Monetary neutrality in one specific class of DGE model with staggered prices

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Abstract

In this paper I show that monetary neutrality proposition holds for one specific parameterization of a dynamic general equilibrium model of monopolistic competition even if nominal rigidity in a form of staggered price setting or partial adjustment price-setting mechanism is present in a model. This parameterization is a result of a zero profit condition for intermediate goods producers and it requires that degree of increasing returns in intermediate goods production is equal to price-marginal costs markup.

Keywords: business cycles, output persistence, nominal rigidities, real rigidities.

1. Introduction

Staggered price setting models are usually used to analyze the dynamics of output and other real variables, such as investment, consumption and real wages in response to a nominal money supply shock. The most appealing feature of this class of models was thought to be their ability to generate output responses to a monetary shock that was more persistent than the underlying frequency of price adjustment (Taylor, 1980). However, recent research in this area, which incorporates staggered prices into the dynamic stochastic general equilibrium paradigm (Ball and Romer, 1990, Chari, Kehoe and McGrattan, 1996, Huang and Liu, 1998) finds that presence of nominal rigidities along is not enough to generate endogenously persistent (i.e. more persistent than the underlying frequency of price adjustment) output response to a monetary shock in a RBC-type model. In this paper I show that in one specific class of such a model money are neutral even if there is price staggering.

The paper is organized as follows. In the next section I present flexible prices version of the model and describe its equilibrium. In section 3 I introduce price staggering into the model and derive monetary neutrality result. Section 4 concludes.

2. The simple flexible prices model

2.1 Description of the model

The model presented here is a simplified version of the model of monopolistic competition developed by Blanchard and Kiyotaki (1987). The economy consist of a large number of homogeneous agents who maximize lifetime utility in the form

$$U_t = \sum_{k=0}^{\infty} \beta^k u(C_{t+k}, L_{t+k}), \tag{1}$$

where β is a discount factor and felicity function u positively depends on the level of agent's consumption of the composite final good C and negatively depends on the amount of labor services supplied to the market L and satisfies standard concavity assumptions.

Final good is produced competitively from the continuum of intermediate goods intermediate goods Y_i where $i\in[0;1]$ subject to constant returns to scale CES production function

$$C = (\int_0^1 Y_i^{\lambda} di)^{1/\lambda}, \tag{2}$$

where *1/(1-λ)* is an elasticity of substitution between different intermediate inputs. Each of the intermediate goods is produced by a different monopolist firm, all of which are owned by agents. For simplicity I assume that there is only one primary factor of production – labor but the main result will hold for the model which allows for two factors of production – labor and capital with possibility of investment although derivations become more complicated. All intermediate firms have access to the same production technology with production function given by

$$Y_i = L_i^{\gamma}, \tag{3}$$

where L_i is amount of labor services used in the production of *i*-th good and γ is a returns to scale parameter which maybe either greater or less than unity. The necessary condition for existence of equilibrium is $\gamma\lambda<1$, otherwise profit-maximizing firm would chose infinite amount of output. One could demonstrate that in the case $\gamma\lambda<1$ equilibrium profit share is equal to $1-\gamma\lambda$. As a staring point it is natural to assume that profit is equal to zero and $\gamma\lambda=1$ and, therefore, production of intermediate goods is subject to increasing returns – the case considered in Kiley (1997), Farmer (2000) and a number of other articles.

I assume that the log of the nominal money supply follows a random walk (or, equivalently, the money growth rate is a white noise process):

$$\log M_t = \log M_{t-1} + \varepsilon_t, \tag{4}$$

where ε_t are i.i.d. normal random variables with zero mean.

Real money balances are determined according to a cash-in-advance constraint

$$C_t = \frac{M_t}{P_t}. \tag{5}$$

2.2 Symmetric equilibrium

Each intermediate firm chose the price it charges for its output in order to maximize real profit which is equal to revenue less labor costs. Therefore, its objective function is given by

$$\max \frac{Y_i P_i}{P} - C(Y_i)\,, \tag{6}$$

where P_i is a price set by the *i*-th firm, P is a price of the final good (composite price index) which given CES production function and no-profit condition in the final good sector is equal to $P = (\int_0^1 P_i^{\lambda/(\lambda-1)} di)^{(\lambda-1)/\lambda}$ and $C(Y_i)$ is a real cost function of production Y_i, which is equal to wL_i, where w is a real wage. When setting its price firm takes aggregate price level and wage rate as given but as a monopolist it takes into account the demand function for its output which follows from (2) and is equal to

$$Y_i = Y(\frac{P_i}{P})^{\frac{1}{\lambda-1}}. \tag{7}$$

Maximization of (6) with respect to (7) results in equating marginal revenue and marginal costs which gives the following price setting rule:

$$P_i = \frac{1}{\lambda} MC_i P \tag{8}$$

which states that intermediate firm sets its price as a constant markup over its marginal costs. Since in symmetric equilibrium all firms charge the same price this price is equal to a price of a final good. So, the symmetric equilibrium condition is

$$P_i = P \text{ for all } i. \tag{9}$$

Therefore, every intermediate firm produces at the point where real marginal costs are equal λ. Since a cost function is given by

$$C(Y_i) = wY_i^{1/\gamma}. \tag{10}$$

Marginal cost are equal to

$$MC(Y_i) = \frac{1}{\gamma} wY_i^{1/\gamma-1}. \tag{11}$$

Agents supply labor to the market up to the point when marginal disutility of work is equal to marginal utility of consumption times real wage:

$$w = -\frac{U_L(L,C)}{U_C(L,C)}. \tag{12}$$

Given that in symmetric everyone work the same amount of time and all firms produce the same amount of output one can write

$$Y_i = C \text{ for all } i \text{ and } L_i = L = C^{1/\gamma} \text{ for all } i. \tag{13}$$

which allows to use (12) to represent real wage w as a function of final good output C. Then from (11) and the condition that $MC_i=\lambda$ one can write:

$$w(C)C^{1/\gamma-1} = \lambda\gamma = 1 \tag{14}$$

which can be used to find equilibrium level of output. Since money does not enter this equation any change in nominal quantity of money has no effect on real variables such as output or real wage rate and lead only to a corresponding change in prices charged by intermediate firms and an aggregate price index. This is a classical dichotomy result.

3. Monetary neutrality in the model with staggered prices

In this section I extend the model described above to allow for nominal price stickiness by introducing staggered price setting mechanism. The model is is similar to that analyzed in Kiley (1997). Namely, it assumes that intermediate firms cannot set their prices in every period. Instead, any firm must set the (nominal) price it charges for its output for two consecutive periods. Therefore, all intermediate firms can be divided into two categories: those who set prices in every odd period and those who set prices in every even period.

As a profit maximizer, any firm which sets price in a given period maximizes present discounted value of the real profit during the time in which price will be fixed. Therefore, objective function of firm i which sets price in period t can be written as:

$$\max \Gamma_t [\frac{P_{it} Y_{it}}{P_t} - C(Y_{it})] + E_t \{\Gamma_{t+1} [\frac{P_{it+1} Y_{it+1}}{P_{t+1}} - C(Y_{it+1})]\}, \qquad (15)$$

where Γ_{t+k} is a subjective discount factor for the *k-th* period into the future which is equal to the product of β^k and the marginal utility of consumption in period $t+k$ MU_{t+k}. When setting price firm takes aggregate price level and wage rate as given but as a monopolist it takes into account the demand function for its output (7). Straightforward maximization gives the following price-setting formula:

$$P_{it} = \frac{1}{\lambda} \frac{MU_t Y_{it} MC_{it} P_t + \beta E_t [MU_{t+1} Y_{it+1} MC_{it+1} P_{t+1}]}{MU_t Y_{it} + \beta E_t [MU_{t+1} Y_{it+1}]} \qquad (16)$$

which is similar to the price-setting formula in the flexible prices model in the sense that a firm chooses its price as a constant markup over its marginal costs but in the case when price is fixed for two periods a firm consider average expected marginal costs over these two periods, not only in a period when price is set. Substituting equations (7) and (11) into (16) one gets the following equation:

$$P_{it} = P_{it}^{(1/\gamma-1)/(\lambda-1)} \frac{MU_t Y_t^{1/\gamma} w_t P_t^{1/(\gamma-1)+1} + \beta E_t [MU_{t+1} Y_{t+1}^{1/\gamma} w_{t+1} P_{t+1}^{1/(\gamma-1)+1}]}{MU_t Y_t P_t^{1/(1-\lambda)} + \beta E_t [MU_t Y_t P_t^{1/(1-\lambda)}]} \qquad (17)$$

in which P_i can be cancelled because $\frac{1/\gamma - 1}{\lambda - 1} = 1$. After simplification and writing more compactly one arrives at the following:

$$1 = \frac{Q_t AC_t + \beta E_t [Q_{t+1} AC_{t+1}]}{Q_t + \beta E_t Q_{t+1}}, \qquad (18)$$

where $Q_t = MU_t C_t P_t^{1/(1-\lambda)}$ and $AC_t = w_t C_t^{1/(1-\lambda)}$ with term *AC* standing for average costs of production**.** Rearranging this and replacing $E_t[Q_t+1(AC_{t+1}-1)]$ with $Q_{t+1}(AC_{t+1}-1)-u_{t+1}$, where u_{t+1} is a one-step ahead forecast error of $Q_{t+1}(AC_{t+1} - 1)$ one gets

$$Q_{t+1}(AC_{t+1}-1) = -1/\beta \, Q_{t+1}(AC_{t+1}-1) + u_{t+1}. \qquad (19)$$

Because $1/\beta > 1$ this means that, if $Q_t(AC_t-1)$ is to be stationary, both $\mathrm{Var}[u_t]=0$ for all t and $Q_0(AC_0 - 1)=0$ should hold or $|Q_t(AC_t - 1)|$ will grow without bounds. Therefore, the unique stationary solution to (19) is the following:

$Q_t(AC_t\text{-}1)=0$ for every t. (20)

But since Q_t is not equal to zero (20) implies that

$AC_t=w_tC_t^{1/\gamma\text{-}1}=1$ (21)

or $w_t=C_t^{1\text{-}1/\gamma}$. (22)

Equation (21) is identical to the equation (14) from the previous section where flexible prices model is analyzed. This means that equilibrium condition in the staggered prices model with $\lambda=1/\gamma$ is the same as that in the flexible prices model. Since the flexible prices model has a unique equilibrium it will be also an equilibrium in the staggered prices model. This equilibrium has a property of a monetary neutrality. This means that money are neutral in the staggered prices model with $\lambda=1/\gamma$ and this is the main result of this paper.

Intuitive explanation of this result can be provided using marginal revenue and marginal costs curves framework. Assume that initially the economy is in its steady state which is the same as flexible prices equilibrium described in the previous section. In this equilibrium every firm equates marginal revenues to marginal costs. From equations (7) and (11) one may see that marginal revenue curve is a straight line with a slope λ-1 (on a logY_i – log MR_i plane) while marginal costs curve is a straight line with a slope $1/\gamma$-1 (also in logs). If $1/\gamma=\lambda$ then these two curves have the same slope and, therefore, they coincide in equilibrium. Now imagine that output changes from its steady state value following a monetary shock. The marginal revenue curve of the firm which sets price shifts to the right by the amount of increase in output. At the same time, because wage also increases, the marginal cost curve shifts upward by an amount of increase of wage which (in logs) is form (22) equal to $(1\text{-}\lambda)$ times the increase in output. But because one unit shift of the straight line with the slope of λ-1 to the right is equivalent to the $(1\text{-}\lambda)$ units upward shift of the same line the marginal cost and revenue curves are again the same in the new equilibrium. But this is exactly the condition of the flexible prices equilibrium. Taking into account the fact that the flexible prices equilibrium is unique and that money is neutral in that environment, I conclude that the assumption that output responds to the monetary shock is false.

4. Conclusions

In this paper I analyzed one special case of a new-Keynesian model of monopolistic competition with staggered price setting which is usually used to examine persistency properties of output in response to a nominal shock. I considered parameterization in which final and intermediate goods production structure is such that the steady state price – marginal costs markup is equal to the degree of increasing returns to scale in the intermediate good production sector which results in a zero profit in steady state. The main result of the paper is that the model parameterized as mentioned above has a classical dichotomy property, i.e. money is neutral and any increase in nominal money stock result in an equal and immediate increase in the aggregate price level.

In this final part of the paper I would like to mention several generalizations of the model for which neutrality proposition will still hold. Firstly, as it was noted in the beginning one can allow for two factors of production in intermediate goods sector – capital and labor thereby introducing non-trivial intertemporal saving-investment decisions through capital accumulation. Money will be neutral in this case if the price-marginal costs markup is equal to the value of returns to both factors together. Secondly, one may introduce a different price-setting rule, namely partial adjustment rule which is among others, considered in Calvo (1983) and Gali (1994). In this case every firm can potentially change its price in every period but only with some positive probability (which is the same for all firms and does not change over time). Monetary neutrality proposition of this paper will hold if staggered price setting rule is replaced by a partial adjustment rule.

It is also worth mentioning that the neutrality proposition is robust in the sense that when price-marginal costs markup ($1/\lambda$) and returns to scale parameter (γ) are not equal but one of then is approaching the other, output response to a monetary shock gets smaller until money become neutral in the case when two parameters are equal. The dynamics of this is the following. As it is shown in Chari, Kehoe and McGrattan (1996) the autoregression root of a process for output is negative in this case, i.e. output is below the trend next period after shock. As $\lambda\gamma$ approach unity the autoregression root gets closer to –1, i.e. the oscillatory dynamics of output amplifies. At the same time initial amplitude of the output increase becomes smaller until it equals zero and money becomes neutral when $\lambda\gamma$ is equal to one.

References

Ball, L and D. Romer. (1990), "Real Rigidities and the Non-neutrality of Money", *Review of Economic Studies* 57:183-203.

Blanchard, O.J and N. Kiyotaki (1987) "Monopolistic Competition and Effects of Aggregate Demand", *American Economic Review* 77(4):647-666.

Calvo, G. (1983), "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12: 383-89.

Chari, V., Kehoe, P. and E. McGrattan. (1996), "Sticky Price Model of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?", *Federal Reserve Bank of Minneapolis Staff Report* 217.

Farmer, R. (2000), "Two New Keynesian Theories of Sticky Prices", *Macroeconomic Dynamics,* 4, 1-34.

Gali, J. (1994), "Monopolistic Competition, Business Cycles and the Composition of Aggregate Demand", *Journal of Economic Theory* 63:73-96.

Huang, K. and Z. Liu. (1998), "Staggered Contracts and Business Cycles Persistence", *Federal Reserve Bank of Minneapolis Discussion paper* 127.

Kiley, M. (1997), "Staggered Price Setting and Real Rigidities", mimeo, *Federal Reserve Board*.

Taylor, J. (1980), "Aggregate Dynamics and Staggered Contracts", *Journal of Political Economy* 88:1-23.