

# **Liability Rules and Evolutionary Dynamics**

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## **Abstract**

We consider the convergence properties of behavior under a comparative negligence rule (CN) and under a rule of negligence with contributory negligence (NCN), assuming bilateral care with three care levels. Using an evolutionary model, we show that CN reduces the proportion of the population using low care more rapidly than does NCN. However NCN increases the proportion of the population using high (efficient) care more rapidly than does CN. As a result, the mean care level increases more rapidly and the mean social cost falls more rapidly under CN than under NCN. (JEL: K 13, C 79)

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## ***1 Introduction***

An economic analysis of law considers tort liability as a tool that can induce injurers to internalize the costs they impose on others. An efficient liability rule should provide incentives for a causative contributor to an accident to minimize the sum of accident and avoidance costs by taking cost-justified precautions. Many authors have discussed the equilibrium and efficiency of different liability rules. (BROWN [1973], LANDES AND POSNER [1987], SHAVELL [1987], ARLEN [1992]).

An important issue is how to choose the best liability rule. In recent years the comparative negligence rule (CN hereafter) has spread widely, replacing the rule of negligence with contributory negligence (NCN hereafter). Eight US states had adopted comparative negligence by 1971, but an additional 34 adopted it between 1971 and 1985. CN was argued to be inferior to NCN because the court must decide on the degree of the negligence by both parties (WHITE [1988]). POSNER [1992] stated that “the modern movement to substitute comparative for contributory negligence” is one of the three “most important counter examples to the efficiency theory of law”. WHITE [1989] tested empirically whether the incentive to take care to avoid accidents is stronger under NCN than under CN.

The above literature only considered whether the rules provide an efficient incentive to take care. WITTMAN, FRIEDMAN, CREVIER AND BRASKIN [1997] addressed another consideration in choosing among liability rules: the speed of convergence to equilibrium levels of care. When behavior is not at equilibrium, Nash equilibrium is seldom achieved instantaneously. WITTMAN, FRIEDMAN, CREVIER AND BRASKIN [1997] undertook an experimental test of convergence to equilibrium under different liability rules. In their laboratory experiment, convergence to equilibrium (measured by mean care level) is much more rapid under comparative negligence than under contributory negligence. They gave no theoretical explanation for their result.

From time to time, society is away from the equilibrium level of care for different reasons. Individuals may not be fully rational. In auto accidents, there may be new drivers who do not correctly perceive the risks and costs, such as when a demographic bulge hits driving age, or a wave of immigration produces a stock of new drivers. At any time some individuals may experiment with new strategies. When a court or legislature changes the rules, it will take time before drivers adapt themselves to that change. Changing technology can change the risk and cost of driving significantly<sup>2</sup>. Any of these factors can require drivers to adjust their behavior to a new optimum, incurring high social costs if the adjustment is slow.

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<sup>2</sup> For example, anti-lock brakes and air bags may reduce the cost of aggressive driving. The increase in the proportion of large sport-utility vehicles and pickup trucks may reduce the benefits of careful driving of their owners and increase the benefits of careful driving for owners of small cars.

The main contribution of this paper is that we use an evolutionary approach to analyze the speed of convergence under different liability rules, using a simple setting of bilateral care with three care levels. Homogenous drivers decide whether to take a high, medium or low levels of care. The high level of care is the social optimum. In this setting of three care levels, the Nash equilibrium under both CN and NCN is the social optimum.

The evolutionary approach assumes that a strategy that does well is imitated, while a strategy that does badly is rejected. We assume that in every period an individual reflects on the payoff from his strategy and shares strategy and payoff information with others. In every period a fraction of those individuals with a lower payoff change their current strategies to more profitable strategies. This eventually leads to the convergence to the equilibrium strategy. Because of inertia and uncertainty the fraction of individuals that change their strategy in any period is relatively small. The greater is the payoff difference, the greater is the incentive to change the strategy.

We show that CN reduces the proportion of the population using low care more rapidly than does NCN. However NCN increases the proportion of the population using high (efficient) care more rapidly than does CN. As a result, the mean care level increases more rapidly and the mean social cost falls more rapidly under CN

than under NCN during the early periods. This is consistent with the result in WITTMAN, FRIEDMAN, CREVIER AND BRASKIN [1997].

The paper is organized as follows: section 2 states the basic assumptions and the Nash equilibrium under different liability rules, section 3 considers evolutionary dynamics and section 4 concludes.

## ***2 Liability Rules and Nash Equilibrium***

We consider a framework of bilateral care with three care levels. Suppose the population has infinitely many drivers. There are only three levels of care for each driver, high ( $h$ ), medium ( $m$ ) and low ( $l$ ), with the costs of taking care as  $c_h$ ,  $c_m$ , and  $c_l$  respectively. For simplicity, we also use  $h, m, l$  to denote the amount of care under three care levels,  $h > m > l > 0$ . Once an accident occurs, a total damage  $D > 0$  is incurred.

A driver does not know what kind of other drivers he will encounter during the day. We model these encounters as random matches. The probability of an accident in a match between two drivers is:  $p_{ij}, i = h, m, l$  and  $j = h, m, l$ , where  $p_{ij}$  is the

probability of an accident when a driver taking care  $i \in \{h, m, l\}$  meets another driver taking care  $j \in \{h, m, l\}$ . We assume that:

**Assumption 1.** Social cost is minimized if all drivers take high care. Social cost is maximized if all drivers take low care. Thus:

$$p_{hh}D + 2c_h < p_{hm}D + c_h + c_m < p_{mm} + 2c_m < p_{hl} + c_h + c_l < p_{ml} + c_m + c_l < p_{ll} + 2c_l. \quad (2.1)$$

From this assumption, we have:  $c_h - c_m < (p_{hm} - p_{hh})D$ ,  $c_h - c_m < (p_{mm} - p_{hm})D$ , and other similar inequalities.

We further assume that:

**Assumption 2.** An increase in the care level reduces more effectively the probability of an accident when the care level of the other driver is relatively low.

This implies that  $p_{hm} - p_{hh} < p_{mm} - p_{hm}$  and other similar inequalities.

Because taking high care minimizes social costs, any driver in an accident who does not take high care is negligent (or contributorily negligent).

Under CN, both drivers share the losses according to their relative negligence, or care shortfall. For example, if an accident happens between a driver taking medium care and a driver taking low care, then the driver taking medium care will bear

$\alpha = \frac{h-m}{h-m+h-l}$  portion of the total loss<sup>3</sup>, while the other party will bear the rest of the loss. Since  $h > m > l$ , we have  $\alpha > \frac{1}{2}$ . The following is the payoff matrix of the game.

Table .1

**Payoff Under CN**

	H	M	L
H	$-\frac{p_{hh}}{2}D - c_h, -\frac{p_{hh}}{2}D - c_h$	$-c_h, -p_{hm}D - c_m$	$-c_h, -p_{hl}D - c_l$
M	$-p_{hm}D - c_m, -c_h$	$-\frac{p_{mm}}{2}D - c_m, -\frac{p_{mm}}{2}D - c_m$	$-p_{ml}D - c_m, -(1-\alpha)p_{ml}D - c_l$
L	$-p_{hl}D - c_l, -c_h$	$-(1-\alpha)p_{ml}D - c_l, -p_{ml}D - c_l$	$-\frac{p_{ll}}{2}D - c_l, -\frac{p_{ll}}{2}D - c_l$

Under NCN, the parties share the liability in the same way as under CN except for the cases when both parties are negligent. In those cases, they both incur half of the total damage. The payoff of the game is the same form as that for CN, with  $\alpha = \frac{1}{2}$ .

Under NCN drivers taking low care and medium care are considered equally negligent.

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<sup>3</sup> This is a typical assumption in comparative negligence, see, e.g., COOTER and ULEN (1997). It may take a more general form as a function of the care levels and (or) the cost of care.

**Proposition 2.1.** If taking high care is socially efficient, then under both CN and NCN, taking high care is a Nash equilibrium.

Proof: Given the assumption about the parameter values, it is easy to check from the payoff matrix that taking high care is a Nash equilibrium.

As we can see from the payoff matrix, the only difference between CN and NCN is the value of  $\alpha$ . When there is an accident between a party taking low care and a party taking medium care, the party who takes low care has to incur a larger fraction of the cost of the accident under CN than under NCN. This provides a greater incentive to abandon low care under CN, as we will discuss in the next section.

### ***3 Evolutionary Dynamics***

In reality it is rare that Nash equilibrium is achieved instantaneously. Nash equilibrium requires that players are rational and know the payoff functions of all players, that they know their opponents are rational and know the payoff functions, that they know their opponents know, etc. In actual life, these requirements may not be met<sup>4</sup>.

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<sup>4</sup> REA [1987] points out that individual may be judgment proof or they could misperceive both the risks and costs. They may not choose the Nash equilibrium care level (they are unresponsive). The author suggests that, in comparison to the negligence rule with contributory negligence, the

This poses a problem: will the Nash equilibrium always be closely approximated at least in the long run? If this is the case, does the outcome converge to the Nash equilibrium rapidly and what is the path of the convergence?

To answer this question, we use an evolutionary approach (MAYNARD SMITH [1982]). The idea of evolutionary games began with the idea that animals are genetically programmed to play different pure strategies, and that the genes whose strategies are more successful will have higher reproductive fitness. The population fractions of strategies whose payoff against the current distribution of opponents' play is relative high will tend to grow at a faster rate, and any steady state must be Nash equilibrium. There is no need for the strong requirement of rationality and common knowledge among players.

Evolution can be taken as a metaphor for learning in economics. Individuals respond to different payoffs by modifying their strategies. If we assume inertia in human behavior and costs associated with switching strategies, then the proportion of the population choosing each strategy changes smoothly. In the following, the proportion of drivers taking care is subject to evolutionary pressure over time. The fraction of the population using better performing strategies will increase relative to

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negligence rule with comparative negligence is more robust to the presence of these unresponsive individuals.

those using lower payoff strategies. Our main focus will not be on the steady state of evolution, but on the relative speed and the path of convergence to the steady state under different liability rules.

We denote  $s_h(t), s_m(t)$  and  $s_l(t)$  the proportion of drivers taking high, medium and low level of care at time  $t$ ,  $s_h + s_m + s_l = 1$ . Given the population composition  $(s_h, s_m, s_l)$ , a driver will meet drivers taking high care with probability  $s_h$  and will meet drivers taking medium and low care with probability  $s_m, s_l$ . Under CN (and also NCN, which corresponds to  $\alpha = \frac{1}{2}$ ), at any time  $t$ , the expected payoff for a driver who takes high care is:

$$\pi_h = -s_h \frac{p_{hh}}{2} D - c_h. \quad (3.1)$$

The expected payoff for a driver who takes medium care is:

$$\pi_m = -(s_h p_{hm} + s_m \frac{p_{mm}}{2} + \alpha s_l p_{ml}) D - c_m. \quad (3.2)$$

The expected payoff for a driver who takes low care is:

$$\pi_l = -(s_h p_{hl} + (1 - \alpha) s_m p_{ml} + s_l \frac{p_{ll}}{2}) D - c_l. \quad (3.3)$$

The mean payoff of the population is:

$$\bar{\pi} = \sum s_i \pi_i . \quad (3.4)$$

The value of  $-\bar{\pi}$  is the social cost.

When  $\pi_h > \bar{\pi}$ , some drivers will find it profitable to switch from the strategy of taking low or medium care to the strategy of taking high care, and  $s_h$  will increase. The payoff differential exerts evolutionary pressure on the population composition. The standard model of the movement of the composition of the population in evolutionary game theory is that of the replicator dynamics<sup>5</sup>, (TAYLOR AND JONKER [1978]), defined as:

$$s_i'(t) = s_i(t)(\pi_i(t) - \bar{\pi}(t)) / M, i = h, m, l, \quad (3.5)$$

where  $s_i'(t)$  is the time derivative of  $s_i(t)$ , and  $M$  is a constant. The rate of the growth (decline) of the proportion of the population using a strategy is proportional to the amount by which that strategy's payoff exceeds (falls below of) the average payoff of the whole population. The standard replicator dynamics can be derived from different models of individual learning behavior (for example, NACHBAR

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<sup>5</sup> This is only for simplicity of the analysis. As we will see later, most of the analysis is still true if we use more general evolutionary dynamics such as a growth monotone dynamics.

[1990])<sup>6</sup>. The NACHBAR [1990] model can be reasonably used in the driving environment.

Evolutionary game theory is widely used in economics, for example in choosing the most likely equilibrium from all possible Nash equilibriums. Replicator dynamics allows us to compare the convergence properties under different liability rules in a given social environment and with a same learning pattern.

Simple calculations using expressions (3.1)-(3.4) show that the dynamics of a population taking high care is exactly the same<sup>7</sup> under both CN and NCN, and can be written as:

$$s_h' = s_h(\pi_h - \bar{\pi}) / M ,$$

with

$$\begin{aligned} \pi_h - \bar{\pi} &= s_m(\pi_h - \pi_m) + s_l(\pi_h - \pi_l) \\ &= s_m \left( (p_{hm} - \frac{p_{hh}}{2})D - (c_h - c_m) \right) + s_m^2 \left( \frac{p_{mm}}{2} - p_{hm} + \frac{p_{hh}}{2} \right) D + \\ & s_m s_l (p_{ml} - p_{hm} - p_{hl} + p_{hh}) D + s_l \left( (p_{hl} - \frac{p_{hh}}{2})D - c_h - c_l \right) + s_l^2 \left( \frac{p_{ll}}{2} - p_{hl} + \frac{p_{hh}}{2} \right) D \end{aligned} \quad (3.6)$$

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<sup>6</sup> In this model, individuals meet randomly somebody else to exchange information about each other's strategy and payoff. The individual with a lower payoff switches his strategy if the switching cost is less than the payoff difference. Assuming the switching cost is independently determined across individuals and is uniformly distributed on [0,M], we get the exact form of replicator dynamics as in (3.5).

<sup>7</sup> Notice that at any given population composition  $\bar{\pi}$  is always the same under CN and NCN.

Under Assumption 1 and Assumption 2, we can check that each term in the above equation is always positive, i.e.,  $s_h' > 0$  at any population composition with  $s_h < 1$ . The proportion of drivers taking efficient cares always strictly increases.

For the dynamics of a population taking medium care, under CN (also under NCN,

which corresponds to  $\alpha = \frac{1}{2}$ ),

$$\begin{aligned}\pi_m - \bar{\pi} &= s_h(\pi_m - \pi_h) + s_l(\pi_m - \pi_l) \\ &= -s_h\left(\frac{P_{mm}}{2}D + c_m - c_l\right) + s_h^2\left(\frac{P_{mm}}{2} - p_{hm} + \frac{P_{hh}}{2}\right)D + s_h s_l(p_{hl} - p_{hm} - p_{ml} + p_{mm})D \\ &\quad + s_l\left(\left(\frac{P_{ml}}{2} - \frac{P_{mm}}{2}\right)D - c_m + c_l\right) + s_l^2\left(\frac{P_{ll}}{2} - p_{ml} + \frac{P_{mm}}{2}\right)D + \left(\frac{1}{2} - \alpha\right)s_l p_{ml}D.\end{aligned}$$

(3.7)

Since  $\frac{P_{hh}}{2} - p_{hm} + \frac{P_{mm}}{2} > 0$ ,  $\frac{P_{ll}}{2} - p_{ml} + \frac{P_{mm}}{2} > 0$ ,  $p_{hl} - p_{hm} - p_{ml} + p_{mm} < 0$  (by

Assumption 2), if  $0 < s_h, s_l < 1$ , we have:

$$\pi_m - \bar{\pi} < -s_h\left(\left(p_{hm} - \frac{P_{hh}}{2}\right)D + c_m - c_l\right) + s_l\left(\frac{P_{ll}}{2} - \frac{P_{ml}}{2} - c_m + c_l\right)D + \left(\frac{1}{2} - \alpha\right)s_l p_{ml}D$$

(3.8)

As  $s_h$  increases to a certain extent,  $s_l$  becomes small and we have  $\pi_m - \bar{\pi} < 0$ , and  $s_m$  will begin to decrease monotonically.

Similarly,  $s_l$  will decrease monotonically as  $s_h$  increases to a certain extent.

From the above analysis, we have:

**Corollary 3.1.** Under evolutionary dynamics, both CN and NCN lead to convergence to the social optimum in the long run.

Proof: As in the above analysis,  $s_h$  always strictly increases, and  $s_m, s_l$  will decrease as  $s_h$  increases to a certain extent. Therefore, the population composition  $(s_h, s_m, s_l)$  must converge to  $(1,0,0)$ , which is the social optimum.

Given any population composition, the difference between the payoff of individuals taking low care and the average payoff of the population is bigger under CN than under NCN. Therefore under CN, the proportion of individuals taking low care decreases faster under CN at any given population composition (the proportion of individuals taking medium care decreases more slowly). We have the following lemma:

**Lemma 3.2.** At any population composition  $(s_h, s_m, s_l)$ , the instantaneous growth rate of the proportion of individuals taking high care is the same under CN and NCN. The instantaneous rate of decrease of the proportion of individuals taking low care is

greater under CN than under NCN by  $(\frac{1}{2} - \alpha)s_m p_{ml} D / M$ . The instantaneous rate of decrease of the proportion of individuals taking medium care is greater under NCN than under CN by  $(\frac{1}{2} - \alpha)s_l p_{ml} D / M$ .

Proof: It is easy to check by using expressions (3.1)-(3.4), and by the definition of replicator dynamics (3.5).

Unfortunately, the above lemma is only a local property and it assumes that we are at the same population composition under CN and NCN. Once the system begins to evolve, the evolution of the system will follow different paths under the two liability rules and the local comparison becomes meaningless. Now we begin to discuss the global convergence property and the convergence path under the two liability rules.

We can look at the path of the dynamics of population composition by looking at the  $(s_h, s_m)$  plane (since  $s_l = 1 - s_h - s_m$ ). From the same starting point at time 0, since the increase rate of  $s_h$  is the same under both liability rules and the decrease rate of  $s_m$  is slower under a comparative negligence rule, we have:

**Lemma 3.3.** Starting from the same point, the path (when  $t$  is small) under CN is above the path under NCN.

We may wonder if this is always true, or the two paths may cross at a later point.

**Lemma 3.4.** After starting from the same point, the path under CN will always be above the path under NCN.

Proof: Suppose that at a later point, the two paths reach a same composition  $s$ . Starting from this point, the path under CN will again be above the path under NCN. So the two paths can never cross.

Therefore the path of convergence under CN in the  $(s_h, s_m)$  plane is always above the one under NCN. See an example in figure 3 of the simulation.

**Lemma 3.5.** For two population compositions  $s_1 = (s_{h1}, s_{m1}, s_{l1})$  and  $s_2 = (s_{h2}, s_{m2}, s_{l2})$ , if  $s_{h1} = s_{h2}$  and  $s_{m1} > s_{m2}$ , then the increase rate of  $s_h$  at point  $s_1$  under CN is less than the increase rate of  $s_h$  at point  $s_2$  under NCN.

Proof: The rate of change of  $s_h$  has exact the same expression under both CN and NCN at any given population composition. From the form of the dynamics and the fact that the path under CN is always above the path under NCN in the  $(s_h, s_m)$  plane, it is sufficient to prove that for a fixed  $s_h$ ,  $\pi_h - \bar{\pi}$  (which is now a function of  $s_m$  only because of the constraint  $s_h + s_m + s_l = 1$ ) is a decreasing function of  $s_m$ .

$$\begin{aligned}
\frac{\partial(\pi_h - \bar{\pi})}{\partial s_m} \Big|_{s_h} &= (p_{hm} - \frac{P_{hh}}{2})D - (c_h - c_m) + 2s_m(\frac{P_{mm}}{2} - p_{hm} + \frac{P_{hh}}{2})D \\
&+ (1 - s_h - 2s_m)(p_{ml} - p_{hm} - p_{hl} + p_{hh})D - (p_{hl} - \frac{P_{hh}}{2})D \\
&+ (c_h - c_l) - 2s_l(\frac{P_{ll}}{2} - p_{hl} + \frac{P_{hh}}{2})D \\
&= (c_m - c_l) - (p_{hl} - p_{hm})D - s_m(p_{ml} - p_{hl} - (p_{mm} - p_{hm}))D - s_l(p_{ll} - p_{hl} - (p_{ml} - p_{hm}))D
\end{aligned}$$

Since  $(c_m - c_l) - (p_{hl} - p_{hm})D < 0$ ,  $p_{ml} - p_{hl} - (p_{mm} - p_{hm}) > 0$ , and

$p_{ll} - p_{hl} - (p_{ml} - p_{hm}) > 0$  (by Assumption 1 and Assumption 2), we have:

$$\frac{\partial(\pi_h - \bar{\pi})}{\partial s_m} \Big|_{s_h} < 0.$$

Intuitively, when  $s_m$  increases (so  $s_l$  decreases), the mean payoff  $\bar{\pi}$  increases.

Therefore, there is less evolutionary pressure for individuals to take a high level of care. According to the lemma, at any level of  $s_h$ , the increase rate of  $s_h$  is larger under NCN than under CN. We have:

**Proposition 3.6.** Globally, the proportion of population taking the efficient level of care increases faster under NCN than under CN.

Locally, under CN there is a stronger evolutionary pressure on the individuals who take low care; while under NCN there is a stronger evolutionary pressure on the individuals who take medium care. CN is more effective in reducing the number of

individuals taking low care while NCN is more effective in reducing the number of individuals taking medium care. Such a difference is significant when the proportion of individuals taking medium or low care is high. When society is close to the social optimum, this difference almost disappears. Globally, since at any given level of  $s_h$  there are more individuals taking low care and the mean payoff of the population is lower under NCN,  $s_h$  increases at a faster rate with NCN.

Which rule is better? It seems that a model of replicator dynamics suggests that NCN is better, since under NCN more individuals take the efficient care at any time. But that is not the whole story. Society cares about minimizing the total present value of social cost. For our case, the social cost at a given time  $t$  can be measured by  $-\bar{\pi}(t)$  and the present value of social cost  $-\bar{\pi}(t)$  is:

$$PV = -\int_0^{\infty} e^{-rt} \bar{\pi}(t) dt, \quad (3.9)$$

which depends not only on how many individuals take efficient care, but also on the care taken by other individuals. Under NCN more individuals take efficient care, but at any level of  $s_h$  more individuals take low care, which might be more costly to the society.

Another measure used in WITTMAN, FRIEDMAN, CREVIER AND BRASKIN [1997] is the mean care level of the population. In our case, the mean care level is:  $\bar{m} = hs_h + ms_m + ls_l$ . Though this measure ignores the social cost associated with each population composition, it is better than looking at only one component of the population composition.

We can compare the speed of the change of social cost and of the change of the mean care level of the population under CN and NCN.

**Proposition 3.7.** At a given population composition, the social cost  $-\bar{\pi}$  falls more rapidly and the mean care level  $\bar{m}$  of the population increases more rapidly under CN than under NCN.

Proof: Under both CN and NCN, we have:

$$\begin{aligned} \bar{\pi} = & -(s_h^2 \frac{p_{hh}}{2} + s_m^2 \frac{p_{mm}}{2} + s_l^2 \frac{p_{ll}}{2} + s_h s_m p_{hm} + s_h s_l p_{hl} + s_m s_l p_{ml})D \\ & - s_h c_h - s_m c_m - s_l c_l \end{aligned} \quad (3.10)$$

Therefore,

$$\begin{aligned} \frac{d\bar{\pi}(t)}{dt} = & -(s_h s_h' p_{hh} + s_m s_m' p_{mm} + s_l s_l' p_{ll} + s_h s_m' p_{hm} + s_h' s_m p_{hm} + s_h' s_l p_{hl} \\ & + s_h s_l' p_{hl} + s_m' s_l p_{ml} + s_m s_l' p_{ml})D - s_h' c_h - s_m' c_m - s_l' c_l \end{aligned} \quad (3.11)$$

At a given population composition  $s = (s_h, s_m, s_l)$ ,  $s_h'$  is the same under both CN and NCN. By equations (3.2) and (3.3), the difference in payoff between taking medium care under CN and under NCN is:  $\Delta_m = (\frac{1}{2} - \alpha)s_l p_{ml} D$ , and the difference in payoff

between taking low care under CN and under NCN is:  $\Delta_l = -(\frac{1}{2} - \alpha)s_m p_{ml} D$ .

Using the replicator dynamics, we have that the difference  $\Delta$  of the time derivative

$\frac{d\bar{\pi}(t)}{dt}$  of the mean social cost between under CN and under NCN is:

$$\begin{aligned} \Delta &= \Delta_m((-p_{mm}s_m^2 - p_{hm}s_h s_m - p_{ml}s_m s_l)D - s_m c_m) \\ &+ \Delta_l((-p_l s_l^2 - p_{hl}s_h s_l - p_{ml}s_m s_l)D - s_l c_l) \\ &= (\frac{1}{2} - \alpha)p_{ml}s_m s_l(((p_{hl} - p_{hm})s_h + (p_{ml} - p_{mm})s_m + (p_{ll} - p_{ml})s_l)D - c_m + c_l) \end{aligned}$$

(3.12)

By Assumption 1 and Assumption 2,  $p_{ml} - p_{mm} > p_{hl} - p_{hm}$ ,  $p_{ll} - p_{ml} > p_{hl} - p_{hm}$ ,

and:

$$\Delta > (\frac{1}{2} - \alpha)p_{ml}s_m s_l((p_{hl} - p_{hm})D - c_m + c_l) > 0.$$

Therefore,  $\bar{\pi}$  increases at a greater rate under NC than under NCN, i.e., the social cost  $-\bar{\pi}$  decreases at a greater rate under NC than under NCN.

Similarly, for the mean care level  $\bar{m}$ , we have:

$$\frac{d\bar{m}(t)}{dt} = h s_h'(t) + m s_m'(t) + l s_l'(t). \quad (3.13)$$

The difference of the time derivative of the mean care level  $\bar{m}$  between under CN and under NCN is:

$$ms_m\Delta_m + ls_l\Delta_l = (m-l)\left(\frac{1}{2} - \alpha\right)s_ms_lP_{ml}D > 0.$$

Therefore, the mean care level  $\bar{m}$  increases at a greater rate under CN than under NCN.

Since CN is more effective in reducing the number of individuals taking low care and NCN is more effective in reducing the number of individuals taking medium care, and taking low care is more costly for the society, at a given population composition, CN minimizes the social cost more effectively (at least for a short period of time). Also for a period of time the mean care level of the society increases more quickly under CN.

We cannot generalize the result in proposition (3.7) (which is a local property) to a global property. After a very long period of time, the result might not be true any more. However in the real world, the inefficiency is caused by periodic shocks, so we may not observe an undisturbed convergence for very long period. The result in proposition 3.7 can be applied in most cases.

We use simulations to further illustrate the difference between CN and NCN. The parameter values are chosen to make it socially optimal for all individuals to take

high care under both CN and NCN. The values of the parameters are:

$p_{ll}D = 11, p_{ml}D = 8.6, p_{hl}D = 6.6, p_{mm}D = 4.8, p_{hm}D = 3.2, p_{hh}D = 2, c_h = 2, c_m = 1, c_l = 0$   
and  $M = 10$ . We take  $h = 10, m = 9$  and  $l = 1$ . We use the discrete counterpart of the

replicator dynamics, that is:

$$s_i(t+1) = s_i(t) + s_i(t)(\pi_i(t) - \bar{\pi}(t)) / M, i = h, m, l. \quad (3.14)$$

Table 2 displays the time path of the composition of population, the social cost and the mean care level under CN and NCN with an assumed initial condition  $s_h(0) = 0.7, s_m(0) = 0.15, s_l(0) = 0.15$ . In this dynamics, both liability rules lead to the social optimum. The convergence paths display the features as we discussed above. The proportion taking high care increases faster under NCN than under CN (figure 1). The proportion taking low care decreases faster under CN. The proportion taking medium care decreases more slowly under CN (figure 2). See figure 3 for the path of  $(s_h, s_m)$ .

Table .2

**Simulation Results**

<b>Comparative Negligence</b>					<b>Negligence with Contributory Negligence</b>					
	<b>h</b>	<b>m</b>	<b>l</b>	<b>social cost</b>	<b>mean care</b>	<b>h</b>	<b>m</b>	<b>l</b>	<b>social cost</b>	<b>mean care</b>
0	0.700	0.150	0.150	-4.4403	8.500	0.700	0.150	0.150	-4.4403	8.500
1	0.752	0.146	0.103	-4.3108	8.932	0.752	0.138	0.110	-4.3290	8.870
2	0.794	0.138	0.068	-4.2159	9.248	0.795	0.126	0.079	-4.2411	9.163
3	0.827	0.128	0.045	-4.1492	9.471	0.831	0.114	0.055	-4.1743	9.387
4	0.854	0.117	0.029	-4.1036	9.624	0.859	0.103	0.038	-4.1250	9.553
5	0.875	0.106	0.018	-4.0729	9.728	0.882	0.092	0.026	-4.0896	9.674
6	0.893	0.096	0.012	-4.0524	9.799	0.900	0.082	0.018	-4.0644	9.760
7	0.907	0.086	0.007	-4.0385	9.847	0.915	0.073	0.012	-4.0468	9.822
8	0.919	0.076	0.005	-4.0291	9.881	0.927	0.065	0.008	-4.0344	9.865
9	0.929	0.068	0.003	-4.0225	9.905	0.937	0.058	0.005	-4.0257	9.896
10	0.938	0.060	0.002	-4.0179	9.922	0.945	0.051	0.003	-4.0196	9.919
11	0.945	0.054	0.001	-4.0145	9.935	0.952	0.045	0.002	-4.0153	9.935
12	0.952	0.048	0.001	-4.0120	9.946	0.958	0.040	0.001	-4.0121	9.947
13	0.957	0.042	0.000	-4.0101	9.954	0.964	0.036	0.001	-4.0098	9.956
14	0.962	0.037	0.000	-4.0085	9.960	0.968	0.031	0.001	-4.0081	9.963
15	0.967	0.033	0.000	-4.0073	9.965	0.972	0.028	0.000	-4.0067	9.969
Present value of social cost:				-33.7903		Present Value of social cc			-33.8837	

h, m, l: proportion of population taking high (10), medium (9), low (1) level of care

Discount factor =0.9

Figure .1

### Proportion of Individuals Taking High Care

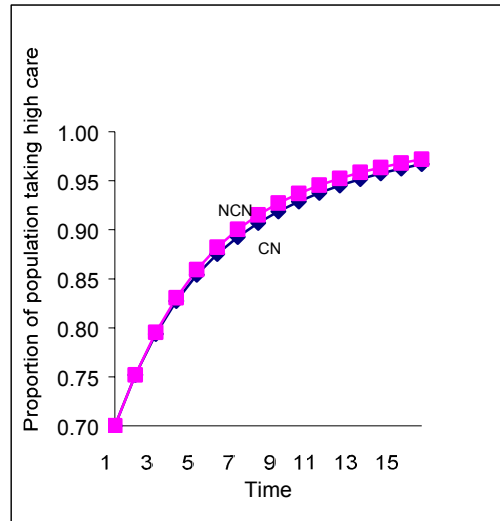


Figure 2

### Proportion of Individuals Taking Medium Care

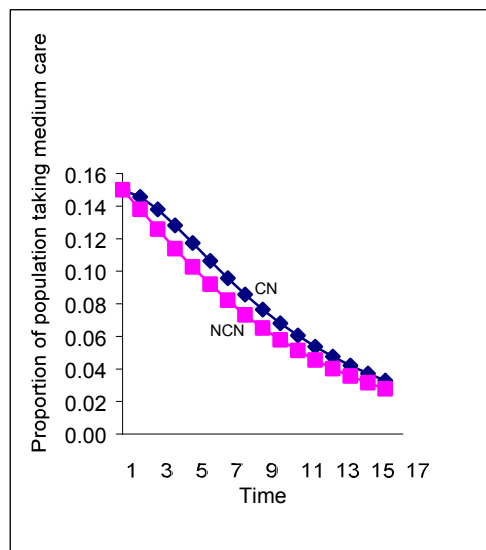
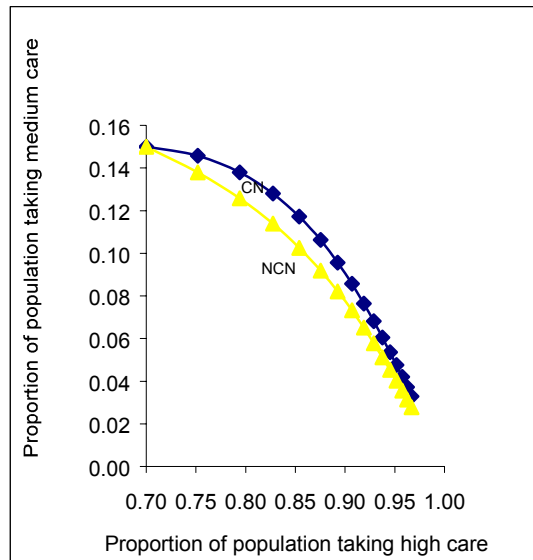


Figure. 2

### The Composition of the Population in the Simulation



We also give the value of mean social cost at each period. Until period 12 the social cost is smaller under CN than under NCN for each period. After that the mean social cost becomes greater under CN for each period. The mean care level is also greater under CN until period 11. After that, the trend also reversed. In the simulation, as time goes on, the proportion of individuals taking low care is very small, so NCN outperforms CN (even though not significantly) because it reduce the proportion taking medium care more effectively.

We calculate the present value of the expected social cost with the discount factor  $\beta = 0.9$ :

$$PV = \sum \beta^t \bar{\pi}(t) \quad (3.15)$$

In 16 periods, the present value of the expected total social cost is -33.7903 under CN and -33.8837 under NCN. The social cost is about 3% lower under CN. With the parameters we have randomly chosen, the cost saving is not very significant.

In reality, we may not see such monotonic convergence. From time to time there will be random shocks to the composition of the population, either because of new entrants or some other factors that drive society away from the equilibrium. The shocks may be very small or relatively large. The evolutionary pressure through imitation and learning leads society to the Nash equilibrium. The constant appearance of shocks makes the rate of convergence and the path of convergence very important, as it leads to different social cost.

Our results are consistent with the experimental results in WITTMAN, FRIEDMAN, CREVIER AND BRASKIN [1997]. The experiments in that paper showed the convergence of the mean care level to the Nash equilibrium under both CN and NCN, and they showed that CN promotes a faster convergence to the Nash equilibrium than NCN. In our analysis, starting from a same population composition, the mean care level increases more rapidly under CN than under NCN for a period of

time. The result may not be maintained in the long run without further shocks, but it is usually true in reality when inefficiency is often caused by periodical shocks.

In WITTMAN, FRIEDMAN, CREVIER AND BRASKIN [1997], individuals are assumed to choose their best response, given the behavior of other individuals in the population. They considered an adjustment dynamics using model  $x_t = \alpha x_{t-1} + \beta B(\hat{x}_{t-1})$ , where  $x_t$  is the state at time  $t$ ,  $B$  is the best response function, and  $\hat{x}_{t-1}$  is a forecast of state at time  $t$  using information at time  $t-1$  ( $x_{t-1}$  is a proxy for all other influences that vary slowly). They consider several possible models by choosing different  $x_t$  and  $\hat{x}_{t-1}$ . One of their findings is that when estimating the population care level ( $x_t$  as the population mean care level), the model in which players give their best response to the mean care level of the last period ( $\hat{x}_{t-1}$ ) provides a good fit of the data (with  $R^2 = 0.824$ ). The model in which players give their best response to the distribution of previous care levels also provides a good fit for the data. Our evolution model considers the distribution of care levels at time  $t$  as a function of last period's distribution of care levels, which is more reasonable for a large population in a social environment.

Instead of using the simple replicator dynamics, we can use a more general evolutionary dynamics model such as a growth monotonic system (see VEGA-REDONDO [1996]):

$$s_i'(t) = s_i(t)F_i(s_h, s_m, s_l), i = h, m, l, \quad (3.16)$$

with the condition: if  $\pi_i(s_h, s_m, s_l) > \pi_j(s_h, s_m, s_l)$ , then  $F_i(s_h, s_m, s_l) > F_j(s_h, s_m, s_l)$ .

Intuitively, in a growth monotonic system, strategies that “do better” grow faster. Under a growth monotonic system, most of our results persist. After starting from the same point, the path in the  $(s_h, s_m)$  plane under CN is always above the path under NCN, using the same reasoning as in Lemma 3.4. (Therefore, at any given  $s_h$  there will be fewer individuals taking low level of care under CN than under NCN). If we further assume that  $\frac{\partial F_h(s_h, s_m, 1 - s_h - s_m)}{\partial s_m} \Big|_{s_h} < 0$ <sup>8</sup> at any given  $s_h$ , we can get the result that  $s_h$  grows faster under NCN as in proposition 3.6. The results about the comparison of mean social cost and mean care level require more restriction on the form of the evolutionary system.

The method we used can also be applied to more than three care levels, and it is still true that CN is more effective in reducing the proportion of individuals taking the most inefficient care levels and NCN is more effective in reducing proportion of individuals taking the medium care levels. This is beyond the scope of this paper.

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<sup>8</sup> The condition is quite natural: at a given  $s_h$ , if  $s_m$  increases a little bit (so  $s_l$  decreases a little bit), the average payoff of the population will increase. The benefit of switching to taking high level of care will decrease and the increasing rate of  $s_h$  will decrease.

## **4 Conclusion**

In this paper we use a simple replicator dynamics model to study the evolutionary dynamics of comparative negligence and negligence with contributory negligence. We compare analytically the convergence properties under these two rules, finding that the proportion of the population taking high care increases more slowly under CN than under NCN. However CN reduces the proportion of the population using low care more rapidly than does NCN. At a given population composition, the mean social cost falls more rapidly and the mean care level increase more rapidly under CN than under NCN. This may explain the modern movements to substitute comparative for contributory negligence.

Intuitively, the advantage of CN is that this rule is more effective in inducing the very careless individuals to abandon their current strategy and to take a more efficient strategy. Very careless individuals impose more cost on society than those slightly careless individuals do. Under comparative negligence the cost is shared according to relative negligence. Very careless individuals always have to incur more cost of an accident under comparative negligence rule than under negligence with contributory negligence rule. This is why comparative negligence reduces social cost more effectively and is better than negligence under contributory negligence.

It is in society's interest that liability rules lead to optimal care and that the optimum be approached rapidly when out of equilibrium. Therefore, when comparing the effects of different liability rules, in addition to considering the efficiency of equilibrium, it is also necessary to consider how rapidly the society converges to that equilibrium.

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