"Uncertainty and the Labor Market"

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ABSTRACT

This paper describes the labor market in general, and the job search process in particular, as stochastic decay processes. this has implications in terms of information, discrimination and the Natural Rate Hypothesis.

1.
Stochastic Job Search and Employment

As Economists, we tend to focus on the effects of prices in the determination of equilibrium in markets. In keeping with our predilections, our main focus in the analysis of the job-search process has been on the concepts of search costs, wage offers, and the reservation wage. However, a component of equal practical importance is the stochastic nature of the job search process. Given the usual simplifying assumptions of Economic model building, homogeneous goods and competitive markets. The probability of an individual receiving a job offer in a dynamic model of a labor market is a function of two variables. One is the number of jobs for which the individual applies, the other is the number of applications received by the firms to which the individual applies. This probability distribution has the functional form:

$$P(h) = 1 - \left(\frac{n-1}{n}\right)^{m} .$$

Where P(h) is the probability of being hired. 'n' is the number of applications that firms receive. And 'm' is the number of applications made by the individual.[1] So, each individual will perceive this probability as a function of the job-search environment (described by 'n') and their own effort (described by 'm'). In this simple version of the model the mean waiting time until employment is the time where:

$$m = \frac{\ln 0.5}{\ln \frac{n-1}{n}}$$

Which is when an individual will expect to become employed (at least in the probabilistic sense of the word).

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[1] Since we have made the usual simplifying assumptions for perfect competition, 'n' is the same for all firms and 'm' will be the same for all individuals. Relaxing this assumption for each individual makes the distribution:

$$P(h) = 1 - \prod_{i=1}^{i=m} \left(\frac{n_i-1}{n_i}\right)$$

Where `i' denotes each firm applied to. This does not however result in any basic change in how the process plays itself out over time.

Adding two more assumptions to this scenario will enable us to describe the functioning of the Labor market in general. Since we are analyzing the process in dynamic terms the necessary assumptions here for a complete description are the rate of creation of job openings and the rate of creation of job seekers. If these two rates are equal to each other, then the Labor market is in equilibrium. And the pool of unemployed Labor remains constant in size. The size of this pool will be determinant under the usual simplifying assumptions that all labor is identical, all jobs are identical, and perfect information is cost-less. Since in that case 'm' will equal 'n' will equal the number of jobs created will equal the number of new entrants into the pool.[2] Unfortunately, in all other scenarios the pool size is indeterminant. Any pool size can be consistent with equilibrium in the Labor market.

For example, if information is not cost-less, then each individual will not apply for all available jobs.[3] This being true, then the number of new Labor suppliers can equal the number of job opening for any values of 'n' and 'm' that are possible. What will occur is that for a given value of 'm', the pool size will adjust to give a value for 'n' that clears the Labor market.

It is also interesting to look at the implications of this in regards to the variability of employment over the Business Cycle. For our purposes, we can regard the Business Cycle as a process that varies the creation of job openings over time. Assume that the process begins with the Labor market in

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[2] The usual competitive assumptions will result in all job seekers applying for all possible jobs. So equilibrium is achieved only when the pool of unemployed labor equals the number of job vacancies in existence. However, this only occurs when m=n=1. Which cannot be viewed as a realistic approximation of the actual work environment. Also in that case the model ceases to be stochastic, P(h)=1 when m=n=1. This can be thought of as occurring in two different ways. One, as each job opening appears, the perfect applicant applies for it and is hired. Two, because the participants have perfect information, the only individual who applies for a given job is the one who will be hired. Everyone else knows they won't be hired, so they don't bother to apply. Perfectly competitive markets are densely deterministic environments. This is the only case where this is necessarily the outcome. However, an equilibrium can occur under many other circumstances. In this model, the concept of a 'Natural Rate of Unemployment' as usually presented does not make sense, because there are always multiple possible sizes for the pool of the unemployed. Perfectly competitive conditions will tolerate no uncleared markets. Therefore an uncleared market is not a competitive market. Therefore the divergence between actual and anticipated inflation can be zero for any unemployment rate (Provided that money is statically neutral.).

[3] The informational assumption of perfect competition is a misunderstood assumption. It is intimately linked with uncertainty. To assume perfect information is to also assume certainty. In the case of this model, assuming perfect information means that m=1=n. So, P(h)=1. Always and everywhere. To assume uncertainty is to also assume that information is scarce (scarce in the economic sense of the term).

equilibrium. Then the rate of job creation declines but the rate of creation of job seekers remains constant. The pool of the unemployed swells, 'n' is greater than 'm.' Then the trend in job creation steepens and eventually reaches a rate that pushes 'n' below 'm' and the pool of the unemployed shrinks. If we make the usual assumptions about the market, the only available state of rest for this process (i.e.- equilibrium) is where the rates of creation of job seekers and of job openings equal each other.[4]

However if we relax our stringent competitive assumptions, the outcome may vary from this desirable state. If we take my prior example of relaxing the informational constraint, then the outcome is not deterministic. If all job seekers do not apply for all possible jobs, then the process of equilibrating the Labor market can be completed without 'n' being pushed below the value for 'm.' In any case where all possible jobs are not applied for, it is likely that the equilibrium reached after a decrease in job creation below the market clearing rate will result in a larger pool of the unemployed. An equilibrium reached after an increase in job creation above the market clearing rate will result in a smaller pool of the unemployed. And there is an asymmetry between increases and decreases in the job creation rate. Notionally, the pool of unemployed workers can increase to any size, but it smallest possible size is zero. It cannot take on negative values. Any cycle with a trend in the rate of job creation above the market clearing rate must eventually result in a bifurcation phenomena, if the trend is maintained for a sufficiently long enough period of time.[5]

However even in light of that caveat, it is a clear conclusion of this stochastic approach to job-search and Labor markets that the alleviation of unemployment requires that the economy "overheat," i.e. - that it overshoot equilibrium in the

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[4] As you may have noticed, I did not motivate the movement back to equilibrium. I believe that we may safely assume that the process of adjustment under the conditions of perfect competition will result in the equilibrium outcome. However, please note that for equilibrium to be reached the economy must "overheat." It is necessary for the rate of job creation to be greater for a period of time than that consistent with equilibrium for the system to reach equilibrium. By definition, if the labor market is in equilibrium, then the rate of job creation is the rate that does not change the unemployment rate.

[5] While it is not clear exactly what would happen in that situation, it must certainly result in a downward movement in the rate of growth of the economy. Which would result in a decrease in the rate of capital accumulation. It certainly would not be an outcome that would be eagerly sought after. So policies that seek to reduce the pool of the unemployed hold their own dangers for the health of the economy. We cannot make any once and for all decisions in terms of employment policy. The nature of the labor market dictates that employment policy be ad hoc, or situation specific, and most of all reversible.

Labor markets. Or in other terms, the rate of growth must remain above the long-run sustainable rate of growth in order to decrease the size of the pool of the unemployed. The sustainable rate of growth can only reduce the pool of the unemployed over the course of generations through the dying off of the excess unemployed. This is clearly not a position that an economist takes except by necessity. It is however the logical conclusion here. This leads to the conclusion that there is no long-run policy that will maintain employment at high levels. Any employment policy is by the nature of the problem of job search a short-run policy.

2.
Queuing in the Labor Market

Assume that there are two types of queues in the Labor Market, the Sorting Queue and the Hiring Queue. The Sorting Queue is the pool of Labor that is already employed. The Hiring Queue is the pool of Labor that is unemployed.

When someone is hired out of the Sorting Queue, the economy is being reshuffled. That person is being reallocated to a job in which, at the margin, they are more productive. This is a process of hiring that acts to move resources into their most highly valued occupation. In general, when a hire occurs in this manner, the number of vacancies is conserved.

If we assume that the Economy continually supports a given number of job vacancies (which seems a reasonable assumption), then an individual is hired out of the Hiring Queue only if a new job is created or an employed person leaves the Labor Force.

This much is straight-forward and intuitive. The difference between these two queues becomes interesting if we consider the fact that the economy is going to support a given number of job vacancies at any particular time, and this number is not necessarily a constant, either as a level or as a proportion of the number of jobs. For if that number were a constant, then the economy would be in state of long run competitive equilibrium, and in such a state, no differentiation between the queues would be possible, because by definition resources would be in their most highly valued use.

If the number of vacancies supported varies over time in a systematic way, whether a job is filled by reshuffling the employed or by hiring the unemployed will be, at least in part, a function of the way the economy supports vacant jobs. The two questions for the employer are; "How long can a particular job be left vacant?" and "What proportion of jobs can be left vacant?" Both of these variables should vary inversely with business conditions. But it is also true that they can vary simply as a matter of a firm's internal policies.

The mechanics of this will be that the longer a job can be left vacant and the greater the proportion of jobs that can be left vacant, then the less likely it will be that any job will be filled from the hiring queue and the more likely it will be that the job will be filled from the sorting queue. This implies that there is a describable probability distribution for this process that varies over time.

Before we consider the probabilities that the job will be filled from one queue or the other we should consider the probability that a job will be filled irregardless of which queue. Should we consider it to be 100%, or should we assign it a probability of less than 100%? If we want to be realistic, we should not assume that jobs must be filled, only that they might be filled. If that view is true then there is a probability, not a certainty that any particular job will be filled. The logical first approximation for what this probability would be is a stochastic decay process that has time as the sole variable. So that the probability that a job will be filled at a particular point in time will be:

$$P(job\ filled) = 1 - \frac{1}{1+t} \quad , \quad t > 0$$

Where 't' is the length of time the job has been vacant. If we wish to vary how quickly the likelihood of a job being filled changes, we must add into the distribution a tuning parameter, 'b,' that will increase or decrease the mean time until the job is filled. If 'b' is between zero and one, that increases the mean time. If 'b' is greater than one, that decreases the mean time. With that change the distribution becomes:

$$P(job\ filled) = 1 - \frac{1}{1+bt} \quad , \quad t > 0 \ , \ b > 0$$

Which represents an approximation which is both more realistic and more in line with the reasoning in this paper.[6]

[6] The mathematical structure of this is exactly analogous to the process of radioactive decay in physics. Thus unemployment is an unstable condition that dissipates over time. Which is what we, as economists, would expect to occur.

Now we proceed to look at the conditional probabilities that the job will be filled from the Sorting Queue or from the Hiring Queue. Of course, our first task will be to make a few simplifying assumptions. First we will assume that all individuals who are hired for vacant positions are engaged in job search. This makes them an identifiable sub-group of the general population. Then if we assume that if employers were indifferent between the Hiring Queue and the Sorting Queue and all job searchers had equal skills then the probability that an employer would hire someone from one queue or the other, is simply the proportion of all job seekers in that particular queue. So those probabilities would be:

$$P(hiring\ queue) = \frac{h}{h+s}\ ,\quad P(sorting\ queue) = \frac{s}{h+s}$$

Where 'h' is the number of individuals in the hiring queue and 's' is the number of individuals in the sorting queue. This situation represents the State of the World where it doesn't matter which queue an individual occupies. Or in Economic terms, the conditions for a perfectly competitive market are satisfied. Armed with this knowledge, we can describe the probabilities when there is bias towards one queue or the other. If we now introduce a parameter 'a' which increases the probability that individuals will be hired out of the sorting queue, then the distributions become:

$$P(sorting\ queue) = \frac{as}{h+s}\ ,\ P(hiring\ queue) = \frac{h+s(1-a)}{h+s}\ ,\ 1<a<\frac{h+s}{s}$$

In this case 'a' is a parameter that biases the hiring process in favor of individuals in the sorting queue. The larger 'a' is, the greater the degree of bias (If we extended the range of 'a' down to zero, then values for 'a' from zero to one would represent bias against individuals in the sorting queue.[7]). The value of 'a' should vary inversely to the value of 'b.' So that the longer a job can remain open, the more likely it is that it will be filled out of the sorting queue.

In order to simplify the mathematical description let us redefine some of the variables to simplify the notation:

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[7] This was not done in order that a treatment of bias against individuals in the sorting queue and in favor of individuals in the hiring queue could be treated in a manner where that bias parameter would be symmetrical to the one we present here.

$$set \ \frac{s}{h+s} = g \ , \ 0<g<1 \ ,$$

$$then \ \frac{h}{h+s} = 1-g \ , \ P(sorting\ queue) = ag \ , \ P(hiring\ queue) = 1-ag$$

Then:

$$P(job\ filled\ out\ of\ sorting\ queue) = (ag)(1-\frac{1}{1+bt})$$

and:

$$P(job\ filled\ out\ of\ hiring\ queue) = (1-ag)(1-\frac{1}{1+bt})$$

and:

$$P(no\ hire) = 1-P(job\ filled\ out\ of\ sorting\ queue)-P(job\ filled\ out\ of\ hiring\ queue) = \frac{1}{1+bt}$$

This completes the description of the stochastic process of hiring with a bias in favor of individuals in the Sorting Queue.

Also, there is no reason that this method could not be applied to the examination of other types of hiring bias. Any scenario where the pool of applicants can be decomposed into identifiable sub-groups by the hiring agent can be examined in this way.

The method could be applied to race, gender, age, or any other set of conditions. All that would be necessary would be that there exist a sub-group that can be described as a proportion, 'g,' of the general pool of applicants. Also, the bias parameter, 'a,' should be statistically estimateable.

3.
Uncertainty, Information, and the Natural Rate Hypothesis

The formulation of unemployment as a stochastic decay process has some implications in the context of the Natural Rate Hypothesis, especially in terms of the question of changes in the Natural Rate. In terms of this model a change in the Natural Rate would be equivalent to holding the time at which P(hire) = 0.5, and varying the values of 'm' and 'n.' Varying these values is equivalent to varying the information content of the economy. If n = m = 1, the World is a world of perfect information, perfect competition, economic efficiency, and complete determinacy. As 'n' and 'm' increase in value, the information is increasingly more imperfect. So an increase in the Natural Rate signals a deterioration in the information content of the economy. An uncertain world is a world of incomplete information.

Within this context it is possible to construct an index that describes the information content described by the Natural Rate. If we take the formula for the mean value as presented in section 1, we can place limiting values on the ratio m/n. First:

$$\left(\frac{n-1}{n}\right)^m = 0.5 \quad \Rightarrow \quad \frac{n-1}{n} = 0.5^{\frac{1}{m}}$$

Then taking the limits on 'n':

$$\lim_{n\to 1} \frac{m}{n}\Big|_{\frac{n-1}{n} = 0.5^{\frac{1}{m}}} = 0 \quad , \quad \lim_{n\to\infty} \frac{m}{n}\Big|_{\frac{n-1}{n} = 0.5^{\frac{1}{m}}} = 1$$

Having found these values of m/n in the extremum, then it follows that when 1 - m/n = 1, information is perfect, and when 1 - m/n = 0, the information content is zero. So that all discussions of the Natural Rate are also discussions on the information content of the economy.[8] So, if we were to use the average number of applicants for vacant jobs as 'n' and the average number of applications made before a job is offered as 'm' then the index would be a rough index of the information content of the labor market. What is called the Natural Rate of Unemployment is a function of the information content of the economy. The better the information, the lower the rate. The worse the information, the higher the rate. So the idea of a non-zero unemployment rate being associated with an environment of full and complete information, is not a logically consistant idea.

[8] As the pool of the unemployed becomes larger, then 'm' and 'n' will also become larger for the mean value. To think otherwise is to make the mistake of interpreting static analysis as a literal version of reality. Both 'm' and 'n' are functions that vary with time. When we speak of the values of 'm' and 'n' that make P(h) = 0.5, we are speaking of a moment in time. Without the passage of time, the concept of uncertainty is devoid of meaning.

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