Expectation Traps and Monetary Policy

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Abstract

We describe a class of monetary economies that generate persistent episodes of high and low inflation. In these economies variations in expectations can lead private agents to take actions which then make it optimal for the monetary authority to validate those expectations. We think these model economies deserve attention because they display several good empirical implications.

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I Introduction

Many countries have gone through prolonged periods of costly, high inflation, as well as prolonged periods of low inflation. The United States and other industrialized countries have experienced relatively low inflation while many emerging market economies have experienced episodes of high inflation as well as episodes of low inflation. A central question in monetary economics is why high inflation episodes occurred and what can be done to prevent them from occurring again.

One tradition for understanding poor inflation outcomes stems from the time inconsistency literature pioneered by Kydland and Prescott (1978) (KP) and Barro and Gordon ( ) (BG). Finite horizon versions of these models do not easily generate the episodes of high and low inflation within countries and the different inflation experiences across countries. For example, one possible explanation for the dramatic differences in inflation rates across countries is that low inflation countries can commit to monetary policies, while high inflation countries cannot. This explanation falls afoul of the observation that high inflation countries have prolonged episodes of relatively low inflation. Stories that countries periodically can commit and periodically cannot seem too facile an explanation. Infinite horizon versions of the KP and BG models have embarrassingly many equilibria, supported by trigger strategies. It is hard to know what observations would be ruled out by such equilibria.

This paper has two purposes. First, we embed the economic forces in KP and BG into a standard general equilibrium model and explicitly rule out trigger strategies. Second, we find that in such a model inflation rates can be high for prolonged periods of time and low for prolonged periods of time. We find that there is modest support in cross-country evidence for key implications of the model.

In this paper we examine whether standard monetary general equilibrium models with benevolent monetary authorities acting under discretion can generate persistent episodes of high and low inflation. Specifically, we ask whether private agents' expectations of high or low inflation can lead these agents to take actions which then make it optimal for monetary authorities to validate these expectations. Following Chari, Christiano and Eichenbaum (1998), we call such an outcome an expectation trap. Chari, Christiano and Eichenbaum (1998) showed that expectation traps could occur in conventional general equilibrium monetary models. They relied, however, on trigger strategies on the part of the monetary authority to support such outcomes. One criticism of trigger strategies is that because of folk theorem-like reasons, virtually any inflation outcome can be rationalized as an equilibrium. A key finding of this paper is that expectation traps can occur, even in the absence of trigger strategies. We think this result deserves attention because it occurs in a model which has several good empirical implications. As we explain below, the model is potentially able to resolve a variety of puzzles in the money demand literature. In addition, the model is consistent with the relative volatility of financial and other variables observed in low versus high inflation episodes.

We build on Lucas and Stokey's (1983) cash-credit good model. In our model, the benefit of unexpected growth in the money supply is a rise in output and the cost is the misallocation of resources arising from a distortion in relative prices. The monetary authority optimally balances the benefit and costs. We obtain the following three results:

- for a large range of parameter values, there are at least two equilibria,

- there is a sign switch in the correlation of the interest rate with other variables across high and low inflation regimes,
- financial variables are more volatile in the high inflation equilibrium than in the low inflation equilibrium, while real variables display similar volatility in both equilibria.

We now briefly explain the economic mechanisms in our benchmark model and the intuition underlying our results. In the model, goods are produced in monopolistically competitive markets. The monopoly power of firms causes the level of economic activity to be inefficiently low. A subset of monopolists set their prices before the monetary authority selects the money growth rate, while the rest of the monopolists set prices afterward. Because of the preset prices, a monetary expansion greater than expected can raise output. Such a monetary expansion tends to raise welfare because output is inefficiently low. A monetary expansion also has costs. In our model, some goods must be purchased with previously accumulated cash. A monetary expansion, by raising prices, reduces the consumption of cash goods and welfare. In addition, because some prices are preset and others are flexible, a monetary expansion changes relative prices and induces an inefficient allocation of resources. These aspects of the model formalize old ideas with an extensive literature.[1]

We consider two versions of our model. In the first, the fraction of goods which are purchased with cash is held fixed. The intuition underlying our second result is that the marginal cost of unanticipated inflation is non-monotone in the expected inflation rate. It turns out that the marginal cost of unexpected inflation is roughly proportional to rM/P, where r is the net nominal interest rate, and M/P denotes real balances. Real balances are bounded and since the nominal interest rate is increasing in the expected inflation rate, it follows that the marginal cost of unanticipated inflation is low at low expected inflation. A key feature of our model is the behavior of money demand at high nominal interest rate. Specifically, rM/P goes to zero as r goes to infinity. This feature implies that the marginal cost of unexpected inflation is low at high levels of expected inflation. We conjecture that the relationship between the marginal cost of inflation and rM/P lies in the fact that a monetary expansion acts as a distorting tax on real balances. Because the marginal cost of inflation has an inverted 'U' shape, as in a Laffer curve, while the marginal benefit is roughly constant, there is more than one value of expected inflation in which the marginal benefit of unanticipated inflation equals the marginal cost.

As the reasoning in the previous paragraph suggests, we find that the properties of money demand at high levels of inflation are crucial to the question of the multiplicity of equilibria. We confirm this reasoning by developing a model in which rM/P does not go to zero as r goes to infinity. In this model we find that there is a unique equilibrium.

In the second version of our model, households can also take defensive measures to protect themselves against expected inflation. Specifically, they can choose the fraction of goods purchased with cash and the fraction purchased with credit. This choice is made before the monetary authority chooses its policy. Cash purchases are costly because households forego interest, while credit purchases require payment of a cost in labor time which differs depending on the type of good. If households expect high inflation, they choose to purchase most goods with credit and few goods with cash while if they expect low inflation they purchase few goods with credit and most goods with cash.

This aspect of our model implies that if households expect high inflation and have chosen to purchase most goods with credit, the marginal cost of unanticipated inflation is small because relatively few goods are purchased with cash. The monetary authority has a strong incentive to inflate. If households expect low inflation, however, they choose to purchase most goods with cash and the marginal costs of unanticipated inflation are high. The monetary authority does not have a strong incentive to inflate. These arguments suggest that there might well be multiple equilibria in our model and, indeed, we find that for a large range of parameter values there are two equilibria.

The multiplicity of equilibria in our model raises the possibility of expectation traps. If private agents expect the monetary authority to pursue an expansionary monetary policy, they set prices sufficiently high, and choose to purchase so few goods with cash, that it is optimal for the monetary authority to validate their expectations. Conversely, if private agents expect low inflation, then the monetary authority optimally validates those expectations. The possibility of expectation traps in our model is promising because it may help account for the observed prolonged periods of high inflation as well as prolonged periods of low inflation. This possibility depends in a crucial way on the properties of money demand. At an abstract level, it should not be surprising that the behavior of the monetary authority depends in an essential way on the determinants of demand for the object they supply, namely money. But, to our knowledge, this connection has not been made as yet in the literature.

The plan of the paper is as follows. Section 2 describes our model. Section 3 analyzes a restricted version of the model, in which the cash-credit good distinction is exogenous. The endogenous case is treated in section 4. The final section concludes.

II A Cash-Credit Goods Model With Financial Intermediation

In this section, we extend Lucas and Stokey's (1983) cash-credit-goods model in a number of ways. Two of our extensions are intended to capture the benefits and costs emphasized in the literature following KP and BG. In our model, a subset of prices are set in advance by monopolistic firms. This feature implies that an unanticipated monetary expansion tends to raise output and welfare, as in KP and BG. We adopt the timing assumption in Svensson ( ) by requiring that households use currency accumulated in the previous period to purchase cash goods. This timing assumption implies that a realization of high inflation reduces the consumption of cash goods relative to credit goods and thereby tends to reduce welfare. Our third extension is intended to capture the idea that when people expect high inflation, they adopt defensive mechanisms to protect themselves. Specifically, in our model each good can be paid for either with cash or with credit. To purchase any good with credit requires a payment of an intermediation cost, which varies across goods. For each good, households trade off the foregone interest from using cash against the intermediation cost. When the inflation rate and the interest rate are expected to be high, households protect themselves by opting to purchase a relatively large number of goods on credit.[2]

Our infinite-horizon economy is composed of a continuum of firms, a representative household and a monetary authority. The sequence of events within a period is as follows. First, the shocks are realized. These are a shock to the production technology, θ, to government consumption, g, and to the payments technology, η. We refer to $s = (\theta, g, \eta)$ as *the exogenous state*, and we assume that s follows a Markov process. Then households choose the fraction, z, of goods to purchase with cash, and a fraction μ of firms (the 'sticky price firms') set their prices. These decisions depend on the exogenous state. Let $Z(s)$ denote the economy-wide average value of z and $P^e(s)$ denote the average price set by sticky price firms. Here, and in what follows, we scale all nominal variables by the beginning-of-period aggregate stock of money.

After that, the monetary authority makes its policy decision. We denote the actual money growth rate by x and the policy rule that the monetary authority is expected to follow by $X(s)$. The state of the economy after the monetary authority makes its decision, the *private sector's state,* is (s, x). Households' and firms' production, consumption and employment decisions depend on the private sector's state.

Notice that we do not include the beginning-of-period aggregate stock of money in either of our states. In our economy, all equilibria are neutral in the usual sense that if the initial money stock is doubled, there is an equilibrium in which real allocations and the interest rate are unaffected and all nominal variables are doubled. This consideration leads us to focus on equilibria which are invariant with respect to the initial money stock. We are certainly mindful of the possibility that there can be equilibria which depend on the money stock. For example, if there are multiple equilibria in our sense, it is possible to construct 'trigger strategy-type' equilibria which are functions of the initial money stock. In our analysis we exclude such equilibria and we normalize the aggregate stock of money at the beginning of each period to unity.

As is customary in defining a Markov equilibrium, we begin with the decisions at the end of the period, and work our way back to the beginning of the period. Accordingly, we first describe the end-of-period problem of households and flexible price firms given (s, x) and future monetary policy, $X(s)$. We then describe the problem of sticky price firms and the household's choice of z. These problems and market clearing allow us to define a private sector equilibrium for arbitrary x. We then describe the monetary authority's problem and define a Markov equilibrium.

A Private Sector at the End of the Period

In this section we discuss the decision problems of households and firms at the end of the period. We begin with the household problem. In each period the household consumes a continuum of differentiated goods as in Blanchard and Kiyotaki ( ) and supplies labor. The preferences of the representative household are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{1}$$

where $0 < \beta < 1$,

$$c_t = \left[\int_0^1 c_t(\omega)^\rho d\omega\right]^{\frac{1}{\rho}}, \ u(c,l) = \frac{\left[c(1-l)^\psi\right]^{1-\sigma}}{1-\sigma},$$

$c_t(\omega)$ denotes consumption of type ω good, l_t denotes labor time, and $0 < \rho < 1$.

Each good in this continuum is one of four types. A fraction μ are produced by sticky price firms and a fraction $1-\mu$ are produced by flexible price firms. The sticky and flexible price firms are randomly distributed over the goods. In addition, each good can be purchased with cash or with credit. Let z denote the fraction of goods the household chooses to purchase with cash. This cash-credit decision is made before households know which goods are produced by sticky or flexible price firms, so that the cash-credit good choice is independent of the type of firm. Thus, a fraction μz of goods are sticky price goods purchased with cash, a fraction $(1-\mu)z$ are flexible price goods purchased with cash, a fraction $\mu(1-z)$ are sticky price goods purchased with credit and a fraction $(1-\mu)(1-z)$ are flexible price goods purchased with credit. It turns out that prices for goods within each type are the same. Utility maximization implies that the amounts purchased of each type of good are the same. Let c_{11} and c_{12} denote quantities of cash goods purchased from sticky and flexible price firms, respectively, and let c_{21} and c_{22} denote the quantities of credit goods purchased from sticky and flexible price goods, respectively. Then we have that

$$c = \left[z\mu c_{11}^\rho + z(1-\mu)c_{12}^\rho + (1-z)\mu c_{21}^\rho + (1-z)(1-\mu)c_{22}^\rho\right]^{\frac{1}{\rho}}. \tag{2}$$

The household divides its labor time, l, into time supplied to goods-producing firms, n, and time devoted to the payments technology according to:

$$l = n + \frac{\eta(\bar{z}-z)^{1+\nu}}{1+\nu}. \tag{3}$$

We discuss the determination of z below.

Let A denote the nominal assets of the household, carried over from the previous period. In the asset market, the household divides A into money holdings, M, and bonds, B, subject to

$$M + B \leq A. \tag{4}$$

Recall that nominal assets, money and bonds are all scaled by the aggregate stock of money. We impose a no-Ponzi constraint of the form $B \leq \bar{B}$, where $\bar{B}$ is a large, finite, upper bound.

The household's cash in advance constraint is

$$M - \left[P^e(s)\mu z c_{11} + \hat{P}(s,x)(1-\mu)z c_{12}\right] \geq 0, \tag{5}$$

where $P^e(s)$ denotes the price set by sticky price firms and $\hat{P}(s,x)$ denotes the price set by flexible price firms. Nominal assets evolve over time as follows:

$$\begin{aligned}(6)\quad 0 \leq\ & W(s,x)n + (1-R(s,x))M - z\left[P^e(s)\mu c_{11} + \hat{P}(s,x)(1-\mu)c_{12}\right] \\ & -(1-z)\left[P^e(s)\mu c_{21} + \hat{P}(s,x)(1-\mu)c_{22}\right] + R(s,x)A + (x-1) + D(s,x) - xA'.\end{aligned}$$

In (6), $W(s,x)$ denotes the nominal wage rate, $R(s,x)$ denotes the gross nominal rate of return on bonds, and $D(s,x)$ denotes profits after lump sum taxes. Finally, B has been substituted out in the asset equation using (4). Notice that A' is multiplied by x. This multiplication reflects the way we have scaled the stock of nominal assets.

Consider the household's asset, goods and labor market decisions for a given value of z. Given that the household expects, in the future the monetary authority to choose policy according to $X(s)$, prices to be set according to $P^e(s)$ and cash credit goods decisions to be made according to $Z(s)$, the household solves the following problem:

$$v(A,z,s,x) = \max_{n,M,A',c_{ij};\ i,j=1,2} u(c,l) + \beta E_{s'}[\max_{z'} v(A',z',s',X(s'))|s] \tag{7}$$

subject to (2), (3), (4), (5), (6), and non-negativity on allocations. The solution to (7) yields decision rules, $d(A,z,s,x)$, where

$$d(A,z,s,x) = [n(A,z,s,x), M(A,z,s,x), A'(A,z,s,x), c_{ij}(A,z,s,x)], \tag{8}$$

$i,j = 1,2$.

We turn now to the decision problems of firms at the end of the period. Each of the differentiated goods is produced by a monopolist using the following production technology:

$$y(\omega) = \theta n(\omega),$$

where $y(\omega)$ denotes output and $n(\omega)$ denotes employment for type ω good. Also, θ is a technology shock that is the same for all goods. The household's problem yields demand curves for each good. The fraction, $1-\mu$, of firms that are flexible price firms set their price, $\hat{P}(s,x)$, to maximize profits subject to these demand curves. Because the household demand curves have constant elasticity, firms set prices as a fixed markup, $1/\rho$, above marginal cost, W/θ, so that:

$$\hat{P}(s,x) = \frac{W(s,x)}{\theta\rho}. \tag{9}$$

Turning to the government, we assume that there is no government debt, government consumption is financed with lump sum taxes, and government consumption is the same for all goods. As a result, the resource constraint for this economy is:

$$\theta n = g + z\left[\mu c_{11} + (1-\mu)c_{12}\right] + (1-z)\left[\mu c_{21} + (1-\mu)c_{22}\right]. \tag{10}$$

Since there is no government debt, bond market clearing requires $B = 0$, $A = 1$. Also, money market clearing requires $M = 1$.

B Private Sector at the Beginning of the Period

At the beginning of the period, after the exogenous shocks are realized, sticky price firms set prices and households make their payment technology decision, z.

As in Blanchard and Kiyotaki (19xx), sticky price firms in our economy must set their price in advance and must produce the amount demanded at that price. These firms, like the flexible price firms, also wish to set their price as a markup, $1/\rho$, over marginal cost, W/θ. In order to do so, they need to forecast the wage rate, W. They do so by taking the wage rate as given by the private sector equilibrium. Thus, the wage they expect to prevail is $W(s, X(s))$, where $P^e(s)$ denotes the average price set by other sticky price firms. Thus, in equilibrium the price set by sticky price firms is given by:

$$P^e(s) = \frac{W(s, X(s))}{\theta\rho} \tag{11}$$

We now discuss the household's payment technology decision. As noted above, each consumption good can be purchased either with cash or with credit. For goods with $\omega > \bar{z}$, (where $\bar{z}$ is a parameter between 0 and 1), the cost of purchasing with credit is zero. Purchasing goods with $\omega \leq \bar{z}$ on credit requires labor time. The household chooses a fraction, $z \leq \bar{z}$, such that goods with $\omega < z$ are purchased with cash and goods with $\omega > z$ are purchased with credit. The labor time required to purchase fraction, z, of goods with cash is given by $\eta(\bar{z} - z)^{1+\nu}/(1+\nu)$, where $\nu > 0$ is a parameter and $\eta > 0$ is the shock to the payment technology. The household's labor time, including time spent working in the market, n, is given in (3). The household chooses z to solve the following problem:

$$z(A, s) = \arg\max_{0 \leq z \leq \bar{z}} v(A, z, s, X(s)). \tag{12}$$

We now define an equilibrium for each possible private sector state, (s, x), and future monetary policy rule, $X(s)$.

Definition For each s and each x, given $X(s)$ a private sector equilibrium is a collection of functions, $P^e(s)$, $Z(s)$, $\hat{P}(s, x)$, $W(s, x)$, $R(s, x)$, $v(A, z, s, x)$, $d(A, z, s, x)$, $z(A, s)$ such that:

1. The functions v and d solve (7), where d is defined in (8),

2. The function, $z(A, s)$ solves (12) and $z(1, s) = Z(s)$,

3. Firms maximize profits, i.e., $\hat{P}(s, x)$ satisfies (9) and $P^e(s)$ satisfies (11),

4. The resource constraint is satisfied, for $d(1, Z(s), s, x)$.

5. The asset markets clear, i.e., $A'(1, s, x) = M(1, s, x) = 1$.

We find it convenient to define another private sector equilibrium concept. A *private sector equilibrium with a fixed payment technology* is a private sector equilibrium with the restriction that z is fixed and is not a choice variable.

C Monetary Authority

The monetary authority chooses x to maximize the discounted utility of the representative household:

$$\max_x v(1, Z(s), s, x), \tag{13}$$

where v is the value function in a private sector equilibrium. Recall that a private sector equilibrium takes as given the evolution of future monetary policy. Thus, in solving (13) the monetary authority implicitly takes as given the evolution of future monetary policy.

D Markov Equilibrium

We now have the ingredients needed to define a Markov equilibrium.

Definition A *Markov equilibrium* is a private sector equilibrium and a monetary policy rule, $X(s)$, such that $X(s)$ solves (13).

Two properties of a Markov equilibrium deserve emphasis. First, the current money growth rate does not affect discounted utility of the household starting from the next period since it does not affect next period's state. Therefore, the monetary authority faces the static problem of maximizing current period utility and we only have to describe how current money growth affects current allocations. Second, inspection of (9) and (11) shows that $\hat{P}(s, X(s)) = P^e(s)$ in a Markov equilibrium. We use these properties below.

In our analysis of Markov equilibrium, we find it convenient to define another Markov equilibrium concept. The *Markov equilibrium with a fixed payment technology* is a Markov equilibrium in which the private sector equilibrium is a private sector equilibrium in which z is exogenously fixed, and beyond the control of agents.

III Analysis with Fixed Payment Technology

In this section we discuss a version of our model in which the payment technology is fixed, in the sense that households cannot alter the value of z. We do this for two reasons. First, understanding the properties of this version of the model helps us develop the properties of the model when z is chosen by households. Second, as mentioned in the introduction, the model with a fixed payment technology is the simplest adaption of a standard monetary model designed to capture the frictions emphasized in KP and BG.

To analyze our model, we decompose the first order condition associated with the monetary authority problem, (13), into benefits and costs of inflation. In our model unexpected inflation has benefits because some prices are sticky and there is a monopoly distortion. Under sticky prices, higher inflation tends to raise output, while the monopoly distortion makes higher output desirable. These are the reasons the monetary authority in our model has a temptation to stimulate the economy. In our model, inflation also has costs because it leads to a reduction in the relative consumption of cash good. In addition, unexpected inflation also leads to a misallocation of resources because some prices are fixed in advance, while others are not.

To analyze a Markov equilibrium we first characterize a private sector equilibrium. We then solve the monetary authority's problem.

A Characterizing Private Sector Equilibrium

We now develop a set of necessary and sufficient conditions for a private sector equilibrium. Omitting arguments of functions for convenience, the first order necessary conditions for household optimization are:

$$
\begin{aligned}
\frac{u_{11}}{u_{12}} &= \frac{u_{21}}{u_{22}} = \frac{\mu}{1-\mu}\frac{1}{q}, \\
\frac{u_{11}}{u_{21}} &= \frac{u_{12}}{u_{22}} = \frac{z}{1-z}R, \\
-u_n &= \frac{\theta\rho u_{22}}{(1-\mu)(1-z)}, \\
\frac{xu_{21}}{P^e\mu(1-z)} &= \beta E_{s'}[v_1(1,s',X(s'))|s],
\end{aligned}
\tag{14}
$$

where $q = \hat{P}/P^e$ and z is fixed. Here, u_{ij} denotes the partial derivative of u with respect to c_{ij}, and v_1 denotes the partial derivative of v with respect to its first argument. In the labor Euler equation, we have used (9).

In addition, the cash in advance constraint can be written as

$$P^e\mu z c_{11} + qP^e(1-\mu)zc_{12} \leq 1.$$

It is easy to show that if $R > 1$, the cash in advance constraint holds with equality and if the cash in advance constraint is slack, $R = 1$. These observations imply that the appropriate complementary slackness condition is

$$\{1 - [P^e\mu z c_{11} + qP^e(1-\mu)zc_{12}]\}[R-1] = 0. \tag{15}$$

The resource constraint is:

$$g + z[\mu c_{11} + (1-\mu)c_{12}] + (1-z)[\mu c_{21} + (1-\mu)c_{22}] = \theta n. \tag{16}$$

Combining (9) and (11) we have that

$$P^e(s) = \hat{P}(s, X(s)). \tag{17}$$

In this last equation we reintroduce the dependence of variables on s and x to emphasize that P^e coincides with $\hat{P}$ only when $x = X(s)$. The conditions in (14), (15), (16) and (17) are necessary and sufficient for a private sector equilibrium.

It is useful to sketch how one might use this characterization result to compute the objects in a private sector equilibrium for given $X(s)$. Pick a particular value of s, say $\tilde{s}$. Set $\bar{x} = X(\tilde{s})$. Posit a value for $P^e(\tilde{s})$. Solve the 7 independent equations provided by (14), (15), and (16) for the seven variables: $\hat{P}(\bar{x}, \tilde{s})$, $c_{ij}(\bar{x}, \tilde{s})$, $R(\bar{x}, \tilde{s})$ and $n(\bar{x}, \tilde{s})$.[3] Adjust $P^e(\tilde{s})$ until (17) is satisfied. This is the private economy value of P^e given $s = \tilde{s}$. Now consider the range of alternative possible values of x. In each case, solve for $\hat{P}(x, \tilde{s})$, $c_{ij}(x, \tilde{s})$, $R(x, \tilde{s})$ and $n(x, \tilde{s})$ using the 7 equations, (14), (15), and (16) and holding P^e to its private economy equilibrium value. This procedure yields the private economy equilibrium value of $P^e(\tilde{s})$ and the functions, $\hat{P}$, c_{ij}, R and n, for fixed $s = \tilde{s}$. By repeating these calculations for the entire range of values of s, we obtain the private economy equilibrium functions, evaluated at $A = 1$. This is sufficient for our purposes. Our ultimate interest lies in identifying a Markov equilibrium, and this does not require evaluating the private economy functions for $A \neq 1$.

Notice that the heart of the previous algorithm involves solving the 7 equations, (14), (15), and (16), for $\hat{P}$, c_{ij}, R and n, conditional on values for s, x, and P^e. These equations have a recursive structure which allows us to drop one unknown and one equation, if we change the choice variable of the monetary authority. In particular, we posit that the variable chosen by the monetary authority is $q = \hat{P}/P^e$ rather than x. This does not substantively change the nature of the monetary authority problem because there is a monotone relationship between $\hat{P}$ and x.[4] With this change, the roles of x and $\hat{P}$ are reversed in the above algorithm. The algorithm becomes one of solving the 7 equations indicated above for x, c_{ij}, R and n conditional on values for s, q, and P^e. Notice that x only appears in the last equation in (14), so that c_{ij}, R and n can be computed as functions of s, P^e, q, and x can be solved for at the last stage. For later reference, it is useful to write these functions out explicitly

$$c_{ij}(s, P^e, q), \ i, j = 1, 2, \ R(s, P^e, q), \ n(s, P^e, q). \tag{18}$$

Given the change in the monetary authority's choice variable, for purposes of identifying a Markov equilibrium it is not necessary to solve for x. As a result, the change in choice variable allows us to drop one equation and one unknown. Moreover, note that the only place where future monetary policy enters is through the last equation in (14). Thus, the change in choice variable also allows us to ignore future monetary policy, $X(s)$. We denote the set, (s, P^e, q), for which there exists a private sector equilibrium by D.[5]

B Markov Equilibrium

Substitute the allocations in (18) into the period utility function, to obtain:

$$U(s, P^e, q) = u\left[c(s, P^e, q), n(s, P^e, q)\right],$$

where c is defined in (2). With the change in the monetary authority's choice variable, the problem becomes

$$\max_{q \in D} U(s, P^e(s), q(s)). \tag{19}$$

Let $q(s)$ denote the solution to this problem. A Markov equilibrium has the property that $q(s) = 1$ for all s. Our analysis is based on the first order necessary condition associated with (19). In the computational examples we verify sufficient conditions numerically.

Monetary authority optimality implies that, in equilibrium[6]:

$$U_q(s, P^e, 1) = u_c c_q + u_n n_q = 0, \tag{20}$$

where U_q is the derivative of U with respect to q. In addition, u_c, u_n are derivatives of the utility function with respect to c and n, respectively, and c_q, n_q are the derivatives of c and n with respect to q. In (20) these derivatives are evaluated at $q = 1$. From here on we suppress the arguments of functions, and evaluate all functions at $q = 1$. In what follows, we show how (20) can be decomposed into a part that reflects the gains of unexpected inflation (i.e., raising q above unity) arising from the presence of monopoly power and the costs associated with price distortions.

In equilibrium, $c_{11} = c_{12} = c_1$, say, and $c_{21} = c_{22} = c_2$, say. Rewrite the expression for U_q by adding and subtracting $\theta u_{22} n_q / [(1-\mu)(1-z)]$:

$$U_q = u_c c_q - \frac{\theta u_{22} n_q}{(1-\mu)(1-z)} + \left[u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)} \right] n_q = 0. \tag{21}$$

In the appendix, we show that the first two terms to the right of the equality in (21) can be written as:

$$u_c c_q - \frac{\theta u_{22} n_q}{(1-\mu)(1-z)} = -\frac{\theta u_{22}}{(1-\mu)(1-z)} \left[(R-1) z c_1 (1-\mu) \right],$$

so that

$$U_q = \frac{u_{22} c_2}{(1-\mu)(1-z)} \left\{ -(1-\mu) z (R-1) \frac{c_1}{c_2} + \left[\frac{u_n (1-\mu)(1-z)}{u_{22}} + \theta \right] \frac{n_q}{c_2} \right\}, \tag{22}$$

Notice that if the government could commit itself to monetary policy, it would follow the Friedman rule and set $R = 1$, so that the first term in braces would be zero. In light of this observation, we call this term the inflation distortion. We call the second term in braces the monopoly distortion for the following reason. In the efficient allocations for our model economy, the marginal rate of substitution between credit good consumption and leisure would be set equal to the marginal product of labor, and the second term would be zero.

It is useful to simplify the expressions in (22). From (14) we have

$$\frac{u_n (1-\mu)(1-z)}{u_{22}} = -\theta \rho,$$

so that

$$U_q = \frac{u_{22} c_2}{(1-\mu)(1-z)} \left\{ -(1-\mu) z (R-1) \frac{c_1}{c_2} + \frac{n_q \theta (1-\rho)}{c_2} \right\}.$$

In the appendix, we show that $n_q\theta(1-\rho)/c_2$ is a function of c_1/c_2. Using our functional forms, from (14) we have that $R = (c_1/c_2)^{\rho-1}$. Let

$$\psi_{ID}\left(\frac{c_1}{c_2}\right) = (1-\mu)z\left[\left(\frac{c_1}{c_2}\right)^{\rho-1} - 1\right]\frac{c_1}{c_2} \text{ and } \psi_{MD}\left(\frac{c_1}{c_2}\right) = \frac{n_q\theta(1-\rho)}{c_2}. \tag{23}$$

Thus, ψ_{ID} is the inflation distortion and ψ_{MD} is the monopoly distortion. Our decomposition of the monetary authority's first order condition is:

$$U_q = \frac{u_{22}c_2}{(1-\mu)(1-z)}\left[-\psi_{ID}\left(\frac{c_1}{c_2}\right) + \psi_{MD}\left(\frac{c_1}{c_2}\right)\right]. \tag{24}$$

A $c_1/c_2 \in [0,1]$ satisfies the necessary condition for a Markov equilibrium if it sets $U_q = 0$. Here, $c_1/c_2 = 0$ corresponds to (infinitely) high R and inflation, and $c_1/c_2 = 1$ corresponds to $R = 1$ and low inflation.

Consider the inflation distortion function, ψ_{ID}. From inspection of (23), it is immediate that

$$\psi_{ID}(0) = \psi_{ID}(1) = 0.$$

That is, there is no inflation distortion when expected inflation rates are high or low. This feature of the model plays an important role in generating a multiplicity of Markov equilibria.

In the appendix, we establish that $\psi_{MD}(0) > 0$. Therefore, $U_q > 0$ at $c_1/c_2 = 0$. Suppose next that $\psi_{MD}(1) > 0$. Then, $U_q > 0$ at $c_1/c_2 = 1$. With one exception, by continuity of U_q there are at least two values of c_1/c_2 such that $U_q = 0$. The exceptional case occurs when the graph of U_q is tangent to the horizontal axis. This case is clearly non-generic. We have established that if $\psi_{MD}(1) > 0$, there are generically two allocations which satisfy the necessary conditions for equilibrium, if there are any.

Suppose next that $\psi_{MD}(1) < 0$. Then, $U_q < 0$ at $c_1/c_2 = 1$. By continuity of U_q there is at least one value of $c_1/c_2 < 1$ such that $U_q = 0$. This value satisfies the necessary conditions for an equilibrium.

Next, we show that if $\psi_{MD}(1) < 0$, then $c_1/c_2 = 1$ satisfies the necessary conditions for an equilibrium. We do this by examining the behavior of U_q when $c_1/c_2 = 1$. In this case, the allocation functions are not differentiable functions of q. The problem is, as we show in the appendix, that for $q > 1$ the cash in advance constraint is binding, while it is not binding for $q < 1$. The allocation functions are different in the two cases because the equations characterizing a private sector equilibrium are different.[7] When the cash in advance constraint is not binding, we replace it by $R = 1$ in (28).

In the appendix, we establish that the right derivative of U with respect to q, denoted by $U_{q\downarrow 1}$, is identical to (22). We also establish that the left derivative of U, denoted by $U_{q\uparrow 1}$, is strictly positive.

These observations imply that a necessary condition for $R = 1$ to be a Markov equilibrium is that the right derivative of U be non-positive. That is, $\psi_{MD}(1) \leq 0$. To see this, notice that, when the right derivative of U is non-positive, the monetary authority has no incentive to raise q. Since the left derivative is strictly positive, the authority also has no incentive to reduce q.

We have established:

Proposition 1: Suppose the first order conditions for an equilibrium are sufficient. Then, generically, there are at least two Markov equilibria with a fixed payment technology.

Next, we show that, in a Markov equilibrium with a fixed payment technology, the equilibrium interest rate is constant.

Proposition 2: In a Markov equilibrium with a fixed payment technology, the interest rate, R, does not depend on the realization of the technology shock, θ, and of government consumption, g.

Proof: Notice from (23) that ψ_{ID} does not depend on θ or g. This is true of ψ_{MD} as well (see (37) and (39)). Thus, c_1/c_2 does not depend on θ or g. The result follows because $R = (c_2/c_1)^{1-\rho}$ (see (28)).

In this subsection we have argued that multiple equilibria are possible in our model. The proof of Proposition 1 suggests that the multiplicity arises because of the Laffer Curve shape of the inflation distortion function, ψ_{ID}. This shape in turn depends on $(R-1)M/P$ going to zero as the interest rate R goes to infinity. That is, it depends on money demand being sufficiently elastic with respect to the interest rate. One conjecture is that if money demand is not very elastic, then the equilibrium is unique. In the Appendix, we develop a model in which rM/P does not go to zero as r goes to infinity. In this model we find that there is a unique equilibrium.

C Numerical Examples

To illustrate the results in Proposition 1, we constructed two deterministic examples. We obtained parameter values for the baseline example as follows. Some of the parameters are obtained from the money demand relationship in our model. To develop this relationship, denote private purchases of consumption goods by

$$c^p = zc_1 + (1-z)c_2.$$

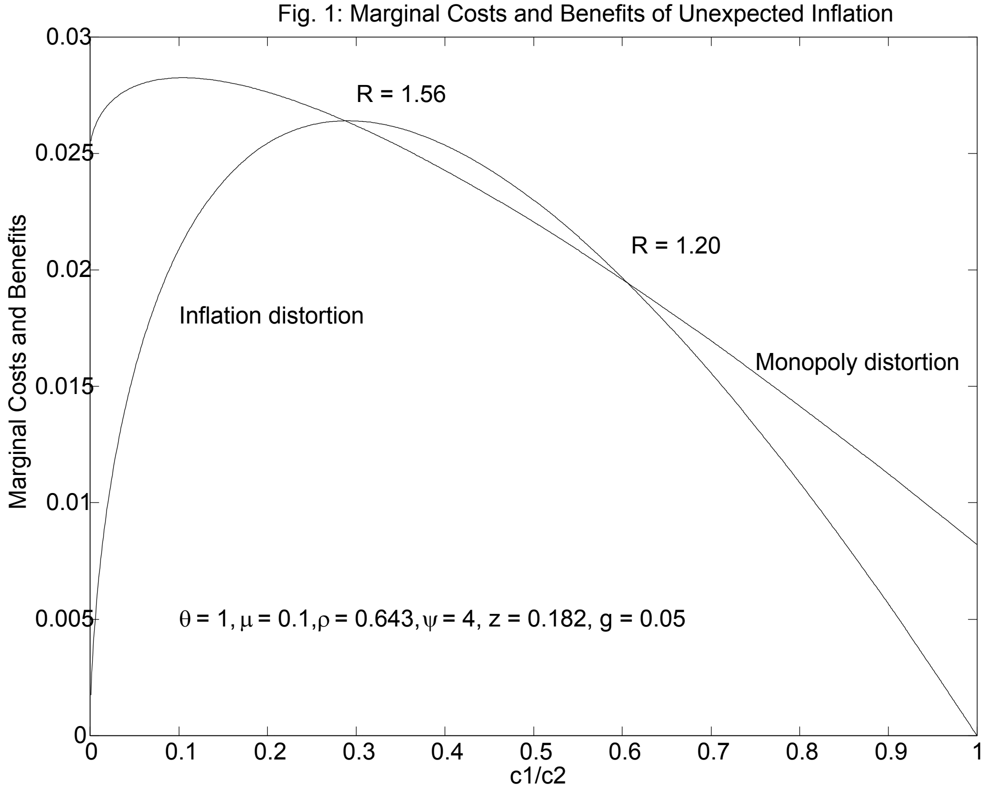
Notice this is the value of consumption goods purchased in markets by the households in the model since both cash and credit goods sell for the same price. Using (28) and that $zc_1 = M/P$ from the cash in advance constraint, we have that, in a Markov equilibrium, the following relationship must hold:

$$\frac{c^p}{M/P} = 1 + \frac{1-z}{z} R^{\frac{1}{1-\rho}}.$$

This relationship can be interpreted as a money demand equation. We regressed $\log(Pc^p/M - 1)$ on $\log R$ and a constant. Under the appropriate orthogonality condition on the regression, the parameters z and ρ can be obtained from the least squares estimates. Based on this results of this procedure, we set $z = \bar{z} = 0.182$ and $\rho = 0.643$.[8] From Christiano and Eichenbaum (1992) we obtained an estimate of $\psi = 4$. We used a value of $\mu = 0.1$, which is somewhat higher than results reported in Parks.[9] We set $g = 0.05$, so that government consumption, as a fraction of output, is roughly consistent with what it is in the data. Finally, we normalized $\theta = 1$.

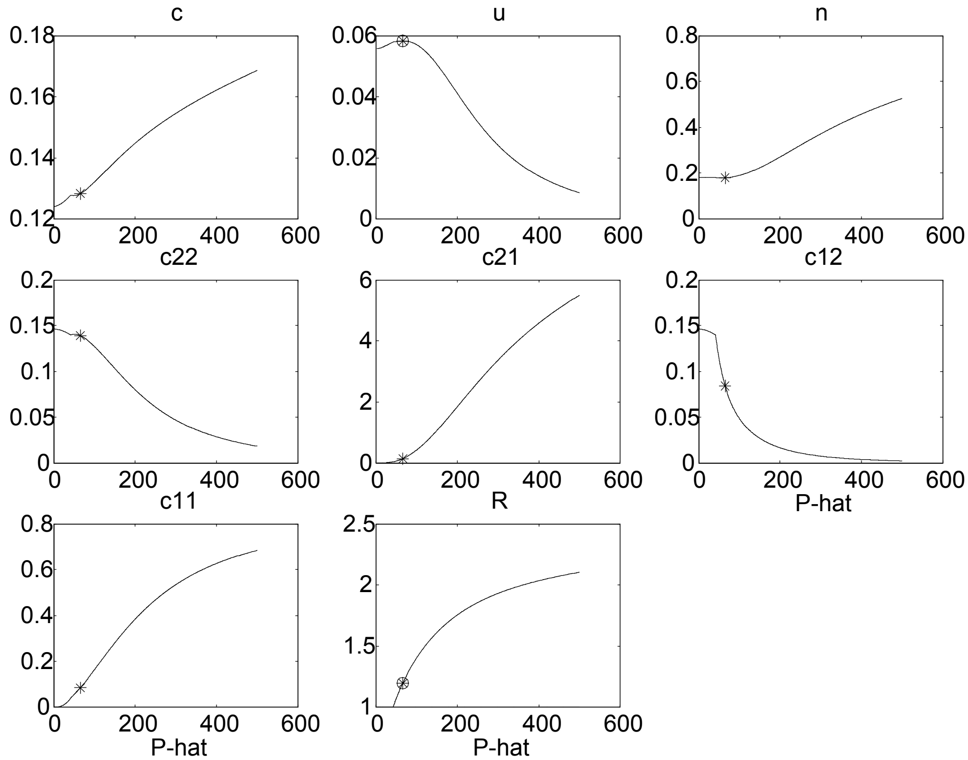
Figure 1 displays the monopoly distortion, ψ_{MD}, and the inflation distortion, ψ_{ID}, for $c_1/c_2 \in (0,1)$. An equilibrium value of c_1/c_2 is one for which the two are equal or, $c_1/c_2 = 1$ and $\psi_{MD} \leq 0$. In this example, the necessary conditions for an optimum are satisfied at $R = 1.20$ and $R = 1.56$. The candidate low inflation equilibrium corresponds to $P^e = \hat{P} = 65$, and the candidate high inflation equilibrium corresponds to $P^e = \hat{P} = 130$. Recall that these prices are scaled by the beginning of period aggregate stock of money. So, the low inflation equilibrium corresponds to the low price (i.e., high real balances) and the high inflation equilibrium corresponds to the high price.

Fig. 1: Marginal Costs and Benefits of Unexpected Inflation



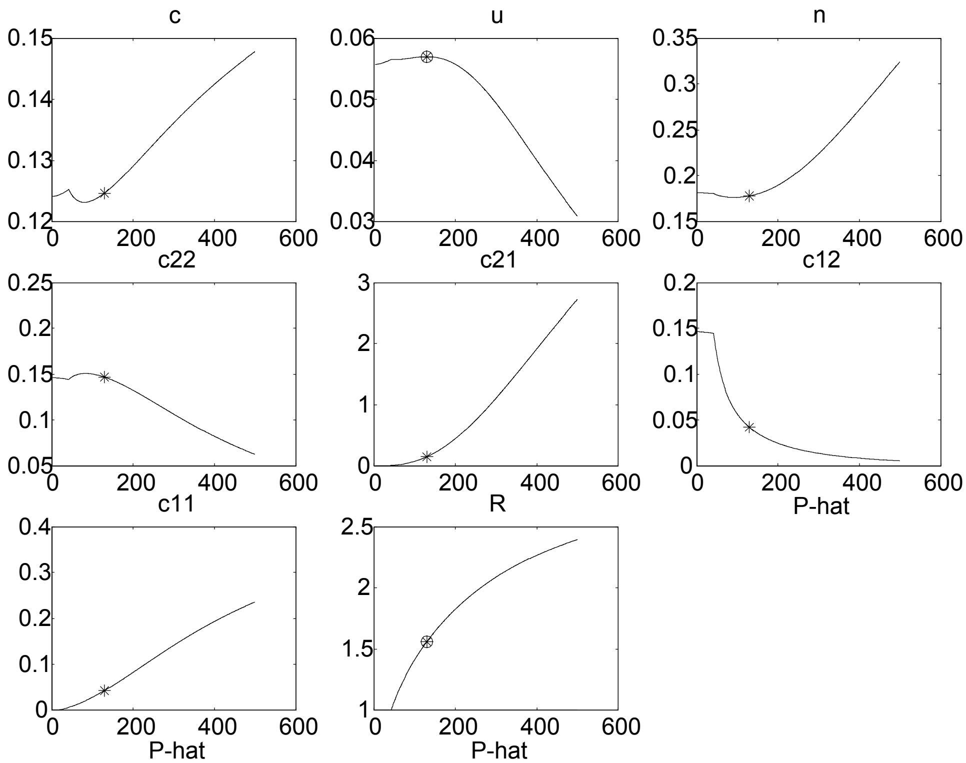
To verify that these are in fact equilibria, we need to confirm that, in each case, $\hat{P} = P^e$ in fact solves the monetary authority's problem, (19). Doing so is also instructive because it makes possible examining the costs and benefits of unanticipated inflation in each case. the monetary authority is choosing the optimal value of $\hat{P}$ in each case. Figure 2 displays the impact on c_{ij}, c, n, R, and utility of variations in $\hat{P}$ over the range, $[0, 500]$ when P^e takes on its value in the candidate low inflation equilibrium. The star in the figure markes the candidate equilibrium identified in Figure 1. There are several things worth noting in this figure. First, note that utility is maximized at the conjectured equilibrium. Thus, it indeed is an equilibrium. Second, note how employment increases with unexpected inflation. Note, too, how the consumption of flexible price goods (c_{12}, and c_{22}) drops, as their price rises with an unexpected inflation. Finally, note the lack of differentiability in the private sector's response to $\hat{P}$ as $\hat{P}$ drops below the point where $R = 1$.[10]

Figure 2: Consequences of Deviation in Low Inflation Equilibrium

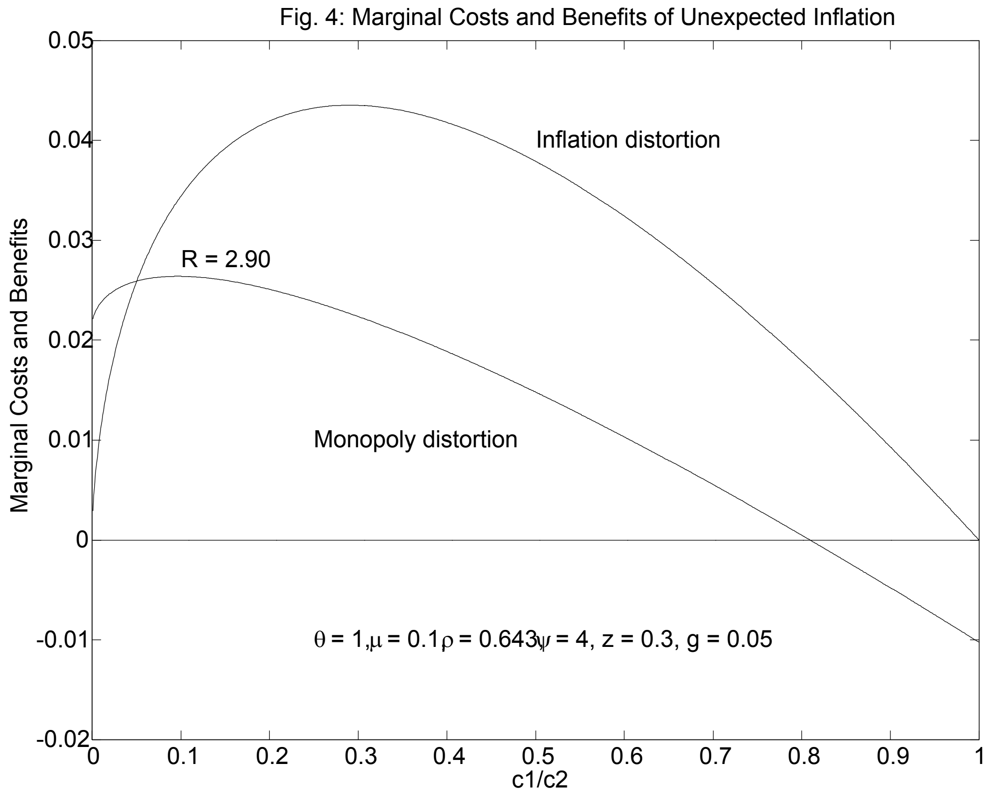


Next, we examine the response of the private economy to $\hat{P} \neq P^e$ when P^e takes on its value in the candidate high inflation equilibrium. The results are displayed in Figure 3 and are similar to those in Figure 2. In particular, it is evident that utility in fact is maximized at $\hat{P} = P^e$, so that this is indeed an equilibrium.

Figure 3: Consequences of Deviation in High Inflation Equilibrium

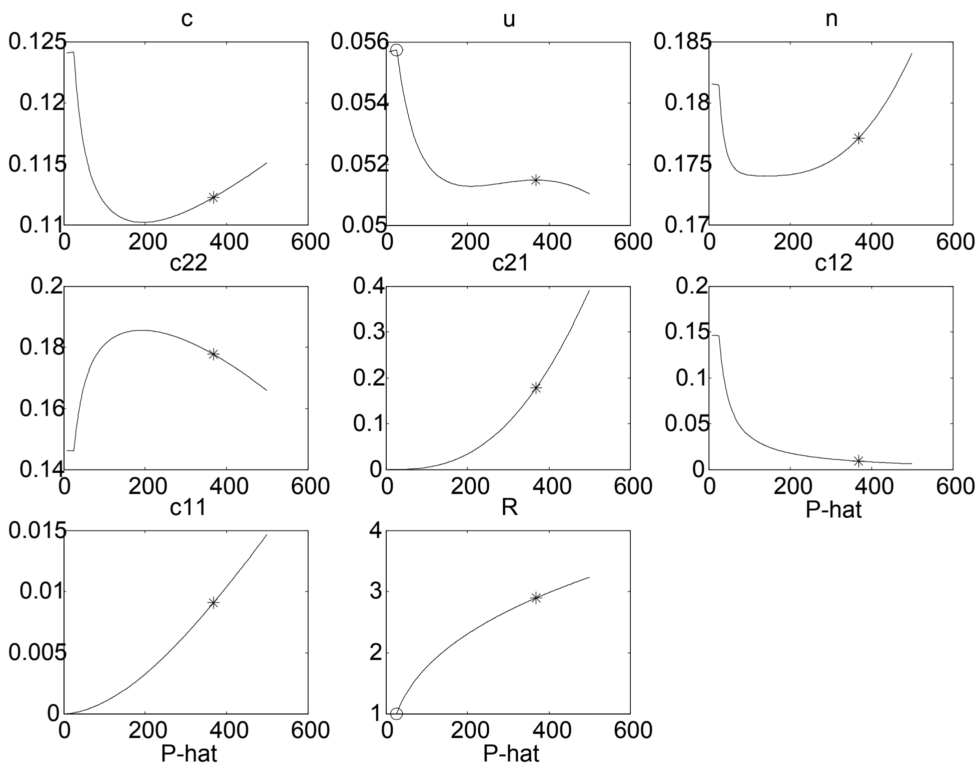


We considered a second example, which illustrates two interesting possibilities. For this example, we simply increased z and $\bar{z}$ to 0.3. The inflation and monopoly distortion curves are presented in Figure 4. Notice that in this example, the monopoly distortion is negative for $c_1/c_2 = 1$. Thus, the Ramsey allocations are a Markov equilibrium.



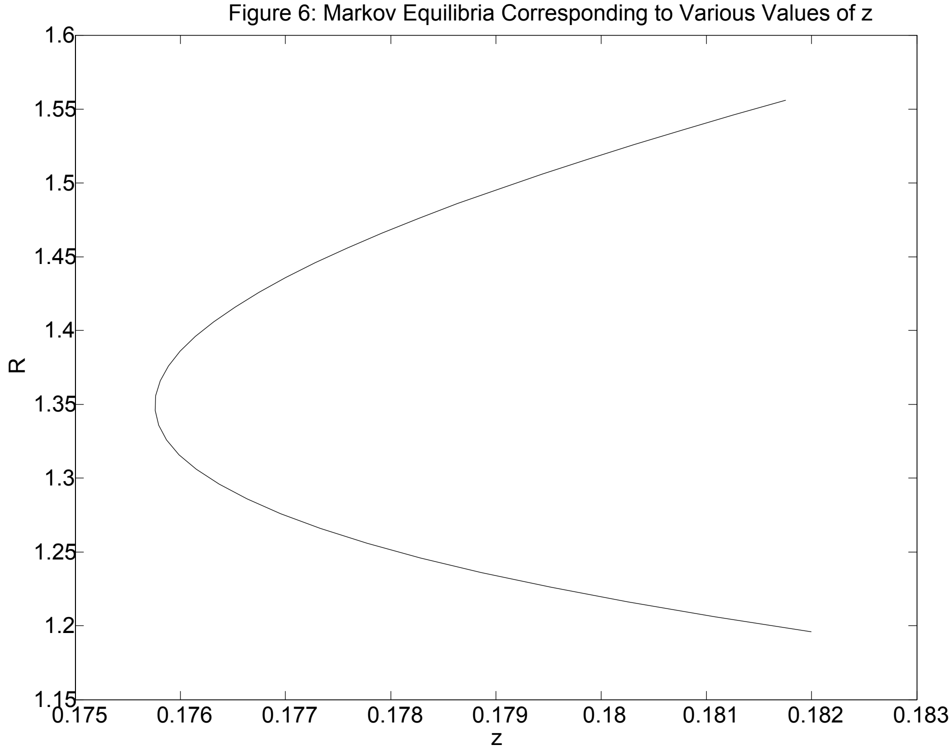
Notice too, that there is a candidate high inflation equilibrium, with $R = 2.90$. Here, $P^e = 368$. To determine if this is actually an equilibrium, we investigate whether $\hat{P} = P^e$ is a global maximum. Figure 5 displays the family of private sector equilibria associated with $\hat{P} \in [0, 500]$. The 'star' in the figure indicates the candidate equilibrium. Note that while $\hat{P} = P^e$ is a local maximum, it is not a global maximum. The global maximum, indicated by a circle in the figure, is at $\hat{P} = 25$. We conclude that in this example, there is a unique Markov equilibrium, and that the inflation bias in that equilibrium is zero. This finding is consistent with Proposition 1, since the condition of that proposition is not satisfied.

Figure 5: Consequences of Deviation in High Inflation Equilibrium



In the following section, we endogenize the variable, z. For this, it is convenient to summarize the Markov equilibria associated with each fixed z. We do this by exploiting the fact that for each fixed $c_1/c_2 \in [0, 1]$ there is either no value of z that sets $U_q = 0$ in (24), or exactly one. We identified a range of values of c_1/c_2 where the value of z that sets $U_q = 0$ satisfies $z \leq \bar{z} = 0.182$. For each of these c_1/c_2, we computed z. We verified numerically that each $z, (c_1/c_2)$ combination obtained in this way satisfies the sufficient conditions for a Markov equilibrium.[11] Figure 6 displays the results. There, we report $R = (c_2/c_1)^{1-\rho}$, rather than c_1/c_2. Note how the collection of z, R that represent Markov equilibria are characterized by a 'horsehoe' shape. For each z lying in an interval beginning for z a little below 0.176 and extending to $\bar{z}$, there are two Markov equilibria.

Figure 6: Markov Equilibria Corresponding to Various Values of z



IV Analysis with Variable Payment Technology

We now turn to the full model, in which the payments technology is determined endogenously. The analysis of the previous section is a basic building block. The R, z combinations that satisfy $U_q = 0$, where U_q is defined in (24), summarize the implications of monetary authority optimization, the Markov equilibrium condition and private economy equilibrium for given z (we refer to c_1/c_2 and R interchangeably, since there is a monotone relationship, $R = (c_2/c_1)^{1-\rho}$, between them). Equation (24) implicitly defines a function (correspondence, actually) mapping from the private payment technology decision (see the horizontal axis in Figure 6) into the monetary policy's choice, here characterized in terms of R (see the vertical axis). Loosely, we can think of this as the *monetary authority's response, given the payment technology, z*. To analyze the full model, we need to take into account the fact that z is actually chosen by households. Their first order condition for z, together with other equations that must hold in a private sector equilibrium, define a function mapping R into z. We can think of this equation as the *private sector's response, given anticipated policy, R*. The reason for the adjective, anticipated, of course, is that the household chooses z before monetary policy is realized. The intersection of these two response functions is a Markov equilibrium for our economy.

In the first subsection below, we derive the private sector's best response function. The following subsection analyzes the full model. That section notes that if technology shocks dominate in the model, then it predicts that in a high inflation equilibrium the interest rate should be negatively correlated with output while it should be positively correlated with output in a low inflation equilibrium. The final section presents cross-country evidence which provides modest support for this implication.

A Implications of Optimal Payment Technology Choice

The first order condition associated with the household's optimal choice of the fraction of goods purchased with cash, z, is:

$$\left(1-\frac{1}{\rho}\right)\frac{1-(c_2/c_1)^{\rho}}{z+(1-z)(c_2/c_1)^{\rho}}=\frac{\psi\eta(\bar{z}-z)^{\nu}}{1-n-(\bar{z}-z)^{1+\nu}\eta/(1+\nu)}. \tag{25}$$

The right side corresponds to the gain, in terms of time released from operating the transactions technology, that occurs when there is a marginal increase in z. The left side summarizes costs. First, there is a drop in the consumption of the goods which are converted into cash goods with a rise in z, as long as $R>1$. This drop gives rise to a fall in utility. Second, the increased consumption of cash goods associated with a rise in z tightens the cash in advance constraint, (5), which is costly when $R>1$. This cost is partially offset by the impact on the household asset evolution equation, (6), of the fact that total expenditures on goods falls with an increase in z when $R>1$. The considerations having to do with the impact on the cash in advance constraint and on the asset evolution equation of an increase in z are captured by Lagrange multipliers, which have already been substituted out from the left side of (25).

It is easy to verify that for each fixed $R\geq 1$ and n, there is at most one value of z that satisfies (25). This can be seen by noting that the term on the left of the equality is non-decreasing, while the term on the right is decreasing, in z. Similarly, as R increases, z falls. This can be seen by noting that the expression on the left of the equality in (25) rises, while the expression on the right remains unchanged, with a rise in R. The intuition here is simple. When R increases, purchasing goods with cash is increasingly costly because it entails foregone interest. Thus, with R high households find it optimal to purchase more goods on credit.

But, this analysis is insufficient because it presumes n in (25) is constant, while n can be expected to change with R. If n fell with a rise in R, then the reasoning in the previous paragraph suggesting that higher R is associated with lower z, is reinforced. However, it appears difficult to establish rigorously that increases in R always lead to a fall in n. Still, by substituting out for n in (25) using the household first order condition for labor and the resource constraint, it is possible to establish that higher R leads to lower z. We do this now.

Evaluating the labor first order condition in (14) and using our functional forms, we obtain:

$$\frac{1}{1-n-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}}=\frac{\theta\rho\,(c/c_2)^{1-\rho}}{\psi c}. \tag{26}$$

Substituting this into (25), and rearranging, we obtain:

$$\left(1-\frac{1}{\rho}\right)\frac{1-(c_2/c_1)^{\rho}}{z+(1-z)(c_2/c_1)^{\rho}}=\frac{\theta\rho\eta(\bar{z}-z)^{\nu}}{(c/c_2)^{\rho}\,c_2}.$$

This is not yet a relationship in terms of R and z only, because of the presence of c_2 in the denominator on the right hand side. However, combining the resource constraint, $\theta n = zc_1 + (1-z)c_2 + g$, with (26) yields an expression for c_2 in terms of c_1/c_2 which, when substituted into the previous expression, yields:[12]

$$\left(1-\frac{1}{\rho}\right)\frac{1-(c_2/c_1)^{\rho}}{z+(1-z)(c_2/c_1)^{\rho}}=\frac{\theta\rho\eta(\bar{z}-z)^{\nu}}{z\left(\frac{c_1}{c_2}\right)^{\rho}+1-z}\times\frac{z\frac{c_1}{c_2}+\frac{\psi z}{\rho}\left(\frac{c_1}{c_2}\right)^{\rho}+(1-z)(1+\frac{\psi}{\rho})}{\theta\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-g},$$

or, after rearranging and making use of the expression for R in terms of c_1/c_2 :

$$\frac{\left(\frac{1}{\rho}-1\right)\left[1-R^{\frac{\rho}{\rho-1}}\right]}{z\left[R^{\frac{1}{\rho-1}}+\frac{\psi}{\rho}R^{\frac{\rho}{\rho-1}}\right]+(1-z)(1+\frac{\psi}{\rho})}=\frac{\rho\eta(\bar{z}-z)^{\nu}}{\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-\frac{g}{\theta}}. \quad (27)$$

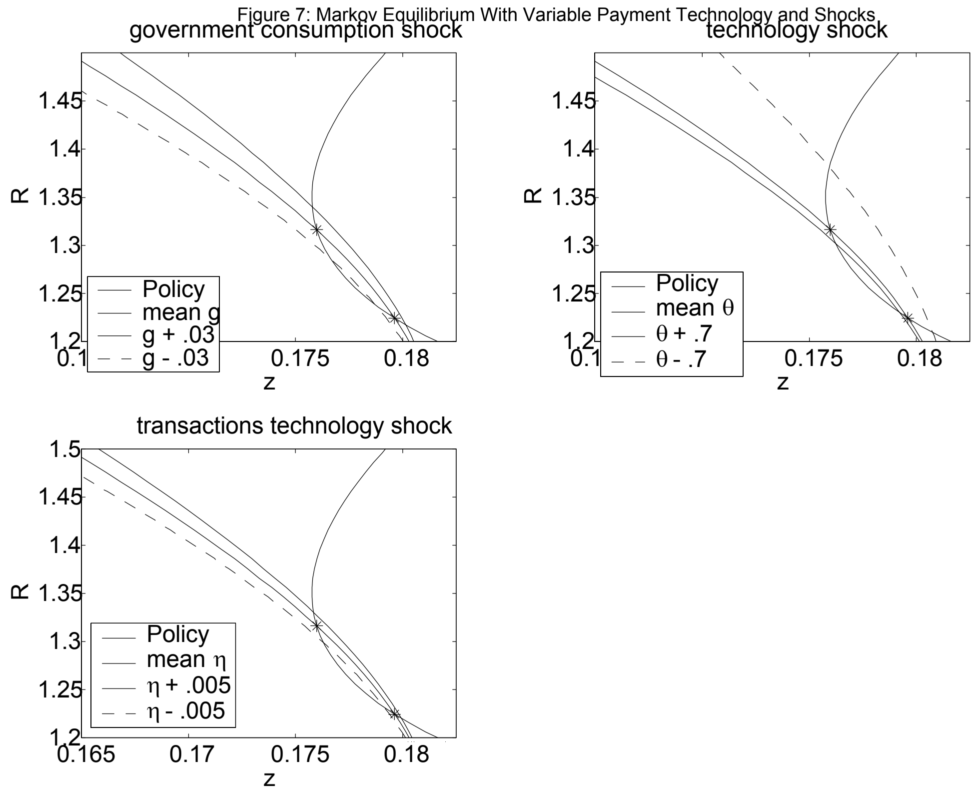
For each fixed R there is at most one z that solves this expression. To see this, note that the left hand side is increasing in z for $R \geq 1$, while the right hand side is decreasing. In addition, z is decreasing as R increases. This can be seen by noting that the left side of the above expression is increasing in R. Thus, taking into account the endogeneity of n does not overturn the intuition underlying the negative relationship between R and z described for (25). With higher R, households optimally choose smaller z. This mapping from R to z is what we have called the private sector's response to anticipated policy.

It is easy to use (27) to deduce the impact on the private sector response of shocks. An increase in g/θ or η shifts the right side of the equality up and so increases z for fixed R.

B Markov Equilibrium with Variable Payment Technology

Figure 7 displays the private sector and monetary authority response functions, for various values of the exogenous shocks. The monetary authority response function is simply taken from Figure 6. Note how the private sector response is downward-sloped, consistent with the analysis in the previous subsection. As suggested in the analysis in the previous subsection, the private sector response shifts with shocks. At the same time, consistent with Proposition

2, the monetary authority response does not respond to shocks.



For each realization of the shocks, the private sector and monetary authority responses intersect twice. Thus, there are two Markov equilibria. In the low interest rate Markov equilibrium households expect the monetary authority to pursue a low inflation policy and choose to purchase a large fraction of goods with cash. The monetary authority, confronted with households who have chosen to purchase a large fraction of goods with cash finds that the marginal cost of inflation is high since a large fraction of goods are purchased with cash. Conversely, in the high interest rate Markov equilibrium, households expect the monetary authority to pursue a high inflation policy and choose to purchase a small fraction of goods with cash. The monetary authority, confronted with households who have chosen to purchase a small fraction of goods with cash, finds that the marginal cost of inflation is low and optimally chooses a relatively high inflation rate.

An interesting feature of our model is that the correlation between R and z switches sign for a given shock, depending whether the economy is in a high inflation or a low inflation equilibrium. Thus, in the low inflation equilibrium interest rates are relatively high when the technology shock is relatively high. In contrast, in the high inflation equilibrium, interest rates are relatively low when the technology shock is relatively high. Output in both equilibria is increasing in the size of the technology shock. Thus, the model displays the feature that in the low inflation equilibrium the correlation between output and the interest is positive, while in the high inflation equilibrium this correlation is negative. The precise sign of the correlation in each equilibrium depends upon the relative sizes of the shocks and their covariances. In this sense, the theory does not impose sharp implications for all stochastic processes generating shocks. Nevertheless, we would expect that if the technology shock were relatively more important than the others, the correlation between output and interest rates would be higher in the low inflation equilibrium than in the high inflation equilibrium.

C A Modest Comparison with the Cross-Country Data

This implication of the model motivates looking at the interest rate, output correlations in the data. For this, we examined data on a cross-section of countries. In particular, we examined annual data from two kinds of countries: those with high inflation and those with low inflation.[13] We defined high inflation countries as those with interest rates that exceed 100 percent in at least one year during the sample for which we have data, and low inflation countries as a subset of OECD countries. We excluded transition economies of Eastern Europe from our sample. In each case, data on output and short-term interest rates were taken from the International Monetary Fund's IFS data base. The output data were logged, and then Hodrick-Prescott filtered. The interest rate data are denominated in annual percentage point terms. These were simply Hodrick-Prescott filtered.[14]

Table 1 reports correlations for the high inflation countries in our sample. In most cases these countries experienced periods of very high inflation and periods of relatively low inflation. We define periods of high inflation to be periods when the nominal interest rate exceeds 50 percent per year, while the other periods are periods of low inflation. Fortunately, these periods turned out - with minor exceptions - to be contiguous. As can be seen from Table 1, there are five countries which have episodes of high and low inflation. With one exception, the correlation between output and interest rates is higher in the low inflation episode than in the high inflation episodes. Table 1 also reports the average value of the correlation between output and the interest rate for all countries in low inflation episodes and in high inflation episodes. Again, the correlation is higher in low inflation episodes than in high inflation episodes.

Table 1: Evidence from High Inflation Economies

Country	Low Inflation		High Inflation		Period of	Period of
	$\rho(y, R)$	mean, R	$\rho(y, R)$	mean, R	Low Inflation	High Inflation
Argentina	-0.43	8.15	-0.57	923928.26	1992 - 2000	1980 - 1991
Brazil	-0.09	21.39	0.02	2362.59	1963 - 1980	1981 - 1995
Brazil	-0.82	25.16	NA	NA	1996 - 2000	NA
Chile	-0.12	25.85	-0.62	73.89	1984 - 2000	1977 - 1983
Israel	0.57	21.93	-0.68	245.70	1972 - 2000	1979 - 1987
Peru	0.44	28.78	-0.57	846.35	1995 - 2000	1986 - 1994
Turkey	NA	NA	-0.45	68.15	NA	1987 - 2000
Uruguay	NA	NA	-0.36	88.10	NA	1976 - 2000
Column mean	-0.08	21.88	-0.46	132516.15	NA	NA

In Table 2, we report on a comparison of the correlation between output and the interest rate between high and low inflation countries. In each case, we use the entire available time series to compute the correlation. As can be seen from this table, the average value of this correlation is negative for high inflation countries and it is marginally positive for the low inflation countries.

Table 2: Evidence from High and Low Inflation Economies

Country	$\rho(y, R)$	mean, R	Sample Period
Argentina	-0.59	527962.50	1980 - 2000
Brazil	0.03	946.05	1963 - 2000
Chile	-0.36	39.86	1977 - 2000
Israel	-0.24	113.47	1979 - 2000
Peru	-0.46	519.32	1979 - 1993
Turkey	-0.41	68.15	1987 - 2000
Uruguay	-0.30	88.10	1976 - 2000
USA	0.20	6.15	1955 - 2000
Austria	0.48	6.09	1967 - 1998
Belgium	0.32	5.22	1953 - 1998
Denmark	-0.31	9.81	1972 - 2000
Finland	0.18	9.68	1978 - 2000
Ireland	0.15	10.65	1971 - 1999
France	0.09	6.96	1950 - 1998
Germany	0.54	5.38	1960 - 2000
Italy	0.09	11.28	1969 - 2000
Japan	0.24	6.21	1957 - 2000
Netherlands	-0.04	5.79	1960 - 1998
New Zealand	0.48	11.11	1985 - 2000
Spain	0.33	11.60	1974 - 2000
Sweden	0.01	8.75	1966 - 2000
Switzerland	0.43	3.40	1969 - 2000
United Kingdom	0.03	7.78	1969 - 2000
Canada	0.40	8.36	1975 - 2000
Mean, High Inflation	-0.33 (-0.29)	75676.78	NA
Mean, Low Inflation	0.21 (0.21)	7.90	NA

We also computed the standard deviation of output and interest rates in the various countries. For the sample of countries in Table 1, we found that $\sigma_y = 0.0234$ and $\sigma_y = 0.0457$ in low and high inflation episodes, respectively. Here, σ_y is the sample standard deviation of logged and then Hodrick-Prescott filtered data, averaged across all episodes. We also found that $\sigma_R = 3.57$ and $\sigma_R = 350900$,in low and high inflation episodes, respectively. Here, σ_R is the sample standard deviation in the interest rate averaged across episodes. Thus, output volatility is a little higher, but interest rate volatility is hugely higher in high inflation episodes.

We turn now to the volatility of the variables in Table 2. There, we find $\sigma_y = 0.0232$ and $\sigma_y = 0.0443$ for low and high inflation countries, respectively. We find $\sigma_R = 1.8535$ and $\sigma_R = 283320$ for low and high inflation countries, respectively. These findings are consistent with the ones reported in Table 1.

We then simulated 500 artificial observations from our model. The mean levels of the government consumption shock, the technology shock, and the money demand shock are 0.05, 1, 0.2 respectively. The autocorrelations of the government consumption shock, the technology shock and the money demand shock are 0.9 each, while the standard deviations of the innovations are 0.001, 0.05, 0.0005 respectively. We filtered the artificial data from the model in the same way that the actual data were filtered. We found that $\sigma_y = 0.017$ in both high and low inflation equilibria, while $\sigma_R = 0.007$ and $\sigma_R = 0.065$ in the low and high inflation equilibria, respectively. The model obviously fails to match the level of volatility in these variables in the data. However, it is interesting that the model predicts the interest rate is an order of magnitude more volatile in the high inflation equilibrium, which output volatility is essentially the same. We also computed the correlation between logged and filtered output and the filtered interest rate. That correlation is 0.731 in the low inflation equilibrium and -0.733 in the high inflation equilibrium.

We interpret the results for correlations and relative volatility across high and low inflation equilibria as providing modest support for the model.

V Stability of High and Low Inflation Equilibria Under Learning

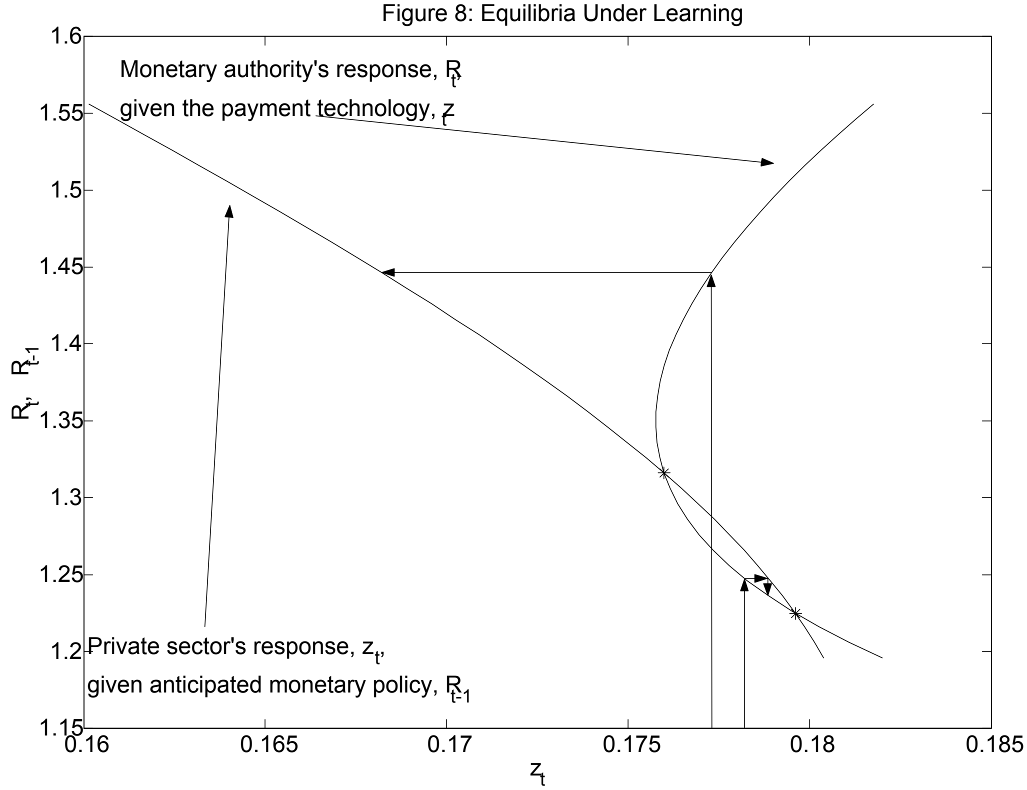
In the previous section we identified two Markov equilibria for our model. Here, we ask whether simple learning mechanisms could be used to rule out one of these equilibria. A variety of learning schemes could be examined. For example, the government might learn about some feature of the private economy or private agents may be uncertain about the government's policy. Our environment is sufficiently simple that a variety of such learning environments can be considered. Here, we consider the case in which households learn about government policy.

When households determine the period t payments technology, z_t, they have to form a guess about monetary policy. Here, we assume that they guess monetary policy will produce an interest rate equal to the one that was realized in the previous period, R_{t-1}. As before, their decision is made after the realization of the period t shocks. So, z_t is the value of z

implied by (27) when R is replaced by R_{t-1}. The monetary authority's response function is what it was before, taking z_t as input and generating R_t as output. It is defined by $U_q = 0$ in (24).

The dynamics of the deterministic version of the economy are displayed in Figure 8. Recall that for every z_t there are two monetary policy responses. The results in the figure suggest that only the lower one is consistent with equilibrium under learning. The high rate of interest produced by jumping to the upper branch produces such a low value of z in the next period that there is not equilibrium then for the model with fixed exogenous payments technology. Now, suppose that the monetary authority's response selects the lower of the two values of R. Then, we see that only the low inflation equilibrium is stable under learning. The high inflation equilibrium is not.

Figure 8: Equilibria Under Learning



VI Conclusion

The results in this paper show that absence of commitment in monetary policy is in principle capable of rationalizing a high inflation bias, as well as prolonged periods of low and high inflation. This work is preliminary. We have provided only modest initial evidence of support for the model from the data. In addition, learning considerations need to be studied further before concluding that all the multiple Markov equilibria for the model are 'reasonable'.

A Appendix 1:

This appendix derives various results used in section B. For later reference, it is useful to know that when utility is given by (**??**), then (14) reduce to:

$$
\begin{aligned}
c_{12} &= c_{11} q^{\frac{-1}{1-\rho}},\ c_{22} = c_{21} q^{\frac{-1}{1-\rho}}, \\
R &= \left(\frac{c_{22}}{c_{12}}\right)^{1-\rho},\ \theta\left(1-n\right) = \frac{\psi}{\rho} c^{\rho} c_{22}^{1-\rho}.
\end{aligned} \tag{28}
$$

Incorporating these results into the cash in advance constraint gives:

$$
c_{11}\left\{\mu + (1-\mu) q^{\frac{-\rho}{1-\rho}}\right\} \leq \frac{1}{zP^e}. \tag{29}
$$

We now develop formulas for c_q and n_q. Consider first c_q. Differentiating (2), and evaluating the derivatives at $q = 1$, we obtain:

$$
c_q = \left(\frac{c}{c_1}\right)^{1-\rho} z\left[\mu c_{11,q} + (1-\mu) c_{12,q}\right] + \left(\frac{c}{c_2}\right)^{1-\rho} (1-z)\left[\mu c_{21,q} + (1-\mu) c_{22,q}\right], \tag{30}
$$

where $c_{ij,q}$ is the derivative of c_{ij} with respect to q. From the resource constraint, we obtain:

$$
n_q = \frac{z\left[\mu c_{11,q} + (1-\mu) c_{12,q}\right] + (1-z)\left[\mu c_{21,q} + (1-\mu) c_{22,q}\right]}{\theta}. \tag{31}
$$

Substituting for c_q and n_q in (21) we obtain

$$
U_q = u_c\left[\left(\frac{c}{c_1}\right)^{1-\rho} - \left(\frac{c}{c_2}\right)^{1-\rho}\right] z\left[\mu c_{11,q} + (1-\mu) c_{12,q}\right] + \left[u_n + \theta u_c (\frac{c}{c_2})^{1-\rho}\right] n_q. \tag{32}
$$

Using (28), we obtain

$$
U_q = u_c\left(\frac{c}{c_2}\right)^{1-\rho} (R-1)\, z\left[\mu c_{11,q} + (1-\mu) c_{12,q}\right] + \left[u_n + \theta u_c (\frac{c}{c_2})^{1-\rho}\right] n_q. \tag{33}
$$

From (29) and (28),

$$
c_{11,q} = c_1 \frac{(1-\mu)\rho}{1-\rho},\ c_{12,q} = -c_1 \frac{1-(1-\mu)\rho}{1-\rho}, \tag{34}
$$

so that

$$
\mu c_{11,q} + (1-\mu) c_{12,q} = -c_1 (1-\mu). \tag{35}
$$

We proceed now to prove four results used in section B. First, we show that n_q is of the form:

$$
n_q = \frac{c_2 \psi_{MD}\left(\frac{c_1}{c_2}\right)}{(1-\rho)\theta}.
$$

Second, we show that $\psi_{MD}(0) > 0$. Third, we show that $U_{q\downarrow 1}$ is identical to (22). Fourth, we show that $U_{q\uparrow 1}$ is strictly positive.

A First result

To establish the first result, we begin by differentiating (28) to obtain,

$$c_{22,q} = c_{21,q} - \frac{c_2}{1-\rho}. \tag{36}$$

Combining this and (35) with (31), we obtain:

$$\frac{(1-\rho)\theta}{c_2} n_q = (1-\rho)\left\{-\frac{c_1}{c_2} z(1-\mu) + (1-z)\left[\frac{c_{21,q}}{c_2} - \frac{1-\mu}{1-\rho}\right]\right\} \tag{37}$$

To get $c_{21,q}$ we work with the labor first order condition in (28), after substituting out for θn from (16) and for c using (2) and for c_{12} and c_{22} from (28) to obtain:

$$\begin{aligned}
\theta &= g + [zc_{11} + (1-z)c_{21}]\left[\mu + (1-\mu)q^{\frac{-1}{1-\rho}}\right] \\
&\quad + \frac{\psi}{\rho}\left[zc_{11}^{\rho} + (1-z)c_{21}^{\rho}\right]\left[\mu + (1-\mu)q^{\frac{-\rho}{1-\rho}}\right] c_{21}^{1-\rho}\frac{1}{q}.
\end{aligned}$$

Totally differentiating this expression, we obtain:

$$c_{21,q} = \frac{-c_{11,q} z\left[1+\psi\left(\frac{c_2}{c_1}\right)^{1-\rho}\right] + [zc_1 + (1-z)c_2]\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}c^{\rho}c_2^{1-\rho}\frac{1-\rho\mu}{1-\rho}}{(1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)\left(\frac{c}{c_2}\right)^{\rho}} \tag{38}$$

Using (34), this reduces to:

$$\frac{c_{21,q}}{c_2} = \frac{-\frac{c_1}{c_2}\frac{(1-\mu)\rho}{1-\rho} z\left[1+\psi\left(\frac{c_1}{c_2}\right)^{\rho-1}\right] + \left[z\frac{c_1}{c_2} + 1 - z\right]\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\frac{1-\rho\mu}{1-\rho}}{(1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)\left(\frac{c}{c_2}\right)^{\rho}}. \tag{39}$$

Substituting (39) into (37) it is easily verified that n_q is of the desired form.

B Second Result

To verify $\psi_{MD}(0) > 0$, it is sufficient to establish that the expression in square brackets in (37) is positive when $c_1/c_2 = 0$. Evaluating this, taking into account (39):

$$\begin{aligned}
\frac{c_{21,q}}{c_2} - \frac{1-\mu}{1-\rho} &= \frac{(1-z)\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}(1-z)\frac{1-\rho\mu}{1-\rho}}{(1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)(1-z)} - \frac{1-\mu}{1-\rho} \\
&= \frac{1}{1-\rho}\frac{1-\mu+\frac{\psi}{\rho}(1-\rho\mu)}{1+\psi+\frac{\psi}{\rho}(1-\rho)} - \frac{1-\mu}{1-\rho} \\
&= \frac{1}{1-\rho}\left\{\frac{1-\mu+\frac{\psi}{\rho}(1-\rho\mu) - (1-\mu)(1+\psi) - (1-\mu)\frac{\psi}{\rho}(1-\rho)}{1+\psi+\frac{\psi}{\rho}(1-\rho)}\right\} \\
&= \frac{1}{1-\rho}\frac{\psi}{\rho}\frac{(1-\rho)\mu}{1+\psi+\frac{\psi}{\rho}(1-\rho)} > 0.
\end{aligned}$$

This establishes the desired result.

C Third Result

To establish the third result, it suffices to establish that the interest rate is increasing in q at the point $c_1/c_2 = 1$. That is, since $R = (c_{22}/c_{12})^{1-\rho}$, we need $c_{21,q} \geq c_{11,q}$ at the point $c_{21} = c_{11} = c_{22} = c_{12} = c$. Substituting for $c_{21,q}$ from (38), we need to show that

$$\frac{-c_{11,q}z\left[1+\psi\right] + c\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}c\frac{1-\rho\mu}{1-\rho}}{(1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)} \geq c_{11,q},$$

or

$$\left\{\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}\frac{1-\rho\mu}{1-\rho}\right\} \geq \frac{c_{11,q}}{c}\left\{z(1+\psi) + (1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)\right\},$$

or, substituting for $c_{11,q}$ and simplifying,

$$\left\{\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}\frac{1-\rho\mu}{1-\rho}\right\} \geq \frac{(1-\mu)\,\rho}{1-\rho}\left\{(1+\psi) + \frac{\psi}{\rho}(1-\rho)\right\}$$

or,

$$1-\mu+\frac{\psi}{\rho}\,(1-\rho\mu) \geq (1-\mu)\,\rho(1+\psi) + (1-\mu)\,\psi(1-\rho).$$

Dividing through by $1-\mu$, we need to show that

$$1+\frac{\psi}{\rho}\frac{1-\rho\mu}{1-\mu} \geq \rho(1+\psi) + \psi(1-\rho)$$

or

$$1+\frac{\psi}{\rho}\frac{1-\rho\mu}{1-\mu} \geq \rho+\psi.$$

Since $\rho \leq 1$ and $(1-\rho\mu)\,/\,[\rho(1-\mu)] = (1/\rho-\mu)\,/(1-\mu) \geq 1$, we have the desired result.

D Fourth Result

To obtain the fourth result, we can see from (33) that, since the first term is zero and the term in square brackets is positive, the result follows if $n_q > 0$. We establish this result here.

Inspecting (30) and (31), and evaluating the derivatives at $c_1 = c_2 = c$, it follows that

$$c_q = \theta n_q.$$

From (28), we have

$$c_{12,q} = c_{11,q} - \frac{c}{1-\rho},\;\; c_{22,q} = c_{21,q} - \frac{c}{1-\rho}$$

Totally differentiating the equation, $R = 1$, i.e., $c_{22} = c_{12}$, we obtain $c_{22,q} = c_{12,q}$. Using this result in the previous equation, we obtain

$$c_{11,q} = c_{21,q}.$$

Using these results in (30), we obtain

$$c_q = \mu c_{11,q} + (1 - \mu) c_{12,q},$$

or

(40)
$$c_q = c_{12,q} + \frac{\mu c}{1 - \rho}.$$

Next, totally differentiating the labor first order condition in (28) and using $c_q = \theta n_q$ and $c_{22,q} = c_{12,q}$, we obtain

(41)
$$c_q = -\frac{\psi(1 - \rho)}{\rho(1 + \psi)} c_{12,q}.$$

Substituting for $c_{12,q}$ from (41) into (40), we obtain

$$c_q = \frac{\mu c}{1 - \rho} \frac{1}{1 + \frac{\rho(1+\psi)}{\psi(1-\rho)}} > 0.$$

Since $n_q = c_q/\theta$, the desired result follows.

B Appendix 2: Model With Inelastic Money Demand

In the previous analysis, we found that the equilibria of our model depend upon the elasticity of substitution in utility between cash and credit goods. In that model, this is the same as the elasticity of demand faced by suppliers. To ensure that their profit function is bounded above, it is necessary that that elasticity be no less than unity. To understand the robustness of our results to situations in which the elasticity of substitution between cash and credit goods is low, we break the link between the elasticity of demand faced by suppliers and the elasticity of substitution between cash and credit goods. We do this by modifying the household's utility function. The market structure of the firm sector, and firm technology remain the same. In addition, the sequence of events in the period is also unchanged. That is, at the beginning of the period a fraction of firms set their prices. Then, the monetary authority selects its action. Finally, the remaining prices and quantities for the period are determined in a private sector equilibrium. We abstract from uncertainty in this section.

A Firms

The firm sector is essentially identical to what it was before. There is a continuum of goods. Each good is produced by a monopolist who faces a demand curve with elasticity denoted here by $1/(1-\lambda)$, where $0 < \lambda < 1$. Some firms ('sticky price firms') set prices before the monetary authority takes it current period action, and other firms ('flexible price firms') set their price afterward. All firms operate competitively in homogeneous factor markets. As before, sticky and flexible price firms set prices as follows:

$$P^e = \frac{W(P^e)}{\lambda},\ \hat{P}(s,x) = \frac{W(s,x)}{\lambda}. \tag{42}$$

where the state s now consists only of the price set by the sticky price firms, P^e. With one exception, all the notation is the same as before. The exception is that ρ has been replaced by λ.

B Households

Preferences are as in (1), with

$$c = \left[zc_1^{\rho} + (1-z)\,c_2^{\rho}\right]^{\frac{1}{\rho}},\quad -\infty < \rho \leq 1$$

and

$$c_1 = \left[\int_0^1 c_1(x)^{\lambda} dx\right]^{\frac{1}{\lambda}},\ c_2 = \left[\int_0^1 c_2(x)^{\lambda} dx\right]^{\frac{1}{\lambda}}. \tag{43}$$

The individual goods, $c_i(x)$, $x \in (0,1)$, $i = 1, 2$, are produced by the firms discussed in the previous section.

To purchase cash goods, $c_1(x)$, households must use cash accumulated in advance:

$$M - \left[P^e \mu_1 c_{11} + \hat{P}(s,x)\,(1-\mu_1)\,c_{12}\right] \geq 0, \tag{44}$$

where c_{1i} denotes consumption of the sticky price cash goods when $i = 1$ and of the flexible price cash goods when $i = 2$ (c_{2i} is the corresponding notation for credit goods.)[15] Also, μ_1 denotes the fraction of cash goods whose prices are sticky (μ_2 is the corresponding fraction sticky price credit goods).

As before, the household begins the period with nominal assets, A. It then goes to the asset market where it faces constraint, (4). The household's nominal assets evolve as follows:

$$\begin{aligned} xA' \leq\ & W(s,x)n + R(s,x)A + (x-1) + D(s,x) - (R(s,x)-1)M \\ & - \left[P^e \mu_1 c_{11} + \hat{P}(s,x)(1-\mu_1)c_{12}\right] - \left[P^e \mu_2 c_{21} + \hat{P}(s,x)(1-\mu_2)c_{22}\right], \end{aligned} \tag{45}$$

where W, R, D and x are as defined before.

The household problem is formally identical to (7), with (6) replaced by (45), (5) replaced by (44), and (2) replaced by:

$$c = \left[z \left[\mu_1 c_{11}^{\lambda} + (1-\mu_1) c_{12}^{\lambda} \right]^{\frac{\rho}{\lambda}} + (1-z) \left[\mu_2 c_{21}^{\lambda} + (1-\mu_2) c_{22}^{\lambda} \right]^{\frac{\rho}{\lambda}} \right]^{\frac{1}{\rho}}. \tag{46}$$

As before, the solution to the household's problem yields decision rules of the form, $n(A, s, x)$, $M(A, s, x)$, $A'(A, s, x)$, and $c_{ij}(A, s, x)$, $i, j = 1, 2$.

Note how our specification of the household problem disentangles the elasticity of substitution between cash and credit goods, c_1 and c_2, from the elasticity of demand for the individual goods, $c_i(x)$, $i = 1, 2$, $x \in (0, 1)$.

C Markov Equilibrium

Our definition of a Markov equilibrium coincides with the definition given in the previous section, with the obvious modifications. For example the labor market clearing condition is:

$$n(1, s, x) = \mu_1 c_{11} + (1-\mu_1) c_{12} + \mu_2 c_{21} + (1-\mu_2) c_{22},$$

where c_{ij} is as previously defined.

D Characterization

This section displays the qualitative properties of the Markov equilibrium of our economy. We proceed as in section on the benchmark model. In particular, we first derive the equations which characterize a Markov equilibrium. For this, we need to first construct the private sector allocation rule, the mapping from government policies to the prices and quantities that define a private sector equilibrium. We then need to express the first order conditions for the monetary authority, who optimizes subject to the private sector allocation rule. In the second section we use our equations to characterize the set of Markov equilibria for the model.

D.1 Private Allocations and Prices

The monetary authority's action, x, is taken at a time when P^e is known. The private sector prices and quantities to be determined are c_{ij}, $i = 1, 2$, q, n, w, R, where $q = \hat{P}/P^e$. As before, we find it convenient to think of the government's policy variable as q instead of x. So, we compute c_{ij}, $i = 1, 2$, n, w, R as a function of q and P^e.

We proceed now to pin down the seven unknowns, c_{ij}, $i = 1, 2$, n, w, R, conditional on q and P^e. For this, we use 7 equations that characterize the equilibrium. As before, the 7 equations depend upon whether or not the cash in advance constraint is binding.

The resource constraint is:

$$g + \mu_1 c_{11} + (1-\mu_1) c_{12} + \mu_2 c_{21} + (1-\mu_2) c_{22} = n. \tag{47}$$

Given our utility function, the first order conditions can be written as follows:

$$c_{12} = c_{11} q^{\frac{-1}{1-\lambda}}, \tag{48}$$

$$c_{22} = c_{21} q^{\frac{-1}{1-\lambda}}, \tag{49}$$

$$R = \frac{z}{1-z} \left(\frac{c_1}{c_2}\right)^{(\rho-\lambda)} \left(\frac{c_{21}}{c_{11}}\right)^{1-\lambda}, \tag{50}$$

$$\lambda = \frac{\psi c^{\rho} c_2^{\lambda-\rho} c_{22}^{1-\lambda}}{(1-n)(1-z)}. \tag{51}$$

The cash in advance constraint is given by:

$$P^e \mu_1 c_{11} + q P^e (1-\mu_1) c_{12} \leq 1. \tag{52}$$

When the cash in advance constraint is not binding, then we impose $R = 1$, i.e.,

$$\frac{z}{1-z} \left(\frac{c_1}{c_2}\right)^{(\rho-\lambda)} \left(\frac{c_{21}}{c_{11}}\right)^{1-\lambda} = 1. \tag{53}$$

As before, we use these equations to define the private sector allocation rules, $c_{ij}(P^e, q)$, $i = 1, 2$, $R(P^e, q)$, $n(P^e, q)$.

D.2 Government Problem

We can summarize the previous discussion as providing functions:

$$c = c(P^e, q), \ \ n = n(P^e, q),$$

where c is obtained by substituting (46) into (2). These functions can be substituted into the utility function,

$$U(P^e, q) = u\left[c(P^e, q), n(P^e, q)\right].$$

Define

$$q(P^e) = \arg\max_{q \in D} U(P^e, q).$$

The function, $q(P^e)$, is the monetary authority's best response, given P^e. Equilibrium requires that $q(P^e) = 1$. This equilibrium requirement allows us to construct the equilibrium price, P^e.

D.3 Qualitative Characteristics of Markov Equilibria

As in section B, we begin by considering equilibria which satisfy $R > 1$. In this case, the allocation rules are differentiable and U_q is given by

$$U_q = u_c c_q + u_n n_q$$

Adding and subtracting $u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho} n_q$, we obtain

$$U_q = u_c\left[c_q - (1-z)\left(\frac{c}{c_2}\right)^{1-\rho} n_q\right] + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \tag{54}$$

Differentiating (46) and evaluating the result at $q = 1$,in which case $c_{ij} = c_i$, $i, j = 1, 2$, we obtain:

$$c_q = \left(\frac{c}{c_1}\right)^{1-\rho} z\left[\mu_1 c_{11,q} + (1-\mu_1)c_{12,q}\right] + \left(\frac{c}{c_2}\right)^{1-\rho}(1-z)\left[\mu_2 c_{21,q} + (1-\mu_2)c_{22,q}\right],$$

which is just (30). We now derive n_q, by differentiating (47):

$$n_q = \mu_1 c_{11,q} + (1-\mu_1)c_{12,q} + \mu_2 c_{21,q} + (1-\mu_2)c_{22,q}. \tag{55}$$

Substituting out for c_q and n_q in the first set of square brackets in (54), we obtain:

$$\begin{aligned} U_q &= u_c\left[\mu_1 c_{11,q} + (1-\mu_1)c_{12,q}\right]\left[\left(\frac{c}{c_1}\right)^{1-\rho} z - \left(\frac{c}{c_2}\right)^{1-\rho}(1-z)\right] \\ &\quad + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \end{aligned}$$

Using the expression for the interest rate, (50), we obtain:

$$\begin{aligned} U_q &= u_c\left[\mu_1 c_{11,q} + (1-\mu_1)c_{12,q}\right](1-z)\left(\frac{c}{c_2}\right)^{1-\rho}(R-1) \\ &\quad + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \end{aligned}$$

Differentiating (52) with respect to q, and evaluating the result at $q = 1$:

$$\mu_1 c_{11,q} + (1-\mu_1)c_{12,q} = -(1-\mu_1)c_1. \tag{56}$$

Substituting this into the preceding expression, we obtain:

$$\begin{aligned} U_q &= -u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}(R-1)\, c_1(1-\mu_1) \\ &\quad + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \end{aligned} \tag{57}$$

Notice that this expression is essentially the same as the corresponding expression, (22), in the benchmark model. In this case, however, it is no longer true that the first term in (57) is zero when $c_1/c_2 = 0$. To see this, it is convenient to write (57) as

$$
(58) \qquad U_q = -\left[u_c c_2(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right]\psi_{ID}\left(\frac{c_1}{c_2}\right) + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q,
$$

where

$$
\psi_{ID}\left(\frac{c_1}{c_2}\right) = \left[\frac{z}{1-z}\left(\frac{c_1}{c_2}\right)^{(\rho-1)} - 1\right]\frac{c_1}{c_2}(1-\mu_1).
$$

When $\rho > 0$, it is possible to use exactly the same kind of argument used in the previous section to demonstrate that there are at least two Markov equilibria. When $\rho < 0$, $\psi_{ID}(c_1/c_2) \to \infty$ as $c_1/c_2 \to 0$. When $\rho = 0$, ψ_{ID} converges to a constant as $c_1/c_2 \to 0$. Therefore, in these cases, it is not possible to use the same argument as earlier to demonstrate multiplicity of equilibria.

Indeed, in the log case ($\rho = 0$), for certain values of the model parameters, it is possible to provide an analytical expression for the best response. This expression shows that there is a unique equilibrium, which also yields the outcome, $R = 1$. We have also constructed robust numerical examples in which it appears that the Markov equilibrium is unique.

To summarize, a key result of the two previous sections is that the issue of multiplicity of equilibria turns on the elasticity of money demand at very high rates of inflation. Specifically, we found that the multiplicity issue depends on the behavior of

$$
(R-1)\frac{c_1}{c_2}.
$$

In both economies, c_1 is proportional to M/P, and when inflation rates are high, c_2 is approximately proportional to aggregate consumption. Thus, the multiplicity of equilibria depends on the behavior of $(R-1)M/(Pc)$ at high inflation rates. This expression is equivalent in a sense to the magnitude of the inflation tax, where the net nominal interest rate is interpreted as the tax rate and the base, of course, is the stock of real balances. One interpretation of our results is that if these inflation tax revenues go to zero, as inflation goes to infinity, there are necessarily multiple equilibria, while if the inflation tax revenues do not go to zero, there are often unique equilibria.

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Notes

[1]Our model formalizes the idea in Kydland and Prescott (1977) and in Barro and Gordon (1983) that an unanticipated monetary expansion raises output and can raise welfare. For models and evidence on the effects of inflation on relative allocations, see Cukierman (1983), Parks (1978), and Vining and Elwertowski (1976).

[2]See Aiyagari, Braun and Eckstein (1998), Cole and Stockman (1992), Dotsey and Ireland ( ), Freeman and Kydland (1994), Ireland (1994), Lacker and Schreft (1996), and Schreft (1992) for similar models of financial intermediation.

[3]Of the first four first order conditions in the first two lines of (14), only three are independent.

[4]This must be established rigorously.

[5]In practice, we obtain the functions in (18) and determine D as follows. For given s, P^e, q, we first solve (14), (16) and the cash in advance constraint as a strict equality for c_{ij}, $i, j = 1, 2$, n, x, R. If R computed in this way satisfies $R \geq 1$, then q, $P^e \in D$. If R violates $R \geq 1$ we resolve the system, replacing the cash in advance constraint by $R = 1$, i.e., $u_{11}/u_{21} = u_{12}/u_{22} = z/(1-z)$. If the cash in advance constraint is satisfied for the resulting values of c_{ij}, $i, j = 1, 2$, n, x, then q, $P^e \in D$. If not, then q, $P^e \notin D$.

[6]We will discuss why we need not be concerned with the case in which this derivative is different from zero at an optimum. A remark about differentiability will be added here.

[7]For details, see an earlier footnote.

[8]The estimation period is 1970Q1 to 1997Q1. For our monetary aggregate, we used $M1$ (FM1) and for our output measure we used GDP (GDP) (Citibase Mnemonics appear in parentheses). For the interest rate, we used the three-month Treasury bill rate (FYGM3).

[9]Parks (1978) reports that a one percentage point rise in aggregate inflation is associated with a rise in the variance of the log of relative prices of 0.015. Consider a version of this model in which the money stock is stochastic and growth rates of money are i.i.d. over time. In this version it is easy to show that a 1% rise in inflation relative to its steady state value is associated with an increase in the variance of relative prices of $\mu(1-\mu)$ suggesting a value of μ of roughly 0.017.

[10]To verify that sufficient conditions for an optimum are satisfied, we proceeded as follows. Corresponding to each of the two candidate equillibria, there is a value of P^e. For each P^e we examined the graph of $U(s, P^e, q)$ for a wide range of values of q. In each case, we verified that $q = 1$ is the global maximum.

[11]For each z and c_1/c_2 that set $U_q = 0$, we computed a candidate Markov equilibrium. We then verified that that candidate equilibrium is an actual equilibrium by confirming that it is consistent with monetary authority maximization. To compute the candidate equilibrium, we combined the optimization conditions for firms with the household first order condition for labor, to obtain an expression that relates z and c_1/c_2 to n :

$$\theta\left[1-n-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right]=\frac{\psi}{\rho}c^{\rho}c_2^{1-\rho}.$$

The levels of c_1 and c_2 can then be computed from the resource constraint, $\theta n = g + zc_1 + (1-z)c_2$. Finally, P is found using the cash in advance constraint, $Pzc_1 = 1$. Setting $P^e = 1/(zc_1)$, we then considered the private sector equilibria associated with a range of $\hat{P}$ about P^e to verify that period utility is maximized at $\hat{P} = P^e$.

[12]To see this, note that after rearranging (26), we obtain

$$\theta n=\theta\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}c_2.$$

Substituting this into the resource constraint, taking into account $c^{\rho} = zc_1^{\rho} + (1-z)c_2^{\rho}$, and rearranging, we obtain:

$$c_2 = \frac{\theta\left(1 - \frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right) - g}{z\frac{c_1}{c_2} + \frac{\psi z}{\rho}\left(\frac{c_1}{c_2}\right)^{\rho} + (1-z)(1+\frac{\psi}{\rho})}$$

[13]Explain data source

[14]The smoothness parameter in the HP filter was set to 100.

[15]As in the previous section, concavity of the utility function guarantees that households optimally choose to consume all sticky price cash goods at the same rate, and similarly for the flexible price cash goods and the sticky and flexible price credit goods.