

**The Balance Curve:  
A Foundation of Macro-Dynamics\***

Masahiro Konno

Private Researcher

Co-op Kabasawa 301

4-7-12 Hongo

Bunkyo-ku Tokyo 113-0033

Japan

E-mail: [gakei-k@mud.biglobe.ne.jp](mailto:gakei-k@mud.biglobe.ne.jp)

February 2002

---

\* I am grateful to Yoshikazu Iikura for comments and encouragement. Of course, all errors are mine.

### **Abstract**

A simple framework is constructed and extended to consider the theory of investment. It shows that there exists in industrialized economies a certain structure that determines the balance curve, which relates the rates of inflation and capital accumulation if demand and productive capacity are to be balanced as a whole at the initial stage of the next period. The balance curve, together with the *modified short-period* Phillips curve, governs the behavior of industrialized economies. In other words, they make it possible to recognize coherently the mechanism of chief economic phenomena such as growth, business cycles, creeping inflation and stagflation, and accordingly to prescribe for “diseases.”

**Key Words:** Balance Curve, Capital Accumulation, Inflation, Modified Short-Period Phillips Curve

**JEL Classification:** E10

## . INTRODUCTION

Suppose that fixed capital investments in a given period are to become productive at the beginning of the next. In this case, it seems likely that there will be a relationship between the rates of inflation and (capital) accumulation if demand and productive capacity are to be balanced as a whole at the initial stage of the next period. We will refer to this relationship as the *balance curve*.

Although the shape of the balance curve is not yet revealed, suppose for the moment that it is given by Fig. 1.<sup>1</sup> Since generally the increase of the rate of inflation decreases quantity demanded, the region above the curve indicates a state of *overcapacity*; the region below it a state of *undercapacity*.

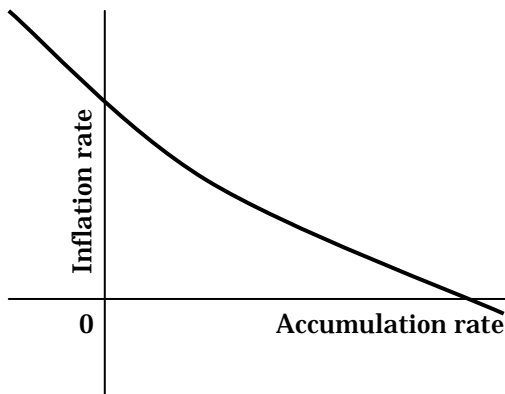


Fig. 1. Balance curve

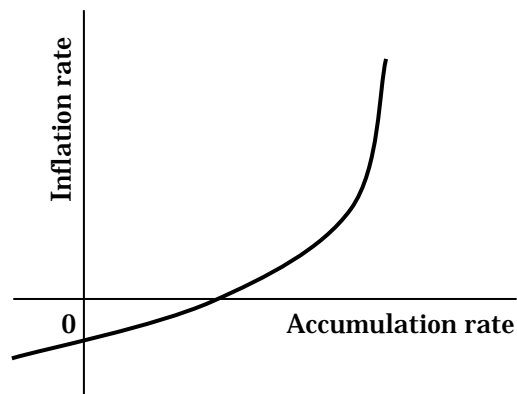


Fig. 2. Modified short-period Phillips curve

Now we will modify the short-run Phillips curve, supposing that the rate of accumulation is parallel with that of real output growth. Fig.2 shows the *modified short-period* Phillips curve.<sup>2</sup> Superimposing it onto Fig. 1, we obtain Fig. 3. The intersection of the two curves gives the *balance point*. Note that the actual rates of inflation and accumulation are always realized on the modified short-period Phillips curve. In the following, we will assume that “short-period expectations are always fulfilled.”<sup>3</sup>

When, as in Fig. 3, the balance curve intersects with the modified short-period Phillips curve from above, the balance point gives a *stable* “moving equilibrium”: The

<sup>1</sup> This curve is different from the aggregate demand curve or its variations, an example of which is the “*DG* line”: an identity based on the definition that nominal GNP growth is equal to the inflation rate plus real GNP growth. See Gordon (1993).

<sup>2</sup> Hicks (1977) has already introduced a similar curve, which he called the “growth-supply curve” (p. 90).

<sup>3</sup> See the footnote of Assumption 8 in Section .

dynamic version of the principle of effective demand is realized. Indeed, if the actual rate of accumulation exceeds the rate given by the balance point, a state of overcapacity appears, which will exert depressing influence; if the former falls below the latter, a state of undercapacity appears, which will stimulate expansion. Hence the actual rates of inflation and accumulation tend to follow the movement of the balance point. For instance, if the modified short-period Phillips curve is stable, an upward shift of the balance curve will be accompanied by an increase in both rates. If, on the other hand, the balance curve is stable, an upward shift of the modified short-period Phillips curve will cause an increase in the rate of inflation and a decrease in the rate of accumulation—stagflation.

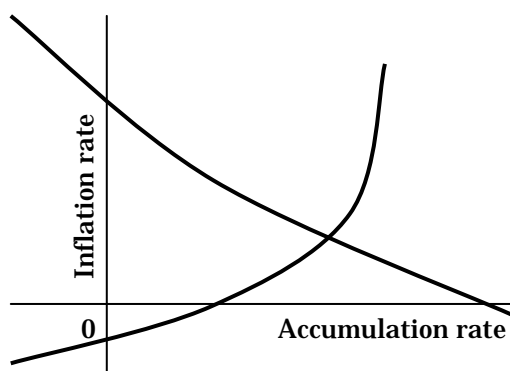


Fig. 3. Balance point

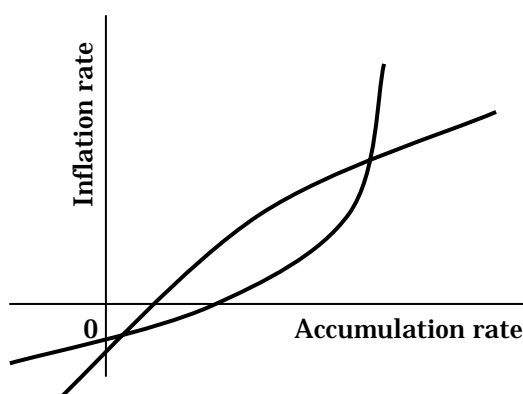


Fig. 4. Two balance points:  
“Knife-edge” or “true inflation”?

When the balance curve is upward-sloping and intersects the modified short-period Phillips curve from below, the balance point gives an *unstable* “moving equilibrium”: Harrod’s “knife-edge” appears. But, as shown by Fig. 4, what is realized will be a stable “moving equilibrium” or a “state of true inflation,” unless the central bank puts on a credit squeeze tight enough to prevent investment.

The virtue of the balance curve as an instrument of economic analysis is easily understood from the above. But questions arise: “Does the balance curve really exist?” “If it does, what shape is it?” and “What factors determine the shape?” To answer these, we will construct a simple framework based on some robust facts of industrialized economies and later extend it for more realistic argument.

Section defines some preliminary concepts. Section constructs a framework and derives the balance curve. Section shows some implications. Section extends the framework and shows its application. Section examines oversimplification. Section concludes.

## . PRELIMINARIES

To begin with, some concepts must be defined.

*Physical definition of individual capital:*<sup>4</sup> In the manufacturing industry, the scale of a factory can be measured with its productive capacity. This implies that the quantity of capital stock can be defined individually by the productive capacity. For instance, in the automobile industry, a unit of capital can be defined as the productive capacity of a factory that produces ten thousand cars per year under normal rate of operation.

Thus, let  $P_t^i$  be the productive capacity of good  $i$  (in period  $t$ ). Then capital stock

$K_t^i$  is proportional to  $P_t^i$ :

$$K_t^i = k^i P_t^i.$$

The value of  $k^i$  depends on the choice of the unit of capital. But the productive capacity may be changed with a change of labor productivity, while sustaining the already existing equipment. Therefore once a unit of capital is defined, the above relation will not always stand. What follows is:

$$K_t^i = k_t^i P_t^i. \quad (1)$$

We will refer to  $k_t^i$  as the *technical capital coefficient* (of capital  $K_t^i$ ).

*Individual demand gap:* Demand gap  $E_t^i$  for good  $i$  can be defined by

$$E_t^i = D_t^i - P_t^i, \quad (2)$$

where  $D_t^i$  denotes the quantity demanded for good  $i$ . It follows from (1) and (2) that

$$E_t^i = D_t^i - K_t^i / k_t^i. \quad (3)$$

*Aggregate demand gap:* Putting  $E_t^i = 0$  in (3) yields  $K_t^i = k_t^i D_t^i$ , which gives the

---

<sup>4</sup> Keynes (1936) believed “the definition of physical unit of capital . . . to be both insoluble and unnecessary” (p. 138). But, as shown below, it is in some measure soluble, and necessary.

quantity of capital  $K_t^i$  that balances demand and productive capacity for given  $k_t^i$  and  $D_t^i$ . Hence, if there are  $n$  demand gaps,  $\sum_{i=1}^n p_t^{K_i} k_t^i D_t^i$  gives the total value of capital stock that balances demand and productive capacity as a whole, where  $p_t^{K_i}$  denotes the replacement cost (supply price) of a unit of capital  $K_t^i$ . It is natural, therefore, to define aggregate demand gap  $AE_t$  by

$$AE_t = \sum_{i=1}^n u_t^i p_t^i D_t^i - \sum_{i=1}^n p_t^{K_i} K_t^i (= \sum_{i=1}^n u_t^i p_t^i E_t^i), \quad (4)$$

where  $p_t^i$  denotes the price of good  $i$  and  $u_t^i$  is given by

$$u_t^i = k_t^i p_t^{K_i} / p_t^i. \quad (5)$$

We will refer to  $u_t^i$  as the *real capital coefficient* (of capital  $K_t^i$ ). The total replacement cost  $\sum_{i=1}^n p_t^{K_i} K_t^i$  is independent of the choice of the unit of each capital asset. So is  $AE_t$ .<sup>5</sup> Note that  $AE_t = 0$  does not necessarily imply that  $E_t^i = 0$  for all  $i = 1, 2, \dots, n$ .

It follows from (5) and (1) that  $u_t^i = p_t^{K_i} K_t^i / p_t^i P_t^i$ , which denotes the ratio of the replacement cost of the capital equipment to the productive capacity measured in terms of value, so that it is independent of the choice of the unit of capital (but depends on length of period). Factories with different values of that ratio must be differentiated, even if they produce the same kind of good.

Putting  $K_t = \sum_{i=1}^n K_t^i$ , we define  $p_t^K$  by

$$p_t^K = \sum_{i=1}^n p_t^{K_i} K_t^i / K_t. \quad (6)$$

---

<sup>5</sup> This is a practical solution to the problem of capital aggregation. Note the expression of the demand side in (4); it is different from the simple summation of expenditures.

<sup>6</sup> As shown in Section ,  $K_t$  has no substance but notational utility.

Then it follows from (4) and (6) that

$$AE_t = \sum_{i=1}^n u_t^i p_t^i D_t^i - p_t^K K_t. \quad (7)$$

*Autonomous expenditure and induced expenditure:* Suppose that nominal consumption  $C_t$  depends on nominal income  $Y_t$ . Then, by the mean value theorem,  $C_t = C_t(Y_t)$  can be expressed as follows:

$$C_t = C_t(Y_{t-1}) + \gamma_t \Delta Y, \quad (8)$$

where  $\Delta Y$  is given by

$$\Delta Y = Y_t - Y_{t-1}. \quad (9)$$

Terms  $C_t(Y_{t-1})$  and  $\gamma_t \Delta Y$  can be referred to as *autonomous consumption* and *induced consumption*, respectively.<sup>7</sup> By the same token, any expenditure that depends on income can be divided *conceptually* into two parts, *autonomous* and *induced*.

One may argue that expenditures should be expressed by real terms. In fact, real terms are indeterminate until the end of the period, so that they are, as shown below, inconvenient for our objective.

*Imaginary gap:* Our objective is concerned with evaluating the effect of investment activities in period  $t$  on aggregate demand gap for consumption goods in period  $t+1$ . One way to do this is to look into each demand gap for consumption good  $i$ :

$$E_{t+1}^i = C_{t+1}^i / p_{t+1}^i - K_{t+1}^i / k_{t+1}^i.$$

In general, however,  $C_{t+1}^i$  and  $k_{t+1}^i$  are indeterminate until the end of period  $t+1$ .

And so is  $E_{t+1}^i$ , even if  $p_{t+1}^i$  and  $K_{t+1}^i$  are given. We should, therefore, focus on the expected state of demand and productive capacity at the initial stage of the period. How then will it be expressed? Unless there are solid reasons otherwise, it is natural to replace  $k_{t+1}^i$  by  $k_t^i$ , and omit induced consumption since the autonomous can be regarded as the purchasing power at the beginning of the period. But the autonomous is also indeterminate, because putting  $a_{t+1}^i = C_{t+1}^i(Y_t) / Y_t$  yields  $C_{t+1}^i(Y_t) = a_{t+1}^i Y_t$ , and  $a_{t+1}^i$  is not necessarily constant. Hence, replacing  $a_{t+1}^i$  by  $a_t^i$ , we obtain *imaginary*

*gap*  $IE_{t+1}^i$ :

$$IE_{t+1}^i = a_t^i Y_t / p_{t+1}^i - K_{t+1}^i / k_t^i, \quad (10)$$

which is expected to show the state of demand gap  $E_{t+1}^i$  at the initial stage of the period. Aggregation of imaginary gaps will be shown later.

*Productive investment and non-productive investment:* Investment in fixed capital to increase productive capacity can be referred to as *productive investment*. On the other hand, investment that has no direct relevance to increase productive capacity, such as housing, can be referred to as *non-productive investment*.

### . A SIMPLIFIED FRAMEWORK

We will be able to show that the balance curve exists in an economy where the following four conditions are fulfilled:

- ( ) The relationship between consumption and income is stable;
- ( ) Fixed capital is used to produce (some) consumption goods;
- ( ) In a “short period,” quantity adjustments are predominant over price adjustments;
- ( ) Investment needs time to become productive.

But it is difficult to deal with them as they are because of their wide implications. Accordingly, it is convenient at first to construct a framework by simplification.<sup>8</sup> Below is an example, in which we can derive the balance curve exactly. It consists of the following nine assumptions (peripheral ones, such as those regarding taxes, will not be stated).

*Assumption 1: The system is closed.*

In this case, (nominal net national) income  $Y_t$  is given by

$$Y_t = C_t + G_t + I_t - CC_t, \quad (11)$$

where  $I_t$ ,  $G_t$  and  $CC_t$  denote gross investment, government expenditures and depreciation (capital consumption allowances), respectively.

*Assumption 2: Induced expenditures are omitted.*

Then it follows that  $C_t = C_t(Y_{t-1})$ . Putting  $a_t = C_t(Y_{t-1})/Y_{t-1}$  yields

$$C_t = a_t Y_{t-1}. \quad (12)$$

We will refer to  $a_t$  as the (*autonomous*) *consumption ratio*. Similarly, we obtain

<sup>7</sup> Note that this terminology is different from the traditional one.

<sup>8</sup> Harcourt (1995) notes: “Our aim should be to produce theories which contain the essential elements of the reality from which they start as the basic ingredients, in a sufficiently simplified form to allow us to see clearly the relationships at work and how exactly they intertwine” (p. 1240).

$$G_t = g_t Y_{t-1}, \quad (13)$$

where  $g_t$  will be referred to as the *ratio of (autonomous) government expenditures*.

Assumption 3: *The prices of goods are determined at the beginning of each period.*

Assumption 4: *Only one kind of consumption goods exists and is produced with only one kind of physical capital assets*

Owing to the above two, we express capital equipment and its unit price by  $K_t$  and  $p_t^K$ , respectively.

Assumption 5: *Investment at a given period is to become productive at the beginning of the next.*

Assumption 6: *Non-productive investment is omitted.*

Assumption 7: *Replacement is equal to depreciation.*

Assumption 8: *“Short-period expectations are always fulfilled,”<sup>9</sup> i.e., the quantity demanded be squared with the quantity of production and inventories are omitted.*

Define  $\Delta K$  by

$$\Delta K = K_{t+1} - K_t. \quad (14)$$

Then, owing to 5,  $p_t^K \Delta K$  gives the net investment in period  $t$ . On the other hand, owing to 6, 7 and 8,  $I_t - CC_t$  in (11) is equal to the net investment in period  $t$ . Hence we have

$$I_t - CC_t = p_t^K \Delta K. \quad (15)$$

Assumption 9: *The system is “completely integrated,” i.e., each firm makes all of its plant and equipment as well as intermediate products for itself.*

Owing to 3, 4, and the above, we have only to deal with one demand gap and can adopt the type of (3) as the aggregate demand gap. And, owing to 8, we have

$$E_t = C_t / p_t^M - K_t / k_t. \quad (16)$$

where  $p_t^M$  and  $k_t$  denote price of the consumption good and technical capital coefficient, respectively. We also have

---

<sup>9</sup> Keynes (1973) says, “I now feel that if I were writing the book [the *General Theory*] again I should begin by setting forth my theory on the assumption that *short-period expectations were always fulfilled*; and then have a subsequent chapter showing what difference it makes when short-period expectations are disappointed” (p. 181, italics added). In this paper, I will omit the corresponding “subsequent chapter.”

$$IE_{t+1} = a_t Y_t / p_{t+1}^M - K_{t+1} / k_t . \quad (17)$$

Assumption 9 can be replaced by the one that “capital goods are made by labor only.”<sup>10</sup> One or the other of them will be needed to get to the bottom of the implication of “the fact that capital is not a self-subsistent entity existing apart from consumption” (Keynes, 1936, p. 106). They facilitate evaluating two effects of investment: capacity effect and *consumption generating effect*, the latter resulting from the income generating effect. It is crucial to distinguish explicitly capital equipment from consumption goods. “One-commodity economy,” in which saving necessarily determines investment, is not applicable to at least industrialized economies. As Keynes (1936) says, “[s]aving, in fact, is a mere residual” (p. 64), though investment activities may depend on the propensity to save. In passing, the “malleability” of capital is far from reality. “In fact, the long-run trend is only a slowly changing component of a chain of short-period situations, it has no independent entity” (Kalecki, 1991, p. 435). In addition, Nikaido (1980) shows that smooth factor substitution does not preclude Harrod’s “knife-edge.”

To sum up, the simplified framework is expressed by the following equations:

$$Y_t = C_t + G_t + I_t - CC_t, \quad (11)$$

$$C_t = a_t Y_{t-1}, \quad (12)$$

$$G_t = g_t Y_{t-1}, \quad (13)$$

$$\Delta K = K_{t+1} - K_t, \quad (14)$$

$$I_t - CC_t = p_t^K \Delta K, \quad (15)$$

$$E_t = C_t / p_t^M - K_t / k_t, \quad (16)$$

$$IE_{t+1} = a_t Y_t / p_{t+1}^M - K_{t+1} / k_t . \quad (17)$$

Each equation above (except for (17)) is nothing new, or rather, arcane; similar ones are shown in Harrod (1939), Samuelson (1939), Domer (1942), Hicks (1950) and others. Implications derived below, however, will be different from the precedents; it is doubtful whether they have been scrutinized in terms of oversimplification. Note that no investment function is supposed; our concern is what investment activities entrepreneurs will learn to take as a whole under certain circumstances, but not what phenomena will result from a strict mode of behavior.

---

<sup>10</sup> See Pasinetti (1973).

Now the question that we aim to solve here is this: How will the relationship between  $\Delta K/K_t$  and  $\Delta p^M/p_t^M$  if  $IE_{t+1} = 0$  to be realized, where  $\Delta p^M$  is given by  $\Delta p^M = p_{t+1}^M - p_t^M$ ? Note that imaginary gap  $IE_{t+1}$  is independent of  $p_{t+1}^K$ .

Define *fixprice imaginary gap*  $FIE_{t+1}$  by

$$FIE_{t+1} = a_t Y_t / p_t^M - K_{t+1} / k_t. \quad (18)$$

If  $p_{t+1}^M = p_t^M$ , then  $IE_{t+1} = FIE_{t+1}$ . It follows from (11)—(18) that

$$FIE_{t+1} = \varphi_t \Delta K / k_t + \sigma_t K_t / k_t + (a_t + g_t) E_t, \quad (19)$$

where  $\varphi_t$  and  $\sigma_t$  are given by

$$\varphi_t = a_t u_t - 1, \quad (20)$$

$$\sigma_t = a_t + g_t - 1, \quad (21)$$

and  $u_t$  is the real capital coefficient of  $K_t$ :

$$u_t = k_t p_t^K / p_t^M. \quad (22)$$

We will refer to  $\varphi_t$  and  $\sigma_t$  as *flow coefficient* and *stock coefficient*, respectively. As shown below, these two coefficients characterize the *underlying structure* of the economy. Note that they are independent of institutional set-ups, that is, their existence is irrelevant to the system, whether it be a market economy or planed economy.

Suppose that  $IE_{t+1} = 0$ . Then  $a_t Y_t = p_{t+1}^M K_{t+1} / k_t$ , so that it follows from (18) that

$$\Delta p^M / p_t^M = k_t FIE_{t+1} / K_{t+1}. \quad (23)$$

Hence it follows from (14) and (19) that

$$\Delta p^M / p_t^M = \varphi_t + \frac{\sigma_t - \varphi_t + (a_t + g_t) k_t E_t / K_t}{1 + \Delta K / K_t}. \quad (24)$$

This shows the relationship between  $\Delta K / K_t$  (the rate of accumulation) and  $\Delta p^M / p_t^M$  (the rate of change of the price of consumption good) for given  $p_t^M$ ,  $p_t^K$ ,  $a_t$ ,  $g_t$ ,  $k_t$ ,  $K_t$  and  $E_t$  if  $IE_{t+1} = 0$  to be realized. The hyperbola (or line) given by (24) can be referred to as the *balance curve*. Note that the right-hand side of (23) is independent of  $p_{t+1}^K$ . If  $E_t = 0$ , then (24) leads to

$$\Delta p^M / p_t^M = \varphi_t + \frac{\sigma_t - \varphi_t}{1 + \Delta K / K_t}. \quad (25)$$

Fig. 5 shows the balance curve in the case where  $\varphi_t < 0 < \sigma_t$  and  $E_t = 0$ . The region above the curve is a state of overcapacity ( $IE_{t+1} < 0$ ); the region below it a state of undercapacity ( $IE_{t+1} > 0$ ). The ratio of depreciation to capital stock is given by  $r_t$ . If  $E_t \neq 0$ , the curve shifts up or down by approximately  $(a_t + g_t)k_t E_t / K_t$ , according to whether  $E_t$  is positive or negative. It is easy to obtain figures in other cases as well.

Meanwhile, it is not so unrealistic to regard the relative price between consumption goods and capital goods as stable, irrespective of the rate of change of the general price level.<sup>11</sup> The relative price will be governed, in the long run, by the difference in the rate of change of productivity between consumption goods sector and capital goods sector. Hence, under normal conditions, the balance curve can be assumed to relate the rates of accumulation and inflation if demand and productive capacity are to be balanced at the initial stage of the next period.

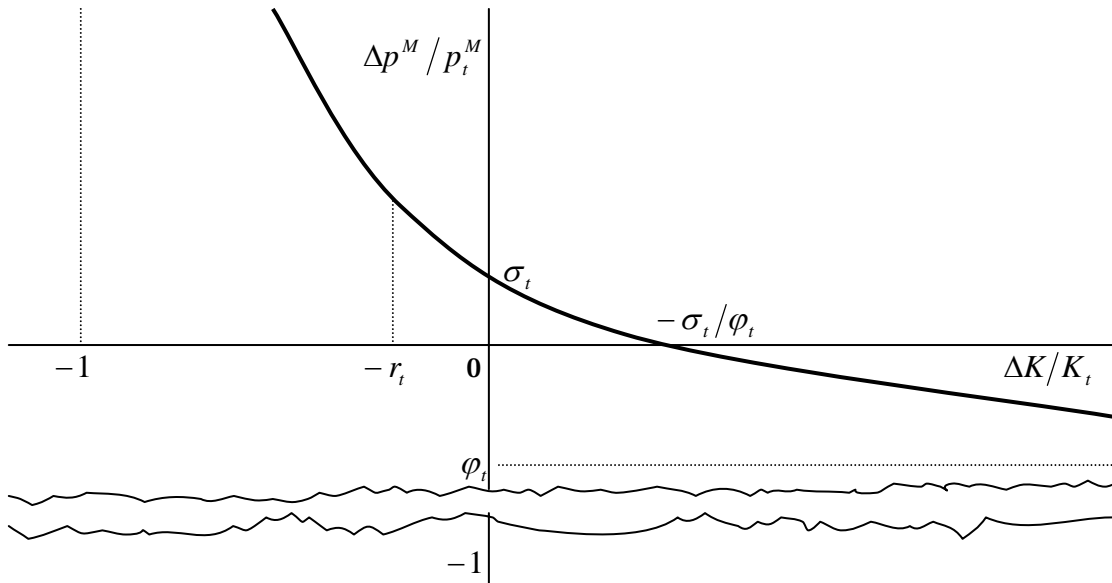


Fig. 5. Balance curve in the case where  $\varphi_t < 0 < \sigma_t$  and  $E_t = 0$ .

**Remark.** If  $E_t = 0$ , it follows from (16), (22) and (12) that

$$a_t u_t = p_t^K K_t / Y_{t-1},$$

which corresponds to the capital coefficient. One may, therefore, consider flow

<sup>11</sup> Keynes (1971) notes: “[M]ovements in our two types of price level [price level of consumable goods and that of investment goods] are connected at one remove and are, generally speaking, in the *same* direction” (p. 162, italics in original).

coefficient  $\varphi_t$  as positive. Reproducible capital stock being the numerator of the capital coefficient, however, consists of not only plant and equipment but also houses, railroads, etc. Hence the above ratio must be far smaller than the actual capital coefficient; the flow coefficient may be negative.

### . SOME IMPLICATIONS

Here we will derive some implications from the simplified framework. For convenience of exposition we will, unless otherwise stated, assume in the following that there are no changes in consumption ratio, technical capital coefficient and relative price over periods. In this case, imaginary gap  $IE_{t+1}$  is equal to demand gap  $E_{t+1}$ , since it follows from (17), (12) and (16) that

$$IE_{t+1} = aY_t / p_{t+1}^M - K_{t+1}/k = C_{t+1} / p_{t+1}^M - K_{t+1}/k = E_{t+1}$$

where suffix  $t$  is deleted from consumption ratio and technical capital coefficient by assumption.

It is now obvious that the balance curve, together with the modified short-period Phillips curve, reins investment activities. If expectations are “rational,” the actual rates of accumulation and inflation will be realized in the neighborhood of the balance point. If not, economic fluctuations will follow; they can also be caused by a shift of either curve. Harrod’s “knife-edge” can appear if  $\sigma_t < \varphi_t$ , i.e., the balance curve is upward sloping.<sup>12</sup>

“Over-investment” will sometimes occur in a state where the balance curve is downward sloping and the rate of accumulation given by the balance point is far less than “Keynes’s Full Employment.” Suppose it does so in period  $t$ , during which a boom may be observed. A large overcapacity will be produced, however, at the beginning of period  $t+1$ , which will cause a downward shift of the balance curve unless the government increases the expenditures to offset it. If the downward shift is drastic, a “multiperiod redundancy situation” as shown by Figs 6 or 7 will be realized; expectations will totally be disappointed and a slump (or contraction) will begin. But as long as the stock coefficient remains positive, overcapacity will decrease with time and disappear in the end (unless “over-investment” occurs again in that process). It is because if  $\varphi_t < 0 < \sigma_t$ ,  $\Delta K_t \leq 0$  and the prices of goods are unchanged, it follows from (19), (21) and (16) that  $E_{t+1} (= IE_{t+1} = FIE_{t+1}) > E_t$ . The duration of the slump (or

---

<sup>12</sup> Keynes (1936) may have excessively emphasized the importance of expectation. See Appendix A for his insight into the underlying structure and the defect of the schedule of the marginal efficiency of capital.

contraction) depends on the degree of “over-investment,” the values of flow coefficient and stock coefficient, and the durable periods of capital equipment.<sup>13</sup> One may consider the recovery to be accelerated if the prices of goods are lowered, because  $p_t^M > p_{t+1}^M$  implies that  $E_{t+1} > FIE_{t+1} (> E_t)$ . In this case, however, we must take into consideration the effect of the change in income distribution caused through wage cuts; it possibly lowers the consumption ratio  $a$  (which is given in this discussion), so that  $E_{t+1} > FIE_{t+1}$  may not hold.<sup>14</sup>

Thus we can explain “the phenomenon of *crisis*—the fact that the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning-point when an upward is substituted for a downward tendency” (Keynes, 1936, p. 318, italics in original).

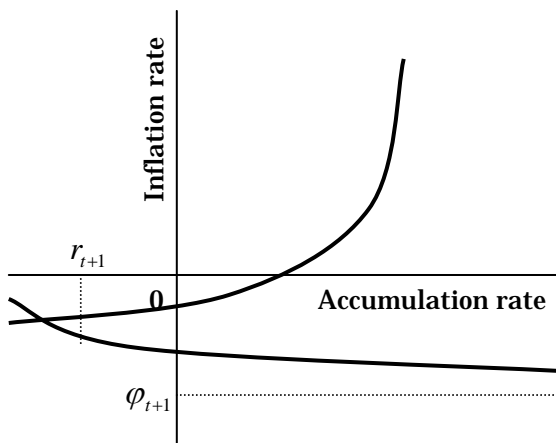


Fig. 6. A “multiperiod redundancy situation” (1)

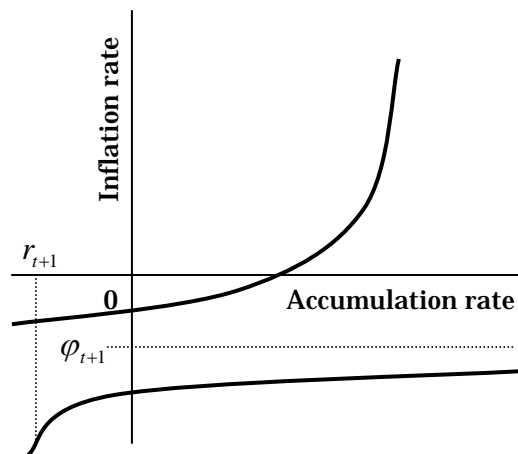


Fig. 7. A “multiperiod redundancy situation” (2)

There are two causes of inflation, given the modified short-period Phillips curve: One is too large a stock coefficient due to a large ratio of government expenditures; the other is a positive flow coefficient. If the latter is the case, a cut in the ratio of government expenditures has little effect on curbing inflation; an extreme one may only engender a “knife-edge.”

<sup>13</sup> Keynes (1936) says, “. . . we have . . . a substantial reason why the duration of the slump should have a definite relationship to the length of life of durable assets and to the normal rate of growth in a given epoch” (p. 318). But what determines “the normal rate of growth in a given epoch”?

<sup>14</sup> In other words, “there is no telling how wage cuts will affect the aggregate level of employment” (Thirlwall, 1999, p. 372).

Further analysis may be possible in this framework. But it should be extended for more detailed argument.

**Remark.** As shown by Fig. 8, the accumulation rate given by the balance point may remain negative over periods if both flow and stock coefficients are negative. In this case, we must take into consideration the real balance effect; the contraction process will end in course of time. See appendix B.

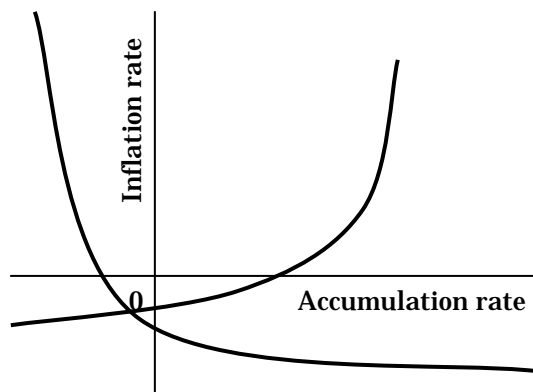


Fig. 8. A state of negative growth

#### . AN EXTENDED FRAMEWORK

Modifying Assumption 4, we introduce here two more expenditures: One is the consumption of *non-manufactured goods*, which indicate goods whose dominant factor of production is labor, whereas manufactured consumption goods are characterized by (16);<sup>15</sup> the other is non-productive investment. As for these expenditures, neither productive capacity nor price levels will be considered; demand and supply are to be adjusted by quantity and/or by price.

Henceforth  $C_t^M$  denotes consumption of manufactured goods;  $C_t^A$  consumption of non-manufactured goods;  $H_t$  non-productive investment. And are assumed to be expressed as follows:

$$C_t^M = a_t^M Y_{t-1},$$

---

<sup>15</sup> Some services that are supplied with facilities, such as a hotel business, can also be characterized by (16). But it may be unreasonable to put information business into this category, even if computers are used.

$$C_t^A = a_t^A Y_{t-1},$$

$$H_t = h_t Y_{t-1}.$$

Putting  $a_t = a_t^M + a_t^A$  yields (12), where  $C_t = C_t^M + C_t^A$ . Demand gap  $E_t$  is defined by

$$E_t = C_t^M / p_t^M - K_t / k_t ;$$

imaginary demand gap  $IE_{t+1}$  by

$$IE_{t+1} = a_t^M Y_t / p_{t+1}^M - K_{t+1} / k_t ;$$

and fixprice imaginary gap  $FIE_{t+1}$  by

$$FIE_{t+1} = a_t^M Y_t / p_t^M - K_{t+1} / k_t .$$

Allowing for these modifications yields

$$FIE_{t+1} = \varphi_t \Delta K / k_t + \sigma_t K_t / k_t + (a_t + g_t + h_t) E_t, \quad (26)$$

where  $\varphi_t$  and  $\sigma_t$  are given by

$$\varphi_t = a_t^M u_t - 1, \quad (27)$$

$$\sigma_t = a_t + g_t + h_t - 1. \quad (28)$$

It is easy to obtain the expression of the balance curve.

We can derive some implications from the above. Here we will mention the following: (a) control of the flow coefficient; (b) control of the stock coefficient; (c) desirable state of the balance curve; (d) necessity of deficit finance.

(a) As shown by (27) and (22), changing the relative price is the only way to control the flow coefficient, given consumption ratio for manufactured consumption goods and technical capital coefficient.<sup>16</sup> That can be done by a change of commodity tax rates on manufactured consumption goods.

(b) As shown by (28), fiscal policy, defined here as the control of the ratio of government expenditures, is *prima facie* a sure means of controlling the stock coefficient. Actually, however, the precariousness of non-productive investments necessitates shrewd control of the ratio of government expenditures, the feasibility of which is both practically and politically doubtful. Fiscal policy is suited for achieving a desirable value of the stock coefficient but not for maintaining it, or rather, "fine

---

<sup>16</sup> In fact, there is a possibility that the flow coefficient depends on the ratio of government expenditures because of induced consumption. See Section .

tuning.”

While a small change of the rates of interest may have no *direct* effect on productive investments in the aggregate, it will at least affect such non-productive investments as housing. This implies that the control of the rates of interest is effective in stabilizing the stock coefficient. It must be noted, however, that a time lag is inevitable and that non-productive investments are not dependent on the rates of interest alone.

Now suppose that the modified short-period Phillips curve is stable and that the central bank holds the key rates of interest for monetary policy constant.<sup>17</sup> Then a shift of the balance curve will be accompanied by a change in the rate of increase of money supply along with changes in the rates of inflation and accumulation. This suggests that money supply can be an indicator of the movement of the balance curve; so that, conversely, we can stabilize the latter by controlling the former. However, the “policy of a steady rate of money growth” can not be a panacea, since both “liquidity-preference” and the modified short-period Phillips curve are not always stable. The search for a “stable money demand function” will be in vain.<sup>18</sup>

Thus, there is no mechanical way to stabilize the stock coefficient, even if the ratio of government expenditures is fixed. While money supply can be a broad criterion, the central bank should control it with flexibility, observing the trend of expenditures and prices.

Socially desirable will be a state where  $\varphi_t < 0 < \sigma_t$ , since such a state gives flexibility to macroeconomic policy. Indeed, Keynesian old doctrine concerning the Phillips curve can be applied if the flow coefficient is negative and the modified short-period Phillips curve is stable. Although a non-negative flow coefficient, if small, seems acceptable, it attenuates the benefit of a downward shift of the modified short-period Phillips curve, to which technical progress and abundant supply of natural resources can lead.

Some may argue that the modified short-period Phillips curve must be vertical in the long run because “supply of labor is a function of anticipated real wage.” Those who would withdraw their labor because of disappointing real wages, however, must have other means of earning. It is because the have-nots have to earn money to live; they cannot afford to weigh labor with “leisure.” In other words, the “natural rate

---

<sup>17</sup> Under the latter condition, we may regard money supply as endogenous.

<sup>18</sup> Keynes (1971) notes: “[T]he total requirements of the monetary circulation are not associated in any stable or invariable manner either with the level of bank rate and its influence on the rate of investment or with the level of prices; so that we shall be misled if we lay much stress on the changes in the total quantity of money when we are trying to trace the causation and the stages of a transition” (pp.196-7).

If a stable one really exists, we can introduce money supply explicitly and make more

hypothesis” will apply only to a system where an adequate income—a “negative income tax”? —is guaranteed to every person.

The collapse of the *original* Phillips curve can be attributed to the upward shift of the modified short-period Phillips curve, three causes of which are conceivable:

- 1) Price hikes of natural resources exemplified by the oil crisis;
- 2) Deterioration of the “state of confidence” under severe inflation caused perhaps by too large a ratio of government expenditures;
- 3) Monetary squeeze in a state with a positive and large flow coefficient.

(d) Non-productive investment  $H_t$  being adopted, net investment  $I_t - CC_t$  is given by

$$I_t - CC_t = p_t^M \Delta K + h_t Y_{t-1}, \quad (29)$$

where depreciation concerned with non-productive investment is omitted. Substituting (12), (13) and (29) into (11) yields

$$Y_{t-1} + \Delta Y = (a_t + g_t + h_t) Y_{t-1} + p_t^K \Delta K, \quad (30)$$

where (9) is used. Compare the coefficients of  $Y_{t-1}$  on each side of (30). Then, if the national budget is balanced, the following will hold:  $1 \geq a_t + g_t + h_t$ , i.e.,  $\sigma_t \leq 0$ . This is because the total sum of expenditures excluding productive investments is usually below that of revenues. Of course, individual household and entrepreneurs can spend on consumption and non-productive investment above their revenues by withdrawing savings and/or by borrowing, but this largely remains a temporary measure. Hence some deficit finance is necessary in principle for the stock coefficient to be positive.<sup>19</sup> And monetary policy can display its own ability on condition that the flow coefficient is negative and the stock coefficient is positive.

### . EXAMINATION OF OVERSIMPLIFICATION

From the methodological point of view, it must be examined whether the simplified framework is oversimplified or not. The existence of the balance curve is dubious if it is based on assumptions *ad hoc*. Although some assumptions have already been relaxed

---

sophisticated analyses.

<sup>19</sup> In some circumstances, the stock coefficient can be positive under a balanced budget, or rather, when there is no government debt. Examples are: when the “propensity to hoard,” or the “speculative motive to liquidity” decreases; when gold is mined under the gold standard system; and it is easy to see that a surplus of current transactions and/or import of capital can lead to an increase in the stock coefficient.

In passing, Wray (1998) and Evdoridis (2000), each from a different perspective,

in the extended framework, Assumptions 1, 2, 4, 5 and 9 that are crucial *prima facie* have not. We will deal with 2, 4 and 5 individually in the following (and need far more space for 1 and 9). All-out generalization is so complex and does not seem to be instructive, though it must be done for the measurement of actual flow coefficients.

On Assumption 2: We will take into consideration only induced consumption for simplicity. In this case, we have only to replace (12) with the following:

$$C_t = a_t Y_{t-1} + \gamma_t \Delta Y, \quad (31)$$

which is obtained from (8) by putting  $a_t = C_t(Y_{t-1})/Y_{t-1}$ . Note that (17) is unchanged. Substituting (31) into (11), we obtain

$$\Delta Y = \{(a_t + g_t - 1)Y_{t-1} + p_t^K \Delta K\} / (1 - \gamma_t), \quad (32)$$

where (9) is used. The multiplier is given by  $1/(1 - \gamma_t)$ . It follows from (14), (16), (32) and (18) that

$$FIE_{t+1} = \varphi_t \Delta K / k_t + \sigma_t K_t / k_t + a_t (a_t + g_t - \gamma_t) E_t / d_t, \quad (33)$$

where  $\varphi_t$ ,  $\sigma_t$  and  $d_t$  are given by

$$\varphi_t = a_t (a_t - \gamma_t) u_t / d_t - 1, \quad (34)$$

$$\sigma_t = (a_t + g_t - 1)(a_t - \gamma_t) / d_t, \quad (35)$$

$$d_t = a_t + g_t \gamma_t - \gamma_t. \quad (36)$$

The expression of the balance curve is easily obtained by the same method as before.

We may assume that  $a_t > \gamma_t$ ,<sup>20</sup> so that  $d_t > 0$  and  $1 > (a_t - \gamma_t) / d_t > 0$ . Hence it is obvious that induced consumption makes no qualitative difference from the simplified framework, except for the dependence of the flow coefficient on the ratio of government expenditures; as shown by (34) and (36), an increase (decrease) in the latter decreases (increases) the former.

Suppose, on the other hand, that investment activities are to balance demand and supply as a whole at the initial stage of the next period. Then it may be more natural to exclude induced consumption from even demand gap (16) and suppose its effect to be dealt with by changes in the rate of operation. If that is the case, it is easy to see that  $\varphi_t$  and  $\sigma_t$  are given by

$$\varphi_t = a_t u_t / (1 - \gamma_t) - 1,$$

---

asserts that growth requires deficit spending.

<sup>20</sup> Compare (31) with the following more general form:  $C_t = \beta_t + \alpha_t Y_{t-1} + \gamma_t \Delta Y$ . It is obvious that  $a_t > \alpha_t$  (unless  $\beta_t \leq 0$ ), so that  $a_t > \gamma_t$ , even if  $\alpha_t = \gamma_t$ . In passing, the ratchet effect is operative when  $a_t > \gamma_t$ .

$$\sigma_t = (a_t + g_t - 1)/(1 - \gamma_t),$$

which clearly shows the role of the multiplier.

Which formalization is appropriate is open; I myself feel that the latter is more plausible.

On Assumption 4: Suppose that there are  $n$  kinds of consumption goods and each demand gap  $E_t^i$  is given by

$$E_t^i = a_t^i Y_{t-1} / p_t^i - K_t^i / k_t^i, \quad (37)$$

where induced consumption is omitted as in the simplified framework. Then, owing to (7), aggregate demand gap  $AE_t$  is given by

$$AE_t = Y_{t-1} \sum_{i=1}^n a_t^i u_t^i - p_t^K K_t. \quad (38)$$

On the other hand, imaginary gap  $IE_{t+1}^i$  is given by (10). Measured by the replacement cost of the capital equipment in *period*  $t$ , the amount of imaginary gap  $IE_{t+1}^i$  is given by

$$(k_t^i p_t^{K_i} / p_{t+1}^i) IE_{t+1}^i = k_t^i p_t^{K_i} a_t^i Y_t / p_{t+1}^i - p_t^{K_i} K_{t+1}^i,$$

where  $p_t^{K_i}$  denotes the replacement cost of a unit of capital  $K_t^i$ . By summation, we obtain *aggregate imaginary gap*  $AIE_t$ :

$$AIE_t = Y_t \sum_{i=1}^n (k_t^i p_t^{K_i} a_t^i / p_{t+1}^i) - \sum_{i=1}^n p_t^{K_i} K_{t+1}^i. \quad (39)$$

Define  $\Delta K$  by

$$\Delta K = \sum_{i=1}^n p_t^{K_i} \Delta K^i / p_t^K,$$

where  $\Delta K^i$  is given by  $\Delta K^i = K_{t+1}^i - K_t^i$ ; and  $p_t^K$  by (6). Then,  $p_t^K \Delta K$  gives net investments in period  $t$  and it follows that

$$\sum_{i=1}^n p_t^{K_i} K_{t+1}^i = p_t^K (K_t + \Delta K). \quad (40)$$

Note that generally  $K_{t+1} (= \sum_{i=1}^n K_{t+1}^i) \neq K_t + \Delta K$ . This implies that  $K_t$  has no

substance but notational utility. It follows (39) and (40) that

$$AIE_{t+1} = Y_t \sum_{i=1}^n (k_t^i p_t^{K_i} a_t^i / p_{t+1}^i) - p_t^K (K_t + \Delta K). \quad (41)$$

Putting  $p_{t+1}^i = p_t^i$  in (41), we obtain *fixprice aggregate imaginary gap*  $FAIE_{t+1}$ :

$$FAIE_{t+1} = Y_t \sum_{i=1}^n a_t^i u_t^i - p_t^K (K_t + \Delta K). \quad (42)$$

It follows from the above that

$$FAIE_{t+1} = \varphi_t p_t^K \Delta K + \sigma_t p_t^K K_t + (a_t + g_t) AE_t, \quad (43)$$

where  $\varphi_t$  and  $\sigma_t$  are given by

$$\varphi_t = \sum_{i=1}^n a_t^i u_t^i - 1, \quad (44)$$

$$\sigma_t = a_t + g_t - 1. \quad (45)$$

Suppose that  $AIE_{t+1} = 0$ . Then, it follows from (41) that

$$Y_t = \frac{p_t^K (K_t + \Delta K)}{\sum_{i=1}^n a_t^i u_t^i p_t^i / p_{t+1}^i}.$$

Substituting this into (42) yields

$$FAIE_{t+1} = p_t^K (K_t + \Delta K) \left( \frac{\sum_{i=1}^n a_t^i u_t^i}{\sum_{i=1}^n a_t^i u_t^i p_t^i / p_{t+1}^i} - 1 \right). \quad (46)$$

Our concern is not the price level of consumption goods but its rate of change. For notational simplicity, therefore, let us represent it by the following:

$$RI_t = \frac{\sum_{i=1}^n a_t^i u_t^i}{\sum_{i=1}^n a_t^i u_t^i p_t^i / p_{t+1}^i} - 1. \quad (47)$$

If  $p_{t+1}^i / p_t^i = \text{constant}$  for all  $i = 1, \dots, n$ , i.e., all the prices of consumption goods change proportionally,  $RI_t$  gives just the value. It follows from (46) and (47) that

$$RI_t = FAIE_{t+1} / p_t^K (K_t + \Delta K).$$

Hence, by (43) we obtain

$$RI_t = \varphi_t + \frac{\sigma_t - \varphi_t + (a_t + g_t)AE/p_t^K K_t}{1 + \Delta K/K_t}.$$

This is the balance curve: the relationship between  $RI_t$  and  $\Delta K/K_t$  that gives  $AIE_{t+1} = 0$ .

Thus we can derive the balance curve irrespective of the diversity of capital assets.

On Assumption 5: Since actual productive periods of capital equipment are usually a year or so,<sup>21</sup> it is appropriate to choose a year as a period of analysis. An investment in capital equipment whose production takes longer than one period should be included in non-productive investments before its operation. The balance curve will be blurred but not nullified by the fact that not every investment becomes productive at the beginning of period.

### . CONCLUDING COMMENTS

Modern economics still misses the structure underlying the industrialized economies, the representation of which is the balance curve: the relationship between the rates of inflation and accumulation if demand and productive capacity are to be balanced as a whole at the initial stage of the next period. Together with the modified short-period Phillips curve, it governs the behavior of the industrialized economies. In other words, they make it possible to recognize coherently the mechanism of chief economic phenomena such as growth, business cycles, creeping inflation and stagflation, and accordingly to prescribe for “diseases.” We should now leave the curves of aggregate demand and supply behind.

### References

- Domer, Evsey D. (1946). “Capital Expansion, Rate of Growth, and Employment.” *Econometrica* 14, pp. 137-47.
- Evdoridis, George I. (2000). “Public sector deficits as the foundation of economic growth.” *Journal of Post Keynesian Economics* 22, pp. 529-47.
- Friedman, Milton (1968). “The Role of Monetary Theory.” *American Economic Review* 58, pp. 1-17.
- Friedman, Milton (1977). *Inflation and Unemployment: The New Dimension of Politics*. Nobel Lecture, Occasional Paper 51, London: Institute of Economics Affairs.
- Gordon, Robert J. (1993). *Macroeconomics*, 6<sup>th</sup> ed. Boston and Toronto: Little, Brown and Company.
- Harcourt, G. C. (1995). “Joan Robinson 1903-1983.” *Economic Journal* 105, pp. 1228-43.
- Harrod, Roy F. (1939). “An Essay in Dynamic Theory.” *Economic Journal* 49, pp. 14-33.
- Hicks, John R. (1950). *A Contribution to the Theory of Trade Cycles*. Oxford: Clarendon Press.

- Hicks, John R. (1973). *The Crisis in Keynesian Economics*. Oxford: Basil Blackwell.
- Hicks, John R. (1977). *Economic Perspectives: Further Essays on Money and Growth*. Oxford: Clarendon Press.
- Kalecki, Michal (1991). *Collected Works of Michal Kalecki, Volume : Capitalism : Economic Dynamics* (ed. J. Osiatynski). Oxford and New York: Clarendon Press.
- Keynes, John M. (1936). *The General Theory of Employment, Interest and Money*, London: Macmillan.
- Keynes, John M. (1971). *A Treatise on Money : The Pure Theory of Money*, Vol. of Keynes, *Collected Writings* (ed. D. Moggridge). London: Macmillan.
- Keynes, John M. (1973). *The General Theory and After: Part , Defence and Development*, Vol. of Keynes, *Collected Writings* (ed. D. Moggridge). London: Macmillan.
- Kregel, J. A. (1980). "Economic Dynamics and the Theory of Steady Growth: a Historical Essay on Harrod's 'knife-edge'." *History of Political Economy* 12:1, pp. 97-123.
- Mayer, Thomas (1960). "Plant and Equipment Lead Times." *Journal of Business* 33, pp. 127-32.
- Nikaido, Hukukane (1980). "Harrodian Pathology of Neoclassical Growth: The Irrelevance of Smooth Factor Substitution." *Zeitschrift für Nationalökonomie* 40, nos. 1-2: 111-134, Springer-Verlag. Reprinted in *Prices, Cycles, and Growth*, pp. 167-92, Cambridge and London: MIT press.
- Pasinetti, Luigi L. (1973). "The Notion of Vertical Integration in Economic Analysis." *Metroeconomica* 25, pp. 1-29.
- Samuelson, Paul A. (1939). "Interaction between the Multiplier Analysis and Principle of Acceleration." *Review of Economic Statistics* 21, pp. 75-8.
- Solow, Robert M. (1956). "A Contribution to the Theory of Economic Growth." *Quarterly Journal of Economics* 70, pp. 65-94.
- Thirlwall, A. P. (1999). "A 'second edition' of Keynes' General Theory." *Journal of Post Keynesian Economics* 21, pp. 367-86.
- Wray, L. R. (1998). *Understanding Modern Money: The Key to Full Employment and Price Stability*. NH: Edward Elgar, Aldershot and Lyme.

#### APPENDIX A: Keynes and the Underlying Structure

Keynes envisaged a few aspects of the underlying structure by some "facts of experience." The quotations in the following are from the *General Theory*.

He mentions his observations of the situation of economy in his era:

. . . , it is an outstanding characteristic of the economic system in which we live that, whilst

---

<sup>21</sup> See Mayer (1960), for example.

it is subject to severe fluctuations in respect of output and employment, it is not violently unstable. . . . Fluctuations may start briskly but seem to wear themselves out before they have proceeded to great extremes, and an intermediate situation which is neither desperate nor satisfactory is our normal lot (pp. 249-50).

He points out four “conditions of stability . . . capable of explaining the observed results,” one of which is:

that a rate of investment, higher (or lower) than prevailed formerly, begins to react unfavourably (or favourably) on the marginal efficiency of capital if it is continued for a period which, measured in years, is not very large (p. 251).

This condition is:

merely based on the presumption that capital-assets are of various ages, wear out with time and are not all very long-lived; so that if the rate of investment falls below a certain level, it is merely a question of time (failing large fluctuations in other factors) before the marginal efficiency of capital rises sufficiently to bring about a recovery of investment above this minimum. And similarly, of course, if investment rises to a higher figure than formerly, it is only a question of time before the marginal efficiency of capital falls sufficiently to bring about a recession unless there are compensating changes in other factors (pp. 253-4).

The above exactly describes the situation shown by Fig. 3 in the text, though “the” presumption is irrelevant to the shape of the balance curve.

But Keynes failed to perceive that there is something structural behind his observations. He says, “[T]hese facts of experience *do not follow of logical necessity*” (p. 250, italics added). They might merely have confirmed him in his preconceived idea inspired, perhaps, by the principle of effective demand or the following presumption:

If there is an increased investment in *any given type of capital* during any period of time, *the marginal efficiency of that type of capital will diminish as the investment in it is increased*, partly because the prospective yield will fall as the supply of that type of capital is increased, and partly because, as a rule, pressure on the facilities for producing that type of capital will cause its supply price to increase (p.136, italics added).

It can be a fallacy of composition if we apply this presumption to “capital in general” as well; and Keynes may not have fully grasped this. Anyway, he could not make sense of Harrod’ instability. Their correspondence shows that in effect he took for granted that the flow coefficient is negative, while Harrod tried to convince him of its being inherently positive.<sup>22</sup>

Thus Keynes lays stress on a psychological factor expectation, tackling the theory of investment with the schedule of marginal efficiency of capital, which “depends partly on the existing quantity of equipment which is one of the given factors, but partly on the state of long-term expectation which cannot be inferred from given factors” (p. 246). Indeed the state of long-term expectation may be so, but there can be something to rein it, because not only the “revision of short-term expectation” but also that of long-term expectation will be “carried on largely in the light of realized results” (p. 50):

. . . the facts of the existing situation enter, in a sense disproportionately, into the formation of long-term expectation; our usual practice being to take the existing situation and to project it into the future, modified only to the extent that we have more or less definite reasons for expecting a change (p. 148).

The crux is the process by which “results” or the “existing situation” is realized; the schedule of marginal efficiency of capital fails to capture or reflect that process. It may be a valid theory of investment from the micro-static perspective, but can only be the complement from the macro-dynamic perspective.

Replace the schedule of marginal efficiency of capital with the underlying structure, and the *General Theory* will still be general.

#### APPENDIX B: Real Balance Effect

Suppose that consumption  $C_t$  is given by

$$C_t = \alpha_t Y_{t-1} + p_t^M b_t, \quad b_t > 0. \quad (\text{B } 1)$$

We can interpret term  $p_t^M b_t$  as showing nominal money balance; and hence  $b_t$  real money balance. In this case, imaginary gap  $IE_{t+1}$  is given by

$$IE_{t+1} = (\alpha_t Y_t + p_{t+1}^M b_t) / p_{t+1}^M - K_{t+1} / k_t. \quad (\text{B } 2)$$

---

<sup>22</sup> See Kregel (1980) for their correspondence and disagreement.

Accordingly, fix-price imaginary gap  $FIE_{t+1}$  is given by

$$FIE_{t+1} = (\alpha_t Y_t + p_t^M b_t) / p_t^M - K_{t+1} / k_t. \quad (\text{B } 3)$$

Substituting (B 1) into (16) yields

$$Y_{t-1} = p_t^M (E_t + K_t / k_t - b_t) / \alpha_t. \quad (\text{B } 4)$$

Therefore, it follows from (11), (B 1), (13), (15) and (B 4) that

$$\begin{aligned} Y_t &= (\alpha_t + g_t) Y_{t-1} + p_t^M b_t + p_t^K \Delta K \\ &= p_t^M (\alpha_t + g_t) (E_t + K_t / k_t - b_t) / \alpha_t + p_t^M b_t + p_t^K \Delta K. \end{aligned} \quad (\text{B } 5)$$

Substituting (B 5) into (B 3) and using (14), we obtain

$$FIE_{t+1} = \varphi_t^* \Delta K_t / k_t + \sigma_t^* K_t / k_t + (\alpha_t + g_t) E_t + (1 - g_t) b_t, \quad (\text{B } 6)$$

where  $\varphi_t^*$  and  $\sigma_t^*$  are given by

$$\varphi_t^* = \alpha_t u_t - 1, \quad (\text{B } 7)$$

$$\sigma_t^* = \alpha_t + g_t - 1, \quad (\text{B } 8)$$

and  $u_t$  is given by (22). Suppose that  $IE_{t+1} = 0$ . Then  $\alpha_t Y_t = p_{t+}^M (K_{t+1} / k_t - b_t)$ , so that it follows from (B 3) that

$$\Delta p^M / p_t^M = k_t FIE_{t+1} / (K_{t+1} - b_t k_t). \quad (\text{B } 9)$$

Substituting (B 5) into (B 9), we obtain the expression of the balance curve:

$$\Delta p^M / p_t^M = \varphi_t^* + \frac{\sigma_t^* - \varphi_t^* (1 - b_t k_t / K_t) + k_t \{ (\alpha_t + g_t) E_t + (1 - g_t) b_t \} / K_t}{1 - b_t k_t / K_t + \Delta K / K_t}, \quad (\text{B } 10)$$

where (14) is used. Since national income is always positive, it follows from (B 4) that if  $E_t \leq 0$ ,

$$1 - b_t k_t / K_t > 0. \quad (\text{B } 11)$$

If  $E_t = 0$ , (B 10) leads to

$$\Delta p^M / p_t^M = \varphi_t^* + \frac{\sigma_t^* - \varphi_t^* (1 - b_t k_t / K_t) + (1 - g_t) b_t k_t / K_t}{1 - b_t k_t / K_t + \Delta K / K_t}. \quad (\text{B } 12)$$

Putting  $\Delta K / K_t = 0$  in (B 12) yields

$$\Delta p^M / p_t^M = \frac{\sigma_t^* + (1 - g_t) b_t k_t / K_t}{1 - b_t k_t / K_t} = \frac{\alpha_t}{1 - b_t k_t / K_t} + g_t - 1, \quad (\text{B } 13)$$

where (21) is used. This is the rate of inflation given by the point where the balance curve intersects the vertical axis.

Now suppose that a state shown by Fig. 8 is realized and the contraction reduces capital stock  $K_t$  to  $b_t k_t$ . Then  $1 - b_t k_t / K_t \rightarrow 0$  (unless  $b_t \rightarrow 0$ ), so that, owing to (B 11), the value given by (B 13) increases and can even be positive. This implies that contraction processes will end in course of time because of the real balance effect (but it will never ensure sustained growth).