Why Love Matters

A Dynamic Model of Marriage and Divorce

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2005

Abstract

Current literature on marriage breakup assumes that divorce only occurs because of mismatch in the marriage market. We question this framework arguing that payoff from marital partnerships is not entirely fixed by marriage market, but depends on effort put by spouses into the growth of their relationship. A search model of marriage and divorce is developed under the assumption that marriage requires accruing specific marital capital at the expenses of private consumption, and each period spouses have the choice of leaving current marriage or investing in it. We obtain a decision rule linking prospective increase in marital capital, distribution of types in the population, and temporal discounting. We find that wealthy agents on average wait longer before getting married, while positive shocks to income may increase the likelihood of divorce. Finally, when marital capital is a public good between spouses, we find noncooperative Nash contributions increasing in income and decreasing in the probability of divorce.

JEL Codes: J12, D13.

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The author thanks Vincenzo Aversa, Shoshana Grossbard, Maria Concetta Chiuri, Alessandro Cigno, Daniela Del Boca, Raquel Fonseca, Massimo Marrelli, Carlo Panico, Erasmo Papagni for their help, suggestions, and corrections. All remaining errors are mine.

1 The Secret of a Happy Marriage

What does predict a happy marriage? Popular wisdom maintains that careful search, sincere love, strengthened by a responsible vision of marriage are the main keys to a joyful marital life and a happy family setting. While legal and religious norms, social customs, and education may promote the development of a conscious and responsible behavior in family life, there is little evidence of institutions which may promote love *per se*. In all societies where matching between partners is not explicitly determined by parents, the search for the loved one is an almost completely individual process, whose outcome is determined by personal traits, attitudes, and luck. The very fact that people do spend time looking for the best partner testifies that must be an activity for which benefits exceed the cost; if search for the partner is not entirely a consumption good in itself, an accurate and patient searcher may expect higher returns from marriage when compared to a capricious and impatient searcher.

So long for the common sense. Economic analysis of marriage instability (Weiss and Willis, 1997), search and divorce models (Cornelius, 2003), and applications of game theory to courting and engagement (Farmer and Horowitz, 2004) have gone further arguing that the quality of a particular match is an experience good whose value becomes known only gradually (Bougheas and Georgellis, 1999). All of these approaches basically assume that information problems in the marriage market are pervasive and partners have expectations about the quality of the match, but cannot fully observe it until the marriage really takes place. In this setting, divorce is merely the outcome of a wrong choice: if the bride or the groom would have obtained the information before the wedding, they would have refrained from accepting the marriage proposal. Along the same line of reasoning, courting and premarital cohabitation would be the indicators that extracting information from a simple meeting is not a reliable signal of future marital bliss and experience from marital life is what matters most.

The trouble with embracing this try-and-see approach to marital life in its entirety is that treats partners as static optimizers, with no change whatsoever in the structure of their behavioral patterns. This is clearly not the case because divorce behavior and marital satisfaction cannot be treated as fully exogenous and independent of a couple's own dynamics. Casual observation, chicagoan family economics (Becker, 1974; Becker, Landes, and Michael, 1977; Grossbard-Shechtman and Izraeli, 1994), and recent dynamic psychological research (Gottman, Murray, Swanson, Swanson, and Tyson, 2002) clearly show that payoff from marriage is not determined forever by the marriage market or premarital agreements–if such contracts are enforceable–but depends crucially on the effort that partners put in the development of their own relationship. Time devoted to marital activities, like holidays, conversations–sometimes even moderate and constructive quarrels–, lazy afternoons spent together listening to the music, or whatever else a couple may find pleasant, produce instantaneous utility *and* contribute to the enhancement of marital satisfaction as long as they increase mutual attachment, devotion, and profound comprehension. Obviously, these activities, no matter how pleasant they may be, surely have monetary and implicit costs in terms of individual consumption which is diverted toward marital activities. In other words, if love is such a precious good–if not the *most* precious of all–because of its scarcity, we should expect that if it is a good which is at least partially reproducible, rational people may increase their utility from choosing a level for which marginal benefits from marital satisfaction equal its marginal value of forgone opportunities.

The basic trade-off between marital and individual activities is the bottom line of this paper. We propose a model of intertemporal search assuming that performance of a marriage relationship is typically affected by two variables: chance and individual effort, where search behavior depends on chance and patience, while individual effort determines marital bliss. This framework of marriage-specific capital accumulation includes the possibility of divorce and subsequent remarriage and allows for the analysis of the effect of divorce risk and income on marital instability. Our model shows that the decision to marry can be reduced to the interplay of three factors: the prospective increase in marital capital, the value of private consumption lost because of marriage, and the difference between the current marriage offer and the average one. One implication for the search spell is that high income agents on average wait longer before getting married and experience happier partnerships. On the side of divorce, we find that shocks to income, rather than its absolute level, may ease marriage breakups and these occur mainly during the first years of marriage. We also find that if marital capital is a public good between spouses, spouses can sustain a Nash equilibrium in which they both contribute with positive amounts of investment.

The next section builds the basic model of search for the mate and investment, while the following expands it to the case of no-fault divorce. Then, we develop an application of *joint* contribution to marital capital and examine the arising strategic questions. The final section summarizes results and presents further directions of research.

2 Searching and Investing in the Ideal Partner

The basic ingredients of the model are (1) a dynamic structure of intertemporal search for the best partner, coupled with (2) a subsequent problem of choosing the optimal level o contribution to marital capital, and (3) an eventual decision of breakup and remarriage. We assume that utility functions of individuals depend upon private consumption and marital satisfaction, the latter being a stock variable that can be incremented via specific investment: in this sense, marital capital is a durable good that is consumed all along the life at a constant rate per period, while private consumption consists of a composite nondurable good. In our model, an individual has two technologies to produce a joyful marriage: the first is finding the best partner among the available ones, the second is investing in the quality of marriage and increasing its return across time.

At first stage, single individuals select their best partner from a stochastic distribution of personal types and then get along together: search does take time and is costly because delaying marriage implies missing opportunities of investment in marital capital. At second stage, partners have the option to increase the initial payoff from marriage accruing capital that increases the value of utility from marriage across time. Then, we frame individual behavior in two different institutional scenarios: in the first divorce is not allowed, while in the second no-fault divorce is an available option. When divorce law permits the marriage to be broken, agents can start a marriage anew bearing the cost of breakup: this consists of direct monetary costs (legal fees, lawyer's parcels, and emotional stress from separation) and the complete waste of marital capital invested so far; in other words, the type of human capital obtained in a given relationship is completely marriage-specific and cannot be transferred to a new marriage. In the divorce-allowed scenario, agents continue to draw offers while married and divorce when net utility from marriage with a new partner exceeds utility from sticking with current marriage, i.e., the model rules out the case of individuals who divorce but do not remarry instantaneously.

2.1 Agents

The parties involved in the model are men and women. All agents live an infinite life made of discrete periods $t = [0, 1, \ldots, \infty]$ in which of them receive a fixed exogenous income whose value equal w, with $w \in \mathbb{R}^+$. No financial market exists, so agents are not allowed to borrow against the future. We make no assumptions on gender differences, because to our extent men and women act the same way, so we generally refer to the *agent* or *individual*, with no regard to sex. All agents prefer marriage to singlehood, even though they may find rational to reject some marriage offers and wait for a better mate.

2.2 The Order of Events

In each period, every agent receives one marriage offer from an agent of opposite sex whose quality is measured by m, with $m \in \mathbb{R}_0^+$; before the matching takes place, m is a stochastic variable extracted from a time-invariant cumulated distribution function $F(\cdot)$; after the matching has taken place, its value becomes common knowledge to both partners who use this information to decide if marry. At $t = 0$ all agents are unmarried and search for a marriageable partner: once married, they must decide how much income is to be devoted to increase in marital capital and how much of it will go in private consumption. Finally, if law permits divorce, they are allowed to break their marriage to form another partnership: otherwise, if law forbids divorce, the marriage decision cannot be reversed. No direct monetary cost is involved in the search activity, nor the frequency of offers can be modified by the effort of the searcher.

2.3 Marital Capital

The variable m represents an index of desiderability of a particular match as it is perceived by one partner, but what agents observe is only its initial value m_0, for this variable evolves across time because of depreciation and voluntary specific investments using resources subtracted from private consumption. Thus, we can frame m as the marriage specific human capital (MSHC). Formally, MSHC follows the law of motion $m : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ defined as

$$m_{t+1} = \gamma m_t + f(b_t) \tag{1}$$

where $1 - \gamma \in (0, 1)$ is the depreciation rate per period and the function $f(b_t)$, with $\partial f / \partial b_t > 0$, represents the increase in MSHC due to the use of resources whose value equal b_t.

2.4 Preferences and Budget Set

The agent's instantaneous utility is provided by the function

$$u = u(c_t, m_t) \tag{2}$$

assumed to be increasing, differentiable, and separable in both arguments (see Chiappori and Weiss, 2000): basically, the agent cannot instantaneously attain a desired level of MSHC, but he does influence it through investment in MSHC. The functional form is invariant of marital status: for unmarried agents, the level of MSHC is zero, so we assume $u = u(w, 0) > 0$ as long as the whole income is used as consumption; furthermore, no matter what the level of MSHC they possess, married agents cannot set private consumption to zero, so we have $u(0, k) = -\infty$, with $k \in \mathbb{R}^{+}$. On the contrary, if agents use the whole income to finance investment in marital capital, the asymptotic value of this capital would be

$$\lim_{t \to \infty} m_t = \frac{w}{1 - \gamma}, \tag{3}$$

but since they cannot set private consumption to zero, expression (3) serves only as the long-run upper bound to marital capital level.

Once an agent gets married, he must decide how to allocate his income between personal consumption c_t and contribution b_t to MSHC. His budget set is described by

$$C_M = \left\{ c_t \in \mathbb{R}^{+}, b_t \in \mathbb{R}_0^{+} : c_t + b_t = w \right\}. \tag{4}$$

2.5 Financial Constraints

The previous assumptions on preferences and budget set, along with the absence of any financial market, indicate that borrowing constraints will be relevant to our results. If agents were not financially constrained, they could increase their utility borrowing or lending against the future to finance private consumption and investment in marital capital; however, our model wants to assess if differences in personal income play any role in determining marriage decision and investment in marital capital, so the assumption of no financial market is crucial.

2.6 Optimization Technique

The model involves two sequential optimal choices: finding the best partner and then selecting the level of marital investment. To solve the model we use dynamic programming, which is particularly suited to solve these type of problems involving a mix of binary and continuous variables. This scheme requires the problem to be solved using backward induction, i.e. solving the problem of the married agent first, and then working backward to identify the best procedure to select the optimal mate, even though the actual order of events is reversed.

3 The Eternal Love Scenario

The constrained optimization program for a married agent can be written as

$$\max \sum_{t=0}^{\infty} \delta^t u(c_t, m_t) \qquad (5)$$
$$s.t. \text{ conditions (1) and (4)}$$

in which δ is the intertemporal discount factor.

To derive conditions for the choice of c and b, we use Bellman's principle of optimality and write the equation for the problem as

$$\begin{aligned} V(M, m_t) &= \max_{\{c_t\}} \{u(c_t, m_t) + \delta V(M, m_{t+1})\} \\ &= \max_{\{c_t\}} \{u(c_t, m_t) + \delta V(M, \gamma m_t + f(w - c_t))\} \end{aligned} \qquad (6)$$

where $V(M, m_t)$ is the maximum value function that depends on marital status ($M = married$, $U = unmarried$) and the level of MSHC. To characterize the optimal choice between investment and consumption, we maximize equation (6) with respect to c_t, then set it to zero to obtain the first order condition

$$\frac{\partial u}{\partial c} = \delta \frac{\partial V}{\partial m} \frac{\partial f}{\partial b}; \qquad (7)$$

in order to get rid of $\partial V / \partial m$, we apply the envelope theorem and maximize (6) with respect to m and obtain

$$\begin{aligned} \frac{\partial V}{\partial m} &= \frac{\partial u}{\partial m} + \delta\gamma \frac{\partial V}{\partial m} \\ &= \frac{\partial u}{\partial m} [1 - \delta\gamma]^{-1} \end{aligned} \qquad (8)$$

that can be plugged into (7) to provide the optimality condition

$$\frac{\partial u / \partial c}{\partial u / \partial m} = \alpha \frac{\partial f}{\partial b} \qquad (9)$$

where $\alpha = \delta / [1 - \delta\gamma]$. Furthermore, to obtain an explicit solution, we make the following two assumptions on preferences and investment technology:

$$u(c_t, m_t) = \log(c_t) + m_t \qquad (10)$$
$$f(b_t) = b_t; \qquad (11)$$

with this structure, optimal levels of consumption and marital capital are

$$c_t = \alpha^{-1} \tag{12}$$

$$m_t = \gamma^t m_0 + \frac{1-\gamma^t}{\gamma}\left(w - \alpha^{-1}\right). \tag{13}$$

Consumption of private goods is constant across time, as well as investment in marital capital, which is equal to $w - \alpha^{-1}$: the fact that only investment grows as income increases whereas consumption stays constant depends on the quasi-linear structure of utility function. Ceteris paribus, individuals endowed with high income invest more in marital capital than low-income ones, as well as more patient agents invest more than impatient ones. This takes us to the following statement:

Proposition 1 *On average, high income individuals experience happier marriages than low income individuals.*

The level of marital capital depends on its own initial value m_0: on one hand, this explains why the search process contributes to marital satisfaction; taking the limit of (13) as t goes to infinite, we get

$$\lim_{t\to\infty} m_t = \frac{w - \alpha^{-1}}{\gamma}:$$

this value does not depend on initial choice of the partner; this implies that on the very long run, investment in marital relationship can help relieving even a marriage started with a very low m_0; on the other hand, the fact that the limit value does not depend on the initial value of m does not make irrelevant the whole process of search, as long as agents discount future even if they face an infinite temporal horizon.

Now, let us turn to the problem of an unmarried individual: his choice set C_U is simply

$$C_U = \{A, R\},$$

because he must decide if (A) accepting the current partner and marry her, or (R) rejecting her and waiting for a better partner. Given that he has no partner, an unmarried individual faces no allocational problem with his budget, for his income is entirely spent in private consumption and then $u(c_t, 0) = \log(w)$.

The payoff associated with acceptance of current offer is the value from problem (6), while the value of waiting depends on the return from continuing search, i.e. from current utility plus expected discounted utility from a marriage with the next partner, that we call m'. Bellman's equation for this problem is:

$$V(U,m) = \max_{C_U}\left\{V(M,m), u(w,0) + \delta\int_0^{\infty} u\left(\alpha^{-1}, m'\right) dF(m')\right\}. \quad (14)$$

Then, an unmarried agent accepts current offer of marriage if

$$V(M,m) \geq u(w,0) + \delta\int_0^{\infty} u\left(\alpha^{-1}, m'\right) dF(m');$$

in our model, the value function can be written as[1]

$$V(M,m_t) = (1-\delta)^{-1}\left[\ln\left(\alpha^{-1}\right) + w\alpha\right] + \frac{\alpha}{\delta}m_t \quad (15)$$

and plugging (1) into (15), condition (14) becomes

$$\frac{\ln(\alpha^{-1}) + w\alpha}{1-\delta} + \frac{(1-\gamma)^t}{\delta}\alpha m_0 + \frac{1-(1-\gamma)^t}{\delta\gamma}(w\alpha - 1) \geq \ln(w) + \delta\int_0^{\infty} u\left(\alpha^{-1}, m'\right) dF(m');$$

given the infinite length of life, comparisons of value function do not vary over time: this implies that at $t = 0$ we can write the condition in the form

$$\frac{\ln(\alpha^{-1}) + w\alpha}{1-\delta} + \frac{\alpha m_0}{\delta} \geq \ln(w) + \delta\left[\ln\left(\alpha^{-1}\right) + \bar{m}\right]$$

where $\bar{m}$ is the average initial level of MSHC dependent on the cumulated distribution function $F(\cdot)$. The previous inequality can be reduced to

$$\left(\frac{\alpha m_0}{\delta} - \delta\bar{m}\right) + \frac{w\alpha}{1-\delta} \geq \ln(w) - \ln\left(\alpha^{-1}\right)\frac{1-\delta(1-\delta)}{1-\delta}. \quad (16)$$

On the left side of the inequality, the first term in parentheses represents the difference between the utility from current marriage offer valued at $t = 0$ minus the discounted level of the average initial marital capital available on the marriage market: this is a *marriage premium* that can be either positive or negative; we call this premium p^m. The second term on the left side shows the possibility of future increase in payoff from marriage due to investment in marriage-specific capital: this is the long-run gain from accumulation of marriage-specific capital, it is always positive, and we call it e^m. On the right

[1] See mathematical appendix.

side of the inequality we find the difference between the value of consumption as a single and as a partner, the latter multiplied by a factor depending on δ: we name all this expression p^c; the term grossly represents the variation in utility–a *consumption premium*–due to a variation in the marital status. Its value can be either negative or positive, because, even though $w > \alpha^{-1}$, the factor $[1 - \delta(1-\delta)] / (1-\delta)$ is positive for $\delta \in (0.5, 1)$; p^c is positive when

$$\frac{\ln(w)}{\ln(\alpha^{-1})} \geq \frac{1 - \delta(1-\delta)}{1-\delta} \tag{17}$$

and its value depends on personal traits (α^{-1}) and income (w). Ceteris paribus, the higher the income, the lower the chance that the previous inequality is reversed; consequently, high income individuals face a positive consumption premium when compared to low income individuals. However, discount factor values close to 1 (very low discounting) could reverse inequality (17): extremely patient individuals put a great value on future streams of utility and are happy to give up their consumption as singles against the payoff from marriage notwithstanding they have very high incomes. These individuals are the most prone to marry, among others. Using our new terminology in terms of premiums for marriage and singlehood, we can write (16) as

$$p^m + e^m \geq p^c: \tag{18}$$

to sum up, when the net value of current marriage offer plus future increase in marriage capital exceeds the value of consumption premium for single individuals, these decide to marry. Given that e^m is always positive, whereas marriage and consumption premiums can be negative or positive, we rewrite (18) in the following form

$$e^m \geq p^c - p^m. \tag{19}$$

The previous discussion is summarized in the following statement:

Proposition 2 *For an individual to decide to marry, the difference between consumption premium and marriage must be lower than the gain from accruing marital capital.*

Now, suppose that $p^m > 0$: this is the case of a current offer whose quality exceeds average quality available on the marriage market. For (19) to hold two situations are possible: (1) p^c is positive but net of p^m is lower than e^m, or (2) p^c is negative. In the first case, an individual decides to marry only if difference in terms of consumption between what he loses when he gets married minus the value from an offer above average is sufficiently small. The individual loses private consumption, but this loss if offset by a high marriage premium. In the second case, when p^c is negative, the term on the right side of (19) is negative, and the individual decides to marry because his evaluation of (17) shows a high propensity to give up private consumption against marriage.

On the contrary, suppose that $p^m < 0$: Can anyone find rational marrying a mate whose quality is below average? Again, for (19) to hold two situations are possible: (1) p^c is positive or (2) p^c is negative but net of p^m is lower than e^m. In the first case, the agent has a positive consumption premium for singlehood but this must be not too large: keeping p^m equal across individuals, those who have lower p^m's have greater chances to accept current offer because they are not forgoing a large amount of private consumption and have great propensity to invest even in a low-quality relationship, as far as they can increase its value greatly across time. In the second case, the agents have a negative consumption premium, so they are very prone to marry and this is the key to explain their decision to marry: they are faced with a below-average marriage offer, but they are very impatient, so they accept it and wait for an increase in its quality. To sum up, our model predicts that all agents, even if their quality is below average, have some chances to get married: they will find partners who hurry to marry, or partners with low income which do not put a big value on their private consumption.

The previous considerations about the decision to marry can be put in a more compact form. Solving inequality (16) for m^0, we obtain the reservation value of marital quality

$$m^* = \frac{\delta}{\alpha}\left[\ln(w) - \frac{[1-\delta(1-\delta)]\ln(\alpha^{-1}) - w\alpha}{1-\delta} + \delta\bar{m}\right]$$

that is time-invariant and whose expression symbolizes the minimum marital quality agents require before accepting a marriage offer. The sign of $dm^*/dw = \delta/\alpha w + \alpha/(1-\delta)$ is strictly positive, so we should expect that, on average,

Proposition 3 *High income individuals are pickier than low-income ones because of a higher reservation quality.*

Alternatively, in terms of waiting times, we can conclude that wealthy agents show a unique behavioral pattern when facing an increase in their income:

Proposition 4 *High income individuals have longer search spells when compared to low income individuals.*

This statement about the impact of income on spell of singlehood conflicts with the Becker-Keeley hypothesis (Becker, 1974; Keeley, 1977): these authors maintain that high-income agents have greater gains from specialization if compared to low-income agents, so the former ones should be more impatient to marry than the latter ones. Our point is that, apart from gains from trade, search behavior is affected by income in direct and indirect ways: given that payoff from marriage is not fixed forever by marriage market (as in Becker, 1973), income not only impacts current reservation value of marriage, but it does explain the future value of marriage because of marriage-specific investments, so patient and wealthy agents can delay marriage even if their *static* gain from trade would suggest otherwise.

4 The No–Fault Divorce Scenario

The problem a married agent confronts with when divorce is legal allows for the additional option of leaving current partner and marrying the best available alternative, since married agents keep searching for better mates even though they are investing in their current marriage. We assume that married agents face the same distribution of marriageable mates from which they drew before getting married. Then, each period a married agent must choose between the following alternatives:

1. Sticking with current marriage, and deciding how to allocate the income between marriage and private consumption (this is basically the problem (5) with some minor additions).

2. Divorcing from current partner, bearing the fixed legal and psychological cost of separation, and marrying anew with their best current available option.

In the same time, a married individual faces the risk that his partner unilaterally decides to divorce: we model this feature assuming that MSHC follows the stochastic law of motion

$$m_{t+1} = \begin{cases} \gamma m_t + f(b_t) & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases} \tag{20}$$

in which $1 - \pi$ is the probability of divorce: even if investment remains deterministic, now its return is stochastic and depends on mate's decision to break up current marriage, assumed exogenous.

The first option open to a married agent, i.e. staying married with his current mate forever, now includes uncertainty from divorce and can be written as

$$\max E\left[\sum_{t=0}^{\infty}\delta^{t}u\left(c_{t},m_{t}\right)\right]$$
$$s.t.\text{ conditions (4) and (20).}$$

Bellman's equation becomes

$$\begin{aligned} V^{d}\left(M,m\right) &= \max_{\{c_t\}}\left\{u\left(c_{t},m_{t}\right)+\pi\delta V^{d}\left(M,m_{t+1}\right)\right\} \\ &= \max_{\{c_t\}}\left\{u\left(c_{t},m_{t}\right)+\pi\delta V^{d}\left(\gamma m_{t}+f\left(w-c_{t}\right)\right)\right\}. \end{aligned}$$

where V^{d} indicates the value function when unilateral divorce is permitted. The condition on maximum utility becomes

$$\frac{\partial u/\partial c}{\partial u/\partial m}=\beta\frac{\partial f}{\partial b} \tag{21}$$

in which $\beta=\delta\pi/\left[1-\pi\delta\gamma\right]$. Equilibrium values for consumption and marital capital are

$$c_{t} = \beta^{-1} \tag{22}$$
$$m_{t} = \left(\pi\gamma\right)^{t}m_{0}+\frac{\pi\left[1-\left(\pi\gamma\right)^{t}\right]}{1-\pi\gamma}\left(w-\beta^{-1}\right). \tag{23}$$

Clearly, when no uncertainty on marriage duration is given ($\pi=1$), the previous results collapse to the ones expressed in the previous section, in which divorce is not allowed. Comparing the magnitudes of these results with those obtained when divorce is forbidden, it is easy checking that personal consumption is higher when divorce is allowed, whereas marital capital's level is higher if divorce is not allowed: in other terms, the agent shifts his income away from the risky investment in marital capital and employs it for personal uses[2], even if the preferences are not assumed to be risk-adverse in marital capital.

[2]Imagine a situation in which marital capital plays a positive role in determining π, i.e. the probability of staying married is not exogenous, but at least partially determined by the current level of marital capital. The Bellman equation for this problem becomes

$$V^{d}\left(M,m\right)=\max\left\{u\left(c,m\right)+\pi\left(m\right)\delta V^{d}\left(M,m\right)\right\}:$$

in this case, if a higher level of marital capital than the one showed in (23) could be enforced, the likelihood of divorce, ceteris paribus, would be lowered. A natural implication

Now, let us turn to the second option which is available to a married agent: each period of his life he is faced with draws from the distribution of marriageable mates and he is allowed to divorce whenever utility from new marriage exceeds that from current marriage, including the fixed cost d. Bellman equation for this stage of the problem is

$$V^d(M, m_t) = \max\left\{V^d(M, m_t), V^d\left(M, m_t^d\right) - d\right\}$$

where m^d is the quality of current marriage offer and d is the cost from divorce, assumed to be $d < w$.

Going back to the problem of an unmarried agent who must decide if accepting current marriage offer, in the unilateral divorce scenario he is aware from the start that a marriage is *not necessarily* forever, so the value function attached to acceptance is not the maximum he can attain from marriage, rather being the maximum he can obtain from an optimal *sequential strategy* of marriages and divorces. Consequently, his Bellman equation is

$$V^d(U, m) = \max_{C_U}\left\{V^d(M, m), u(w, 0) + \delta\int_0^\infty u\left(\beta^{-1}, m'\right) dF(m')\right\} \quad (24)$$

where the inclusion of $V^d(M, m)$ takes into account the possibility of subsequent divorce.

A married agent divorces if

$$V^d\left(M, m^d\right) - V^d(M, m_t) \geq d$$

and plugging value functions in it, we obtain the condition

$$m_t^d - m_t \geq \left(\delta\pi\beta^{-1}\right) d$$

that expresses the fact that new marriages take place only when the premium for divorce is positive and higher than a given fraction of fixed cost. Plugging in the value of β we get

$$m_t^d - m_t \geq (1 - \delta\pi\gamma)\, d\text{:}$$

if we had $\pi = 0$, i.e. certainty of divorce, an agent would decide to break up his marriage a starting a marriage anew if the difference between the current

is that introduction of divorce self-fulfills expectations of divorce itself, via reduced level of marital investment. However, when such a relation is known to the agents, it would be taken into account when deciding the optimal allocation of income between consumption and investment, but the general statement that individuals compensate increased risk from divorce nonetheless remains valid.

level of marital capital and the value of the new marriage offer exceeds the costs from divorce; in this case, the new offer would permit the agent to make a leap to a higher level of marital capital, but just for one period given that marriages last just one period. If the probability of divorce is less than one, people require that difference between m_t^d and m_t must be less than d to divorce. This result is somewhat puzzling: how can a higher probability of divorce ease the decision of staying married? We assumed that π is not marriage-specific, but is taken as the default risk depending on structural conditions of marriage market; ceteris paribus, when this value increases, this impacts both already formed marriages as well as potential marriages, but the next available marriage becomes more profitable because its expected return is higher and then the cost required to break up current marriage must go down.

Our result is that, at the margin, income does not enter the decision of divorce: only monetary costs and structural parameters play a role. In fact, income plays a role only if it varies over time: for example, consider the case of an agent who knows that registers an increase in his future income, namely w'; in this case, the decision rule for divorce becomes

$$\frac{\delta\pi}{1-\delta\pi}\left(w'-w\right)+\left(m_t^d-m_t\right)\geq\left(1-\delta\pi\gamma\right)d\text{:}$$

ceteris paribus, a positive shock to income should ease divorce. Furthermore, for an optimal divorce decision we must also have $m_t^d \geq m_t$ because the same increase in income can be used to boost current marriage instead of starting a marriage from scratch, so it does make sense to start a new marriage only if current offer's quality exceeds current level of marital capital. However, income shocks enter in the decision rule and can facilitate divorce because they increase the speed of accumulation of capital and allow agents to make a discrete leap to a partner of superior quality.

Previous discussion is summarized in the following statement:

Proposition 5 *A married individual decides to divorce when the difference between the value of new marriage option minus the value of current marriage is greater or equal than a given fraction of divorce expenses. The absolute level of income does not enter the decision, but a positive shock to income does.*

5 Strategic Investments

So far, we have considered the case of an isolated individual: even when he marries, the payoff from marriage depends only upon his choice and no interaction whatsoever takes place between partners. A natural extension of the model applies to the case of strategic interaction between husband and wife. If we conceive m as a *public* good representing marital bliss, the substitutability between private consumption and contribution to public good uncovers a chance of conflict between the partners. Let us assume that m is common to both partners, i.e. husband and wife must decide every period the *joint* investment in marital bliss, and be this function equal to

$$b_t = b_t\left(b_t^h, b_t^w\right)$$

where b_t^h and b_t^w are, respectively, the contributions of husband and wife. To make calculations easy, assume that investment function has the form

$$b_t = \log\left(b_t^h + b_t^w\right) \tag{25}$$

in which contributions are perfect substitutable factors in the production of marital bliss. Using this functional specification in the condition for the optimal choice of private consumption (21), we can write it as

$$c_t^{-1} = \beta\left(b_t^h + b_t^w\right)^{-1};$$

from this condition, we can derive the optimal response function for the Cournot-Nash game of contribution. This function is

$$b_t^h\left(b_t^w\right) = \frac{\beta}{1+\beta}\left(w - b_t^w \beta^{-1}\right) \tag{26}$$

in which we have $db_t^h/db_t^w < 0$. To find the Cournot-Nash value of investment, we assume that partners possess identical preference structure, so equation (26) can be solved assuming symmetry, i.e. $b_t^h = b_t^w$. The equilibrium value of contribution is

$$b_t^* = b_t^h = b_t^w = \frac{\beta}{2+\beta} w \tag{27}$$

that is increasing in personal income. This reduced form allows us to derive predictions about the effect of an increase of π on the contributions. We evaluate it and find that

$$\frac{db_t^*}{d\pi}\left[\frac{\beta}{2+\beta}w\right] = \frac{2\delta w}{\left[\delta\pi\left(2\gamma-1\right)-2\right]^2} > 0:$$

in this case of perfect substitutability, an decrease in the exogenous probability of divorce is compensated by an increase in the contribution of each partner, nothwithstanding the fact that preferences are linear in marital capital and do not show risk aversion. The inverse relation between risk of divorce and investment is completely due to the structure of joint contribution to marital capital.

Proposition 6 *In the noncooperative game of contribution, with perfect substitutability of partners, a Cournot-Nash equilibrium exists. The value of optimal contributions is increasing in the level of income and decreasing in the probability of divorce.*

Now we ask: Is the contribution Pareto-efficient? To answer this question, we must compare the optimal contribution of a single individual to the joint contribution obtained through the Cournot-Nash approach. A single individual operating with the same technology expressed in (25) would have an optimal contribution equal to

$$b = \frac{\beta}{1+\beta} w \tag{28}$$

which is less than twice the optimal contribution of spouses acting separately as Cournot-Nash competitors: consequently, the level of marriage capital increases as partners can invest competitively. This result is counterintuitive for Cournot competition because in oligopoly theory the profits obtained under monopoly exceeds the sum of profits obtained by firms acting separately. However, as indicated by Weiss (1997), marriage can increase welfare of spouses when financial markets are not competitive: in this case, given the financial constraint, the level of contribution does depend on personal income, and its pooling between partners results in an overall higher utility, even if this level is obtained through Cournot competition. To test how absence of financial markets constrains marital choices, consider the case of an agent whose income is twice the income of a regular agent: Does this agent act as two partners or does he show a higher level of investment? Assuming the same investment technology, he would invest $2\beta w/(1+\beta)$, which is twice the value expressed in (28); this is higher than $2\beta w/(2+\beta)$, which is the sum of contributions of two partners adopting a Cournot-Nash behavior. This suggests the possibility of trading between partners given that the quantity

$$\frac{2\beta}{(1+\beta)(2+\beta)} w$$

is lost due to strategic interactions. In the long run, folk theorem (Lundberg and Pollak, 1994) implies that partners can enforce higher levels of investment via mutual bargaining. From our model, we see that this incentive to bargain is increasing in the level of income.

6 Discussion

The model presented above suggests that inclusion of marital capital in the search framework, when contrasted with past literature, results in a wider economic prospective to analyze marriage and search behavior: there is no need to assume systematic information problems in the marriage market and delusions after the wedding to explain the existence of divorce, because rational agents may consider it as a chance to obtain a discrete increase in their stream of intertemporal utility. We showed that marriage decision depends on the quality of current marriage offer compared to the average one, as well as on its expected increase due to specific investment, and on the difference of utility from consumption as single and as married. On average, patient searchers marry high quality mates, and high income individuals tend to marry later because the minimum quality of marriage they require is higher if compared to the quality required by low-income individuals. Income does not impact directly the divorce decision, but positive variations in the level of income favor divorce because they give the individual a better chance to bear the fixed costs of separation and attaining a leap in the level of marital capital. We showed that a source of interaction between partners comes from determination of joint contribution to marriage when marital capital is a public good: in this case, income impacts positively on Cournot-Nash equilibrium contributions and financial constraints create incentives to income pooling in marriage; furthermore, the infinitely repeated game of contributions may support cooperative solutions and higher levels of marital welfare.

Are our results consistent with the current evidence on marriage and divorce? Keeley (1979), building on the theoretical work of Becker (1973), assumes that postponing marriage is more costly for high-income individuals as they give up profitable opportunities of household specialization, so it should be expected that such individuals marry earlier. Then, he finds a negative sign for wage in the equation explaining age at marriage, but an alternative functional specification provided by Bergstrom and Schoeni (1996), using a different dataset, suggests that "age-at-first-marriage decreases with earnings for those with low earnings, but that increases with earnings for those with high earnings."

The evidence on divorce (Becker, Landes, and Michael, 1977; Peters, 1986; Burgess, Propper, and Aassve, 2003) shows that separations mainly occur in the first years of marriage: in our model, this happens because during the first stages of accumulation of marriage specific capital, the act of switching from a marriage to another is not very costly because of two reasons: (1) the level of marriage-specific capital accrued so far is not large, then the cost of wasting it is not very relevant and (2) it is likely that comparison between the level of marital capital reached inside marriage and the available offers available in the marriage market shows gains from divorce and remarriage. As time passes by, however, the level of accrued marital capital increases and terms of trade between current and new partner deteriorate against the new one, so we should expect that the hazard rate of divorce is decreasing in the number of years spent married.

Further developments of the model may include rewriting the maximization program for a finite temporal horizon: the question is relevant because some activities, as children births and rearing, are not completely independent of age at marriage. As pointed out in the literature on differential fecundity (Siow, 2001; Giolito, 2004), given that women are biologically constrained because of fertility, their search patterns may differ systematically from those of men and then result in asymmetric satisfaction from marriage: as the quality of matching is increasing in the time spent searching, women who desire to have children, on average have shorter spells and then lower quality from marriage. This phenomenon becomes explicit through divorce agreements, because separation halts the compensation women enjoy to stay married, showing an increase in men's consumption and a decrease in women's consumption. An implication of this asymmetric biological constraint to marital search is that medical care and melioration of general living conditions modify fertility and consequently search spells: in this sense, the recent increase of age at marriage and first pregnancy may reflect the trade-off between desired fertility and quality of marriage.

A Mathematical Appendix

Value Function

In this appendix, we present the formal derivation of the value function for married agents, whether divorce is allowed or it is not. Because of the infinite time horizon, the search for the value function must start from a guess and then verifying if it satisfies the requirements of the problem; for the deterministic case our tentative function is

$$V(m_t) = K + Lm_t; \tag{29}$$

where K and L are two real constants to be found. If our guess is correct, then the Bellman equation (6) becomes

$$K + Lm_t = \max_{\{c_t\}} \{\ln(c_t) + m_t + \delta K + \delta L [\gamma m_t + w - c_t]\}; \tag{30}$$

the first order condition for a maximum with respect to c_t is

$$c_t = (\delta L)^{-1}$$

that, once inserted in the law of motion of capital, provides

$$\begin{aligned} K + Lm_t &= \ln\left((\delta L)^{-1}\right) + m_t + \delta K + \delta L\left[\gamma m_t + w - (\delta L)^{-1}\right] \\ &= \ln\left((\delta L)^{-1}\right) + \delta K + \delta L w - 1 + m_t + \delta\gamma L m_t; \end{aligned} \tag{31}$$

next, if the value function we have selected fits the problem, then the previous equation must hold for each m_t. Consequently, we group terms on the right side of the equation in order to get an expression that looks like the left side. For the two sides to be equal, the following recursive non-linear system of equations must hold for L and K

$$\begin{aligned} L &= 1 + \delta\gamma L \\ K &= \ln\left((\delta L)^{-1}\right) + \delta K + \delta L w - 1 \end{aligned}$$

whose solutions are

$$\begin{aligned} L^* &= \frac{1}{1-\delta\gamma} \\ K^* &= (1-\delta)^{-1}\left[\ln\left(\frac{1-\delta\gamma}{\delta}\right) + \frac{\delta w}{1-\delta\gamma} - 1\right] \end{aligned}$$

then the value function is

$$V(m_t) = (1-\delta)^{-1}\left[\ln\left(\frac{1-\delta\gamma}{\delta}\right) + \frac{\delta w}{1-\delta\gamma} - 1\right] + \frac{1}{1-\delta\gamma} m_t.$$

Retrieving the abbreviation $\alpha = \delta / [1 - \delta\gamma]$ stated on page 7, we can write the value function in a more compact fashion

$$V(m_t) = (1 - \delta)^{-1} \left[\ln\left(\alpha^{-1}\right) + \alpha w - 1\right] + \alpha\delta^{-1} m_t. \tag{32}$$

When divorce is permitted, the problem is analogous and the value function becomes

$$V(m_t) = (1 - \delta\pi)^{-1} \left[\ln\left(\beta^{-1}\right) + \beta w - 1\right] + \beta(\delta\pi)^{-1} m_t \tag{33}$$

where $\beta = \delta\pi / [1 - \pi\delta\gamma]$.

Positivity of (27)

Plugging $\beta = \delta\pi / [1 - \pi\delta\gamma]$ into the equilibrium value of personal contribution, the following expression for the equilibrium contribution is obtained

$$b_t^* = \frac{\beta}{2 + \beta} w = \frac{\delta\pi}{2 - \delta\pi(2\gamma - 1)} w;$$

its value is positive when

$$\frac{2}{2\gamma - 1} > \delta\pi.$$

Recalling that $\delta\pi < 1$, for the previous condition to be true we need only $\gamma > 1/2$.

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