GOVERNMENT FINANCING AND INTEREST RATES IN A THREE ASSETS SIDRAUSKI-BASED MODEL

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ABSTRACT

In this paper we formulate a Sidrauski-based model with three assets in which we introduce public bonds into the utility function of agents, with the purpose of analyzing some related questions with regards to the consequences of the financial activity of the government and the determination of the interest rates. The results obtained permit us to conclude that, within this framework, ① financial decisions of government will not influence the steady state levels of consumption and capital, and ② the inflation rate affects the real interest rate on bonds negatively.

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1.-INTRODUCTION

In 1967 M. Sidrauski, using the framework developed by Ramsey (1928), formulated a two assets growth model in which the intertemporal conditions that determine the optimal path of money, capital and consumption of the economy are analyzed. The Sidrauski model is based on the following hypotheses:

- There are three kinds of agents: households, firms and public sector (or "government").
- The households are identical. They are assumed to have an infinite horizon and perfect foresight. They must choose, in every period of time, their level of consumption and savings. Savings are distributed among two kinds of assets: money and capital.
- There are many identical firms, which behave competitively. Output is produced using capital and labor (rented from the families) with a constant returns to scale technology.
- There is only one type of good, which may be consumed or accumulated, i.e. added to the stock of capital.
- The only activity of the government is to provide money, which ends up in the hands of agents by means of lump-sum transfers.
- The agents demand money for the liquidity services that money provides. In the model, the "mechanism" that induces agents to hold money is explicitly considered by introducing liquid balances in the households' utility function.

The Sidrauski model has reached a great diffusion in the literature and many important economic consequences have been subsequently obtained from its framework. In particular, it has been used by authors such as Fischer (1972), Dornbusch and Frenkel (1973), Calvo (1979), Drazen (1981), Siegel (1983), Danthine and Donaldson (1986), Dutkovsky and Foote (1988), Barr and Cuthberson (1991), etc.

One of the main results of the model is that, assuming the rate of time preference constant, in the long term the real interest rate will run independent of the inflation rate. This conclusion would still hold true if the existence of other assets, besides money and capital, were permitted without considering any other additional assumptions, because in this case all the assets, except money, would be treated as perfect substitutes, and consequently their real rates would not be affected by the inflation rate. In our opinion, this fact contradicts the evidence suggesting that inflation does not influence all interest rates equally, which would mean that the implicit assumption of perfect substitutability is not supported by data.

We think that one reason which may help to explain this different behavior of the assets returns is the varying characteristics in terms of liquidity that the different assets have. In this paper, we will try to reflect this argument within the framework of a version of the Sidrauski model with three assets (money, bonds, and capital) in which bonds and capital will not be treated as perfect substitutes.

In particular, we shall formulate an augmented version of the model by permitting the public sector to make use of other sources of finance (besides money issue) and incur other kinds of expenditure, not only transfers to households. We will allow government to borrow from the private sector, thus assuming that a third type of financial asset will appear: the public bonds, which we shall consider they offer a positive rate of return (therefore higher than that of money), but likewise are more liquid than capital.

Ultimately, in this paper we have three purposes:

1.-First, to introduce adequately into the framework of the Sidrauski model this liquidity attribute of the public bonds.

2.-Second, to examine the effects on the economy's equilibrium of certain government's financial decisions.

3.-Third, to analyze the effects of inflation on the interest rate on bonds.

In the following section we will briefly characterize the behavior of the different economic agents in the model (government, households and firms) and, subsequently (in section 3) we will obtain some results about the topics referred on purposes two and three above.

2.-THE BEHAVIOR OF THE ECONOMIC AGENTS

2.1-Public Sector: Their activity is displayed through a series of expenditures, which must be financed by means of alternative sources of income. In particular the government must fulfil in each period the following budget constraint:

$\frac{dM_t}{dt} + \frac{dB_t}{dt} + T_t = G_t + R_{B_t} \cdot B_t$, where M_t and B_t are, respectively, money and public bonds supplies, T_t are the government taxes (net of transfers), G_t is the public consumption, and R_{B_t} is the nominal interest rate on bonds. Dividing both sides by P_t (price level) and N_t (household size), denoting real per-capita variables by lowercase letters, and omitting time indexes we have:

$$\frac{dm}{dt} + (\pi + n) \cdot m + \frac{db}{dt} + n \cdot b + \tau = g + r_B \cdot b, \qquad (1)$$

where r_B is the real interest rate on bonds, n is the rate of population growth, and π is the inflation rate.

2.2-Households: It is assumed that households decide their levels of consumption and savings following an optimizing behavior, and that they distribute their wealth among three kinds of assets: money, capital and public bonds. As previously mentioned, in the Sidrauski model money demand was exclusively based on the liquidity services of money, while demand for capital was justified by the return that capital goods yield to their holders. In our model another kind of asset incorporates in the agents' portfolios: the public bonds. In order to explain its demand we will consider that bonds, besides offering a positive rate of return, present certain characteristics of liquidity which stand between those of money and capital. We will assume that bonds are more liquid than capital in their ability to reduce both risk and transaction costs, which will lead agents to demand bonds not only because of the "return motive" but also due to their liquidity.[1] This would imply that bonds are supposed to enter the households' utility function. In particular, we will consider that bonds holdings multiplied by a parameter "z" ($0 < z < 1$) add to money holdings as an argument of the utility function, so that, the higher the liquidity of bonds is, the higher the value of z will be. In other words, the household liquidity will not be only determined by m but rather it will rise to $m + z \cdot b$.

Then each household maximizes, in any instant of time s:

$$U_s = \int_s^{\infty} u(c_t, m_t + z \cdot b_t) \cdot e^{-\theta(t-s)} \cdot dt\,, \qquad (2)$$

where $u(\,)$ is the instantaneous utility function (with $u_1',\ u_2' > 0;\ u_{11}'',\ u_{22}'' < 0$), c_t and m_t are real consumption and liquid balances per-capita, and θ is the rate of time preference. Household budget constraint is given by the expression:

$C_t + \dfrac{dM_t}{dt} + \dfrac{dB_t}{dt} + \dfrac{dK_t}{dt} = W_t + R_{B,t} \cdot B_t + R_{K,t} \cdot K_t - T_t$, where C_t, M_t, B_t and K_t are household consumption, and household money, bonds and capital holdings, respectively; $R_{K,t}$ is the nominal interest rate on capital; W_t are the salary incomes of the household. Dividing both sides by P_t and N_t and omitting time indexes we have:[2]

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[1] This argument has been proposed by authors such as Carmichael and Stebbing (1983), Fried and Howit (1983), and Mitchel (1985). Fried and Howitt state that the Sidrauski model and its variants recognise the liquidity of money (by allowing the representative agent's production or utility function to depend upon his holdings of real balances) but they do not recognise the liquidity of bonds.

[2] It is assumed that equilibrium conditions in the assets markets are fulfilled, so, in order to simplify, we denote by M, B and K both assets demands and assets supplies.

$$c + \frac{dm}{dt} + \frac{db}{dt} + \frac{dk}{dt} = w + (r_K - n)\cdot k + (r_B - n)\cdot b - \tau - m\cdot(\pi + n) \;\Rightarrow$$

$$\Rightarrow \; \frac{da}{dt} = w - c + (r_K - n)\cdot a + (r_B - r_K)\cdot b - \tau - (r_K + \pi)\cdot m, \qquad (3)$$

where r_k is the real rate on capital and a is the household wealth ($a = m + b + k$).

Households must maximize *(2)*, subject to *(3)*. The present value Hamiltonian function associated with this problem is: $H = \left\{ U(c, m + z\cdot b) + \lambda \cdot \left[\frac{da}{dt} \right] \right\} \cdot e^{-\theta t}$.

Denoting the costate variable by λ, the first order conditions to solve the household maximization problem will be as follows:

$$u_1'(c, m+z\cdot b) = \lambda, \qquad (4)$$

$$u_2'(c, m+z\cdot b) = (r_K + \pi)\cdot\lambda, \qquad (5)$$

$$u_2'(c, m+z\cdot b) = \frac{(r_K - r_B)}{z}\cdot\lambda, \qquad (6)$$

$$\frac{d\lambda}{dt} - \theta\cdot\lambda = -\lambda\cdot(r_K - n), \qquad (7)$$

$$\lim_{t\to\infty} a_t\cdot\lambda_t\cdot e^{-\theta t} = 0. \qquad (8)$$

2.3-Firms: Aggregate output (Y) is produced using labor (L) and capital (K). The productive sector of the economy will be treated as if it were one competitive firm, with a production function displaying constant returns to scale.[3] So, $Y_t = F(L_t, K_t)$, or, alternatively, in per capita terms, and eliminating time indexes, $y = f(k)$. Finally, from the profit maximization conditions we obtain:

$$f'(k) = r_K, \qquad (9)$$

$$f(k) - k\cdot f'(k) = w. \qquad (10)$$

[3]The constant returns assumption implies that the number of firms is of no consequence.

3.-SOME RESULTS

3.1-Effects of government's financial decisions

The equilibrium adjustment path of the capital and private consumption is represented by the following equations:

$$\frac{dk}{dt} = f(k) - n\cdot k - c(k, \lambda, m, b) - g \,, \tag{11}$$

$$\frac{d\lambda}{dt} = [n + \theta - f'(k)]\cdot\lambda \,. \tag{12}$$

Equation *(11)* is obtained by combining, after some manipulations, equations *(1)*, *(3)* and *(10)*, and equation *(12)* is derived by substituting *(9)* into *(7)* and using *(4)* to express the private consumption as a function of *k*, *λ*, *m* and *b*.[4] It can be observed that money and debt affect the private consumption and consequently enter the equation of capital accumulation. This implies that, considering a given level of government spending, the way it was financed will have consequences for the equilibrium path of consumption and capital. In this respect, by total differentiation of *(4)* we obtain:

$$\frac{dc}{dm} = -\frac{u''_{12}}{u''_{11}} \,, \tag{13}$$

$$\frac{dc}{db} = -z\cdot\frac{u''_{12}}{u''_{11}} \,. \tag{14}$$

We observe that the impact of changes in *m* or *b* on the private consumption will depend on the values of u'_{11} and u'_{12}, and thus, given that $u'_{11} < 0$, the sign of *(13)* and *(14)* will be the same as that of u'_{12}. In general, it is assumed that money and consumption are

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[4] Note that, in order to simplify, we use the rate of change of the costate variable instead of the rate of change of private consumption. It can be easily demonstrated that equation *(12)* is equivalent to $dc/dt = c\cdot\sigma(c)\cdot[f'(k) - n - \theta]$, where σ is the instantaneous elasticity of substitution of private consumption.

Edgeworth-Pareto complementary, i.e. $u'_{12} > 0$.[5] In this case, if money or bonds stock increase, this would have a short term positive effect on private consumption and a negative effect on the rate of capital accumulation. This influence of bonds comes exclusively from their liquidity attribute, thus the greater the value of the parameter z, the greater the influence will be.

However, these results do not hold true in the steady state, in which we have the same results as in the Sidrauski (and the Ramsey) models: $c^* = f(k^*) - n \cdot k^* - g$ and $f'(k^*) = n + \theta$, where we denote by c^* and k^* the steady state values of private consumption and capital. In this situation, neither money nor bonds affect consumption and capital.

These conclusions can be resumed in the following proposition:

Proposition 1: Money and public bonds influence the equilibrium trajectories of private consumption and capital by means of their effect on the consumption marginal utility, with the higher the liquidity of bonds, the greater the influence of these bonds. This relationship disappears in the steady state.

3.2-Assets substitutability and interest rates

From equations *(3)* and *(4)* we obtain:

$$(r_K + \pi) = \frac{(r_K - r_B)}{z} \Rightarrow (r_K + \pi) \cdot z = (r_K - r_B) \Rightarrow r_B = r_K - (r_K + \pi) \cdot z \Rightarrow$$

$$\Rightarrow r_B = (1 - z) \cdot r_K - z \cdot \pi. \qquad (15)$$

We conclude from *(15)* that the real interest rate on capital will be in any case higher than that on bonds. The interpretation is obvious: given that the liquidity of bonds is greater, their equilibrium return must be lesser. We could say that liquidity carries a price in terms of foregone interest.

[5] See Fischer (1972) and Calvo (1979).

If we concentrate on the steady state, differentiating *(15)*, gives

$$dr_B = (1 - z)\cdot dr_K - z\cdot d\pi. \qquad (16)$$

Given that k (and, consequently, r_K) are not affected by the inflation rate, we can conclude that, in the steady state $\frac{dr_B}{d\pi} = -z < 0$, i.e. the real interest rate on bonds and the inflation rate are inversely related. Alternatively, reasoning with the nominal interest rates ($R = r + \pi$):

$$R_B = (1\text{-}z)\cdot r_K + (1\text{-}z)\cdot\pi \Rightarrow \frac{dR_B}{d\pi} = (1\text{-}z) < 1. \qquad (17)$$

Moreover, differentiating the definition of m ($m = \frac{M}{P \cdot N}$) and denoting by σ the rate of money growth, we obtain the rate of growth of real per-capita liquid balances, $\frac{dm}{dt} = (\sigma - n - \pi)\cdot m$, from which we derive the steady state inflation rate $\pi^* = (\sigma - n)$. Introducing this relation into *(16)* it follows that $dr_B = (1 - z)\cdot dr_K - z\cdot\frac{d\pi}{d\sigma}\cdot d\sigma \Rightarrow \frac{dr_B}{d\sigma} = -z$.

We will resume these latter results in the following proposition:

<u>Proposition 2</u>: The interest rate on capital will be in all cases higher than that on bonds, verifying that the higher the liquidity of bonds (determined by the value of the parameter z), the greater the difference between both interest rates. On the other hand, in the steady state an increase in the inflation rate (of an amount equal to $\Delta\pi > 0$) will lead to an increase in the interest rate on bonds at an amount less than $\Delta\pi$, the greater being this increase, the lesser the liquidity of bonds. Consequently, given that in the steady state a modification of σ implies a modification of π by the same amount, in the steady state the rate of money growth will affect negatively the real interest rate on bonds.

REFERENCES

BARR, D. G. and K. CUTHBERTSON (1991). "Neoclassical Consumer Demand Theory and the Demand for Money", *The Economic Journal*, 101, pp. 855-876.

CALVO, G. A. (1979). "On Models of Money and Perfect Foresight", *International Economic Review*, 20, pp. 83-103.

CARMICHAEL J. and P. W. STEBBING (1983). "Fisher's Paradox and the Theory of Interest", *American Economic Review*, 73, pp. 619-630.

DANTHINE, J.P. and J. B. DONALDSON (1986). "Inflation and Asset Prices in an Exchange Economy". *Econometrica*, 54, pp. 585-605.

DORNBUSCH, R. and J. A. FRENKEL (1973): "Inflation an Growth", *Journal of Money, Credit and Banking*, 5, pp. 141-156.

DRAZEN, A. (1981). "Inflation and Capital Accumulation Under a Finite Horizon", *Journal of Monetary Economics*, 8, pp. 247-260.

DUTKOWSKY, D. H. and W. G. FOOTE (1988). "The Demand for Money: A Rational Expectations Approach", *The Review of Economics and Statistics*, 70, pp. 83-92.

FISCHER, S. (1972). "Money, Income, Wealth, and Welfare", *Journal of Economic Theory*, 4, pp. 289-311.

FRIED, J. and P. HOWITT (1983). "The Effects of Inflation on Real Interest Rates", *American Economic Review*, 73, pp. 968-980.

MITCHELL, D. W. (1985). "Expected Inflation and Interest Rates in a Multi-asset Model: A Note", *Journal of Finance*, 40, pp. 595-599.

PATINKIN, D. (1965). *Money, Interest and Prices: An Integration of Money and Value Theory*, 2nd edition, New York, Harper and Row.

RAMSEY, F. P. (1928). "A Mathematical Theory of Saving", *Economic Journal*, 38, pp. 543-559.

SIDRAUSKI, M. (1967). "Rational Choice and Patterns of Growth in a Monetary Economy", *American Economic Review*, 57, pp. 534-544.

SIEGEL, J. J. (1983). “Technological Change and the Superneutrality of Money”, *Journal of Money, Credit and Banking*, 15, pp. 363-367.