The consequences of staggered wage setting for the credibility of monetary policy

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ABSTRACT: This paper introduces staggered wage contracts *à la* Taylor (1979) into a standard model of monetary policy credibility. The overlapping wage structure is shown to considerably exacerbate the time consistent inflation rate in Markov perfect equilibrium. If the central bank can commit to its monetary policy for one-period ahead, this reduces but does not eliminate the inflationary bias. Even if it can commit for a length of time equal to the nominal contract length (i.e., two-periods), this does not generally lead to a zero inflation outcome, and may even lead to negative inflation if the central bank's rate of time discount is sufficiently high.

KEYWORDS: staggered wage contracts, time consistency, commitment

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1 Introduction

New-Keynesian economists have often blamed short-run nominal wage rigidity for causing unemployment in the wake of adverse nominal shocks. Indeed, the impact of aggregate demand on inflation and employment depends, among other factors, on the length of wage contracts and on the speed of adjustment of contracts to new information about the state of the economy. Where there are long-term staggered contracts and an absence of synchronised wage setting, changes in monetary policy will have a much greater effect on employment. Hence, the role for active monetary stabilisation, under staggering, has been defended by Fischer (1977) and Taylor (1979), within a context of time-varying and fixed wage contracts, respectively. Furthermore, many authors have argued that the staggered timing of price adjustment makes output losses inevitable during deflation because of the inflationary momentum caused by overlapping price and wage decisions. Nominal wage and price rigidities reflect the empirically common practice of setting wages and prices in money terms for several periods in advance.[1] But multi-period contracts are often incomplete, that is, they are not contingent on nominal wage and price developments elsewhere in the economy.

In contrast with the New-Keynesian theory, new classical economists argue that credibility problems are central to the disinflationary process, so that disinflation would be costless if the government announced credible commitments. But, if multi-period contracts lead to more lasting effects of monetary policy surprises, they will enhance the credibility (time consistency) problem of monetary policy. In other words, credibility problems and the microfoundations of wage and price setting cannot be easily separated for the quantification of the costs of disinflationary policy. Despite the importance of this problem, to our knowledge, only Levine and Pearlman (1994) have formally incorporated wage setting dynamics into the time consistency literature.[2] Borrowing heavily from Calvo (1983), they analyse real and nominal wage inertia which arise from the ex-

[1]For a detailed discussion of nominal wage rigidity in OECD countries, see Layard *et al.* (1991).
[2]The research reported here was conducted independently of Levine and Pearlman (1994).

istence of contracts extending over many periods. In this setting they study delegation to a conservative central bank. They show that the optimal *conservatism* of the central banker increases as nominal and real wage inertia increases, due to an increased credibility problem. They further argue that the feasibility of a monetary union requires the characteristics of labour markets to be fairly similar.

Levine and Pearlman relate their analysis to Rogoff's (1985) paper on delegation. A more recent strand of the literature following Walsh (1995), however, argues that the use of central bank contracts can simultaneously achieve optimal responsiveness to shocks *and* avoid the time consistency bias. A question which has not been addressed in this literature is the relationship between the length of a central bank contract and that of wage contracts; the implicit assumption is that the former is at least as long as the latter, allowing wage setters to be confident that the government cannot "cheat" by changing the terms of the contract in the period in which nominal wages are fixed. We shall consider what happens when this relationship is reversed, so that the central bank contract is shorter than the labour contract. Thus in contrast to Levine and Pearlman's investigation of delegation, our interest is in contract length—or equivalently, given that we abstract from supply shocks, our interest is in the length of time the government is able to commit to its monetary policy.

The model is an extension of the Taylor (1979) model of overlapping wage contracts. We analyse the interaction of staggered wage setting and credibility problems. Section 2 presents the model. In Section 3 we obtain the discretionary equilibrium, where the government cannot precommit to a rate of inflation, and compare it to the *static* model with one period contracts (i.e., the standard model). Our findings indicate that the existence of long-term staggered contracts increases the credibility problem of monetary policy. This result mirrors that obtained by Levine and Pearlman (1994), for an economy where nominal rigidities arise from the presence of Calvo contracts. We next look at what happens when the period of commitment of the government is increased. In the Calvo framework contracts last with positive probability for any length of time and it is difficult to interpret the length of commitment of the government relative to wage setters. An advantage of the Taylor framework is that this interpretation is much more straightforward. In Section 4 we focus on the feasibility of precommited equilibria. It will be shown that, as wage contracts last for more than one period—where by a period is meant the length of time between the decisions on monetary policy taken by the central banker—the central banker will be unable to overcome the time consistency problem. This is the key result of this chapter. Section 5 concludes.

2 Wage contract models

2.1 The one-period wage-setting model

Prior to studying the staggered wage case, consider the following one period wage setting model. The economy evolves over an infinite number of periods. All variables are in logs and real variables represent deviations from their long-run equilibrium levels.

Wage setters select their nominal wage, x_t, in order to target the real wage. Assuming that the wage setters' target real wage is zero, the contract wage determination is given by:

$$x_t = \hat{p}_t, \tag{1}$$

where p_t denotes the price at period t and $\hat{}$ the rational expectations operator.

A contract is assumed to specify a fixed nominal wage which will apply for the duration of the contract. The contractually determined money wage will be set equal to the expected market - clearing money wage, based on individuals' expectations of the average level of prices prevailing in the market. Once wages have been set, the actual supply and demand conditions become known. Then, the level of employment equals the actual quantity of labour demanded, i.e., it is assumed that labour demand, the short side of the market, always dominates.

The general level of prices depends upon the underlying nominal wage and demand fluctuations,

$$p_t = x_t + \rho y_t, \tag{2}$$

where ρ represents the output elasticity of the price level. Alternatively, Equation (2) can be seen as a downward-sloping labour demand curve, where $1/\rho$ is the price elasticity of labour demand. Aggregate demand is an increasing function of real money balances,

$$y_t = m_t - p_t, \tag{3}$$

where m_t is the logarithm of the nominal money supply. The model is completed by specifying an objective function for the government. As is customary in this literature, it is assumed that the government sets its monetary policy, after the wage contract, x_t, has been determined, in order to minimise a social loss function of the form

$$V_t = \sum_{s=t}^{\infty} \delta^{s-t} \left[(1-\lambda)(y_t - \tilde{y})^2 + \lambda(p_t - p_{t-1})^2 \right], \tag{4}$$

where $0 \leq \lambda \leq 1$ captures the weight that the government puts on targeting inflation versus output or, equivalently, employment. The socially desired level of output, $\tilde{y}$, is assumed to be greater than $\bar{y}$, the long run equilibrium level or 'natural rate' of output, which in our model equals zero. (Possible factors which might cause equilibrium output to lie below the socially desired level include distortions in the labour market such as income taxation, unemployment insurance and monopolistic unions.) Zero is the most preferred level of inflation and $0 < \delta < 1$ is the parameter representing the society's inter-temporal preferences.

Solving for the discretionary equilibrium one finds that output is at its natural rate

$$y_t = 0 \tag{5}$$

and inflation is positive, given by

$$p_t - p_{t-1} = \frac{(1-\lambda)\tilde{y}}{\lambda\rho}. \tag{6}$$

2.2 The dynamic model

Borrowing from Taylor (1979), wage contracts last for two periods and decision dates overlap:. We assume that half of the contracts are set in period t, half in $t+1$. The contract wage determination is assumed to be

$$x_t \;=\; \frac{\hat{p}_t \;+\; \hat{p}_{t+1}}{2}. \tag{7}$$

As before, the wage contract is assumed to specify a fixed nominal wage which will apply for the duration of the contract. Hence, workers have to form expectations about current and future prices.

The aggregate price level at t is determined by the average wage prevailing at time t, $w_t \;=\; (x_{t-1} + x_t)$, together with output fluctuations,

$$p_t \;=\; \frac{x_{t-1} \;+\; x_t}{2} \;+\; \rho y_t, \qquad \rho > 0. \tag{8}$$

Hence, p_t is homogeneous of degree one in past and current contracts, while x_t is homogeneous of degree one in current and future prices. The degree of nominal inertia is symmetric in lag and lead contracts.

Aggregate demand depends upon real money balances,

$$y_t \;=\; m_t \;-\; p_t. \tag{9}$$

Using (8) and (9), we find

$$p_t \;=\; \frac{1}{1+\rho}\left[\rho m_t \;+\; \frac{1}{2}\left(x_t + x_{t-1}\right)\right], \tag{10}$$

$$y_t \;=\; \frac{1}{1+\rho}\left[m_t \;-\; \frac{1}{2}\left(x_t + x_{t-1}\right)\right]. \tag{11}$$

Finally, the objective function for the government is as in (4) above.

Due to the presence of long-term staggered contracts, the central bank plays a *dy-*

namic rather than a repeated game with the wage-setters. The wage contracts set in the period t will hold until $t+1$, no matter what the choice of m_t is. We focus on equilibria in Markov strategies, that is, strategies that only depend on the past history of the game through an appropriately defined payoff relevant state variable. Following this approach, we concentrate on Markov perfect equilibria (MPE), which have been defined as *a profile of Markov strategies that yields a Nash equilibrium in every period* (Fudenberg and Tirole (1991), p. 501). In general there will be other equilibria. For instance, the MPE restriction rules out punishment strategies, which could be used to sustain a low inflation equilibrium as in Barro and Gordon (1983).[3]

3 The time consistent policy rule

Given the homogeneity properties of the system, the payoff relevant state variable at period $t+1$ is the *real* contract wage determined at t.[4] We define the state variable, z_t, to be the inverse of the period t wage contract written, in real terms, i.e.,

$$z_t \quad := \quad p_t - x_t. \tag{12}$$

Because of the linear-quadratic structure of the model, we look for a solution with linear decision rules and a quadratic value function for the policy maker. The wage-setters choose the wage contract at every period t as a function of the state variable z_{t-1}:

$$x_t \quad = \quad \alpha_0 \ + \ \alpha_1 z_{t-1} \ + \ p_{t-1}, \tag{13}$$

The constants α_0 and α_1 need to be determined and the term p_{t-1} is introduced in order to satisfy the homogeneity properties of x_t, i.e., it is the real wage which is a function of z_{t-1}.

[3] See Obstfeld (1990, 1991) for examples of monetary games focused on Markov perfect equilibria.

[4] It would be possible to treat p_t and x_t as separate state variables, in which case we would expect to obtain other solutions in addition to those identified here; however it is only the difference between these two variables which is payoff relevant, and so z_t is the appropriate state variable given the MPE definition.

Moreover, we denote by

$$V(z_{t-1}) \;=\; \beta_0 \;+\; \beta_1 z_{t-1} \;+\; \beta_2 z_{t-1}^2 \tag{14}$$

the present discounted value of social losses from t onwards, as a function of the state variable z_{t-1}, where the coefficients β_0, β_1 and β_2 are to be determined. Bellman's principle of optimality implies

$$V(z_{t-1}) \;=\; Min_{m_t}\left[(1-\lambda)(y_t-\tilde{y})^2 \;+\; \lambda(p_t-p_{t-1})^2 \;+\; \delta V(z_t)\right], \tag{15}$$

The central bank sets m_t after observing z_{t-1} and the current wage contract x_t. After substituting (14), (10) and (11) into (15), the best response of the central bank to the wage-setters' reaction function is to choose m_t to minimise

$$\begin{aligned} V(z_{t-1}) = \underset{m_t}{Min}(1-\lambda)&\left[\frac{1}{1+\rho}\left(m_t-\frac{x_t}{2}-\frac{x_{t-1}}{2}\right)-\tilde{y}\right]^2 \\ &+\lambda\left[\frac{\rho}{1+\rho}m_t+\frac{1}{2(1+\rho)}(x_t+x_{t-1})-p_{t-1}\right]^2 \\ &+\delta(\beta_0+\beta_1 z_t+\beta_2 z_t^2). \end{aligned} \tag{16}$$

where

$$z_t \;\equiv\; p_t-x_t = \;\frac{1}{1+\rho}\left[\rho m_t \;+\; \frac{1}{2}\left(x_t+x_{t-1}\right)\right]-x_t.$$

The optimal monetary policy at t is characterised by the first order condition to this problem, which, after substituting x_t from the wage-setters' reaction function (equation (13)), can be written as

$$\begin{aligned} m_t &= 2A(1+\rho)(1-\lambda)\tilde{y}-A(1+\rho)\delta\rho\beta_1 \\ &\quad+B\alpha_0+Cx_{t-1}+Dp_{t-1}+B\alpha_1 z_{t-1}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} A &= \frac{1}{2\left(1-\lambda+\lambda\rho^2+\delta\rho^2\beta_2\right)}, \\ B &= A\cdot[\beta_2\delta\rho\,(1+2\rho)+1-\lambda-\lambda\rho], \\ C &= A\cdot(1-\lambda-\lambda\rho-\beta_2\delta\rho) \\ D &= A\cdot[\,(\beta_2\delta\rho+\rho\lambda)\,(1+2\rho)+1-\lambda]. \end{aligned}$$

Note that $C + D = 1$, so that the homogeneity properties of the model are satisfied. Thus, the optimal monetary policy can be expressed as:

$$\begin{aligned} m_t = {} & 2A(1+\rho)(1-\lambda)\tilde{y} - A(1+\rho)\delta\rho\beta_1 \\ & + B\alpha_0 + p_{t-1} + (B\alpha_1 - C)z_{t-1}. \end{aligned} \tag{18}$$

Substituting (18) into (10) and (11) and making use of (13), we get the following:

$$\begin{aligned} p_t - p_{t-1} = {} & 2A(1-\lambda)\rho\tilde{y} - A\delta\rho^2\beta_1 \\ & + \frac{\alpha_0}{1+\rho}\left(\rho B + \frac{1}{2}\right) + \left(\frac{\rho\alpha_1 B - \rho C + \frac{\alpha_1 - 1}{2}}{1+\rho}\right) z_{t-1}, \end{aligned} \tag{19}$$

$$\begin{aligned} y_t = {} & 2A(1-\lambda)\tilde{y} - A\delta\rho\beta_1 \\ & + \frac{\alpha_0}{1+\rho}\left(B - \frac{1}{2}\right) + \left(\frac{\alpha_1 B - C - \frac{\alpha_1 - 1}{2}}{1+\rho}\right) z_{t-1}. \end{aligned} \tag{20}$$

To guarantee consistency of the solution and get expressions for β_1 and β_2, we substitute the optimal monetary rule given by (18) and the wage setters' reaction function (13) back into (16) and equate coefficients on the state variables in the value function given by (14). Furthermore, the parameters α_0 and α_1 in (13) must be consistent with those deriving from substituting (19) into (7). This leads to a system of equations for α_0, α_1, β_0, β_1 and β_2, of which the four relevant ones are presented in Appendix 6.1.

We now derive the steady state of the system where the rate of inflation is constant, i.e., $p_t - p_{t-1} = \bar{k}$, $x_t - x_{t-1} = \bar{k}$, and (hence) $z_t = z_{t-1}$. The long-run (steady state) equilibrium level for the state variable of the system will be characterised by: [5]

$$z_t = \frac{(-1+\lambda)\tilde{y}(1-\lambda+2\lambda\rho^2+\delta\lambda\rho^2)}{2\lambda\rho\left[1-\lambda+2\lambda\rho^2+(\lambda-1)\delta\right]}. \tag{21}$$

Similarly, the inflationary bias that corresponds to the steady state will be given by:

$$\bar{k} = \frac{(1-\lambda)\tilde{y}(1-\lambda+2\lambda\rho^2+\delta\lambda\rho^2)}{\lambda\rho\left[1-\lambda+2\lambda\rho^2+(\lambda-1)\delta\right]}, \tag{22}$$

which is unambiguously larger than the inflationary bias associated with the *static* econ-

[5] The state variable jumps immediately to its steady-state value, whereas there is a one period lag before inflation and output reach their steady-state levels.

omy, given by Equation (5.6). Indeed, given $1 \geq \lambda \geq 0$, it will be always the case as:

$$\frac{1-\lambda+2\lambda\rho^2+\delta\lambda\rho^2}{1-\lambda+2\lambda\rho^2+(\lambda-1)\delta} > 1,$$

so that,

$$\bar{k} > \frac{(1-\lambda)\bar{y}}{\lambda\rho}.$$

This result mirrors that obtained by Levine and Pearlman (1994) for an economy where nominal rigidities arise from the presence of Calvo contracts.

The steady-state level of output is given by Equation (20), once z_{t-1} has been substituted by its steady state value. As is usual in a time consistent equilibrium, the long-run level of output turns out to equal its natural equilibrium level, which in our model equals zero.[6]

In order to discuss the welfare implications of the existence of overlapping contracts, we compare the time consistent equilibrium of our staggered economy with that obtained from the so called *static* economy, in which wages are set only for a single period. A comparison between the two economies leads to the following results.

First, in the long-run equilibrium, the existence of overlapping contracts does not make any difference in terms of output. As was pointed out above, in both cases, the nominal wage is set at a sufficiently high level so that the government finds it too costly to allow for higher inflation. Despite the presence of multi-period contracts, monetary policy has no long-run real effect. This is a simple consequence of wage setters' expectations being fulfilled in the steady state.

Second, for the two economies, the steady state level of inflation responds in the same direction to changes in the parameters as in the static economy (see Appendix 6.3 for details). Thus, the inflationary bias decreases with ρ, the output elasticity of prices, and with λ, the monetary authority's concern about inflation (the point emphasised by Rogoff (1985), that calling for a more conservative central banker will reduce the

[6]See Appendix 6.2 for further details on the algebra for the steady state.

credibility problem). Also, the smaller $\tilde{y}$, the equilibrium output loss due to the labour market distortion, the smaller the inflation in the equilibrium. Finally, for the staggered contract economy, expected inflation increases with δ, the government's discount factor. Furthermore, the static and dynamic cases coincide for $\delta = 0$ since the short-run incentive to inflate is the same in either case, and only the short-run matters.

Third, as pointed out above, the existence of multi-period contracts enhances the inflationary bias. To see why, suppose, contrary to the result, that we start off at the inflationary bias π^s associated with the *static* (one period contract) equilibrium. By definition of the time consistent solution, the benefits *at* t of increasing demand in terms of increasing y_t are exactly offset by the increased inflation. However, consider the effects of a small, positive monetary surprise at t, increasing inflation above π^s, but thereafter returning to π^s. Consider the path of real wages: Because the wage contract fixes the nominal wage to be received for the next two periods, for any equilibrium with positive inflation, the current old's real wage is below zero, while the young's is above. Therefore, if wage setters anticipate the *static* equilibrium rate of inflation, π^s, the real wage received by the workers will evolve as in Figure 1 below. However, the monetary surprise at t implies that not only during period t, but also during period $t+1$, the old's real wage will be lower than otherwise. If we went back to the *static* equilibrium rate of inflation, π^s, from $t+1$ onwards, y_{t+1} would be increased at effectively no cost (hence, a first-order gain), whereas the small increase in y_t and $p_t - p_{t-1}$ due to the monetary surprise at time t represents a zero first-order loss at t by definition of π^s being optimal for the static case. Thus, when nominal wages are fixed for two periods, the short-run incentive to inflate is not different from the *static* case. But, given that an inflation surprise at t will not only increase current output, but also next period output, the (present-value) benefit from this one-off inflation increase is larger than in the static case.[7]

[7] A similar argument suggests that the inflation bias associated with the *static* case would not be sufficient to stop the government inflating in a model with long-term, time-varying contracts *à la* Fischer.

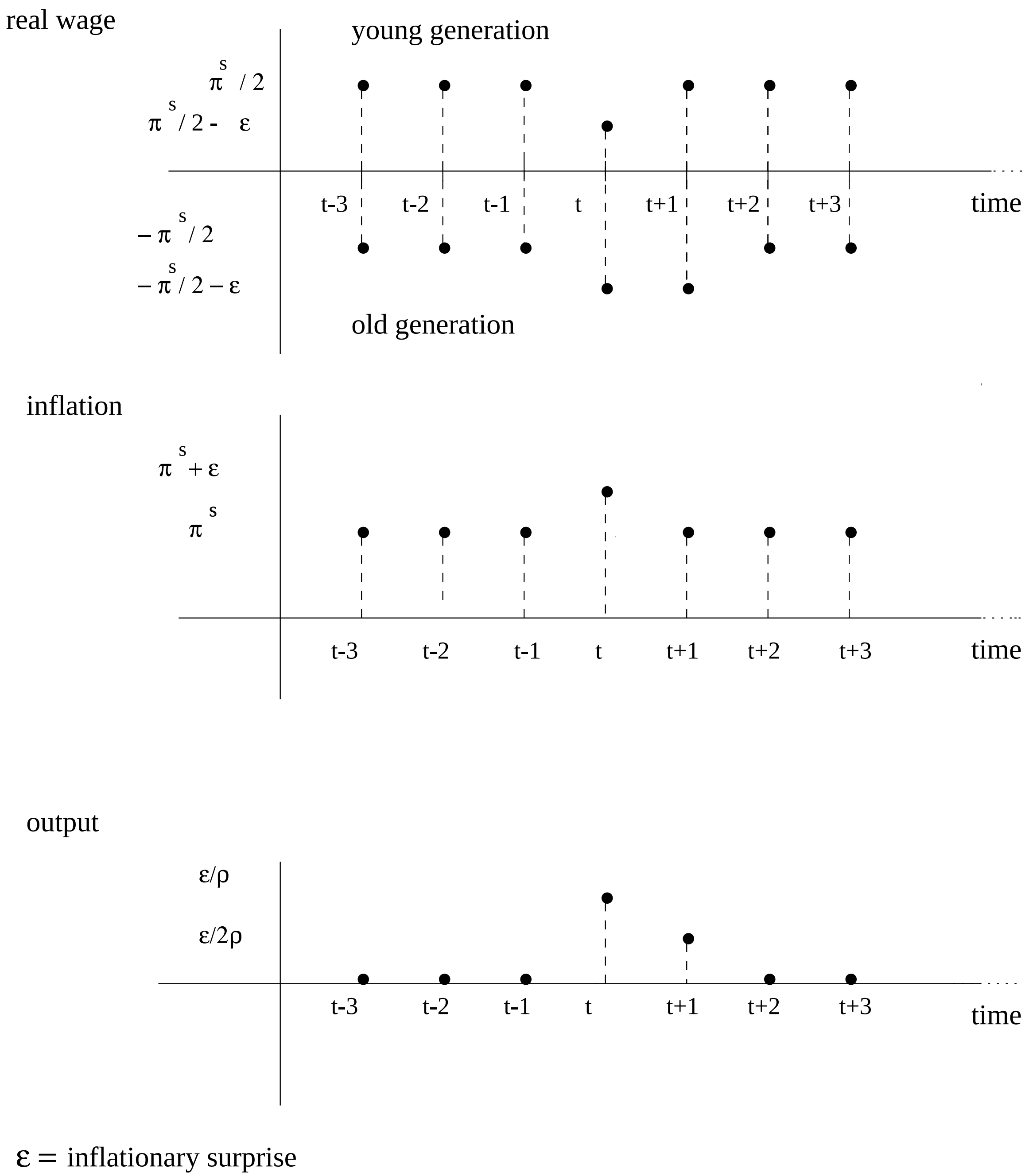

Figure 1: The effects of a monetary surprise at time t

4 The Role for Precommitment

To avoid thetime-consistency problem, the policy maker could design a permanent constitutional reform that prescribes zero inflation but allows the central bank to respond optimally to disturbances. There are some practical drawbacks, however, in legislating a fully state-contingent rule. For example, for the rule to be effective, it must be set in such a way that it is very difficult to change. This raises the danger that the policy-maker may be unable to foresee the nature of the shocks buffeting the economy many years in advance. Furthermore, the existence of transaction costs may make it prohibitively costly to specify the inflation rate to be chosen optimally in every possible state of nature.

In a deterministic framework – as in the one presented here – no monetary policy stabilisation is required; the optimal policy rule is easy to legislate, and a policy maker, who has a zero inflation bliss point, could deliver *sensible* monetary policy. Indeed, in the one period wage setting model – here also referred to as the *static* economy –it is sufficient for the second-best to be attained that the policy maker be able to precommit period by period. Precommitment solutions, however, appear to have different properties in the presence of staggered wage setting.

In this section we discuss two partial commitment scenarios for an economy with long-term staggered contracts. In the first one, the central bank commits to the money supply m_t only for the current period. We show that, as the wage contracts last for two periods, the monetary authority's precommitment to m_t, but not to m_{t+1}, is not enough entirely to remove the inflation bias. Therefore, we also analyse the precommitment solution when the government is able to announce in advance the monetary policy that will be implemented for the same duration as nominal wage contracts. In other words, we allow the central bank to choose (commit to) the money supply one period in advance, so that at the beginning of time t the central bank also makes the announcement for m_{t+1}. We show that, even with this precommitment structure, the outcome will not generally be zero inflation. In other words, even before the wage setters write their contracts, at the beginning of time t, the policy-maker in equilibrium will generally prefer to announce a monetary policy, m_{t+1}, that does not deliver zero inflation.

4.1 Precommitment to m_t

At the beginning of each period, before wage setters determine x_t, the government announces m_t. The optimal rule then solves (15) subject to (10), (11) and[8]

$$x_t \;=\; \alpha_0\text{'} + \alpha_1\text{'} z_{t-1} + \alpha_2\text{'}(m_t - x_{t-1}) + p_{t-1}. \tag{23}$$

Formally, the only difference from the previous model is that, within a period, the order of movements between the wage setters and the policy-maker has been reversed. Following the MPE restrictions, we look for a wage setters' reaction function that includes the new variable, $(m_t - x_{t-1})$, in order to capture all the relevant information available to wage setters at the time of writing their contracts. Note that the present discounted social losses from t onwards are still given by an equation of the form of (14), so that the optimal rule is given by a feedback rule for m_t on z_{t-1}. Following a similar procedure as before, the F.O.C. yield:

$$m_t - x_{t-1} \;=\; \frac{-J}{A} - \left(\frac{K}{A} - 1\right) z_{t-1}; \tag{24}$$

$$y_t \;=\; \frac{-\alpha_0\text{'}}{2(1+\rho)} + \left[\frac{2-\alpha_2\text{'}}{2(1+\rho)}\right] (m_t - x_{t-1}) - \left[\frac{\alpha_1\text{'}+1}{2(1+\rho)}\right] z_{t-1}, \tag{25}$$

$$p_t - p_{t-1} \;=\; \frac{\alpha_0\text{'}}{2(1+\rho)} + \left[\frac{2\rho+\alpha_2\text{'}}{2(1+\rho)}\right] (m_t - x_{t-1}) + \left[\frac{\alpha_1\text{'} - 1 - 2\rho}{2(1+\rho)}\right] z_{t-1}, \tag{26}$$

$$z_t \;=\; -\alpha_0\text{'} + \frac{\alpha_0\text{'}}{2(1+\rho)} + \left[\frac{2\rho+\alpha_2\text{'}}{2(1+\rho)} - \alpha_2\text{'}\right] (m_t - x_{t-1}) + \left[\frac{\alpha_1\text{'} - 1 - 2\rho}{2(1+\rho)} - \alpha_1\text{'}\right] z_{t-1}, \tag{27}$$

[8] Henceforth, the parameters $\alpha_{i'}$ ($\beta_{i'}$) denote the coefficients that characterise the equilibrium wage setters' reaction function (present discounted social losses) under commitment to m_t.

where J, A and K are given in Appendix 7.1.

In order to characterise the solution to our problem we need to identify the parameters α_0', α_1', α_2', β_1', and β_2'. For α_0' and α_1', the coefficients in Equation (23) must be equal to those obtained after substituting Equation (26) into (7). Next, we substitute the expressions (25) and (26) into (15) and ensure that the coefficients on z_{t-1} are identical to those of Equation (14).

After substituting out the values for α_0', α_1' and α_2' in (27), we can write (see Appendix 7.2):

$$z_t = \frac{-(1-\lambda)\tilde{y}}{4\lambda\rho}. \tag{28}$$

Our solution for the state variable mirrors that obtained in no commitment case, in the sense that the state variable is always at its unique rational expectations equilibrium level. Therefore, Equation (28) gives us the steady state value for z_t, which, in turn, implies a steady state inflationary bias of:

$$\bar{k} = \frac{(1-\lambda)\tilde{y}}{2\lambda\rho}. \tag{29}$$

The steady-state level of output remains at the natural rate, i.e., $y = 0$.

A comparison between Equations (29) and (6) enables us to conclude that the equilibrium inflation rate equals half the inflation bias associated with the *static* case. The reason for this is intuitive: In any period t, the policy maker has the usual incentive to exploit the output benefits derived from surprise inflation, but now only with respect to the half of the wage contracts that will not be revised at t. As usual, wage setters at $t-1$ anticipate this temptation and increase their wages so as to prevent any monetary surprise, but this increase needs only to be half as large as in the static model.[9] In the presence of long-term staggered contracts, in order to remove completely the inflation

[9] One might think that this structure of precommitment could be sufficient to remove the current inflationary bias in the presence of long-term, time-varying contracts *à la* Fischer. If wage contracts last for more than one period, however, precommitment to m_t only will not be sufficient to avoid the incorporation of positive expected inflation in the future.

bias, it seems that the government must be able to precommit for at least the length of nominal contracting. We turn to this matter in the next subsection.

4.2 Precommitment to m_{t+1}

In this section, we analyse the precommitment solution when the government is able to announce in advance the monetary policy that will be implemented for the whole duration of the contract written at $t-1$. In other words, before the wage contracts are written, at the beginning of period $t-1$, the government commits to m_{t-1} and m_t. It should be noted that the government cannot follow this structure of precommitment for more than the first period: when time t arrives, m_t has been already determined by the previous announcement, and all that is left is the announcement of the monetary policy for next period, m_{t+1}. This will lead to a sort of sequence of overlapping monetary decisions.

The optimal rule under commitment to m_{t+1} is derived as the solution to the following problem:

$$V(z_{t-1}, m_t - x_{t-1}) \;=\; \underset{m_{t+1}}{Min}\left[(1-\lambda)(y_t - \tilde{y})^2 \;+\; \lambda(p_t - p_{t-1})^2 \;+\delta V(z_t, m_{t+1} - x_t)\right], \tag{30}$$

subject to (10), (11) and [10]

$$x_t \;=\; \alpha_0" + \alpha_1"(m_t - x_{t-1}) + \alpha_2"(m_{t+1} - x_{t-1}) + x_{t-1}. \tag{31}$$

In this case, the wage setters know both m_t and m_{t+1} at the time of writing their contracts. Following the MPE restrictions, we look for a wage setters' reaction function that includes the new state variable $(m_t - x_{t-1})$, and also $(m_{t+1} - x_{t-1})$ as the workers make use of all relevant information available when the contracts are written at the beginning of period t. Since there are now two state variables, the present discounted value of social

[10] Henceforth, the parameters α_i" (β_i") denote the coefficients that characterise the equilibrium wage setters' reaction function (present discounted social losses) under commitment to m_{t+1}.

losses can now be rewritten as follows:

$$V(z_{t-1}, m_t - x_{t-1}) = \beta_0" + \beta_1"z_{t-1} + \beta_2"(m_t - x_{t-1}) + \beta_3"z_{t-1}(m_t - x_{t-1}) + \beta_4"z_{t-1}^2 + \beta_5"(m_t - x_{t-1})^2. \tag{32}$$

Since we assume that $V_t(z_{t-1}, m_t - x_{t-1})$ is linear quadratic in z_{t-1} and $(m_t - x_{t-1})$, the optimal monetary rule can be written as a linear feedback rule for m_{t+1} on z_{t-1} and $(m_t - x_{t-1})$.Following the same procedure as before, we are unable to obtain a closed form solution for the equilibrium inflation rate, but we do get some economic intuition through the numerical simulations presented in Table 1.[11]

Table 1: Inflation in the steady state

	π
$\delta = .95$	.01441
$\delta = .5$	.06
$\delta = \frac{1}{3}$	0
$\delta = 0$	$-.03$

Other values $\rho = .25$, $\lambda = .5$, and $\tilde{y} = .03$.

	π
$\lambda = 1$	0
$\lambda = .8$	.0041
$\lambda = .05$	.01805

Other values $\rho = .25$, $\delta = .95$, and $\tilde{y} = .03$.

	π
$\rho = 1.5$	.0011
$\rho = .75$	.0033
$\rho = .05$	.0867

Other values $\delta = .95$, $\lambda = .5$, and $\tilde{y} = .03$.

	π
$\tilde{y} = 1$	.4805
$\tilde{y} = .05$	.024
$\tilde{y} = 0$	0

Other values $\rho = .25$, $\lambda = .5$, and $\delta = .95$.

If the government does not suffer from a credibility problem, either because it is only concerned about inflation ($\lambda = 1$), or because it lacks the incentives to create monetary surprises, ($\tilde{y} = 0$), the equilibrium inflation rate equals zero. However, when the parameter λ is not equal to 1 or when $\tilde{y}$ is different from zero, the long-run level of inflation does not correspond to zero. In these cases the value of society's discount factor seems to play

[11]Some infinite horizon monetary policy games, based on MPE, have been shown to possess a multiplicity of Perfect Nash Equilibria, see, e.g., Lockwood and Philippopoulos (1994). Our simulations suggest that this is also the case for this example. The results presented in this subsection correspond to the most intuitive equilibrium. A Mathematica program for these results is available on request, as are details of other calculations reported in this section.

an important role.

For example, assuming that we start off at zero inflation, a very impatient monetary authority would choose to announce future cuts in the growth of money, so as to induce a short-run increase in current output. In other words, a monetary institution that heavily discounted the future would choose to announce, at the beginning of time t, a contractionary monetary policy for $t+1$. In extreme cases, when the policy-maker is very *myopic*, a deflationary policy would be announced. Our simulations show that a negative steady state inflation rate might exists for small values of δ.

The intuition is clear: suppose that at time t, m_t has been already determined by, let us say, a previous government committed to zero inflation, i.e., $m_t = m_{t-1} = p_{t-1}$. The announcement of a decrease in the growth of money for tomorrow, $m_{t+1} - m_t < 0$, leads to a disinflationary process, for labour market equilibrium to be preserved at $t+1$.[12] As a result, wages written at the beginning of time t, x_t, will not be as high as expected at time $t-1$, when x_{t-1} and m_t were chosen. Therefore, the current level of prices, p_t, will be smaller than expected at $t-1$. But, if $p_t < p_{t-1} = m_t$, m_t will be too large and there will be gains in terms of current output which, for a small cut in money supply growth, would imply a first-order welfare increase compared to the second-order welfare cost of deflation.

Similar scenarios for costless, credible disinflations, which occur without any increase in unemployment, can be found in the literature. In Taylor (1983) credible disinflation can be costless, but only if it is slow, in a model with precommitment and time-varying wage contracts. Buiter and Miller (1985) use Calvo's (1983) model of staggering to show that immediate disinflation can be costless. The closest example to our analysis is provided by Ball (1994), who finds that fully credible disinflation will cause a *boom* in a model *à la* Taylor. Our analysis, however, departs from these works in two important respects. First, trying to explain whether the inflationary inertia arising from the staggered timing

[12]Note that, in this particular example, *disinflation* (decrease in the growth rate of prices) effectively means *deflation* (decrease in the price level), since we are assuming a zero inflation starting point at time t.

of price adjustments can be blamed for the costs of disinflationary policy, these authors assume perfect credibility from the side of the policy-maker. In our model, however, disinflationary policy is not assumed to be credible but it is shown to be the optimal and, therefore, credible choice of a very impatient policy-maker. Second, our model predicts that, for a very impatient central banker, not only disinflationary policy but also a deflationary one may be optimal and credible.

A very different picture is obtained when society is relatively more patient – values of δ closer to 1, as the long-run equilibrium is inflationary. The intuition for this seems to be as follows. Suppose that the policy maker has maintained zero monetary growth in the past, so inflation is zero. Suppose that at time t it announces a (surprise) increase in m_{t+1} (m_t is already committed to of course), and suppose that this new level of money supply is committed to thereafter[13]. As we have just argued, this will cause a deflation at time t and a loss of output. From time $t+1$ onwards, however, there will be positive output gains as the price level asymptotically approaches its new higher steady-state level, with real balances and hence output above their steady-state levels. One can check that the infinite sum of these positive output deviations is always greater than the loss of output at t. For a small increase in m_{t+1}, the welfare effects of inflation at each date are all second-order, while the output changes lead to first-order welfare effects, which, for small changes are proportional to the output changes. As the policy maker becomes very patient, the overall discounted welfare change will converge to the undiscounted sum of welfare changes, which thus has the same sign as the undiscounted sum of output changes. Since the latter is positive, the central banker gains by increasing inflation above zero.[14]

Finally, delegating monetary policy to a central banker with the appropriate discount factor will lead to zero inflation: in our numerical simulations, if δ was equal to 1/3, the zero inflation rule could be achieved, when $\rho = 0.25$, $\lambda = 0.5$ and $\tilde{y} = 0.03$. As usual,

[13] Clearly this does not correspond to equilibrium dynamics, but it illustrates the incentives which exist for positive inflation.

[14] In fact, in simulations, the discounted sum of output deviations is positive for relatively low discount factors.

of course, the delegation of monetary policy to a policy-maker without any concern over output, $\lambda = 1$, or the elimination of the distortions tempting the policy-maker to inflate, $\tilde{y} = 0$, will also deliver a zero inflation outcome.

5 Conclusions

What we show in this paper is how the credibility problem of monetary policy acquires special significance in the presence of staggered wage setting. Moreover, some of the solutions proposed in the literature to overcome this problem, such as delegation or pre-commitment, appear to be less efficient when the extensive use of overlapping contracts is accounted for. There are several reasons:

Taylor (1980) demonstrates that, under staggered wage setting, one-off shocks to the economy are capable of generating unemployment persistence. Yet, persistence in unemployment increases the costs of delegating monetary policy to a very conservative central bank. Conservative central bankers are a *reasonably good*, *third-best* solution in a model *à la* Rogoff, but they seem not to perform so well for economies with distorted labour markets.

Precommitment to the *ex-ante* optimal monetary policy (*second-best*) though difficult to implement in a stochastic environment, would eliminate the inflationary bias of non credible monetary policy in a model with one period wage setting. Yet, allowing for nominal rigidities seriously reduces the efficiency of this mechanism. As shown before, period by period precommitment is not sufficient to reach the *second-best* solution, if wage setters write their contracts for more than one period. If monetary decisions are taken at the beginning of each period, when, let us say, workers are setting contracts for two periods, at the beginning of every second period, then, the central banker will face credibility problems if committed only period by period. This result is also relevant to the literature on optimal central bank inflation contracts, for it emphasises the need for such incentive contracts to have sufficient length. Finally, in an attempt to solve this difficulty, a longer-term but still feasible commitment was analysed. Our findings suggest that, in the presence of long-term staggered wage contracts, the monetary authority will in general not deliver zero inflation.

One major limitation of the current paper was the absence of supply shocks. This allowed us to concentrate on the classical time-consistency problem in a tractible fashion, and is justifiable given our concentration on the question of commitment. In order, however, to extend the discussion to the issue of delegation (to a conservative central banker) in the overlapping contracts context, it would be necessary to introduce such shocks, as has been done in Levine and Pearlman (1994) with Calvo contracts. It is hoped to consider this extension in future work.

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6 APPENDIX: The time consistent equilibrium

6.1 Solving for α_1, β_2, β_1 and α_0

Our consistency conditions lead to the following relationships

For α_1:

$$\alpha_1 = \frac{\left(1+\rho+\rho D+\frac{1}{2}+\alpha_1\rho B+\frac{\alpha_1}{2}\right)\left(\alpha_1\rho B+\frac{\alpha_1}{2}-\rho C-\frac{1}{2}\right)}{(1+\rho)\left[2(1+\rho)-\rho C-\frac{1}{2}+\alpha_1\rho B+\frac{\alpha_1}{2}\right]}.$$

Making use of the relationship $C+D=1$, and simplifying, one can write

$$\alpha_1 = -\left(\frac{\frac{1}{2}+\rho C}{\frac{1}{2}+\rho-\rho B}\right) = \frac{\lambda-1}{1-\lambda+2\lambda\rho^2}. \tag{1}$$

For β_2:

$$\begin{aligned}\beta_2 = {} & \frac{1}{(1+\rho)^2}\ \lambda\left(\rho D+\alpha_1\rho B+\frac{\alpha_1}{2}-\frac{1}{2}-\rho\right)^2 \\ & + \frac{1}{(1+\rho)^2}\ (1-\lambda)\left(D-\frac{1}{2}+\alpha_1 B-\frac{\alpha_1}{2}\right)^2 \\ & + \frac{\delta\beta_2}{(1+\rho)^2}\ \left[\rho D+\frac{1}{2}-(\alpha_1+1)(1+\rho)+\alpha_1\rho B+\frac{\alpha_1}{2}\right]^2.\end{aligned}$$

Using $C+D=1$, substituting α_1 and simplifying, one finds

$$\beta_2 = \frac{(1-\lambda)\lambda(1-\lambda+\lambda\rho^2)}{(1-\lambda+2\lambda\rho^2)^2}. \tag{2}$$

For α_0:

$$\begin{aligned}\alpha_0 = {} & \frac{\left[2(1+\rho)+\rho D+\frac{1}{2}+\alpha_1\rho B+\frac{\alpha_1}{2}\right]}{\left[2(1+\rho)-\frac{1}{2}-\rho C+\alpha_1\rho B+\frac{\alpha_1}{2}\right]} \\ & \times\left[2A\rho(1-\lambda)\tilde{y}-A\delta\rho^2\beta_1+\alpha_0\left(\frac{\rho B+\frac{1}{2}}{1+\rho}\right)\right],\end{aligned}$$

that, after simplifying, becomes

$$\alpha_0 = \frac{2A(1-\lambda+3\lambda\rho^2)}{1-\lambda+4\lambda\rho^2}\,[2\rho(1-\lambda)\tilde{y}-\delta\rho^2\beta_1+\alpha_0(1-\lambda+2\delta\rho^2\beta_2)]. \tag{3}$$

For β_1:

$$\begin{aligned}\beta_1 = {} & \frac{2(\lambda-1)}{1+\rho}\left(C-\frac{1}{2}+\frac{\alpha_1}{2}-\alpha_1 B\right) \\ & \times\left[[2A(1-\lambda)-1]\tilde{y}-A\delta\rho\beta_1+\alpha_0\left(\frac{B-\frac{1}{2}}{1+\rho}\right)\right] \\ & -\frac{2\lambda}{1+\rho}\left(\rho C+\frac{1}{2}-\frac{\alpha_1}{2}-\alpha_1\rho B\right) \\ & \times\left[2A(1-\lambda)\rho\tilde{y}-A\delta\rho^2\beta_1+\alpha_0\left(\frac{\rho B+\frac{1}{2}}{1+\rho}\right)\right] \\ & -\frac{\delta\beta_1}{1+\rho}\left(\rho C+\frac{1}{2}+\alpha_1(1+\rho)-\alpha_1\rho B-\frac{\alpha_1}{2}\right) \\ & -\frac{2\delta\beta_2}{1+\rho}\left(\rho C+\frac{1}{2}+\alpha_1(1+\rho)-\alpha_1\rho B-\frac{\alpha_1}{2}\right) \\ & \times\left[2A(1-\lambda)\rho\tilde{y}-A\delta\rho^2\beta_1+\alpha_0\left(\frac{\rho B+\frac{1}{2}}{1+\rho}-1\right)\right].\end{aligned}$$

After making some simplifications, following similar steps as before,

$$\beta_1 = \frac{2(1-\lambda)\lambda}{1-\lambda+2\lambda\rho^2}\left(-\rho\tilde{y}-\frac{\alpha_0}{2}\right). \tag{4}$$

To obtain the reduced form for α_0 and β_1, we eventually get:

$$\alpha_0 = \frac{(-1+\lambda)(-1+\lambda-3\lambda\rho^2)(1-\lambda+2\lambda\rho^2+\delta\lambda\rho^2)\tilde{y}}{\lambda\rho\,(1-\lambda+2\lambda\rho^2)\,(1-\delta-\lambda+\delta\lambda+2\lambda\rho^2)}, \tag{5}$$

and

$$\beta_1 = \frac{2(-1+\lambda)\lambda\rho\tilde{y}}{1-\lambda+2\lambda\rho^2}+\frac{(1-\lambda)^2(-1+\lambda-3\lambda\rho^2)(1-\lambda+2\lambda\rho^2+\delta\lambda\rho^2)\tilde{y}}{\rho(1-\lambda+2\lambda\rho^2)^2\,(1-\delta-\lambda+\delta\lambda+2\lambda\rho^2)}. \tag{6}$$

6.2 The steady state

6.2.1 The equilibrium inflation rate

In the steady state, the following must be satisfied: $x_t = x_{t-1}+\bar{k}$, $p_t = p_{t-1}+\bar{k}$ and $z_{t-1} = z_t$, where $\bar{k}$ represents the inflationary bias associated with the steady state.

Hence, making use of Equations (12) and (13), one can write

$$z_t \;=\; p_t - x_t \;=\; p_t - \alpha_0 - \alpha_1 z_{t-1} - \; p_{t-1}. \tag{7}$$

Therefore, substituting out Equation (19) and simplifying,

$$z_t \;=\; 2A\rho(1-\lambda)\tilde{y} - A\delta\rho^2\beta_1 + \frac{\alpha_0}{1+\rho}(\rho B - \frac{1}{2} - \rho). \tag{8}$$

The reduced form for the state variable can then be obtained:

$$z_t \;=\; \frac{(-1+\lambda)\tilde{y}(1-\lambda+2\lambda\rho^2+\delta\lambda\rho^2)}{2\lambda\rho\,[1-\lambda+2\lambda\rho^2+(\lambda-1)\delta]}. \tag{9}$$

On the other hand, given that Equation (7) implies that, in the steady state,

$$x_t \;=\; \frac{p_t + p_t + k}{2}, \tag{10}$$

one can write

$$z_t \;=\; -\frac{k}{2}, \tag{11}$$

which leads to

$$\bar{k} \;=\; \frac{(1-\lambda)\tilde{y}(1-\lambda+2\lambda\rho^2+\delta\lambda\rho^2)}{\lambda\rho\,[1-\lambda+2\lambda\rho^2+(\lambda-1)\delta]}. \tag{12}$$

6.3 The response of the equilibrium inflation rate to changes in the structural parameters

The derivatives of the steady state inflationary bias with respect to λ, $\tilde{y}$, ρ y δ can be calculated to be as follows:

$$D[\bar{k},\lambda] \;=\; \frac{-\left[(1-\delta)(1-\lambda)^2+4\lambda\rho^2(1-\lambda)+4\lambda^2\rho^4+2\delta\lambda^2\rho^4\right]\;\tilde{y}}{\lambda^2\rho(1-\delta-\lambda+\delta\lambda+2\lambda\rho^2)^2} \leq 0, \tag{13}$$

$$D[\bar{k},\tilde{y}] \;=\; \frac{(1-\lambda)(1-\lambda+2\lambda\rho^2+\delta\lambda\rho^2)}{\lambda\rho(1-\delta-\lambda+\delta\lambda+2\lambda\rho^2)} \geq 0, \tag{14}$$

$$D[\bar{k},\rho] = \frac{(1-\lambda)\tilde{y}}{\lambda\rho^2(1-\delta-\lambda+\delta\lambda+2\lambda\rho^2)^2} \times \Big[(1-\lambda)^2(-1+\delta) - 4\lambda\rho^2(1-\lambda) - \delta\rho^2\lambda(1+\delta)(1-\lambda) - 2\lambda^2\rho^4(2+\delta)\Big] \leq 0,$$

and

$$D[\bar{k},\delta] \;=\; \frac{(1-\lambda)\tilde{y}\,[(1-\lambda)^2 + 3\lambda\rho^2(1-\lambda) + 2\lambda^2\rho^4]}{\lambda\rho(1-\delta-\lambda+\delta\lambda+2\lambda\rho^2)} \geq 0. \tag{15}$$

7 Precommitment to m_t

7.1 The values of J, A and K.

$$\begin{aligned} J = &-2\alpha_0' + \alpha_0'\alpha_2' - \alpha_2'\beta_1'\delta + \alpha_0'\alpha_2'\beta_2'\delta + 2\alpha_0'\lambda + 2\beta_1'\delta\rho - 3\alpha_2'\beta_1\delta\rho - 2\alpha_0'\beta_2'\delta\rho \\ &+ 4\alpha_0'\alpha_2'\beta_2'\delta\rho + 2\alpha_0'\lambda\rho + 2\beta_1'\delta\rho^2 - 2\alpha_2'\beta_1'\delta\rho^2 - 4\alpha_0'\beta_2'\delta\rho^2 + 4\alpha_0'\alpha_2'\beta_2'\delta\rho^2 \\ &- 4\tilde{y} + 2\alpha_2'\tilde{y} + 4\lambda\tilde{y} - 2\alpha_2'\lambda\tilde{y} - 4\rho\tilde{y} + 2\alpha_2'\rho\tilde{y} + 4\lambda\rho\tilde{y} - 2\alpha_2'\lambda\rho\tilde{y}, \end{aligned} \tag{16}$$

$$\begin{aligned} A = &\,4 - 4\alpha_2' + \alpha_2'^2 + \alpha_2'^2\beta_2'\delta - 4\lambda + 4\alpha_2'\lambda - 4\alpha_2'\beta_2'\delta\rho + 4\alpha_2'^2\beta_2'\delta\rho \\ &+ 4\alpha_2'\lambda\rho + 4\beta_2'\delta\rho^2 - 8\alpha_2'\beta_2'\delta\rho^2 + 4\alpha_2'^2\beta_2'\delta\rho^2 + 4\lambda\rho^2, \end{aligned} \tag{17}$$

$$\begin{aligned} K = &\,2 - 2\alpha_1' - 3\alpha_2' + \alpha_1'\alpha_2' + \alpha_2'^2 + \alpha_2'\beta_2'\delta + \alpha_1'\alpha_2'\beta_2'\delta + \alpha_2'^2\beta_2'\delta - 2\lambda + 2\alpha_1'\lambda \\ &+ 2\alpha_2'\lambda - 2\beta_2'\delta\rho - 2\alpha_1'\beta_2'\delta\rho + 4\alpha_1'\alpha_2'\beta_2'\delta\rho + 4\alpha_2'^2\beta_2'\delta\rho - 2\lambda\rho + 2\alpha_1'\lambda\rho \\ &+ 2\alpha_2'\lambda\rho - 4\alpha_1'\beta_2'\delta\rho^2 - 4\alpha_2'\beta_2'\delta\rho^2 + 4\alpha_1'\alpha_2'\beta_2'\delta\rho^2 + 4\alpha_2'^2\beta_2'\delta\rho^2. \end{aligned} \tag{18}$$

7.2 Solving for α_0', α_1', α_2', β_1' and β_2'

Under commitment, our consistency conditions lead to the following relationships:

$$\alpha_2\text{'} = \frac{2\rho}{1+2\rho}; \tag{19}$$

$$\alpha_1\text{'} = -1, \tag{20}$$

$$\alpha_0\text{'} = \frac{(1-\lambda)\,(1+\rho)\,\tilde{y}}{2\lambda\,\rho\,(1+2\rho)}, \tag{21}$$

$$\beta_1\text{'} = \frac{(-1+\lambda)\,(1-\lambda+8\lambda\rho^2)\tilde{y}}{2\rho\,(1-\lambda+4\lambda\rho^2)}, \tag{22}$$

$$\beta_2\text{'} = \frac{\lambda\,(1-\lambda)}{1-\lambda+4\lambda\rho^2}. \tag{23}$$

8 Notes to Referees: Precommitment to m_{t+1}

Suppose that m has been constant at m_0 in the past. Then at date $t = 0$, the government announces that from $t = 1$, m will be constant at m^* – recall that m_0 will be fixed from the previous period. The announcement can be considered a surprise as we assume that $x_{t-1} = m_0$ for $t < 0$, i.e., the wage setters do not anticipate $m = m^*$.

To solve for the solution, *taking this money path as given*, we first look for the solution for a given money stock (at m^*) and work backwards to $t = 0$ (which has $m = m_0$), taking $x_{-1} = m_0$.

From Equation (7) and (10) in the main text,

$$x_t = \frac{\hat{p}_t + \hat{p}_{t+1}}{2}, \tag{24}$$

and

$$p_t = \frac{1}{1+\rho}\left[\rho m_t + \frac{1}{2}\left(x_t + x_{t-1}\right)\right], \tag{25}$$

so in equilibrium one can write

$$\begin{aligned} x_t &= \frac{1}{2(1+\rho)} \cdot [\rho m^* + \frac{1}{2}\left(x_t + x_{t-1}\right) + \rho m^* + \frac{1}{2}\left(x_{t+1} + x_t\right)] \\ &= \frac{1}{4(1+\rho)} \cdot (x_{t-1} + 2x_t + x_{t+1} + 4\rho m^*). \end{aligned} \tag{26}$$

Or, alternatively,

$$x_{t+1} - (2 + 4\rho)x_t + x_{t-1} + 4\rho m^* = 0, \tag{27}$$

a second order difference equation with single stable root that equals

$$\eta = 1 + 2\rho - 2\sqrt{\rho(1+\rho)}, \tag{28}$$

so that,

$$x_t = K\eta^t + m^* \qquad t = 0, 1, 2, 3, ..., \tag{29}$$

where K is a constant to be determined.

Note that Equation (29) holds at $t = 0$, even though m_0 is different from m^*, since Equation (27) holds for $t = 1$, and this includes x_0 which must be on this stable path.

To solve for K, the following conditions have been used. At $t = 0$,

$$x_0 = K + m^*. \tag{30}$$

The solution for x_0 must also satisfy Equations (24) and (25) which now lead to (cf. Equation (27)):

$$x_1 - (2 + 4\rho)x_0 + x_{-1} + 2\rho(m_0 + m^*) = 0. \tag{31}$$

Then, from Equation (29),

$$x_1 = \eta K + m^* = \eta x_0 + (1 - \eta)m^*, \tag{32}$$

so in Equation (31), after rearrangement,

$$x_0 = \frac{m^*(1 - \eta + 2\rho) + m_0(1 + 2\rho)}{2 + 4\rho - \eta}, \tag{33}$$

and

$$K = \frac{m_0 - m^*}{2 + 4\rho - \eta}. \tag{34}$$

To determine whether a patient government might have an incentive to deviate from a zero inflation starting point in the direction of positive inflation, despite the output loss in period $t = 0$, we can look at, for $(m^* - m_0)$ very small, the sum –over time– of output deviations from 0, i.e., $y_0 + y_1 + y_2 + \dots$ (see main text). For $t = 1, 2, 3, ...,$

$$p_t = \frac{1}{1+\rho} \cdot \left[\frac{(K\eta^{t-1} + m^*) + (K\eta^t + m^*)}{2} + \rho m^* \right], \tag{35}$$

$$p_t = m^* + \frac{K\eta^{t-1}(1+\eta)}{2(1+\rho)} \tag{36}$$

and

$$y_t = m^* - p_t = \frac{-K\eta^{t-1}(1+\eta)}{2(1+\rho)} \tag{37}$$

Likewise, for $t = 0$, after rearrangement,

$$p_0 = \frac{1}{1+\rho} \cdot \left[\frac{m_0(2 + 4\rho - \eta + 4\rho - \delta\rho^2 - 2\rho\eta + 1 + 2\rho) + m^*(1 - \eta + 2\rho)}{2(2 + 4\rho - \eta)} \right] \tag{38}$$

$$y_0 = m_0 - p_0 = \frac{(1 + 2\rho - \eta)(m_0 - m^*)}{2(1+\rho)(2 + 4\rho - \eta)}. \tag{39}$$

(Nb. if $m^* < m_0$, we get $y_0 > 0$ as argued in the text.).

$$\begin{aligned} \sum_{t=1}^{\infty}(y_t) &= \frac{-K(1+\eta)}{2(1+\rho)(1-\eta)} \\ &= frac(m^* - m_0)(1 + 2\rho)(1+\eta)(2 + 4\rho - \eta)2(1-\eta)(1+\rho) \end{aligned} \tag{40}$$

If $m^* > m_0$, the output gain in Equation (40) is bigger than the output loss in Equation (39), if and only if,

$$(1-\eta)(1 + 2\rho - \eta) - (1+\rho)(1+2\rho)(1+\eta) < 0, \tag{41}$$

which holds for all $\rho \geq 0$. Hence, clearly, this inflation policy is beneficial for sufficiently patient policy makers.