Time-Space Model of Business Fluctuations

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* This article represents the personal view of author.

ABSTRACT

Here author made an attempt to extend the Continuous-Time Model of Business Fluctuations on the space domain. Research methodology is based on Time-Space Model of Wave Propagation developed by author to describe fluctuation processes in physics.

Journal of Economic Literature Classification Numbers: E 32.

Keywords: Business Fluctuations

1. Introduction

Recent Continuos-Time Model of Business Fluctuations (e.g. see [2, 3]) describes the nature of business fluctuations in the time domain only. Later author developed (see [4]) the Time-Space Model of Wave Propagation, which successfully describes some fluctuation problem in physics. Author updated the Business Fluctuations model here to describe such fluctuations both in time and in space.

2. Model Assumptions

The following is held in the time domain. At each point in space I assume:

(a) The derivative of commodity's price with respect to time is directly proportional to the amount of commodity's deficit at this time (where deficit means the commodity's demand minus its supply).

(b) The second derivative of commodity's production with respect to time is directl proportional to the derivative of commodity's price with respect to time.

(c) The derivative of commodity's demand with respect to time is inversel proportional to the derivative of commodity's price with respect to time.

The following is held in the space domain. At each point in time I assume:

(d) The derivative of commodity's price with respect to direction is directl proportional to the amount of commodity's deficit at this place.

(e) The second derivative of commodity's production with respect to direction is directly proportional to the derivative of commodity's price with respect to direction.

(f) The derivative of commodity's demand with respect to direction is inversel proportional to the derivative of commodity's price with respect to direction.

For simplicity, I will describe the one-dimensional space.

3. Model Description

Let me consider an economical model with one commodity. I denote $V_d(x,t)$ the volume of commodity's demand in point x at time t, and $V_p(x,t)$ the volume of commodity's production in point x at time t.

I assume that system was at rest until time $t_0 = 0$, i.e. production rate $r_p(x,t) = \frac{dV_p(x,t)}{dt}$ coincided with demand rate $r_d(x,t) = \frac{dV_d(x,t)}{dt}$ in every point in time and in space, and whole commodity was consumed.

Therefore

$$r_d(x,t) = r_p(x,t) = const \, , \tag{1}$$

$$V_d(x,t) = V_p(x,t) = 0 \, , \tag{2}$$

for $-\infty < t < 0$ and for all x.

At time $t_0 = 0$, I assume that commodity's demand rate has been increased in the point $x_0 = 0$.

Hence

$$r_d(0,0) = r_p(0,0) + \Delta_r \, . \tag{3}$$

Let's look how the initial commodity's deficit rate Δ_r will be propagated in the space and time.

According to our assumptions we can write the following expressions.

In the time domain,

$$\frac{dP(x_1,t)}{dt} = \lambda_t (V_d(x_1,t) - V_p(x_1,t)), \qquad (4)$$

$$\frac{d^2 V_p(x_1,t)}{dt^2} = \mu_t \frac{dP(x_1,t)}{dt}, \qquad (5)$$

$$\frac{dV_d(x_1,t)}{dt} = -\nu_t \frac{dP(x_1,t)}{dt}, \qquad (6)$$

where $x_1 \geq 0$, $\lambda_t, \mu_t, \nu_t > 0$ are constants, and $P(x_1,t)$ is commodity's price in point x_1 at time t.

We can rewrite equations (4) – (6) to describe dynamics of commodity's deficit $V_{df}(x_1,t)$ in time,

$$\frac{d^2 V_{df}(x_1,t)}{dt^2} + \nu_t \lambda_t \frac{dV_{df}(x_1,t)}{dt} + \mu_t \lambda_t V_{df}(x_1,t) = 0, \qquad (7)$$

where $V_{df}(x,t) = V_d(x,t) - V_p(x,t)$, $r_{df}(x,t) = \frac{dV_{df}(x,t)}{dt}$.

Equation (7) is the ordinary differential equation of the second order, and can be resolved by standard methods (see [1, 5]).

Note that the value $V_{df}(x_1,t) \to 0$ when $t \to +\infty$ for all $x_1 > 0$.

In the space domain,

$$\frac{dP(x,t_1)}{dx} = \lambda_x (V_d(x,t_1) - V_p(x,t_1)), \qquad (8)$$

$$\frac{d^2V_p(x,t_1)}{dx^2} = \mu_x \frac{dP(x,t_1)}{dx}, \quad (9)$$

$$\frac{dV_d(x,t_1)}{dx} = -\nu_x \frac{dP(x,t_1)}{dx}, \quad (10)$$

where $t_1 \geq 0$, $\lambda_x, \mu_x, \nu_x > 0$ are constants.

We also can rewrite equations (8) – (10) to describe dynamics of commodity's deficit $V_{d\,f}(x,t_1)$ in space,

$$\frac{d^2V_{d\,f}(x,t_1)}{dx^2} + \nu_x\lambda_x \frac{dV_{d\,f}(x,t_1)}{dx} + \mu_x\lambda_x V_{d\,f}(x,t_1) = 0. \quad (11)$$

Equation (11) is also the ordinary differential equation of the second order (see [1, 5]).

Here again the value $V_{d\,f}(x,t_1) \to 0$ when $x \to +\infty$ for all $t_1 > 0$.

I said before about intention to represent value $V_{d\,f}(x,t)$ of commodity's deficit, which propagated from initial value of commodity's deficit rate Δ_r.

Let me show how the value $V_{d\,f}(x_1,t_1)$ is obtained for $x_1 > 0$ and $t_1 > 0$.

If I denote c the velocity of deficit spreading, then commodity's deficit (surplus) in point x_1 starts to appear at time $t_0 = {x_1}/{c}$.

Therefore, if $t_1 \leq t_0$ then $V_{d\,f}(x_1,t_1) \equiv 0$.

If $t_1 = t_0 + \Delta t$ (where $\Delta t > 0$) we may find at first $V_{d\,f}(0,\Delta t)$ from equation (7), and initial values $V_{d\,f}(0,0) = 0$ and $r_{d\,f}(0,0) = \Delta_r$. Then we find $V_{d\,f}(x_1,\Delta t)$ from equation (11), and initial values $V_{d\,f}(0,\Delta t)$ and $\frac{dV_{d\,f}(0,\Delta t)}{dx} = 0$. This $V_{d\,f}(x_1,\Delta t)$ is the commodity's deficit in point x_1 at time Δt for ideal *'instant'* velocity of deficit spreading that is equal to the commodity's deficit in point x_1 at time $(t_0 + \Delta t)$ for deficit spreading with real velocit c.

3. Dynamics of Prices

Similarly to (7), we can express the dynamics of commodity's price in time,

$$\frac{d^2 P(x_1,t)}{dt^2} + \lambda_t \nu_t \frac{dP(x_1,t)}{dt} + \lambda_t \mu_t P(x_1,t) + C_1 = 0, \tag{12}$$

where $C_1 = \lambda_t \left(r_p(x_1,t_0) - \mu_t P(x_1,t_0) \right)$. Note $r_p(x_1,t_0)$, $P(x_1,t_0)$ are values in point x_1 at initial time t_0.

Therefore, $P(x_1,t) \rightarrow \left(P(x_1,t_0) - \frac{1}{\mu_t} r_p(x_1,t_0) \right)$ when $t \rightarrow +\infty$ for all $x_1 > 0$.

And it follows the dynamics of commodity's price in space,

$$\frac{d^2 P(x,t_1)}{dx^2} + \lambda_x \nu_x \frac{dP(x,t_1)}{dx} + \lambda_x \mu_x P(x,t_1) + C_2 = 0, \tag{13}$$

where $C_2 = \lambda_x \left(r_p(x_0,t_1) - \mu_x P(x_0,t_1) \right)$. Note $r_p(x_0,t_1)$, $P(x_0,t_1)$ are values at time t_1 in initial point x_0.

Therefore, $P(x,t_1) \rightarrow \left(P(x_0,t_1) - \frac{1}{\mu_x} r_p(x_0,t_1) \right)$ when $x \rightarrow +\infty$ for all $t_1 > 0$.

4. Conclusive Remarks

I exposed here only some features (which are applicable to the phenomenon of business fluctuations) of the Time-Space Model of Wave Propagation. For more insight into the problem, one should address the original article [4] or subsequent papers.

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