Transmission mechanism, Equilibrium and Multiplier

Abstract:

Since science tries to deal with reality, it normally works with more or less imperfectly understood aproximations. In some cases, economists turn the theoretical assumptions, their convenient points of analytical emphasis, into axioms and take these axioms literally in their further research. Although approximations are necessary and efficient in modeling the simple economy, when we approach complicating ones, mathematical analysis of some models may show ridiculous results. The multiplier theory, a fundamental part of macroeconomics, is a case in point.

Here I first show a deduction of the "monetary transmission mechanism", and then analyze the main faults (misconceptions of multiplier theory) and discuss the multiplier theory. Finally, I propose my own idea of the multiplier theory and “monetary transmission mechanism.”

Prelude

The IS-LM model "shows that monetary policy influences income by changing the interest rate” and that "an increase in the money supply lowers the interest rate, which stimulates investment and thereby expands the demand for goods and services”, and Rudiger Dornbush, professor of MIT, who goes further to analyze another factor during the transmission------ the increase in consumption demand but attribute this only to the lower interest rate, summarizes the stages in the transmission mechanism as the following table:

(1)	(2)	(3)	(4)
Change in real Money supply	Portfolio adjustments lead to a change in asset prices and interest rates	Spending adjusts to the change in interest rate	Output adjust to the change in aggregate demand

In fact, consumption can also be influenced by the increasing income during the transmission process, with the increase of output that will in turn induce an increase in the national income (Y), the consumption demand will change consequently, according to the equation as follows:

$$\mathbf{C=c+cY_d} \qquad \mathbf{Y_d=Y+TR_0-(T+tY)}$$

Robert J. Gordon wrote more:" Finding themselves with more money than they need…. This raises the prices of stocks and reduces the interest rate. The initial decline in interest rate is called `liquidity effect` of a monetary expansion. The lower interest rate raises the desired level of autonomous consumption and investment spending requiring an increase in production. This is the `income effect of a monetary expansion." As regards the income effect, Let's look at the figure 1: after liquidity effect, the economy stands at point E_3 and then with the increase in income the interest rate will also rise. So the income effect is only started by the initial change of interest rate, and the whole process is an interactive course between the increasing income and rising interest rate.

However, these theories do not tell us the route from E_1 to E_2 (Figure 1, a case of monetary policy). Some academicians argue that the mechanism is like camera bellows, and we cannot and need not know the process. But we should at least prove that the economy is able to transmit from E_1 to E_2.

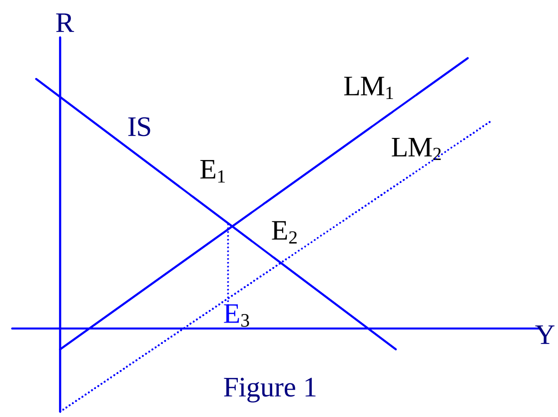


Figure 1

The cobweb model in microeconomics shows that although the equilibrium point is determined by the intersection of the supply curve and demand curve. However, after a shock to the supply, the economy will not always result in a convergent point, and even bounds larger and larger round the equilibrium.

Statistics shows that although the economy sometimes fluctuates sharply around the equilibrium, it will generally become stable, and such case never exists. I think the problem partly lies in the assumption. The model implies that everyone must behave rigidly. In fact, individuals are unpredictable, and a mathematical model cannot simplify their behavioral.

Moreover, some academicians made similar misleading deduction of the monetary transmission mechanism. Deduction 1 shows their theory. I personally think that although the multiplier theory is a wonderful tool to help us explain the change of economy, it also set our students` minds in a rigid way of thinking.

I developed an alternative deduction, which may give our students a new idea, though it has its approximations and does not fully resolve the problem.

Part 1: Mathematical deduction of monetary transmission mechanism (IS-LM model) -------Deduction 1:

In a simple three-department economy, which has no foreign trade:

Y is output or national income; C_0 is autonomous consumption; I is investment spending ($I=I_0-br$, where r is the interest rate and b measures interest response of investment, I_0 denotes autonomous investment spending); G is government purchase of goods and services; TR_0 is transfer to the private sector; t is tax rate; c is marginal propensity to consume out of disposable income; h is the interest elasticity of money demand; k is the elasticity of output. We derive two curves:

IS curve**:**

(1) $\mathbf{Y_0= (C_0+ cTR_0 +I_0 +G_0- br)/(1-c (1-t))=(A_0 -br)/(1-c (1-t))}$ $(A_0= C_0+ cTR_0 +I_0 +G_0)$

LM curve**:**

(2) $\mathbf{M/p=kY-hr}$,

Equilibrium point:

(3) $\mathbf{Y_0= (C_0+ cTR_0 +I_0 + G_0+ b\, Mp / h)/[1-c (1-t)+bk/h]}$

When faced with recession, the government should apply monetary and fiscal policy tools and LM-IS model shows how this policy will work**:**

An increase in the real money stock shifts the LM schedule to the right (Figure 1) .The asset markets adjust immediately, and interest rates decline between point E_1 and E_3 .The lower interest rate stimulate investment, and spending and income rise until a new equilibrium is reached at point E_2. Once all adjustments have taken place, a rise in the real money stock raises equilibrium income and lowers the equilibrium interest rates.

With the increase of output that will in turn induce an increase in the national income (Y), the consumption demand will change consequently, according to the equation as follows:

(4) $\mathbf{C=c+cY_d}$ $\mathbf{Y_d=Y+TR_0- (T+tY)}$

They demonstrated the transmission mechanism as follows:

[I] Initiative effects:

(1)The government increases real money stock by □M/p.

(2)Good markets adjust slowly and Y will not change in a short period, and thus interest rate will be down by □M/hp (**M/p=kY-hr ,□M/p=kY-h□r** .Otherwise the equation M/p=kY-hr will not hold after an increase on its left side), and the change in the economy reflects the common sense that asset market always adjust quickly.

(3)Lower interest rate will stimulate the investment demand and consequently increase the aggregate demand. Finally, the output and national income will rise.

The above stages can be demonstrated by the following graph:

□M/p —□R=□M/hP→ I ↑ —□I=b□M/hP =□AD→ AD ↑ —□AD=□Y→ Y ↑

[II] Induced effects:

1. Round 1

Y ↑ —□R=k□Y/h→ R ↑ —□I= -bk□Y/h→ I ↓ —$□Y_I$ = -bk□Y/h→ ↓

Notes: M/p=kY-hr, M/p will remain stable after the monetary policy is applied, and therefore, in order to hold the equation, r will rise as a result of the increase in Y. And this increase can be denoted as Y_i

Y ↑ —$□Y_d$= (1-t)□Y→ Y_d ↑ —$□Y_d$= (1-t)□Y, $□C=c□Y_d$→ C ↑ —□AD=□C→ AD ↑ → Y ↑ (This increase in Y is induced by increased consumption demand, and thus can be denoted as Y_c)

In round 1, the combination of effects induced by the initiative increase in Y is Y_c+Y_i, we may denote Y_c+Y_i by Y_1, then $\mathbf{Y_1=[c(1-t)-bk/h]□Y}$ .Similarly, In round 2, the combination of effects induced by the change of Y in round 1(denoted by Y_1) is $\mathbf{[c(1-t)-bk/h]Y_1}$,or to be more exact, $\mathbf{[c(1-t)-bk/h]^2□Y}$(denoted by Y_2).And in round 3, $\mathbf{[c(1-t)-bk/h]^3□Y}$……….

If **| c(1-t) -bk/h| < 1** , the successive terms in the series become progressively smaller, we can write out the successive rounds of increased output, starting with the initial increase in output, we may obtain a geometric series: **□Y** , $\mathbf{[c(1-t)-bk/h]^1□Y}$, $\mathbf{[c(1-t)-bk/h]^2□Y}$ …..

Moreover, the total change can be obtained by addi∑ng them up:.

(5) $$\sum Y_i=□Y\{1+[c(1-t)-bk/h]^1+[c(1-t)-bk/h]^2+\cdots\}=\sum□Y/[1-c(1-t)+bk/h]=b□M/hp/[1-c(1-t)+bk/h]$$

Problems:

1. This deduction seeks to use the multiplier theory to explain the mechanism. It implies that an increase in demand will immediately induce a same amount of increase in Y. If the economy is in recession how can the output gain such a sharp increase? Maybe only the demand can behave like this..

2. Now let’s look back to the initial effects: It’s doubtable that Y will increase until it meets the surplus demand induced by government policy while no other effects happen, when Y finishes the route, the economy enters into stage 2. Macro economy is the aggregate behaviors of individuals, including not only two farmers. Before the initial effect completes, other factors may have already changed.

3. The initiative increase in Y involves both increasing and decreasing effect in the national income, if Y_c and

Y_i can be added up together in a round, it must be given the assumption that the process of Y_c and Y_i both start and finish within exactly the same period, or the geometric series will fail. I personally think that it is the key reason why the first deduction fails. More details will be discussed in part 2.

4. The linear discrete dynamical systems can behave in some surprising ways.(Figure 3.2) When |c(1-t) -bk/h| < 1,a sequence obtained from a linear discrete dynamical system bounces around the equilibrium point and the bounces get smaller so that the sequence approaches the equilibrium point; but when |c(1-t) -bk/h| >1, it bounces around the equilibrium point but the bounces get larger so that the sequence does not approach the equilibrium point. If |c(1-t) -bk/h|>1 the initial effect is **b□M/hP**, while the predicted equilibrium output is **[C_0+ cTR_0 +I_0 + G_0+ b(M+□M)/ph]/[1-c (1-t)+bk/h]**, and the predicted total increase of outcome is **b□M/ph[1-c (1-t)+bk/h].**Obviously**, b□M/ph[1-c (1-t)+bk/h]< b□M/hP.** That means after the initial increase output exceeds that of the equilibrium. We can also figure out that in the second period, the output is below that of the equilibrium, and in the third period, the output exceeds that of the equilibrium again…. Just like a cobweb graph. If the economy is in recession and output is below that of the sufficient employment, how can the output gain such a sharp increase immediately after the increase in money stock? Maybe only the demand can behave like this. I do not think the geometric series can properly explain the mechanism.

Part 2: Multiplier

Here is a story: Suppose there is one government, two farmers: *Jack* who grows wheat and *Mike* who grows rice, in an economy. The government purchases $100`s worth of wheat. Jack uses $100c to buy rice, and then Mike earns $100c and uses $$100c^2$ to buy wheat, when suddenly wheat is out of stock, and Mike could only buy $$30c^2$ worth of wheat, and keep $$70c^2$ in hand. Jack earns $$30c^2$ and can only buy $$30c^3$ worth of rice. A few days later, Jack increases his production, and Mike could draw his excessive money and buy $$70c^2+\$30c^4$ worth of wheat. If the production is no longer out of stock afterwards, we can also derive a geometric series: $70c^2+30c^4$, $c(70c^2+30c^4)$, $c^2(70c^2+30c^4)$,……and add them up:$100+100c+30c^2+30c^3+70c^2+30c^4+c(70c^2+30c^4)+c^2(70c^2+30c^4)+\ldots=100+100c+(70c^3+30c^3)+(70c^2+30c^2)+(30c^4+70c^4)\ldots=100+100c+100c^2+100c^3+100c^4+\ldots=100/(1-c)$

We can deduce that if grain is frequently out of stock, there will be no geometric series, and the above formula will be more complex. However, if we add them up, the total increase in the simple economy is the same as predicted by the multiplier theory. When we study the multiplier process, the geometric series only give us a logical deduction. The number value of the increase of each round is indefinite.

The multiplier process 1 + c + c^2 + c^3 + ... + c^n = 1/(1 - c), for 0< c < 1 applies only for an economy with a fixed price level and a sufficiently large excess capacity to produce. In general, the multiplier 1/1(1 - c) tells us the distance by which the AD curve shifts rightward at a given price level. If we think about output being say Boeing 747s,the analysis is more complex simply because it takes time to build a plane.

IS curve represents the contrail of the equilibrium points of the good market. If we consider the transmission process, we must note that we are not dealing with an equilibrium course. The space between E1 and E2 is below the IS curve, and thus implies that the aggregate demand exceeds the aggregate supply. The GNP in this economy will increase by $100/(1-c) because the cumulative demand of each round is $100/(1-c). We can deduce the multiplier process in another way: (1) Increase purchase of good (2) demand in economy is □G ($100).(3)In the first period, income or output increases by □Y_1 (we need not know how much, but we suppose □Y_1<=□G, so output does not meet the demand in the first round).(4) consumption demand increases by c□Y_1. Note that output increases more than demand: □Y_1>c□Y_1.(5)Output can not meet demand, so it will still increase by □Y_2, and consumption demand increases by c□Y_2… The output growth will not stop until it meet the demand. Thus we can derive the equation: **Error!**$\mathbf{Y_i}$=□G+ c□Y_1 +c□Y_2+c□Y_3+…=□G+ c**Error!**$\mathbf{Y_i}$ **,** and obtain the number value of **Error!**$\mathbf{Y_i}$ by solving the equation:(1-c) **Error!**$\mathbf{Y_i}$=□G, **Error!**$\mathbf{Y_i}$=□G /(1-c) .Figure 2 shows the route of the multiplier process. This process can be demonstrated by a click on the icon:  multiplier process.exe

If the economy can adjust quick enough, then Jack can buy $100c^2 worth of rice immediately after he earns $100c, and each round is a process from one small unstable equilibrium to another. However, if the economy adjusts slowly, will the multiplier process fail? As the story suggests, if we add total change of each round, and change the equation into another form, we still obtain a geometric series. However, geometric series cannot always give us a satisfactory explanation of multiplier, especially when the multiplier proceeds in two independent courses.

Many academicians argue that the concept of multiplier is a most important tool to apprehend the influence upon economy imposed by the change of variables of a model. Let us look at the government purchase multiplier for example:

Initial change in government purchase:	ΔG
First change in consumption:	$MPC \times \Delta G$
Second change in consumption:	$MPC^2 \times \Delta G$
Third change in consumption:	$MPC^3 \times \Delta G$
$\Delta Y=(1+ MPC+ MPC^2+ MPC^3+\ldots)\Delta G$	$\Delta Y/\Delta G=1/(1-MPC)$
The multiplier is 1/(1-MPC)	

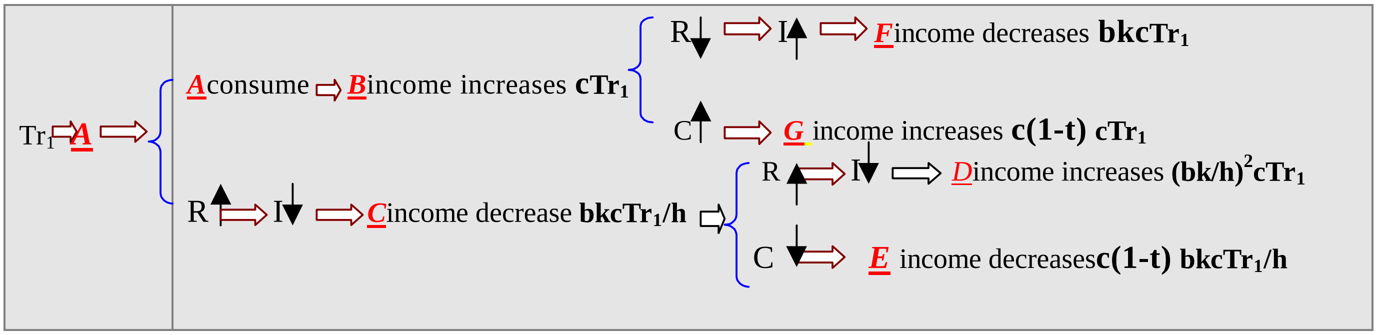
Many academicians argue that the multiplier is the wonderful tool of describing the cumulative change of the increase in autonomous spending. However, the concept of multiplier will not always give us a satisfactory explanation of the subtle change of economy. We may obtain from the table above the total increase in income of each round but we do not know when this will happen, and it is of little possibility that we can find the amount of increase in a given period. The following table shows the effect of a sire of random selected units of government transfer to the public (Tr_1 Tr_2…Tr_6…each of which stands for one dollar of transfer, and each will be transmitted from one person to another in a period) in a time series:

Time G_i	**Period 1**	**Period 2**	**Period 3**	**Period 4**	**Period 5**	**Period 6**	**Period 7**	..	∑
Tr_1	cTr_1	$c(1-t)^1 cTr_1$	$[c(1-t)]^2 cTr_1$	$[c(1-t)]^3 cTr_1$	$[c(1-t)]^4 cTr_1$	$[c(1-t)]^5 cTr_1$	$[c(1-t)]^6 cTr_1$	..	$cTr_1/c(1-t)$
Tr_2	0	cTr_2	$[c(1-t)]^1 cTr_2$	$[c(1-t)]^2 cTr_2$	$[c(1-t)]^2 cTr_2$	$[c(1-t)]^4 cTr_2$	$[c(1-t)]^4 cTr_2$	..	$CTr_2/c(1-t)$
Tr_3	0	0	0	0	cTr_3	$[c(1-t)]^1 cTr_3$	$[c(1-t)]^2 cTr_3$	..	$cTr_3/c(1-t)$
Tr_4	0	0	0	cTr_4	$[c(1-t)]^1 c\, Tr_4$	$[c(1-t)]^2 c\, Tr_4$	$[c(1-t)]^3 cTr_4$	..	$cTr_4/c(1-t)$
Tr_5	0	0	cTr_5	$c(1-t)^1 cTr_5$	$[c(1-t)]^2 cTr_5$	$[c(1-t)]^3 cTr_5$	$[c(1-t)]^4 cTr_5$	..	$cTr_5/c(1-t)$
Tr_6	0	0	0	0	0	0	cTr_6	..	$cTr_6/c(1-t)$
Erro	c	$c[1+ c(1-t)]$	$c\{1+c(1-t)+[c(1-t)]^2\}$	…	…	…	…		
…	…		…	…	…	…	…	..	…

Round is a logical concept while period involves a time series. Suppose the government increases the transfer to the public by $100, we know that the output will increase by 100c/[1-c(1-t)], if there is no asset market. We can calculate the effect of each dollar of transfer in a given round, but we do not know when it will happen.

From the table, we see that Tr_3 will be transmitted from one person to another more slowly than Tr_1, so we may obtain the total increase and the income in each period, but the value is not definite. I personally think it depends on the behavioral of individuals and we don't know what he will do in a given period, in other words, the model does not tell us about that, although the total increase in each period may have some trend due to the macroeconomic circumstances.

Here I have my own idea .The effect of each dollar of government transfer to the public (Tr_i) in a given period will not be seen apparently, since r is determined by the aggregate effect of each dollar of income. For example, Tr_1 increases a person's income, and thus induces the effect of increase of r, but r will not necessarily rise, because we do not know the effect of other dollars of Tr. I personally think that when we trace the route of each dollar of Tr , and analyze the aggregate effect , the result gained must be the same as the analysis of the macro economy using IS-LM model, if the model is correct. Suppose Tr_i is a dollar of money in government purchase □Tr, $\sum Tr_i$ =□Tr , Tr_1 passes though the 6 selected persons ***ABCDEF*** :



When each Tr_i is transferred to a person within the same period, then we obtain the following table:

	Period 0	Period 1	Period 2	Period 3
			I: $(bk/h)^2\ cTr_1$	I: $-(bk/h)^3\ cTr_1$
		I: $(-bk/h)\ cTr_1$		C: $-[c(1-t)]^2\ cTr_1$
				I: $c(1-t)\ (bk/h)^2\ cTr_1$
Tr_1 →	cTr_1		C: $-c(1-t)bk/h\ cTr_1$	C: $-[c(1-t)]^2 bk/h\ cTr_1$
				$c(1-t)\ (bk/h)^2\ cTr_1$
			$-c(1-t)bk/h\ cTr_1$	$-[c(1-t)]^2 bk/h\ cTr_1$
		C: $c(1-t)\ cTr_1$		$-[c(1-t)]^2 bk/h\ cTr_1$
			$[c(1-t)]^2\ cTr_1$	$[c(1-t)]^3\ cTr_1$
	Σ	$[c(1-t)-bk/h]\ cTr_1$	$[c(1-t)-bk/h]^2\ cTr_1$	$[c(1-t)-bk/h]^3\ cTr_1$……

If $|c(1-t)-bk/h|<1$ we draw the same conclusion as we use a geometric series, but the narrow assumption is doubtful. However, it does not follow that the multiplier theory is incorrect. The multiplier is introduced in the theories that deal with the good market where the government purchase multiplier is discussed and the ones that deal with the asset market where monetary multiplier is discussed. When we apply the multiplier theory to explain IS-LM model, which is a theory that involves both the asset market and good market, then many problems come up. Therefore, I draw the conclusion that the geometric series cannot sufficiently explain the whole multiplier process of such models as combine two markets, in which individual behave on asset market and good market separately and independently.

Part 3: Deduction 2:

Notes:

AD=C+I+G= $\mathbf{C_0 + cTR_0 + I_0 + G_0 + c[(1-t)Y+TR]-br}$

From **M/p=kY-hr,** we derive **r= kY/h- M/hp**

AD= $C_0 + cTR_0 + I_0 + G_0 + c[(1-t)Y+TR] - bkY/h + bM/hp$ = $\mathbf{C_0 + cTR_0 + I_0 + G_0 + cTR + bM/hp} + \mathbf{[c(1-t)-bk/h]Y}$ (1)

When we account GNP, we need not distinguish the effect of contemporary demand and the previous demand. In fact, the previous demand influences the contemporary output. If we want to analyze the transmission mechanism in details, I personally think that we should distinguish the two effects. The equation AD=C+I+G is used to account the aggregate demand of a year, if we want to account the AD of a month, we can use a similar equation: $AD_m=C_m+I_m+G_m$, but c, b, k, h ,t are constant. So an increase in national income□**Y,** will stimulate consumption demand by **[c(1-t)-bk/h]□Y** , the change is the same as we use the equation for GNP. When $AD_t > AS_{t-1}$, output will increase; when $AD_t > AS_{t-1}$, output will decrease; when $AD_t = AS_{t-1}$, the economy reach its equilibrium point)

LM-IS model account for the equilibrium income under the circumstance that the aggregate supply curve is horizontal .In other words, demand determine the national income when output is below that of the sufficient employment. In order to judge whether the output will increase we may consider the aggregate demand (AD) and supply (AS). When AD>AS, output will increase; when AD<AS, output will decrease; when AD=AS, the economy reach its equilibrium point.

Initiative effects□the government increase money stock by□M

□M/p —the asset market adjust quickly □R=□M/hP→ R ↑ —□I=b□M/hP→ I ↑ → AD ↑

Since the good market adjust slowly□there exists a surplus demand for good. AD-AS is denoted by GAP. GAP=□I=b□M/hP .Now AD>AS ,the output will rise as a result of the stimulation of demand .we suppose that the output in the first short period of time increases by □Y_1, then the change in income will be□Y_1 ,the income increased(or sometimes it will decrease) in each period will induce the increase in consumption and the decrease in investment spending in the next period. Apparently, □Y_1, the increase in the first period ,will induce an increase of c(1-t)□Y_1 in consumption and an decrease of bk□Y_1/h in investment in the second period .The following table shows the detail of the transmission mechanism, in which GAP_i denotes the gap between AD and AS in each period.

Period	**Effect on C**	**Effect on I**	**Combination effects of each period**	**GAP_i**
1	c(1-t)□Y_1	(-bk/h)□Y_1	c(1-t)□Y_1-bk/h□Y_1	b□M/hP-[1-c(1-t) +bk/h]□Y_1
2	c(1-t)□Y_2	(-bk/h)□Y_2	c(1-t)□Y_2-bk/h□Y_2	b□M/hP-[1-c(1-t) +bk/h] (□Y_1+□Y_2)
□□□	□□□	□□□	□□□	□□□
m	c(1-t)□Y_m	(-bk/h)□Y_m	c(1-t)□Y_m-bk/h□Y_m	b□M/hP-[1-c(1-t) +bk/h]**Error!**□Y_i
m+1	c(1-t)□Y_{m+1}	(-bk/h)□Y_{m+1}	c(1-t)□Y_{m+1}-bk/h□Y_{m+1}	b□M/hP-[1-c(1-t) +bk/h]**Error!**□Y_i
….	…	…	…	…
k	c(1-t)□Y_k	(-bk/h)□Y_k	c(1-t)□Y_k-bk/h□Y_k	b□M/hP-[1-c(1-t) +bk/h] **Error!**□Y_i

When GAP_i=0, the economy reaches its equilibrium point. From the equation **b□M/hP-[1-c(1-t) -bk/h] Error!□Y_i=0**, we obtain the total increase in income:

(6) **Error!□Y_i=(b□M)/ {[1-c(1-t)-bk/h]hP}**.

The ultimate result is identical with the one predicted by IS-LM model.

As for a four-department economy, which includes foreign trade, we can simply add a equation to modify the deduction=g-mY-nr

Then the equilibrium income determined by IS-LM model is

(7) **Y_0= (C_0+ cTR_0 +I_0 + G_0+g) / [1-c(1-t)+m+(b+n)k/h],**

and the above table will be more complex to demonstrate the mechanism, but the result is similar.

Further discussion on the formula c(1-t)-bk/h: The range of c , t and k is (0 , 1), while the range of b and k is (0,+∞).Thus c(1-t)-bk/h□(-∞,1).If c(1-t)-bk/h□(0,1),with the increase of AS, AD will consequently increase but at a lower rate (when AS increases by□Y, AD only increases by [c(1-t)-bk/h]□Y.) If c(1-t)-bk/h□(-∞,0), with the increase of AS by□Y, AD will decrease by [-c(1-t)+bk/h]□Y. So AS will ultimately meet AD.

The above deduction also implies that the effects on consumption and investment both complete within the same period, but in fact this is not the necessary assumption. Suppose the effects on consumption and investment process in two different periods□suppose the effect on I is one period later than that on C□, in the long run, we can still add up the aggregate effect of each period. Even if the time difference between the two effects is not stable, we can still get the approximate result. Look at the following tables:

(1) First we suppose that the effect on asset market lags behind that on the good market by only one period

Period	Effect on C	Effect on I	Aggregate effect on AD of each period
1	c(1-t)□Y_1		c(1-t)□Y_1
2	c(1-t)□Y_2	(-bk/h)□Y_1	c(1-t)□Y_2-bk/h□Y_1
□□□	□□□	(-bk/h)□Y_2	□□□
m	c(1-t)□Y_m	□□□	□□□
m+1	c(1-t)□Y_{m+1}	(-bk/h)□Y_m	c(1-t)□Y_{m+1}-bk/h□Y_m
….	…	(-bk/h)□Y_{m+1}	…
k	c(1-t)□Y_k	□□□	c(1-t)□Y_k-bk/h□Y_k
k+1		(-bk/h)□Y_k	

.

(2) Then we suppose that the time lag is *m* periods

Period	**Effect on C**	**Effect on I**	**Aggregate effect on AD of each period**	**GAP$_i$**
1	$c(1-t)\square Y_1$	0	$c(1-t)\square Y_1$	$b\square M/hP-[1-c(1-t)]\square Y_1$
2	$c(1-t)\square Y_2$	0	$c(1-t)\square Y_2$	$b\square M/hP-[1-c(1-t)]\ (\square Y_1+\square Y_2)$
□	□□□	□□□	□□□	□□□
m	$c(1-t)\square Y_m$	$-bk/h\square Y_1$	$c(1-t)\square Y_m-bk/h\square Y_1$	$b\square M/hP-[1-c(1-t)]$**Error!**$\square Y_i-bk/h\square Y_1$
m+1	$c(1-t)\square Y_{m+1}$	$-bk/h\square Y_2$	$c(1-t)\square Y_{m+1}-bk/h\square Y_2$	$b\square M/hP-[1-c(1-t)]$**Error!**$\square Y_i-bk/h(\square Y_1+\square Y_2)$
…	…	…	…	…
k	$c(1-t)\square Y_k$	$-bk/h\square Y_{k-m+1}$	$c(1-t)\square Y_k-bk/h\square Y_{k-m+1}$	$b\square M/hP-[1-c(1-t)]$ **Error!**$\square Y_i-bk/h$**Error!** $\square Y_i$
…	…	…	…	…

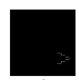
In any given round, $\square Y_k > c(1-t)\square Y_k-bk/h\square Y_{k-m+1}$,the increase of output is more than that of demand.

Therefore, equilibrium can be reached ultimately. When $b\square M/hP-[1-c(1-t)]$ **Error!**$\square Y_i-bk/h$**Error!** $\square Y_i$

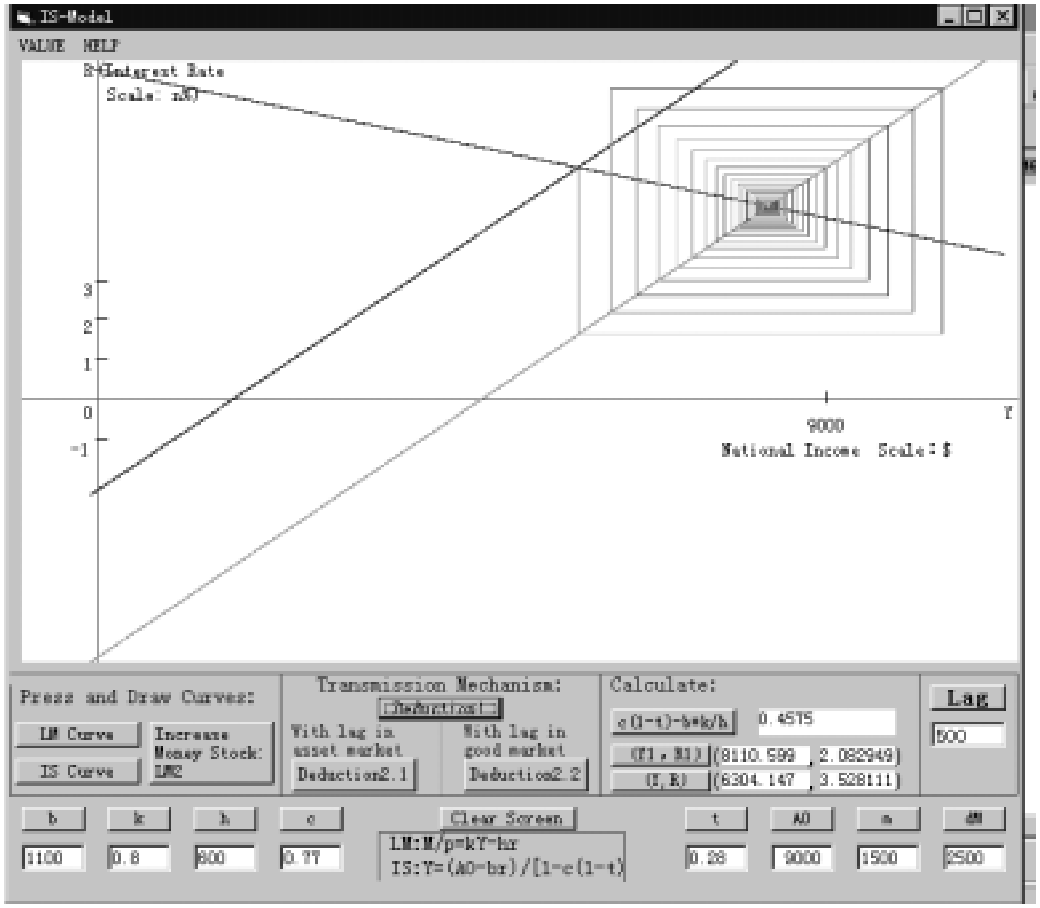
= 0,

we obtain the total increase in Y approximately:

(8) $$\sum \square Y_i = (b\square M)/\{[1-c(1-t)-bk/h]hP\}$$.

If you want a vivid demonstration, give the icon a double-click  mechanism.exe. And you will find that time lags will not significantly influence the approach to the equilibrium.(Even if you change the number value for lag from 0 to 100, the route of the change of economy is much the same.)

A guide to the program



1. The program is designed by Visual Basic 6.0,
2. Press the "Value "menu and choose one of the selections for the parameters of LM - IS model.
3. Press " LM curve " and "IS curve" button to draw LM curve and. IS curve,.
4. Press " Increase money stock "button to draw LM_2(a new LM curve after increased money stock)
5. Press "c(1-t)-bk/h" to see the range of c(1-t)-bk/h . c(1-t)-bk/h<-1, -1<c(1-t)-bk/h<0 or 0<c(1-t)-bk/h<1 ?
6. Press "Deduction 1"botton to show the graph of deduction 1.There are three modes of graph according to the range of c(1-t)-bk/h.
7. "Deduction 2.1"and "Deduction 2.2"show my deduction of the monetary transmission mechanism. "Deduction 2.1" shows the case that the effect on asset market lags behind that on the good market, and "Deduction 2.2" shows the case that the effect on good market lags behind that on the asset market
8. "(Y1,R1)"shows the coordinate of the new equilibrium point, while "(Y,R)"shows the coordinate of the initial equilibrium point.
9. Press" lag" button and input number values for time lag to see if the time lag has any effect.
10. You may input your number values for the parameters by clicking on "b" "k" etc.

References:

(1) Macroeconomics, 3rd ed. Mankwi N. George, 1997,worth publishers

(2) Macroeconomics 5th ed. Gordon Robert J., 1990 ,Harper Collins College Publishers

(3) Macroeconomics 6th ed. Dornbusch Rudiger , 1994,Mc Graw-Hill

Graphy and Figures:

1.

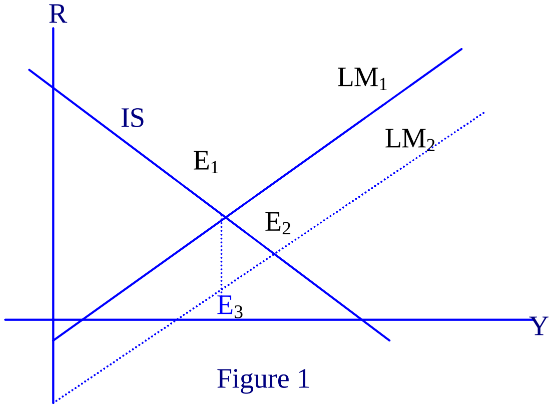
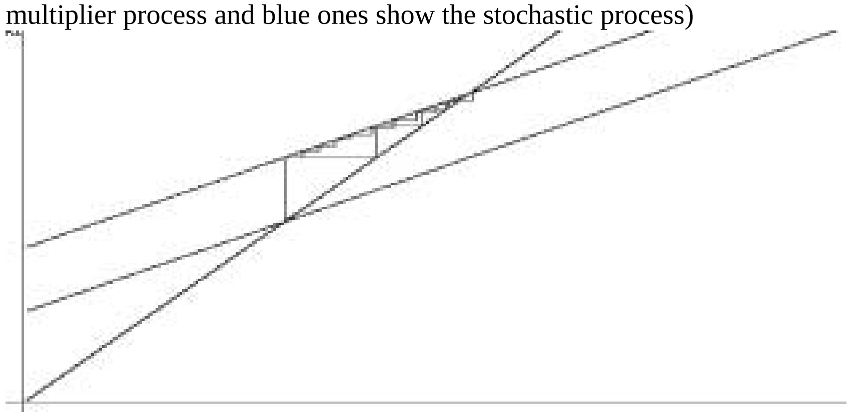


Figure 1

2.Multiplier process in a simple economy (red lines shows the multiplier process and blue ones show the stochastic process)



3.Figures of Deduction 1 Figure 2

(3.1) 0<c(1-t) -bk/h<1

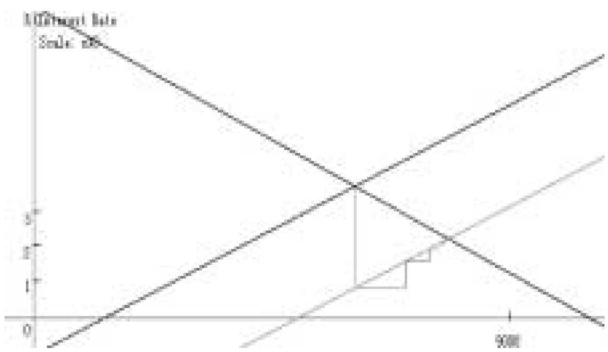


Figure 3.1

(3.2) |c(1-t) -bk/h|>1

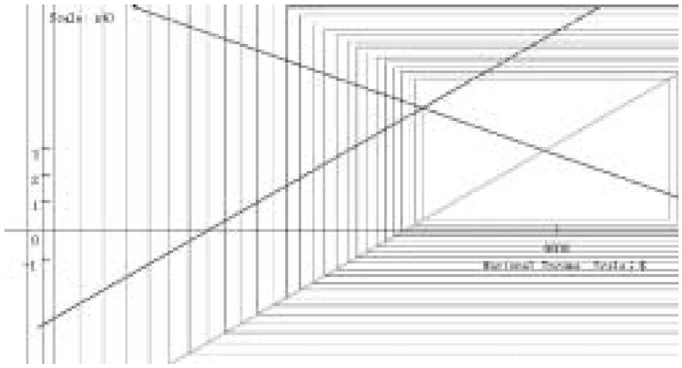


Figure 3.2

(3.3) $-1<c(1-t)-bk/h<0$

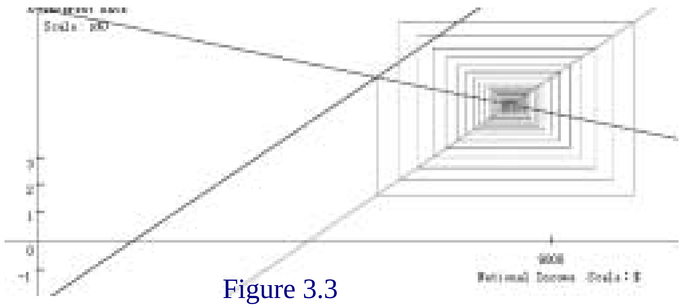


Figure 3.3

4. Deduction 2

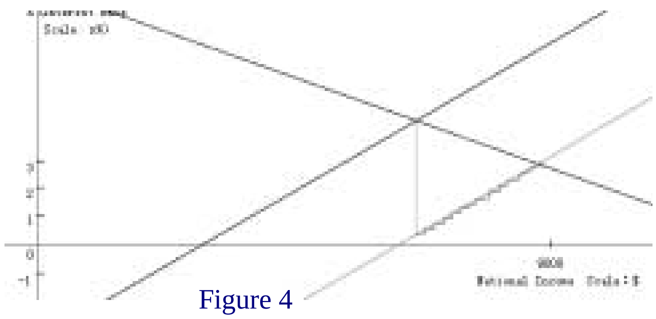


Figure 4

Tables

Table 1

(1)	(2)	(3)	(4)
Change in real Money supply	Portfolio adjustments lead to a change in asset prices and interest rates	Spending adjusts to the change in interest rate	Output adjust to the change in aggregate demand

Table 2

Time / G_1	**Period 1**	**Period 2**	**Period 3**	**Period 4**	**Period 5**	**Period 6**	**Period 7**	..	∑
Tr_1	cTr_1	$c(1-t)^1cTr_1$	$[c(1-t)]^2cTr_1$	$[c(1-t)]^3cTr_1$	$[c(1-t)]^4cTr_1$	$[c(1-t)]^5cTr_1$	$[c(1-t)]^6cTr_1$	..	$cTr_1/c(1-t)$
Tr_2	0	cTr_2	$[c(1-t)]^1cTr_2$	$[c(1-t)]^2cTr_2$	$[c(1-t)]^2cTr_2$	$[c(1-t)]^4cTr_2$	$[c(1-t)]^4cTr_2$	..	$CTr_2/c(1-t)$

Tr_3	0	0	0	0	cTr_3	$[c(1-t)]^1cTr_3$	$[c(1-t)]^2cTr_3$	..	$cTr_3/c(1-t)$
Tr_4	0	0	0	cTr_4	$[c(1-t)]^1c\ Tr_4$	$[c(1-t)]^2c\ Tr_4$	$[c(1-t)]^3cTr_4$	..	$cTr_4/c(1-t)$
Tr_5	0	0	cTr_5	$c(1-t)\ ^1cTr_5$	$[c(1-t)]^2cTr_5$	$[c(1-t)]^3cTr_5$	$[c(1-t)\ ]^4cTr_5$	..	$cTr_5/c(1-t)$
Tr_6	0	0	0	0	0	0	cTr_6	..	$cTr_6/c(1-t)$
Erro	c	$c[1+ c(1-t)]$	$c\{1+c(1-t)+[c(1-t)]^2\}$	…	…	…	…		
…	…		…	…	…	…	…	..	…

Table 3

Initial change in government purchase:	□G
First change in consumption:	MPC×□G
Second change in consumption:	MPC^2×□G
Third change in consumption:	MPC^3×□G
□Y=(1+ MPC+ MPC^2+ MPC^3+…)□G	□Y/□G=1/(1-MPC)
The multiplier is 1/(1-MPC)	

Table 4

	Period 0	**Period 1**	**Period 2**	**Period 3**
Tr_1 →	cTr_1	I: (-bk/h) cTr_1	I: $(bk/h)^2$ cTr_1	I: - $(bk/h)^3$ cTr_1 C: $-[c(1-t)]^2$ cTr_1
			C: -c(1-t)bk/h cTr_1	I: c(1-t) $(bk/h)^2$ cTr_1 C: $-[c(1-t)]^2$bk/h cTr_1
		C: c(1-t) cTr_1	-c(1-t)bk/h cTr_1	c(1-t) $(bk/h)^2$ cTr_1 $-[c(1-t)]^2$bk/h cTr_1
			$[c(1-t)]^2$ cTr_1	$-[c(1-t)]^2$bk/h cTr_1 $[c(1-t)]^3$ cTr_1
	∑	[c(1-t)-bk/h] cTr_1	$[c(1-t)- bk/h]^2$ cTr_1	$[c(1-t)- bk/h]^3$ cTr_1…...

Table 5

Period	**Effect on C**	**Effect on I**	**Aggregate effect on AD of each period**
1	c(1-t)□Y_1		c(1-t)□Y_1
2	c(1-t)□Y_2	(-bk/h)□Y_1	c(1-t)□Y_2-bk/h□Y_1
□□□	□□□	(-bk/h)□Y_2	□□□
m	c(1-t)□Y_m	□□□	□□□
m+1	c(1-t)□Y_{m+1}	(-bk/h)□Y_m	c(1-t)□Y_{m+1}-bk/h□Y_m
….	…	(-bk/h)□Y_{m+1}	…
k	c(1-t)□Y_k	□□□	c(1-t)□Y_k-bk/h□Y_k

k+1		(-bk/h)□Y_k	

Table 6

Period	Effect on C	Effect on I	Aggregate effect on AD of each period	GAP_i
1	c(1-t)□Y_1	0	c(1-t)□Y_1	b□M/hP-[1-c(1-t)]□Y_1
2	c(1-t)□Y_2	0	c(1-t)□Y_2	b□M/hP-[1-c(1-t)] (□Y_1+□Y_2)
□	□□□	□□□	□□□	□□□
m	c(1-t)□Y_m	-bk/h□Y_1	c(1-t)□Y_m-bk/h□Y_1	b□M/hP-[1-c(1-t)]**Error!**□Y_i-bk/h□Y_1
m+1	c(1-t)□Y_{m+1}	-bk/h□Y_2	c(1-t)□Y_{m+1}-bk/h□Y_2	b□M/hP-[1-c(1-t)]**Error!**□Y_i-bk/h(□Y_1+□Y_2)
…	…	…	…	…
k	c(1-t)□Y_k	-bk/h□Y_{k-m+1}	c(1-t)□Y_k-bk/h□Y_{k-m+1}	b□M/hP-[1-c(1-t)] **Error!**□Y_i-bk/h**Error!**□Y_i
…	…	…	…	…

Code:

The code of “mechanism.exe”

```
Dim b As Single
Dim k As Single
Dim h As Single
Dim c As Single
Dim t As Single
Dim A0 As Single
Dim Mp As Single
Dim dM As Single

Private Sub Command17_Click()
Dim a(-1000 To 40000) As Double
Y = (A0 + (b * Mp / h)) / (1 - c * (1 - t) + b * k / h) 'Mp stands for the real money stock,Y is the national income on point E1.
r = (k * A0 - Mp + Mp * c * (1 - t)) / (h * (1 - c * (1 - t)) + b * k)
dr = -dM / h 'dr is the initial decrease in interest rate induced by the increased money stock
p.Line (Y, (r + dr) * 600)-(Y, r * 600) ' draw the line E1E3
p.ForeColor = RGB(255, 0, 0) 'define the color
r = r + dr
ddemand = -b * dr 'the increased demand induced by lower r
demand = Y + ddemand
i = 0
Do
i = i + 1
a(i) = Rnd * 200 ' To generate random increase in Y,in fact ,in our real economy the increase is not simply random ,just to show the indefinite process of economy
demand = demand + c * (1 - t) * a(i) - b * k / h * a(i - timelag) '(c * (1 - t) - b *k / h) * a is demand induced by the increase inY
output = Y + a(i)
p.Line (Y + a(i), r * 600)-(Y, r * 600) '*600 in order to match the scale
Y = Y + a(i)
p.Line (Y, (r + a(i) * k / h) * 600)-(Y, r * 600)
r = r + a(i - timelag) * k / h
delay1
Loop Until output - demand > 0 'the mutiplier process will not stop  until output- demand > 0`
End Sub
```

```
Private Sub Command19_Click()
Dim a(-1000 To 40000) As Double
Y = (A0 + (b * Mp / h)) / (1 - c * (1 - t) + b * k / h) 'Mp stands for the real money stock,Y is the national income on point E1.
r = (k * A0 - Mp + Mp * c * (1 - t)) / (h * (1 - c * (1 - t)) + b * k)
dr = -dM / h 'dr is the initial decrease in interest rate induced by the increased money stock
p.Line (Y, (r + dr) * 600)-(Y, r * 600) ' draw the line E1E3
p.ForeColor = RGB(255, 0, 0) 'define the color
r = r + dr
ddemand = -b * dr 'the increased demand induced by lower r
demand = Y + ddemand
i = 0
Do
i = i + 1
a(i) = Rnd * 200 ' To generate random increase in Y,in fact ,in our real economy the increase is not simply random ,just to show the indefinite process of economy
demand = demand + c * (1 - t) * a(i - timelag) - b * k / h * a(i) '(c * (1 - t) - b *k / h) * a is demand induced by the increase inY
output = Y + a(i)
p.Line (Y + a(i), r * 600)-(Y, r * 600) '*600 in order to match the scale
Y = Y + a(i)
p.Line (Y, (r + a(i) * k / h) * 600)-(Y, r * 600)
r = r + a(i) * k / h
delay1
Loop Until output - demand > 0 'the mutiplier process will not stop  until output- demand > 0`
End Sub

Private Sub Form_Load()
p.Scale (-840, 5000)-(12000, -4000)
End Sub
Private Sub command2_click()
For Y = 0 To 11500 Step 0.5
a = 1 / (1 - c * (1 - t))
r = (a * A0 - Y) / (a * b)
p.Line (Y, 600 * r)-(Y - 1, ((a * A0 - Y + 1) / (a * b)) * 600)
p.ForeColor = vbBlue
Next Y
End Sub
Private Sub Command4_Click()
For Y = 1 To 11600 Step 0.5
delay2
r = -(Mp + dM) / h + k * Y / h
p.Line (Y, 600 * r)-(Y - 1, (-(Mp + dM) / h + k * (Y - 1) / h) * 600)
p.ForeColor = RGB(200, 100, 1000)
p.BorderStyle = dot
Next Y
End Sub
Private Sub Command1_Click()
For Y = 1 To 11500 Step 0.4
r = -Mp / h + k * Y / h
p.Line (Y, 600 * r)-(Y - 1, (-Mp / h + k * (Y - 1) / h) * 600) '*600 in order to match the scale
p.ForeColor = vbBlue

Next Y
End Sub
```

```
Private Sub Command3_Click()
g = (A0 + (b * Mp / h)) / (1 - c * (1 - t) + b * k / h)
o = (k * A0 - Mp + Mp * c * (1 - t)) / (h * (1 - c * (1 - t)) + b * k)
Text8 = g
Text9 = o
End Sub
Private Sub Command16_Click()
Y = (A0 + (b * (Mp + dM) / h)) / (1 - c * (1 - t) + b * k / h)
r = (k * A0 - Mp - dM + (Mp + dM) * c * (1 - t)) / (h * (1 - c * (1 - t)) + b * k)
Text12.Text = Y
Text13.Text = r
End Sub


Private Sub cmd2_Click()
Text6.Text = " 3000"
Text1.Text = " 350"
Text2.Text = "0.75"
Text3.Text = "1000"
Text4.Text = "0.9"
Text5.Text = "0.2"
Text10.Text = "2800"
Text7.Text = "1200"
Text14.Text = 0
A0 = Val(Text6.Text)
b = Val(Text1.Text)
k = Val(Text2.Text)
h = Val(Text3.Text)
c = Val(Text4.Text)
t = Val(Text5.Text)
dM = Val(Text10.Text)
Mp = Val(Text7.Text)
End Sub

Private Sub cmd3_Click()
Text6.Text = " 3000"
Text1.Text = " 350"
Text2.Text = "0.75"
Text3.Text = "1000"
Text4.Text = "0.9"
Text5.Text = "0.2"
Text10.Text = "2800"
Text7.Text = "1200"
Text14.Text = 100
A0 = Val(Text6.Text)
b = Val(Text1.Text)
k = Val(Text2.Text)
h = Val(Text3.Text)
c = Val(Text4.Text)
t = Val(Text5.Text)
dM = Val(Text10.Text)
Mp = Val(Text7.Text)
End Sub

Private Sub cmd4_Click()
Text6.Text = " 10000"
Text1.Text = "1100"
```

```
Text2.Text = "0.8"
Text3.Text = "700"
Text4.Text = "0.5"
Text5.Text = "0.54"
Text10.Text = "2800"
Text7.Text = "2500"
Text14.Text = 100
A0 = Val(Text6.Text)
b = Val(Text1.Text)
k = Val(Text2.Text)
h = Val(Text3.Text)
c = Val(Text4.Text)
t = Val(Text5.Text)
dM = Val(Text10.Text)
Mp = Val(Text7.Text)
End Sub

Private Sub cmd5_Click()
Text6.Text = " 9000"
Text1.Text = "1100"
Text2.Text = "0.8"
Text3.Text = "600"
Text4.Text = "0.77"
Text5.Text = "0.28"
Text10.Text = "2500"
Text7.Text = "1500"
Text14.Text = 500
A0 = Val(Text6.Text)
b = Val(Text1.Text)
k = Val(Text2.Text)
h = Val(Text3.Text)
c = Val(Text4.Text)
t = Val(Text5.Text)
dM = Val(Text10.Text)
Mp = Val(Text7.Text)
End Sub

Private Sub cmd6_Click()
form2.Show
End Sub
Private Sub cmd7_Click()
form3.Show
End Sub
Private Sub Command14_Click()
a = c * (1 - t) - b * k / h
Text11 = a
End Sub
Private Sub command15_Click()
p.Cls
End Sub

Private Sub Command13_Click()
a = 0
Y = (A0 + (b * Mp / h)) / (1 - c * (1 - t) + b * k / h)
r = (k * A0 - Mp + Mp * c * (1 - t)) / (h * (1 - c * (1 - t)) + b * k)
dr = -dM / h
```

```
p.Line (Y, (r + dr) * 600)-(Y, r * 600)
p.ForeColor = RGB(Rnd * 255, Rnd * 255, Rnd * 255)
p.BorderStyle = dot
r = r + dr
dy = -b * dr
delay1
p.Line (Y + dy, r * 600)-(Y, r * 600)
p.ForeColor = RGB(Rnd * 255, Rnd * 255, Rnd * 255)
p.BorderStyle = dot
dr = k * dy / h
Y = Y + dy
delay1

Do
a = a + 1
p.Line (Y, (r + dr) * 600)-(Y, r * 600)
p.ForeColor = RGB(Rnd * 255, Rnd * 255, Rnd * 255)
p.BorderStyle = dot
dy = (c * (1 - t) - b * k / h) * dy
r = r + dr
delay1
p.Line (Y + dy, 600 * r)-(Y, r * 600)
p.ForeColor = RGB(Rnd * 255, Rnd * 255, Rnd * 255)
p.BorderStyle = dot
delay1
Y = Y + dy
r = r
dr = dy * k / h
Loop Until Abs(Y - (A0 + (b * Mp / h)) / (1 - c * (1 - t) + b * k / h)) < 80 Or Y > 20000 Or a > 60 'in order to stop the program within 45 rounds
End Sub

Private Sub Command6_Click()
Text2.Text = InputBox("please input k!", "k", "k")
k = Val(Text2.Text)
End Sub
Private Sub Command7_Click()
Text3.Text = InputBox("please input h!", "h", "h")
h = Val(Text3.Text)
End Sub
Private Sub Command8_Click()
Text4.Text = InputBox("please input c!", "c", "c")
c = Val(Text4.Text)
End Sub
Private Sub Command9_Click()
Text5.Text = InputBox("please input t!", "t", "t")
t = Val(Text5.Text)
End Sub
Private Sub Command12_Click()
Text10.Text = InputBox("please input dM!", "dM", "dM")
dM = Val(Text10.Text)
End Sub
Private Sub Command10_Click()
Text6.Text = InputBox("please input A0!", "A0", "A0")
A0 = Val(Text6.Text)
End Sub
Private Sub Command11_Click()
```

```
Text7.Text = InputBox("please input M/p!", "M/p", "M/p")
Mp = Val(Text7.Text)
End Sub
Private Sub Command5_Click()
Text1.Text = InputBox("please input b!", "b", "b")
b = Val(Text1.Text)
End Sub
Private Sub Command18_Click()
Text14.Text = InputBox("please input an integer for the number value of time lag!", "timelag", "0")
timelag = Val(Text14.Text)
End Sub
Private Sub delay2()
For a = 1 To 500
Next a
End Sub
Private Sub delay1()
For a = 1 To 1000000
Next a
End Sub
```