this carries over also for the effects of technological progress.

Proposition 3.2.1 *Under assumption (3.1), an increase in the rate of technological progress g, leads to an immediate decline in the X_t and Z_t.*

Proof. The proof follows from the fact that in the system of equations (29) -(30), $\frac{\partial a_2}{\partial g} < 0$, $\frac{\partial b_1}{\partial g} < 0$, $\frac{\partial b_2}{\partial g} < 0$, $\frac{\partial c_1}{\partial g} < 0$, and $\frac{\partial c_2}{\partial g} < 0$.

Proposition 3.2.2 *Under assumptions (3.1) and (3.2), an increase in the rate of technological progress, g leads to a declines in the steady state values of both the aggregate life cycle savings output ratio and the aggregate bequest output ratio.*

Quantitatively,

$$\frac{\partial \bar{X}}{\partial g} = \frac{\beta(1-p)(1-\theta)}{(1-\beta p)} \frac{\bar{w}}{\bar{y}} \Lambda'(g) < 0$$

and,

$$\frac{\partial \bar{Z}}{\partial g} = \Omega'(g)\Lambda(g) + \Omega(g)\Lambda'(g) < 0$$

since $\Omega'(g) < 0$ and $\Lambda'(g) < 0$.

Figure 5 illustrates this case. A one time shock to technological progress moves both lines, $\Delta X_t = 0$ and $\Delta Z_t = 0$. In the case of the former, the intercept falls while the slope goes up. In the case of the latter, both the slope and the intercept decline. Corresponding to the increase in g from g_0 to g_1, the steady state moves from E_0 to E_1. A permanent increase in the rate of technological progress moves the economy to a new convergent path. The transitional dynamics are somewhat more complex and a simple visual inference can be misleading. Figure 6 (which is similar to figure 5) shows that the stable path makes a parallel shift from AA to BB. From any point on AA, a rise

[15] Except at $t = 0$.

[16] If one were to transform this coupled dynamical system in (X_t, Z_t) to an uncoupled one in $(Y1_t, Y2_t)$, this would mean that movement to the steady state would take place along the horizontal axis only. If at time $t = 0$, the economy is placed somewhere outside of the horizontal axis, then the economy would move there in the next period and all future dynamics take place along this axis. For a simple explanation of this transformation the reader is referred to Galor (1996).

in g implies that the bequest ratio should fall in the short run. However, simply by looking at the phase space, it would seem that another scenario (not shown in the figure) is possible where the steady state values for both ratios decline but the new stable path is above (and to the left of) AA (i.e. E_1 lies above the old convergent path but remains south-west of E_0). This would imply that an increase in g in the short run actually *increases* the bequest ratio. However, this is not only counter intuitive but also mathematically not possible as shown in proposition 3.2.1. Further, in appendix C, the eigenvector for λ_1 is derived and one can show that such dynamics can be ruled out.

While the mathematics may seem complex, intuitively it is not difficult to see why all the results carry over from the certainty case. In the certainty case it was shown that in any time period, t, the life cycle savings rate and the bequest rate were both linear functions of the bequest ratio in time period, $t-1$. With the introduction of uncertainty, in the aggregate, all that really happens is that for a fraction of individuals in society, life cycle savings end up becoming bequests as well. Therefore there is only a shift in the composition of savings. Beyond that there is really no substantial complication in the story. When examining the effects of g on the steady state in section 3.1, I remarked that the effects on the life cycle savings was likely to be greater than the on bequests. With the introduction of unintentional bequests, the effects of a change in g is now obviously less adverse for life cycle savings since a fraction of it always ends up being becoming bequests.

3.2.1 A Note on Changes in Life Expectancy

An important concern in the study of domestic capital accumulation is the issue of increasing life expectancy and the consequent aging of society. As aging increases, a few offsetting effects may occur. For example, life cycles savings will rise while accidental bequests decline. Capital accumulation could go in either direction. The short run effect would be higher life cycle savings which would imply greater accumulation. However the higher life cycle savings are completely consumed in the next period. At the same time accidental bequests are lower. Accidental bequests however do not translate completely into retirement consumption. They are allocated to life cycle savings and intentional bequests. Therefore the reduction in accidental bequests implies that the medium run capital accumulation rate will be lower. Additionally, in terms of fiscal policy concerns, increasing expenditures on social security means lower capital accumulation rates. The framework presented in this section lends itself to studying the effect of changes in mortality rates on domestic capital accumulation in the presence of technological progress. A decline in p in the model is equivalent to increasing life expectancy.

In contrast to the effects of changes in g, the effects of a mortality decline is more involved. It turns out that a couple of different possibilities emerge for both the steady state and the transition path. Depending upon parameter values, in the long run the economy may actually face a lower life cycle savings ratio despite a mortality decline.[17] If the initial mortality rate is already less than half ($p < 1/2$), then further declines imply that the steady state bequest ratio will unambiguously decline while the life cycle savings ratio may either decline or increase depending.[18] Therefore, there arises the possibility that the aggregate savings ratio might fall. A decline in mortality implies that individuals allocate more of their resources to life cycle savings and relatively less to intergenerational transfers and first period consumption. The result that steady state aggregate bequest ratio declines should not then come as a surprise: the fact that the share of education expenditures is fixed by the rate of technological progress and there is a reallocation of resources towards life cycle savings means that intentional bequest ratio will decline. Further, the accidental bequest ratio also declines given the increased life expectancy. What comes more as a surprise is the possibility of the decline in the life cycle savings rate. A conjecture is that as higher amounts are allocated to life cycle savings and intentional bequests decline, the amount transferred to the next generation declines (the ratio that is) and this decline in the ratio leads to the possibility of a lower life cycle savings ratio by the next generation since all else being equal the amount that they inherit falls relatively. In the long run thus there can remain the possibility of life cycle savings ratio also declining.[19] This scenario is depicted in figure 7a. As mortality declines from p_0 to p_1, the steady state ratios move from E_0 to a lower equilibrium level of E_1. The dynamics are almost similar to the case of an increase in technological progress except that the nature of the shift of the stable path is different (see appendix C for further details).

[17] The effects of mortality decline in savings analyzed in this section do not rely crucially on the assumption of skill augmenting technological progress and one can assume a constant productivity level. However the parameter assumptions,in particular, on the interest rate, to ensure that the system is stable, will need to be made even stronger.

[18] As in footnote 17, again, this is a stronger sufficient condition. The bequest ratio can decline with a weaker necessary and sufficient condition.

[19] Zhang, Zhang and Lee (1999) also rely on this argument to explain lower rates of capital accumulation with declining mortality.

The traditional result that life cycle savings rate increases also remains as a possibility, accompanied however, with a fall in the bequest ratio. In this case what happens to the aggregate ratio depends upon various parameter values and is more of an empirical issue. Figure 7b shows the dynamics when life cycle savings ratio increases in the steady state. The movement of the stable path from AA to BB is similar to that in Figure 7a. In all of the above cases however the effect of the shock to life cycle savings ratio remains unclear in the transition path. For most values of the life cycle savings ratio, a shock causes a change in the same direction the ratio was already moving in (as opposed to the bequest ratio, which in the short run unambiguously declines).

4 Conclusion

The objective of this paper was to motivate the idea that in a world where skills are becoming predominantly more important, one needs to think about how the composition of capital accumulation shifts. Given that it is generally agreed many developed countries in the world have been undergoing a period of rapid skill-biased technological progress, it is definitely worthwhile to think about the relative importance of human capital accumulation vis-a-vis physical capital accumulation. The fact that the savings ratio as measured by national accounts has been declining during the same period, makes the question all the more interesting. Though skill biased technological progress need not be the only explanation for declining savings, the fact that it might account for atleast 25% of the decline, makes the issue important enough to ponder over. To motivate the argument, I presented a simple model where altruistic parents derive utility from the amount they spend on their children. Given the nature of the problem I am studying this seems to be the most obvious formulation. Prima facie, the theory that people will reallocate between two types of transfers given the changing payoffs is appealing. However one needs to investigate how important this reallocation is in explaining declining savings. Clearly national income accounts are inadequate since they do not provide ways to measure indirect expenditures in human capital. For example, as mentioned earlier there are a number of complementary expenditures that enhance human capital and further there are "time costs" that are also not included. One needs to devise ways to capture these numbers. Though the model presented here is based on household optimization, to measure human capital investment in the economy one should add investment by firms in " on the job training". Further one needs to look at microeconomic data to better capture the fact that a lot of this reallocation is likely to take place in households who have children going to school. Kendrick (1976) and Jorgenson and Fraumeni (1990) tackled the challenge of estimating a more broad measure of human capital investment and found that it claimed a large share of national output but clearly one needs to update and improve on those lines of research.

Regarding the model itself, there are other interesting lines along which it could be extended. For example, I have not included public education which forms a major chunk of educational expenditures in the U.S. An increase in public education expenditures implies that the incentive to reallocate towards private educational expenditures is less. However, as a matter of empirical observation the share of educational expenditures in public spending has not risen during the past two decades. There are issues about the quality of public education too. If parents feel that quality of public education is not improving which puts students at public schools at a disadvantage, there emerges an incentive to complement public schooling with private expenditures. It also suggests that there might be some not so obvious measures of education expenditures in public spending and revenue. For example, when families move to better neighborhoods where the quality of public schooling is better, they pay higher home prices, part of which should conceivably enter as an educational expenditure. Recent evidence from Florida suggests that the introduction of school quality rating into communities led to major housing price effects.[20] Also I have not considered the heterogeneity of agents. This can affect the results to the degree that borrowing constraints are an important consideration in education financing. However these could be incorporated and the results are unlikely to change substantially. The fact that enrollments have risen in college substantially indicates that there is a desire to acquire skills and increase spending on education. The model has also abstracted from saving and borrowing for education. One can argue that in a model with more than two or three periods, parents would allocate more towards educational savings following a positive shock to returns to education. This argument though theoretically plausible has two problems. First, such a shock is more likely to lead parents to start spending more on education right away causing an unplanned decrease in savings. Secondly, as an empirical matter people have saved far less for college than they typically have to borrow.[21] Clearly all these

[20]See Figlio and Lucas (2000).

[21]I am grateful to Caroline Hoxby for drawing my attention to this fact and related data. A lot of interesting evidence on the drastic increase in borrowing for higher education can be found in the web site for the National Post-secondary Student Aid Study (http://nces.ed.gov/pubsearch/getpubcats.asp?sid=013). For example, The percentage of dependent undergraduates with family incomes of $50,000 or more who ever borrowed from federal loan programs

issues discussed above present avenues for further research.

Finally, the model provides some insights into the empirical debate over bequests and savings. Though the savings rate in the economy has been declining for the past two decades, there has not been any effort to estimate whether this has come largely at the cost of life cycle savings or intergenerational transfers. Also it suggests that in empirical studies, simply looking at intentional bequests is not the best way to analyze intergenerational altruism. It is perfectly reasonable to observe insignificant bequests in households with children without that implying the lack of intergenerational altruism.

Appendix

A. A static closed economy with CES production function

In this section, the robustness of the result derived in section 2 is examined in the context of a closed economy exhibiting a constant elasticity of substitution production function. However since the CES function does make the analysis a little more complicated, the analysis is restricted to static version of the model. Also, to get a closed form solution the human capital production function is assumed to be linear in educational expenditures, i.e. $h_t = e_{t-1}$. Output is now

$$Y_{t+1} = [(A_K K_{t+1})^\rho + (A_H H_{t+1})^\rho]^{\frac{1}{\rho}}$$

with $\rho < 1$, which is a standard assumption and implies that elasticity of substitution between capital and labor, $\frac{\rho}{1-\rho}$ is greater than one.[22] The production function suggests that wages and rents are respectively equal to

$$w_{t+1} = \frac{\partial Y_{t+1}}{\partial L_{t+1}} = [(A_K K_{t+1})^\rho + (A_H H_{t+1})^\rho]^{\frac{1}{\rho}-1} A_H^\rho h_{t+1}^\rho L^{\rho-1} \tag{39}$$

$$r_{t+1} = \frac{\partial Y_{t+1}}{\partial K_{t+1}} = [(A_K K_{t+1})^\rho + (A_H H_{t+1})^\rho]^{\frac{1}{\rho}-1} A_K^\rho K_{t+1}^{\rho-1} \tag{40}$$

As before, population is normalized to 1, i.e. $L = 1$.

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increased between 1992-93 and 1995-96 at both public and private, not-for-profit 4-year institutions. Further, in 1992-93, 21 percent of dependent undergraduates at public 4-year institutions from families making between \$ 50,000 and \$ 59,999 had ever borrowed. By 1995-96, 44 percent of undergraduates from families in that income range had borrowed.

[22] Duffy and Papageorgiou (2000) recently showed that this assumption is valid for developed economies.

The individual's utility maximization problem as in section 1, is now

$$\max_{c_t,\omega_t} U_t^t = \alpha \ln c_t^t + \gamma \ln \omega_t \tag{41}$$

subject to,

$$w_t + r_t b_{t-1} = c_t + \omega_t \tag{42}$$

As before Utility maximization implies,

$$c_t = \alpha(w_t + r_t b_{t-1})\omega_t = \gamma(w_t + r_t b_{t-1}) \tag{43}$$

Since this is a closed economy,

$$Y_t = w_t + r_t K_t$$

Also now

$$K_{t+1} = b_t^* = \omega_t - e_t^*$$

where b_t^* and e_t^* are the optimal amounts allocated to bequests and educational expenditures respectively. That is capital per capita in the next period is the bequest per capita allocated in the earlier period. Also income in any period is now determined by wages and an interest rate both of which are endogenous.

Therefore $e_t^* = \arg\max \left[[(A_K(\omega_t - e_t)^\rho + (A_H H(e_t)_{t+1})^\rho]^{\frac{1}{\rho}} \right]$

The first order condition for this is:

$$\frac{\partial Y_{t+1}}{\partial e_t} = 0$$

$$\Rightarrow \frac{\partial w_{t+1}}{\partial e_t} + \frac{\partial r_{t+1}}{\partial e_t} K_{t+1} - r_{t+1} = 0$$

Taking the derivatives of equations (39) and (40) above and then rearranging gives the following expression:

$$[(A_K K_{t+1})^\rho + (A_H H_{t+1})^\rho]^{\frac{1}{\rho}-2} (1-\rho) \left[A_H^\rho h_{t+1}^{\rho-1} \frac{\partial h_{t+1}}{\partial e_t} \right] \left[A_H^\rho h_{t+1}^\rho + A_K^\rho K_{t+1}^{\rho-1} K_{t+1} \right]$$

$$+ [(A_K K_{t+1})^\rho + (A_H H_{t+1})^\rho]^{\frac{1}{\rho}-1} \rho \left[A_H^\rho h_{t+1}^{\rho-1} \frac{\partial h_{t+1}}{\partial e_t} \right]$$

$$= [(A_K K_{t+1})^\rho + (A_H H_{t+1})^\rho]^{\frac{1}{\rho}-1} \, A_K^\rho K_{t+1}^{\rho-1}$$

This simplifies to,

$$H^\rho h_{t+1}^{\rho-1} \frac{\partial h_{t+1}}{\partial e_t} = A_K^\rho K_{t+1}^{\rho-1}$$

Substituting $h_{t+1}(e_t) = e_t$ and $K_{t+1} = \omega_t - e_t$,

$$A_H^\rho e_t^{\rho-1} = A_K^\rho (\omega_t - e_t)_{t+1}^{\rho-1}$$

$$\Rightarrow \left(\frac{e_t}{\omega_t - e_t} \right) = \left(\frac{A_H}{A_K} \right)^{\rho/(1-\rho)} \tag{44}$$

This means that the amount of expenditures allocated to education relative to bequests increases as the technology complementing human capital increases relative to the technology complementing physical capital. Since $\omega_t = \gamma y_t$, the share of educational expenditures must rise while savings must fall.

B. Derivation of the Dynamical System in Section 4

The fact that there exists uncertainty in the model in this section implies that even if I start from an economy of perfectly homogenous agents in period zero, in period $t > 0$ there will be agents will be heterogenous in their wealth due to differences in their dynasty's mortality history. To be precise, in period $t > 0$, agents can be divided into 2^t types. However the important thing to recognize is that I are interested in the economy wide savings ratio and hence I aggregate over households. It turns out that this can be done easily if I assume that individuals are homogenous in period zero. Here I present a stylized version of the method. The variables X_t and Z_t used here are the same as the ones in the text, however I suppress the constants since they will not play any role in the derivation. Since individuals are all alike in period 0 they all make the same life cycle savings,education and bequest decision. However a fraction $1 - p$ of them die before retirement and p of them die at the end of retirement. This means that the wealth distribution for the next generation is dualistic. For the , $1 - p$ fraction of the population whose ancestors lived the entire two periods, the life cycle savings ratio and the bequest ratio are the following:

$$X_1^{1-p} = \phi Z_0$$

$$Z_1^{1-p} = \varphi Z_0$$

For the remaining p fraction of the population, the corresponding decisions are,

$$X_1^p = \phi(X_0 + Z_0)$$

$$Z_1^p = \varphi(X_0 + Z_0)$$

Therefore the average life cycles savings ratio and bequest ratio in the economy for period 1 is,

$$X_1 = pX_1^p + (1-p)X_1^{1-p}$$

$$X_1 = p(\phi(X_0 + Z_0)) + (1-p)\phi Z_0$$

$$\Rightarrow X_1 = p\phi X_0 + \phi Z_0$$

Similarly, the average bequest ratio in the economy is,

$$Z_1 = pZ_1^p + (1-p)Z_1^{1-p}$$

$$\Rightarrow Z_1 = p\varphi X_0 + \varphi Z_0$$

In period two the wealth distribution will be of the following nature Individuals either inherit intentional bequests Z_1^p or Z_1^{1-p}. Of the fraction p that inherit the former, a fraction p^2 will actually inherit $Z_1^p + X_1^p$ and a fraction $p(1-p)$ will inherit only Z_1^p. Therefore for these two groups I have,

$$Z_2^{p,p} = \varphi(Z_1^p + X_1^p)$$

$$Z_2^{p,1-p} = \varphi Z_1^p$$

where the superscripts denote their ancestral 'mortality outcomes'. Of the remaining fraction $1-p$, a fraction $(1-p)p$ will inherit $Z_1^{1-p} + X_1^{1-p}$ and the remaining $(1-p)(1-p)$ fraction inherit Z_1^{1-p}. Therefore,

$$Z_2^{1-p,p} = \varphi(Z_1^{1-p} + X_1^{1-p})$$

$$Z_2^{1-p,1-p} = \varphi Z_1^{1-p}$$

Given these four expressions I can now calculate the average bequest ratio in $t = 2$,

$$Z_2 = p^2\varphi(Z_1^p + X_1^p) + p(1-p)\varphi Z_1^p + (1-p)p\varphi(Z_1^{1-p} + X_1^{1-p}) + (1-p)^2\varphi Z_1^{1-p)}$$

$$\Rightarrow Z_2 = p^2 \varphi X_1^p + (1-p) p \varphi X_1^{1-p} + p \varphi Z_1^p + (1-p) \varphi Z_1^{1-p}$$

which simplifies to

$$Z_2 = p\varphi X_1 + \varphi Z_1$$

given the expressions for X_1 and Z_1 above. One can similarly show that

$$X_2 = p\phi X_1 + \phi Z_1$$

I have shown that for the first two periods the structure of the system is self repeating despite the increasing heterogeneity of the economy. With a little bit more effort one can show that this is true for all t. One can now apply this logic to the model in section 2. The terms corresponding to $b_1, b_2, c_1 and c_2$ in the main body of the text here are $p\phi, p\varphi, \phi, \varphi$ respectively.

C. Analysis of the Eigenvector associated with λ_1

In order to derive the eigenvector, I first transform the non-homogenous dynamical system described in equations (20) and (21) into a homogenous one and derive the eigenvector from that of the latter[23]. The system in (20) and (21) can be rewritten in matrix form as,

$$\begin{bmatrix} X_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 & c_1 \\ b_1 & c_2 \end{bmatrix} \begin{bmatrix} X_t \\ Z_t \end{bmatrix}$$

which can be rewritten as,

$$V_{t+1} = A + BV_t$$

Every non homogeneous dynamical system can be rewritten as a homogenous one. In effect the above system can be transformed into a homogenous system,

$$W_{t+1} = BW_t$$

where,

$$W_t = V_t - \begin{bmatrix} \overline{X} \\ \overline{Z} \end{bmatrix}$$

This system has the same eigenvalue as V_t. Therefore the stable eigenvector associated with λ_1 is the solution to,

$$[B - \lambda_1 I]\, W_t = 0$$

By making all the substitutions I get,

$$Z_t = \frac{\gamma}{\beta(1-p)} X_t - \frac{\gamma}{\beta(1-p)} \bar{X} + \bar{Z}$$

$$\Rightarrow Z_t = \frac{\gamma}{\beta(1-p)} X_t - \frac{\bar{w}}{\bar{y}} \frac{\theta(1+g)^{\frac{1}{1-\theta}}}{1+r}$$

The above is the equation of the stable path in the dynamical system. One can see that a decrease in p lowers the slope while an increase in g lowers the intercept.

[23]See Galor (1996)

Figure 1
Share of 18-29 Cohort Enrolled in College

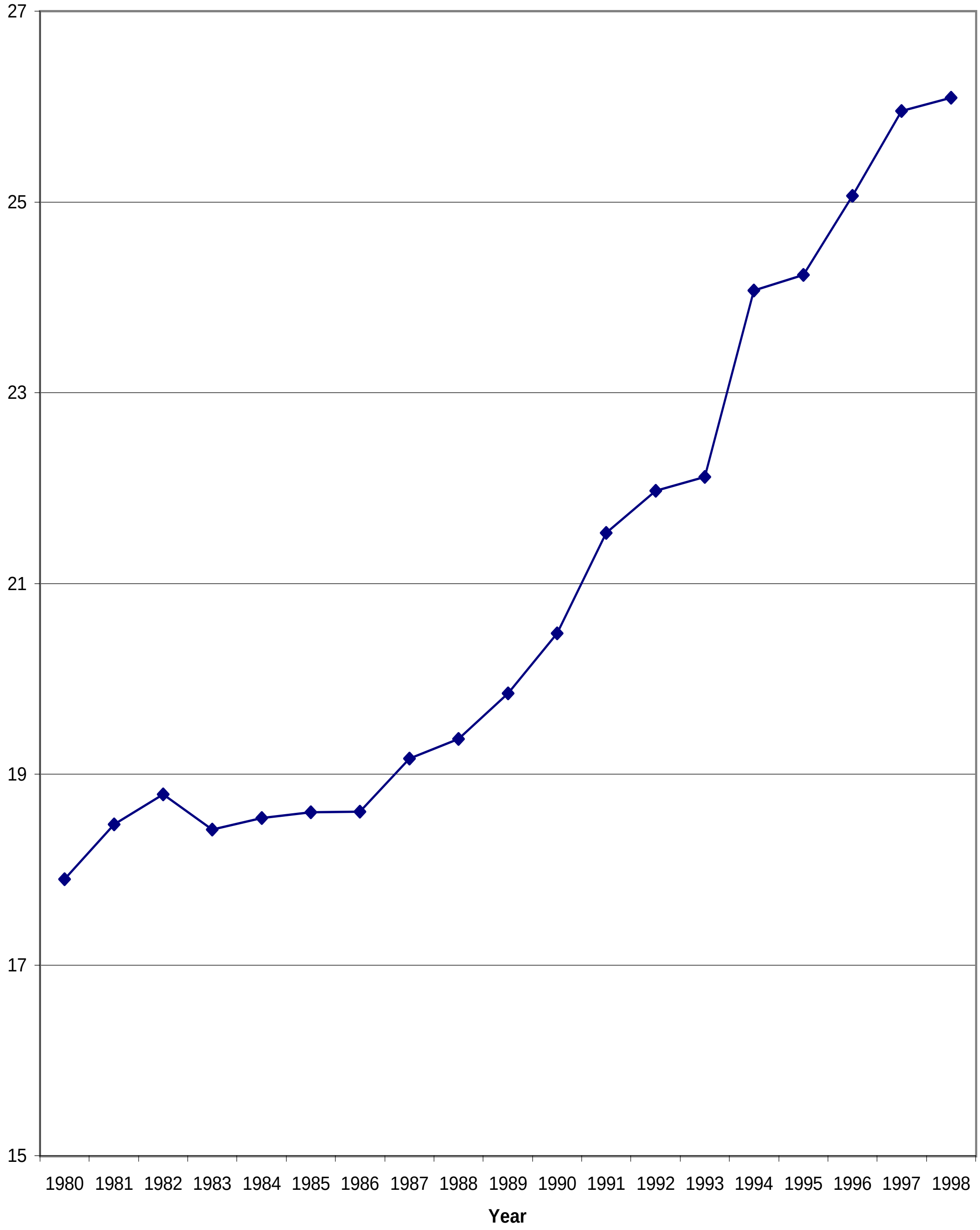


Figure 2: Returns to Education and Personal Savings through the 20th Century

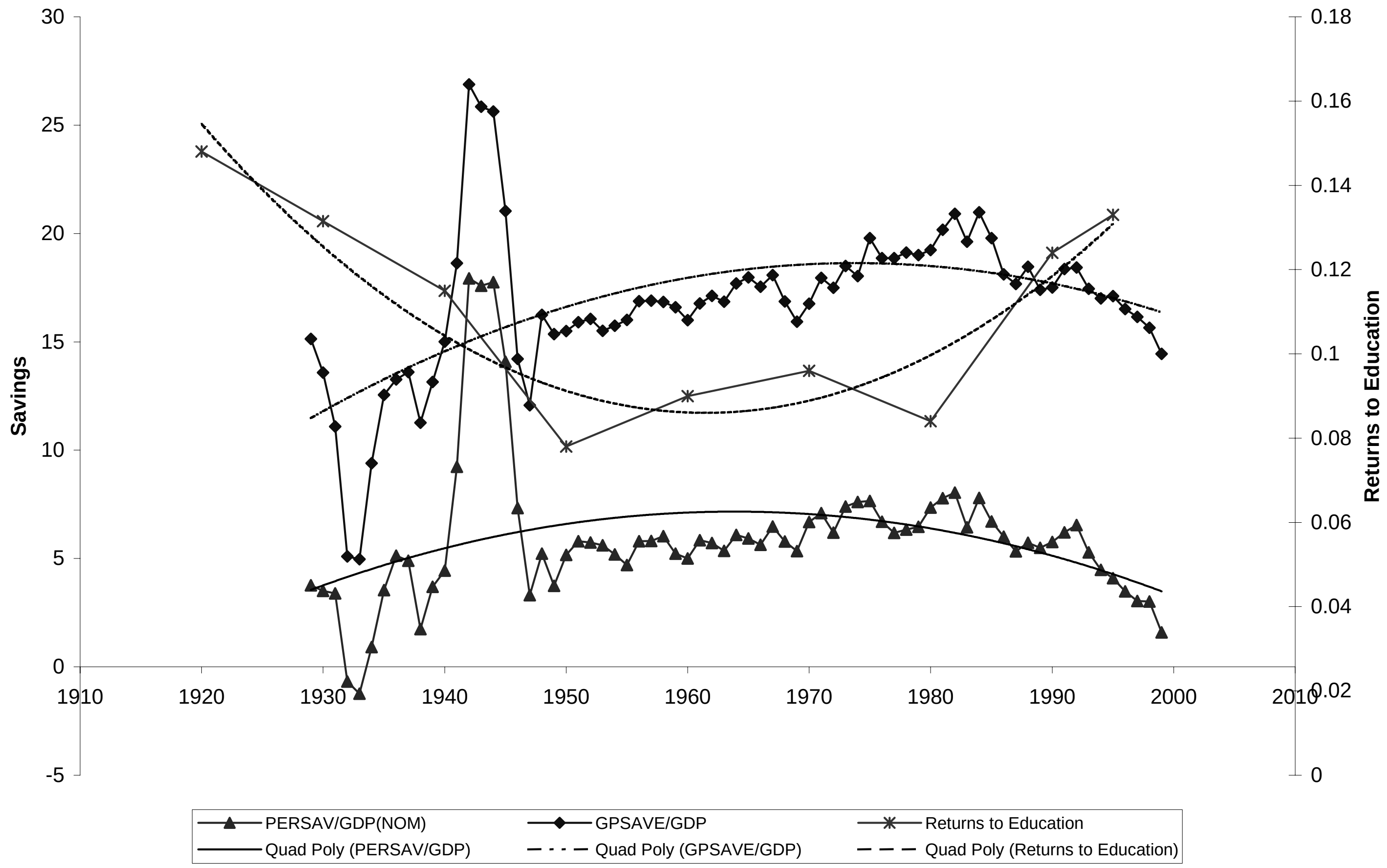


Figure 3
Effect of an increase of g on Z_t
($g_2 > g_1$)

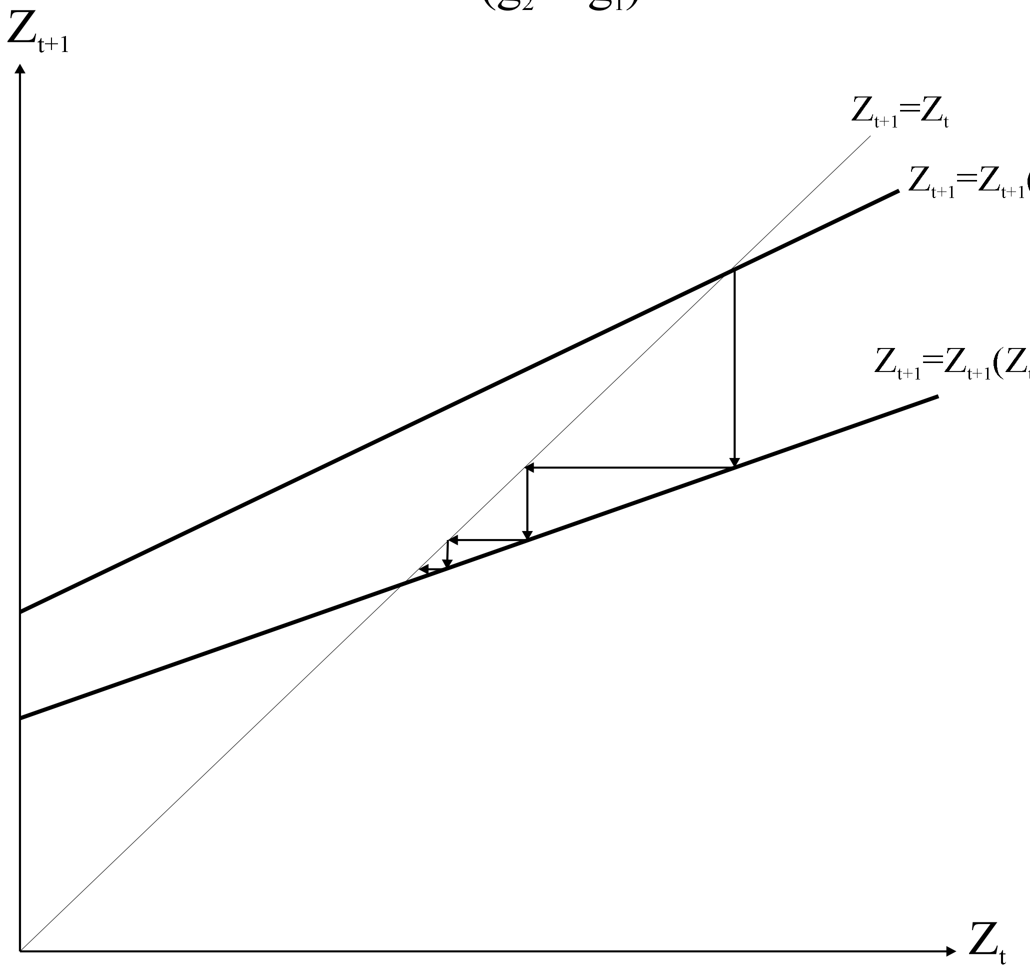


Figure 4
Dynamics with Uncertainty

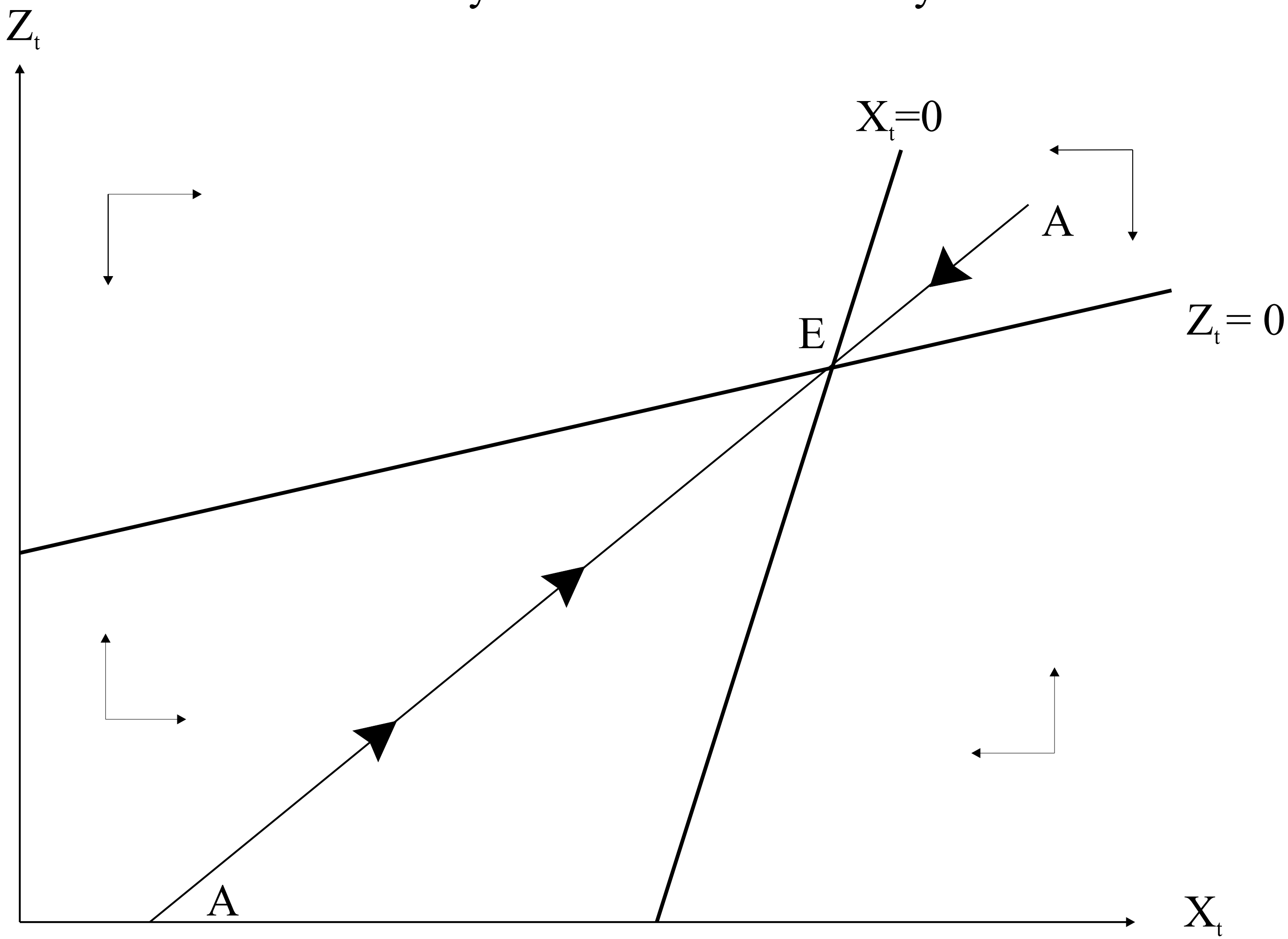


Figure 5

Effects on X_t=0 and Z_t= 0 after an increase in g

(g_1>g_0)

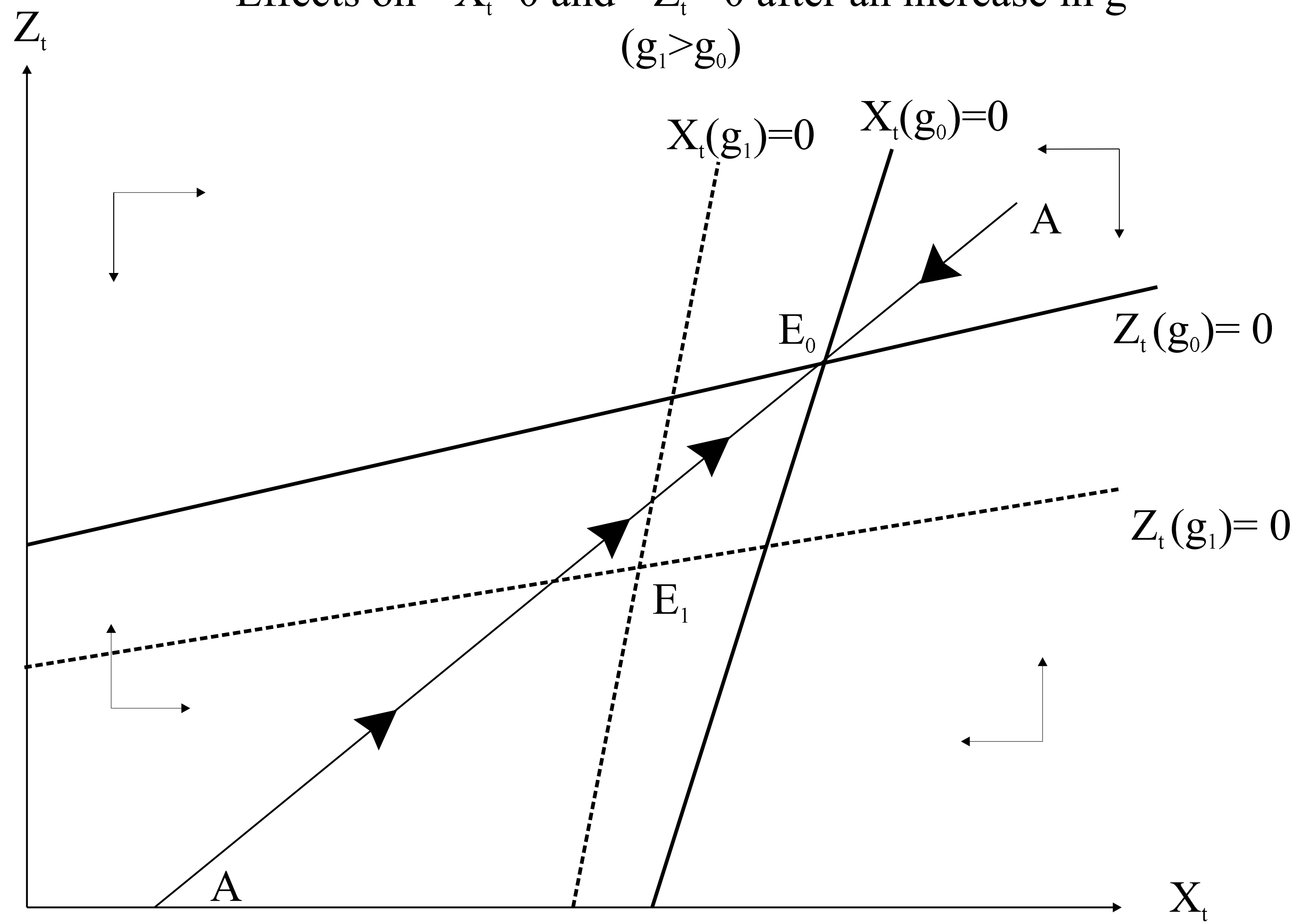


Figure 6
Dynamics after an increase in g ($g_1 > g_0$)

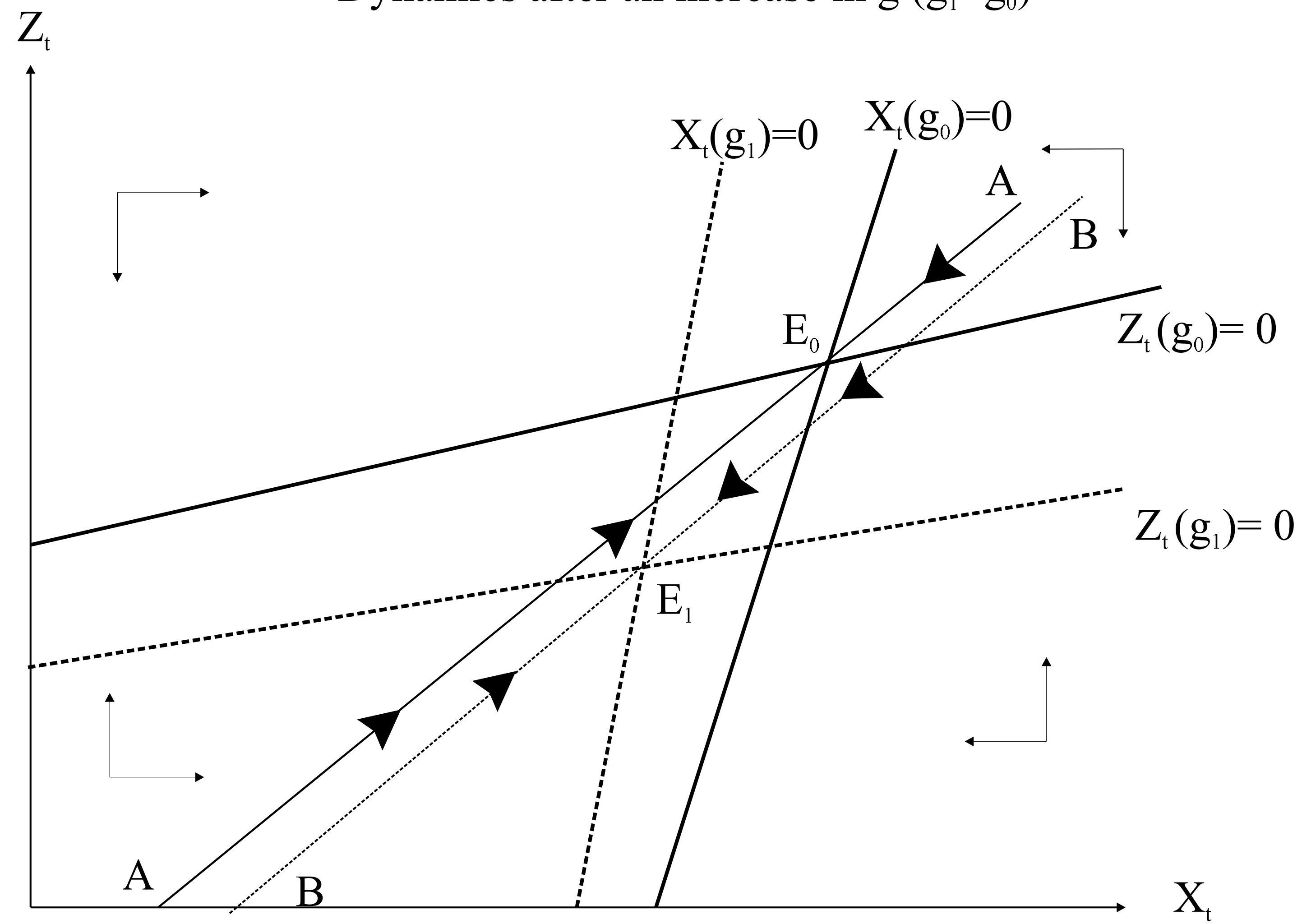


Figure 7a

Dynamics After a Decrease in Mortality ($p_1 < p_0 < 0.5$)

Case 1: Both X_t and Z_t fall

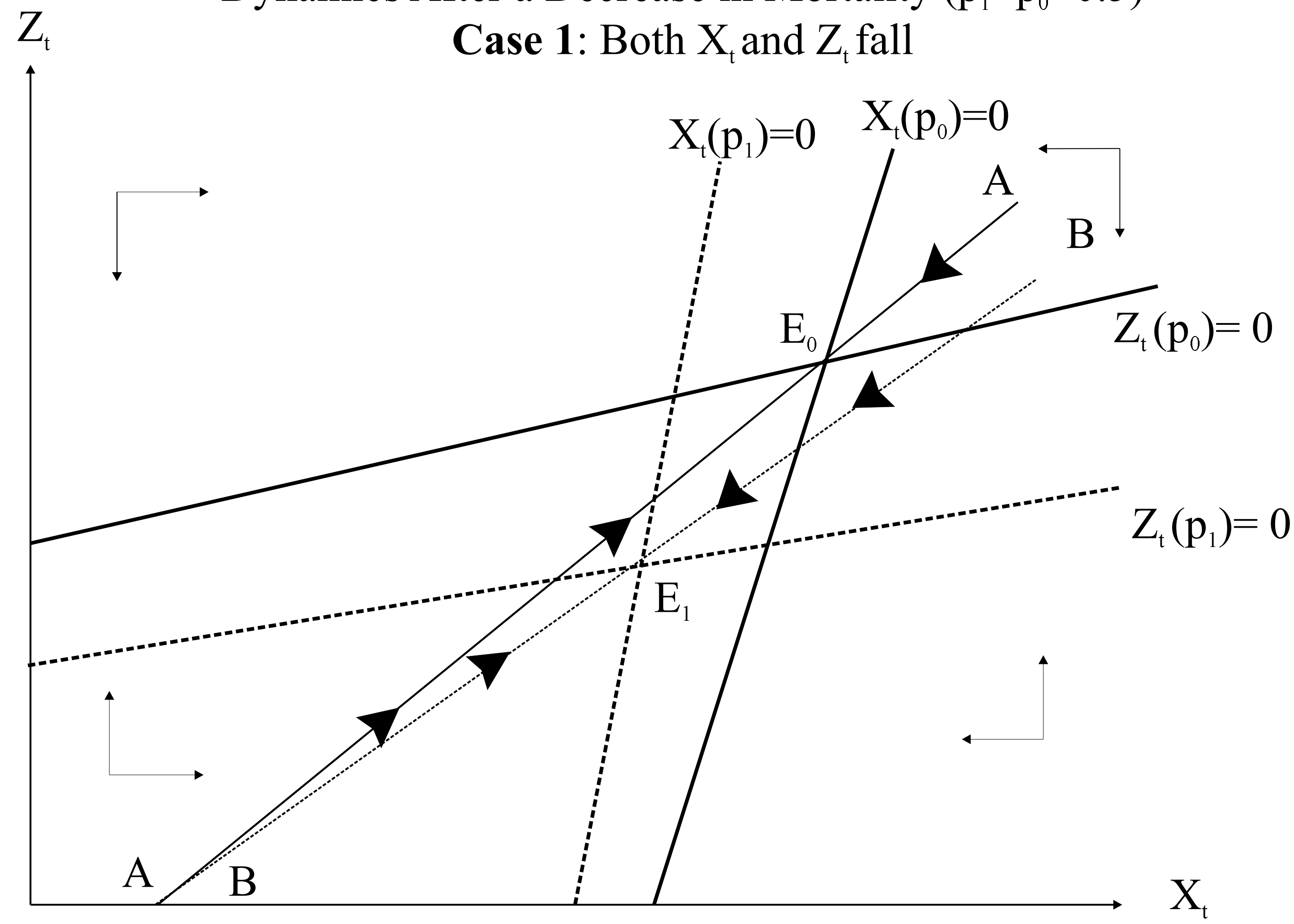
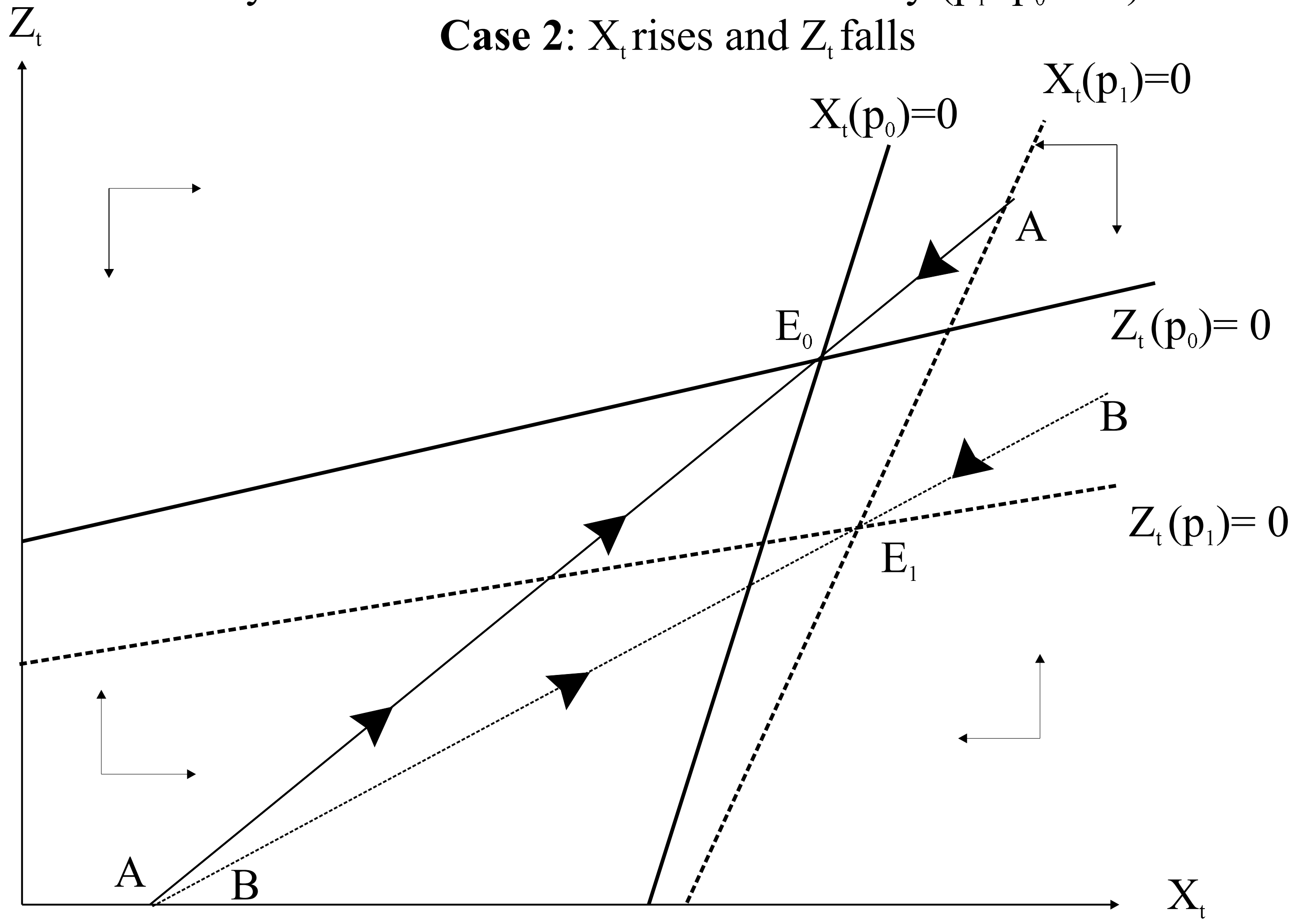


Figure 7b

Dynamics After a Decrease in Mortality ($p_1<p_0<0.5$)

Case 2: X_t rises and Z_t falls



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